IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received November 17, 2020, accepted December 8, 2020, date of publication December 16, 2020, date of current version December 31, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3044973

A Levenberg–Marquardt Backpropagation Neural Network for the Numerical Treatment of Squeezing Flow With Heat Transfer Model

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This work was supported by the Deanship of Scientific Research (DSR), King Abdulaziz University, under Grant KEP-Msc-17-130-41.

ABSTRACT In this paper, the computational strength in terms of soft computing neural networks backpropagated with the efficacy of Levenberg-Marquard training (NN-BLMT) is presented to study the squeezing flow with the heat transfer model (SF-HTM). The governing system of PDEs is reduced to an equivalent system of nonlinear ODEs using similarity transformations. NN-BLMT dataset for all problem scenarios progresses through the standard Adam numerical method by the influence of Prandtl number, Eckert number, and thermal slip. The processing of NN-BLMT training, testing, and validation, is employed for various scenarios and cases to find and compare approximation solutions with reference results. For the fluidic system SF-HTM, convergence analysis based on mean square errors, histogram presentations, and statistical regression plots is considered for the proposed computing infrastructure's performance in terms of NN-BLMT. Matching of the results for the fluid flow system SF-HTM based on proposed and reference results in terms of convergence up-to 10^{-07} to 10^{-03} proves the worth of proposed NN-BLMT.

INDEX TERMS Squeezing flow, heat transfer, soft computing infrastructure, neural networks backpropagated, Levenberg-Marquard training.

I. INTRODUCTION

The squeezing flow is generated due to two approaching parallel surfaces in relative motion. In recent years, the squeezing flow of Newtonian and non-Newtonian fluids has attained considerable importance due to its widespread engineering discipline applications. Lubricated squeezing flow is one of the applications in polymer processing; i.e., it was also interested in the behavior of materials under the compression and extension. The flow fields are derived from the radially symmetric stream function solution. Their flow regimes are identified as super-lubricated, apparent slip, and lubrication failure. Then we can that the result of the experiment depends on the viscosity ratio, relative thicknesses of the films, and the applied stress, respectively.

Domairry and Aziz [1] studied squeeze flow between infinite parallel disks with the effect of suction or injection by performed He's homotopy perturbation method (HPM). Siddique *et al.* [2] investigated the squeezing flow in the presence of magnetic effects and using the homotopy perturbation method (HPM) to obtained velocity functions. Qayyum *et al.* [3] be examined the unsteady flow of a Jeffrey fluid between parallel disks and dissect the effects of velocity, porosity, and squeezing on the flow. Stefan [4] presented a study for squeezing flow based on lubrication approach and published his article in 1874

The effect of heat transfer for a nanofluid flow squeezed between parallel plates is one of the most important studies topics due to its engineering applications and scientific. Sheikholeslami *et al.* [5] investigated the heat transfer of nanofluid squeezing flow by using the homotopy perturbation method (HPM). They also calculated the effect of nanofluid's thermal conductivity and viscosity by the Maxwell-Garnetts (MG) and Brinkman models. Syed Tauseef *et al.* [6] Presented a solution of heat transfer squeezing flow of a non-Newtonian fluid by employed the differential transform method. Hayat *et al.* [7] extended the work on a stretching surface by considering the steady laminar boundary layer flow and heat transfer past a stretching sheet and taking upper convective Maxwell (UCM) a

The associate editor coordinating the review of this manuscript and approving it for publication was Lei Wang.

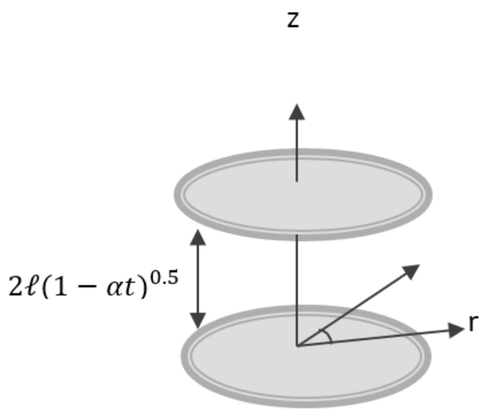


FIGURE 1. Geometry of the problem.

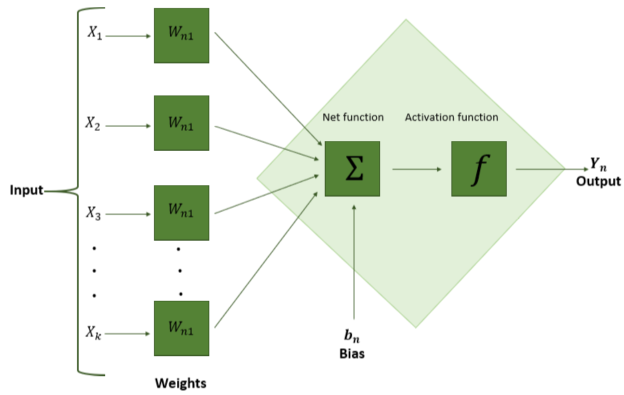


FIGURE 2. A Single neural model structure.

rheological model. Q. K. Ghori *et al.* [8] solved squeezing magneto-hydrodynamic (MHD) flow with the effect of heat transfer via variational iteration method(VIM). Such vide practical applications and important industrial characteristics attracted the attention of many researchers [9]–[14].

All these numerical methods are applied to solve the problem in different scenarios, and each has advantages and disadvantages. Although stochastic numerical computing based on artificial intelligence has been developed to solve stiff nonlinear problems, these solvers are not yet used to analyze this squeezing flow model's governing system. Stochastic numerical computing solvers are generated basically by taking advantage of computing-based on artificial neural networks (ANN) modeling and its optimization of the process to solve different problems system of ordinary or partial differential equations.

There are many modern applications of stochastic numerical computing solvers in various fields such as nonlinear systems emerging in fluid dynamics [15]–[17], biological mathematics [18], [19], financial system model [20], neuro-fuzzy model [21], pantograph system [22]–[24] plasma physics [25], fuel catching fire model [26], magneto-hydrodynamics [27] electrical conduction solids [28], and atomic physics [29] are little under significant examples of these solutions. Such facts inspire the authors to explore and incorporate the soft computing architectures as an alternative, precise, and feasible computational approaches for solving the fluid mechanics' systems associated with the squeezing flow system.

The main purpose of this study is to analyze the effect of physical parameters associated with the squeezing flow system under the influence of heat transfer by using an intelligent computing technique based on the Levenberg-Marquard algorithm. Whereas, Levenberg-Marquard (LM) inherits accuracy and fastness from the Newton method. Moreover, it also has the steepest descent method convergence capability [30]. Some structures of our discussion are noted as follows:

- Levenberg-Marquard (LM) based backpropagated neural networks are used to offer a lot of diverse applications of Computational Intelligence that analyzes the squeezing flow with the heat transfer system for different scenarios.
- The governing PDEs for squeezing flow and heat transfer model is reduced to an equivalent system of nonlinear ODEs by using similarity transformations.
- NN-BLMT dataset for all scenarios of squeezing flow problem progresses through the standard Adam numerical method.
- The processing of NN-BLMT that is training, testing, and validation is employed on the squeezing flow model for various scenarios and cases to find an approximation of proposed solutions and comparison with reference results.
- For the fluidic system SF-HTM, convergence analysis based on mean square errors, histogram presentations,

TABLE 1. Description of scenarios along with cases for squeezing flow model.

Scenario	Case	Physical quantities of interest		
		PR	EC	γ
1	1	1.0	0	0.00
	2	1.0	0	0.05
	3	1.0	0	0.10
	4	1.0	0	0.17
2	1	0.3	1.0	0.1
	2	0.8	1.0	0.1
	3	1.2	1.0	0.1
	4	1.5	1.0	0.1
3	1	1.0	0.5	0.1
	2	1.0	0.9	0.1
	3	1.0	1.2	0.1
	4	1.0	1.5	0.1

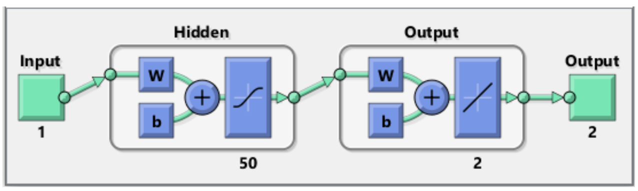


FIGURE 3. Neural networks architecture.

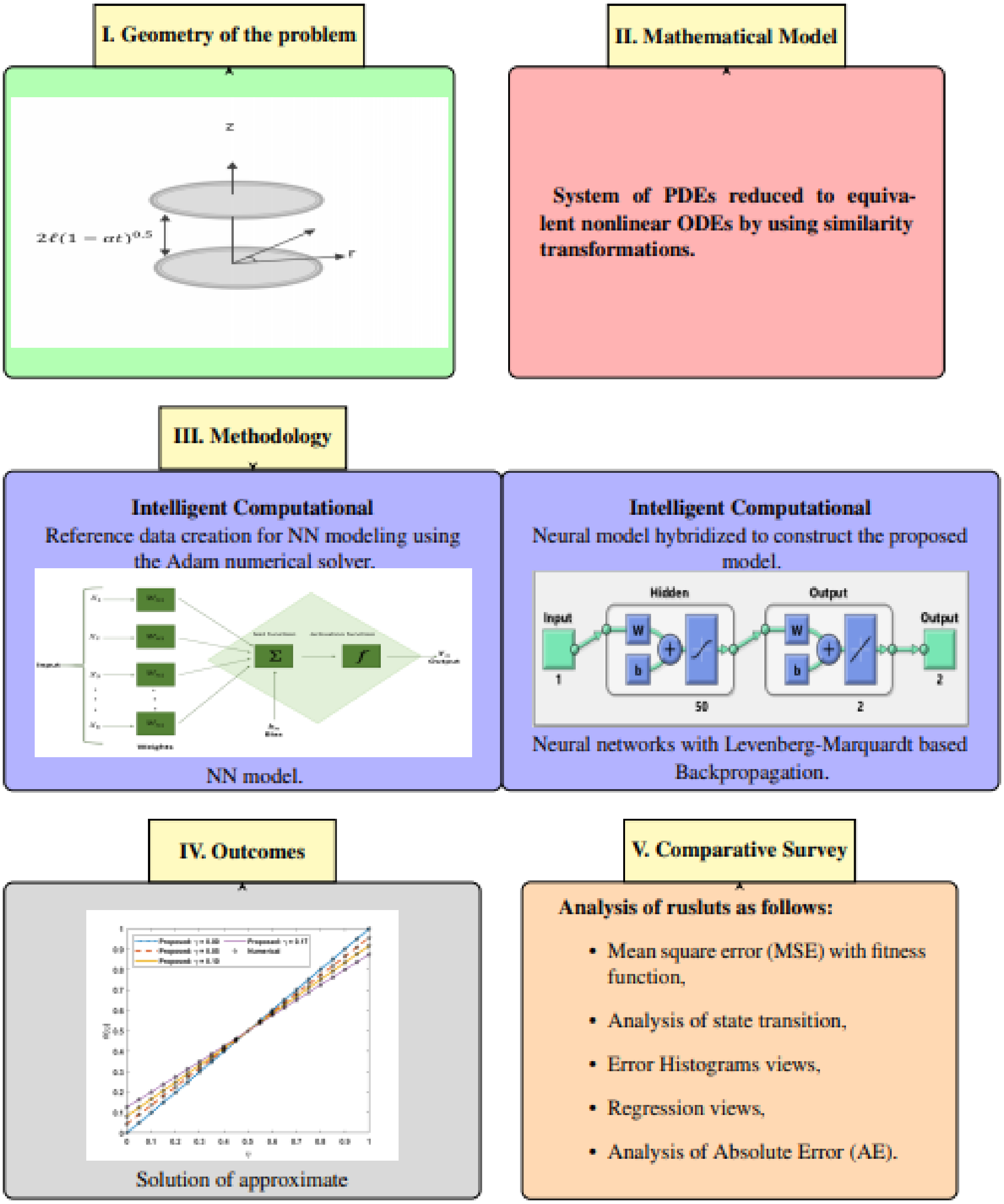


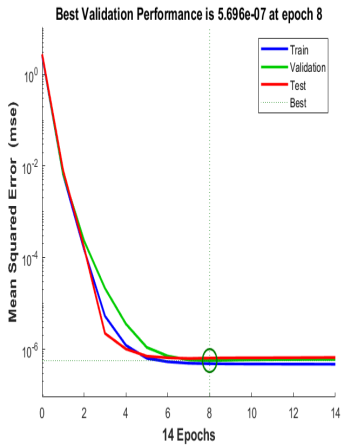
FIGURE 4. The overall process flow diagram of proposed NN-BLMT for squeezing flow with heat transfer model.

and statistical regression plots are employed to ensure the performance of NN-BLMT for the smooth and detailed analysis related to the dynamics of squeezing flow under the impact of heat transfer.

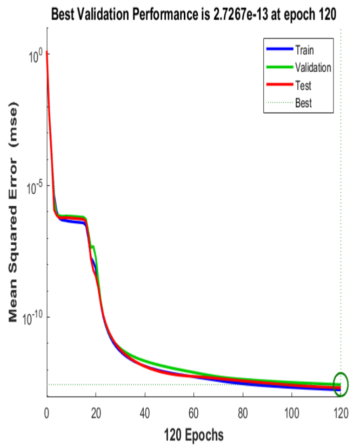
Mathematical modeling of the squeezing flow model has been presented in Section II. The method for the analysis of SF-HTM has been discussed in section III. The numerical and graphical results with discussion and comparison for the fluid flow SF-HTM through proposed technique NN-BLMM with numerical reference results are given in section IV. Finally, concluding remarks for the study on the proposed methodology and flow dynamics in terms of squeezing flow under the impact of heat transfer are presented in section V.

II. MATHEMATICAL MODEL

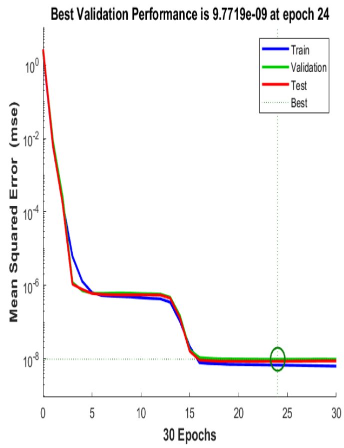
An incompressible Couette fluidic system SF-HTM is considered, and geometrical representation is shown via (Fig.1). The Couette flow based on parallel plates in influenced by the variation of the distance between the plates $z = \pm\ell(1-\alpha t)^{\frac{1}{2}} = \pm h(t)$. Thermal energy is produced along with the momentum of the fluid flow.



(a) Outcomes of MSE for Scenario 1



(b) Outcomes of MSE for Scenario 2



(c) Outcomes of MSE for Scenario 3

FIGURE 5. Performance of the NN-BLMT of Case 3 for squeezing flow model.

The governing system for SF-HTM [31] is described as

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0, \quad (1)$$

$$\rho\Big(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\Big)=-\frac{\partial p}{\partial r}+\mu\Big(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}\Big)-\frac{\sigma B_0^2}{1-\alpha t}u, \quad (2)$$

$$\rho\Big(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\Big)=-\frac{\partial p}{\partial z}+\mu\Big(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\Big), \quad (3)$$

$$\rho C_p\Big(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\Big)=K\Big(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}-\frac{u}{r^2}\Big)+2\mu\Big[\Big(\frac{\partial u}{\partial r}\Big)^2+\Big(\frac{u}{r}\Big)^2+\Big(\frac{\partial w}{\partial r}\Big)^2+\frac{1}{2}\Big(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\Big)^2\Big], \quad (4)$$

along with associated B.Cs

$$u=\beta_1\frac{\partial u}{\partial z},\quad w=0,\quad T=\gamma_1\frac{\partial T}{\partial z}+T_0\ at\ z=0,$$
$$u=-\beta_1\frac{\partial u}{\partial z},\quad w=\frac{dh(t)}{dt},$$
$$T=-\gamma_1\frac{\partial T}{\partial z}+T_1\ at\ z=h(t), \quad (5)$$

where T denotes the temperature, u and w represent the radial and axial velocities along r and z axes, respectively.

After some mathematical simplification, we get

$$g''''-S(\eta g'''+3g''-2gg''')-m^2g''=0. \quad (6)$$
$$\theta''+PRS(2g\theta'-\eta\theta')+PREC(g''^2+12\delta^2g'^2)=0. \quad (7)$$

With the following boundary conditions

$$g(0)=0,\quad g'(0)-\beta g''(0)=0,\quad \theta(0)-\gamma\theta'(0)=0,$$
$$g(1)=\frac{1}{2},\quad g'(1)+\beta g''(1)=0,\quad \theta(1)+\gamma\theta'(1)=1. \quad (8)$$

III. SOLUTION METHODOLOGY

The proposed soft computing infrastructure based on NN-BLMT provided in the single neural representation, as shown in Fig. (2). This proposed model depends on the framework of the fitting tool "nftool" which is available in the neural networks toolbox in Matlab.

The numerical attempt based on NN-BLMT is presented for squeezing flow with the heat transfer model given in Eqs. (6-8). The proposed NN-BLMT is performed for three scenarios by variation of PR, EC, γ with four different cases for each scenario and fixed values of $S=1.0$, $m=1.0$, $\beta=0.1$, $\delta=0.1$ as shown in Table (1).

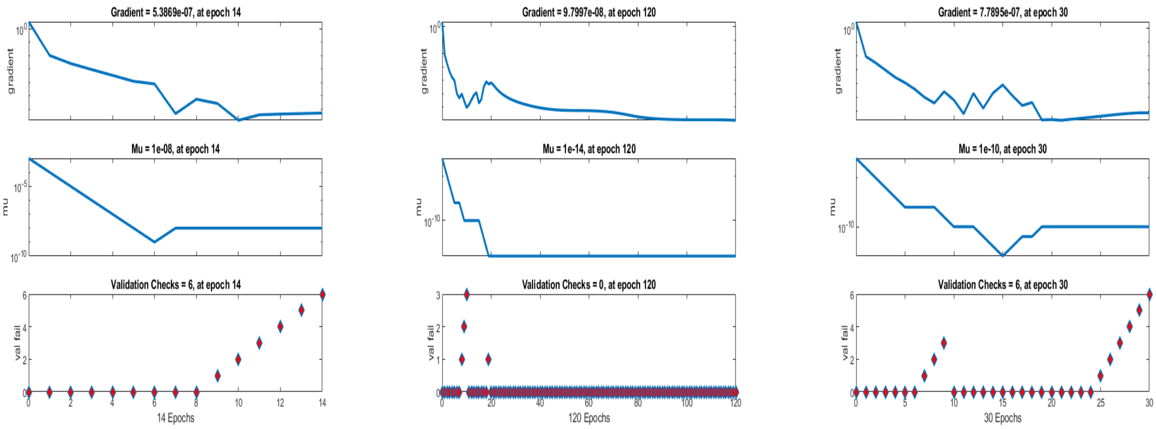
A summary of the proposed NN-BLMT workflow is presented in Fig. (4). The supervised neural network in the NN-BLMT is used to obtain the output to get a more accurate calculation repeatedly. Choose 0.001 as stepsize, which means a 1001 data set of points between 0 and 1 created by using the Adam numerical solver for the solution of ODEs in Mathematica. Select 80% of points for training while choosing the validation and testing 10%, 10%, respectively.

The number of neurons is considered 50 for the computational accuracy. The suggested structure of the NN-BLMT consists of two layers of neural networks (hidden, output), as shown in Fig. (3).

IV. INTERPRETATION OF RESULTS

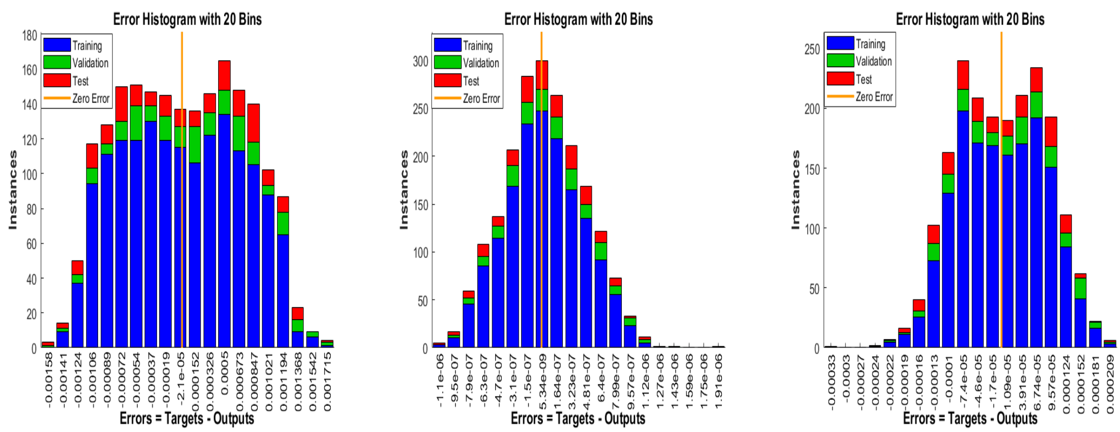
After using NN-BLMT for cases in the squeezing flow with the heat transfer model based on all three scenarios for the performance, states, and error histograms given in Figs. (5-7), respectively. Regression illustrations are given in Fig. (8). Finally, a comparison of result finding by NN-BLMT with the reference solution in Fig. (9). Furthermore, convergence is monitored for each of the training, validation, testing, performance, gradient, backpropagation measures, and the time that each of the four cases mentioned in the Table (2).

The convergence for each of the training, validation, testing is given in Fig. (5) for the variants based on three scenarios in the squeezing flow with the heat transfer model. It is easy to notice that the best performance is achieved



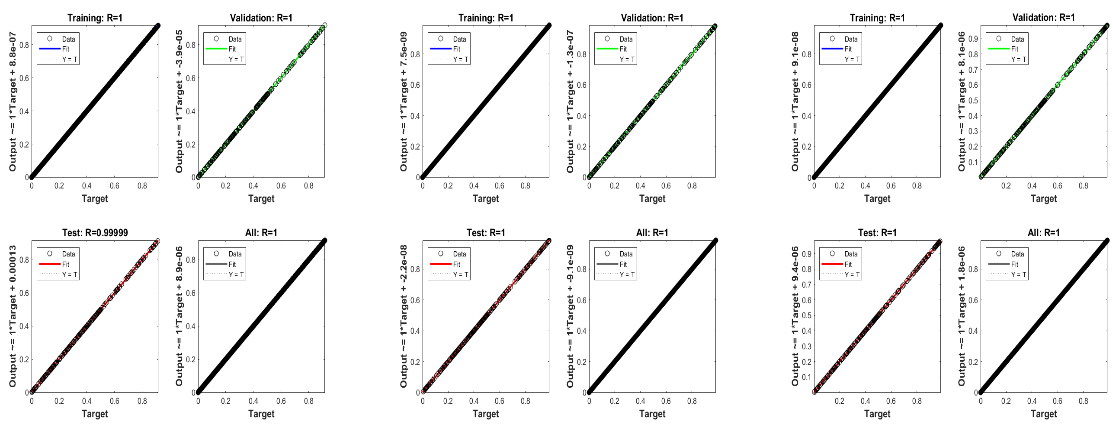
(a) The state transition outcomes of Scenario 1 (b) The state transition outcomes of Scenario 2 (c) The state transition outcomes of Scenario 3

FIGURE 6. **Performance of NN-BLMT in terms of Gradient, Mu, and validation checks of Case 3 for squeezing flow model.**



(a) Error-Histogram of Scenario 1 (b) Error-Histogram of Scenario 2 (c) Error-Histogram of Scenario 3

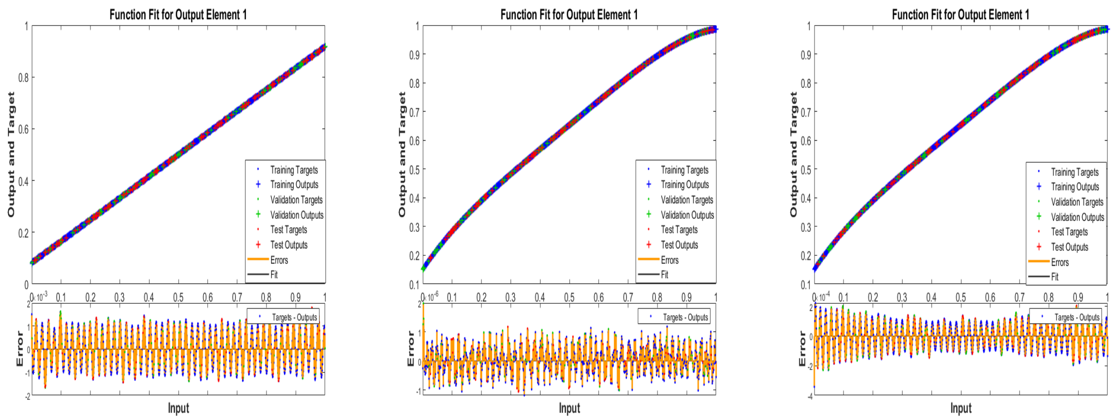
FIGURE 7. **Error-Histogram views of NN-BLMT for Case 3 for squeezing flow model.**



(a) The regression of Scenario 1 (b) The regression of Scenario 2 (c) The regression of Scenario 3

FIGURE 8. **Regression views of NN-BLMT results for Case 3 for squeezing flow model.**

at 8,120,24 epochs, while MSE is almost 10^{-06}, 10^{-10} to 10^{-14}, 10^{-08} respectively. The gradient and backpropagation measures are 5.39×10^{-07}, 9.80×10^{-08}, 7.79×10^{-07} and 10^{-08}, 10^{-14}, 10^{-10} as shown in Fig. (6) The error dynamics

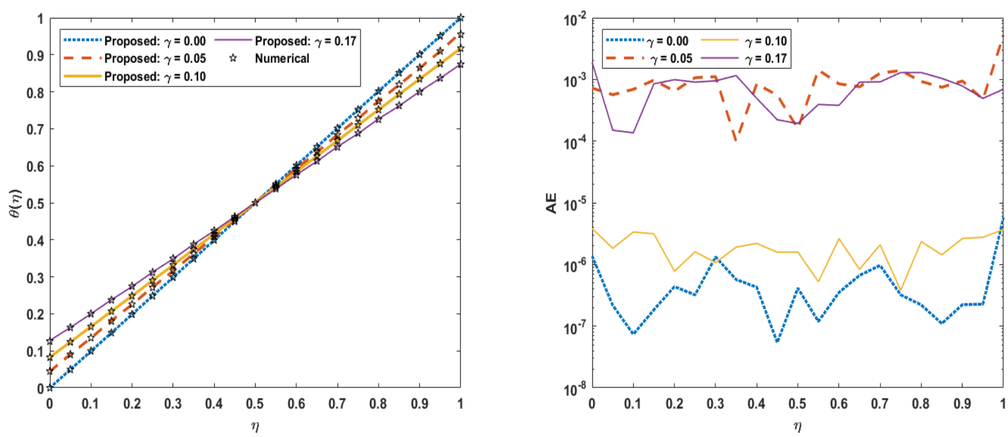


(a) The plot of Scenario 1 (b) The plot of Scenario 2 (c) The plot of Scenario 3

FIGURE 9. Comparison of reference solutions with NN-BLMT outcomes for Case 3 for squeezing flow model.

TABLE 2. Total numerical analysis of NN-BLMT for Squeezing flow model.

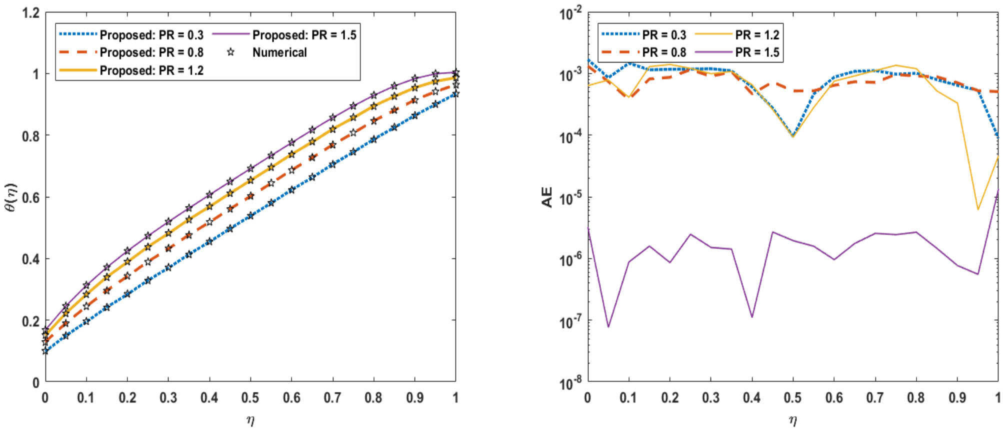
Scenarios	Cases	MSE			Performance	Grad	Mu	Epochs	Time
		Training	Validation	Testing					
(1)	1	2.29566E-12	3.12352E-12	3.18886E-12	2.30E-12	9.93E-08	1.00E-13	129	0
	2	5.70423E-7	6.07855E-7	7.67025E-7	5.09E-07	1.21E-06	1.00E-08	11	0
	3	4.86303E-7	5.69596E-7	6.45569E-7	4.75E-07	5.39E-07	1.00E-08	14	0
	4	3.84989E-7	4.57780E-7	5.52347E-7	3.72E-07	1.15E-05	1.00E-09	15	0
(2)	1	4.97845E-7	5.86784E-7	6.15383E-7	4.53E-07	1.79E-06	1.00E-08	11	0
	2	3.00616E-13	4.42020E-13	5.72846E-13	3.01E-13	9.98E-08	1.00E-14	76	0
	3	1.71790E-13	2.72673E-13	2.08614E-13	1.72E-13	9.80E-08	1.00E-14	120	0
	4	5.72458E-7	6.99869E-7	6.13350E-7	5.24E-07	5.69E-06	1.00E-08	11	0
(3)	1	2.94521E-13	6.20642E-13	4.20637E-13	2.95E-13	9.73E-08	1.00E-14	78	0
	2	4.90871E-7	6.46487E-7	7.11071E-7	4.69E-07	7.15E-06	1.00E-09	12	0
	3	6.77065E-9	9.77185E-9	8.57043E-9	6.34E-09	7.79E-07	1.00E-10	30	0
	4	5.72458E-7	6.99869E-7	6.13350E-7	5.24E-07	5.69E-06	1.00E-08	11	0



(a) Variation of γ (b) Analysis on AE

FIGURE 10. Results comparison of fluid system SF-HTM through NN-BLMM with reference numerical results for scenario 1.

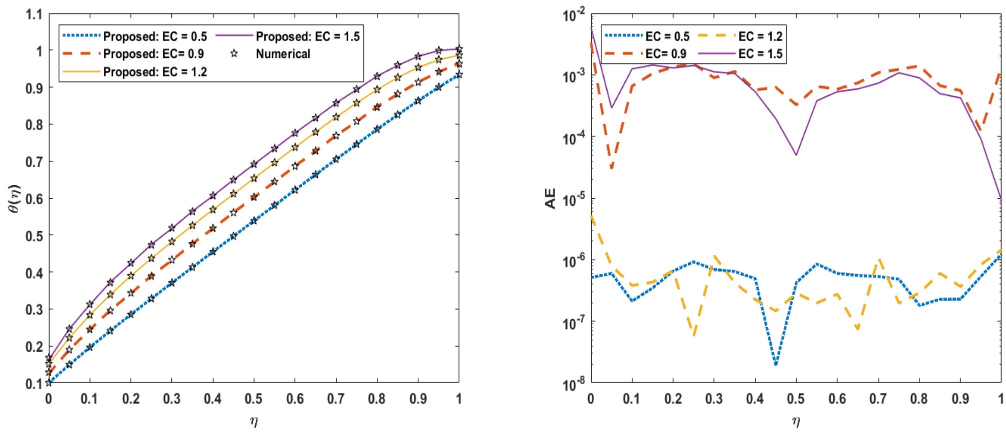
estimate may be easier by error histograms for each input point, and the results are presented in Fig. (7). The error is approaching to reference zero error line, i.e., almost -2.1×10^{-05}, 5.34×10^{-07}, and 1.09×10^{-05} for three scenarios depending on the squeezing flow with the heat transfer model. As shown in Fig. (8), the correlation value R approaches statically around the unit, and it is the desired value for training, testing, and validation. A comparison of the NN-BLMT result with the reference solution is shown in Fig. (9). The maximum error for the proposed fluidic problem SF-HTM in terms of testing, training and validation is even less than 2×10^{-3}, 2×10^{-06}, and 2×10^{-04}, respectively.



(a) Variation of PR

(b) Analysis on AE

FIGURE 11. **Results comparison of fluid system SF-HTM through NN-BLMM with reference numerical results for scenario 2L.**



(a) Variation of EC

(b) Analysis on AE

FIGURE 12. **Results comparison of fluid system SF-HTM through NN-BLMM with reference numerical results for scenario 3.**

The considered problem SF-HTM is evaluated through the efficiency of designed NN-BLMT presented in Table (2) is almost 10^{-12} to 10^{-07}, 10^{-13} to 10^{-07}, 10^{-13} to 10^{-07} respectively. The significance of these results is the consistency of the performance measured through NN-BLMT for solving squeezing flow with the heat transfer model.

Eventually, the outcomes for SF-HTM through NN-BLMT are given for the temperature profile $\theta(\eta)$ in terms of different scenarios which are shown in Figs.(10(a) -12(a)), respectively. The outcomes are compatible with the results we obtained through the standard Adam numerical method for all cases of each scenario. While the absolute error from reference solutions for all scenarios is shown in Figs. (10(b) -12(b)) respectively. It can be noticed that AE is around 10^{-07} to 10^{-03}, 10^{-07} to 10^{-03}, 10^{-07} to 10^{-02} for scenarios, respectively. All these results show the accuracy, effectiveness, and smoothness of the proposed NN-BLMT computing algorithm for solving the variants of squeezing flow with the heat transfer model.

V. CONCLUSION

The computational strength in terms of supervised learning method NN-BLMT is exploited to obtain a numerical solution for SF-HTM after the transformation of PDEs based on SF-HTM into a system of ODEs by using similarity variables conversions. The standard Adam numerical method is used for the present dataset for the squeezing flow problem. The data containing training, testing, and validation for NN-BLMT depending on various scenarios are determined by 80%, 10%, and 10%, respectively. The close agreement of both proposed and reference results is 10^{-07} to 10^{-03}. This means that the proposed model provides highly accurate results for the fluid flow system under consideration. The efficacy and performance of the proposed NN-BLMT for the solution of SF-HTM appears via mean squared error functions, performance measures, regression metrics, and histograms.

In the future, new types of platforms based on artificial intelligence will be fully developed to solve the problems of fluid mechanics [32], [33].

Nomenclature

μ	Dynamic viscosity
w	Axial velocities
β	Dimensionless velocity
δ	Dimensionless number
γ	Thermal slip
ρ	Fluid density
σ	Electrical conductivity
C_p	Specific heat at constant pressure
EC	Eckert number
K	Thermal conductivity
m	Hartman number
p	The pressure
PR	Prandtl number
S	Squeeze number
T	Temperature of fluid
u	Radial velocities
BLMT	Backpropagated Levenberg-Marquard training
NN	Neural Network

ACKNOWLEDGMENT

The authors acknowledge with thanks DSR technical and financial support.

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