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Fuzzy Model Predictive Control With Enhanced Robustness for Nonlinear System via a Discrete Disturbance Observer

JIANZHONG ZHU[1,2] AND SING KIONG NGUANG[3], (Senior Member, IEEE)
[1]Department of Electric Power Engineering, Nanjing Institute of Technology, Nanjing 211167, China
[2]Key Laboratory of Energy Thermal Conversion and Control of Ministry of Education, School of Energy and Environment, Southeast University, Nanjing 210096, China
[3]Department of Electrical and Computer Engineering, The University of Auckland, Auckland 1010, New Zealand

Corresponding author: Jianzhong Zhu (zjz@njit.edu.cn)

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ABSTRACT This paper addresses the tracking accuracy and robustness enhancement problems of fuzzy model based predictive control (MPC) for a class of nonlinear systems subjecting to lumped disturbances composed of bounded unknown disturbances and a model-plant mismatch. Main features of the proposed method are: 1) A fuzzy disturbance observer and an auxiliary controller are jointly developed to meet a certain control objective that minimizes the peak bound of the errors caused by the lumped disturbances, which eventually leads to desired offset-free tracking performance. 2) A pre-computed robust positively invariant set whose central is the nominal state is derived with the premise of input-to-state stability. 3) Tightened constraints for the guarantee of recursive feasibility of MPC is computed off-line and the quasi-min-max fuzzy MPC is elaborately designed according to a piecewise Lyapunov function. Furthermore, characteristics of robustness enhancement and low on-line computational burden are obtained as compared with the existing offset-free MPCs, and further the impacts of estimation error arising from sampling time and admissible target set on the system performance are also discussed. Two simulation examples verify the effectiveness of the proposed approach ensuring the satisfaction of constraints.

INDEX TERMS Fuzzy discrete disturbance observer, robust model predictive control, T-S fuzzy models, input-to-state stability.

I. INTRODUCTION

Model predictive control (MPC), which can predict the future process behavior and optimize the control input with the consideration of various constraints, has been extensively studied in the past decades [1]. However, for the general MPC, the control performance is significantly challenged when the industrial process characterizes large nonlinearity over a wide operating range and subjects to unknown strong disturbances and uncertainties [2], such familiar example of this sort of processes is the continuous stirred tank reactor (CSTR) in chemical plant [3], or boiler-turbine unit in power plant [4]. First, tracking offset is an unavoidable problem which will trouble the MPC and cause the tracking performance degradation. Although the disturbance rejection methods, such as disturbance estimation with feedforward compensation, have been regarded as effective strategies to counteract the disturbances, the recursive feasibility which is the intrinsic attribute of MPC for stability becomes a difficult issue to be managed since the separate frame design procedure. Second, for nonlinear systems in the presence of disturbances, the requirement of robustness becomes serious matters that the controller designed on a single nominal model is never probably representing the dynamics of the true plant. Third, within the framework of the existing disturbance observer based model predictive fuzzy control strategies, the original control input constraints for the model-based predictive control are undermined by the disturbance compensation in the composite control law.

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A. LITERATURE REVIEW

It is well known that Takagi-Sugeno (T-S) model has been widely used to approximate the system's nonlinearity [5]–[7].

Many theoretical results on stability analysis and controller synthesis has been obtained T-S fuzzy models, in which sufficient conditions of stability are converted into a set of linear matrix inequalities (LMIs) based on a common Lyapunov function (CLF) [8] or a piecewise Lyapunov function (PLF) [9]. Considering the presence of uncertainty and disturbance in application, the stabilization with performance indexes of H_2 [10] and H_∞ [11] is achieved to satisfy the robust requirements. To solve the control problem of the nonlinear dynamic systems with persistent bounded disturbances, a fuzzy observer-based or filter-based fuzzy controller was developed to minimize the upper bound of L_∞ gain of the closed-loop system [12], [13]. The tracking accuracy was further enhanced in [14], in which the modeled disturbances were estimated by a novel fuzzy disturbance observer and then compensated in the compound control law, in term of the semiglobally input-to-state practically stability of the closed-loop system. More recently, a disturbance observer-based integral slide-mode control scheme was proposed on the T-S fuzzy model to deal with the control problem of nonlinear system subjecting to non-periodic form of disturbance [15]. For more complicated chaotic processes, a fuzzy logic controller was proposed with stability analysis in [16]. Despite such great achievements, there are still some performance improvements to be made in industrial process with various constraints, such as transient control performance and/or economic efficiency.

Among the advanced control methods, MPC has strong ability to address the issues of transient performance and deal with various constraints during design stage, appearing in [17]–[19]. T-S fuzzy model-based predictive control with the issues of stability and optimization has been designed on the basis of PLFs in [20]. Xia *et al.* developed new sufficient stability conditions for the fuzzy MPC through the technique of slack matrices [21]. Afterwards, T-S fuzzy model-based predictive control has been successfully applied in many processes, such as energy-efficient office building [22] and power generation [23]. Although the nonlinear behavior or parameter variation of the system have been considered in the aforementioned methods, the appearance of disturbances, which is ubiquitous in the industry process, can cause severe degradation of the control performance [24]. Thus, for uncertain discrete-time T-S fuzzy systems with the consideration of input constraints and disturbance, a robust model predictive controller was developed in accordance of input-to-state stability (ISS) and the robust positively invariant (RPI) set for T-S systems was further investigated [25], [26].

It should be noted that the disturbances mentioned above are usually assumed to be smooth and centered around zero, regarded as general disturbances. When there are strong type disturbances intruding into the loop, field engineers will be confused by the problem of tracking offset. Pannocchia and Bemporad [27] proposed an offset-free MPC by estimating the integrated disturbances through a steady-state Kalman filter, in which the target set-point was optimally computed by formulating the estimated disturbance into the steady-state equations. Based on the T-S fuzzy model, Wu *et al.* [23] has extended the offset-free MPC to nonlinear system and applied it to a boiler-turbine unit. As an alternative, disturbance observer (DOB) provides another possibility for the estimation of disturbance [28]. In [29], a DOB based MPC for linear systems was proposed, in terms of ISS stability, to cancel out the disturbances effect from the control input via feedforward channel. Because the control design framework is on the basis of continuous time, the time interval reserved for control computation is expected to be as small as possible and the disturbance varying rate is required to be slow. In [30], the perturbations of a mobile robot treated as input addictive disturbances were estimated by an extended state observer (ESO) and compensated through the control law, thus the tracking performance of the distributed model predictive control was improved. The input disturbance compensation may violate the constraints of the model predictive control, which may lead to the performance degradation or even instability of the model predictive control. For nonlinear controlled plant, a fuzzy RMPC with ESO was proposed in [31], where the lumped disturbances could be estimated through the ESO and alleviated by an appropriate disturbance compensator. It was observed from the simulation that a better transient response in disturbance compensation could be achieved. However, due to the RMPC and ESO were designed individually, the disturbance compensation broke the optimality of the constrained predictive control system. Moreover, the recursive feasibility of the control scheme was not considered.

Recently, in [32], a novel method called tube-based MPC for the linear systems with bounded disturbances was provided, where the robustness can be enhanced by using an auxiliary controller. A disturbance RPI set was computed to approximate the adverse effect of disturbances, through which strong stability result can be obtained. Defined from the RPI set, the center of the "tube" is the trajectory of the corresponding nominal system driven by the conventional constrained MPC, while all possible trajectories of the perturbed system are constrained in it. The state of the nominal system, which is free from the disturbance, can guarantee the MPC to be recursively feasible. In [33], the linear tube-based MPC design approach was extended to nonlinear system. And in [34], a robust MPC was developed for a class of hybrid systems using the technique of RPI set, in which the ISS stability of the control system can be guaranteed. In [35], state observer was further considered in the tube-based MPC design.

Consequently, within the framework of conventional tube-based MPC, disturbances with features of finite energy or slow variation can be well handled. However, the tracking offset is still an urgent problem to be solved when the disturbances are strong and there is still a lot of research to be done to meet the challenges of large nonlinear systems.

B. NOVELTY AND CONTRIBUTION

In light of tube-based MPC, this paper proposes a disturbance observer based fuzzy model-based predictive

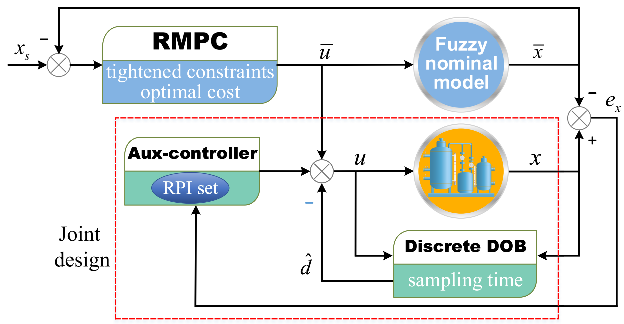

FIGURE 1. Block diagram of the overall system.

control (DOBFMPC) approach, which is shown in FIGURE 1 to address the aforementioned control challenges for industry process. Three controllers are involved in the DOBFMPC framework, namely, RMPC, discrete DOB and aux-controller. With the evolution of the system state, all the controllers and the fuzzy nominal model are updated according to the scheduling signal. The nominal input $\bar{u}$ generated by the RMPC on the fuzzy nominal model fulfills a major role in tracking the set-point x_s, which can guarantee the optimality of the system without disturbances. The estimated disturbance $\hat{d}$, as a feed-forward compensating signal, is the output of discrete DOB designed with the consideration of sampling time. The remaining control input generated by the aux-controller is used to alleviate the deviation between real plant state x and nominal state $\bar{x}$. It should be noted that the ultimate bound of the error state e_x is determined by both the discrete DOB and the aux-controller, in which the impact factors are the disturbance variation and the selection of sampling time. For this reason, the DOB and aux-controller are jointly designed as shown in FIGURE 1 to meet a certain control objective that minimizes the peak bound of the error state. The RPI set used to constrain the error state and the tightened constraints acted on the RMPC optimization can be computed off-line. Consequently, DOBFMPC consisted of RMPC, discrete DOB and aux-controller is proposed for the fuzzy system, which improves the tracking performance and robustness of the control system.

To our best knowledge, a unified framework of discrete fuzzy DOB and fuzzy MPC for nonlinear systems subject to disturbances has never been reported in literature so far. Contributions and qualitative improvements of the proposed DOBFMPC approach compared with the previous literatures can be summarized as follows:

1) Compared with [30], [31], this paper proposes a novel offset-free MPC strategy with enhanced robustness in terms of aux-controller, and with the satisfactory of recursive feasibility that the global stability is achieved.

2) Unlike the tube-based MPC [32], [34] and the approach in [29], a discrete-time control strategy with full consideration of sampling time is developed for a nonlinear plant. The disturbance rejection's ability is improved and model-plant mismatch arising from the T-S fuzzy modeling is compensated.

3) Moreover, a novel RPI set solution for T-S fuzzy system is derived by constructing a new ISS-Lyapunov condition.

The rest of the paper is organized as follows. Section 2 presents relevant concepts, definitions and the control structure. Integrated design of the disturbance observer and aux-controller is provided in Section 3. The robust MPC design is presented in Section 4. Simulations are carried out in Section 5. Finally, some conclusions of the proposed approach are provided in Section 6.

Notation: Given two sets A and B, the Minkowski sumation is defined as $A \oplus B = \{a+b | a \in A, b \in B\}$ and the Pontryagin difference is defined as $A \ominus B = \{a | a \oplus B \subset A\}$. $||w||$ denotes the Euclidean norm and the norm $||w||_\infty = \sup_{0 \le i \le N-1} ||w(i||)$.

II. PRELIMINARIES

A. DEFINITIONS

Based on the definition of $\mathcal{K}-$ functions, $\mathcal{K}_\infty-$ functions and $\mathcal{KL}-$ functions in [37], the input-to-state stability of a discrete nonlinear system is given as follows

Definition 1 (ISS [37]): A discrete nonlinear system $x(k+1) = f(x(k), d(k))$ is called input-to-state stable if there exist $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$ such that for any bounded disturbance $d(k)$ and any initial state $x(0)$, the behavior of $x(k)$ satisfies

$$\|x(k)\| \le \beta(\|x(0)\|, k) + \gamma(\sup_{0 \le i \le k-1} \|d(i)\|) \tag{1}$$

Definition 2 (ISS-Lyapunov Function [37]): For a discrete nonlinear system $x(k+1) = f(x(k), d(k))$, a positive definite function $V(x)$ is called an ISS-Lyapunov function if there exist $\alpha_1,\ \alpha_2,\ \alpha_3 \in \mathcal{K}_\infty$ and $\gamma \in \mathcal{K}$ such that

$$\alpha_1(\|x\|) \le V(x) \le \alpha_2(\|x\|)$$
$$V(x(k+1)) - V(x(k)) \le -\alpha_3(\|x(k)\|) + \gamma(\|d(k)\|)$$

Lemma 1 [37]): A discrete nonlinear system $x(k+1) = f(x(k), d(k))$ is ISS, if an ISS-Lyapunov function can be found for it.

Definition 3 (RPI): Consider a nonlinear system $x(k+1) = f(x(k), d(k))$. A set Ω is called a RPI set for the closed-loop system with control law π, if $\forall x(k) \in \Omega$, $x(k+1) \in \Omega$, $\forall d \in \mathbb{D}$ where $\mathbb{D}$ is a compact set.

Assumption 1: The lumped disturbance $d(k) := [d_1(k), \dots d_m(k)] \in \mathbb{W}$ is bounded and the disturbance deviation $\Delta d(k) := d(k) - d(k-1)$ satisfies $\Delta d(k) \in \mathbb{D} = \{\Delta d : F_d \Delta d \le T_s \cdot \sigma\}$ where the matrix F_d and the vector σ are assumed to be constant with $\sigma > 0$. The state and control input of the system are subject to the constraints $x(k) \in \mathbb{X}$ and $u(k) \in \mathbb{U}$, respectively, where the origin is contained in the interior of $\mathbb{X}$ and $\mathbb{U}$ which are assumed to be compact.

B. T-S FUZZY MODEL

We consider a class of continuous nonlinear system with multiple disturbances described by

$$\dot{x} = f(x, u, w) \tag{2}$$

where $x = [x_1, \dots, x_n] \in \mathbb{R}^n$ is the state, $u = [u_1, \dots, u_m] \in \mathbb{R}^m$ is the control input and $w = [w_1, \dots, w_m] \in \mathbb{R}^m$ is the disturbance satisfying the matching condition that the disturbances enter the plant through the same input distribution matrix as the control input.

Using the approximation-based modeling method, a nonlinear system can be represented by the following discrete-time T-S fuzzy model with the sampling time T_s.

$$\begin{aligned} &R^l : \textbf{If } \nu_1 \text{ is } M_1^l \quad \text{and } \ldots \nu_\upsilon \text{ is } M_\upsilon^l \\ &\textbf{Then } x(k+1) = A_l x(k) + B_l(u(k) + d(k)) \end{aligned} \tag{3}$$

where $l \in \mathbb{N}_{L+}$ and L is the number of inference rules. $M_1^l, \ldots, M_\upsilon^l$ are fuzzy set and $\nu := [\nu_1, \nu_1, \ldots, \nu_\upsilon]$ are scheduling signals. $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$ and $d(k) \in \mathbb{R}^m$ represent the vector of state, input and lumped disturbances, respectively.

Remark 1: The lumped disturbances are mainly composed of plant uncertainties, model-plant mismatches, modeling errors and external disturbances.

The discrete-time perturbed dynamic fuzzy system used as the model for the controller design below can be rewritten as

$$x(k+1) = A_\mu x(k) + B_\mu(u(k) + d(k)) \tag{4}$$

where $A_\mu := \sum_{l=1}^{L} \mu_l(\nu) A_l$ and $B_\mu := \sum_{l=1}^{L} \mu_l(\nu) B_l$, $\mu_l(\nu)$ is the normalized membership function.

III. LOCAL ROBUST CONTROLLER DESIGN WITH DISTURBANCE OBSERVER

To alleviate the influence of unknown disturbances on the model predictive control system, fuzzy DOB and aux-controller are jointly designed in this section. The fuzzy DOB is designed to estimate the value of the disturbances and make a direct compensation for the MPC, while the aux-controller is designed to limit the estimation error. To reduce the conservativeness of the MPC, the aux-controller and DOB are designed synthetically so that the estimation error RPI set is as small as possible, which will be used to calculate the tightened constraints for the RMPC

A. FUZZY DISCRETE DISTURBANCE OBSERVER

In this subsection, a discrete fuzzy disturbance observer (FDOB) is firstly developed to estimate the lumped disturbances. Extending the study in [26] for fuzzy system, the dynamics of the FDOB can be constructed as

$$\begin{cases} \theta(k+1) = \theta(k) + L_\mu \left\{ (A_\mu - I_n)x(k) + B_\mu(u(k) + \hat{d}(k)) \right\} \\ \hat{d}(k) = L_\mu x(k) - \theta(k) \end{cases} \tag{5}$$

where $\theta(k)$ is the observer state variable, L_μ is the observer gain and I_n is an identity matrix.

Denoting the estimation error $e_d(k) := d(k) - \hat{d}(k)$, and then the estimation error dynamics can be given as

$$e_d(k+1) = (I_m - L_\mu B_\mu) e_d(k) + \Delta d(k+1) \tag{6}$$

Lemma 2: Suppose that the pair $(I_m, \ B_\mu)$ is observable and the observer gain L_μ are chosen such that the system (6) is stable, the estimation error will asymptotically converge to a bound in the order of $\mathcal{O}(T_s)$.

Proof: The proof can be referred to [37] and [38].

The stability of (6) can be guaranteed in condition of the pair $(I_m, \ B_\mu)$ is observable which implies that B_μ is of full column-rank, i.e., $\text{rank}(B_\mu) = m$. In most control problem, such condition can be satisfied by choosing appropriate control inputs. Further, various methods can be applied here to choose the observer gain L_μ, such as pole-placement or the method provided in [38]. In this paper, the observer gain is elaborately designed to meet a global goal, which is presented in the following section.

Corollary 1: Suppose $\lim_{k\to\infty} \Delta d(k) = 0$, the lumped disturbances $d(k)$ can be accurately estimated as k goes to infinity.

Proof: Since $(I_m, \ B_\mu)$ is observable, then $I_m - L_\mu B_\mu$ is Schur (i.e., the corresponding eigenvalues are located strictly inside the unit disk), the system (6) is ISS from $\Delta d(k)$ to state $e_d(k)$. Then, there exist $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$, such that

$$\|e_d(k)\| \leq \beta(\|e_d(0)\|, k) + \gamma(\sup_{0 \leq i \leq k-1} \|\Delta d(i)\|).$$

Considering that $\lim_{k\to\infty} \Delta d(k) = 0$ and $\beta(\|e_d(0)\|, k) \to 0$ as $k \to \infty$, the state $\|e_d(k \to \infty)\| \to 0$, i.e., $\lim_{k\to\infty} d(k) - \hat{d}(k) = 0$.

Remark 2: If $\Delta d(k \to \infty) = 0$ in (6), the estimation error will be $e_d(k \to \infty) = 0$. Therefore, the disturbance observer (5) can accurately estimate the disturbance. It should be noted that the smaller the sampling time T_s, the smaller the estimation error, then the estimation error can be limited due to the assumption of the disturbance changing rate in *Assumption 1*. To prohibit the undesirable transient response, it is necessary to initialize the observer state to $\theta_0 = L_\mu x(0)$ where $x(0)$ is the initial state of the plant, since the developed observer gain L_μ may be large.

In [38], the approach of pole placement was used to determine the observer gain, which was simple but not suited for the fuzzy system. Moreover, as it is pointed out in Section I, both the aux-controller and the DOB have great influences on the performance of the control system. Thus, a novel design approach is proposed in the next subsection, which jointly determines the aux-controller and the DOB gain.

B. JOINT DESIGN OF DISCRETE DOB AND AUX-CONTROLLER

In this subsection, we first suppose that there are no disturbances appeared in the system, and the system (4) can be expressed in the following fuzzy nominal form

$$\bar{x}(k+1) = A_\mu \bar{x}(k) + B_\mu \bar{u}(k) \tag{7}$$

where $\bar{x}(k)$ is the nominal state satisfying $\bar{x}(k) \in \bar{\mathbb{X}}$, and $\bar{u}(k)$ is the nominal input satisfying $\bar{u}(k) \in \bar{\mathbb{U}}$.

Remark 3: Due to the existence of $d(k)$, the compact sets satisfy $\bar{\mathbb{X}} \subset \mathbb{X}$, $\bar{\mathbb{U}} \subset \mathbb{U}$, where the set $(\bar{\mathbb{X}}, \bar{\mathbb{U}})$ is prerequisite for the implementation of MPC.

To force the system state $x(k)$ close to the nominal state $\bar{x}(k)$ in the presence of lumped disturbances $d(k)$, a composite control law $u(k)$ for the system (4) is proposed

$$u(k) = \bar{u}(k) + K_\mu e_x(k) - \hat{d}(k) \tag{8}$$

where K_μ is the gain of the aux-controller, which can bring the enhanced robustness characteristic for the control system.

Combining (4), (7) and (8), the dynamic error state e_x can be expressed as

$$e_x(k+1) = A_\mu e_x(k) + B_\mu(\tilde{u}(k) + d(k)) \tag{9a}$$

$$\tilde{u}(k) = K_\mu e_x(k) - \hat{d}(k) \tag{9b}$$

Substituting (9b) into (9a) yields

$$e_x(k+1) = (A_\mu + B_\mu K_\mu)e_x(k) + B_\mu e_d(k) \tag{10}$$

Combining (6) and (10), the closed-loop of the augmented error system can be constructed as

$$\begin{bmatrix} e_x(k+1) \\ e_d(k+1) \end{bmatrix} = \begin{bmatrix} A_\mu + B_\mu K_\mu & B_\mu \\ 0 & I_m - L_\mu B_\mu \end{bmatrix} \begin{bmatrix} e_x(k) \\ e_d(k) \end{bmatrix} + \begin{bmatrix} 0 \\ I_m \end{bmatrix} \Delta d(k+1) \tag{11}$$

Theorem 1: Suppose that *Assumption 1* is satisfied for the system (9a). If K_μ and L_μ are selected in such a way that $A_\mu + B_\mu K_\mu$ and $I_m - L_\mu B_\mu$ are Schur matrices, the system (11) under the proposed control law (9b) is ISS and the error state $e_x(k)$ is bounded in the order of $\mathcal{O}(T_s)$.

Proof:

1) Since both $A_\mu + B_\mu K_\mu$ and $I_m - L_\mu B_\mu$ are Schur matrices,

$$\begin{bmatrix} A_\mu + B_\mu K_\mu & B_\mu \\ 0 & I_m - L_\mu B_\mu \end{bmatrix}$$

is also a Schur matrix. Thus, the closed-loop system (11) is ISS [37].

2) Since the closed-loop system (11) is ISS, there exist $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$, such that

$$\left\| \begin{bmatrix} e_x(k) \\ e_d(k) \end{bmatrix} \right\| \leq \beta\left(\left\| \begin{bmatrix} e_x(0) \\ e_d(0) \end{bmatrix} \right\|, k\right) + \gamma\left(\sup_{0\leq i\leq k-1} \|\Delta d(i)\|\right) \tag{12}$$

Considering *Assumption 1* and the *Lemma 2*, the state $e_x(k)$ is bounded to $\|\Delta d(k)\|_\infty$, which is not related to the $\|d(k)\|_\infty$.

Thus, we finish the proof.

Remark 4: From *Theorem 1*, the upper bound of $e_x(k)$ is determined by the selection of K_μ, L_μ and $\|\Delta d(k)\|_\infty$. Furthermore, the sampling time T_s is also a non-negligible factor as indicated in *Lemma 2*.

Lemma 3: Suppose that *Assumption 1* is satisfied for system (9a) and $\lim_{k\to\infty} \Delta d(k) = 0$. If K_μ and L_μ are calculated to ensure that $A_\mu + B_\mu K_\mu$ and $I_m - L_\mu B_\mu$ are Schur matrices, the lumped disturbances $d(k)$ can be attenuated from the state $e_x(k)$ under the control law (9b) as k goes to infinity.

Proof: For system (10), because $I_m - L_\mu B_\mu$ is Schur matrix and $\lim_{k\to\infty} \Delta d(k) = 0$, one can get $e_d(k \to \infty) = 0$ from the *Corollary 1*. Because $A_\mu + B_\mu K_\mu$ is Schur matrix, then $\lim_{k\to\infty} e_x(k) = 0$. Thus, the impact of disturbances can be eliminated from the state $e_x(k)$.

C. MAIN RESULTS

Theorem 1 already obtained only provides a design criterion for making the system (9a) ISS stable. In this subsection, the problem of obtaining the aux-controller gains and observer gains will be solved with the technique of LMI. In [39], a minimal RPI was sought to express the minimal influence of the disturbances for the linear system. For the fuzzy system considered in this paper, the future behavior of the system is unknown, which brings difficulties for the solution of minimal RPI set using the reachable set method [40]. Thus, an alternative method is proposed here to find the minimal RPI set.

Reconsidering the disturbance deviation constraint $\mathbb{D}$, one can find an outer ellipsoid $\mathcal{E}(P_d) = \{\Delta d \in \mathbb{R}^m : \Delta d^T P_d \Delta d \leq 1,\ P_d > 0\}$ that approximates $\mathbb{D}$.

Considering system (11) and choosing $e_x(k)$ as the controlled output, one can get

$$\begin{cases} *20c \begin{bmatrix} e_x(k+1) \\ e_d(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} A_\mu + B_\mu K_\mu & B_\mu \\ 0 & I_m - L_\mu B_\mu \end{bmatrix}}_{A_\mu^{cl}} \times \begin{bmatrix} e_x(k) \\ e_d(k) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ I_m \end{bmatrix}}_{B_\mu^{cl}} \Delta d(k) \\ e_x(k) = \underbrace{\begin{bmatrix} I_n & 0 \end{bmatrix}}_{C_\mu^{cl}} \begin{bmatrix} e_x(k) \\ e_d(k) \end{bmatrix} \end{cases} \tag{13}$$

Denoting the error state $e(k) := \begin{bmatrix} e_x(k)^T & e_d(k)^T \end{bmatrix}^T$, the following lemma shows that there exists a controlled output bound for system (13) with the input bound $\mathcal{E}(P_d)$.

Define $P_\mu := \sum_{i=1}^{L} \mu_i(\nu)P_i$ and $P_\mu^+ := \sum_{i=1}^{L} \mu_i(\nu^+)P_i$ in which P_i is the piecewise Lyapunov matrix for the i-th local model of the fuzzy system, the following Lemma is derived.

Lemma 4: Consider the system (13) and suppose $e(0) = 0$. If there exist matrices $P_\mu = (P_\mu)^T$ and $P_\mu^+ = (P_\mu^+)^T$, scalars $\rho > 0$, $\chi > 0$ and $0 < \tau < 1$, such that

$$\begin{bmatrix} (A_\mu^{cl})^T P_\mu^+ A_\mu^{cl} - \tau P_\mu & (A_\mu^{cl})^T P_\mu^+ B_\mu^{cl} \\ (B_\mu^{cl})^T P_\mu^+ A_\mu^{cl} & (B_\mu^{cl})^T P_\mu^+ B_\mu^{cl} - \rho P_d \end{bmatrix} < 0 \tag{14a}$$

$$(\tau - 1)P_\mu + (C_\mu^{cl})^T C_\mu^{cl} < 0 \tag{14b}$$

$$\rho - \chi < 0 \tag{14c}$$

Then $e_x(k)^T e_x(k) < \chi$ is satisfied for $\Delta d(k)^T P_d \Delta d(k) \leq 1$, $\forall k \geq 1$ and the system (13) is ISS.

Proof: Given in the Appendix.

It should be emphasized that inequalities (14a), (14b), (14c) are a bilinear matrix inequality (BMI) problem which is difficult to solve. Through the analysis of the formula (28), we can find that the parameter τ plays the role of decay rate of the closed-loop system. To solve the BMI problem to reach the minimization of the upper bound χ, we can do a line search over $0 < \tau < 1$ or resort to PENBMI toolbox [41].

Remark 5: Lemma4 will be transformed to the result in [42] where an RPI set is identified for linear systems, if we add the condition $\tau + \rho = 1$ into the LMIs. The value of ρ will be decreased during the minimization of χ, when considering the constraint of (14c).

To derive the feedback controller gains, the observer gains and the upper bound χ, the following *Theorem 2* is given for the closed-loop system (13).

Theorem 2: Considering fuzzy system (13) and suppose $e(0) = 0$, if there exist positive-definite matrices X_i (or X_k), positive-definite matrix Q, matrices G_j, F_j, H_m, scalars $\rho > 0$, $\chi > 0$ and $0 < \tau < 1$, such that the following minimization is feasible

$$\min_{Q,G_j,F_j,H_m,\rho,\tau} \chi \tag{15}$$

subject to (16a)-(16e)

$$\begin{bmatrix} -X_k & 0 & 0 & 0 & A_iG_j + B_iF_j & B_i & 0 \\ * & -Q & 0 & 0 & 0 & Q - H_mB_i & 0 \\ * & * & -X_k & 0 & 0 & 0 & 0 \\ * & * & * & -Q & 0 & 0 & Q \\ * & * & * & * & -\tau(G_j + G_j^T - X_i) & 0 & 0 \\ * & * & * & * & * & -\tau Q & Q - B_i^T H_m^T \\ * & * & * & * & * & * & -\rho P_d \end{bmatrix} < 0 \tag{16a}$$

$$\begin{bmatrix} (\tau - 1)I_n & X_i \\ X_i & -I_n \end{bmatrix} < 0 \tag{16b}$$

$$\rho - \chi < 0 \tag{16c}$$

where $i, j, k, m \in \mathbb{N}_{L+}$. Then, the system (13) is ultimate bounded corresponding to the control law (8b), where the aux-controller gain $K_\mu = \sum_{j=1}^{L} \mu_j(\nu)K_j$ with $K_j = F_jG_j^{-1}$, and the observer gain $L_\mu = \sum_{m=1}^{L} \mu_m(\nu)L_m$ with $L_m = Q^{-1}H_m$ are obtained.

Proof: Given in the Appendix.

Remark 6: The positive matrix $P_\mu = diag\{R_\mu,\ Q\}$ is used to construct a partial piecewise Lyapunov function, which is less conservative than that using a common Lyapunov function. The reason for applying the common Lyapunov function Q is that the disturbance estimation error region can be constructed by a fixed ellipsoid set.

Remark 7: The upper bound χ we obtained is determined by the chosen of $\|\Delta d(k)\|_{P_d} \leq 1$ whose volume is influenced by the weight matrix P_d. In practice, the determination of P_d should be in accordance to the plant characteristics and operation environment, where the larger the compact set volume, the more conservative the controller design will be, and vice versa.

In terms of *Theorem 2*, the aux-controller and the disturbance observer are elaborately designed, and system state $x(k)$ can be regulated converging to the tube centered at the nominal state $\bar{x}(k)$ while k goes to infinity. The remaining work is to design a robust MPC to regulate the nominal state $\bar{x}(k)$ to the set-point value.

IV. ROBUST FUZZY MPC

The remaining unsolved issue of the composite control law (8) is the nominal control input $\bar{u}$ which mainly determines the tracking performance of the system. Since the scheduling signal can be measured at the sampling instant, quasi-min-max RMPC [19], acknowledged as an efficient controller whose first control action, can be designed with full consideration of the model, and it will be redesigned for the fuzzy nominal model (7) with piecewise Lyapunov function.

A. SOLUTION OF THE TIGHTENED CONSTRAINTS

In this subsection, the tightened constraints for the RMPC will be calculated. Obviously, it is difficult to achieve the optimal control performance by designing the predictive controller under the original constraints ($\mathbb{X}$, $\mathbb{U}$). As stated in *Remark 3*, the existence of lumped disturbances $d(k)$ causes tighter constraints for the state $\mathbb{X}$ and control input $\mathbb{U}$ of the RMPC. Therefore, the tightened constraints ($\bar{\mathbb{X}}$, $\bar{\mathbb{U}}$) are required to be calculated first.

Denoting $\Omega_x := \{e_x \in \mathbb{R}^n : \|e_x\|_\infty < \chi^{1/2}\}$, the tightened state constraint can be determined from the calculation of $\bar{\mathbb{X}} = \mathbb{X} \ominus \Omega_x$. The region Ω_x is a RPI set of state e_x, which can be used to approximate the adverse effect of the disturbances [39].

The tightened input constraint $\bar{\mathbb{U}}$ can be determined corresponding to the tightened state constraint $\bar{\mathbb{X}}$. Denoting Ω_d as the RPI set of the disturbance estimation error state e_d, the constraint for the estimated disturbance can be defined as $\tilde{\mathbb{W}} := \mathbb{W} \oplus \Omega_d$.

Reconsider the composite control law (8) for the nonlinear system (4). The nominal control law is $\bar{u}(k) = u(k) + \hat{d}(k) - K_\mu e_x(k)$, the original input constraints $u \in \mathbb{U}$ can be guaranteed by satisfying the tightened constraint $\bar{u} \in \bar{\mathbb{U}}$ for the fuzzy nominal system. The tightened input constraint can be calculated from $\bar{\mathbb{U}} = \mathbb{U} \ominus \left(\Theta \oplus (-\tilde{\mathbb{W}})\right)$ where $\Theta = K_\mu\Omega_x$, and Θ can be solved following approach given in [27]:

$$\Theta = Co\{K_i\Omega_x,\ \forall i = 1, 2, \ldots, L\}$$

where Co denotes the convex hull.

Substituting $P_\mu = \begin{bmatrix} R_\mu & 0 \\ 0 & Q \end{bmatrix}$ into (29), we have

$$e_x(k)^T R_\mu e_x(k) + e_d(k)^T Q e_d(k) \leq \frac{\rho}{1 - \tau} \tag{17}$$

Since $e_x(k)^T R_\mu e_x(k) > 0$, we can obtain $e_d(k)^T Q e_d(k) \leq \rho/(1 - \tau)$. Therefore, denoting $\Omega_d := \{e_d \in \mathbb{R}^q :$

$e_d(k)^T Q e_d(k) \leq \rho/(1-\tau)\}$, we can determine the constraint for the estimated disturbance. It is noted that the $\tilde{\mathbb{W}}$ can also be obtained by artificially expanding the presupposed disturbance bounded region instead of using the conservative condition $e_d(k)^T Q e_d(k) \leq \rho/(1-\tau)$.

B. QUASI-MIN-MAX FMPC

The purpose of the fuzzy MPC is to regulate nominal fuzzy system (6) from an initial point $(\bar{x}(0), \bar{u}(0))$ to a target point (x_s, u_s). Suppose that the set point (x_s, u_s) is admissible under the tightened constraints. The following infinite horizon objective function is considered in the MPC design

$$\begin{aligned} J_0^{\infty}(k) &= \sum_{i=0}^{\infty} \left\{ \|\bar{x}(k+i|k) - x_s\|_{Q_0}^2 + \|\bar{u}(k+i|k) - u_s\|_{R_0}^2 \right\} \\ &= \|\bar{x}(k|k) - x_s\|_{Q_0}^2 + \|\bar{u}(k|k) - u_s\|_{R_0}^2 + J_1^{\infty}(k) \end{aligned} \tag{18}$$

where $Q_0 > 0$, $R_0 > 0$ are the state weighting matrix and input weight matrix, respectively. Considering a piecewise Lyapunov function $V(\bar{x}(k+i|k) - x_s) = \|\bar{x}(k+i|k) - x_s\|_{P_\mu}^2$ where $P_\mu := \sum_{l=1}^{L} \mu_l(\nu) P_l$ with positive matrices P_l, the fuzzy nominal model (7) can be robustly stabilized by satisfying the following constraint

$$\begin{aligned} &V(\bar{x}(k+i+1|k) - x_s) - V(\bar{x}(k+i|k) - x_s) \\ &\quad \leq -\|\bar{x}(k+i|k) - x_s\|_{Q_0}^2 - \|\bar{u}(k+i|k) - u_s\|_{R_0}^2 \leq 0 \end{aligned} \tag{19}$$

Summing (19) from $k = 1$ to ∞ yields $J_1^{\infty}(k) \leq V(\bar{x}(k+1|k) - x_s)$.

Then, the minimization of the objective function is turned to optimization problem

$$\min \ \zeta \tag{20}$$

subject to

$$\begin{aligned} &\|\bar{x}(k|k) - x_s\|_{Q_0}^2 + \|\bar{u}(k|k) - u_s\|_{R_0}^2 \\ &+ \big\|A_\mu(k|k)(\bar{x}(k|k) - x_s) \\ &+ B_\mu(k|k)(\bar{u}(k|k) - u_s)\big\|_{P_\mu}^2 \leq \zeta \end{aligned} \tag{21}$$

where $A_\mu(k|k) = \sum_{i=1}^{L} \mu_i(k|k) A_i$ and $B_\mu(k|k) = \sum_{i=1}^{L} \mu_i(k|k) B_i$.

Denote $\bar{\mathbb{U}}_0 := \bar{\mathbb{U}} \ominus u_s$ as the input constraints and $\bar{\mathbb{X}}_0 := \bar{\mathbb{X}} \ominus x_s$ as the state constraints which both centered at the origin. The peak bound of $\bar{\mathbb{X}}_0$ and $\bar{\mathbb{U}}_0$ can be defined as $(\bar{x}_{0,\min}, \bar{x}_{0,\max})$ and $(\bar{u}_{0,\min}, \bar{u}_{0,\max})$, respectively.

Considering the steady-state point $(\bar{x}_s, \bar{u}_s)$, the constraints of the nominal MPC state and input can be expressed as

$$\left|\bar{x}^t(k+i|k) - x_s^t\right| \leq \bar{x}_{0,boundary}^t, \quad t = 1, \cdots, n, \ i \geq 0 \tag{22a}$$

$$\left|\bar{u}^t(k+i|k) - u_s^t\right| \leq \bar{u}_{0,boundary}^t, \quad t = 1, \cdots, m, \ i \geq 1 \tag{22b}$$

$$\bar{u}_{0,\min} \leq \bar{u}_0(k|k) \leq \bar{u}_{0,\max}, \ \bar{u}_0(k|k) = \bar{u}(k|k) - u_s \tag{22c}$$

where $\bar{x}_{0,boundary} := \min(\left|\bar{x}_{0,\min}\right|, \left|\bar{x}_{0,\max}\right|)$ and $\bar{u}_{0,boundary} := \min(\left|\bar{u}_{0,\min}\right|, \left|\bar{u}_{0,\max}\right|)$.

The control inputs of the quasi-min-max MPC can be split into two parts, i.e., $\left\{\bar{u}_0(k|k), \ F_\mu(\bar{x}(k+i|k) - x_s)\right\}_{i\geq 1}$, where $\bar{u}_0(k|k)$ is a free control action constrained by (22c) and the remaining control laws rely on the feedback gain F_μ which is constrained by (22b).

Using LMI-based approach, the optimization problem (19) can be solved through the following theorem.

Theorem 3: For the nominal fuzzy system (7), the control inputs $\left\{\bar{u}_0(k|k), \ F_\mu(\bar{x}(k+i|k) - x_s)\right\}_{i\geq 1}$ minimize the worst case objective function (17) if there exist a decision variable $\bar{u}_{0,s}$, general matrices Y_j, G_j, positive matrices S_i (or S_l) and symmetric matrices $\mathcal{U}$, $\mathcal{X}$, such that the following minimization problem is feasible

$$\min_{\bar{u}_0(k|k), G_j, Y_j, S_i} \zeta \tag{23}$$

subject to (24a)-(24e)

$$\begin{bmatrix} 1 & * & * & * \\ \begin{array}{c} A_\mu(k|k)(\bar{x}(k|k) - x_s) \\ + B_\mu(k|k)\bar{u}_0(k|k) \end{array} & S_l & * & * \\ Q_0^{1/2}(\bar{x}(k|k) - x_s) & 0 & \zeta I & * \\ R_0^{1/2}\bar{u}_0(k|k) & 0 & 0 & \zeta I \end{bmatrix} \geq 0 \tag{24a}$$

$$\begin{bmatrix} G_j^T + G_j - S_i & * & * & * \\ A_i G_j + B_i Y_j & S_l & * & * \\ Q_0^{1/2} G_j & 0 & \zeta I & * \\ R_0^{1/2} Y_j & 0 & 0 & \zeta I \end{bmatrix} \geq 0 \tag{24b}$$

$$\begin{aligned} &\begin{bmatrix} \mathcal{X} & * \\ (A_i G_j + B_i Y_j)^T & G_j^T + G_j - S_j \end{bmatrix} \geq 0, \\ &\mathcal{X}_{tt} \leq (\bar{x}_{0,boundary}^t)^2, \ t = 1, \cdots, n \end{aligned} \tag{24c}$$

$$\begin{aligned} &\begin{bmatrix} \mathcal{U} & * \\ Y_j^T & G_j^T + G_j - S_j \end{bmatrix} \geq 0, \\ &\mathcal{U}_{tt} \leq (\bar{u}_{0,boundary}^t)^2, \ t = 1, \cdots, m \end{aligned} \tag{24d}$$

$$\bar{u}_{0,\min} \leq \bar{u}_{0,s} \leq \bar{u}_{0,\max} \tag{24e}$$

where $\mathcal{X}_{tt}$ and $\mathcal{U}_{tt}$ are the tth diagonal element of the corresponding matrix, and $i, j, l \in N_{L+}$, then the feedback gain can be calculated from $F_\mu := \sum_{j=1}^{L} \mu_j(\nu) Y_j G_j^{-1}$.

Proof: The LMIs (24a)-(24e) are equivalent to the expression of (21), (19), (22a), (22b), (22c), respectively. Proofs of *Theorem 3* can be found in [6], [43] and thus are not repeated here.

Remark 8: After the optimization of the worst case infinite horizon objective, we only apply the free control action $\bar{u}_0(k|k)$ to the plant and the existence of the feedback gain F_μ can guarantee the stability of control strategy [19]. The system matrices $(A_\mu(k|k), \ B_\mu(k|k))$ of (24a) are then updated at the sampling instant according to the scheduling signal ν measured from the plant.

In conclusion, the algorithm of the proposed DOBFMPC is summarized as below

Theorem 4: (Recursive feasibility and Stability) The proposed fuzzy MPC with disturbance rejection can asymptoti-

Algorithm 1

Offline:

Calculate the disturbance observer gain $L_{i=1,\cdots,L}$ and auxiliary feedback gain $K_{i=1,\cdots,L}$ from inequalities (16a)-(16c). Compute the tightened constraints of $\bar{\mathbb{U}}$, $\bar{\mathbb{X}}$.

Online:

Step 1: Initialize the system state $x(0)$ and assign it to the fuzzy nominal model state $\bar{x}(0)$ such that the optimization problem of (23) is initially feasible for the target (x_s, u_s). Also, initialize $\theta_0 = L_\mu x(0)$ for the fuzzy disturbance observer.

Step 2: Solve the optimization problem (23), with the inequalities (24a)-(24e), according to the current state $\bar{x}(k|k)$ to obtain the control action $\bar{u}_0(k|k)$ and evolve the fuzzy disturbance observer (5) to get the current estimated lumped disturbance $\hat{d}(k)$.

Step 3: Feed the plant with $u(k) = \bar{u}_0(k|k) + u_s - \hat{d}(k) + K_\mu(x(k) - \bar{x}(k|k))$ following the composite control law Eq. (8).

Step 4: Measure the real state $x(k+1)$ at the next time instant and compute the subsequent state $\bar{x}(k+1)$ of the nominal fuzzy system (7) under the control action $\bar{u}_0(k|k) + u_s$.

Step 5: Replace $(x(k), \bar{x}(k|k))$ with $(x(k+1), \bar{x}(k+1))$ and set $k = k+1$, then go to *Step 2*.

cally steer the system state $x(k)$ from a feasible initial state $x(0)$ to the RPI set Ω_x whose center is the admissible target state x_s.

Proof:

1) The optimization problem (23) is solved considering the fuzzy nominal model (7) and the tightened constraints at each sampling instant, and the external disturbances are isolated from the solution. Therefore, the recursive feasibility can be guaranteed if the problem (23) is feasible at the initial state $x(0)$ with the constraints $x(0) \in \bar{\mathbb{X}}$ [30].

2) In terms of Theorem 2, $\|e_x(k)\|_\infty < \chi^{1/2}$ for all $k \geq 1$. Thus, we have $\|x(k) - x_s\| \leq \beta(\|\bar{x}(0) - x_s\|, k) + \chi^{1/2}$ since $x(k) = \bar{x}(k) + e_x(k)$ for all $k \geq 1$.

Considering $\beta(\cdot, k)$ is a $\mathcal{KL}$-function, thus $\beta(\|\bar{x}(0) - x_s\|, k) \to 0$ as $k \to \infty$. Then, we have $\|x(k) - x_s\| \leq \chi^{1/2}$ as $k \to \infty$.

Thus, the system state $x(k)$ will be eventually driven to the set $x_s \oplus \Omega_x$, where $\Omega_x = \{x \in \mathbb{R}^n : \|x\|_\infty < \chi^{1/2}\}$.

C. SET-POINT TRACKING DISCUSSION

In this section, some treatments to improve the tracking performance of the system are given when the set-point of the system is not reachable due to tightened constraints. In the above strategy, the target set-point is assumed to be fixed and admissible. In case of operating point change, due to the existence of the tightened constraints $\bar{\mathbb{X}}$ and $\bar{\mathbb{U}}$, the target set-point may be unreachable which was revealed in [44]. To improve the feasibility region of the target set-point, the following approaches can be adopted:

1) Decrease the sampling time as Remark 2 stated. The RPI set Ω_x and Ω_d will be smaller than that designed under a larger sampling time since the deviation in disturbance will be smaller at two sampling instants.

2) Introduce an artificial steady-state point (x_s', u_s') to evolve the perturbed system to the neighborhood of the desired steady point (x_s, u_s). The artificial steady point, which can be determined through minimizing the deviation to the desired steady point through optimization computation, is an admissible tracking point [31].

Considering the fuzzy nominal system (7), (x_s', u_s') can be obtained by solving the following quadratic programming:

$$
\begin{aligned}
&\min_{x_s', u_s'} \left(\left(x_s' - x_s\right)^T Q_s \left(x_s' - x_s\right) + \left(u_s' - u_s\right)^T R_s \left(u_s' - u_s\right) \right) \\
&\text{s.t. } (I - A_\mu)x_s' - B_\mu u_s' = 0 \\
&\quad \bar{u}_{\min} \leq u_s' \leq \bar{u}_{\max} \\
&\quad \bar{x}_{\min} \leq x_s' \leq \bar{x}_{\max}
\end{aligned}
$$

where $(\bar{x}_{\min}, \bar{x}_{\max})$ and $(\bar{u}_{\min}, \bar{u}_{\max})$ are the peak bound of tightened constraints $\bar{\mathbb{X}}$ and $\bar{\mathbb{U}}$, respectively, $Q_s = Q_s^T > 0$ and $R_s = R_s^T > 0$ are symmetric weighting matrices.

3) Note that the lumped disturbances of the system come from two aspects: internal disturbances and external disturbances. The amplitude and rate of external disturbances are difficult to be intervened, but the internal disturbances can be intervened by changing the variation of set-point from fast step variation to slow ramp variation. Both the disturbances $d(k)$ and $\Delta d(k)$ can be reduced since the disturbances coming from the modelling error or/and uncertainties can be limited to a smaller bound between the two neighboring setpoints.

Remark 9: Reducing the sampling time will inevitably bring an additional computational load, if the computation is limited, we can resort to other two approaches. For process control, engineers usually take the third approach, e.g., a ramping power reference input is adopted into the boiler-turbine unit so that the internal disturbance derivation caused by model uncertainties is reduced.

V. SIMULATION RESULTS

Two simulation examples are given in this section to demonstrate the efficiency of the proposed DOBFMPC. The simulations are carried out under Matlab R2017a environment using the Yalmip toolbox [45].

A. NUMERICAL EXAMPLE

The first example is designed to validate the disturbance rejection performance of the DOBFMPC. A second-order perturbed discrete fuzzy system [25] is considered in this example:

$$
\begin{aligned}
&Rule1: \text{If } x_1(k) \text{ is } M_1, \text{ then} \\
&x(k+1) = A_1 x(k) + B_1(u(k) + d(k)) \\
&Rule2: \text{If } x_1(k) \text{ is } M_2, \text{ then}
\end{aligned}
$$

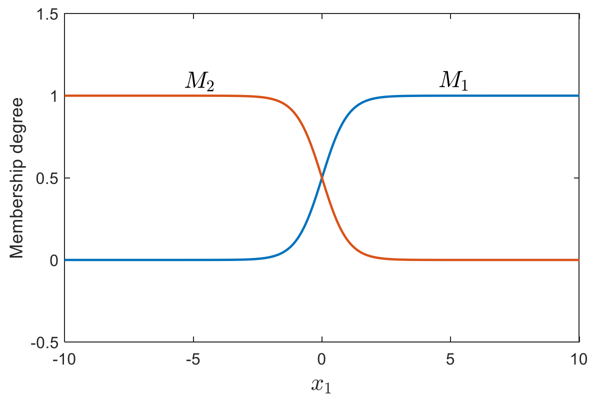

FIGURE 2. Membership functions of the numerical example.

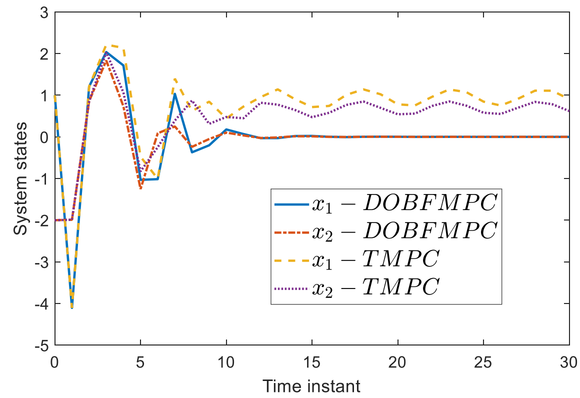

FIGURE 3. System states of the proposed DOBFMPC compared with TMPC.

$$x(k+1) = A_2x(k) + B_2(u(k) + d(k))$$

where

$$A_1 = \begin{bmatrix} -0.5 & 2 \\ -0.1 & 1.1 \end{bmatrix}, A_2 = \begin{bmatrix} -0.19 & 0.5 \\ -0.1 & -0.2 \end{bmatrix}, B_1 = \begin{bmatrix} 4.1 \\ 4.8 \end{bmatrix}$$
$$\text{and } B_2 = \begin{bmatrix} 3 \\ 0.1 \end{bmatrix}.$$

The membership functions are given as

$$M_1(x_1) = \frac{1}{1+\exp(-2x_1)}, \quad M_2(x_1) = 1 - M_1(x_1)$$

which are shown in FIGURE 2. The system input satisfies $|u(k)| \leq 0.8$ and the disturbance $d(k)$ satisfies $d(k) = 0.2(1-e^{-0.25k})$, $k = 1, \ldots, \infty$ where $|d(k)| \leq 0.2$ and $|\Delta d(k)| \leq 0.05$.

The weighted parameters for the RMPC are $Q_0 = diag\{2, 1\}$ and $R_0 = 0.01$. The disturbance region is approximated as $\mathcal{E}(P_d) = \{\Delta d^T P_d \Delta d \leq 1,\ P_d = 400\}$.

Following the algorithm scheme in Section IV Part B, parameters of the proposed approach can be obtained. More details are presented here. By doing a rough line search with an step of ± 0.1 around initial $\tau_0 = 0.5$, scalar $\tau = 0.7$ is obtained by solving the inequalities (16a)-(16c); scalar $\tau = 0.67$ is finally chosen by carrying out same procedure with the step of ± 0.01. Meanwhile, the controller gains are obtained; and the peak error state $\|e_x\|_\infty < \chi^{1/2} = 0.3093$ and the estimation error RPI set $\Omega_d = \{e_d(k)^T Q e_d(k) \leq 1.5282,\ Q = 400\}$ can be determined. Then, the disturbance estimation bound $|e_d| < 0.0618$ and auxiliary control bound $\left\|K_\mu e_x\right\|_\infty < 0.1079$ can be obtained from the computation. Thus, a tightened input constraints $|\bar{u}(k)| \leq 0.8-0.2-0.1079 -0.0618 = 0.4303$ can be found.

The control target is to drive the state from $x(0) = [1, -2]^T$ to the origin. A tube-based MPC (TMPC) developed in [33] is used for comparison and the control performance is shown in FIGURE 3. It can be observed that asymptotic stability can be achieved by the proposed DOBFMPC in the presence of disturbance, while the TMPC cannot achieve satisfactory

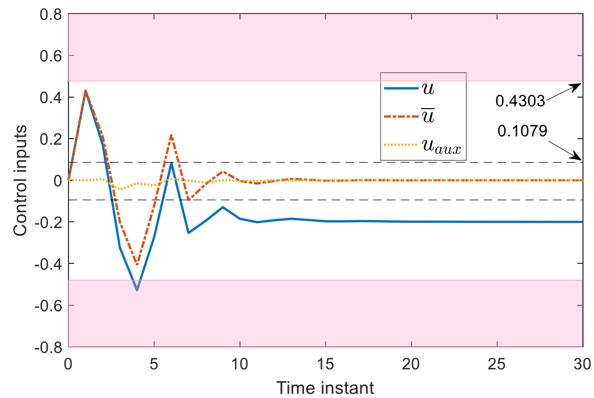

FIGURE 4. Control inputs of the proposed approach (dashed marked: u_{aux} boundary; inner edge of red region: $\bar{u}$ boundary; outer edge of red region: u boundary).

results. The input constraint is not violated during the simulation as shown in FIGURE 4, where the tightened input boundaries are marked with red region and the aux-controller input (abbreviated as u_{aux}) is constrained into the dashed mar- ked region. The disturbance observer estimates the disturbance with a relatively small error as shown in FIGURE 5, where the estimation error is bounded into a dashed marked region. The evolution of system states encircles nominal states with the peak norm whose boundaries are marked with a red region in FIGURE 6.

B. BOILER-TURBINE UNIT

The second simulation is done on a 160MWe oil-fired sub-critical power plant model to show the strength of the proposed DOBFMPC in case of model-plant mismatches. The schematic diagram of the power plant is shown in FIGURE 7. The plant is mainly composed by two parts, i.e. the boiler and turbine. The basic working principle of the power plant is energy conversion. The chemical energy stored in the fossil fuel is transformed into thermal energy of the steam through the combustion and heat transferring in the boiler, the steam is then expanded through the turbine, converting its thermal

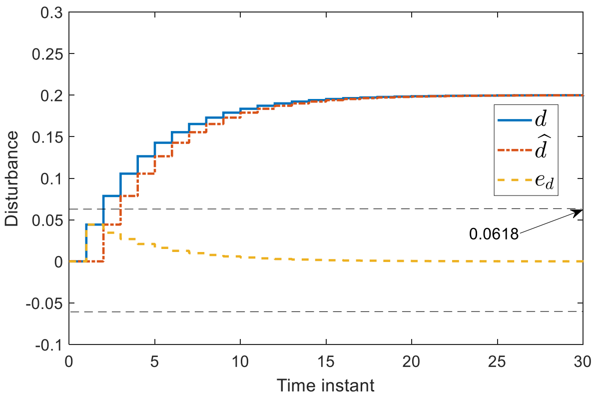

FIGURE 5. **Disturbance and the estimated disturbance (dashed marked: e_d boundary).**

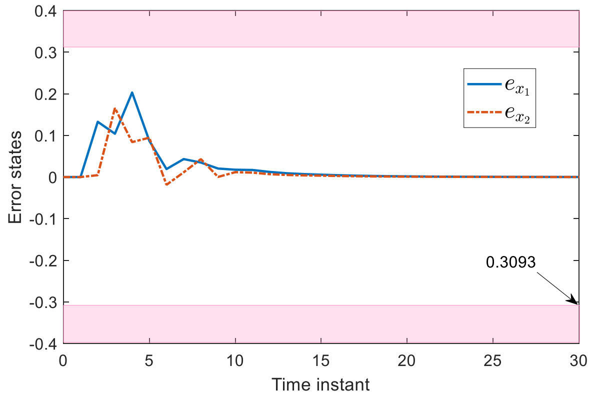

FIGURE 6. **Evolution of error states e_x (inner edge of red region: $\|e_x\|_\infty$).**

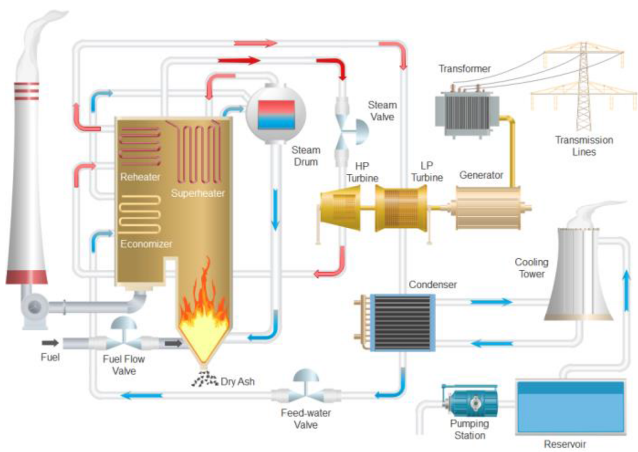

FIGURE 7. **Typical schematic view of boiler-turbine (red arrows: steam flow; blue arrows: water flow; black arrows: fuel flow).**

energy into mechanical rotational energy, finally the energy is transformed into electric power through the coaxially connected turbo-generator.

For the boiler-turbine unit control, the presence of unknown disturbance may cause the deterioration of the tracking performance. In [46], a nonlinear disturbance rejection ability, inherited from a sliding mode disturbance observer, was achieved for the control of the unit with the premise of slow-tracking demand. In recent works [31], disturbance observer and FMPC are designed individually. It was assumed that the disturbances were accurately estimated and then cancelled from true plant to obtain nominal model. The FMPC was designed on the nominal fuzzy model but considering the original input constraints coming from the valve physical limitation. It is obviously to see that the input constraints for the FMPC calculation should be tightened due to that the estimated disturbances are one part of the control compensation. Also, the observation error is not considered through the whole control design. In this section, we develop the proposed DOBFMPC into the control of boiler-turbine unit with the consideration of model-plant mismatch, and the sampling error causing from the selection of the sampling time is further considered.

The primary target of boiler-turbine unit control is to adjust the power output to meet the demand of the grid, meanwhile the steam pressure should be guaranteed within an appropriate range for the safe operation of the plant. During the operation, there are strict constraints due to the physical properties of the actuators.

The mathematical model of the boiler-turbine unit [47] is given as follows:

$$\begin{cases} \dot{P} = -0.0018u_2P^{9/8} + 0.9u_1 - 0.15u_3 \\ \dot{E} = (0.073u_2 - 0.016)P^{9/8} - 0.1E \\ \dot{\rho}_f = (141u_3 - (1.1u_2 - 0.19)P)/85 \end{cases} \tag{25}$$

where state variables P, E, and ρ_f denote drum pressure (kg/cm^2), electric power (MW), and water-steam density (kg/m^3), respectively. The inputs variables u_1, u_2 and u_3 represent the valve opening degree of fuel flow, steam control, and feed-water flow, respectively, which are constrained within the interval [0,1]. Several typical operating points of this boiler-turbine unit are shown in TABLE 1.

Denoting the state variables as $x = [x_1,\ x_2,\ x_3]^T := [P, E\ ,\ \rho_f]^T$ and considering the external disturbances, modeling errors, the dynamics of the boiler-turbine can be rewritten as:

$$\dot{x} = A_bx + B_b(u + d) \tag{26}$$

where d is the lumped disturbance, and

$$A_b = \begin{bmatrix} 0 & 0 & 0 \\ -0.016P^{1/8} & -0.1 & 0 \\ 0.19/85 & 0 & 0 \end{bmatrix},$$

$$B_b = \begin{bmatrix} 0.9 & -0.0018P^{9/8} & -0.15 \\ 0 & 0.073P^{9/8} & 0 \\ 0 & -1.1P/85 & 141/85 \end{bmatrix}.$$

Choosing the steam pressure as the scheduling signal, the membership functions of the fuzzy boiler-turbine system model can be expressed as [47]:

$$\mu_1(P) = \frac{P_{\max} - P}{P_{\max} - P_{\min}},\ \mu_2(P) = \frac{P - P_{\min}}{P_{\max} - P_{\min}} \tag{27}$$

TABLE 1. Typical operating points of the model.

	#1	#2	#3	#4	#5	#6	#7
x_1	75.6	86.4	97.2	108	118.8	129.6	135.4
x_2	15.27	36.65	50.52	66.65	85.06	105.8	127
x_3	492.0	469.2	450.8	427.8	397.9	355.5	276.1
u_1	0.119	0.209	0.271	0.34	0.418	0.502	0.6
u_2	0.380	0.552	0.621	0.69	0.759	0.821	0.895
u_3	0.123	0.256	0.340	0.433	0.543	0.658	0.788

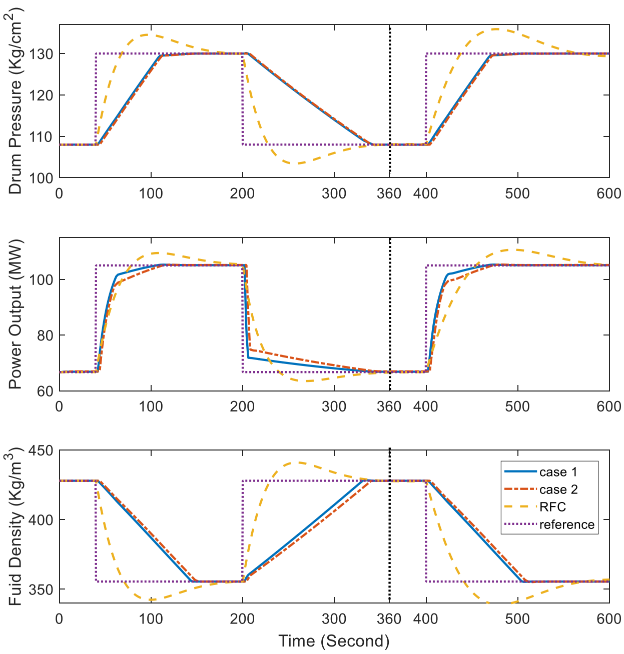

FIGURE 8. Response of the boiler-turbine unit states with different sampling time.

where $P_{\min}$ and $P_{\max}$ are set to be the steam pressure value from the operating conditions #1 and #7 in TABLE 1, respectively.

Replacing the steam pressure P with $P_{\min}$ and $P_{\max}$, respectively, we achieve the final T-S fuzzy model

$$\dot{x} = \sum_{i=1}^{2} \mu_i A_i x + \sum_{i=1}^{2} \mu_i B_i (u + d) \tag{28}$$

Suppose that the plant is operated at initial state $x_s = [108, 66.7, 427.8]^T$, then at $t = 40$s, 200s and 400s, the setpoints change to $[130, 105, 355.5]^T$, $[108, 66.7, 427.8]^T$ and $[130, 105, 355.5]^T$ respectively. After a long operation of the boiler-turbine unit, equipment wear and furnace ash will cause model mismatch. To show the robust performance of the proposed approach, a severe model-plant mismatch is considered in the simulation that at $t = 360$s, all coefficients of the plant model (24) are reduced to 70% of their original values. The parameters of the DOBFMPC are set as:

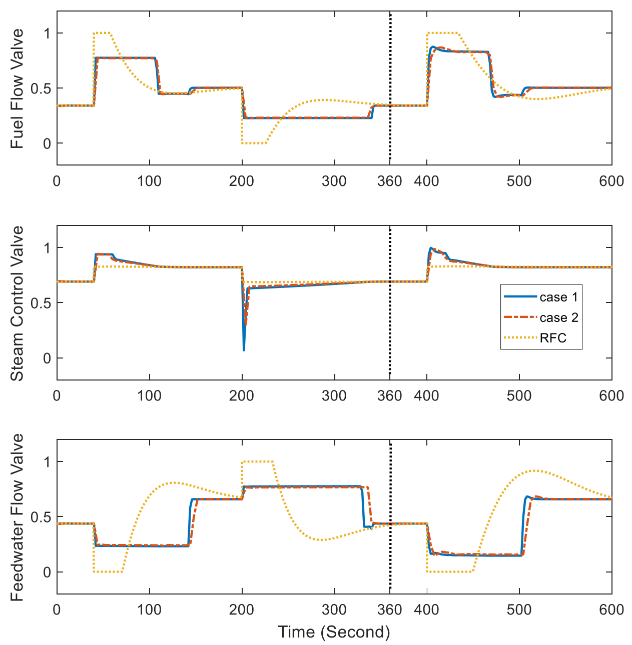

FIGURE 9. Control inputs of the boiler-turbine unit with different sampling time.

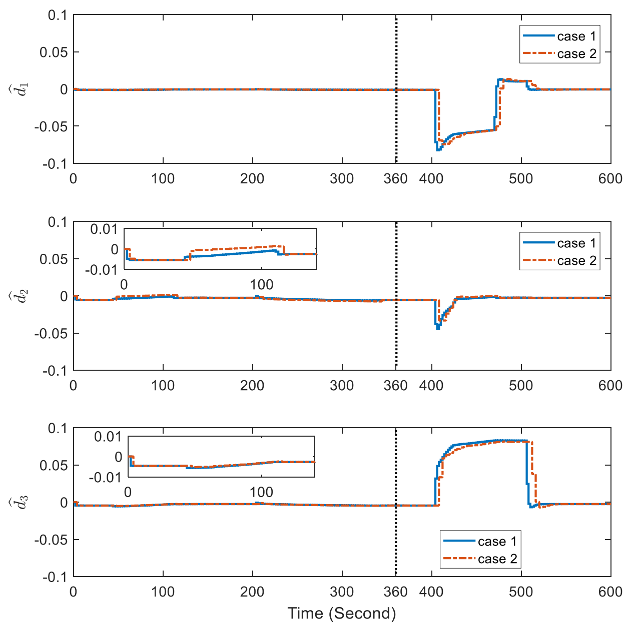

FIGURE 10. Estimated disturbances of the discrete DOB with different sampling time.

$Q_0 = diag\{100, 200, 10\}$, $R_0 = diag\{1, 1, 1\}$ and $|d(k)| \leq [0.1, 0.05, 0.1]^T$. Taking into account the operating environment and the computational burden, the sampling range is suggested to be from 1s to 8s. To show the impact of the sampling time, two DOBFMPCs developed with different sampling times are compared and the results are shown in FIGUREs 8-11.

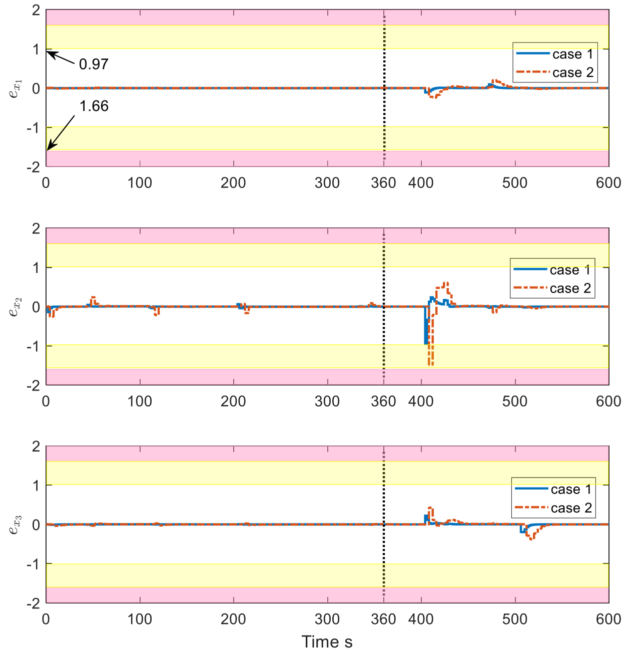

FIGURE 11. State errors from the plant to nominal model with different sampling time (inner edge of yellow region: $\|e_x\|_\infty$ of case 1; inner edge of red region: $\|e_x\|_\infty$ of case 2).

Case 1: Set $T_s = 2s$ and suppose the rate of the disturbances are bounded to $|\Delta d(k)| \leq [0.05,\ 0.01,\ 0.05]^T$. Choosing $\tau = 0.63$, $P_d = diag\{400, 10000, 400\}$ and solving the inequalities (16a)-(16c), the peak error state $\|e_x\|_\infty < \chi^{1/2} = 0.97$ can be obtained. Then, the tightened input constraints $|\bar{u}(k)| < [0.87,\ 0.89,\ 0.86]^T$ can be computed.

Case 2: Set $T_s = 4s$ and suppose the rate of the disturbances are bounded to $|\Delta d(k)| \leq [0.1,\ 0.02,\ 0.1]^T$. Choosing $\tau = 0.63$, $P_d = diag\{100, 2500, 100\}$ and solving the inequalities (16a)-(16c), the peak error state $\|e_x\|_\infty < \chi^{1/2} = 1.66$ can be obtained. Then, the tightened input constraints $|\bar{u}(k)| < [0.85,\ 0.88,\ 0.84]^T$ can be computed.

For a comparison, the robust fuzzy model control (RFC) method presented in [47] is carried out here in which an anti-windup strategy is adopted to prevent the windup caused by the saturation of the actuators.

The system response shown from FIGURE 8 illustrates that the target states can be tracked with no offset even in the presence of severe model-plant mismatch. In spite of the usage of anti-windup strategy, the RFC method performs large overshoot performance, while the proposed method shows the strong ability of dealing with input constraints. Through the simulation verification, without the control input constraints, the RFC exhibits outstanding performance. However, the controller designed in this paper incorporates input constraints in the design process which is also one of the advantages of model predictive control. We can also discover that the proposed approach designed with smaller sampling time has better control performance where the disturbances coming from the fuzzy modeling error and the model-plant mismatch. Also, the different disturbance changing rate in terms of sampling time influences the observer error and further the control performance. For the same step change of the set-point, the control inputs of the two cases shown in FIGURE 9 are different in cases of modeling mismatch and no mismatch, but the output responses of the boiler-turbine system are similar owing to the help of disturbance compensator and the aux-controller. Comparatively, the RPC method obviously shows a greater overshoot output response when modelling mismatch occurs, which can be further observed from the control inputs. It should be stressed that input tightened constraints calculated offline keep the control input satisfying constraints during the whole simulation. The control inputs of the RFC are constrained by the limitation of the actuators.

The estimations of equivalent disturbances are illustrated in FIGURE 10, where the modeling errors shown before $t = 360s$ are small and subsequently, the perturbations caused by model-plant mismatch increase significantly. Thus, model-plant mismatch, which poses a challenge to control performance, cannot be ignored in the field of process control. FIGURE 11 shows that due to the suppression effect of aux-controller, the state errors (e_x) between nominal model states and true plant states are kept within a peak norm bound where the boundaries of case 1 are marked with yellow color region and the red color region for case 2. Furthermore, we can find that case 2 with larger sampling time behaves larger state error boundary than that in case 1. Therefore, reducing the sampling time helps improve the control performance which is in accordance to *Remark 4*.

VI. CONCLUSION

This paper proposes a novel DOBFMPC approach for nonlinear system in the presence of disturbance and various constraints. To reject the unknown disturbance, a fuzzy DOB is developed to estimate the disturbance where the disturbance estimation error is further considered to enhance the robustness by an aux-controller. The fuzzy DOB and the aux-controller are jointly designed, so that the minimization of disturbance positively invariant set can be achieved. Tightened constraints are calculated to guarantee the recursive feasibility of RMPC in an optimal way. Simulation results show that the proposed DOBFMPC strategy can effectively drive the state of the system to the target set-point with satisfactory transient response.

APPENDIX

Proof of Lemma 4: Multiplying $\begin{bmatrix} e(k)^T & \Delta d(k)^T \end{bmatrix}$ and its transpose from both side of (14a), respectively, we have

$$e(k+1)^T P_\mu^+ e(k+1) - \tau e(k)^T P_\mu e(k) < \rho \Delta d(k)^T P_d \Delta d(k) \tag{29}$$

Then denoting $V(k) := e(k)^T P_\mu e(k)$, $W(k) := \tau^{-k+1} V(k)$ and multiplying (29) with τ^{-k}, we have $W(k+1) - W(k) < \tau^{-k} \rho \Delta d(k)^T P_d \Delta d(k)$. Summing the inequality from sampling instant 0 to $k-1$ yields $W(k) - W(0) <$

$\rho \sum_{i=0}^{k-1} \tau^{-i} \Delta d(i)^T P_d \Delta d(i)$. Due to that $\Delta d(i)^T P_d \Delta d(i) \leq 1$ and $e(0) = 0$, we can get $W(k) < \rho \sum_{i=0}^{k-1} \tau^{-i}$. Using $0 < \tau < 1$ and $W(k) = \tau^{-k+1} V(k)$, one has $V(k) < \rho \sum_{i=0}^{k-1} \tau^{k-1-i} = \rho \sum_{i=0}^{k-1} \tau^{i} \leq \rho/(1-\tau)$. Thus, inequality (29) implies

$$e(k)^T P_\mu e(k) \leq \frac{\rho}{1-\tau} \tag{30}$$

Combining (14b) and (14c) into a single inequality matrix and considering $P_d > 0$, one has

$$\begin{bmatrix} (\tau-1)P_\mu + (C_\mu^{cl})^T C_\mu^{cl} & 0 \\ 0 & (\rho-\chi)P_d \end{bmatrix} < 0 \tag{31}$$

Multiplying $\begin{bmatrix} e(k)^T & \Delta d(k)^T \end{bmatrix}$ and its transpose from both side of (31), respectively, one can obtain

$$\begin{aligned} e_x(k)^T e_x(k) < (1-\tau)e(k)^T P_\mu e(k) \\ + (\chi-\rho)\Delta d(k)^T P_d \Delta d(k) \end{aligned} \tag{32}$$

Taking $\Delta d(k)^T P_d \Delta d(k) \leq 1$ into account, inequality (32) can be written as

$$e_x(k)^T e_x(k) < (1-\tau)e(k)^T P_\mu e(k) + (\chi-\rho) \tag{33}$$

Substituting inequality (30) into (33), we have

$$e_x(k)^T e_x(k) < \chi \tag{34}$$

Considering $(\tau-1)P_\mu + (C_\mu^{cl})^T C_\mu^{cl} < 0$, the condition $0 < \tau < 1$ implies $(\tau-1)P_\mu < (\tau-1)P_\mu + (C_\mu^{cl})^T C_\mu^{cl} < 0$ and thus $P_\mu > 0$.

From (14a), it hence holds that

$$\begin{aligned} \left(A_\mu^{cl}\right)^T P_\mu^+ A_\mu^{cl} - P_\mu &< \left(A_\mu^{cl}\right)^T P_\mu^+ A_\mu^{cl} - P_\mu + (1-\tau)P_\mu \\ &= \left(A_\mu^{cl}\right)^T P_\mu^+ A_\mu^{cl} - \tau P_\mu < 0. \end{aligned}$$

In conclusion, A_μ^{cl} is Schur stable with the satisfaction of $V(k) = e(k)^T P_\mu e(k) > 0, e(k)^T \left(\left(A_\mu^{cl}\right)^T P_\mu^+ A_\mu^{cl} - P_\mu\right) e(k) < 0$. Then, the system (13) is ISS.

Thus, we finish the proof.

Proof of Theorem 2: In terms of *Lemma 4*, we have the following proof procedures

1) Following inequality $-G^T X^{-1} G \leq X - G^T - G$ and multiplying $diag\{I, I, I, I, (G_j^T)^{-1}, I, I\}$ and its transpose from both sides of (22a), respectively, then, taking $K_j = Y_j G_j^{-1}$, $L_m = Q^{-1} H_m$ into account, one has

$$\begin{bmatrix} -X_k & 0 & 0 & 0 & A_i + B_i K_j & B_i & 0 \\ * & -Q & 0 & 0 & 0 & Q - QL_m B_i & 0 \\ * & * & -X_k & 0 & 0 & 0 & 0 \\ * & * & * & -Q & 0 & 0 & Q \\ * & * & * & * & -\tau X_i^{-1} & 0 & 0 \\ * & * & * & * & * & -\tau Q & Q - B_i^T L_m^T Q \\ * & * & * & * & * & * & -\rho P_d \end{bmatrix} < 0 \tag{35}$$

Denoting $\mu_i(\nu)$ as μ_i, it holds that

$$\sum_{i=1}^{L}\sum_{j=1}^{L}\sum_{m=1}^{L} \mu_i \mu_j \mu_m \times \begin{bmatrix} -X_k & 0 & 0 & 0 & A_i + B_i K_j & B_i & 0 \\ * & -Q & 0 & 0 & 0 & Q - QL_m B_i & 0 \\ * & * & -X_k & 0 & 0 & 0 & 0 \\ * & * & * & -Q & 0 & 0 & Q \\ * & * & * & * & -\tau X_i^{-1} & 0 & 0 \\ * & * & * & * & * & -\tau Q & Q - B_i^T L_m^T Q \\ * & * & * & * & * & * & -\rho P_d \end{bmatrix} < 0 \tag{36}$$

Denoting $R_i := X_i^{-1}$ and multiplying $diag\{I, Q^{-1}, I, Q^{-1}, I, I, I\}$ from both side of (36), one has

$$\begin{bmatrix} -X_k & 0 & 0 & 0 & A_\mu + B_\mu K_\mu & B_\mu & 0 \\ * & -Q & 0 & 0 & 0 & I_q - L_\mu B_\mu & 0 \\ * & * & -X_k & 0 & 0 & 0 & 0 \\ * & * & * & -Q & 0 & 0 & Q \\ * & * & * & * & -\tau R_\mu & 0 & 0 \\ * & * & * & * & * & -\tau Q & Q - B_\mu^T L_\mu^T Q \\ * & * & * & * & * & * & -\rho P_d \end{bmatrix} < 0$$

Using the Schur complement, the upper inequality can be written as

$$\begin{bmatrix} A_\mu + B_\mu K_\mu & B_\mu & 0 \\ 0 & I_q - L_\mu B_\mu & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_q \end{bmatrix}^T \begin{bmatrix} X_k^{-1} & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & X_k^{-1} & 0 \\ 0 & 0 & 0 & Q \end{bmatrix} \begin{bmatrix} A_\mu + B_\mu K_\mu & B_\mu & 0 \\ 0 & I_q - L_\mu B_\mu & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_q \end{bmatrix} + \begin{bmatrix} -\tau R_\mu & 0 & 0 \\ 0 & -\tau Q & Q - B_\mu^T L_\mu^T Q \\ 0 & 0 & -\rho P_d \end{bmatrix} < 0 \tag{37}$$

Denoting $P_k = \begin{bmatrix} R_k & 0 \\ 0 & Q \end{bmatrix}$ and considering $A_\mu^{cl} = \begin{bmatrix} A_\mu + B_\mu K_\mu & B_\mu \\ 0 & I_q - L_\mu B_\mu \end{bmatrix}$ and $B_\mu^{cl} = \begin{bmatrix} 0 \\ I_q \end{bmatrix}$, we have

$$\begin{bmatrix} (A_\mu^{cl})^T P_k A_\mu^{cl} & 0 \\ 0 & (B_\mu^{cl})^T P_k B_\mu^{cl} \end{bmatrix} + \begin{bmatrix} -\tau P_\mu & (A_\mu^{cl})^T P_k B_\mu^{cl} \\ * & -\rho P_d \end{bmatrix} < 0 \tag{38}$$

Noticing $P_\mu^+ = \sum_{k=1}^{L} \mu_k(\nu^+)P_k$, the inequality (16a) holds.

2) Considering $C_\mu^{cl} = \begin{bmatrix} I_n & 0 \end{bmatrix}$ and substituting $P_\mu = \begin{bmatrix} R_\mu & 0 \\ 0 & Q \end{bmatrix}$ into (14b), we have

$$\begin{bmatrix} (\tau - 1)R_i + I_n & 0 \\ 0 & (\tau - 1)Q \end{bmatrix} < 0 \tag{39}$$

Since $\tau < 1$ and $Q > 0$, the above inequality is equivalent to $(\tau - 1)X_i^{-1} + I_n < 0$. Using the Schur complement, it holds that

$$\begin{bmatrix} (\tau - 1)X_i^{-1} & I_n \\ I_n & -I_n \end{bmatrix} < 0 \tag{40}$$

Multiplying $diag\{X_i, I\}$ from both sides of (40), the inequality (16b) holds.

3) In terms of *Lemma 4*, if (16a), (16b) and (16c) hold, the upper bound of e_x can be found, i.e., $\|e_x\|_\infty < \chi^{1/2}$.

The proof of *Theorem 2* is completed.

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JIANZHONG ZHU received the B.S. degree in automation engineering from the University of Technology of Nanjing, Nanjing, China, in 2004, and the M.S. degree in automation engineering from the University of Aeronautics and Astronautics of Nanjing, Nanjing, in 2007. He is currently pursuing the Ph.D. degree in thermal power engineering with Southeast University, Nanjing.

His research interests include modeling, model predictive control, and disturbance rejection control.

SING KIONG NGUANG (Senior Member, IEEE) received the B.E. (Hons.) and Ph.D. degrees from the Department of Electrical and Computer Engineering, University of Newcastle, Callaghan, Australia. He is currently a Chair Professor with the Department of Electrical and Computer Engineering, The University of Auckland, Auckland, New Zealand. He has published over 300 refereed journal and conference papers on nonlinear control design, nonlinear control systems, nonlinear time-delay systems, nonlinear sampled-data systems, biomedical systems modeling, fuzzy modeling and control, biological systems modeling and control, and food and bio-product processing. He has/had served on the editorial board of a number of international journals. He is the Chief-Editor of the *International Journal of Sensors, Wireless Communications and Control*.