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Nonlinear PI Control for High-Order Agents With Unknown High-Frequency Gain Signs Under Switching Topologies

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ABSTRACT Existing results on dealing with unknown high-frequency gain signs (UHFGSs) mainly adopt the Nussbaum-type functions. A new kind of control algorithms with nonlinear PI functions are presented to cope with UHFGSs. It is rigorously proven that the proposed algorithms with properly selected nonlinear PI functions can guarantee consensus for high-order multi-agent systems (MASs) under switching topologies with uniformly quasi-strongly δ-connected graphs (UQSGs). Furthermore, we also investigate the output leaderless consensus of heterogeneous agents with UHFGSs. Finally, the numerical examples are illustrated to show the validity of the proposed results.

INDEX TERMS Switching topologies, high-order agents, nonlinear PI functions.

I. INTRODUCTION

For the last several decades, the cooperative control problems of designing algorithms that achieve consensus in multi-agent systems (MASs) have attracted much attention in the field of system control [1]–[4]. The important feature of these control algorithms is that, while the agents make use of only local information to implement their own local controllers, the resulting global algorithms can achieve consensus with the global networked agents [5]–[8].

Recently, many efforts were devoted to designing distributed algorithms of MASs with unknown high-frequency gain signs (UHFGSs) [9]–[11] that guarantee state or output consensus, since practical applications may not have access to the high-frequency gain signs in advance [12], [13]. Therefore, the consideration of UHFGSs when designing consensus control algorithms is certainly necessary. One of the challenging problems in this context is the problem of dealing with UHFGSs. In [14], the Nussbaum-type functions was first proposed to solve the stability problem of dynamics with UHFGSs, and then the method was widely adopted to deal with various systems with UHFGSs [15]–[18].

More recently, the problem of MASs with UHFGSs is a demanding topic, since MASs has attracted significant attention. Due to the constraints imposed on the multiple control inputs with UHFGSs, such problems are very difficult to solve, where the critical challenge is dealing with the problem of multiple Nussbaum-type functions. The early studies on dealing with this issue are assuming that UHFGSs are all identical. For the nonidentical UHFGSs in some of the literature, partially known UHFGSs are required in [19], and global network information is used in [20]. To break this limit, the work of [21] can construct a partial Lyapunov function in which one Nussbaum-type function exists for each agent, and then the problem of multiple Nussbaum-type functions is solved. According to this method, the leaderless consensus of MASs with nonidentical UHFGSs was achieved for linear MASs under switching topologies [22] and for nonlinear MASs [23]. In addition, the control algorithms were illustrated in [24]–[26] to cope with the problem of nonidentical UHFGSs by using the output regulation.

Note from the above literature that the Nussbaum-type function is the key technique to solve the problem of UHFGSs. However, a major drawback of the Nussbaum-type function was that designed controllers suffer from large control overshoot [27], and this would limit the application of these controllers in practical engineering applications. Suppressing therefore the large control overshoot when designing distributed control algorithms is a demanding task. One method is introducing nonlinear PI functions

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[28]–[30], where the consensus of linear MASs with nonidentical UHFGSs was achieved under switching topologies with balanced/unbalanced subgraphs.

Motivated by the aforementioned literature, in this paper we will design a class of control algorithms for high-order agents with UHFGSs under switching topologies, where the nonlinear PI functions will be used to deal with the UHFGSs. It will be proven that the proposed algorithms with properly selected nonlinear PI functions can guarantee the consensus of MASs under switching topologies with uniformly quasi-strongly δ-connected graphs (UQSGs). Moreover, the results are extended to the output leaderless consensus of heterogeneous agents with UHFGSs. The contributions are presented as follows.

1) A new class of nonlinear PI functions are employed in order to deal with nonidentical UHFGSs and ensure that the designed algorithms can achieve leaderless consensus. Different from existing results [30], the proposed algorithms in this paper are suitable for the consensus of first-order dynamics with UHFGSs.
2) The considered results are extended to the case of output leaderless consensus of heterogeneous agents with UHFGSs. To our best knowledge, this is the first result that achieves output leaderless consensus for networks of high-order agents under switching topologies with UQSGs. As we know, the communication network is the mildest condition for MASs with nonidentical UHFGSs, which implies a major improvement.

The following sections of this paper are organized as follows. Section II gives some preliminaries and the problem is formulated, while the algorithms with nonlinear PI function are presented in Section III. Two simulation examples are provided in Section IV, and this paper is concluded in Section V.

II. PRELIMINARIES AND PROBLEM FORMULATION

Notation: The notations $\mathcal{L}_\infty$ and $\mathcal{L}_2$ represent the bounded and square integrable signals, respectively. $\{t_j\}_{j\in I}$ is the discontinuity points with $I = \{1, 2, \ldots\} \subseteq \mathbb{N}_+$. The vectors and matrices use bold symbols.

A. PRELIMINARIES

Lemma 1 [29]: The piecewise right continuous differentiable function is denoted by $\rho : [0, \infty) \to \mathbb{R}$, and the discontinuity points is represented by $\{t_j\}_{j\in I}$. Assume that the bounded derivative exists for ρ, and $\lim_{t\to\infty} \int_0^t \rho(\tau)d\tau \in \mathcal{L}_\infty$. Suppose $t_{j+1} - t_j > \epsilon$ for some $\epsilon > 0$ with $j \in I$, then $\lim_{t\to\infty} \rho(t) = 0$.

Lemma 2 [29]: Suppose the function $M : [0, t_f) \to \mathbb{R}$ is piecewise right continuous, and the function $S : [0, t_f) \to \mathbb{R}$ is continuous, piecewise differentiable. If

$$\dot{S}(t) = [\alpha_1 + \alpha_2 S(t)\cos(S(t))]\, M(t)$$

in which the parameters α_1 and $\alpha_2 \neq 0$ are real, it is obtained $|S(t) - S(0)| \leq 2(\pi + |\alpha_1/\alpha_2|)$ for $t \in [0, t_f)$.

The basic concepts of directed graph are omitted in this paper, and the detailed introduction can be referred in the work of [22]. Some related definitions on directed graphs are revisited in the following. A directed graph is defined as $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t), \mathcal{A}(t))$, where the set of nodes is $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, the set of edges is $\mathcal{E}(t) \subseteq \mathcal{V} \times \mathcal{V}$ and the adjacency matrix is $\mathcal{A}(t) = [a_{ik}(t)] \in \mathbb{R}^{N\times N}$. For a center node, it means other nodes are reachable from this node. The graph $\mathcal{G}(t)$ is quasi-strongly connected if it has a center. If we give a constant $\delta > 0$, the definition of δ-arc is the arc with the property that $\int_{t_1}^{t_2} a_{ji}(t)dt \geq \delta$ in the interval $[t_1, t_2)$. The definition of δ-path is a path in the interval $[t_1, t_2)$ with the δ-arc.

Definition 1 [31]: The UQSGs is that for the interval $[t, t+T)$ where $T > 0$ and $t \geq 0$, the δ-arcs of $\mathcal{G}(t)$ contains a quasi-strongly connected graph.

Lemma 3 [31]: Consider N first-order agents. The i-th dynamic of agent is

$$\dot{\xi}_i(t) = -\sum_{k=1}^{N} a_{ik}(t)\,(\xi_i(t) - \xi_k(t)) + \varrho_i(t) \tag{1}$$

with $i = 1, 2, \ldots, N$, in which the function $\varrho_i(t)$ is a continuous on $[0, \infty)$. Suppose $\mathcal{G}(t)$ is the UQSGs and it is satisfied that $\varrho_i \in \mathcal{L}_\infty$ with $\lim_{t\to\infty} \varrho_i(t) = 0$, then $\lim_{t\to\infty} (\xi_i(t) - \xi_k(t)) = 0$ with $i, k = 1, 2, \ldots, N$.

B. PROBLEM FORMULATION

Consider a network of N agents with nth-order and the dynamics are

$$x_i^{(n)}(t) = b_i u_i(t) \tag{2}$$

for all $i = 1, 2, \ldots, N$, where the order $n > 1$, $x_i^{(m)}(t) \in \mathbb{R}$, $m = 1, 2, \ldots, n-1$ are the high-order states, $b_i \in \mathbb{R}$ is the gain and its sign is unknown, and the control input is $u_i(t) \in \mathbb{R}$.

Assumption 1: The value of $b_i \neq 0$, is unknown, and its sign is unknown, constant.

The objective is to design nonlinear PI control algorithms for MASs (2) with Assumption 1 under UQSGs such that consensus is reached, i.e.,

$$\begin{cases} \lim_{t\to\infty} (x_i(t) - x_k(t)) = 0 \\ \lim_{t\to\infty} \left(x_i^{(m)}(t) - x_k^{(m)}(t)\right) = 0 \end{cases} \tag{3}$$

where $m = 1, 2, \ldots, n-1$, $i, k = 1, 2, \ldots, N$.

III. MAIN RESULTS

We first introduce nonlinear PI control algorithms for MASs under UQSGs, and then the results are extended to output leaderless consensus of heterogeneous agents. Before stating the main results of this section, we introduce the following lemma.

Lemma 4 [32]: Let $x(t)$ be the smooth function, and suppose the initial conditions $x(0), x^{(m)}(0) \in \mathcal{L}_\infty$,

$m = 1, 2, \ldots, n-1$. Let

$$w(x,t) = \left(\gamma + \frac{d}{dt}\right)^{n-1} x(t), \tag{4}$$

where the constant $\gamma > 0$. If $|w(x,t)| \le k$ with $k > 0$ and $t \ge 0$, then it is satisfied that

$$\left\| x^{(m)}(t) \right\| \le \frac{2^m k}{\gamma^{n-m-1}}, \quad t \ge T_0 \tag{5}$$

for $m = 1, 2, \ldots, n-1$, in which the finite time T_0 depends on the initial conditions. Moreover, if $\lim_{t\to\infty} w(x,t) = w^* \in \mathcal{L}_\infty$, then $\lim_{t\to\infty} x(t) = x^* \in \mathcal{L}_\infty$ and $\lim_{t\to\infty} x^{(m)}(t) = x_m^* \in \mathcal{L}_\infty$. If $\lim_{t\to\infty} w(x,t) = 0$, then $\lim_{t\to\infty} x(t) = 0$ and $\lim_{t\to\infty} x^{(m)}(t) = 0$, $m = 1, 2, \ldots, n-1$.

To facilitate the technical development, we define the following states:

$$\begin{aligned} z_i(t) &= \left(\gamma + \frac{d}{dt}\right)^{n-1} x_i(t) \\ &= C_{n-1}^0 \gamma^{n-1} x_i(t) + C_{n-1}^1 \gamma^{n-2} \dot{x}_i(t) \\ &\quad + \cdots + C_{n-1}^{n-2} \gamma x_i^{(n-2)}(t) + C_{n-1}^{n-1} x_i^{(n-1)}(t) \end{aligned} \tag{6}$$

and

$$\begin{aligned} q_i(t) &= C_{n-1}^0 \gamma^{n-1} \dot{x}_i(t) + C_{n-1}^1 \gamma^{n-2} \ddot{x}_i(t) \\ &\quad + \cdots + C_{n-1}^{n-3} \gamma^2 x_i^{(n-2)}(t) + C_{n-1}^{n-2} \gamma x_i^{(n-1)}(t) \end{aligned} \tag{7}$$

where $\gamma > 0$, and C_i^js' are coefficients of the binomial expansion.

A. NONLINEAR PI CONSENSUS CONTROL FOR MASs

In this subsection, we will propose nonlinear PI control algorithms for MASs with UHFGSs under UQSGs, and one of the main results is summarized:

Theorem 1: Consider the MASs (2) with Assumption 1 under UQSGs defined in Definition 1. The consensus objective (3) is achieved if the distributed control algorithms are designed

$$u_i(t) = R_i(t) \cos(R_i(t)) \left[\kappa \phi_i(t) + q_i(t) + \dot{e}_i(t)\right] \tag{8}$$

with PI term

$$R_i(t) = \frac{1}{2}\phi_i^2(t) + \kappa \int_0^t \phi_i^2(\tau) d\tau \tag{9}$$

and

$$\begin{cases} \phi_i(t) = z_i(t) + e_i(t) \\ \dot{e}_i(t) = \sum_{k=1}^N a_{ik}(t)\left(z_i(t) - z_k(t)\right) \end{cases} \tag{10}$$

where $\kappa > 0$ is a constant.

Proof: Denote by $\boldsymbol{x_{cl}} = \left[\boldsymbol{z}^T, \boldsymbol{e}^T, \boldsymbol{\varphi}^T, \boldsymbol{h_c}^T\right]^T$ is a state vector with $\boldsymbol{z} = [z_1, z_2, \ldots, z_N]^T$, $\boldsymbol{e} = [e_1, e_2, \ldots, e_N]^T$, $\boldsymbol{\varphi}^T = [\boldsymbol{\varphi_1}, \boldsymbol{\varphi_2}, \ldots, \boldsymbol{\varphi_N}]^T$ with $\boldsymbol{\varphi_i} = \left[x_i, \dot{x}_i, \ldots, x_i^{(n-2)}\right]^T$, and $\boldsymbol{h_c} = [h_{c1}, h_{c2}, \ldots, h_{cN}]^T$ where for each $i = 1, 2, \ldots, N$,

$$h_{ci} = \kappa \int_0^t \phi_i^2(\tau) d\tau. \tag{11}$$

The closed-loop systems (2) with (8), (9) and (10) are

$$\begin{cases} \dot{z}_i = W_i(\boldsymbol{x_{cl}}) \left[\kappa(z_i + e_i) + q_i + \boldsymbol{\varepsilon_i}^T \boldsymbol{L(t)z}\right] + q_i \\ \dot{e}_i = \boldsymbol{\varepsilon_i}^T \boldsymbol{L(t)z} \\ \dot{\boldsymbol{\varphi}}_i = \boldsymbol{A\varphi_i} + \boldsymbol{B} z_i \\ \dot{h}_{ci} = \kappa (z_i + e_i)^2 \end{cases} \tag{12}$$

and

$$\begin{cases} W_i(\boldsymbol{x_{cl}}) = b_i R_i \cos(R_i) \\ R_i = \dfrac{1}{2}(z_i + e_i)^2 + h_{ci} \\ q_i = \boldsymbol{\beta}^T (\boldsymbol{A\varphi_i} + \boldsymbol{B} z_i) \end{cases} \tag{13}$$

with

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -C_{n-1}^0 \gamma^{n-1} & -C_{n-1}^2 \gamma^{n-3} & \cdots & -C_{n-1}^{n-2}\gamma \end{bmatrix} \tag{14}$$

and

$$\boldsymbol{B} = \begin{bmatrix} 0 & 0 & \cdots & 1 \end{bmatrix}^T, \tag{15}$$

where

$$\boldsymbol{\beta} = \begin{bmatrix} C_{n-1}^0 \gamma^{n-1} & C_{n-1}^1 \gamma^{n-2} & \cdots & C_{n-1}^{n-2}\gamma \end{bmatrix}^T, \tag{16}$$

and $\boldsymbol{\varepsilon_i}$ is the i-th column of the identity matrix. In view of (12), we can know that the dynamics $\dot{\boldsymbol{x}}_{cl} = f(\boldsymbol{x_{cl}}, t)$ is piecewise continuous and locally Lipschitz with mapping f. Therefore, $\boldsymbol{x_{cl}}(\cdot)$ has a unique and continuous solution, and its solution satisfies $\left[0, t_f\right)$ [33]. Thus, we have

$$\dot{R}_i(t) = \left[1 + b_i R_i(t) \cos(R_i(t))\right] \phi_i(t) \left[\kappa \phi_i(t) + q_i(t) + \dot{e}_i(t)\right]$$

with $t \in \left[0, t_f\right)$. In view of Lemma 2, it is obtained

$$|R_i(t) - R_i(0)| \le 2\left(\pi + 1/|b_i|\right).$$

It is seen that $R_i(t)$ is bounded with $\left[0, t_f\right)$. According to (9), we have the boundedness of $\phi_i(t)$ and $\int_0^t \phi_i^2(\tau)d\tau$. Therefore, we have h_{ci} is bounded. Moreover, we see from (10) that

$$\dot{e}_i(t) = -\sum_{k=1}^N a_{ik}(t)\left(e_i(t) - e_k(t)\right) + \varrho_i^e(t) \tag{17}$$

with

$$\varrho_i^e(t) = \sum_{k=1}^N a_{ik}(t)\left(\phi_i(t) - \phi_k(t)\right) \tag{18}$$

where $i = 1, 2, \ldots, N$. For the equation (18) with the boundedness of $\phi_i(t)$, it is obtained $\varrho_i^e(t)$ is bounded.

According to Lemma 4.3 in [31] with (17), the boundedness of $\dot{e}_i(t)$ and $e_i(t)$ can be obtained. This implies the boundedness of $z_i(t)$ with (10). Also, in view of Lemma 4 and (6), since $z_i(t)$ is bounded, we can obtain that $x_i(t)$ and $x_i^{(m)}(t)$, $m = 1, 2, \ldots, n-1$ are all bounded. Therefore, it is easy to get the boundedness of $\boldsymbol{\varphi_i(t)}$ and $q_i(t)$. Therefore, the boundedness of $\boldsymbol{x_{cl}}$ is obtained and its solution is suitable for $t_f = \infty$ since the bounds are unchanged with $t_f = \infty$, that is, $R_i(t)$, $z_i(t)$, $q_i(t)$, $e_i(t)$, $\dot{e}_i(t)$, $h_{ci}(t)$, $\phi_i(t)$, $\int_0^t \phi_i^2(\tau)d\tau \in \mathcal{L}_\infty$. Therefore, we can obtain from (8) that $u_i(t) \in \mathcal{L}_\infty$, which implies $x_i^{(n)}(t) \in \mathcal{L}_\infty$. Thus, it is obtained from (10) that $\dot{\phi}_i(t) = \dot{z}_i(t) + \dot{e}_i(t) = x_i^{(n)}(t) + q_i(t) + \dot{e}_i(t) \in \mathcal{L}_\infty$. In view of the Barbalat's lemma with $\dot{\phi}_i(t) \in \mathcal{L}_\infty$, $\phi_i(t) \in \mathcal{L}_\infty$ and $\int_0^t \phi_i^2(\tau)d\tau \in \mathcal{L}_\infty$, we obtain $\lim_{t\to\infty} \phi_i(t) = 0$.

Since $\phi_i(t) \in \mathcal{L}_\infty$ and $\lim_{t\to\infty} \phi_i(t) = 0$, the equation (18) implies that $\varrho_i^e(t) \in \mathcal{L}_\infty$ and $\lim_{t\to\infty} \varrho_i^e(t) = 0$. Thus, we can see that (17) is the form of (1) with $\xi_i(t) = e_i(t)$ and $\varrho_i(t) = \varrho_i^e(t)$. Since we have assume that $\mathcal{G}(t)$ is UQSGs, according to Lemma 3, it has $\lim_{t\to\infty} (e_i(t) - e_k(t)) = 0$ with $i, k = 1, 2, \ldots, N$. According to (10) with $\lim_{t\to\infty} \phi_i(t) = 0$, it has $\lim_{t\to\infty} (z_i(t) - z_k(t)) = \lim_{t\to\infty} [(\phi_i(t) - \phi_k(t)) - (e_i(t) - e_k(t))] = 0$ with $i, k = 1, 2, \ldots, N$. Since $\lim_{t\to\infty} (z_i(t) - z_k(t)) = 0$, in view of Lemma 4, it is obtained $\lim_{t\to\infty} \left(x_i^{(m)}(t) - x_k^{(m)}(t)\right) = 0$ with $m = 1, 2, \ldots, n-1$ and $i, k = 1, 2, \ldots, N$. Therefore, it is easy to get from (6) that $\lim_{t\to\infty} (x_i(t) - x_k(t)) = 0$ with $i, k = 1, 2, \ldots, N$. The proof is completed. ■

Remark 1: Different from the proposed algorithm in [30] including $\dot{z}_i(t)$, which limits that the proposed algorithms cannot be applied to the first-order agents, the proposed algorithms (8)-(10) in Theorem 1 in this paper can be suitable for the cooperative control of first-order agents with UHFGSs. Therefore, the state $z_i(t) = x_i(t)$ with $q_i(t) = 0$ when $n = 1$.

B. OUTPUT CONSENSUS OF HETEROGENEOUS AGENTS

We deal with the case of output consensus of heterogeneous MASs with UHFGSs under UQSGs.

Consider N heterogeneous agents with dynamics

$$\begin{cases} x_{\rho_i,i}^{(\rho_i)}(t) = b_i u_i(t) \\ y_i(t) = x_{\rho_i,i}(t) \end{cases} \tag{19}$$

with $i = 1, 2, \ldots, N$, where ρ_i ($\rho_i > 1$) is the order of the ith agent, and the high-order states of agent i are $x_{\rho_i,i}^{(m)}(t) \in \mathbb{R}$, $m = 1, 2, \ldots, \rho_i - 1$. The signals b_i, $u_i(t) \in \mathbb{R}$ are the same as defined in (2), and $y_i(t) \in \mathbb{R}$ is the output of the agents.

The aim is to construct nonlinear PI control algorithms for heterogeneous MASs (19) with Assumption 1 under UQSGs such that output leaderless consensus is reached, that is,

$$\lim_{t\to\infty} (y_i(t) - y_k(t)) = 0 \tag{20}$$

for all $i, k = 1, 2, \ldots, N$.

For the MASs (19), our main results are as follows.

Theorem 2: Consider N heterogeneous agents (19) with Assumption 1 and the UQSGs. The output consensus (20) is guaranteed if the algorithms are

$$u_i(t) = S_{\rho_i,i}(t) \cos(S_{\rho_i,i}(t)) \times \left[\kappa\phi_{\rho_i,i}(t) + q_{\rho_i,i}(t) + \dot{e}_{\rho_i,i}(t)\right] \tag{21}$$

with PI term

$$S_{\rho_i,i}(t) = \frac{1}{2}\phi_{\rho_i,i}^2(t) + \kappa \int_0^t \phi_{\rho_i,i}^2(\tau)d\tau \tag{22}$$

and

$$\begin{cases} \phi_{\rho_i,i}(t) = z_{\rho_i,i}(t) + e_{\rho_i,i}(t) \\ \dot{e}_{\rho_i,i}(t) = \sum_{k=1}^N a_{ik}(t)\left(z_{\rho_i,i}(t) - z_{\rho_i,k}(t)\right) \end{cases} \tag{23}$$

where

$$\begin{aligned} z_{\rho_i,i}(t) &= \left(\gamma + \frac{d}{dt}\right)^{\rho_i - 1} x_{\rho_i,i}(t) \\ &= C_{\rho_i-1}^0 \gamma^{\rho_i-1} x_{\rho_i,i}(t) + C_{\rho_i-1}^1 \gamma^{\rho_i-2} \dot{x}_{\rho_i,i}(t) \\ &\quad + \cdots + C_{\rho_i-1}^{\rho_i-2}\gamma x_{\rho_i,i}^{(\rho_i-2)}(t) + C_{\rho_i-1}^{\rho_i-1} x_{\rho_i,i}^{(\rho_i-1)}(t) \end{aligned} \tag{24}$$

and

$$\begin{aligned} &q_{\rho_i,i}(t) \\ &= C_{\rho_i-1}^0 \gamma^{\rho_i-1} \dot{x}_{\rho_i,i}(t) + C_{\rho_i-1}^1 \gamma^{\rho_i-2} \ddot{x}_{\rho_i,i}(t) \\ &\quad + \cdots + C_{\rho_i-1}^{\rho_i-3}\gamma^2 x_{\rho_i,i}^{(\rho_i-2)}(t) + C_{\rho_i-1}^{\rho_i-2}\gamma x_{\rho_i,i}^{(\rho_i-1)}(t) \end{aligned} \tag{25}$$

with $\kappa > 0$, $\gamma = 1$, and C_i^js' are coefficients of the binomial expansion.

Proof: In view of (11)-(18), it is similarly obtained $\phi_{\rho_i,i}(t) \in \mathcal{L}_\infty$ and $\lim_{t\to\infty} \phi_{\rho_i,i}(t) = 0$. According to (23), it is obtained

$$\dot{e}_{\rho_i,i}(t) = -\sum_{k=1}^N a_{ik}(t)\left(e_{\rho_i,i}(t) - e_{\rho_i,k}(t)\right) + \varrho_{\rho_i,i}^e(t) \tag{26}$$

with

$$\varrho_{\rho_i,i}^e(t) = \sum_{k=1}^N a_{ik}(t)\left(\phi_{\rho_i,i}(t) - \phi_{\rho_i,k}(t)\right). \tag{27}$$

It is known that $\phi_{\rho_i,i}(t) \in \mathcal{L}_\infty$ and $\lim_{t\to\infty} \phi_{\rho_i,i}(t) = 0$. According to (27), we have $\varrho_{\rho_i,i}^e(t) \in \mathcal{L}_\infty$ and $\lim_{t\to\infty} \varrho_{\rho_i,i}^e(t) = 0$. Suppose $\xi_i(t) = e_{\rho_i,i}(t)$ and $\varrho_i(t) = \varrho_{\rho_i,i}^e(t)$ with Lemma 3, we obtain $\lim_{t\to\infty} \left(e_{\rho_i,i}(t) - e_{\rho_i,k}(t)\right) = 0$. In view of (23) with $\lim_{t\to\infty} \phi_{\rho_i,i}(t) = 0$, it is obtained $\lim_{t\to\infty} \big(z_{\rho_i,i}(t) - z_{\rho_i,k}(t)\big) = 0$ for all $i, k = 1, 2, \ldots, N$. Note from (23) that $\lim_{t\to\infty} \dot{e}_{\rho_i,i}(t) = 0$. Furthermore, in view of Lemma 4.3 in [31], since $\varrho_{\rho_i,i}^e(t) \in \mathcal{L}_\infty$ with (26), it is seen that $\dot{e}_{\rho_i,i}(t) \in \mathcal{L}_\infty$ and $e_{\rho_i,i}(t) \in \mathcal{L}_\infty$, which implies $z_{\rho_i,i}(t) \in \mathcal{L}_\infty$. Along with $\lim_{t\to\infty} \dot{e}_{\rho_i,i}(t) = 0$, it implies that $\lim_{t\to\infty} e_{\rho_i,i}(t)$ exists and is a constant. Therefore, it is known from (23) that $\lim_{t\to\infty} z_{\rho_i,i}(t) = z^*$ is a constant due to $\lim_{t\to\infty} \phi_{\rho_i,i}(t) = 0$.

Since $z_{\rho_i,i}(t) \in \mathcal{L}_\infty$ and $\lim_{t\to\infty} z_{\rho_i,i}(t) = z^*$, according to Lemma 4 with (24), it is obtained $x_{\rho_i,i}(t) \in \mathcal{L}_\infty$, $\lim_{t\to\infty} x_{\rho_i,i}(t) = x_{\rho_i,i}^*$, $x_{\rho_i,i}^{(m)}(t) \in \mathcal{L}_\infty$ and $\lim_{t\to\infty} x_{\rho_i,i}^{(m)}(t) = x_{\rho_i,im}^*$ for each $m = 1, 2, \ldots, \rho_i - 1$. Since $\lim_{t\to\infty} x_{\rho_i,i}(t) = x_{\rho_i,i}^*$, $\dot{x}_{\rho_i,i}(t) \in \mathcal{L}_\infty$, $\ddot{x}_{\rho_i,i}(t) \in \mathcal{L}_\infty$, according to the Barbalat's lemma, we obtain $\lim_{t\to\infty} \dot{x}_{\rho_i,i}(t) = 0$. Similarly, it is easy to obtain $\lim_{t\to\infty} x_{\rho_i,i}^{(m)}(t) = 0$ with $m = 2, 3, \ldots, \rho_i - 2$. Furthermore, due to $x_{\rho_i,i}^{(m)}(t) \in \mathcal{L}_\infty$ for all $m = 1, 2, \ldots, \rho_i - 1$, we know that $q_{\rho_i,i}(t)$ is bounded. In view of (21), it is easy to know $u_i(t) \in \mathcal{L}_\infty$. Thus, it has $x_{\rho_i}^{(n)}(t) = b_i u_i(t) \in \mathcal{L}_\infty$. Since $\lim_{t\to\infty} x_{\rho_i,i}^{(\rho_i-2)}(t) = 0$,

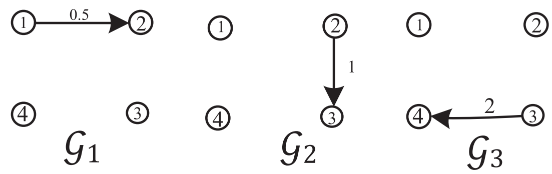

FIGURE 1. The switching topologies $\mathcal{G}(t)$ described by the UQSGs.

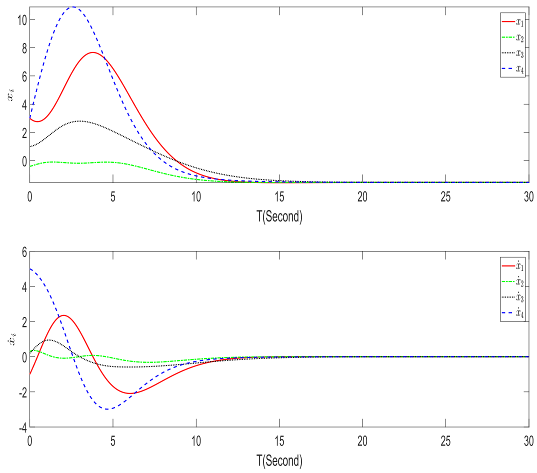

FIGURE 2. The state trajectories of x_i and $\dot{x}_i$ $(i = 1, \ldots, 4)$.

$x_{\rho_i,i}^{(\rho_i-1)}(t) \in \mathcal{L}_\infty$ and $x_{\rho_i,i}^{(\rho_i)}(t) \in \mathcal{L}_\infty$ for all $i = 1, 2, \ldots, N$, we can also obtain that $\lim_{t\to\infty} x_{\rho_i,i}^{(\rho_i-1)}(t) = 0$. Thus, we have $\lim_{t\to\infty} x_{\rho_i,i}^{(m)}(t) = 0$ for each $m = 1, 2, \ldots, \rho_i - 1$. Along with (24), $\gamma = 1$ and $\lim_{t\to\infty}\left(z_{\rho_i,i}(t) - z_{\rho_k,k}(t)\right) = 0$, we have $\lim_{t\to\infty}\left(x_{\rho_i,i}(t) - x_{\rho_k,k}(t)\right) = 0$, and this implies $\lim_{t\to\infty}\left(y_i(t) - y_k(t)\right) = \lim_{t\to\infty}\left(x_{\rho_i,i}(t) - x_{\rho_k,k}(t)\right) = 0$. ■

Remark 2: In view of Theorem 2, we deal with the output consensus of heterogeneous MASs with UHFGSs under UQSGs, which extends the results in Theorem 1 to the more general case.

IV. SIMULATION EXAMPLES

We give two examples to verify the performance of the proposed algorithms for state and output consensus of MASs under UQSGs, respectively. Furthermore, the considered UQSGs is illustrated in Fig. 1.

Example 1: The effectiveness of proposed algorithms (8)-(10) is verified by one simulation example. The network topology of MASs with four six-order dynamics described by Fig. 1, where the transitions are $\mathcal{G}_1 \to \mathcal{G}_2 \to \mathcal{G}_3 \to \mathcal{G}_1 \to \cdots$. The switching sequence is

$$\mathcal{G}(t) = \begin{cases} \mathcal{G}_1, & \text{when } t \mod 2 \in [0, 0.5) \\ \mathcal{G}_2, & \text{when } t \mod 2 \in [0.5, 1) \\ \mathcal{G}_3, & \text{when } t \mod 2 \in [1, 2)\,. \end{cases}$$

The initial states $\left[x_i^{(5)}(0), x_i^{(4)}(0), x_i^{(3)}(0), \ddot{x}_i(0), \dot{x}_i(0), x_i(0)\right]^T$, $i = 1, \ldots, 4$, are $[5, -2, 1, 2, -1, 3]^T$, $[-2, 4, -2.5, 0.5, 0.3, -0.4]^T$, $[1, -3, 0.3, 1, 0.2, 1]^T$ and $big[-1, 3, -2, -0.4, 5, 3]^T$, and the UHFGSs are $b_1 = 1$, $b_2 = 3$, $b_3 = 2$, $b_4 = -1$. In control algorithms (8)-(10), the parameter

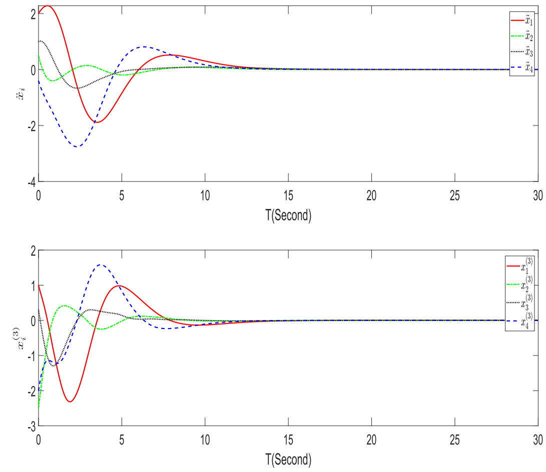

FIGURE 3. The state trajectories of $\ddot{x}_i$ and $x_i^{(3)}$ $(i = 1, \ldots, 4)$.

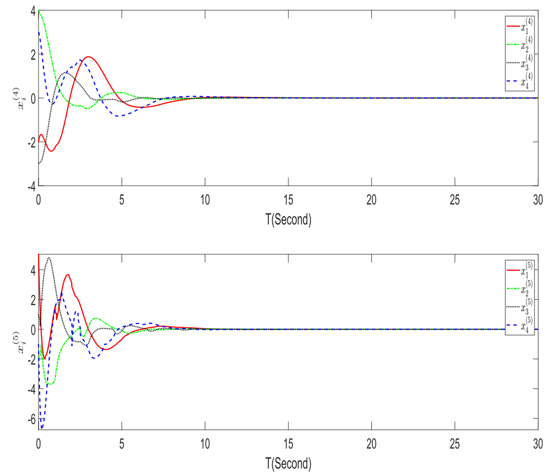

FIGURE 4. The state trajectories of $x_i^{(4)}$ and $x_i^{(5)}$ $(i = 1, \ldots, 4)$.

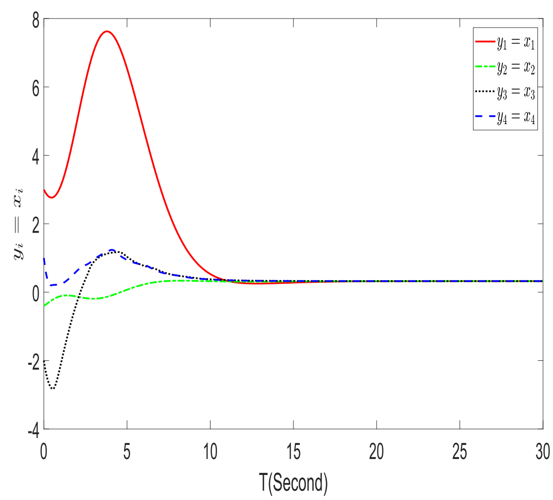

FIGURE 5. The output y_i $(i = 1, \ldots, 4)$ for heterogeneous agents.

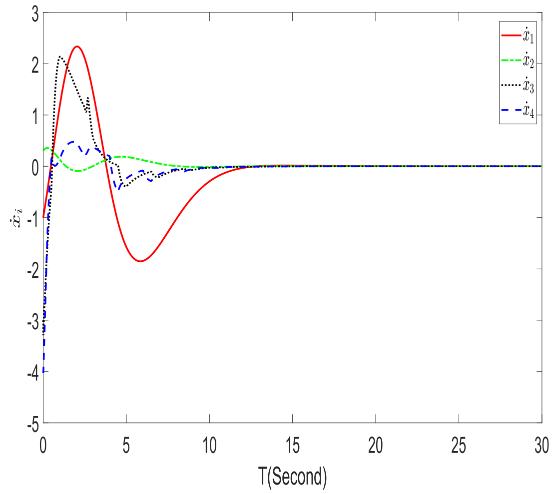

FIGURE 6. **The state trajectories of $\dot{x}_i$ ($i = 1, \ldots, 4$) for heterogeneous agents.**

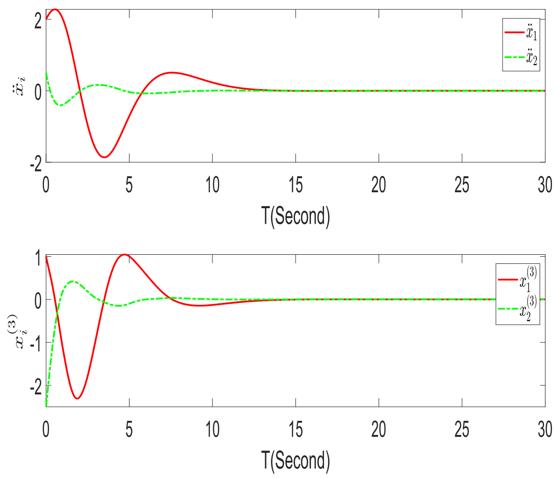

FIGURE 7. **The state trajectories of $\ddot{x}_i$ and $x_i^{(3)}$ ($i = 1, 2$) for heterogeneous agents.**

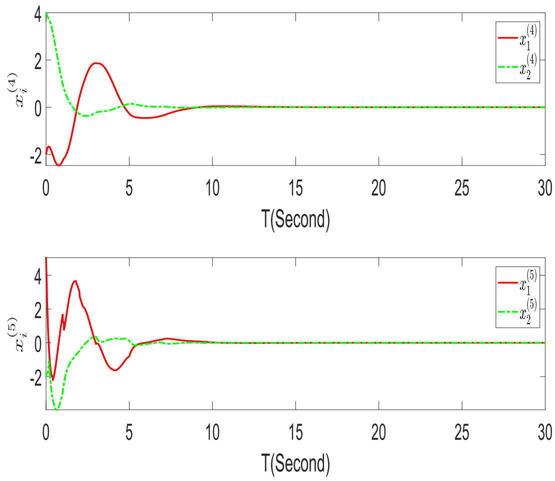

FIGURE 8. **The state trajectories of $x_i^{(4)}$ and $x_i^{(5)}$ ($i = 1, 2$) for heterogeneous agents.**

$\kappa = 1$. It is observed from Fig. 2-4 that under the UQSGs, the consensus objective can be reached.

Example 2: In this example, we verify the proposed algorithms (21)-(23) by one simulation example. The topology is selected as the switching topologies shown in Fig. 1, where agent 1 and agent 2 are six-order dynamics, and agent 3 and agent 4 are second-order dynamics. The initial states $\left[x_i^{(5)}(0), x_i^{(4)}(0), x_i^{(3)}(0), \ddot{x}_i(0), \dot{x}_i(0), x_i(0)\right]^T$, $i = 1, 2$, are randomly given as $[5, -2, 1, 2, -1, 3]^T$ and $[-2, 4, -2.5, 0.5, 0.3, -0.4]^T$, respectively. The initial conditions of agent 3 and agent 4 $[\dot{x}_i(0), x_i(0)]^T$, $i = 3, 4$, are given as $[-3, -2]^T$ and $[-4, 1]^T$. The parameter of control algorithms (21)-(23) is $\kappa = 1$, and the nonidentical unknown high-frequency gain signs are $b_1 = 1$, $b_2 = 3$, $b_3 = 2$, $b_4 = -1$. It can be observed from Fig. 5 that the output consensus of heterogeneous agents is achieved.

V. CONCLUSION

This paper has proposed the nonlinear PI control algorithms for consensus of high-order agents with UHFGSs under switching topologies. It has been proven that the proposed algorithms with properly selected nonlinear PI functions can guarantee the consensus for MASs under the UQSGs. Moreover, the existing results have been extended to the output consensus of heterogeneous agents with UHFGSs. At last, two examples have been given to present the efficiency of the proposed algorithms. Further research direction will be the cooperative control of MASs with complex dynamics.

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