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SPECIAL SECTION ON RECENT ADVANCES ON HYBRID COMPLEX NETWORKS: ANALYSIS AND CONTROL

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Asynchronous Control for Markov Switching Lur'e Systems With Round-Robin Protocol

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ABSTRACT This paper addresses the asynchronous control design problem for discrete-time Markov switching Lur'e systems. First, to schedule the information transmission and data collisions by the limited shared channel, the round-robin protocol is adopted in scheduling the information exchange order. Second, considering that the system mode cannot be identified to controller, a mode observer is employed to evaluate the system mode by menas of a hidden Markov model. To address this issue, the mode-dependent stochastic Lur'e type Lyapunov functional is analyzed, and several sufficient criteria with prescribed performance are attained to guarantee the stochastically stable of the resulting discrete-time MSLSs. Finally, the effectiveness and applicability of the gained technique is verified by a practical example.

INDEX TERMS T-S fuzzy system, fading channel, static output feedback control, actuator fault.

I. INTRODUCTION

The past years have witnessed extensive study of nonlinear systems, due to the complexity of the physical systems [1], [2]. Accordingly, many efficient tools have been exhibited to approximate the resulting systems, such as Takagi-Sugeno model [3]–[5], Inter-type 2 method [6], Lur'e systems [7], and so on. Among them, Lur'e systems (LSs) consist of both linear terms and bounded nonlinear ones. Compared with the normal nonlinear systems, LSs have been proved to be more general. Up to now, considerable attention has been paid to the issues of LSs, such as stability, robust control, distributed filtering [7], [8].

Over the past decades, because of the existence of randomly occurring phenomena, for instance, sudden environment changes, component failure, et al, the structure and parameters of the hybrid systems become variable. To depict the above mentioned changes, Markov switching systems (MSSs) have been developed. In recent years, many fruitful achievements have been reported on MSSs [9]–[13]. However, as implied in [9]–[13], most of the existing results are concerned on linear systems. In reality, more nonlinear systems are involved, and the investigation of nonlinear models is more realistic. Recently, the Markov switching Lur'e systems (MSLSs) have been developed, each mode is consisted by a linear term and a mode-dependent nonlinearity [8], [14]. In [15], the quadratic Lyapunov functional (LF) was adopted in analysis of discrete-time MSLS. To further reduce the conservatism, based on a sector condition assumption, sufficient criteria was proposed by means of a Lur'e-type LF. Lately, a cone-bounded nonlinearity was considered in the analysis and synthesis of MSLS [8], [16]. However, research on MSLSs is far away from maturity and deserves further study.

On the other hand, owing to the advantage of network communication, considerable attention has been shifted to networked systems [17]–[23]. Meanwhile, many unfavorable network-induced factors are also emerged, for instance, quantization effects [18], cyber attack [19], time delays [20], data collisions [21], fading channel [23], and many control issues have been addressed [24]–[26]. For mitigating the communication burden and saving limited resources, many communication protocols (try-once-discard protocol [27], event-triggered scheme [28], stochastic communication law [29]) are introduced to schedule the information transmission via a shared channel. Note that these protocols aiming at reducing transmission volume at each time. Differently, to mitigates data collision by scheduling transmission frequency, the round-robin protocol (RRP) is recognized as a time-periodic multiple strategy to access the network [30].

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Nevertheless, to our knowledge, the asynchronous control issue for discrete-time MSLSs with RRP has not been studied yet. It is an interesting issue worth further exploring, which inspires us to shorten this a gap.

Motivated by the above discussion, this work is dedicated to the asynchronous control for discrete-time MSLS by the means of RRP. Different from previous work, to schedule the information transmission and data collisions by the limited shared channel, the RRP is adopted in scheduling the information exchange order. Meanwhile, to tackle the difficulty in acquiring system mode information, a detector is introduced and an asynchronous controller is established. By formulating the mode-dependent stochastic Lur'e type LK, several sufficient criteria are attained to guarantee the stochastically stable of the resulting discrete-time MSLSs. At last, the effectiveness of this work is verified by a practical simulation.

Notations: In this research, $\mathbb{R}^n$ symbolizes the n-dimensional Euclidean. diag$\{\ldots\}$ means the block diagonal matrix. sym$\{Z\}$ indicates $Z^\top + Z$. $\delta(\cdot)$ means Kronecker product. $\mathscr{E}\{\cdot\}$ is the mathematic expectation.

II. PRELIMINARY

Consider a type of discrete-time Markov switching Lur'e system (DTMSLS) described by:

$$\begin{cases} \delta(k+1) = A(\varphi_k)\delta(k) + B(\varphi_k)u(k) \\ \qquad +D(\varphi_k)\zeta(\varphi_k, H(\varphi_k)\delta(k)) \\ \qquad +G(\varphi_k)\omega(k), \\ z(k) = F(\varphi_k)\delta(k), \end{cases} \tag{1}$$

where $\delta(k) \in \mathbb{R}^{n_\delta}$, $z(k) \in \mathbb{R}^{n_z}$, $u(k) \in \mathbb{R}^{n_u}$ are, respectively, the state vector, output vector and control input. $\zeta(\varphi_k, H(\varphi_k)\delta(k)) \in \mathbb{R}^{n_h}$ represents a mode-dependent memoryless nonlinear function (MDNS). $\omega(k) \in \mathbb{R}^{n_w}$ describes the disturbance taking values in $l_2[0,\infty)$. The sequence $\{\varphi_k, k \geq 0\}$ renders a discrete-time Markov chain (DTMC) having values over a set $\mathcal{L}_p = \{1, 2, \ldots, L_p\}$. In this work, the transition probabilities (TPs) of DTMSLS (1) are elicited as

$$\alpha_{ij} = \Pr\{\varphi_{k+1} = j \mid \varphi_k = i\},$$

where $\alpha_{ij} \geq 0$, and $\sum_{j\in\mathcal{L}_p} \alpha_{ij} = 1, \forall i, j \in \mathcal{L}_p$, and TP matrix $\Pi = [\alpha_{ij}]_{\mathcal{L}_p\times\mathcal{L}_p}$.

For technique convenience, $\forall i \in \mathcal{L}_p$, $A(\varphi_k)$, $B(\varphi_k)$, $D(\varphi_k)$, $F(\varphi_k)$, $G(\varphi_k)$, $H(\varphi_k)$, and $\zeta(\varphi_k, H(\varphi_k)\delta(k))$ are denoted by $A_i, B_i, D_i, F_i, G_i, H_i$ and $\zeta_i(H_i\delta(k))$, respectively.

In DTMSLS (1), it is assumed that ith MDNF $\zeta(i, H_i\delta(k))$ satisfies the Assumption 1 as below.

Assumption 1: The ith MDNF $\zeta_i(H_i\delta(k))$ satisfies a cone-bounded sector function, and can be decentralized into $\varphi_k = i \in \mathcal{L}_p$, the ith MDNF $\zeta_i(H_i\delta(k))$ satisfies: 1) $\zeta_i(0) = 0$ and 2) there exists the diagonal matrices $\Omega_i \in \mathbb{R}^{n_h\times n_h} \geq 0$, $\forall q = 1, 2, \ldots, n_h$, one has

$$\zeta_{i,(q)}(H_i\delta(k))[\zeta_{i,(q)}(H_i\delta(k)) - \Omega_i H_i\delta(k)]_{(q)} \leq 0.$$

By Assumption 1, for $\forall i \in \mathcal{L}_p$, the following condition holds:

$$\zeta_i^\top(H_i\delta(k))\Theta_i[\zeta_i(H_i\delta(k)) - \Omega_i H_i\delta(k)] \leq 0, \tag{2}$$

where matrices $\Theta_i \geq 0$ ($\forall i \in \mathcal{L}_p$). Obviously, by inequality (2), one concludes $\forall i \in \mathcal{L}_p$

$$\begin{aligned} 0 &\leq \zeta_i^\top(H_i\delta(k))\Theta_i\zeta_i(H_i\delta(k)) \leq \zeta_i^\top(H_i\delta(k))\Theta_i\Omega_i H_i\delta(k) \\ &\leq \delta^\top(k)H_i^\top\Omega_i\Theta_i\Omega_i H_i\delta(k). \end{aligned} \tag{3}$$

In reality, the network-induced phenomenon exists in the data transmission, such as data collisions, information quantization, *et al.* To improve the communication of network, one assumes that the sensors are divided into n_u, then can be written as $u(k) = [u_1(k)\; u_2(k)\; \ldots\; u_{n_u}(k)]^\top$. To omit the data collisions in the limited communication resources, the RRP is adopted to decide which sensor is permitted to access the network. More specifically, only one node is activated by means of RRP with respect to $\vartheta_k \in \{1, 2, \ldots, n_u\}$. Here, the RRP is scheduled in the following principle:

$$\vartheta_k = \mathrm{mod}(k-1, n_u) + 1. \tag{4}$$

where mod$(k-1, n_u)$ implies the remainder on division of $k-1$ by n_u.

In the RRP scheduling, the chosen node ϑ_k satisfies $\vartheta_{k+n_u} = \vartheta_k$. Setting $\vartheta_k = k$, by the means of zero-holder (ZOH) approach, a compensation strategy is adopted for the unselected signals. Therefore, $\forall m = 1, 2, \ldots, n_u$, one has

$$u_m(k) = \begin{cases} v_m(k), & \text{if } \vartheta_k = m \\ u_m(k-1), & \text{otherwise} \end{cases} \tag{5}$$

where $u_m(k)$ stands for the measurement signal after being sent out of mth sensor.

Letting $u(k) = [u_1(k)\; u_2(k)\; \ldots\; u_{n_u}(k)]^\top$, $\delta_x^y = \delta(x-y)$, $\Psi_m \triangleq \mathrm{diag}\{\delta_m^1, \delta_m^2, \ldots, \delta_m^{n_u}\}$ $(m = 1, 2, \ldots, n_u)$, where $\delta(x-y) = \begin{cases} 1, & \text{if } x = y \\ 0, & \text{otherwise} \end{cases}$. Thus, $\forall\vartheta_k = 1, 2, \ldots, n_y$, a compensator (6) is elicited as

$$u(k) = \Psi_{\vartheta_k} v(k) + (I - \Psi_{\vartheta_k})u(k-1). \tag{6}$$

On the other hand, due to the difficulty in acquiring system mode information, in most circumstance, the DTMC φ_k is unmeasured. To solve this issue, a mode detector is applied, in which the output is denoted by another DTMC τ_k. Obviously, the DTMC τ_k runs asynchronous with φ_k. Thus, a HMM is presented to model the aforementioned asynchronous phenomena, for any $i \in \mathcal{L}_p, t \in \mathcal{L}_c = \{1, 2, \ldots, L_c\}$, such that the detection probability matrix $\Phi = [\beta_{it}]_{\mathcal{L}_p\times\mathcal{L}_c}$ is obtained:

$$\beta_{it} = \mathrm{Prob}\{\tau_k = t \mid \varphi_k = i\}, \tag{7}$$

where $\beta_{it} \in [0, 1]$, and $\sum_{t\in\mathcal{L}_c} \beta_{it} = 1$.

Following the above discussion, the purpose of this work is provide a suitable control scheme for the DTMSLS (1). In this work, the control strategy is inferred as

$$v(k) = K(\tau_k)\delta(k), \tag{8}$$

where $K(\tau_k)$ represents the controller gains to be solved.

For $\varphi_k = i$, $\tau_k = t$, the overall closed-loop DTMSLS (9) can be derived:

$$\begin{cases} \delta(k+1) = (A_i + B_i\Psi_{\vartheta_k}K_t)\delta(k) \\ \qquad +B_i(I - \Psi_{\vartheta_k})u(k-1) \\ \qquad +D_i\overline{\zeta}_i(H_i\delta(k)) + G_i\omega(k), \\ z(k) = F_i\delta(k) \end{cases} \tag{9}$$

Before proceeding further, one introduces the definitions for DTMSLS (9).

Definition 1 [18]: The DTMSLS (9) with $\omega(k) = 0$ is called stochastic stable (SS), if for any (δ_0, ϑ_0), one has

$$\mathscr{E}\left\{\sum_{k=0}^{\infty} \| \delta(k) \|^2 | \ \delta_0, \vartheta_0\right\} < \infty.$$

This work aiming at exploring the $\mathcal{H}_\infty$ asynchronous control problem for DTMSLS (9) with RRP such that

(i) DTMSLS (9) is SS in mean square;

(ii) Under zero initial condition, the controlled output $z(k)$ meets

$$\sum_{k=0}^{\infty}\mathscr{E}\left\{\| \delta(k) \|^2\right\} < \gamma^2\sum_{k=0}^{\infty}\mathscr{E}\left\{\| \varrho(k) \|^2\right\}.$$

III. MAIN RESULTS

Theorem 1 The DTMSLS (9) is called SS, if there exists matrices $P_{i\vartheta_k} > 0$, $\Theta_{i\vartheta_k} > 0$, $R_{i\vartheta_k} > 0$ and $Q_{it\vartheta_k} > 0$ $(i, j \in \mathcal{L}_p, t \in \mathcal{L}_c, \vartheta_k = \{1, 2, \ldots, n_u\})$ such that

$$-P_{i\vartheta_k} + \sum_{t\in\mathcal{L}_c}\beta_{it}Q_{it\vartheta_k} < 0, \tag{10}$$

$$\begin{bmatrix} \Sigma_{it}^{(1)} & \Sigma_{it}^{(2)\top} \\ * & -\text{diag}\{I, \Xi_i^{-1}, \mathcal{P}_{i\vartheta_{k+1}}^{-1}, \overline{\Theta}_{i\vartheta_{k+1}}^{-1}, \mathcal{R}_{\vartheta_{k+1}}^{-1}\} \end{bmatrix} < 0, \tag{11}$$

where

$$\Sigma_{it}^{(1)} = \begin{bmatrix} -Q_{it\vartheta_k} & 0 & 0 & 0 \\ * & -R_{i\vartheta_k} & 0 & 0 \\ * & * & -\Xi_i - \Theta_{i\vartheta_k} & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\Sigma_{it}^{(2)} = \begin{bmatrix} \Sigma_{1it}^{(2)\top} & \Sigma_{2it}^{(2)\top} & \Sigma_{3it}^{(2)\top} & \Sigma_{3it}^{(2)\top} & \Sigma_{4it}^{(2)\top} \end{bmatrix}^\top,$$

$$\Sigma_{1it}^{(2)} = [F_i \ \ 0 \ \ 0 \ \ 0], \ \Sigma_{2it}^{(2)} = [\Omega_i H_i \ \ 0 \ \ 0 \ \ 0],$$

$$\Sigma_{3it}^{(2)} = \left[A_i + B_i\Psi_{\vartheta_k}K_t \ B_i(I - \Psi_{\vartheta_k}) \ D_i \ G_i\right],$$

$$\Sigma_{4it}^{(2)} = [\Psi_{\vartheta_k}K_t \ \ (I - \Psi_{\vartheta_k}) \ 0 \ 0],$$

$$\mathcal{P}_{i\vartheta_{k+1}} = \sum_{j\in\mathcal{L}_p}\alpha_{ij}P_{j\vartheta_{k+1}}, \ \mathcal{R}_{i\vartheta_{k+1}} = \sum_{j\in\mathcal{L}_p}\alpha_{ij}R_{j\vartheta_{k+1}},$$

$$\overline{\Theta}_{i\vartheta_{k+1}} = \sum_{j\in\mathcal{L}_p}\alpha_{ij}H_j^\top\Omega_j\Theta_{j\vartheta_{k+1}}\Omega_j H_j.$$

Proof: In this work, establishing the Lyapunov functional for DTMSLS (9) as follows:

$$V(\delta_k, \varphi_k, \vartheta_k) = \sum_{s=1}^{3} V_s(\delta_k, \varphi_k, \vartheta_k), \tag{12}$$

where

$$V_1(\delta_k, \varphi_k, \vartheta_k) = \delta^\top(k)P_{\varphi_k\vartheta_k}\delta(k),$$
$$V_2(\delta_k, \varphi_k, \vartheta_k) = \zeta^\top(\varphi_k, H(\varphi_k)\delta(k))\Theta_{\varphi_k\vartheta_k}\Omega(\varphi_k)H(\varphi_k)\delta(k),$$
$$V_3(\delta_k, \varphi_k, \vartheta_k) = \overline{u}^\top(k-1)R_{i\vartheta_k}\overline{u}(k-1).$$

With calculating the difference of $V(\lambda_k, \varphi_k, \vartheta_k)$, which implies

$$\mathscr{E}\{\Delta V(\lambda_k, \varphi_k, \vartheta_k)\} = \mathscr{E}\{V(\lambda_{k+1}, \varphi_{k+1} = j, \vartheta_{k+1} \mid \lambda_k, \varphi_k = i, \vartheta_k)\} - V(\lambda_k, \varphi_k, \vartheta_k).$$

For the first term of (12), $\mathscr{E}\{\Delta V_1(\lambda_k, \varphi_k, \vartheta_k)\}$ infers

$$\begin{aligned} &\mathscr{E}\{\Delta V_1(\delta_k, \varphi_k, \vartheta_k)\} \\ &= \mathscr{E}\{\delta^\top(k+1)\mathcal{P}_{i\vartheta_{k+1}}\delta(k+1) - \delta^\top(k)P_{\varphi_k\vartheta_k}\delta(k)\} \\ &= \mathscr{E}\Bigg\{\sum_{t\in\mathcal{L}_c}\beta_{it}\delta^\top(k)\mathcal{A}_{it\vartheta_k}^\top\mathcal{P}_{i\vartheta_{k+1}}\mathcal{A}_{it\vartheta_k}\delta(k) \\ &\quad + \sum_{t\in\mathcal{L}_c}\beta_{it}sym\{\delta^\top(k)\mathcal{A}_{it\vartheta_k}^\top\mathcal{P}_{i\vartheta_{k+1}}B_i(I - \Psi_{\vartheta_k})u(k-1)\} \\ &\quad + \sum_{t\in\mathcal{L}_c}\beta_{it}sym\{\delta^\top(k)\mathcal{A}_{it\vartheta_k}^\top\mathcal{P}_{i\vartheta_{k+1}}D_i\zeta_i(H_i\delta(k))\} \\ &\quad + \sum_{t\in\mathcal{L}_c}\beta_{it}sym\{\delta^\top(k)\mathcal{A}_{it\vartheta_k}^\top\mathcal{P}_{i\vartheta_{k+1}}G_i\omega(k)\} \\ &\quad + \sum_{t\in\mathcal{L}_c}\beta_{it}u^\top(k-1)(B_i(I - \Psi_{\vartheta_k}))^\top\mathcal{P}_{i\vartheta_{k+1}} \\ &\quad \times B_i(I - \Psi_{\vartheta_k})u(k-1) \\ &\quad + \sum_{t=1}^{N_c}\beta_{it}sym\{u^\top(k-1)(B_i(I - \Psi_{\vartheta_k}))^\top \\ &\quad \times\mathcal{P}_{i\vartheta_{k+1}}\mathcal{D}_i\zeta_i(H_i\delta(k))\} \\ &\quad + \sum_{t=1}^{N_c}\beta_{it}sym\{u^\top(k-1)(B_i(I - \Psi_{\vartheta_k}))^\top\mathcal{P}_{i\vartheta_{k+1}}G_i\omega(k)\} \\ &\quad + \sum_{t=1}^{N_c}\beta_{it}\zeta_i^\top(H_i\delta(k))D_i^\top\mathcal{P}_{i\vartheta_{k+1}}D_i\zeta_i(H_i\delta(k)) \\ &\quad + \sum_{t=1}^{N_c}\beta_{it}sym\{\zeta_i^\top(H_i\delta(k))D_i^\top\mathcal{P}_{i\vartheta_{k+1}}G_i\omega(k)\} \\ &\quad + \sum_{t=1}^{N_c}\beta_{it}\omega^\top(k)G_i^\top\mathcal{P}_{i\vartheta_{k+1}}G_i\omega(k)\Bigg\} \\ &\quad -\delta^\top(k)P_{i\vartheta_k}\delta(k). \end{aligned} \tag{13}$$

Besides, for the second term of (12), one has

$$\begin{aligned} &\mathscr{E}\{\Delta V_2(\delta_k, \varphi_k, \vartheta_k)\} \\ &\quad \leq \mathscr{E}\Big\{\delta^\top(k+1)H^\top(\vartheta_{k+1})\Omega(\vartheta_{k+1})\Theta_{\varphi_{k+1}\vartheta_{k+1}} \end{aligned}$$

$$
\begin{aligned}
&\times\Omega(\vartheta_{k+1})H(\vartheta_{k+1})\delta(k+1)\\
&-\zeta_i^\top(H_i\delta(k))\Theta_i\zeta_i(H_i\delta(k))\}\\
=\ &\mathscr{E}\Big\{\delta^\top(k)\mathcal{A}_{it\vartheta_k}^\top\overline{\Theta}_{i\vartheta_{k+1}}\mathcal{A}_{it\vartheta_k}\delta(k)\\
&+sym\{\delta^\top(k)\mathcal{A}_{it\vartheta_k}^\top\overline{\Theta}_{i\vartheta_{k+1}}B_i(I-\Psi_{\vartheta_k})u(k-1)\}\\
&+sym\{\delta^\top(k)\mathcal{A}_{it\vartheta_k}^\top\overline{\Theta}_{i\vartheta_{k+1}}D_i\zeta_i(H_i\delta(k))\}\\
&+sym\{\delta^\top(k)\mathcal{A}_{it\vartheta_k}^\top\overline{\Theta}_{i\vartheta_{k+1}}G_i\omega(k)\}\\
&+u^\top(k-1)(B_i(I-\Psi_{\vartheta_k}))^\top\overline{\Theta}_{i\vartheta_{k+1}}\\
&\times B_i(I-\Psi_{\vartheta_k})u(k-1)\\
&+sym\{u^\top(k-1)(B_i(I-\Psi_{\vartheta_k}))^\top\\
&\times\overline{\Theta}_{i\vartheta_{k+1}}\mathcal{D}_i\zeta_i(H_i\delta(k))\}\\
&+sym\{u^\top(k-1)(B_i(I-\Psi_{\vartheta_k}))^\top\overline{\Theta}_{i\vartheta_{k+1}}G_i\omega(k)\}\\
&+\zeta_i^\top(H_i\delta(k))D_i^\top\overline{\Theta}_{i\vartheta_{k+1}}D_i\zeta_i(H_i\delta(k))\\
&+sym\{\zeta_i^\top(H_i\delta(k))D_i^\top\overline{\Theta}_{i\vartheta_{k+1}}G_i\omega(k)\}\\
&+\omega^\top(k)G_i^\top\overline{\Theta}_{i\vartheta_{k+1}}G_i\omega(k)\Big\}\\
&-\zeta_i^\top(H_i\delta(k))\Theta_{i\vartheta_k}\zeta_i(H_i\delta(k)).
\end{aligned}\tag{14}
$$

On the other hand, for the last term of (12), one derives

$$
\begin{aligned}
&\mathscr{E}\{\Delta V_3(\delta_k,\varphi_k,\vartheta_k)\}\\
&=\mathscr{E}\Big\{\delta^\top(k)K_t^\top\Psi_{\vartheta_k}^\top\mathcal{R}_{i\vartheta_{k+1}}\Psi_{\vartheta_k}K_t\delta(k)\\
&\quad+sym\{\delta^\top(k)K_t^\top\Psi_{\vartheta_k}^\top\mathcal{R}_{i\vartheta_{k+1}}(I-\Psi_{\vartheta_k})u(k-1)\}\\
&\quad+u^\top(k-1)(I-\Psi_{\vartheta_k})^\top\mathcal{R}_{i\vartheta_{k+1}}(I-\Psi_{\vartheta_k})\\
&\quad\times u(k-1)-u^\top(k-1)R_{i\vartheta_k}u(k-1).
\end{aligned}\tag{15}
$$

Recalling Assumption 1, one has

$$\zeta_i^\top(H_i\delta(k))\Xi_i(\zeta_i^\top(H_i\delta(k))-\Omega_iH_i\delta(k))\le 0.\tag{16}$$

With respect to (16), one further achieves

$$
\begin{aligned}
0&\le -sym\{\zeta_i^\top(H_i\delta(k))\Xi_i(\zeta_i^\top(H_i\delta(k))-\Omega_iH_i\delta(k))\}\\
&\le -\zeta_i^\top(H_i\delta(k))\Xi_i\zeta_i^\top(H_i\delta(k))\\
&\quad+\delta^\top(k)H_i^\top\Omega_i\Xi_i\Omega_iH_i\delta(k).
\end{aligned}\tag{17}
$$

Letting $\overline{\eta}(k)=[\delta^\top(k)\ u^\top(k-1)\ \zeta_i^\top(H_i\delta(k))]^\top$, and combining (12)-(17), one also has

$$
\begin{aligned}
&\mathscr{E}\{\Delta V(\delta_k,\varphi_k,\vartheta_k)\}\le\Delta V(\delta_k,\varphi_k,\vartheta_k)\\
&\quad+sym\left\{\zeta_i^\top(H_i\delta(k))\Xi_i(\zeta_i^\top(H_i\delta(k))-\Omega_iH_i\delta(k))\right\}\\
&\le\delta^\top(k)\left(\sum_{t\in\mathcal{L}_c}\beta_{it}Q_{it\vartheta_k}-P_{i\vartheta_k}\right)\delta(k)\\
&\quad+\overline{\eta}^\top(k)\left(\overline{\Sigma}_{it}^{(1)}+\overline{\Sigma}_{it}^{(2)\top}(\mathcal{P}_{i\vartheta_{k+1}}+\Theta_{i\vartheta_{k+1}})\overline{\Sigma}_{it}^{(2)}\right)\overline{\eta}(k),
\end{aligned}\tag{18}
$$

where

$$
\overline{\Sigma}_{it}^{(1)}=\begin{bmatrix}-Q_{it\vartheta_k}+H_i^\top\Omega_i\Xi_i\Omega_iH_i & 0 & 0\\ * & -R_{i\vartheta_k} & 0\\ * & * & -\Xi_i-\Theta_{i\vartheta_k}\end{bmatrix},
$$

$$
\overline{\Sigma}_{it}^{(2)}=\begin{bmatrix}\mathcal{A}_{it\vartheta_k} & B_i(I-\Psi_{\vartheta_k}) & D_i\end{bmatrix}.
$$

When $\omega(k)=0$, the inequality (18) can be written as

$$
\begin{aligned}
&\mathscr{E}\{\Delta V_1(\delta_k,\varphi_k,\vartheta_k)\}\\
&\le\delta^\top(k)\left(\sum_{t\in\mathcal{L}_c}\beta_{it}Q_{it\vartheta_k}-P_{i\vartheta_k}\right)\delta(k)\\
&\quad+\overline{\eta}^\top(k)\left(\overline{\Sigma}_{it}^{(1)}+\overline{\Sigma}_{it}^{(2)\top}(\mathcal{P}_{i\vartheta_{k+1}}+\overline{\Theta}_{i\vartheta_{k+1}})\overline{\Sigma}_{it}^{(2)}\right)\overline{\eta}(k)\\
&\le\delta^\top(k)\left(\sum_{t\in\mathcal{L}_c}\beta_{it}Q_{it\vartheta_k}-P_{i\vartheta_k}\right)\delta(k)\\
&\le-\chi\mathscr{E}\{\|\ \delta(k)\ \|^2\},
\end{aligned}\tag{19}
$$

where $\chi=\min_{i\in\mathcal{N}_p,\vartheta_k\in\{1,2,\dots,n_y\}}\{\lambda_{\min}(P_{i\vartheta_k})-\sum_{t=1}^{L_c}\beta_{it}Q_{it\vartheta_k}\}$. Apparently, it follows from (18) that $\chi>0$. Consequently, it can be concluded that

$$
\begin{aligned}
\mathscr{E}\left\{\sum_{k=0}^{\infty}\|\ \delta(k)\ \|^2\right\}&<-\frac{1}{\chi}\mathscr{E}\left\{\sum_{k=0}^{\infty}\Delta V(\delta_k,\varphi_k,\vartheta_k)\right\}\\
&\le\frac{1}{\chi}\mathscr{E}\{V(\delta_0,\varphi_0,\vartheta_0)\}<\infty.
\end{aligned}\tag{20}
$$

By Definition 1, the DTMSLS (9) with $\omega(k)=0$ is SS.

In what follows, the analysis of H_∞ performance for DTMSLS (9) with $\omega(k)\neq 0$ will be attained. Define the H_∞ performance index:

$$\mathscr{J}(T)=\mathscr{E}\left\{\sum_{k=0}^{T}z^\top(k)z(k)-\gamma^2\omega^\top(k)\omega(k)\right\}.\tag{21}$$

By combing (12)-(17) and (21), $\mathscr{J}(T)$ can be reformulated as

$$
\begin{aligned}
\mathscr{J}(T)&\le\mathscr{E}\Big\{\sum_{k=0}^{T}[z^\top(k)z(k)-\gamma^2\omega^\top(k)\omega(k)\\
&\quad+\Delta V(\delta_k,\vartheta_k)]\Big\}\\
&\le\lambda^\top(k)\left(\sum_{t\in\mathcal{L}_c}\beta_{it}Q_{it\vartheta_k}-P_{i\vartheta_k}\right)\delta(k)\\
&\quad+\eta^\top(k)\Big(\Sigma_{it}^{(1)}+\Sigma_{it}^{(2)\top}\mathcal{P}_{i\vartheta_{k+1}}\Sigma_{it}^{(2)}\\
&\quad+\Sigma_{it}^{(2)\top}\overline{\Theta}_{i\vartheta_{k+1}}\Sigma_{it}^{(2)}\Big)\eta(k),
\end{aligned}\tag{22}
$$

where $\eta(k)=[\overline{\eta}^\top(k)\ \omega^\top(k)]^\top$.

In addition, by utilizing Schur complement to (10) and (11), we have

$$\mathscr{J}(T)<0.\tag{23}$$

Letting $T\to\infty$, it is readily obtained that

$$\sum_{k=0}^{\infty}\mathscr{E}\{\|\ z(k)\ \|^2\}\le\gamma^2\sum_{k=0}^{\infty}\mathscr{E}\{\|\ \omega(k)\ \|^2\}.\tag{24}$$

Therefore, by Definition 1, it is easy to see that the DTMSLS (9) is SS with $\mathcal{H}_\infty$ performance index γ. This completes the proof. □

Theorem 2: For a given scalar γ, the DTMSLS (9) is called SS, if there exists matrices $\overline{P}_{i\vartheta_k}>0$, $\widetilde{\Theta}_{i\vartheta_k}>0$, $\overline{R}_{i\vartheta_k}>0$,

$\overline{Q}_{it\vartheta_k} > 0$, $\widetilde{\Xi}_i > 0$ $(i, j \in \mathcal{L}_p, t \in \mathcal{L}_c, \vartheta_k = \{1, 2, \ldots, n_u\})$, matrices $Z, \overline{K}_t$, such that

$$-\overline{P}_{i\vartheta_k} + \sum_{t\in\mathcal{L}_c} \beta_{it}\overline{Q}_{it\vartheta_k} < 0, \tag{25}$$

$$\begin{bmatrix} \widetilde{\Sigma}_{it}^{(1)} & \widetilde{\Sigma}_{it}^{(2)\top} \\ * & \widetilde{\Sigma}_{it}^{(3)} \end{bmatrix} < 0, \tag{26}$$

where

$$\begin{aligned}
\widetilde{\Sigma}_{it}^{(3)} &= \text{diag}\{-I, \widetilde{\Xi}_i - sym\{Y\}, \overline{\mathscr{P}}_{i\vartheta_{k+1}}, \overrightarrow{\Theta}_{i\vartheta_{k+1}}, \overline{\mathscr{R}}_{i\vartheta_{k+1}}\},\\
\overline{\mathscr{P}}_{i\vartheta_{k+1}} &= \text{diag}\{\overline{P}_{1\vartheta_{k+1}} - sym\{Z\}, \overline{P}_{2\vartheta_{k+1}} - sym\{Z\},\\
&\qquad \ldots, \overline{P}_{L_p\vartheta_{k+1}} - sym\{Z\}\},\\
\overrightarrow{\Theta}_{i\vartheta_{k+1}} &= \text{diag}\{\widetilde{\Theta}_{1\vartheta_{k+1}} - sym\{Y\}, \widetilde{\Theta}_{2\vartheta_{k+1}} - sym\{Y\},\\
&\qquad \ldots, \widetilde{\Theta}_{L_p\vartheta_{k+1}} - sym\{Y\}\},\\
\overline{\mathscr{R}}_{i\vartheta_{k+1}} &= \text{diag}\{\overline{R}_{1\vartheta_{k+1}} - sym\{Z\}, \overline{R}_{2\vartheta_{k+1}} - sym\{Z\},\\
&\qquad \ldots, \overline{R}_{L_p\vartheta_{k+1}} - sym\{Z\}\},\\
\widetilde{\Sigma}_{it}^{(1)} &= \begin{bmatrix} -\overline{Q}_{it\vartheta} & 0 & 0 & 0 \\ * & -\overline{R}_{i\vartheta_k} & 0 & 0 \\ * & * & -\overline{\Xi}_i - \overline{\Theta}_{i\vartheta_k} & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix},\\
\widetilde{\Sigma}_{it}^{(2)} &= \begin{bmatrix} \overline{\Sigma}_{1it}^{(2)\top} & \overline{\Sigma}_{2it}^{(2)\top} & \widetilde{\Sigma}_{3it}^{(2)\top} & \widetilde{\Sigma}_{4it}^{(2)\top} & \widetilde{\Sigma}_{5it}^{(2)\top} \end{bmatrix}^\top,\\
\widetilde{\Sigma}_{3it}^{(2)\top} &= \begin{bmatrix} \sqrt{\alpha_{i1}}\overline{\Sigma}_{3it}^{(2)\top} & \sqrt{\alpha_{i2}}\overline{\Sigma}_{3it}^{(2)\top} & \ldots & \sqrt{\alpha_{iL_p}}\overline{\Sigma}_{3it}^{(2)\top} \end{bmatrix},\\
\widetilde{\Sigma}_{4it}^{(2)\top} &= \Big[\sqrt{\alpha_{i1}}H_1^\top\Omega_1\overline{\Sigma}_{3it}^{(2)\top} \quad \sqrt{\alpha_{i2}}H_2^\top\Omega_2\overline{\Sigma}_{3it}^{(2)\top}\\
&\qquad \ldots \quad \sqrt{\alpha_{iL_p}}H_{L_p}^\top\Omega_{L_p}\overline{\Sigma}_{3it}^{(2)\top} \Big],\\
\widetilde{\Sigma}_{5it}^{(2)\top} &= \begin{bmatrix} \sqrt{\alpha_{i1}}\overline{\Sigma}_{4it}^{(2)\top} & \sqrt{\alpha_{i2}}\overline{\Sigma}_{4it}^{(2)\top} & \ldots & \sqrt{\alpha_{iL_p}}\overline{\Sigma}_{4it}^{(2)\top} \end{bmatrix},\\
\overline{\Sigma}_{1it}^{(2)} &= [F_iZ \;\; 0 \;\; 0 \;\; 0],\ \overline{\Sigma}_{2it}^{(2)} = [\Omega_iH_iZ \;\; 0 \;\; 0 \;\; 0],\\
\overline{\Sigma}_{3it}^{(2)} &= \begin{bmatrix} A_iZ + B_i\Psi_{\vartheta_k}\overline{K}_t & B_i(I - \Psi_{\vartheta_k})Z & D_iY & G_i \end{bmatrix},\\
\overline{\Sigma}_{4it}^{(2)} &= [\Psi_{\vartheta_k}\overline{K}_t \;\; (I - \Psi_{\vartheta_k})Z \; 0 \; 0].
\end{aligned}$$

In this case, the desired asynchronous controller gain can be acquired as

$$K_t = \overline{K}Z^{-1}. \tag{27}$$

Proof: First, let $\overline{P}_{it\vartheta_k} = Z^\top P_{it\vartheta_k}Z$, $\overline{Q}_{it\vartheta_k} = Z^\top Q_{it\vartheta_k}Z$, $\overline{R}_{i\vartheta_k} = Z^\top R_{i\vartheta_k}Z$, $\widetilde{\Theta}_{i\vartheta_k} = Z^\top\Theta_{i\vartheta_k}Z$, $\widetilde{\Xi}_i = Z^\top\Xi_iZ$. In fact, for any matrix Z, one has

$$\begin{aligned}
\mathscr{P}_{i\vartheta_k} &= \overline{P}_{i\vartheta_k} - sym\{Z\}\\
&= ZP_{i\vartheta_k}Z^\top - sym\{Z\} \geq -P_{i\vartheta_k}^{-1}.
\end{aligned} \tag{28}$$

Similarly, one has

$$\begin{aligned}
\mathscr{R}_{i\vartheta_k} &= \overline{R}_i - sym\{Z\} \geq -R_{i\vartheta_k}^{-1},\\
\widehat{\Xi}_i &= \widetilde{\Xi}_i - sym\{Z\} \geq -\Xi_i^{-1},\\
\widehat{\Theta}_{i\vartheta_k} &= \widetilde{\Theta}_{i\vartheta_k} - sym\{Z\} \geq -\Theta_{i\vartheta_k}^{-1}.
\end{aligned} \tag{29}$$

By the equation (27), one has $\overline{K}_t = K_tZ$. Then, pre- and post multiplication by diag$\{Z^\top, Z^\top, Z^\top, I, I, I, I, I, I\}$ and its transpose, which implies (10) and (11) that (25) and (26) hold, respectively. The proof is finished. □

Remark 1: Remarkably, in (5), the compensation scheme is adopt to improve the signal transmission. By neglecting the compensation scheme, $\forall m = 1, 2, \ldots, n_u$, the received measurement signal (RMS) is reduces to

$$u_m(k) = \begin{cases} u_m(k), & \text{if } \vartheta_k = m \\ 0, & \text{otherwise} \end{cases} \tag{30}$$

Hereafter, the RMS is degraded as:

$$u(k) = \Psi_{\vartheta_k}u(k). \tag{31}$$

Consequently, the closed-loop DTMSLS (9) can be formulated as

$$\begin{cases} \delta(k+1) = (A_i + B_i\Psi_{\vartheta_k}K_t)\delta(k) \\ \qquad\qquad +D_i\zeta_i(H_i\delta(k)) + G_i\omega(k), \\ z(k) = F_i\delta(k) \end{cases} \tag{32}$$

To exploit the analysis and control synthesis of system (32), the sufficient conditions are summarized in corollary 1.

Corollary 1: For a given scalar γ, the DTMSLS (32) is called SS, if there exists matrices $\overline{P}_{i\vartheta_k} > 0$, $\widetilde{\Theta}_{i\vartheta_k} > 0$, $\overline{Q}_{it\vartheta_k} > 0$, $\widetilde{\Xi}_i > 0$ $(i, j \in \mathcal{L}_p, t \in \mathcal{L}_c, \vartheta_k = \{1, 2, \ldots, n_u\})$, matrices $Z, \overline{K}_t$, such that (33) holds and

$$\begin{bmatrix} \widetilde{\Sigma}_{it}^{(1)} & \widetilde{\Sigma}_{it}^{(2)\top} \\ * & \widetilde{\Sigma}_{it}^{(3)} \end{bmatrix} < 0, \tag{33}$$

where

$$\begin{aligned}
\widetilde{\Sigma}_{it}^{(3)} &= \text{diag}\{-I, \widetilde{\Xi}_i - sym\{Z\}, \overline{\mathscr{P}}_{i\vartheta_{k+1}}, \overrightarrow{\Theta}_{i\vartheta_{k+1}}\},\\
\overline{\mathscr{P}}_{i\vartheta_{k+1}} &= \text{diag}\{\overline{P}_{1\vartheta_{k+1}} - sym\{Z\}, \overline{P}_{2\vartheta_{k+1}} - sym\{Z\},\\
&\qquad \ldots, \overline{P}_{L_p\vartheta_{k+1}} - sym\{Z\}\},\\
\overrightarrow{\Theta}_{i\vartheta_{k+1}} &= \text{diag}\{\widetilde{\Theta}_{1\vartheta_{k+1}} - sym\{Z\}, \widetilde{\Theta}_{2\vartheta_{k+1}} - sym\{Z\},\\
&\qquad \ldots, \widetilde{\Theta}_{L_p\vartheta_{k+1}} - sym\{Z\}\},\\
\widetilde{\Sigma}_{it}^{(1)} &= \begin{bmatrix} -\overline{Q}_{it\vartheta} & 0 & 0 \\ * & -\overline{\Xi}_i - \overline{\Theta}_{i\vartheta_k} & 0 \\ * & * & -\gamma^2 I \end{bmatrix},\\
\widetilde{\Sigma}_{it}^{(2)} &= \begin{bmatrix} \overline{\Sigma}_{1it}^{(2)\top} & \overline{\Sigma}_{2it}^{(2)\top} & \widetilde{\Sigma}_{3it}^{(2)\top} & \widetilde{\Sigma}_{4it}^{(2)\top} \end{bmatrix}^\top,\\
\widetilde{\Sigma}_{3it}^{(2)\top} &= \begin{bmatrix} \sqrt{\alpha_{i1}}\overline{\Sigma}_{3it}^{(2)\top} & \sqrt{\alpha_{i2}}\overline{\Sigma}_{3it}^{(2)\top} & \ldots & \sqrt{\alpha_{iL_p}}\overline{\Sigma}_{3it}^{(2)\top} \end{bmatrix},\\
\widetilde{\Sigma}_{4it}^{(2)\top} &= \Big[\sqrt{\alpha_{i1}}H_1^\top\Omega_1\overline{\Sigma}_{3it}^{(2)\top} \quad \sqrt{\alpha_{i2}}H_2^\top\Omega_2\overline{\Sigma}_{3it}^{(2)\top}\\
&\qquad \ldots \quad \sqrt{\alpha_{iL_p}}H_{L_p}^\top\Omega_{L_p}\overline{\Sigma}_{3it}^{(2)\top} \Big],\\
\overline{\Sigma}_{1it}^{(2)} &= [F_iZ \;\; 0 \;\; 0],\ \overline{\Sigma}_{2it}^{(2)} = [\Omega_iH_iZ \;\; 0 \;\; 0],\\
\overline{\Sigma}_{3it}^{(2)} &= \begin{bmatrix} A_iZ + B_i\Psi_{\vartheta_k}\overline{K}_t & D_i & G_i \end{bmatrix}.
\end{aligned}$$

IV. A NUMERICAL EXAMPLE

To verify the applicability of the derived results, a practical F-404 aircraft engine model (FAEM) is considered [31], [32]. Note that in FAEM, the signals are transmitted via a wireless communication with RRP. In this model, the system matrix

$A(k) = \begin{bmatrix} -1.46 & 0 & 2.4280 \\ 0.1643 & -0.4 & -0.3788 \\ 0.3107 & 0 & -2.23 \end{bmatrix}$. Obviously, it is easy to discretize the FAEM into

$$A(k) = \begin{bmatrix} 0.5227 & 0 & 0.5009 \\ 0.0458 & 0.8187 & -0.0783 \\ 0.0641 & 0 & 0.3638 \end{bmatrix}.$$

As stated in [31], [32], in FAEM, $\delta_1(k)$, $\delta_2(k)$ stand for horizontal position, $\delta_3(k)$ symbolizes the altitude. By absorbing the external disturbance and other unexpected factors, the resulting parameters are elicited as

$$A_1 = \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0.1 & 0.5 & 0 \\ 0.35 & 0 & 0.65 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.22 & 0.3 & -0.2 \\ 0.01 & 0.22 & 0.59 \\ -0.3 & 0 & -0.09 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.81 & 0 & 0.05 \\ 0 & 0.58 & 0 \\ 0.17 & 0 & 0.3 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 0.1 & 0.1 & -0.18 \\ 0.22 & 0.22 & 0.33 \\ -0.35 & 0.5 & 0.1 \end{bmatrix},$$

and other parameters are given as

$$\begin{aligned} D_1 &= [0.6\ 0.2\ 0.1]^\top, D_2 = [0.4\ 1.2\ 0.7], \\ D_3 &= [1.1\ 0.6\ 1.2], \\ H_1 &= [0.5\ 0.3\ 0.1], H_2 = [0.4\ -0.5\ 0.3], \\ H_3 &= [1.5\ -0.9\ 0.4], \\ F_s &= \mathrm{diag}\{1.1, 1.3, 1.2\}(s = 1, 2, 3), \Omega_1 = 0.8, \Omega_2 = 1.5, \\ \Omega_1 &= 0.8, G_1 = 0.15, G_2 = 0.3, G_3 = 0.2. \end{aligned}$$

Similarly, the TPM of DTMSLS (1) is selected as

$$\Pi = \begin{bmatrix} 0.55 & 0.45 \\ 0.2 & 0.8 \end{bmatrix}$$

Recalling the NDNF $\zeta_i(H_i\delta(k))$ in [16], the parameters of DTMSLS (1) are selected as:

$$\begin{aligned} D_1 &= [0.6\ 0.8\ 1]^\top,\ D_2 = [0.4\ 0.7\ 1]^\top, \\ D_3 &= [0.4\ 0.5\ -0.3]^\top,\ H_1 = [0.5\ 0.3\ 0.1], \\ H_2 &= [0.4\ -0.5\ 0.3],\ \ H_3 = [1.5\ -0.9\ 0.4], \\ \Omega_1 &= 0.8,\ \Omega_2 = 1.5,\ \Omega_3 = 0.7. \end{aligned}$$

More specifically, the Lur'e nonlinear functions are given by $\zeta_1(H_1\delta(k)) = 0.3\Omega_1\delta(k)(1 + \cos(H_1\delta(k)))$, $\zeta_2(H_2\delta(k)) = 0.3\Omega_2\delta(k)(1 - \exp(-0.1(H_2\delta(k))^2))$, $\zeta_3(H_3\delta(k)) = 0.3\Omega_3\delta(k)(1 - \sin(H_3\delta(k)))$. In the following subsection, three cases will be exploit for DTMSLS (9): Case 1: $\mathcal{L}_p > \mathcal{L}_c$; Case 2: $\mathcal{L}_p = \mathcal{L}_c$; Case 3: $\mathcal{L}_p < \mathcal{L}_c$.

Case 1: $\mathcal{L}_p > \mathcal{L}_c$, i.e., $\mathcal{L}_p = \{1, 2\}$, $\mathcal{L}_c = \{1\}$, $\Phi = [1\ 1]^\top$.

In view of LMIs of Theorem 2, the controller gains K_t can be attained as below:

$$K_1 = \begin{bmatrix} -0.1226 & -1.3304 & -0.1018 \\ -3.4558 & 0.0071 & -0.7832 \\ 2.3613 & -0.9765 & 0.6108 \end{bmatrix}.$$

Letting $x(0) = [-0.2\ 0.5\ 0.3]^\top$, $\omega(k) = 0.25\exp(-0.2k)\sin(5k)$, the resulting plant mode and the controller mode are shown in Fig. 1. Based on the achieved controller gains, the state evolution $\delta(k)$ is plotted in Fig. 2, from which one can concludes the closed-loop DTMSLS is convergence.

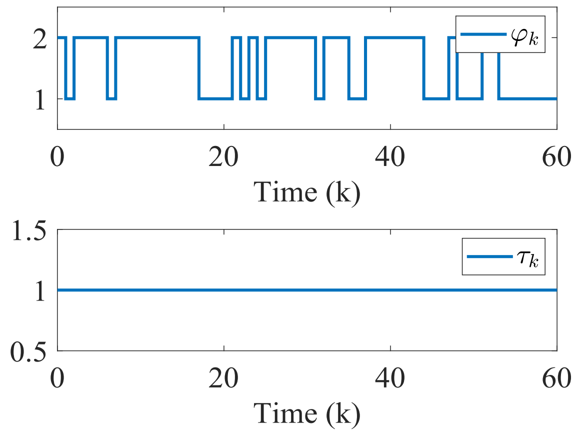

FIGURE 1. The switching sequences of DTMCs ϑ_k and τ_k.

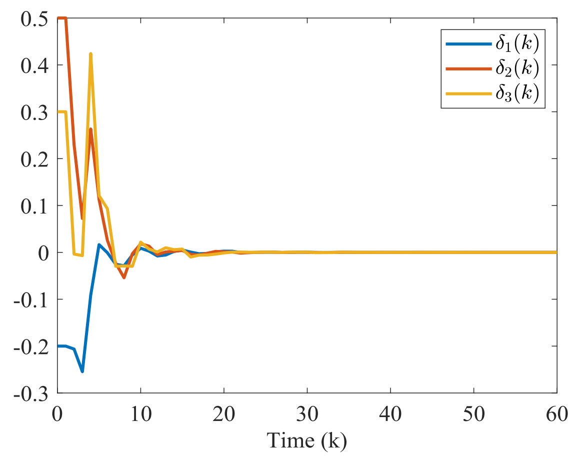

FIGURE 2. The state evolution $\delta(k)$.

Case 2: $\mathcal{L}_p = \mathcal{L}_c$, i.e., $\mathcal{L}_p = \mathcal{L}_c = \{1, 2\}$, $\Phi = \begin{bmatrix} 0.35 & 0.65 \\ 0.9 & 0.1 \end{bmatrix}$.

In view of LMIs of Theorem 2, the controller gains K_t can be attained as below:

$$K_1 = \begin{bmatrix} 0.4907 & -1.5784 & 0.0366 \\ -3.3455 & -0.0252 & -0.8267 \\ 2.3567 & -1.1673 & 0.6355 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -1.5211 & -0.5028 & -0.4689 \\ -3.4639 & 0.0670 & -0.8231 \\ 2.1005 & -0.7704 & 0.6331 \end{bmatrix}.$$

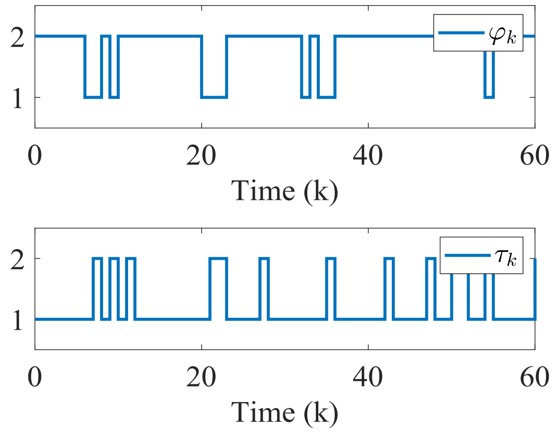

FIGURE 3. The switching sequences of DTMCs ϑ_k and τ_k.

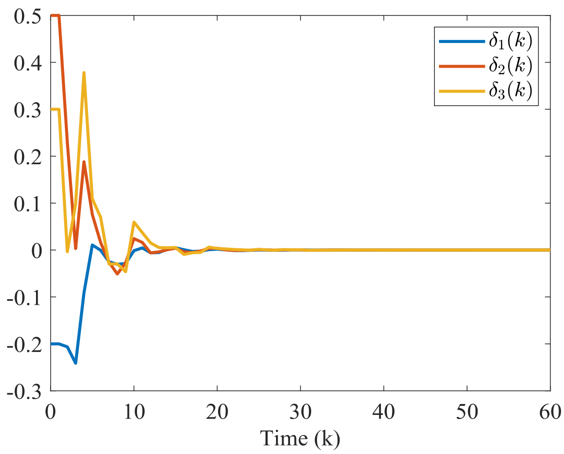

FIGURE 4. The state evolution $\delta(k)$.

Similar to Case 1, the resulting plant mode and the controller mode are shown in Fig. 3. Based on the achieved controller gains, the state evolution $\delta(k)$ is plotted in Fig. 4, from which one can concludes the closed-loop DTMSLS is convergence.

Case 3: $\mathcal{L}_p < \mathcal{L}_c$, i.e., $\mathcal{L}_p = \{1, 2\}$, $\mathcal{L}_c = \{1, 2, 3\}$, $\Phi = \begin{bmatrix} 0.25 & 0.4 & 0.35 \\ 0.5 & 0.2 & 0.3 \end{bmatrix}$.

In view of LMIs of Theorem 2, the controller gains K_t can be attained as below:

$$K_1 = \begin{bmatrix} 0.1430 & -1.4382 & -0.0566 \\ -3.4437 & 0.0137 & -0.7987 \\ 2.4425 & -1.0904 & 0.6054 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -0.4979 & -1.0959 & -0.1897 \\ -3.4883 & 0.0176 & -0.7777 \\ 2.2985 & -0.8844 & 0.6200 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} -0.1648 & -1.2727 & -0.1220 \\ -3.4670 & 0.0135 & -0.7864 \\ 2.3576 & -0.9659 & 0.6227 \end{bmatrix}.$$

Similarly, the resulting plant mode and the controller mode are shown in Fig. 5. Based on the achieved controller gains,

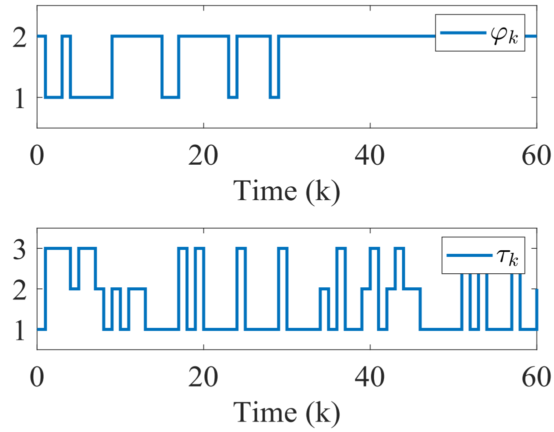

FIGURE 5. The switching sequences of DTMCs ϑ_k and τ_k.

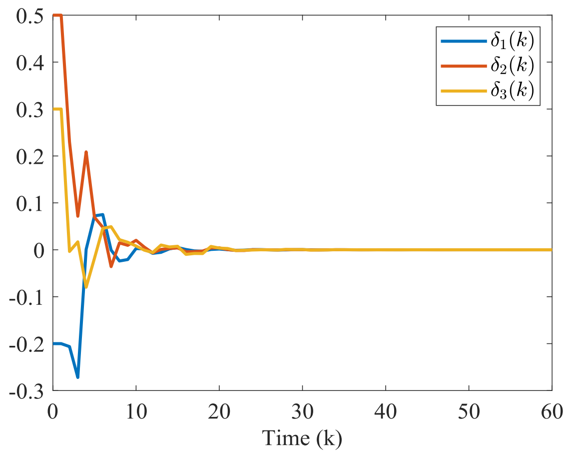

FIGURE 6. The state evolution $\delta(k)$.

the state evolution $\delta(k)$ is plotted in Fig. 6, from which one can concludes the closed-loop DTMSLS is convergence.

V. CONCLUSION

This paper has investigated the asynchronous control design problem for discrete-time MSLSs. To schedule the information transmission and data collisions by the limited shared channel, the RRP is adopted in scheduling the information exchange order. Then, the mode-dependent stochastic Lur'e type LF is analyzed, and several sufficient criteria with prescribed performance are attained to guarantee the stochastically stable of the resulting discrete-time MSLSs. Finally, the effectiveness and applicability of the gained technique is verified by a practical FAEM. Furthermore, how to extend the derived results to multi-agent systems is our future research topic.

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