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A Multi-Parameter Chaotic Fusion Approach for Traffic Flow Forecasting

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ABSTRACT Since traffic is a time-dependent and complicated non-linear system. Chaos Theory has been specifically designed to identify chaotic behaviour and properties for such systems. Previous researches have been restricted to single traffic parameter, which fails to get actual behaviour of the traffic systems. In this work, we have devised the multi-parameters chaos prediction method to describe the tendency of traffic from different aspects for prediction of single parameter chaotic time series. The fusion method is considered the relationship of different traffic parameters according to the phase space reconstruction. Taking the exit ramp of Xishanping expressway in Beibei district, Chongqing Municipality as the example, the feasibility and reliability of traffic state prediction were tested. The trial results indicates that more features of realistic traffic conditions are reflected by fusing multiple traffic parameters, and gain real accuracy improvements in traffic prediction. Compared with three single-parameter time series prediction methods, the mean absolute relative error of the multi-parameter prediction method decreased by 2.42, 2.39 and 0.8 respectively, the mean absolute relative error decreased by 2.33, 3.25 and 1.27, and the equal coefficient reached 0.9528 with a slight increase. Proposed method will definitely generates the opportunity for next generation of traffic control that are better able to detect the dynamic states of traffic, and therefore more effectively prevent the traffic congestion and pollution in the urban areas worldwide.

INDEX TERMS Traffic parameter, traffic condition, traffic prediction, phase space reconstruction.

I. INTRODUCTION

With the upward development of the economy, as the vehicle ownership increased rapidly, results in an increase in the road traffic on daily basis. The accurate prediction of the changing trends of upcoming traffic flow is a primary basis for remitting traffic congestion problems. It has academic significance and practical value to early detection, prevention, and treatment of traffic flow.

Traffic management system is an open, non-linear time-varying and complex system [1], [2]. It influences by objective factors such as weather, geography, and other human activities. These factors overall influence the changing states of traffic flow conditions [3]. Currently, several literatures demonstrate that traffic state predictions can

enhance the stability of traffic flow [4]. According to the description of transport services [5], Traffic conditions can be divided into four states: 1- Smooth, 2- Slow, 3- Crowded, and 4- Blocked. This not only preserves sequential relationship between traffic flow condition change, but also effective for the model processing. The common traffic state prediction time series [6] uses neural network [7], support vector machine [8] and the grey theory [9]. These are based on single traffic parameter prediction model which cannot completely reflect the intricate movements of transport system [10]. It does not predict the actual time-varying and complex road traffic environment demonstrated in the recent years [11]. Chaos theory provides a totally new way for transportation systems inherent randomness and complexity of traffic state prediction angle [12], [13]. In the study of chaos theory, chaotic time series modeling and forecasting has become a chaotic field of information processing in a hotspot [14].

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This purposes to make the predicative model to approach the actual physical processes better, so as to improve the reliability and usefulness of forecasts. Theoretically, a reasonable selection of embedded dimension helps to achieve ideal forecast results through one-parameter time series. Literature [15] integrates VNNTF neural model into chaos theory algorithm, and on the basis of considering chaotic characteristics of traffic flow time series, designs VNNTF model of traffic flow chaotic time series. The algorithm can reduce the network training time and improve the prediction accuracy. Literature [16] uses the time series of three traffic indicators to reconstruct the phase space, and combines the multi-source data with the Bayesian estimation theory to design the traffic flow prediction neural network based on RBF, which greatly improves the accuracy of the short-term traffic flow prediction. On the basis of the traffic flow prediction model based on Kalman filter and the theory of phase space reconstruction, Literature [17] carries out the phase space reconstruction of the traffic flow time series, and establishes the traffic flow prediction model based on Kalman filter based on the phase space reconstruction. The simulation results show that the improved model has smaller error and higher accuracy.

However, considering the practical problems, since the chaotic behaviour of complex systems often described by a number of parameters, the single parameter time series can't guarantee a complete reconstruction of the original system. Taking this into account, the multi-parameter time series contains more information of the driving force than one parameter time series. It can also reconstruct a more accurate phase space [18]. The multi-parameter time series can obtain high accuracy in phase space detection. Thus, if the integration through multi-parameter can reflect more features of traffic flow states, than it can also express more traffic parameters based on the state change process in the form of a one-dimensional time series to predict the traffic condition in the next period. Therefore, it is meaningful to study the reconstruction of phase space of multivariable time series.

It can be seen from the above studies that the chaotic fusion method based on multiple parameters is still rare. Based on the reconstruction of the phase space, this paper considers the relationship between different traffic parameters and proposes a multi-variable phase space reconstruction method for short-term traffic prediction. On the basis of chaotic model, an increase in the amount of reconstruction phase space systems' information makes trajectories closer to the real traffic availability in the phase space phase point. This increase impacts the establishment of more traffic parameters. There exists a coupling relationship among the transport parameters. The prediction method selects multiple trajectories to the reference points for each parameter and for each high-dimensional phase-point integration. The results of this experiment show that this method has high prediction accuracy after predicting one or more steps for the integration of new parameters.

II. MULTI-PARAMETER TIME SERIES PHASE SPACE RECONSTRUCTION AND COMPUTATION

A. MULTI-PARAMETER TIME SERIES RECONSTRUCTION

According to Takens Theorem [19], the evolutionary information of a power system is implicit in any component of the development. The parameters for a single time series $\{x_i\}$, ($i = 1, 2, \dots, n$), observed some delay τ and find the right m -dimensional vector can reconstruct a phase space that can restore the original dynamics of the system, which could be expressed by the equation (1).

$$X = \begin{bmatrix} x_1 & x_{1+\tau} & \cdots & x_{1+(m-1)\tau} \\ x_2 & x_{2+\tau} & \cdots & x_{2+(m-1)\tau} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n-(m-1)\tau} & x_{n-(m-2)\tau} & \cdots & x_n \end{bmatrix} \quad (1)$$

where, $k = (1, 2, \dots, n-(m-1)\tau)$ represents a phase points m -dimensional phase space. With the change of k , dynamical behaviour of the evolution of the power system can be expressed by space relative point $X(k)$ of the trajectory express [20] Similarly, based on the literature [21], the existing M parameter time series $\{y_i, j\}$ ($i = 1, 2, \dots, M$; $j = 1, 2, \dots, N$) multi-parameter space reconstruction, For M -dimensional time series: X_1, X_2, \dots, X_M , where $X_i = (x_i^1, x_i^2, x_i^3, \dots, x_i^N)$, $i = (1, 2, \dots, M)$. A time delayed reconstruction can be made as the multi-parameter phase space model (2).

$$Y_M(k) = (y_i^k, y_i^{k-\tau}, \dots, y_i^{k-(m_i-1)\tau_i}) \quad (2)$$

where, $i = (1, 2, \dots, M)$; $k = (j, j+1, \dots, N)$; $j = \max_{1 \leq i \leq M} (m_i - 1)\tau_i + 1$, τ_i and m_i are the time delay and the embedding dimension, respectively.

Among them, the delay time τ_i ($i = 1, 2, \dots, M$) with a minimum mutual information calculated, embedding dimension m_i ($i = 1, 2, \dots, M$) is calculated in reference [22]. In the multi-parameter reconstruction of phase space-time sequences, the method of calculating between chaotic time series and the largest Lyapunov exponent is the same. The calculation results of literature[22] and literature[23] shown that using several parameters to calculate the maximum Lyapunov exponent sequence needed length is short, with accuracy higher than the calculated result of a single parameter timing.

B. THE PHASE FUSION OF HIGH-DIMENSIONAL PHASE SPACE

The literature [24] presented with the point of a high-dimensional phase space on the fusion algorithm and proved that the new space trajectory fusion contains the main features of the original space trajectory. It also showed the feature of its attractor originally as a single parameter space reconstruction that can be approximated close to the real state of the system. Taking the use of method from the literature [24] to integrate multiple traffic parameters of the state of high-dimensional phase space, and to evaluate the state of fusion is called multi-parameter state.

Suppose that M is normalized after a traffic parameter mapping to a multi-dimensional phase space, phase points need to integrate the set as follows:

$$D = \left[y_1^{k+(q-1)\tau}, y_2^{k+(q-1)\tau}, \dots, y_M^{k+(q-1)\tau} \right] \quad (k = 1, 2, 3, \dots, M; q = 1, 2, 3, \dots, m) \quad (3)$$

$$\text{set: } y_i = (y_i^k, y_i^{i,k+\tau}, \dots, y_i^{k+(m-1)\tau}) (i = 1, 2, \dots, M) \quad (4)$$

Then equation (3) can be abbreviated as follows:

$$D_k = [y_1, y_2, \dots, y_M] \quad (5)$$

In equation (5), where y_i ($i = 1, 2, \dots, M$) represent an arbitrary set of M phase space point z_k represents the fusion of phase points, then the Bayes estimate as the equation (6).

$$p(z_k | y_1, y_2, \dots, y_M) = \frac{p(z_k; y_1, y_2, \dots, y_M)}{p(y_1, y_2, \dots, y_M)} \quad (6)$$

Assume parameter z_k normal $N(z_0, \sigma_0^2)$ distribution, and the distribution D_k obey $N(z_k, \sigma_h^2)$, remember $a = 1/p(y_1, y_2, \dots, y_M)$, so

$$\begin{aligned} p(z_k | y_1, y_2, \dots, y_M) &= a \prod_{h=1}^M \frac{1}{\sqrt{2\pi\sigma_h}} \exp \left[-\frac{1}{2} \left(\frac{y_h - z_k}{\sigma_h} \right)^2 \right] \\ &\quad \times \frac{1}{\sqrt{2\pi\sigma_0}} \exp \left[-\frac{1}{2} \left(\frac{z_k - z_0}{\sigma_0} \right)^2 \right] \\ &= a \exp \left[-\frac{1}{2} \sum_{h=1}^M \left(\frac{y_h - z_k}{\sigma_h} \right)^2 - \frac{1}{2} \left(\frac{z_k - z_0}{\sigma_0} \right)^2 \right] \quad (7) \end{aligned}$$

As can be seen from the equation (7), the exponent part is a quadratic function with respect to z_k , so it is still submitting to a normal distribution,

$$p(z_k | y_1, y_2, \dots, y_M) = \frac{1}{\sqrt{2\pi\sigma}} \exp \left[-\frac{1}{2} \left(\frac{z_k - z}{\sigma} \right)^2 \right] \quad (8)$$

According to equation (7) and (8) can obtain equation (9)

$$z = \left(\sum_{h=1}^M \left(\frac{y_h}{\sigma_h^2} + \frac{z_0}{\sigma_0^2} \right) \right) / \left(\sum_{h=1}^M \left(\frac{1}{\sigma_h^2} + \frac{1}{\sigma_0^2} \right) \right) \quad (9)$$

Then, $D_k = [y_1, y_2, \dots, y_M]$ Optimal integration phase points Bayes estimation can be expressed as the equation (10).

$$\hat{z}_k = \int_{\Omega} z_k \frac{1}{\sqrt{2\pi\sigma}} \exp \left[-\frac{1}{2} \left(\frac{z_k - z}{\sigma} \right)^2 \right] dz_k = z \quad (10)$$

According to the above process, to integrate all phases to obtain a new spatial phase point, and then acquire a new m -dimensional phase space at the end,

$$Z = [Z_1, Z_2, \dots, Z_M]^T \quad (11)$$

where,

$$Z_i = [z_i, z_{i+\tau}, \dots, z_{i+(m-1)\tau}]^T \quad (i = 1, 2, \dots, M) \quad (12)$$

where in equation (12), Z_i represents a new phase after the fusion point, i is any coordinate point in the time series, and M is the number of phase space. In order to ensure that all parameters can be fully reconstructed and not distorted in the same phase space we do this: taking the largest embedding dimension number m_i and the new phase space that is reconstructed with minimal delay time, the multi-parameter characteristics of each state reconstructed in the new phase space can include the reconstruction of a single parameter. To continue, in reference to more traffic parameters, although the characteristics of different parameters vary, yet they are reflected from different angles of the same traffic flow state. Therefore, we use multiple traffic parameters that evaluated the traffic conditions than those that more fully reflect the actual situation of traffic flow with single traffic parameters.

III. MULTI-PARAMETER TIME-SERIES FORECASTING TRAFFIC STATE

Take consideration of the state time series of more traffic parameters $\{z_i\}$, ($i = 1, 2, \dots, k$), and based on the equation (12) to make the time series phase space reconstruction. We arbitrarily selected two adjacent initial positions (usually taken initial point and its nearest neighbours), and calculate the initial distance $d(0)$ between the two points. These points are appropriately selected step size and duration. Next, we calculate elapsed time t after the evolutionary distance between the two points. Then select the Lyapunov exponent as the quantization index divergence and initial orbit Chaos estimated system, based on the physical significance of the largest Lyapunov exponent there is shown in the equation (13).

$$\lambda_1 = \frac{1}{k\Delta t} \ln \frac{d(k\Delta t)}{d(0)} \quad (13)$$

where, λ_1 largest Lyapunov exponent for the system. If the prediction center Z_T and most immediately by the evolution of the initial distance d between times points after Z_t, Z_T and $Z_{(t+k)}$ were to become Z_t and $Z_{(t+k)}$, then the maximum Lyapunov exponent prediction algorithm as shown in equation (14).

$$\begin{aligned} Z_{T+k} &= Z_{t+k} \pm \sqrt{\sum_{i=1}^{m-1} (Z_{T+k}(i) - Z_{t+k}(i))^2 - ((Z_T - Z_t) e^{\lambda_1 k \Delta t})^2} \quad (14) \end{aligned}$$

where, in addition to $Z_{(T+k)}$ unknown outside the rest are a known quantity. Therefore, we can obtain the predicted values required according to equation (14).

The algorithm process as Figure 1:

The specific steps shown in Figure 1 can be divided into the following four steps:

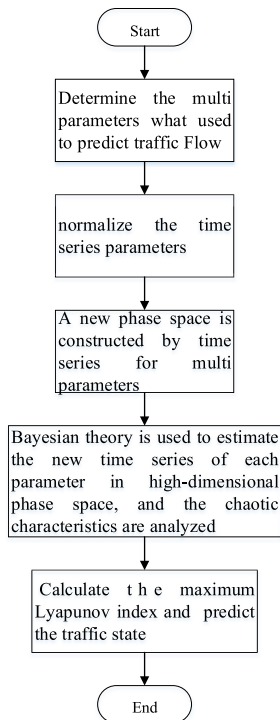


FIGURE 1. Flow chart of prediction algorithm.

(1) Respectively, the time series M argument $\{X_i\}$, $(1, 2, \dots, M)$ were normalized, and determine the parameters of chaotic time series reconstruction embedding dimension m_i ($i = 1, 2, \dots, M$) and the delay time ($i = 1, 2, \dots, M$);

(2) A new phase space dimension m and the delay time, the M parameter time-sequentially reconstructed into a new phase space, which contains all the M parameters obtained new information fusion phase space;

(3) Estimate the parameters of a new time series $\{z_i\}$ use Bayes theory in high-dimensional phase space, ($i = 1, 2, \dots, m$), and analyse the chaotic properties;

(4) The maximum Lyapunov exponent prediction algorithm parameters for its new time series forecasting.

IV. TEST VERIFICATION AND EFFECT ANALYSIS

The Xishanping Expressway Exit Ramp, which located in the Beibei District of Chongqing Municipality, is selected to test forecasting decisions of traffic state with respect to feasibility and reliability. The forecasting results are true for real-time control of traffic signals, which have a key role, but also true for highly affected traffic guidance. Thus, microwave detectors and a video monitor automatically installed on this intersection that gives the average traffic utilization in three parameters that include the average occupancy, average speed and time series. It also gives research trends at the junction traffic state in the same time sequence and timely measure to avoid traffic congestion. Figure shows a set of test data traffic every five minutes, each generating 288 data records per day, drawing its two days (December 27th, 2016 on Friday

and 28th on Saturday) time series of measured parameters and corresponding traffic states. Their traffic status can be shown in Figure 2.

As different traffic parameters describe traffic flow status differently and cannot describe reflect the actual situation completely and accurately. Therefore, one should consider various parameters of traffic flow from different angles in estimation of traffic state. In accordance with the multi-parameter integration chaotic time series prediction algorithm that analyses first test data from two days, and the normalization process is shown by equation (15).

$$u(n) = \frac{x(n) - \min(x(n))}{\max(x(n)) - \min(x(n))} \quad (15)$$

The correlation dimension of traffic parameters could reflect not only the scaling variation properties of spatial correlation of individuals, but also the spatial and scale occupation degree of individuals. All of these using the correlation dimension algorithm (referred to as GP algorithm) time series of three sets of traffic parameters separately analysis, and correlation integral therewith draw any radius r $\ln C(r) - \ln r$ graph (Figure 3).

According to the definition of correlation dimension, we found a best fit straight line from the curve in Figure 3, and then the correlation dimension D can be got by compute the slope of the line. In order to test this method, we collected the three traffic parameters simultaneously within two days on the same road link, and the traffic status are observed by these different correlation dimension. According to the results, the correlation dimension D_i ($i = 1, 2, 3$) with embedded dimension m_i ($i = 2, 3, 4, \dots, 15$) of changes has shown in Table 1.

As can be seen from Table 1, along with embedding dimension m_i increases, the correlation dimension D_i gradually stabilized, to determine the parameters of embedding dimension, minimum delay time and the corresponding maximum Lyapunov exponent as shown in Table 2, all show.

In order to render all the characteristics of the three traffic parameters as completely as possible, select the maximum and minimum embedding dimension m_z delay reconstruction parameters as phase space,

$$m_z = \max(m_i), \tau_z = \min(\tau_i), \quad (i = 1, 2, 3) \quad (16)$$

Because of phase space (16) to determine the state of the transportation system reconstruction parameter value $m_z = 9$, or $m_z = 5$, then according to equation (3)-(12) phase sequence of three sets of traffic status in the new phase space to do the optimal fusion results which shown in Figure 4.

Based on the comparative analysis of Figure 2 and Figure 4, and then combined with video images of the state, then one finds that after the integration of new traffic state time series with the real state of the detection of road traffic does coincide. This provides an information base for further analysis of this new state time series properties and its self-correlation test. After experimental basis, take 10 when

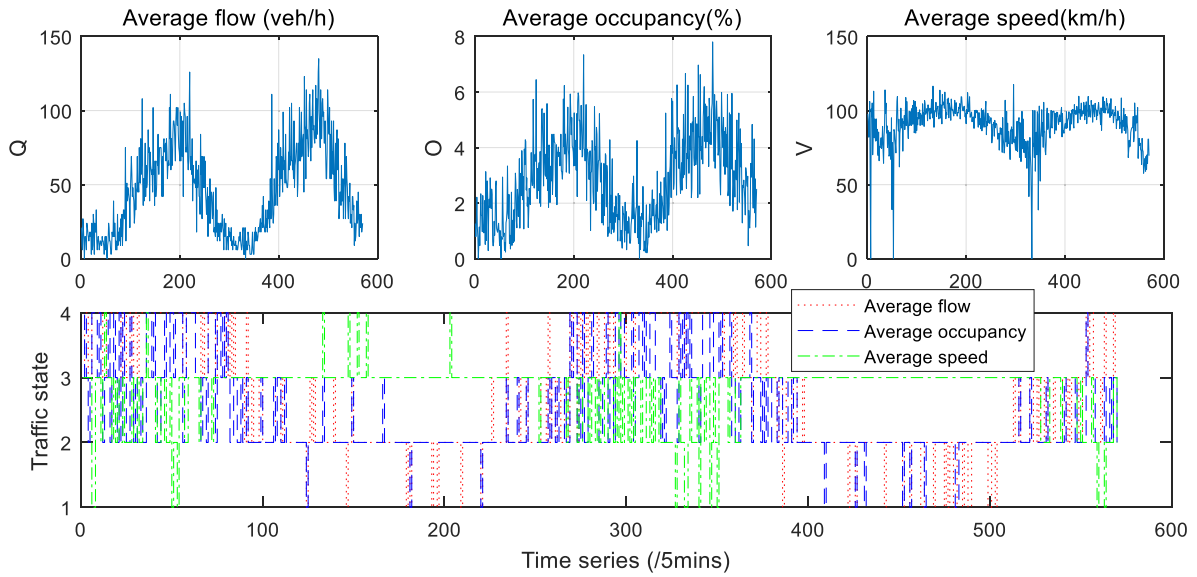


FIGURE 2. Traffic status discrimination result after fuzzy comprehensive evaluation.

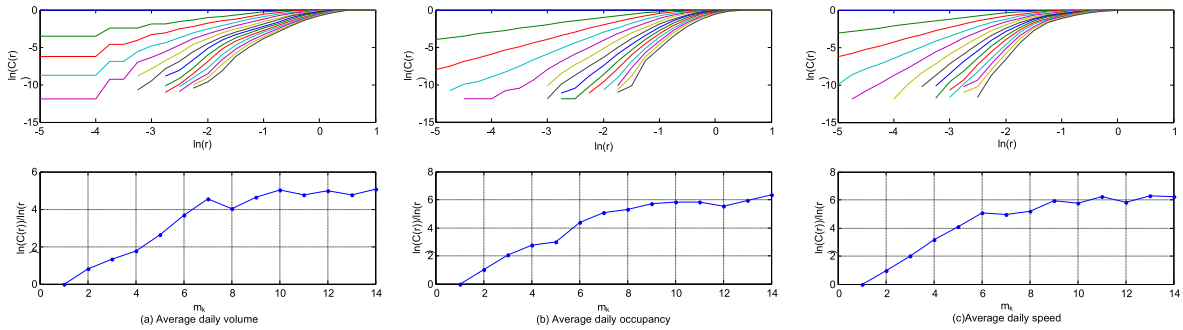


FIGURE 3. Correlation dimension of different traffic parameters.

TABLE 1. Correlation dimension with embedding dimension DI and MI change.

m_i	D_1	D_2	D_3
2	0.7886	1.0103	0.9718
3	1.3417	2.0419	1.9798
4	1.7687	2.7692	3.1652
5	2.6174	2.9795	4.112
6	3.6706	4.4049	5.0687
7	4.5402	5.0678	4.9687
8	4.0146	5.317	5.1736
9	4.6396	5.6863	5.9305
10	5.0047	5.8237	5.7574
11	4.7446	5.8201	6.2447
12	4.9806	5.5396	5.8491
13	4.7639	5.945	6.2893
14	5.0474	6.371	6.219
15	4.8942	5.8783	6.1293

embedding dimension, delay time is 15:00 (Figure 5), the correlation dimension stabilized.

After the fusion of more traffic parameters, to examine chaos characteristic of the new state time series calculates

the new state time-series using small amount of data gives maximum Lyapunov index of 0.01794. The results after fusing multiple traffic parameters show the new state time series those still showing Chaos characteristics. Then, this

TABLE 2. Calculated for system reconfiguration parameters.

Traffic Parameters	m_i	τ_i	λ_1
Average traffic, $Q_s(t)$	7	8	8.2540
Average occupancy, $O_s(t)$	9	5	2.8113
Average Speed, $V_s(t)$	6	7	6.8691

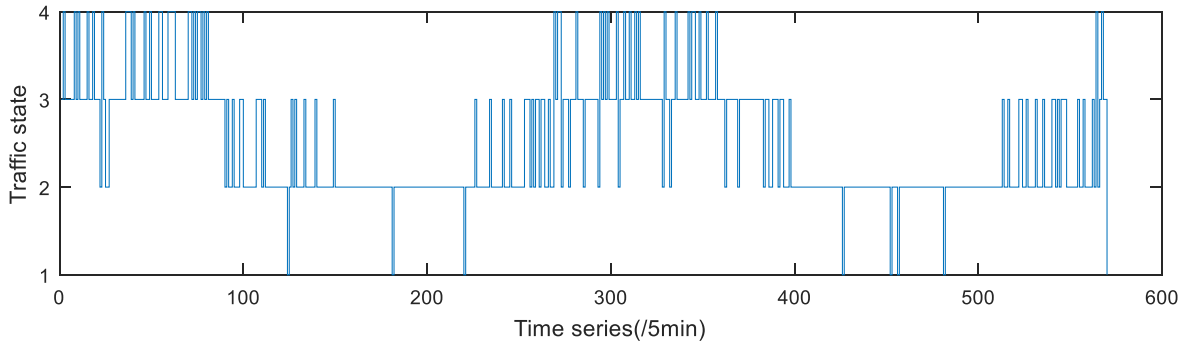


FIGURE 4. The fusion trend of phase space in more traffic parameters.

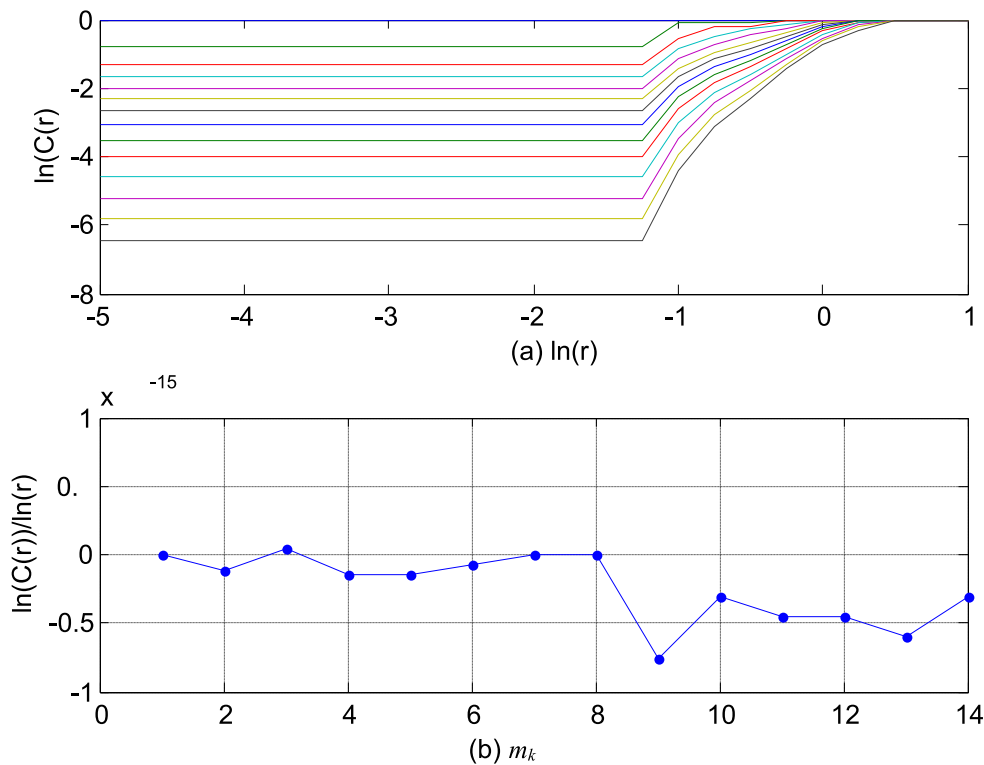


FIGURE 5. Linear variation of the correlation dimension.

can be used to predict chaos theory and to calculate the new state of the maximal Lyapunov exponent of short step whose accuracy is also higher than the calculated result of a single parameter timing. On the basis of the new traffic state that is fused by the equation (14) to obtain time series, and then we choose the front 555 samples to predict the

multiple continuous traffic status. This is done after forecasting 15 samples that are shown in the comparison of results and the actual state of Figure 6.

As it can be seen in Figure 6, we proposed a good forecasting method to predict more traffic effects, based on chaos fusion parameters to predict results that can reflect the trends

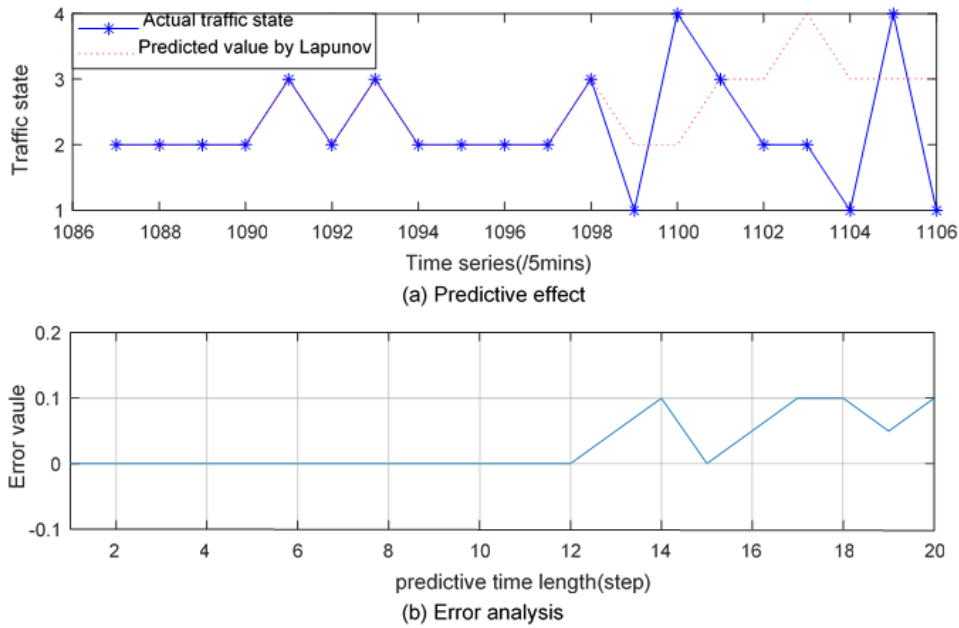


FIGURE 6. Compare traffic state predictions by traffic parameter map.

TABLE 3. Comparison between the prediction errors.

Error indicator	Average traffic $Qs(t)$	average occupancy $Os(t)$	Average Speed $Vs(t)$	Fusion Time Series $Zs(t)$
MAE /veh	10.55	10.52	8.93	8.13
MARE /%	14.17	15.09	13.11	11.84
UC	0.9357	0.9356	0.9505	0.9528

and patterns of traffic conditions change more appropriately. However, due to the impact of transport systems uncertainties, after the predicting 12 steps, traffic state prediction error significantly increased. Experimental results have shown that a case where the sampling period is 5 minutes, which is the best predictor, step 12 corresponds to a maximum length of 60 minutes prediction. This shows that there is plenty of time for congestion impending intervention, as much as possible to ensure a smooth road.

For comparative analysis, three traffic parameters are, a. one-dimensional chaotic traffic state time series prediction comparative index error calculation using the mean absolute error MAE (Mean represent actual deviation predicted and observed absolute value); b. The mean absolute relative error MARE (representing the actual deviation predicted and measured values of the absolute value of the mean percentage accounted for observations); and, c. the equal coefficient UC (represented fit predicted and measured values, generally above 0.9 is a better fit). These predicted the actual values compared with the test results in Table 3. Examples of performance prediction.

As can be seen from Table 3, compared to the direct use of a single parameter time series forecasting method to predict

the results of multi-parameter prediction method to predict than a single point in this three error specification method is better. Its MAE and MARE are significantly reduced, but the UC has also increased. Therefore, multi-parameter prediction method of chaotic state space based on the fusion of prediction accuracy ensures that the road traffic state prediction is reliable

In short, multi-parameter test results confirmed the status of traffic flow prediction method in predicting fusion and that it meet the needs of road traffic management, both long-term and for prediction accuracy.

V. CONCLUSION

In this paper, we have proposed a multi-parameters chaos prediction method to describe the tendency of traffic from different aspects for prediction of single parameter chaotic time series in high-dimensional phase space of the corresponding phase point integration. Multi-parameter maps should include new characteristic of each parameter. We used chaotic time series prediction algorithm (largest Lyapunov exponent) with road traffic to predict changes in the state by using single parameter. The experimental results show that when the sampling period of 5 minutes is the best predictor, the maximum

prediction time corresponding to step 12 is 60 minutes. There is a large margin of error from steps 12 to 20. It can be seen that the longer the predicted time, the greater the error value, and the higher the accuracy of the predicted value in the short time range. Both were analysed and it can be seen that the chaotic time series prediction method based on multi-parameter integration has better predicted effect in the short time range, also the prediction performance and accuracy is improved in traffic state prediction with good promotion and utilization value.

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