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Direct Neural Network Adaptive Tracking Control for Uncertain Non-Strict Feedback Systems With Nonsymmetric Dead-Zone

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ABSTRACT In this paper, combined with the approximation of neural network, a novel direct adaptive alleviating tracking control algorithm is presented for a class of non-strict feedback uncertain nonlinear systems. Here, both nonlinear uncertainties and nonsymmetric dead-zone inputs are considered. First, according to some coordinate transforms and variable separation methods, the non-strict feedback form is converted into the normal form. Second, the relationship of state vector and error functions are established, and the inputs of dead-zone are compensated with adaptive approaches. This novel direct scheme assumes that the approximation error and optimal approximation norms of NN are to be bounded by unknown constants and can alleviate the number of online adjusted parameters so as to improve the robust control performance of the systems. At last, under Lyapunov theorem analysis, the uniformly ultimately boundness of all the signals in the closed-loop systems can be guaranteed and the dead-zone inputs can be compensated, the effectiveness of this algorithm is well demonstrated by simulation results.

INDEX TERMS Adaptive neural networks, non-strict feedback form, nonsymmetric dead-zone, uncertain nonlinear systems.

I. INTRODUCTION

During the last decades, stability theory for uncertain systems with nonlinearities were discussed constantly [1]–[11], diverse adaptive approximation-based fuzzy or NN control schemes have been designed for uncertain systems with nonlinearities [5], [9]–[19]. Note that many of the these mentioned approximation-based fuzzy [7]–[10], [14], [17] or NN [3], [5], [6], [11]–[13] approaches were based on strict-feedback uncertain nonlinear systems [5], [12], [13] or pure-feedback nonlinear systems [8], [14]–[16], rather than uncertain non-strict feedback systems. In fact, the functions of non-strict feedback uncertain systems contain all the state variables of the system, that is to say, the above two structures strict feedback and pure-feedback forms are included in the non-strict feedback ones. So, the non-strict ones are more challenge and general for practical control systems.

Recently, many adaptive researches and control strategies based on backstepping techniques and approximation of fuzzy or NN have been proposed for non-strict feedback uncertain nonlinear systems [17]–[23]. Combing with input saturation and output constraint, [17] discussed fuzzy control for non-strict feedback systems. [18] extended NN scheme to the non-strict feedback with backlashlike hysteresis uncertain systems. Considering a class of discrete-time systems, [19]established states NN reinforcement learning adaptive control approach. Based on finite-time adaptive control approaches, [20], [21] analyzed fuzzy states feedback control and output feedback dynamic surface control for non-strict feedback respectively. [22] extended NN adaptive command filter control to stochastic time-delayed systems with unknown input saturation. Neural control methods for

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full-state constraints and unmodeled dynamics in non-strict feedback uncertain systems are designed [23]. But, many of these papers did not consider the unknown dead-zone inputs, especially for the more complex uncertain nonlinear non-symmetric dead-zones.

Unknown dead-zone input as one of the nonlinearities often occurs in the process of the practical engineering, which is a source of instability and limitation of performance of systems. Recently, the investigations of input dead-zone has attracted a great deal of attention [24]–[30]. Decentralized control for large-scale systems with actuator faults and tracking control for switched stochastic actuator dead-zone systems were discussed in [24], [25]. [26] studied the non-backstepping VUFC algorithm for pure-feedback form. Based on switched nonlinear systems, [27] [28] extended time-varying tan-type barrier Lyapunov function adaptive fuzzy control and adaptive neural quantized control for states constrained systems and MIMO asymmetric actuator systems. Adaptive neural control [29] and fuzzy decentralized control [30] were proposed for unknown control directions systems and strong interconnected nonlinear systems in unmodeled dynamics. Based on robust optimal control method, [36] discussed the event-triggered physically interconnected mobile Euler-Lagrange systems.

Although many researchers have extensively studied for non-strict feedback for nonlinear systems or for systems with unknown dead-zones, to the authors' best knowledge, very few investigators concentrated on non-strict feedback systems with uncertain nonlinearities and non-symmetric unknown nonlinear dead-zone inputs, and many adaptive parameters need to be adjusted in the recursive process of these backstepping or approximation-based approaches, due to updating parameters of NN optimal weight vector or the optimal approximation vector of fuzzy logic systems, which would affect the systems control performance and the online computation burden. As far as we know, for non-strict feedback nonlinear systems, no reports on novel alleviating computation NN control approach in the literature can be found. All of these motivate this paper.

Motivated by the above considerations, aiming at alleviating the computation, this paper consider a novel adaptive NN tracking control for a class of non-strict feedback systems with nonsymmetric dead-zone inputs. Neural networks (NN) are utilized to approximate the unknown nonlinearities and nonlinear functions, and a robust NN state-feedback tracking control method is developed in the framework of backstepping design technique. This approach can not only compensate the effect of the non-symmetric dead-zone inputs but also improve the robust performance of the system by updating estimations of unknown bounds. Compared with the related existing literature, the main advantages and contributions of this paper proposed are listed below.

1) This established control scheme can compensate non-symmetric dead-zone inputs, uncertainties and solve the problems of included non-affine structure states non-strict feedback simultaneously. Although the previous results in [17]–[23] also studied the same control design problem for non-strict feedback nonlinear systems, they do not consider uncertain non-symmetric dead-zones and have computing burden problem.

2) Based on NN novel alleviating computation control approach, at each design step, F-norm parameters and unknown constants are used to approximate the bound of optimal weight vector of NN and the approximation error. Thus, this approach needs to adjust only one parameter rather than the elements of the optimal approximation vectors of NN. As a result, compared with the traditional back-stepping-based and approximation-based scheme for nonlinear systems [4]–[14], [17], [23], [27], [29], [37], [38], the approach needs to adjust fewer parameters and the computational burden is significantly alleviated.

The rest of this paper is organized as follows. Preliminaries and problem formulation and are explained in Sect. 2. A novel adaptive NN tracking control design procedure is presented in Sect. 3. Simulation is demonstrated in Sect. 4 to illustrate the availability of the approach, Sect. 5 gives the conclusion.

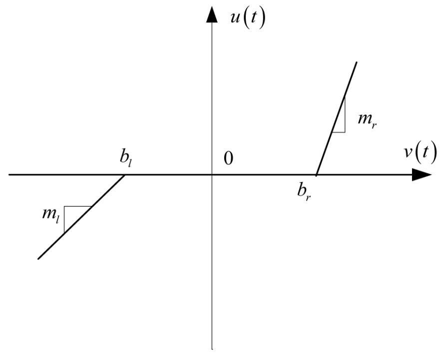

FIGURE 1. Nonlinear dead-zone model.

II. PROBLEM STATEMENTS AND PRELIMINARIES

A. PRELIMINARIES FORMULATION AND SYSTEM DESCRIPTIONS

In this paper, we focus on a class of uncertain nonlinear time-varying non-strict feedback systems with unknown nonlinearities and non-symmetrical dead-zone inputs as follows:

$$\begin{cases} \dot{x}_1 = g_1(x_1)x_2 + f_1(x) + \triangle_1(t), \\ \dot{x}_i = g_i(\overline{x}_i)x_{i+1} + f_i(x) + \triangle_i(t) \\ \qquad\qquad (i = 2, \ldots, n-1) \\ \dot{x}_n = g_n(x)u(t) + f_n(x) + \triangle_n(t) \\ y = x_1. \end{cases} \tag{1}$$

where the non-symmetrical dead-zone with input $v(t)$ and output $u(t)$ as shown in Fig. 1, and the dynamic model of unknown non-strict feedback dead-zone nonlinear systems [26] can be described as:

$$u(t) = D(v(t)) = \begin{cases} m_r(v(t)) & \text{if } v(t) \geqslant b_r, \\ 0 & \text{if } b_l < v(t) < b_r, \\ m_l(v(t)) & \text{if } v(t) \leqslant b_l. \end{cases} \tag{2}$$

where $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T \in R^i$ $(i = 1, 2, \ldots, n)$, $x = [x_1, x_2, \ldots, x_n]^T \in R^n$, and $y \in R$ are the state vector and output of the systems respectively. $u(t) \in R$ is the input of the system (output of the dead-zones); $v(t) \in R$ is the input to dead-zone. In this paper, $f_i(\cdot), i = 1, 2, \ldots, n$ and $g_i(\cdot), i = 1, 2, \ldots, n$ are unknown smooth nonlinear functions with $f_i(0) = 0, g_i(0) = 0$; $\triangle_i(\cdot), i = 1, 2, \ldots, n$ are smooth uncertain disturbance. $m_r(\cdot)$, $m_l(\cdot)$ for dead-zone are unknown nonlinear smooth functions; b_r, b_l represent unknown right and left slopes of dead-zone and dead-zone breakpoint parameters respectively.

The control objective is to design robust adaptive NN controllers $v(t)$ for the non-strict feedback systems (1), such that the following can be observed:

1) The system output $y(t) = x_1$ can track the desired trajectory reference signal $y_d(t)$ very small;

2) All the signals in the closed-loop systems are uniformly ultimately bounded. Where $y_d(t)$ and its kth order derivative $y_d^{(k)}(t)$ $(k = 1, 2, \ldots, n)$ are assumed to be bounded and continuous.

Similar to [32], [32], to facilitate control system design, we need the following Assumptions for the dead-zone of the control problem investigated in this paper.

Asusmption 1 [26], [28]: The dead-zone outputs $u(t)$ is assumed to be not available and the parameters b_r and b_l are assumed to be unknown constants, but their signs are known, i.e., $b_r > 0$ and $b_l < 0$.

Remark 1: As stated in [25], [27], [32], [33], this non-strict feedback nonlinear model with unknown dead-zone input is a typical model for a hydraulic servo valve or a servo motor in many practical industrial mechanical processes. However, many results in these papers were based on traditional backstepping technique as well as the approximation features of FlSs or NN [17], [23], as we known that in the recursive process of these approximation and backstepping-based approaches, as the order increased, the design procedure can cause 'explosion of complexity' [25], [27], [33], many adaptive parameters were needed to be adjusted [29]–[33] even together with dynamic surface control (DSC) method [12], therefore, the online computation burden is rather heavy, especially in dealing with MIMO or non-strict feedback nonlinear systems [17], [23]. Different from these results [29]–[33], or the optimal control method to compensate the dead-zone [36], in this paper, we will explore a direct novel alleviating computation NN control method for nonlinear non-strict feedback systems.

Asusmption 2 [26]: Assume that the dead-zones' left and right growth functions $m_r(\cdot)$, $m_l(\cdot)$ are smooth, and there exist unknown positive constants $k_{l0}, k_{l1}, k_{r0}, k_{r1}$, such that

$$0 < k_{l0} \leqslant m_l'(v(t)) \leqslant k_{l1}, \forall v(t) \in (-\infty, b_l],$$
$$0 < k_{r0} \leqslant m_r'(v(t)) \leqslant k_{r1}, \forall v(t) \in [b_r, +\infty),$$

where $m_r'(v(t)) = \frac{dm_r(z)}{dz}|_{z=v}, m_l'(v(t)) = \frac{dm_l(z)}{dz}|_{z=v}$.

In general, for convenience, $m_r(v(t))$ and $m_l(v(t))$ in above Eqs. are assumed to be true for $v(t) \in (-\infty, m_l]$ and for $v(t) \in [m_r, +\infty)$ respectively.

According to the differential mean value theorem, there exist $\xi_l \in (-\infty, b_l]$ and $\xi_r \in [b_r, +\infty)$ such that

$$m_l(v(t)) - m_l(b_l) = m_l'(\xi_l(v(t)))(v(t) - b_l),$$

for $\xi_l(v(t)) \in (v(t), b_l)$ or $(b_l, v(t))$. and

$$m_r(v(t)) - m_r(b_r) = m_r'(\xi_r(v(t)))(v(t) - b_r),$$

for $\xi_r(v(t)) \in (v(t), b_r)$ or $(b_r, v(t))$.

Now define vectors $\Phi(t)$ and $\Theta(t)$ as follows:

$$\Phi(t) = [\varphi_r(t), \varphi_l(t)]^T,$$
$$\Theta(t) = [m_r'(\xi_r(v(t))), m_l'(\xi_l(v(t)))]^T,$$

and where

$$\varphi_r(t) = \begin{cases} 1 & \text{if } v(t) > b_l, \\ 0 & \text{if } v(t) \leqslant b_l, \end{cases}$$
$$\varphi_l(t) = \begin{cases} 1 & \text{if } v(t) < b_r, \\ 0 & \text{if } v(t) \geqslant b_r, \end{cases}$$

Based on Assumption 2, the dead-zone model (2) can be redefined as follows:

$$u(t) = D(v(t)) = \Theta^T(t)\Phi(t)v(t) + d(v), \tag{3}$$

$d(v)$ can be calculated from Assumption 2 and above equations:

$$d(v(t)) = \begin{cases} -m_r'(\xi_r(v(t)))b_r, & \text{if } v(t) \geqslant b_r, \\ -[m_l'(\xi_l(v(t))) & \\ +m_r'(\xi_r(v(t)))]v(t), & \text{if } b_l < v(t) < b_r, \\ -m_l'(\xi_l(v(t)))b_l, & \text{if } v(t) \leqslant b_l, \end{cases} \tag{4}$$

where $\xi_l(v) \in (v, b_l)(v < b_l)$; $\xi_l(v) \in (b_l, v)(b_l < v < b_v)$; $\xi_r(v) \in b_r, v(b_r < v)$; $\xi_r(v) \in (v, b_r)(b_l < v < b_v)$.

Asusmption 3: Assume that the signs of $g_i(\bar{x}_i)$, $(i = 1, 2, \ldots, n)$ are known, and there exist positive parameters g_{i0} and g_{i1}, satisfying $|g_i(\cdot)| \geqslant g_{i0} > 0$, $\forall\, \bar{x}_i \in \Omega_i \subset R^i$, and $|g_i(\cdot)| \leqslant g_{i1}$, $\forall\, \bar{x}_i \in \Omega_i \subset R^i$. Without loss of generality, we assume that $g_{i0} < g_i(\cdot) < g_{i1} < \infty$ [3].

Remark 2: In this paper, dead-zone output $u(t)$ is assumed to be not available, parameters b_l and b_r are assumed to be unknown but with $b_r > 0$ and $b_l < 0$ [33]–[35]. In addition, according to Assumption 2, we conclude that $|d(v)| \leqslant p^*$, and p^* is an unknown positive constant and can be chosen as $p^* = (k_{l1} + k_{r1})\max\{b_r, -b_l\}$. There exist positive constant β_0, satisfying $\beta_0 \leqslant \min\{k_{l0}, k_{r0}\}$. For unknown external disturbance $\triangle_i(t)$, there exist positive parameters d^* satisfying $|\triangle_i(t)| \leqslant d^*$ [33]–[35].

Asusmption 4 [17], [23]: There exist strictly increasing smooth functions $\phi_i(\cdot) : R^+ \to R^+$, with $\phi_i(0) = 0$, such that

$$|f_i(x)| \leqslant \phi_i(\|x\|), \quad i = 1, 2, \ldots, n \tag{5}$$

Remark 3 [17], [23]: According to Assumption 4, we conclude that if there exist $a_i \geqslant 0 (i = 1, 2, \ldots, n)$, the function $\phi_i(\|x\|)$ in the Assumption 4 can be deduced that $\phi_i(\sum_{i=1}^n a_i) \leqslant \sum_{i=1}^n \phi_i(na_i)$, Because $\phi_i(s)$ is smooth and $\phi_i(0) = 0$, the following inequality holds $\phi_i(\sum_{i=1}^n a_i) \leqslant \sum_{i=1}^n na_ih_i(na_i)$, where $h_i(s)$ is a smooth function, satisfying $\phi_i(s) < sh_i(s)$, such a property will be used to cope with the structure of non-feedback [17], [23].

B. RADIAL BASIS FUNCTION NEURAL NETWORK(RBF NN)

In this paper, we will exploit RBF neural networks to approximate the unknown nonlinearities for system (1).

Such as, an unknown smooth nonlinear function $\psi(Z) : R \to R$ will be approximated on a compact set Ω by the following RBF neural network

$$\psi(Z) = W^{*T}\xi(Z) + \varsigma \tag{6}$$

where $W^* = [W_1, W_2, \ldots, W_m]^T \in R^m$ is an optimal constant weight vector, and $\xi(Z) = [\xi_1(Z_1), \ldots, \xi_m(Z_m)]^T : \Omega \to R^m$ is a vector-valued function defined in R^m, denoted the components of $\xi_i(Z)$ by $\rho_i(Z_i)$,$i = 1, \ldots, m$. $\rho_i(Z_i)$ is called a basis function with the neural number $m > 1$, commonly chosen as Gaussian function, $\rho_i(Z_i) = \exp\left[-(Z_i - \varrho_i)^2/\eta^2\right]$, where $\varrho_i \in \Omega$,$i = 1, \ldots, m$ are constant vectors called the center of the basis function, and $\eta > 0$ is a real number called the width of basis function.

As pointed out in [5] and [6], according to the approximation property of the RBF network, for a continuous real-valued function $\psi(Z) : \Omega \to R$, Ω is a compact, and any $\varsigma_H > 0$, by appropriately choosing $\varrho_i \in \Omega$and η, $i = 1, \ldots, m$, for some sufficiently large integer m, there exists an ideal weight vector $W^* \in R^m$ such that the RBF network $W^T\xi(Z)$ can approximate the given function $\psi(Z)$with the approximation error bounded by ς_H.

$$\sup_{Z\in\Omega} |\psi(Z) - W^T\xi(Z)| \leqslant \varsigma$$

where $\varsigma = \psi(Z) - W^T\xi(Z)$ and ς denotes the neural network inherent approximation error with $|\varsigma| \le \varsigma_H$ [5], [6].

Remark 4: In this paper, based on F-norm approximation of NN, the proposed direct novel alleviating NN tracking control could algorithm guarantee that the adaptive adjusted parameters here are only one no matter how many states in the design procedure. Thus, this new approach can alleviate the online computation burden and improve the robust control performance.

III. ADAPTIVE ROBUST RBF NN CONTROL DESIGN AND PERFORMANCE ANALYSIS

Different from the similar backstepping-based results in feedback form with unknown dead-zone inputs in [1]–[6], [12]–[16], [29]–[34], [38], in this section, we will discuss a novel alleviating computation adaptive NN approximation-based tracking control approach in details for the nonlinear non-strict feedback plant in (1). The concrete design procedure contains n steps. First, from step 1 to step $n-1$, virtual controllers α_i and adaptive laws $\hat{\theta}_i$, $\hat{\delta}_i$, $(i = 1, 2, \ldots, n-1)$ will be constructed, in step n, actual controller $v(t)$ will be designed to ensure that the whole system is stable and the adaptive laws $\hat{\theta}_i$, $\hat{\delta}_i$ will be given in the following design procedure.

A. ADAPTIVE NN DESIGNING PERFORMANCE

The coordinate transformation is given as follows:

$$\begin{cases} z_1 = x_1 - y_d \\ z_i = x_i - \alpha_{i-1}, \quad (i = 1, 2, \ldots n) \end{cases} \tag{7}$$

where α_{i-1} $(i = 1, 2, \ldots, n)$ are virtual controllers, which will be determined in $i-1th$ steps. To make the system achieve the desired performance, the system (1) is considered to be a series of subsystems. Different from designing a fractional order controller, here, based on backstepping design technique, NN approximation and the alleviating algorithm, we will give the detailed feasible virtual control signals controller, NN adaptive laws and actual controller design procedure in the following steps.

The first feasible virtual control signal α_1 and adaptation laws $\hat{\theta}_1$, $\hat{\delta}_1$are considered as follows:

$$\begin{aligned} \alpha_1 = {} & (\frac{1}{1+g_{11}})[-c_1z_1 - \frac{\hat{\theta}_1^2 z_1||\xi_1(Z_1)||^2}{\hat{\theta}_1|z_1|||\xi_1(Z_1)|| + \tau_1^{(1)}} \\ & - \frac{\hat{\delta}_1^2 z_1}{\hat{\delta}_1|z_1| + \tau_1^{(2)}}] \end{aligned} \tag{8}$$

$$\dot{\hat{\theta}}_1 = -\rho_1^{(1)}\hat{\theta}_1 + \gamma_1^{(1)}|z_1|||\xi_1(Z_1)|| \tag{9}$$

$$\dot{\hat{\delta}}_1 = -\rho_1^{(2)}\hat{\delta}_1 + \gamma_1^{(2)}|z_1| \tag{10}$$

where parameters $c_1 > 0$, $\tau_1^{(1)} > 0$, $\tau_1^{(2)} > 0$, $\rho_1^1 > 0$, $\rho_1^2 > 0$, $\gamma_1^{(1)} > 0$ and $\gamma_1^{(2)} > 0$ are positive design constants to be designed. $\hat{\theta}_1$, $\hat{\delta}_1$ are adaptive adjusted parameters to be designed later. $Z_1 = [z_1^T, \hat{\theta}_i^T, y_d, y_d^{(1)}]^T \in R^4$, $\xi_1(Z_1)$ is basis function of NN.

The ith feasible virtual control signal α_i and adaptation laws $\hat{\theta}_i$, $\hat{\delta}_i$are considered as follows:

$$\begin{aligned} \alpha_i = {} & (\frac{1}{1+g_{i1}})[-c_iz_i - \frac{\hat{\theta}_i^2 z_i||\xi_i(Z_i)||^2}{\hat{\theta}_i|z_i|||\xi_i(Z_i)|| + \tau_i^{(1)}} \\ & - \frac{\hat{\delta}_i^2 z_i}{\hat{\delta}_i|z_i| + \tau_i^{(2)}} - z_{i-1}] \end{aligned} \tag{11}$$

$$\dot{\hat{\theta}}_i = -\rho_i^{(1)}\hat{\theta}_i + \gamma_i^{(1)}|z_i|||\xi_i(Z_i)|| \tag{12}$$

$$\dot{\hat{\delta}}_i = -\rho_i^{(2)}\hat{\delta}_i + \gamma_i^{(2)}|z_i| \tag{13}$$

where parameters $c_i > 0$, $\tau_i^{(1)} > 0$, $\tau_i^{(2)} > 0$ and $\rho_i^{(1)} > 0$, $\rho_i^{(2)} > 0$, $\gamma_i^{(1)} > 0$ and $\gamma_i^{(2)} > 0$ are positive design constants to be designed later. $\hat{\theta}_i$, $\hat{\delta}_i$ are adaptive adjusted parameters to be designed later. $Z_i = [z_i^T, \bar{\hat{\theta}}_i^T, (\bar{y}_d^{(i)})]^T \in R^{3i+1}$, $\bar{\hat{\theta}}_i = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_i]^T \in R^i$, $\bar{y}_d^{(i)} = [y_d, y_d', \ldots, y_d^{(i-1)}] \in R^{(i+1)}$, $\xi_i(Z_i)$ is basis function of NN.

Finally, the independent actual controller $v(t)$ and the *nth* adaptive laws $\hat{\theta}_n$, $\hat{\delta}_n$ are designed as follows:

$$v(t) = (\frac{1}{1+g_{n1}})\frac{1}{\beta_0}[-c_n z_n - \frac{\hat{\theta}_n^2 z_n||\xi_n(Z_n)||^2}{\hat{\theta}_n|z_n|||\xi_n(Z_n)|| + \tau_n^{(1)}} - \frac{\hat{\delta}_n^2 z_n}{\hat{\delta}_n|z_n| + \tau_n^{(2)}} - z_{n-1} - p^*] \tag{14}$$

$$\dot{\hat{\theta}}_n = -\rho_n^{(1)}\hat{\theta}_n + \gamma_n^{(1)}|z_n|||\xi_n(Z_n)|| \tag{15}$$

$$\dot{\hat{\delta}}_n = -\rho_n^{(2)}\hat{\delta}_n + \gamma_n^{(2)}|z_n| \tag{16}$$

where parameters $c_n > 0$, $\tau_n^{(1)} > 0$, $\tau_n^{(2)} > 0$, $\rho_n^{(1)} > 0$, $\rho_n^{(2)} > 0$, $\gamma_n^{(1)} > 0$ and $\gamma_n^{(2)} > 0$ are positive constants to be designed later. $\hat{\theta}_n$, $\hat{\delta}_n$ are adaptive adjusted parameters to be designed later. $Z_n = [z_n^T, \bar{\hat{\theta}}_n^T, (\bar{y}_d^{(n-1)})]^T \in R^{3n+1}$, $\bar{\hat{\theta}}_n = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_n]^T \in R^i$, $\bar{y}_d^{(i)} = [y_d, y_d', \ldots, y_d^{(n-1)}] \in R^{(n+1)}$, $\xi_n(Z_n)$ is basis function of NN.

The following four lemmas will be used for control design in this Section.

Lemma 1 [17]: For any arbitrary $\omega \in R$, and $\varepsilon > 0$, the inequality holds, $0 \leqslant |\omega| - \omega\tanh(\frac{\omega}{\varepsilon}) \leqslant \sigma\varepsilon$. with $\sigma = 0.2785$.

Lemma 2 [35]: for any positive variable $a, b \in R^n$, $a > 0, b > 0$, inequalities $a - \frac{a^2}{a+b} \leqslant b$ hold.

Lemma 3 (Young's Inequality [35]): for any vectors $x, y \in R^n$, the inequality $x^T y \leqslant \frac{a^p}{p}||x||^p + \frac{1}{qa^q}||y||^q$ hold, where $a > 0, p > 1, q > 1$, and $(p-1)(q-1) = 1$.

Lemma 4: For the coordinate transformations $z_i = x_i - \alpha_i - 1$, for $i = 1, 2, \ldots, n$, the following result holds.

$$\|x\| \leqslant \sum_{i=1}^{n} |z_i||\psi_i(z_i, \hat{\theta}_i, \hat{\delta}_i)| + |d_h| \tag{17}$$

where $\psi_i(z_i, \hat{\theta}_i, \hat{\delta}_i) = -c_i - \frac{\hat{\theta}_i^2}{\hat{\theta}_i|z_i| + \tau_i^{(1)}} - \frac{\hat{\delta}_i^2}{\hat{\delta}_i|z_i| + \tau_i^{(2)}} - z_{(i-1)}$, for $i = 1, 2, \ldots, n-1$, and $\psi_n(\cdot) = 1$, $d_h = y_d + p^*$.

Proof: Let $\alpha_0 = y_d$, form the virtual controller α_i, $i = 1, 2, \ldots, n$ in (8), (11) and the fact that $\|\xi(Z_i)\| \leqslant 1$, for $i = 1, 2, \ldots, n$, then, $\|x\|$ becomes,

$$\begin{aligned}\|x\| &\leqslant \sum_{i=1}^{n}|x_i| = \sum_{i=1}^{n}|z_i + \alpha_i| \leqslant \sum_{i=1}^{n}(|z_i| + |\alpha_i|) \\ &\leqslant \sum_{i=1}^{n}|z_i| + \sum_{i=1}^{n-1}(c_i + z_{i-1} + \frac{\hat{\theta}_i^2}{\hat{\theta}_i|z_i| + \tau_i^{(1)}} \\ &\quad + \frac{\hat{\delta}_i^2}{\hat{\delta}_i|z_i| + \tau_i^{(2)}} - z_{(i-1)}) + |y_d| \\ &\leqslant \sum_{i=1}^{n}|\psi_i(z_i, \hat{\theta}_i, \hat{\delta}_i)||z_i| + |d_h|\end{aligned}$$

This complete the proof. □

Remark 4: Lemma 4 gives the relationship between $\|x\|$ and error signals z_i, $(i = 1, 2, \ldots, n)$, together with (5), plays an important role in this paper, due to the nonlinear function $f_i(x)$ contains the whole state variables in the *ith* differentiate equation, which cannot be estimated by RBF NN directly. Then, it provide a variable separation approach to decompose the function $f_i(x)$ into a sum bounded functions with respect to z_i, $(i = 1, 2, \ldots, n)$.

The main results are presented by the following theorem.

Theorem 1: Consider the closed-loop system with unknown dead-zone input of the plant (1) and (2), the virtual controllers α_1 in (8),α_i in (11) and adaptive laws $\dot{\hat{\theta}}_1$ in (9), $\dot{\hat{\delta}}_1$ in (10), $\dot{\hat{\theta}}_i$ in (12), $\dot{\hat{\delta}}_i$ in (13), $\dot{\hat{\theta}}_n$ in (15), $\dot{\hat{\delta}}_n$ in (16), and the actual controller $v(t)$ in (14), under Assumptions 1-4. Suppose that for $i = 1, 2, \ldots, n$, the unknown functions $H_i(Z_i)$ can be approximated by RBF NN system $W_i^T\xi(Z_i)$ in the sense that the approximation error ς_i is bounded, then based on the bounded initial conditions, according to the Lyapunov stable analysis methods.

1) It can guarantee that all the signals in the closed-loop system are ultimately uniformly bounded(UUB).

2) The output $y = x_1$ can track the reference signals y_d and make sure that the tracking error convergence to a small neighborhood of zero.

Proof: There will contain n steps.

***Step1*:** Consider the first part in plant (1) $\dot{x}_1 = g_1(x_1)x_2 + f_1(x) + \triangle_1(t)$. Define the first tracking error variable $z_1 = x_1 - y_d$, and along its trajectory, we have $\dot{z}_1 = \dot{x}_1 - \dot{y}_d = g_1(x_1)x_2 + f_1(x) + \triangle_1(t) - \dot{y}_d$.

Define the first smooth Lyapunov function as follows:

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2\gamma_1^{(1)}}\tilde{\theta}_1^2 + \frac{1}{2\gamma_1^{(2)}}\tilde{\delta}_1^2 \tag{18}$$

where $\tilde{\delta}_1 = \hat{\delta}_1 - \delta_1$ and $\tilde{\theta}_1 = \hat{\theta}_1 - \theta_1$, parameters $\gamma_1^{(1)}$, $\gamma_1^{(2)}$ will be designed in the following analysis.

The time derivative of V_1 is

$$\begin{aligned}\dot{V}_1 &= z_1\dot{z}_1 + \frac{\tilde{\theta}_1\dot{\hat{\theta}}_1}{\gamma_1^{(1)}} + \frac{\tilde{\delta}_1\dot{\hat{\delta}}_1}{\gamma_1^{(2)}} \\ &= z_1(g_1(x_1)(z_2 + \alpha_1) + f_1(x) + \triangle_1(t) - \dot{y}_d) \\ &\quad + \frac{\tilde{\theta}_1\dot{\hat{\theta}}_1}{\gamma_1^{(1)}} + \frac{\tilde{\delta}_1\dot{\hat{\delta}}_1}{\gamma_1^{(2)}}\end{aligned} \tag{19}$$

According to Assumption 4 and Lemma 1-4, we conclude that,

$$\begin{aligned}z_1 f_1(x) &\leqslant |z_1|\phi_1(\|x\|) \\ &\leqslant |z_1|\phi_1(\sum_{j=1}^{n}(z_j\psi_j) + |d_h|) \\ &\leqslant |z_1|\sum_{j=1}^{n}[\phi_1(|z_j\psi_j|) + |d_h|] \\ &\leqslant \sum_{j=1}^{n}\frac{z_1^2}{2} + \sum_{j=1}^{n}\frac{1}{2}z_j^2\bar{\phi}_1^2(|z_j\psi_j|) \\ &\quad + |z_1|\phi_1((n+1)|d_h|)\end{aligned} \tag{20}$$

where $\bar{\phi}_1(|z_j\psi_j|) = (n+1)|\psi_j|h_1((n+1)z_j\psi_j)$

And together with Lemma 1, we conclude another inequality.

$$|z_1|\phi_1((n+1)|d_h|) \leqslant z_1 U_1 \tanh(\frac{z_1 U_1}{\varepsilon_1}) + \sigma\varepsilon_1 \tag{21}$$

where $U_1 = \phi_1((n+1)|d_h|)$

Substituting the above inequalities (20), (21), the virtual control law (8), and adaptive laws (9), (10) into the derivative of Lyapunov functionV_1 (18), we obtain,

$$\begin{aligned}\dot{V}_1 \leqslant{}& z_1(g_1(x_1)x_2 + \delta_1(t) - \dot{y}_d) + \sum_{j=1}^{n}\frac{1}{2}z_j^2\bar{\phi}_1^2(|z_j\psi_j|)\\ &+\frac{nz_1^2}{2} + |z_1|\phi_1((n+1)|d_h|) + \frac{\tilde{\theta}_1\dot{\hat{\theta}}_1}{\gamma_1^{(1)}} + \frac{\tilde{\delta}_1\dot{\hat{\delta}}_1}{\gamma_1^{(2)}}\end{aligned} \tag{22}$$

Step *i:* ($2\leqslant i\leqslant n-1$), in this step, we will construct the *ith* Lyapunov function.

Define the ith smooth Lyapunov function as follows:

$$V_i = \frac{1}{2}z_i^2 + \frac{1}{2\gamma_i^{(1)}}\tilde{\theta}_i^2 + \frac{1}{2\gamma_i^{(2)}}\tilde{\delta}_i^2 \tag{23}$$

where $\tilde{\delta}_i = \hat{\delta}_i - \delta_i$ and $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$, parameters $\gamma_i^{(1)}, \gamma_i^{(2)}$ will be designed in the following analysis. where $z_i = x_i - \alpha_{i-1}$, consider equation $\dot{x}_i = g_i(\bar{x}_i)x_{i+1} + f_i(x) + \triangle_i(t)$ in plant (1).

The time derivative of V_i at t is.

$$\dot{V}_i = z_i(g_i(\bar{x}_i)x_{i+1} + f_i(x) + \triangle_i(t) - \dot{\alpha}_{i-1}) + \frac{\tilde{\theta}_i\dot{\hat{\theta}}_i}{\gamma_i^{(1)}} + \frac{\tilde{\delta}_i\dot{\hat{\delta}}_i}{\gamma_i^{(2)}} \tag{24}$$

where $\alpha_{i-1} = \alpha(x_1, \ldots, x_{i-1}; \hat{\theta}_1, \ldots, \hat{\theta}_{i-1}; \hat{\delta}_1, \ldots, \hat{\delta}_{i-1}; y_d, y_d', \ldots, y_d^{(i-1)})$ and the derivative of α_{i-1} is as follows:

$$\begin{aligned}\dot{\alpha}_{i-1} ={}& \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}f_k(x) + \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}(g_k(\bar{x}_k)x_{k+1} + \triangle_k(t))\\ &+\sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_k}\dot{\hat{\theta}}_k + \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\delta}_k}\dot{\hat{\delta}}_k + \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(k-1)}}y_d^{(k)}\end{aligned} \tag{25}$$

Substituting above equation (25) into the derivative of the Lyapunov function (24), we have.

$$\begin{aligned}\dot{V}_i ={}& z_i(g_i(\bar{x}_i)x_{i+1} - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}(g_k(\bar{x}_k)x_{k+1}))\\ &+z_i(\delta_i(t) - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}\delta_k(t)) - z_i\sum_{k=1}^{i}\frac{\partial\alpha_{i-1}}{\partial x_k}f_k(x)\\ &+z_i(\sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_k}\dot{\hat{\theta}}_k + \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\delta}_k}\dot{\hat{\delta}}_k + \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(k-1)}}y_d^{(k)})\\ &+\frac{\tilde{\theta}_i\dot{\hat{\theta}}_i}{\gamma_i^{(1)}} + \frac{\tilde{\delta}_i\dot{\hat{\delta}}_i}{\gamma_i^{(2)}}\end{aligned} \tag{26}$$

where $\frac{\partial\alpha_{i-1}}{\partial x_i} = -1$.

Based on Assumption 4, Remark 3, and Lemma 4, we conclude the following inequality,

$$\begin{aligned}&-z_i\sum_{k=1}^{i}\frac{\partial\alpha_{i-1}}{\partial x_k}f_k(x)\\ &\leqslant \sum_{k=1}^{i}\left|z_i\frac{\partial\alpha_{i-1}}{\partial x_k}\right||f_k(x)|\\ &\leqslant \sum_{k=1}^{i}\left|z_i\frac{\partial\alpha_{i-1}}{\partial x_k}\right|\phi_k(\|x\|)\\ &\leqslant \sum_{k=1}^{i}\sum_{j=1}^{n}\left|z_i\frac{\partial\alpha_{i-1}}{\partial x_k}\right||z_j|\bar{\phi}_k(|z_j\psi_j|)\\ &\quad+\sum_{k=1}^{i}\left|z_i\frac{\partial\alpha_{i-1}}{\partial x_k}\right|\phi_k((n+1)d_h)\\ &\leqslant \sum_{k=1}^{i}\sum_{j=1}^{n}\frac{1}{2}z_i^2\left(\frac{\partial\alpha_{i-1}}{\partial x_k}\right)^2 + \sum_{k=1}^{i}\sum_{j=1}^{n}\frac{1}{2}z_j^2\bar{\phi}_k^2(|z_j\psi_j|)\\ &\quad+\sum_{k=1}^{i}\left|z_i\frac{\partial\alpha_{i-1}}{\partial x_k}\right|\phi_k((n+1)|d_h|)\end{aligned} \tag{27}$$

where $\bar{\phi}_k(|z_j\psi_j|) = (n+1)|\psi_j|h_k((n+1)|z_j\psi_j|))$.

Based on Lemma 1, we have

$$\begin{aligned}&\sum_{k=1}^{i}\left|z_i\frac{\partial\alpha_{i-1}}{\partial x_k}\right|\phi_k((n+1)|d_h|)\\ &\qquad\leqslant z_iU_i\tanh\left(\frac{z_iU_i}{\varepsilon_i}\right) + \sigma\varepsilon_i\end{aligned} \tag{28}$$

where $U_i = \sum_{k=1}^{i}\frac{\partial\alpha_{i-1}}{\partial x_k}\phi_k((n+1)|d_h|)$, and ε_i, σ are positive constants to be designed.

By substituting inequalities (27) and (28) back into (26), we have,

$$\begin{aligned}\dot{V}_i \leqslant{}& -z_i\left(\sum_{k=1}^{i}\frac{\partial\alpha_{i-1}}{\partial x_k}(g_k(\bar{x}_k)x_{k+1})\right) + \frac{\tilde{\theta}_i\dot{\hat{\theta}}_i}{\gamma_i^{(1)}} + \frac{\tilde{\delta}_i\dot{\hat{\delta}}_i}{\gamma_i^{(2)}}\\ &+z_i\left(\sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_k}\dot{\hat{\theta}}_k + \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\delta}_k}\dot{\hat{\delta}}_k + \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(k-1)}}y_d^{(k)}\right)\\ &-z_i\left(\sum_{k=1}^{i}\frac{\partial\alpha_{i-1}}{\partial x_k}\triangle_k(t)\right) + z_iU_i\tanh\left(\frac{z_iU_i}{\varepsilon_i}\right) + \sigma\varepsilon_i\\ &+\sum_{k=1}^{i}\frac{n}{2}z_i^2\left(\frac{\partial\alpha_{i-1}}{\partial x_k}\right)^2 + \sum_{k=1}^{i}\sum_{j=1}^{n}\frac{1}{2}z_j^2\bar{\phi}_k^2(|z_j\psi_j|)\end{aligned} \tag{29}$$

Step *n*: Choose the nth Lyapunov function candidate

$$V_n = \frac{1}{2}z_n^2 + \frac{1}{2\gamma_n^{(1)}}\tilde{\theta}_n^2 + \frac{1}{2\gamma_n^{(2)}}\tilde{\delta}_n^2 \tag{30}$$

In this step, based on plant $\dot{x}_n = g_n(\bar{x}_n)u + f_n(x) + \triangle_n(t)$, we are going to construct the actual controller $v(t)$. According to coordination, we have $\dot{z}_n = \dot{x}_n - \dot{\alpha}_{n-1} = g_n(\bar{x}_n)u + f_n(x) + \triangle_n(t) - \dot{\alpha}_{n-1}$, where

$\alpha_{n-1} = \alpha(x_1, \ldots, x_{n-1}; \hat{\theta}_1, \ldots, \hat{\theta}_{n-1}; \hat{\delta}_1, \ldots, \hat{\delta}_{n-1}; y_d, y_d', \ldots, y_d^{(n-1)})$ and the derivative of α_{n-1} is as follows:

$$\dot{\alpha}_{n-1} = \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k}(f_k(x) + g_k(\bar{x}_k)x_{k+1} + \triangle_k(t)) + \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_k}\dot{\hat{\theta}}_k + \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\delta}_k}\dot{\hat{\delta}}_k + \sum_{k=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(k-1)}} y_d^{(k)} \tag{31}$$

Along the trajectory of V_n, we require

$$\begin{aligned}\dot{V}_n &= z_n\dot{z}_n + \frac{\tilde{\theta}_n\dot{\hat{\theta}}_n}{\gamma_n^{(1)}} + \frac{\tilde{\delta}_n\dot{\hat{\delta}}_n}{\gamma_n^{(2)}} \\ &= z_n(g_n(\bar{x}_n)(\Theta^T(t)\Phi(t)v(t) + d(v)) \\ &\quad - z_n\left(\sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k}(g_k(\bar{x}_k)x_{k+1}) + \triangle_k(t)\right) \\ &\quad - z_n\left(\sum_{k=1}^{n} \frac{\partial \alpha_{n-1}}{\partial x_k} f_k(x)\right) + \frac{\tilde{\theta}_n\dot{\hat{\theta}}_n}{\gamma_n^{(1)}} + \frac{\tilde{\delta}_n\dot{\hat{\delta}}_n}{\gamma_n^{(2)}} \\ &\quad + z_n\left(\sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_k}\dot{\hat{\theta}}_k + \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\delta}_k}\dot{\hat{\delta}}_k\right) \\ &\quad + z_n\left(\sum_{k=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(k-1)}} y_d^{(k)}\right)\end{aligned} \tag{32}$$

Based on Assumption 4, Remark 3, and Lemma 4, we obtain,

$$\begin{aligned}-z_n\sum_{k=1}^{n} \frac{\partial \alpha_{n-1}}{\partial x_k} f_k(x) &\leqslant \sum_{k=1}^{n} \left| z_n \frac{\partial \alpha_{n-1}}{\partial x_k}\right| |f_k(x)| \\ &\leqslant \sum_{k=1}^{n} \left| z_n \frac{\partial \alpha_{n-1}}{\partial x_k}\right| \phi_k(\|x\|) \\ &\leqslant \sum_{k=1}^{n}\sum_{j=1}^{n} \left| z_n \frac{\partial \alpha_{n-1}}{\partial x_k}\right| |z_j|\bar{\phi}_k(|z_j\psi_j|) \\ &\quad + \sum_{k=1}^{n} \left| z_n \frac{\partial \alpha_{n-1}}{\partial x_k}\right| \phi_k((n+1)d_h) \\ &\leqslant \sum_{k=1}^{n}\sum_{j=1}^{n} \frac{1}{2} z_n^2 \left(\frac{\partial \alpha_{n-1}}{\partial x_k}\right)^2 + \sum_{k=1}^{n}\sum_{j=1}^{n} \frac{1}{2} z_j^2 \bar{\phi}_k^2(|z_j\psi_j|) \\ &\quad + \sum_{k=1}^{n} \left| z_n \frac{\partial \alpha_{n-1}}{\partial x_k}\right| \phi_k((n+1)|d_h|)\end{aligned} \tag{33}$$

where $\bar{\phi}_k(|z_j\psi_j|) = (n+1)|\psi_j|h_k((n+1)|z_j\psi_j|))$.

Based on Lemma 1, we have,

$$\sum_{k=1}^{n} \left| z_n \frac{\partial \alpha_{n-1}}{\partial x_k}\right| \phi_k((n+1)|d_h|) \leqslant z_n U_n \tanh\left(\frac{z_n U_n}{\varepsilon_n}\right) + \sigma\varepsilon_n \tag{34}$$

where $U_n = \sum_{k=1}^{n} \frac{\partial \alpha_{n-1}}{\partial x_k}\phi_k((n+1)|d_h|)$, and ε_n, σ are positive constants to be designed.

Substitute these inequalities (33), (34) into $\dot{V}_n$ (32), we have:

$$\begin{aligned}\dot{V}_n &\leqslant z_n(g_n(\bar{x}_n)(\Theta^T(t)\Phi(t)v(t) + d(v)) \\ &\quad - z_n\left(\sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k}(g_k(\bar{x}_k)x_{k+1}) + \triangle_k(t)\right) \\ &\quad + \sum_{j=1}^{n} \frac{n}{2} z_i^2 \left(\frac{\partial \alpha_{n-1}}{\partial x_k}\right)^2 + \sum_{k=1}^{n}\sum_{j=1}^{n} \frac{1}{2} z_j^2 \bar{\phi}_k^2(|z_j\psi_j|) \\ &\quad + z_n U_n \tanh\left(\frac{z_n U_n}{\varepsilon_n}\right) + \sigma\varepsilon_n + \frac{\tilde{\theta}_n\dot{\hat{\theta}}_n}{\gamma_n^{(1)}} + \frac{\tilde{\delta}_n\dot{\hat{\delta}}_n}{\gamma_n^{(2)}} \\ &\quad - z_n\left(\sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_k}\dot{\hat{\theta}}_k + \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\delta}_k}\dot{\hat{\delta}}_k\right) \\ &\quad - z_n\left(\sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(k-1)}} y_d^{(k)}\right)\end{aligned} \tag{35}$$

Now, we choose the whole Lyapunov function for the plant (1), $V = \sum_{i=1}^{n} V_i$, the derivative of V is concluded based on the above analysis.

$$\begin{aligned}\dot{V} &\leqslant z_1\left(g_1(x_1)x_2 + \triangle_1(t) - \dot{y}_d + U_1 \tanh\left(\frac{z_1 U_1}{\varepsilon_1}\right)\right) \\ &\quad + \sum_{j=1}^{n} \frac{1}{2} z_j^2 \bar{\phi}_1^2(|z_j\psi_j|) + \frac{n z_1^2}{2} + \sigma_1\varepsilon_1) + \frac{\tilde{\theta}_1\dot{\hat{\theta}}_1}{\gamma_1^{(1)}} + \frac{\tilde{\delta}_1\dot{\hat{\delta}}_1}{\gamma_1^{(2)}} \\ &\quad - \sum_{i=2}^{n-1} z_i \left[\sum_{k=1}^{i} \frac{\partial \alpha_{i-1}}{\partial x_k}(g_k(\bar{x}_k)x_{k+1} + \triangle_k(t))\right] \\ &\quad - \sum_{i=2}^{n-1} z_i \left[\sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}_k}\dot{\hat{\theta}}_k + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\delta}_k}\dot{\hat{\delta}}_k\right] \\ &\quad - \sum_{i=2}^{n-1} z_i \left[\sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(k-1)}} y_d^{(k)}\right] + \sum_{i=2}^{n-1} \sigma\varepsilon_i \\ &\quad + \sum_{i=2}^{n-1} z_i \left[U_i \tanh\left(\frac{z_i U_i}{\varepsilon_i}\right) + \sum_{k=1}^{i} \frac{n}{2} z_i \left(\frac{\partial \alpha_{i-1}}{\partial x_k}\right)^2\right] \\ &\quad + \sum_{i=2}^{n-1} \left[\sum_{k=1}^{i}\sum_{j=1}^{n} \frac{1}{2} z_j^2 \bar{\phi}_k^2(|z_j\psi_j|) + \frac{\tilde{\theta}_i\dot{\hat{\theta}}_i}{\gamma_i^{(1)}} + \frac{\tilde{\delta}_i\dot{\hat{\delta}}_i}{\gamma_i^{(2)}}\right] \\ &\quad + z_n(g_n(\bar{x}_n)(\Theta^T(t)\Phi(t)v(t) + d(v)) \\ &\quad - z_n\left(\sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k}(g_k(\bar{x}_k)x_{k+1}) + \triangle_k(t)\right) \\ &\quad - z_n\left(\sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_k}\dot{\hat{\theta}}_k + \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\delta}_k}\dot{\hat{\delta}}_k\right) \\ &\quad + \sum_{j=1}^{n} \frac{n}{2} z_n^2 \left(\frac{\partial \alpha_{n-1}}{\partial x_k}\right)^2 + \sum_{k=1}^{n}\sum_{j=1}^{n} \frac{1}{2} z_j^2 \bar{\phi}_k^2(|z_j\psi_j|)\end{aligned}$$

$$+ z_n U_n \tanh\left(\frac{z_n U_n}{\varepsilon_n}\right) + \sigma\varepsilon_n + \frac{\tilde{\theta}_n \dot{\hat{\theta}}_n}{\gamma_n^{(1)}} + \frac{\tilde{\delta}_n \dot{\hat{\delta}}_n}{\gamma_n^{(2)}} - z_n \left(\sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(k-1)}} y_d^{(k)}\right) \tag{36}$$

Note that,

$$\begin{aligned}\sum_{j=1}^{n} \frac{1}{2} z_j^2 \bar{\phi}_1^2(|z_j\psi_j|) &+ \sum_{k=1}^{n}\sum_{j=1}^{n} \frac{1}{2} z_j^2 \bar{\phi}_k^2(|z_j\psi_j|) \\ &+ \sum_{i=2}^{n-1}\sum_{k=1}^{i}\sum_{j=1}^{n} \frac{1}{2} z_j^2 \bar{\phi}_k^2(|z_j\psi_j|) \\ &= \sum_{i=1}^{n}\sum_{k=1}^{i}\sum_{j=1}^{n} \frac{1}{2} z_j^2 \bar{\phi}_k^2(|z_j\psi_j|) \\ &= \sum_{i=1}^{n} z_i^2 \sum_{k=1}^{n} C(n,k) \bar{\phi}_k^2(|z_j\psi_j|)\end{aligned}$$

where $c(n,k) = (n-k+1)/2$

Substituting this equality in to above $\dot{V}$, we get,

$$\begin{aligned}\dot{V} \leqslant\ & z_1\left(g_1(x_1)(z_2+\alpha_1) + \triangle_1(t) - \dot{y}_d + U_1 \tanh\left(\frac{z_1U_1}{\varepsilon_1}\right)\right) \\ &+ \frac{nz_1^2}{2} - \sum_{i=2}^{n-1} z_i(g_k(\bar{x}_k)(z_{k+1}+\alpha_k) + \triangle_i(t)) \\ &- \sum_{i=2}^{n} z_i \left[\sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k}(g_k(\bar{x}_k)x_{k+1} + \triangle_k(t))\right] \\ &- \sum_{i=2}^{n} z_i \left[\sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}_k}\dot{\hat{\theta}}_k + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\delta}_k}\dot{\hat{\delta}}_k\right] \\ &- \sum_{i=2}^{n} z_i \left[\sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(k-1)}} y_d^{(k)}\right] \\ &+ \sum_{i=2}^{n} z_i \left[U_i \tanh\left(\frac{z_iU_i}{\varepsilon_i}\right) + \sum_{k=1}^{i} \frac{n}{2} z_i \left(\frac{\partial \alpha_{i-1}}{\partial x_k}\right)^2\right] \\ &+ \sum_{i=1}^{n} \frac{\tilde{\theta}_i\dot{\hat{\theta}}_i}{\gamma_i^{(1)}} + \sum_{i=1}^{n} \frac{\tilde{\delta}_i\dot{\hat{\delta}}_i}{\gamma_i^{(2)}} + \sum_{i=1}^{n} \sigma\varepsilon_i \\ &+ z_n(g_n(\bar{x}_n)(\Theta^T(t)\Phi(t)v(t) + d(v)) \\ &+ \sum_{i=1}^{n} z_i^2 \sum_{k=1}^{n} C(n,k)\bar{\phi}_k^2(|z_j\psi_j|)\end{aligned} \tag{37}$$

To facilitate the adaptive controller design, we will use RBF neural networks to approximate the nonlinearities, now define $H_1(\cdot), H_i(\cdot), H_n(\cdot)$ as follows:

$$\begin{aligned}H_1(Z_1) = \triangle_1(t) - \dot{y}_d + U_1 \tanh\left(\frac{z_1U_1}{\varepsilon_1}\right) + \frac{nz_1}{2} \\ + z_1 \sum_{i=1}^{n} C(n,k)\bar{\phi}_k^2(|z_1\psi_1|)\end{aligned} \tag{38}$$

For H_i, $(i = 2, 3, \ldots, n)$, we define,

$$\begin{aligned}H_i(Z_i) = &-\sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k}(g_k(\bar{x}_k)x_{k+1} + \triangle_k(t)) \\ &- \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}_k}\dot{\hat{\theta}}_k - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\delta}_k}\dot{\hat{\delta}}_k - \sum_{k=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(k-1)}} y_d^{(k)} \\ &+ U_i \tanh\left(\frac{z_iU_i}{\varepsilon_i}\right) - \sum_{k=1}^{i} \frac{n}{2} z_i \left(\frac{\partial \alpha_{i-1}}{\partial x_k}\right)^2 \\ &+ z_i \sum_{k=1}^{n} C(n,k)\bar{\phi}_k^2(|z_i\psi_i|)\end{aligned} \tag{39}$$

Then, substituting $H_i(Z_i)$, $(i = 1, 2, \ldots, n)$ into $\dot{V}$ in (37), we obtain:

$$\begin{aligned}\dot{V} \leqslant\ & z_1(g_1(x_1)(z_2+\alpha_1) + H_1) \\ &+ \sum_{i=2}^{n-1} z_i(g_i(\bar{x}_i)(z_{i+1}+\alpha_i) + H_i) \\ &+ z_n(g_n(\bar{x}_n)(\Theta^T(t)\Phi(t)v(t) + d(v) + H_n) \\ &+ \sum_{i=1}^{n} \frac{\tilde{\theta}_i\dot{\hat{\theta}}_i}{\gamma_i^{(1)}} + \sum_{i=1}^{n} \frac{\tilde{\delta}_i\dot{\hat{\delta}}_i}{\gamma_i^{(2)}} + \sum_{i=1}^{n} \sigma\varepsilon_i\end{aligned} \tag{40}$$

According to the definition of $H_i(Z_i)$ and the Lemma1-4 and Assumption 1-4, it can conclude $H_i(Z_i)$ are also smooth functions, then, based on the universe approximation lemma, we can use RBF NN to approximate the unknown smooth function $H_i(Z_i)$ on the compact space Ω_1, and $H_i(Z_i)$ can be rewritten as

$$H_i(Z_i) = W_i^{*T}\xi_i(Z_i) + \varsigma_i \tag{41}$$

where Z_i is the input of the NN system, W_i^{*T} and ς_i denote the ideal optimal approximation parameter vector and the NN approximator error, respectively. For simplification, we define

$$||W_i^{*T}|| = \frac{1}{1+g_{i1}}\theta_i$$

Throughout this paper, in order to alleviate online approximation parameters, we assume the following Assumption:

Assumption 5: Based on the definition of θ_i, on the compact Ω_i, we assume that the optimal approximation parameter vector W_i^{*T} and the NN approximator errors ς_i, satisfy:

$$||W_i^T|| \leqslant \theta_i, |\varsigma_i| \leqslant \delta_i \tag{42}$$

where $i = 1, 2, \ldots, n$, parameters $\theta_i \geqslant 0$ and $\delta_i \geqslant 0$ are unknown constants. Z_i, W_i^* and ς_i will be defined later. $\hat{\theta}_i \geqslant 0$, $\hat{\delta}_i \geqslant 0$ will be used to denote estimations of the θ_i and δ_i respectively. Throughout this paper, $\tilde{(\cdot)} = \hat{(\cdot)} - (\cdot)$.

Remark 5: There are a lot of significant results regarding adaptive fuzzy or NN control or FNN control algorithms for nonlinear systems with unknown dead-zones. However, many of these approximation control methods go through updating the estimations of each optimal parameter of FLSs [7]–[11], [20]–[23] NN, FNN directly, resulting the heavy online computation burden due to the rules of fuzzy, the hidden nodes of NN, or FNN are rather large generally. In this paper, Assumption 5 relaxes the conditions that the approximation errors or external disturbance are bounded with only unknown constants rather than known constants or satisfying square integrable condition. Only estimations $\hat{\theta}_i$, $\hat{\delta}_i$, $(i = 1, 2, \ldots, n$, of parameters θ_i, δ_i need to adaptively adjusted. Thus, this novel proposed approach reduces the adjusted parameters and alleviate the on-line computation burden.

According to approximation functions, $H_i, (i = 1, 2, \ldots, n)$, virtute controllers $\alpha_1, \ldots, \alpha_{n-1}$, and actual controller $v(t)$, and the adaptive laws $\hat{\theta}_i, \hat{\delta}_i, (i = 1, 2, \ldots, n)$ back into $\dot{V}$. Based on Young's inequalities, we obtain the following inequalities:

$$\begin{aligned} &z_i H_i \leqslant |z_i|\theta_i^{*T}\|\xi_i(Z_i)\| + |z_i||\delta_i|, \quad (i = 1, 2, \ldots, n-1) \\ &z_n(g_n(\bar{x}_n)(\Theta^T \Phi v(t) + d(v) + H_n) \\ &\quad \leqslant z_n g_{n1}\beta(v(t) + p^*) + z_n\theta_n^{*T}\|\xi_n(Z_n)\| + |z_n||\delta_n| \end{aligned}$$

For $i = 1, 2, \ldots, n$, based on the Lemma 4, we could conclude the following inequalities hold:

$$\begin{aligned} &z_i\theta_i^{*T}\xi_i(Z_i) - \frac{\hat{\theta}_i^2 z_i^2||\xi_i(Z_1)||^2}{\hat{\theta}_i|z_1|||\xi_i(Z_i)|| + \tau_i^{(1)}} + \frac{\tilde{\theta}_i\dot{\hat{\theta}}_i}{\gamma_i^{(1)}} \\ &\quad \leqslant |z_i|(\hat{\theta}_i - \tilde{\theta}_i)||\xi_i(Z_i)|| - \frac{\hat{\theta}_i^2 z_i^2||\xi_i(Z_i)||^2}{\hat{\theta}_i|z_i|||\xi_i(Z_i)|| + \tau_i^{(1)}} \\ &\qquad + \frac{1}{\gamma_i^{(1)}}\tilde{\theta}_i[-\rho_i^{(1)}\hat{\theta}_i + \gamma_i^{(1)}|z_i|||\xi_i(Z_i)||] \\ &\quad \leqslant \tau_i^{(1)} - \frac{\rho_i^{(1)}}{\gamma_i^{(1)}}\tilde{\theta}_i\hat{\theta}_i \leqslant \tau_i^{(1)} - \frac{\rho_i^{(1)}}{\gamma_i^{(1)}}\tilde{\theta}_i^2 + \frac{\rho_i^{(1)}}{\gamma_i^{(1)}}\theta_i^2 \end{aligned} \tag{43}$$

Similarly, based on the adaptation laws (24),(25) and Young's inequality, we have

$$\begin{aligned} &z_i\delta_i - \frac{\hat{\delta}_i^2 z_i^2}{\hat{\delta}_i|z_i| + \tau_i^{(2)}} + \frac{1}{\gamma_i^{(2)}}\tilde{\delta}_i\dot{\hat{\delta}}_i \\ &\quad \leqslant |z_i|(\hat{\delta}_i - \tilde{\delta}_i) - \frac{\hat{\delta}_i^2 z_i^2}{\hat{\delta}_i|z_i| + \tau_i^{(2)}} \\ &\qquad + \frac{1}{\gamma_i^{(2)}}\tilde{\delta}_i(-\rho_i^{(2)}\hat{\delta}_i + \gamma_i^{(2)}z_i) \\ &\quad \leqslant \tau_i^{(2)} - \frac{\rho_i^{(2)}}{\gamma_i^{(2)}}\tilde{\delta}_i^2 + \frac{\rho_i^{(2)}}{\gamma_i^{(2)}}\delta_i^2 \end{aligned} \tag{44}$$

By substituting inequality (43), (44) back into (40), we acquire,

$$\begin{aligned} \dot{V} \leqslant &-\sum_{i=1}^{n}[c_i z_i^2 + \frac{\rho_i^{(1)}}{\gamma_i^{(1)}}\tilde{\theta}_i^2 + \frac{\rho_i^{(2)}}{\gamma_i^{(2)}}\tilde{\delta}_i^2] \\ &+ \sum_{i}^{n}[\frac{\rho_i^{(1)}}{\gamma_i^{(1)}}\theta_i^2 + \frac{\rho_i^{(2)}}{\gamma_i^{(2)}}\delta_i^2] + \mu_i \end{aligned} \tag{45}$$

where $\mu_i = \sum_{i=1}^{n}[\tau_i^{(1)} + \tau_i^{(2)} + \sigma\varepsilon_i]$ is a constant.

If we choose the appropriate adjusted parameters and constants $\tau_i^{(1)}, \tau_i^{(2)}, \sigma_i, \varepsilon_i, \rho_i^{(1)}, \rho_i^{(2)}, \gamma_i^{(1)}, \gamma_i^{(2)}, p^*, d^*, \theta_i, \delta_i$ and based on the Assumptions 1-4, Lemmas 1-4 and the RBF NN approximations, together with the virtual and actual controllers, we will have the following inequalities.

$$\dot{V} \leqslant -\mu\sum_{i=1}^{n}[\frac{1}{2}z_i^2 + \frac{1}{2\gamma_i^{(1)}}\tilde{\theta}_i^2 + \frac{1}{2\gamma_i^{(2)}}\tilde{\delta}_i^2] + \alpha \tag{46}$$

where $\mu = \min_{1\leqslant i\leqslant n}\{2c_i, 2\rho_i^{(1)}, 2\rho_i^{(2)}\}$ and

$$\alpha = \sum_{i=1}^{n}[\frac{\rho_i^{(1)}}{\gamma_i^{(1)}}\theta_i^2 + \frac{\rho_i^{(2)}}{\gamma_i^{(2)}}\delta_i^2 + \mu_i]$$

Then, we obtain

$$\dot{V} \leqslant -\mu V + \alpha \tag{47}$$

Multiplying both sides of the above Eq. by $e^{\mu t}$ and it can be rewritten as

$$d\left(V(t)e^{\mu t}\right)/dt \ \leq \ell e^{\mu t} \tag{48}$$

Then, integrating the above equation over $[0, t]$, we can obtain

$$0 \leq V(t) \leq \frac{\ell}{\mu} + \left[V(0) - \frac{\ell}{\mu}\right]e^{-\mu t} \tag{49}$$

If we note that $0 < e^{-\mu t} < 1$ and $(\ell/\mu)e^{-\mu t} > 0$, then, we can know the above Eq.holds as

$$0 \leq V(t) \leq \ell/\mu + V(0)$$

and we can conclude that

$$|z_i| \leq \sqrt{\frac{\ell}{\mu} + \left[V(0) - \frac{\ell}{\mu}\right]e^{-\mu t}}$$

Therefore, it can be shown that all the signals $z_i, \tilde{\theta}_i, \tilde{\delta}_i$ $(i = 1, 2, \ldots, n)$ in the closed-loop systems (1) are bounded. There exists $T > 0$, for $T > \sqrt{2\mu/\ell}$, satisfying $|z_1| \leqslant T$ for all $t \geqslant T$, the tracking error $z_1 = x_1 - y_d$ converges to a neighborhood of zero. The proof is completed. □

Remark 6: Compared with many approximation control approaches, which involve updating the estimations of each optimal parameter of FLSs NN, and FNN directly [2]–[4], [6]–[11], [19], [22], [29]–[31], due to the hidden nodes of NN, or FNN and the rules of fuzzy are rather large generally, which result in the heavy online computation burden. Based on Assumption 5, at each design procedure for each system in this paper, fewer parameters need to be adjusted, we only need to approximate the unknown constant for the norm of the optimal parameter. So, this new approach can improve the robust control performance and alleviate the online computation burden.

IV. SIMULATION EXAMPLE

In this section, based on a practical one-link robot simulation system and its figure model can be seen in [38], the effectiveness of the presented control technique will be illustrated. The dynamics of one-link manipulator with the inclusion of motor [20], [38] can be described by the following equations:

$$\begin{cases} D\ddot{q} + B\dot{q} + N\sin(q) = \tau + \tau_d \\ M\dot{\tau} + H\tau = u - K_m\dot{q} \end{cases}$$

where τ_d is the torque disturbance, τ represents the torque produced by the electrical system [20], [38], q is the link position, $\dot{q}$ is velocity, and $\ddot{q}$ is acceleration. $D = 100kg/m^2$ is the mechanical inertia. u is the control input used to represent the electromechanical torque. $B = 1Nms/ras$ is the coefficient of viscous friction at the joint,$K_m = 2NM/A$ is the back-emf coefficient, $H = 0.1F$ is the armature resistance, $N = 10$ is a positive constant related to the mass of the load and the coefficient of gravity [38], and $M = 20\ H$is the armature inductance [38].

When we introduce the variable change $x_1 = q, x_2 = \dot{q}$, and $x_3 = \tau$, and assume that the system exist unknown disturbance and unknown functions, $x = [x_1, x_2, x_3]^T$ is the state of the system, and $y = x_1$ is system output, u is the input of the system and the output of the dead-zone. Then, above one-link system can be re-expressed as

$$\begin{cases} \dot{x}_1 = g_1(x_1)x_2 + f_1(x) + \triangle_1(t), \\ \dot{x}_2 = g_2(\bar{x}_2)x_3 + f_2(x) + \triangle_2(t) \\ \dot{x}_3 = g_3(x)u(t) + f_3(x) + \triangle_3(t) \\ y = x_1. \end{cases} \tag{50}$$

where $g_1(x_1) = 1+0.6x_1^2, f_1(x) = ((B/D)x_1+x_2)x_3$, $\triangle_1(t) = \exp(-(B/D)(x_1+x_2))$, $g_2(\bar{x}_2) = (N/D + \cos(x_1x_2))x_2$, $f_2(x) = (N/M)x_1x_2^2 + x_2x_3^2$, $\triangle_2(t) = \frac{1}{K_m}\sin(x_2)$, $g_3(x) = x_2 + x_1x_2x_3^2$, $f_3(x) = ((K_m/M) + \sin(x_1x_2))x_3$, and $\triangle_3(t) = \frac{1}{K_m}\sin(x_3)$. Then, we obtain the following third-order uncertain non-strict feedback nonlinear system with unknown dead-zone input:

$$\begin{cases} \dot{x}_1 = (0.01x_1 + x_2)x_3 + \exp(-0.01(x_1+x_2)) \\ \qquad + (1 + 0.6x_1^2)x_2 \\ \dot{x}_2 = 0.5x_1x_2^2 + x_2x_3^2 + (0.1 + \cos(x_1x_2))x_2 + \dfrac{1}{2}\sin(x_2) \\ \dot{x}_3 = (x_2 + x_1x_2x_3^2)u + (0.1 + \sin(x_1x_2))x_3 + \dfrac{1}{2}\sin(x_3) \\ y = x_1 \end{cases} \tag{51}$$

Choose the initial values $x_1(0) = y(0) = x_2(0) = 0.5, x_3(0) = 0.7$. The unsymmetrical dead-zone inputs satisfies

$$u = D(v) = \begin{cases} m_r(v(t) - b_r) & v(t) \geqslant b_r \\ 0 & b_l < v(t) < b_r \\ m_l(v(t) - b_l) & v(t) \leqslant b_l \end{cases}$$

the dead-zone break points are chosen as: $m_r = b_r = 0.8, m_l = b_l = 2.5$.

The objective of simulation is to apply the proposed novel adaptive NN tracking control approach for this three-order system, satisfy 1) the whole signals in this closed-loop system are bounded, 2) the output $y = x_1$ can track the reference signal $y_d = 0.25\sin(t)$ very well.

Based on the novel adaptive robust NN tracking control approach in Sec3, the designed adaptive NN virtual controller α_i adaptive laws θ_i, δ_i and actual controller $v(t)$ are chosen as follows:

$$\begin{aligned} \alpha_i = &(\frac{1}{1+g_{i1}})[-c_iz_i - \frac{\hat{\theta}_i^2 z_i||\xi_i(Z_i)||^2}{\hat{\theta}_i|z_i|||\xi_i(Z_i)|| + \tau_i^{(1)}} \\ &- \frac{\hat{\delta}_i^2 z_i}{\hat{\delta}_i|z_i| + \tau_i^{(2)}} - z_{i-1}], \quad i = 1, 2 \end{aligned} \tag{52}$$

$$\begin{aligned} v(t) = &(\frac{1}{1+g_{31}})\frac{1}{\beta_0}[-c_3z_3 - \frac{\hat{\theta}_3^2 z_3||\xi_3(Z_3)||^2}{\hat{\theta}_3|z_3|||\xi_3(Z_3)|| + \tau_3^{(1)}} \\ &- \frac{\hat{\delta}_3^2 z_3}{\hat{\delta}_3|z_3| + \tau_3^{(2)}} - z_2 - p^*] \end{aligned} \tag{53}$$

$$\dot{\hat{\theta}}_i = -\rho_i^{(1)}\hat{\theta}_i + \gamma_i^{(1)}|z_i|||\xi_i(Z_i)||, \quad i = 1, 2, 3 \tag{54}$$

$$\dot{\hat{\delta}}_i = -\rho_i^{(2)}\hat{\delta}_i + \gamma_i^{(2)}|z_i|, \quad i = 1, 2, 3 \tag{55}$$

where $z_1 = x_1 - y_d$, $z_2 = x_2 - \alpha_1$, $z_3 = x_3 - \alpha_2$, for $i = 1, 2, 3$, $Z_i = [z_i^T, \bar{\hat{\theta}}_i^T, (\bar{y}_d^{(i)})]^T$, $\bar{\hat{\theta}}_i = [\hat{\theta}_1, \ldots, \hat{\theta}_i]^T$, $\bar{y}_d^{(i)} = [y_d, y_d', \ldots, y_d^{(i-1)}]$, $\xi_i(Z_i) = [\xi_{i1}(Z_i), \ldots, \xi_{i9}(Z_i)]^T$, $i = 1, 2, 3$ is basis function of NN.

$\xi_{ij}(Z_i) = \exp[\frac{-(Z_i-\varrho_{ij})^T(Z_i-\varrho_{ij})}{\eta_{ij}^2}], (j = 1, 2, \ldots, 9)$.

The initial conditions and design parameters are selected as follows: $\hat{\theta}_1(0) = \hat{\theta}_2(0) = \hat{\theta}_3(0) = 0$, $\hat{\delta}_1(0) = \hat{\delta}_2(0) = \hat{\delta}_3(0) = 0.5$, $c_1 = c_2 = c_3 = 0.5$, $\tau_1^{(1)} = \tau_1^{(2)} = 0.1$, $\tau_2^{(1)} = 1$, $\tau_2^{(2)} = 0.8$, $\tau_3^{(1)} = 10$, $\tau_3^{(2)} = 0.5$, $\rho_1^{(1)} = 1$, $\rho_1^{(2)} = 0.9$, $\rho_2^{(1)} = 1.5$, $\rho_2^{(2)} = 9$, $\rho_3^{(1)} = 1.8$, $\rho_3^{(2)} = 0.3$, $\gamma_1^{(1)} = 0.9$, $\gamma_1^{(2)} = 0.8$, $\gamma_2^{(1)} = \gamma_2^{(2)} = 1.2$, $\gamma_3^{(1)} = \gamma_3^{(2)} = 0.3$. $\beta = 9$. and NN center parameters are chosen as $\varrho_{i1} = -7$, $\varrho_{i2} = -5$, $\varrho_{i3} = -3$, $\varrho_{i4} = -1$, $\varrho_{i5} = 0$, $\varrho_{i6} = 1$, $\varrho_{i7} = 3$, $\varrho_{i8} = 5$, $\varrho_{i9} = 7$, $\eta_{ij}^2 = 3$.

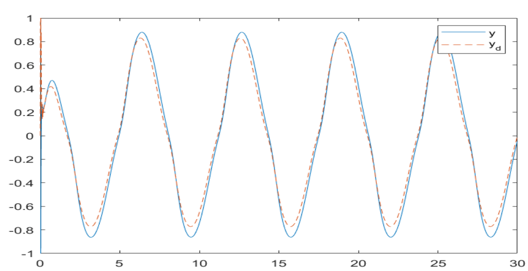

FIGURE 2. Output y and reference signal y_d.

The effective simulation results are shown in Figs. 2-5. Fig. 2 plots the trajectory of output y and the tracking signal y_d. A good tracking performance is achieved and the trajectories of signals are bounded. We conclude that the adjusted parameters $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3$ and adaptive signals $\hat{\delta}_1, \hat{\delta}_2, \hat{\delta}_3$

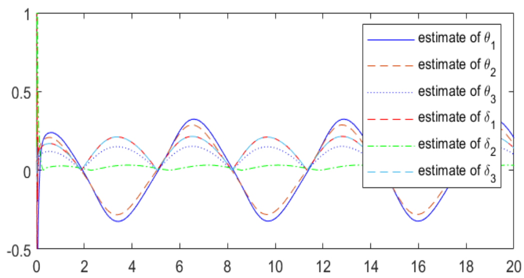

FIGURE 3. Adaptive adjusted parameters $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3$ and adaptive signals $\hat{\delta}_1, \hat{\delta}_2, \hat{\delta}_3$.

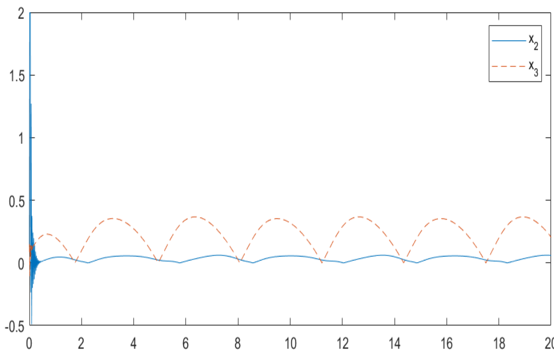

FIGURE 4. Trajectory of states signals x_2 and x_3.

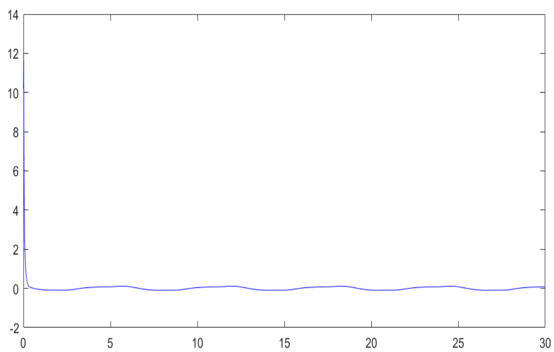

FIGURE 5. Trajectory of input signal v.

are ultimately uniformly bounded in Figs. 3. From Figs.4 we can see control signals are bounded, and the dead-zone output (or system input) v is also bounded, and is located in a small convergence of zero in Figs.5.

From the above simulation results, it can be clearly shown that the proposed control method can guarantee that all the signals in the closed systems are UUB, the proposed controller design method is effective. Compared with related results [20], [22], [23], [38], we need only one adaptive law to compensate the unknown dead-zones in non-feedback form. This method can reduce the online computing burden and simplify the design procedure considerably.

V. CONCLUSION

In this paper, a novel NN alleviating tracking control approach has been proposed for a class of uncertain non-strict feedback systems with both asymmetrical dead-zones inputs and unknown nonlinear functions. Compared with the existing results, we consider not only asymmetrical dead-zones, but also non-strict feedback structure. This presented scheme adopts variable separation technique and adaptive method to cope with the non-strict feedback structure and the unknown dead-zones, the unknown functions have been approximated by NN. By using two unknown parameters as the approximation error and the bound of the norm of the optimal approximation vectors of the NN, the number of adjusted parameters is alleviated. Furthermore, based on Lyapunov theorem analysis, it has been shown that all the signals in the closed-loop systems are UUB and the tracking error is controlled into a small compact set. Finally, simulation results illustrate the feasibility and effectiveness of this approach. In the future, we could explore these method to more complex systems, such as switched or stochastic nonlinear non-strict feedback system.

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