IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received November 17, 2020, accepted November 27, 2020, date of publication December 8, 2020, date of current version December 18, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3043275

Fuzzy Identification of Nonlinear Dynamic System Based on Input Variable Selection and Particle Swarm Optimization Parameter Optimization

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This work was supported by the National Science Foundation of Hebei Province under Grant F2019203505.

ABSTRACT Input variable selection is an essential step in the development of data-driven models. In order to establish a fuzzy model with high identification accuracy for complex nonlinear systems (such as variable load pneumatic loading system) in engineering, a novel fuzzy identification method is proposed, which is based on the selection of important input variables. Firstly, the simplified Two Stage Fuzzy Curves and Surfaces method is used to rank the original input variables according to their significance, and the variables which are most relevant to the output are selected as the input of the T-S fuzzy model. Then, the Fuzzy c-Means clustering algorithm and Particle Swarm Optimization algorithm are used to identify the antecedent parameters, and the Recursive Least Square method is used to identify the consequent parameters. The validity of the proposed fuzzy identification method is verified by two benchmark problems, and the results show that the accuracies of identified models have been improved significantly compared with the other existing models. Finally, the proposed approach is implemented to the practical data of an actual variable load pneumatic loading system, and preponderant trajectory matching performance is achieved.

INDEX TERMS Fuzzy identification, fuzzy c-means, Gaussian function, input variable selection, particle swarm optimization algorithm, T-S fuzzy modeling.

I. INTRODUCTION

With the ever-increasing demands of precise system modeling and comprehensive dynamic description, advanced mathematical models are greatly needed in industrial practice. Input Variable Selection (IVS) is an essential step in the development of data-driven models. For an unknown system, there may be numerous mixed input and state variables. As the input-output relationship is unknown in advance, any element may affect the output or can be considered as an input. The method of IVS is usually used to identify the most important input from a large set of candidate input variables, where the importance is defined as having the maximum correlation with the output. Consequently, the optimal input variable set contained the fewest input variables required to describe the behavior of the output variable, with minimum redundancy [1]. In addition, reducing the number of model input variables is also important for minimizing the calculation requirements, solving the dimension disaster, reducing the output variability caused by the local minimum on the error surface and clarifying the physical behavior of the system. The T-S fuzzy model identification based on experimental data is considered as an efficacious method of mathematical modeling of practical systems, which has been extensively applied in chemical, petroleum and electrical industries. Recent researches in this field have largely focused on optimization algorithms that are used to train (or calibrate) the models. However, the research on the selection of the suitable inputs needed for optimal model performance is still widely ignored. Generally speaking, fuzzy modeling is always based on fixed input variables determined by experience. Therefore, it is obvious that a more robust IVS method is needed for fuzzy modeling of nonlinear systems, which means the method not only does not rely on

The associate editor coordinating the review of this manuscript and approving it for publication was Qi Zhou.

prior knowledge or assumptions of the system, but also can characterize the nonlinear and interdependent relationship between candidate inputs.

In recent years, the development and discussion of IVS algorithms mainly focus on classification, machine learning and many other fields where artificial neural network (ANN) models are applied. Comprehensive discussions on the taxonomy of such IVS methods can be found in [1]–[4]. IVS algorithms can be broadly classified into two categories: model-free algorithms and model-based algorithms. The difference between them depends on whether the IVS is treated as preprocess or interweaving with a learning task [2]. Model-based IVS algorithms take IVS task as a part of model architecture optimization or directly incorporate IVS algorithm into model training algorithm, which mainly includes wrapper and embedded algorithms [3], such as genetic programming, input omission and combined neural path strength of ANN traffic prediction model. The main characteristic of these wrapper methods and embedded algorithms is large computation, which are usually applied in the case of relatively small number of training samples and more candidate input variables. Considering application, the scope is narrowed to model free filtering type IVS algorithm (also known as filter algorithm). In contrast to model-based IVS, filter algorithms offer a fast, model-free approach for variable selection, meanwhile, these algorithms are well suitable for some applications where independence from a specific ANN architecture is required [4]. And the ability to identify an optimal set of input variables prior to training an ANN eliminates the computational burden associated with training and model selection, which can reduce the overall effort of ANN development. At present, the filter algorithms based on the information theory measure, such as mutual information, partial mutual information, conditional mutual information, and so on, have been widely used in the actual ANN system [5].

For the IVS methods in the fuzzy identification of nonlinear system, there are few literatures. A method of Two Stage Fuzzy Curves and Surfaces (TSFCS) is proposed for the input structure identification of nonlinear systems [6]. The first stage fuzzy curves are local averages of the output for each input, and the second stage fuzzy curves are local estimates of the variance. The second stage fuzzy surfaces are the two-dimensional analogs of the first stage fuzzy curves. In [7], a control structure based on robust model is proposed, which includes the dynamic model of system fuzzy logic and robust fuzzy control rules. Firstly, the fuzzy logic model is systematically constructed from the input-output data. In order to identify the significant input variables among a finite number of candidates, the output clusters are projected onto the space of each of the input candidates. Then, a robust fuzzy controller is designed by using the sliding mode control theory. Finally, the method is applied to the trajectory tracking control of the four degree of freedom manipulator, and by comparing with the high gain PID controller, the method has good tracking performance. In [8], the improved modified mountain clustering is combined with Structure Tree (ST) to establish a dynamic system model. In order to find the best input candidate of Takagi-Sugeno-Kang (TSK) fuzzy identification model, a ST is constructed, whose nodes correspond to various possible combinations of candidate variables. Two search algorithms, loop and genetic algorithm, are used to search the best node of the ST, that is, the best subset of input variables, so that the final input variables of TSK fuzzy model can be determined.

The Takagi-Sugeno (T-S) fuzzy model is one of the most important fuzzy models [9]. For large-scale nonlinear systems, event-based decentralized adaptive fuzzy output-feedback finite-time control problems were investigated. Fuzzy logic system was used to model the unknown auxiliary functions, and then a state observer was established to estimate the unmeasured state [10]. For a class of completely unknown nonlinear systems with considering fixed-time tracking control, fuzzy logic systems are utilized to model these unknown nonlinear systems [11]. In general, the construction of T-S fuzzy model includes two aspects: structure identification and parameter estimation. The structure identification can be divided into structure identification I and structure identification II. Structural identification I includes the selection and determination of input variables, and structure identification II includes the determination of number of fuzzy rules and the division of fuzzy space and so on. In the construction of fuzzy model, the proportion of the structure identification I, structure identification II and parameter identification is 100:10:1 [12]. It can be seen that as a part of structure identification, IVS is very important to improve the model accuracy. For an unknown system which contains many mixed input and state variables, if all variables are considered, the number of fuzzy rules will increase exponentially. For the identification problem based on fuzzy rules, selecting important input variables can solve the contradiction between improving model accuracy and reducing the size of rule base. In the past researches of fuzzy identification, most of the research results were for the case that the input variables being known or determined for a nonlinear dynamic system. Although the selection of input variables is relatively important in the fuzzy structure identification, there is still no research result, which motives us to carry out this task.

On the other hand, parameter identification includes antecedent parameter identification and consequent parameter identification. In antecedent parameter identification, the division of fuzzy antecedent is to determine the premise membership function. Membership function is a way to describe the membership of a fuzzy subset, which plays an important role in determining the fuzzy transformation. The commonly used membership functions include Gaussian function, triangle function and fuzzy clustering type function. In practical application, the shape of Gaussian membership function curve is more suitable to describe the fuzzy subset. However, when the bell Gaussian function is chosen as the fuzzy logic function, the two parameters of its center and width cannot be determined automatically. Fortunately, this problem can be solved by using appropriate optimization algorithm.

Fuzzy c-means clustering (FCM) algorithm [13] is a method for automatically classifying sample points, which can determine the cluster center by automatic search. In the literatures, it is mostly used for fuzzy space division and antecedent parameter estimation [14]. However, the FCM algorithm is easy to converge to the local minimum of the objective function, which means different initial values may lead to different results. In order to solve this problem, many scholars first use fuzzy c-means clustering algorithm to initially determine the rough clustering center, and then use some optimization algorithms with global search ability to further fine tune, such as genetic algorithm (GA) [15], differential evolution algorithm (DEA) [16] and particle swarm optimization (PSO) [17], which can effectively improve the identification accuracy. But, overcomplicated iterative optimization algorithm will increase the complexity of the model and the amount of calculation, and affect the convergence speed, which is very unfavorable for the on-line identification and real-time control of actual systems.

Based on the above analysis, for the fuzzy identification of nonlinear dynamic systems, a novel fuzzy identification method considering the selection of important input variables is proposed in this paper. First of all, the IVS method of simplified TSFCS is used to quickly select important input variables from a large number of optional input variables. With less input variables, higher identification accuracy can be obtained, which can effectively reduce the complexity of the model and improve the convergence speed. Then, the FCM algorithm and PSO algorithm are combined to optimize the center and width of Gaussian membership function. Thus, the premise parameters of the T-S fuzzy model are determined. Considering the fact that the FCM algorithm is efficient, intuitive and easy to implement and PSO algorithm has the advantages of simple principle, wide universality and fast convergence speed compared with other optimization algorithms, the combination of the two algorithms and Gaussian membership function can avoid complex iterative optimization algorithm and further improve the identification accuracy of the model. Finally, the recursive least square (RLS) is used to identify the conclusion parameters of the fuzzy model. The research results of this paper are very instructive to solve the contradiction between the number of rules and the accuracy of model in the fuzzy identification of nonlinear systems. In summary, the innovations of this article are as follows:

(1) This paper applies the Two Stage Fuzzy Curves and Surfaces variable selection method to the fuzzy identification system for the first time.

(2) The FCM algorithm and PSO algorithm are used to optimize the two parameters of the Gaussian function to further improve the accuracy of fuzzy modeling.

This paper is organized as follows. The Section 2 introduces the T-S fuzzy model related intelligent optimization algorithm. In Section 3, the fuzzy identification method which combines the improved IVS algorithm with parameter optimization is discussed in detail. In Section 4, the proposed method is verified by two international standard cases and applied to approximate the dynamic characteristics of an actual variable load pneumatic loading system. Finally, the conclusion is given at the end of the paper.

II. PRELIMINARIES

A. T-S FUZZY MODEL

The T-S fuzzy model makes it possible to approximate the nonlinear system into several locally linear subsystems. T-S fuzzy model is a rule-based model in which the preconditions of rules are fuzzy variables and the conclusion is a linear function of input and output. It is based on local linearity and achieves global nonlinearity through fuzzy reasoning. Let S be a set of N input-output data pairs $(x_1, x_2, \ldots, x_r, y)$, T-S fuzzy model is generally defined as:

$$\begin{aligned} R_i: \ & If\ x_1\ is\ A_1^i,\ and \ldots and,\ x_r\ is\ A_r^i \\ & Then\ y^i = p_0^i + p_1^i x_1 + p_2^i x_2 + \ldots + p_r^i x_r \end{aligned} \tag{1}$$

where R_i is the i-th fuzzy rule, $i = 1, 2, \ldots, c$; c is the number of fuzzy rule; $\boldsymbol{p}^i = [p_0^i, p_1^i, p_2^i, \ldots, p_r^i] \in R^{r+1}$ are the polynomial coefficients that form the consequent parameters of the i-th fuzzy rule; $\boldsymbol{x} = [x_1, x_2, \ldots, x_r]$ is the input vector of the fuzzy model; y^i is local output variable of i-th fuzzy rule. $A_j^i (j = 1, 2, \ldots, r)$ is the j-th fuzzy set of the i-th rule; $\mu_j^i(x_j)$ is the fuzzy membership grade of x_j belonging to fuzzy set A_j^i, which is usually determined by a bell-shaped Gaussian membership function as

$$\mu_j^i(x_j) = exp(-(\frac{x_j - c_j^i}{\rho_j^i})^2) \tag{2}$$

where c_j^i and ρ_j^i represent the center and width of the fuzzy set in the i-th rule, respectively.

Each fuzzy rule has a matching degree, which represents the contribution of i-th rule to the total T-S fuzzy model:

$$\begin{aligned} \omega^i &= \mu_1^i(x_1) \times \mu_2^i(x_2) \times \cdots \times \mu_r^i(x_r) \\ &= \bigcap_{j=1}^{r} \mu_j^i(x_j) \end{aligned} \tag{3}$$

The global estimated output of T-S model is a weighted average of the others output of local models according to the expression:

$$\hat{y} = \sum_{i=1}^{c} \bar{\omega}^i y^i \tag{4}$$

where $\bar{\omega}^i$ is validity function of i-th rule, and

$$\bar{\omega}^i = \omega^i / \sum_{j=1}^{c} \omega^j \tag{5}$$

where, $\forall\ i = 1, \ldots, c,\ 0 \leq \bar{\omega}^i \leq 1$ and $\sum_{i=1}^{c} \bar{\omega}^i = 1$.

B. FUZZY C-MEANS CLUSTERING ALGORITHM

The FCM algorithm can be expressed as minimizing the following objective function:

$$J_m(U,V)=\sum_{k=1}^{N}\sum_{i=1}^{c}(u_{ik})^m(d_{ik})^2 \quad (6)$$

satisfying

$$\sum_{i=1}^{c}u_{ik}=1,\ 1\le k\le N,$$
$$u_{ik}\ge 0,\ 1\le k\le N,\quad i=1,\ldots,c$$

where u_{ik} is the fuzzy membership degree of the k-th data vector $\boldsymbol{x}_k=(x_{k1},x_{k2},\ldots,x_{kr})$ belonging to the i-th clustering center $\boldsymbol{v}_i=(v_{i1},v_{i2},\ldots,v_{ir})$. N is the total number of eigenvectors and c is the number of the cluster center. $m>1$ is weight index of membership functions. If m is too small or too large, the identification accuracy will be influenced. In practice, m is usually chosen as 2, $m=2$. U is a fuzzy partition matrix, which contains the membership degree of each eigenvector for each cluster, and $V=(\boldsymbol{v}_1,\boldsymbol{v}_2,\ldots,\boldsymbol{v}_c)^T$ is the center matrix of clustering, and $\boldsymbol{v}_i$ is calculated by the following equation:

$$\boldsymbol{v}_i=\sum_{k=1}^{N}(u_{ik})^m\boldsymbol{x}_k/\sum_{k=1}^{N}(u_{ik})^m,\quad i=1,\ldots,c \quad (7)$$

The fuzzy membership function can be obtained by the following formula:

$$u_{ik}=1/\sum_{j=1}^{c}(\frac{d_{ik}}{d_{jk}})^{2/(m-1)} \quad (8)$$
$$d_{ik}=\|\boldsymbol{x}_k-\boldsymbol{v}_i\|\ge 0,\ \forall i\ and\ k$$
$$If\ d_{ik}=0,\ then\ u_{ik}=1,\ u_{jk}=0,\ j\neq i \quad (9)$$

The initial value of the cluster center matrix V is given randomly, and the fuzzy partition matrix U is calculated according to (8) for all the eigenvectors. The initialization V of is obtained by randomly selecting the eigenvalues of each cluster center $\boldsymbol{v}_i$, which should be within the set of the listed eigen data. The stop condition is achieved by setting ε. Offline calculation method is as follows:

(1) Random number generator is used to give the initial value to the clustering center matrix V, and the clustering center was recorded, and set $l=0$;

(2) The initial value of the fuzzy partition matrix $U^{(l=0)}$ is calculated by using (8) and (9);

(3) Increase $l=l+1$, and use (7) to update cluster center V;

(4) Equations (8) and (9) are used to renewal the fuzzy partition matrix $U^{(l)}$;

(5) If $\|U^{(l)}-U^{(l-1)}\|<\varepsilon$, stop, otherwise repeat steps (3)-(5).

C. PSO ALGORITHM

The PSO algorithm proposed by Kennedy *et al.* is a heuristic global optimization algorithm, which has the advantages of evolutionary computation and swarm intelligence.

PSO algorithm can be described as: let particles search in D-dimensional space, and the number of particles is N. The position of k-th particle is $B_k=(b_{k1},b_{k2},\ldots,b_{kD})$, the velocity of the particle is $V_k=(v_{k1},v_{k2},\ldots,v_{kD})$, each particle is a solution to the optimization problem, and the particle finds a new solution by constantly changing its position and speed. The optimal solution of the k-th particle searched is $P_k=(p_{k1},p_{k2},\ldots,p_{kD})$, the optimal position experienced by the whole group is $P_g=(p_{g1},p_{g2},\ldots,p_{gD})$. The velocity and position of each particle varies in line with (10) and (11):

$$v_{kd}(t+1)=\omega v_{kd}(t)+c_1r_1(p_{kd}(t)-b_{kd}(t))+c_2r_2(p_{gd}(t)-b_{kd}(t)) \quad (10)$$
$$b_{kd}(t+1)=b_{kd}(t)+v_{kd}(t+1) \quad (11)$$

where r_1 and r_2 are random numbers between [0, 1]; c_1 and c_2 are normal numbers, which are called accelerators; w is the inertia weight. The range of velocity and position variation in d-dimension of each particle is $[-v_{d,max},v_{d,max}]$ and $[-x_{d,max},x_{d,max}]$. If the maximum velocity of the particle, $v_{d,max}$, is too high, it might cause the particle to fly through the best solution; if the maximum velocity is too small and make the search speed too slow, it may lead to fall into local optimal solution. Inertia weight w can well control the search range of particles. When w is large, particles are searched in a wide range. When w is small, particles are excavated in a small range.

III. THE PROPOSED T-S FUZZY MODELING APPROACH

The T-S fuzzy modeling method proposed in this paper adopts a simple and reasonable IVS algorithm to determine the system inputs, instead of the traditional method of determining system input variables empirically. From the aspects of the number of input variables and the number of fuzzy rules, the influence of the selection of important input variables on the identification accuracy of T-S fuzzy model is studied in detail.

A. INPUT VARIABLE SELECTION BASED ON TWO STAGE FUZZY CURVES AND SURFACES

The IVS method of TSFCS in [6] is more suitable to deal with the situation that the input variables are interdependent, which can be used to automatically and quickly identify the important input variables for a system with a large number of inputs. However, because the importance index of input variables is obtained by interpolation calculation, the algorithm has a certain error. In this paper, Considering that the correlation between input variables is relatively small, which is the T-S fuzzy model benchmark problem, the IVS method of TSFCS is simplified as a Two Stage Fuzzy Curves (TSFC) method, which can give the weight of the correlation degree between each input variable and output. On this basis, the importance index of all input variables will be obtained. Important input variables can be selected according to input variable indicators. The detailed algorithm is shown in Fig. 1.

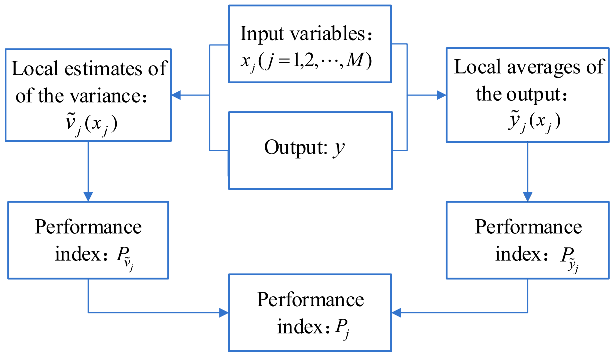


FIGURE 1. The flow diagram of the TSFC algorithm.

1) THE FIRST STAGE FUZZY CURVES

The first stage fuzzy curves are based on a simple idea: the better the approximation effect of the output, the more important the inputs. Suppose that the fuzzy system has M possible input variables: $x_1, x_2, \ldots, x_M$, one output y, and N pairs of input and output data $(x_{k1}, x_{k2}, \ldots, x_{kM}, y_k)$.

First, for each input $x_i (i = 1, 2, \ldots, M)$, a Gaussian membership function $\mu_{ki}(x_i)$ in $x_i - y$ space is defined as follows:

$$\mu_{ki}(x_i) = exp(-(\frac{x_{ki} - x_i}{b_i})^2),\ k = 1, 2, \ldots, N \tag{12}$$

where (x_{ki}, y_k) presents a data point in $x_i - y$ space, b_i is the width of Gaussian function, which is often taken as

$$b_i = 0.2 * (max_{1\le k\le N}(x_{ki}) - min_{1\le k\le N}(x_{ki})) \tag{13}$$

Then, using (12), we construct a function $\tilde{y}_i(x_i)$ of each input x_i, which is a fuzzy curve:

$$\tilde{y}_i(x_i) = \frac{\sum_{k=1}^{N}[y_k \cdot \mu_{ki}(x_i)]}{\sum_{k=1}^{N}\mu_{ki}(x_i)} \tag{14}$$

The fuzzy curve $\tilde{y}_i$ can be viewed as weighted local averages of y_k along x_i axis, and the size of the local neighborhood is determined by b_i.

If the degree of correlation between the variable x_i and the output y is higher than that between x_j and y, then $\forall k$, the value of $\tilde{y}_i(x_{ki})$ will be closer to the output value of y_k than $\tilde{y}_j(x_{kj})$. So we define a performance index $P_{\tilde{y}_i}$ for the candidate input variable x_i as

$$P_{\tilde{y}_i} = \frac{1}{Nv_y}\sum_{k=1}^{N}(\tilde{y}_i(x_{ki}) - y_k)^2 \tag{15}$$

where $v_y = \frac{1}{N}\sum_{k=1}^{N}(y_k - \bar{y})^2$ is the variance of $y_1, y_2, \ldots, y_N$, $\bar{y}$ is the mean of $y_1, y_2, \ldots, y_N$. The smaller $P_{\tilde{y}_i}$ is, the more important x_i is to y. Hence, an ascending sequence of the performance index functions $P_{\tilde{y}_i}$ $(i = 1, 2, \ldots, M)$ gives a list of the variables x_i to figure out the more important variables.

2) THE SECOND STAGE FUZZY CURVES

If $P_{\tilde{y}_i}$ is equal to $P_{\tilde{y}_j}$, for $i \neq j$, the important input variables cannot be identified. Therefore, the second stage fuzzy curve is employed to solve these problems.

On the basis of the first stage fuzzy curves, the second stage fuzzy curves can be given as follow:

$$\tilde{v}_i(x_i) = \frac{\sum_{k=1}^{N}[(\tilde{y}_i(x_i) - y_k)^2 \cdot \mu_{ki}(x_i)]}{\sum_{k=1}^{N}\mu_{ki}(x_i)} \tag{16}$$

If $\tilde{v}_i(x_{ki})$ is very difference from v_y, $\forall k$, then it means the input x_i is very important. On the contrary, if there is always $\tilde{v}_i(x_k i) \approx v_y$, the correlation between x_i and the output y is very low. The second performance index function $P_{\tilde{v}_i}$ describing the importance of variables can be defined as follow:

$$P_{\tilde{v}_i} = \frac{1}{N \cdot (v_y)^2}\sum_{k=1}^{N}(\tilde{v}_i(x_k i) - v_y)^2 \tag{17}$$

In contrast to $P_{\tilde{y}_i}$ in (15), the larger $P_{\tilde{v}_i}$ is, the more important x_i is. Combining $P_{\tilde{y}_i}$ and $P_{\tilde{v}_i}$ into a single performance index P_i as follow:

$$P_i = \frac{P_{\tilde{y}_i}}{1 + P_{\tilde{v}_i}} \tag{18}$$

This results in a range $0 < P_i < 1$, and the smaller P_i is, the more important the corresponding variable x_i is. An ascending sequence of the performance indexes P_i will give a list of the variables x_i sorted by their importance.

B. THE PROPOSED T-S FUZZY MODELING APPROACH

In this section, a new T-S fuzzy modeling method based on IVS is proposed, which is composed of three parts: (1) premise structure identification based on selection of important input variables. The IVS method of TSFC is adopted to determine significant input variables of T-S fuzzy model; (2) premise parameters identification based on FCM and PSO. FCM clustering algorithm and PSO algorithm are combined with Gaussian function to identify the antecedent parameters. The clustering center obtained by FCM algorithm is treated as the center of Gaussian membership function, which makes up for the defect that the center of Gaussian membership function cannot be determined automatically. On this basis, PSO algorithm is served to optimize the width of Gaussian function to obtain the final antecedent parameters; (3) conclusion parameters identification based on RLS algorithm. RLS is used for identification of the consequent parameters. The fuzzy model identification approach proposed is illustrated in Fig. 2. The specific identification steps are as follows:

Step 1: Selection of important input variables

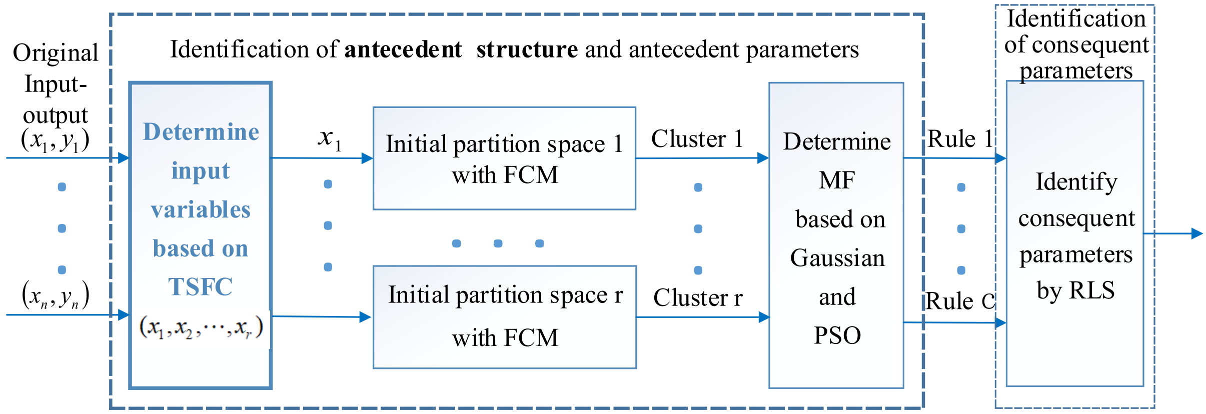


FIGURE 2. T-S fuzzy model identification approach based on selecting important input variables.

Using the simplified TSFC method, the first stage fuzzy curve performance index $P_{\bar{y}_i}$ and the second stage fuzzy curve performance index $P_{\tilde{y}_i}$ of the candidate input variables x_i are calculated by (15) and (17) respectively, and then the composite performance index function P_i is calculated by (18) and ranked in ascending order. The list of variables x_i sorted by importance can be obtained, in the light of which the important input variables are selected and the number of input variables is determined.

Step 2: Calculating the fuzzy c-means clustering center

First, the initial fuzzy clustering number c is determined, and the initial value of the clustering center matrix V is assigned by the random number generator. According to (7) and (8), the clustering center v_i and the fuzzy membership function matrix U are calculated respectively. According to the cycle stop condition ε, the final clustering center v_i is obtained as the center of Gaussian function.

Step 3: Optimizing the width of Gaussian membership function by PSO algorithm

Under the condition that the center of the Gaussian function remains unchanged, PSO algorithm is used to optimize the width of Gaussian function ρ_j^i. The learning factors $c1$, $c2$ are both set as 2, and the inertia weight ω is updated by the following formula

$$\omega = \omega_{min} + DT \cdot \frac{\omega_{max} - \omega_{min}}{MaxDT} \tag{19}$$

where DT is the number of iterations. Let $MaxDT = 100$ is the maximum number of iterations, and $\omega_{min} = 0.4$, $\omega_{max} = 0.9$. The most appropriate modeling width is obtained after optimization.

Step 4: Obtaining the antecedent membership function $\mu_j^i(x_j)$

In this step, the parameters (center and width) of Gauss membership function are identified. The center of the Gaussian function $c_j^i = v_{ij}$ is taken as the clustering center $\mathbf{v}_i = (v_{i1}, v_{i2}, \ldots, v_{ir})$ calculated by step 2; the appropriate width of the Gaussian function ρ_j^i is optimized by step 3, which are substituted into (2) to obtain the antecedent membership function $\mu_j^i(x_j)$.

Step 5: Identification of consequent parameters by RLS

From (4), the following formula can be obtained

$$\begin{aligned} \hat{y}_k &= \sum_{i=1}^{c} \bar{\omega}_k^i \cdot (p_0^i + p_1^i x_{k1} + p_2^i x_{k2} + \ldots + p_r^i x_{kr}) \\ &= [\bar{\omega}_k^1 \ \bar{\omega}_k^1 x_{k1} \ \ldots \ \bar{\omega}_k^1 x_{kr} \ \ldots \ \bar{\omega}_k^c \ \bar{\omega}_k^c x_{k1} \ \ldots \ \bar{\omega}_k^c x_{kr}] \\ &\quad \times [p_0^1 \ p_1^1 \ \ldots \ p_r^1 \ \ldots \ p_0^c \ p_1^c \ \ldots \ p_r^c]^T \end{aligned} \tag{20}$$

Substituting N pairs of input and output data into (20), we get a matrix equation

$$Y = XP \tag{21}$$

where $P = [p_0^1 \ p_1^1 \ \ldots \ p_r^1 \ \ldots \ p_0^c \ p_1^c \ \ldots \ p_r^c]^T$ is the $L = (r+1)c$ dimensional consequent parameter vector; r is number of input variables and c is fuzzy rule number. X and Y are matrices of $N \times L$ and $N \times 1$, X_k is the k-th row vector of X, and y_k is the k-th component of Y; $P^* = (X^T X)^{-1} X^T Y$ is least square estimation of P. In order to iteratively optimize the consequent parameter matrix P and avoid matrix inverse, then the recursive algorithm is:

$$P_{k+1} = P_k + \frac{S_k \cdot X_{k+1}^T \cdot (y_{k+1} - X_{k+1} \cdot P_k)}{1 + X_{k+1} \cdot S_k \cdot X_{k+1}^T} \tag{22}$$

$$S_{k+1} = S_k - \frac{S_k \cdot X_{k+1}^T \cdot X_{k+1} \cdot S_k)}{1 + X_{k+1} \cdot S_k \cdot X_{k+1}^T} \tag{23}$$

where $K = 0, 1, 2, \ldots, N-1$, S_k is the matrix of $L \times L$.

Step 6: Calculate the performance index MSE

Initial condition is: $P_0 = 0$, $S_0 = \alpha I$. α is always going to be more than 10,000. I is the identity matrix of $L \times L$. By using (22) and (23), the optimal conclusion parameter and the minimum mean square error MSE in the sense of error square are obtained:

$$MSE = \sum_{k=1}^{N} (y_k - \hat{y}_k)^2 / N \tag{24}$$

where $\hat{y}_k$ is the predicted output and y_k is the target output. In the follow experiments, the prediction error of the fuzzy model is given by series $e_k = y_k - \hat{y}_k$.

If the MSE meets the identification accuracy, the identification algorithm ends; otherwise, add c and go to Step 2.

IV. EXPERIMENTS AND APPLICATIONS

In this paper, two well-known simulation examples and a practical application system are cited to confirm that the performance of the proposed identification method is superior to some previous methods, which mainly includes the prediction performance and generalization of the model. In the simulation example, the importance of selecting important input variables is proved by selecting different number of input variables. The prediction performance of the model in the simulation example is testified by comparing with other methods in the literature. In order to verify the generalization of the model, the data sample set is divided into two parts: training and testing. The training data are used to build the fuzzy model, and the testing data is used to check the generalization of the model.

A. MACKEY-GLASS CHAOTIC SYSTEM

The Mackey-Glass chaotic differential delay equation in [18] is recognized as a benchmark problem, which is widely used to compare the learning and generalization capabilities of different models. The Mackey-Glass system is generated by the following differential equation:

$$\frac{dx(t)}{dt} = \frac{0.2x(t-17)}{1+x^{10}(t-17)} \tag{25}$$

The purpose of the Mackey-Glass chaotic time series prediction problem is to predict the value of $x(t+1)$ using the past values. 1000 datasets obtained from the Mackey-Glass chaotic time series are used to construct the T-S fuzzy model, where $x(0) = 1.2$ and $0.42 \leq x(t) \leq 1.31$.

1) INPUT VARIABLE SELECTION BASED ON TWO STAGE FUZZY CURVES AND SURFACES

In the existing literatures, the past values which have been chosen as input variables are $x(t-1)$, $x(t-2)$, $x(t-3)$, $x(t-4)$, $x(t-5)$ and $x(t-6)$ in most cases. In this paper, we use the IVS method of TSFC in Section 3 to compare the importance of input variables and select input variables. Letting

$$x_i = x(t-i), \quad i = 1, 2, \ldots, 18$$
$$y = x(t+1)$$

By calculating the performance index P_i of every variable x_i, we can get a list of the variables according to their significance (see TABLE 1). The smaller P_i is, the more important is, so the six variables: $x(t-1)$, $x(t-2)$, $x(t-3)$, $x(t-4)$, $x(t-5)$ and $x(t-18)$ are most important.

TABLE 1. List of performance index for Mackey-Glass chaotic system.

x_i	x_1	x_2	x_3	x_4	x_5	x_{18}
input	$x(t-1)$	$x(t-2)$	$x(t-3)$	$x(t-4)$	$x(t-5)$	$x(t-18)$
P_i	0.0611	0.1164	0.1967	0.3003	0.4203	0.4960

2) PARAMETER IDENTIFICATION BASED ON FUZZY C-MEANS CLUSTERING ALGORITHM AND PARTICLE SWARM OPTIMIZATION ALGORITHM

In order to check the robustness of the fuzzy model, 1000 sets of data are divided into two parts, and the first 500 data pairs are used to build the model, the remaining 500 data pairs are used to test the model. Here, the variables in Table 1 are selected as the input of the T-S fuzzy model and the number of fuzzy rules c is set as 2. The performance of the fuzzy model is shown in Fig. 3, where Fig.3 (a) and Fig.3 (c) exhibit the original output and the predicted output of the model, and Fig.3 (b) and Fig.3 (d) depict the errors.

Table 2 lists the model identification results in two different cases of selecting conventional input variables empirically and selecting important input variables by TSFC algorithm. For the former, the errors of training and testing are 3.6048×10^{-5} and 3.5941×10^{-5}, respectively. However, the errors of 1.2329×10^{-6} and 1.2424×10^{-6} for training and testing are achieved with the same number of variables for the latter. The comparison results show that with the same number of input variables, the identification accuracy of the fuzzy model considering the selection of important input variables is much better than that of the model with empirical input variables. The application of TSFC method effectively improves the performance of the model.

At the same time, the performance obtained by using the method in this paper and the obtained by using the method in existing literatures are shown in Table 3. It can be seen from Table 3 that when the number of fuzzy rules c is small ($c = 2$), the model performance obtained by using the proposed method are much better than the ones obtained by using other methods in the literatures.

B. BOX-JENKINS SYSTEM

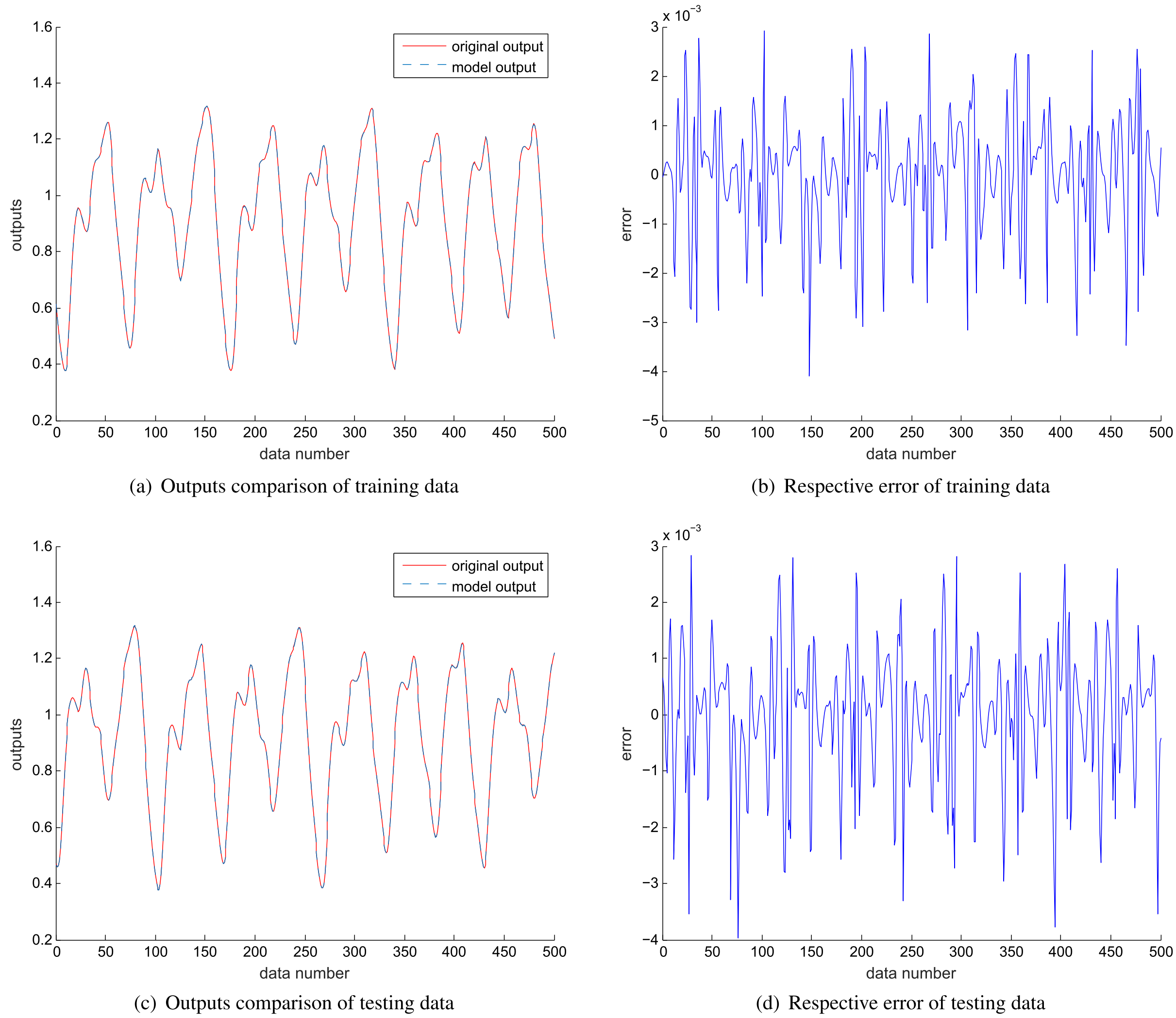
In this section, we use the Box-Jenkins data set [25], which consists of 296 input-output measurements of a gas-furnace process. At each sampling time k, the input $x(k)$ of this process is the gas flow rate and the output $y(k)$ is the output CO_2 concentration.

1) INPUT VARIABLE SELECTION BASED ON TWO STAGE FUZZY CURVES AND SURFACES

According to the method of TSFC, letting

$$x_i = \begin{cases} u(k-i+1), & i = 1, 2, \ldots, 5 \\ y(k-i+5), & i = 6, 7, 8 \end{cases}$$

From (18), we can get list of the performance index P_i of every input variable x_i (see Table 4). The six variables: $y(k-1)$, $u(k-4)$, $y(k-2$, $u(k-3)$, $y(k-3))$ and $u(k-2)$ are the most important input variables.



(a) Outputs comparison of training data

(b) Respective error of training data

(c) Outputs comparison of testing data

(d) Respective error of testing data

FIGURE 3. Comparison of our model and the original system for Mackey-Glass chaotic system.

TABLE 2. Comparison of the results of selecting input variables for Mackey-Glass chaotic system.

Model	Input variables	No. of rules	MSE1 (Training)	MSE2 (Testing)
FCM-PSO	$x(t-1), x(t-2), x(t-3), x(t-4), x(t-5), x(t-6)$	2	3.6048×10^{-5}	3.5941×10^{-5}
FCM-PSO with IVS	$x(t-1), x(t-2), x(t-3), x(t-4), x(t-5), x(t-18)$	2	1.2329×10^{-6}	1.2424×10^{-6}

TABLE 3. Comparison of different models for Mackey-Glass chaotic system.

Model	No. of rules	Training MSE	Testing MSE $\times 10^{-4}$
J. S. R. Jang et al. [19]	25	-	7.3×10^{-4}
J. C. Duan and F. L. Chung [20]	25	5.76×10^{-4}	6.401×10^{-4}
F. Guo et al. [21]	-	9.61×10^{-4}	10.24×10^{-4}
M. S. Mojtaba [22]	-	4.84×10^{-4}	7.29×10^{-4}
W. Zou et al. [23]	10	5.0216×10^{-4}	6.7449×10^{-4}
F. C. Liu [24]	2	6.377×10^{-5}	6.518×10^{-5}
Our Model	2	1.2329×10^{-6}	1.2424×10^{-6}

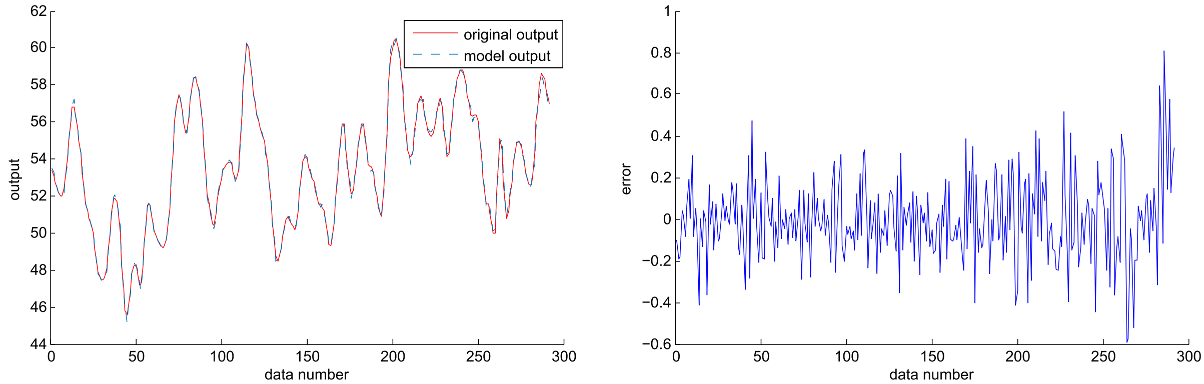
2) PARAMETER IDENTIFICATION BASED ON FUZZY C-MEANS CLUSTERING ALGORITHM AND PARTICLE SWARM OPTIMIZATION ALGORITHM

There are two situations in this experiment: Case 1 and Case 2. In Case 1, all 296 sets of data are used as training data. In Case 2, the first 148 data pairs are taken as the training dataset and the remaining 148 data pairs are taken as testing dataset.

TABLE 4. List of performance index for Box-Jenkins system.

x_i	x_6	x_5	x_7	x_4	x_8	x_3
input	$y(k-1)$	$u(k-4)$	$y(k-2)$	$u(k-3)$	$y(k-3)$	$u(k-2)$
P_i	0.0705	0.1494	0.1605	0.2604	0.3061	0.4296

In the most literatures, the following variables: $u(k)$, $u(k-1)$, $u(k-2)$, $y(k-1)$, $y(k-2)$ and $y(k-3)$ are chosen as the set of input variables for predicting. Here, the important variables in the previous part: $y(k-1)$, $u(k-4)$, $y(k-2$, $u(k-3)$, $y(k-3))$ and $u(k-2)$ are used as the candidate



(a) Outputs comparison (b) Respective error

FIGURE 4. Comparison of our model and the original system for Box-Jenkins system (Case 1).

TABLE 5. Comparison of the results of selecting input variables for Box-Jenkins system (Case1).

Model	Input variables	No. of rules	MSE
FCM-PSO	$u(k), u(k-1), y(k-1), y(k-2)$	3	0.0679
FCM-PSO with IVS	$u(k-4), u(k-3), y(k-1), y(k-2)$	3	0.0462
FCM-PSO	$u(k), u(k-1), u(k-2), y(k-1), y(k-2), y(k-3)$	3	0.0453
FCM-PSO with IVS	$u(k-4), u(k-3), u(k-2), y(k-1), y(k-2), y(k-3)$	3	0.0413

input variables, and all or some of them as input variables are selected as the inputs of the model. The number of fuzzy rules is set as 3.

In Case 1, Table 5 lists the results of model identification. The number of fuzzy rule is 3, and the numbers of the input variable are chosen as 4 and 6, respectively. When the number of input variables is 4, using empirical input variables and important input variables, the obtained MSE of the model is 0.0679 and 0.0462, respectively; when the number of input variables is 6, selecting regular input variables and important input variables, the obtained MSE of the model is 0.0453 and 0.0413, respectively. The results show that with the same number of input variables, the application of the important input variable selection method based on the TSFC algorithm effectively improves the performance of the model. At the same time, the comparison between the results of this model and the existing literature is shown in Table 6. It can be seen from Table 6 that the modeling accuracy is better than the identification results in other literatures when the number of fuzzy rule is less.

Case 2 is used to test the generalization of the model. The performance results and the approximation error of the fuzzy model established by applying the training data and the testing data are shown in Fig. 5. It can be seen that the output of the constructed T-S fuzzy model can closely approximate the actual model output. Table 7 lists the model identification results in the case of selecting conventional input variables based on experience and in the other case of selecting important input variables based on TSFC algorithm. When the number of input variable is 4, choosing conventional input variables, the training and testing errors of the model are 0.0254 and 0.1539 respectively, and choosing important input variables, the training and testing errors

TABLE 6. Comparison of different models for Box-Jenkins system (Case1).

Model	No. of inputs	No. of rules	Training MSE
M. Sugeno and K. Tanaka [26]	6	2	0.068
L. Wang and R. Langari [27]	6	2	0.066
G. E. Tsekoura [14]	6	8	0.075
F. C. Liu [24]	6	2	0.0561
C. S. Li et al [28]	6	4	0.0498
C. S. Li et al [29]	6	3	0.0560
C. S. Li et al [30]	6	3	0.0534
Our Model	6	3	0.0413

are 0.0130 and 0.1461 respectively. When the number of input variables is 6, the training MSE and the testing MSE of the model are 0.0141 and 0.1434 by using conventional input variables, and the training MSE and the testing MSE are 0.0114 and 0.1369 by using important input variables. At the same time, Table 8 shows the detailed comparison results of the identification method proposed in this paper and the ones of the identification methods proposed in other methods. It can be concluded that when the number of fuzzy rule is close to the models in literatures, the training performance index and testing performance index of the T-S fuzzy model constructed in this paper are better than or close to the performance indices obtained by using these existing methods. The model has good generalization ability.

C. THE VARIABLE LOAD PNEUMATIC LOADING SYSTEM

The variable load pneumatic loading system has the advantages of low cost, high output/mass ratio, no pollution, convenient maintenance and so on, which is widely used in the field of industrial automation [36]. Because of the complexity of gas flow, the compressibility of gas, the nonlinearity of valve,

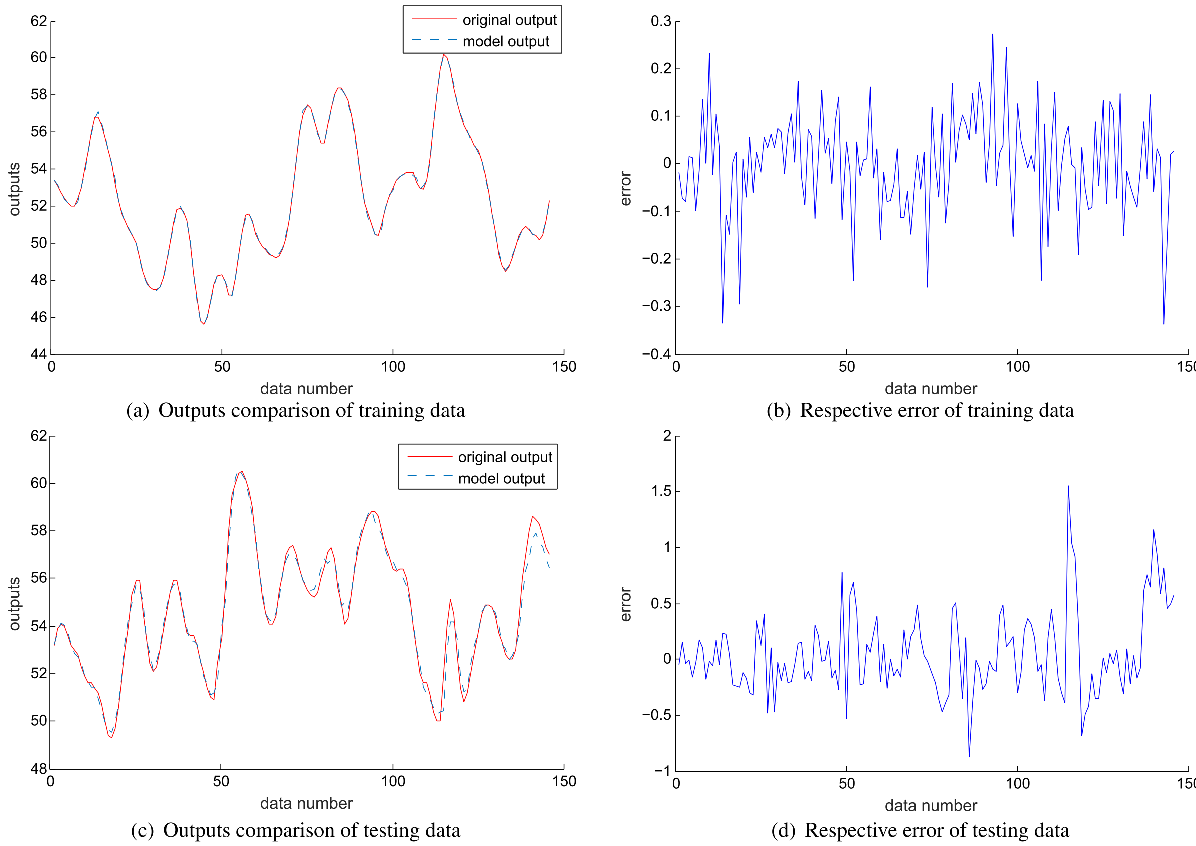


FIGURE 5. Comparison of our model and the original system for Box-Jenkins system (Case 2).

TABLE 7. Comparison of the results of selecting input variables for Box-Jenkins system(Case2).

Model	Input variables	No. of rules	MSE1 (Training)	MSE2 (Testing)
FCM-PSO	$u(k), u(k-1), y(k-1), y(k-2)$	3	0.0254	0.1539
FCM-PSO with IVS	$u(k-4), u(k-3), y(k-1), y(k-2)$	3	0.0130	0.1461
FCM-PSO	$u(k), u(k-1), u(k-2), y(k-1), y(k-2), y(k-3)$	3	0.0141	0.1434
FCM-PSO with IVS	$u(k-4), u(k-3), u(k-2), y(k-1), y(k-2), y(k-3)$	3	0.0114	0.1369

TABLE 8. Comparison of different models for Box-Jenkins system (Case2).

Model	No. of rules	MSE1(Training)	MSE2(Testing)
Y. Lin and G. A. Cunningham [31]	4	0.071	0.261
E. Kim et al. [32]	2	0.034	0.244
G. E. Tsekoura [14]	7	0.022	0.236
C. S. Li et al. [29]	3	0.0159	0.1255
C. S. Li et al. [30]	3	0.0150	0.1470
M. N. Luo et al. [33]	2	0.0254	0.1243
S. Q. Yan et al. [34]	2	0.0168	0.1402
C. S. Li et al. [35]	3	0.0149	0.1324
Our Model	3	0.0114	0.1369

the friction characteristics of cylinder and the vulnerability of system parameters to environment, the modeling and control of pneumatic loading system has become a very challenging work.

Generally speaking, there are two ways to establish the system model: one is that the operation law of the system is completely known and the model is built according to the physical law; the other one is to identify the system model from the operation and experimental data of the system. In this paper, data-driven fuzzy modeling method is used to build the model of the variable load pneumatic loading system.

Fig. 6 is the structure diagram of the pneumatic loading system for test. The system includes stabilized pressure air source, pneumatic couplet, SMC ITV2050 pilot electric proportional pressure valve, SMC CDQ2A50 single rod double acting cylinder with cylinder diameter of 40mm and stroke of 50mm and other pneumatic components. The measurement and control system includes MCL-L pull pressure sensor for real-time pressure measurement, Advantech PCI1710 data acquisition card for analog input, and Advantech PCI17 20 for control output. The system controller is IPC-610H industrial computer.

In this paper, during the dynamic range of the system, the pseudo-random sequence is used as the excitation signal, which continuously acts on the system in the open-loop

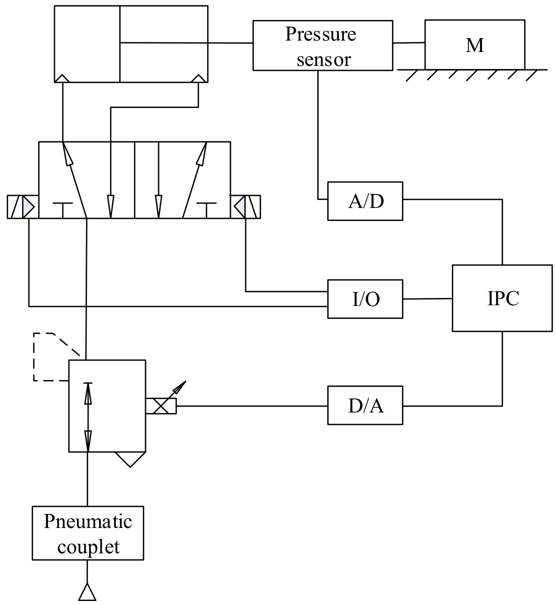


FIGURE 6. Structural diagram of the variable load pneumatic loading system.

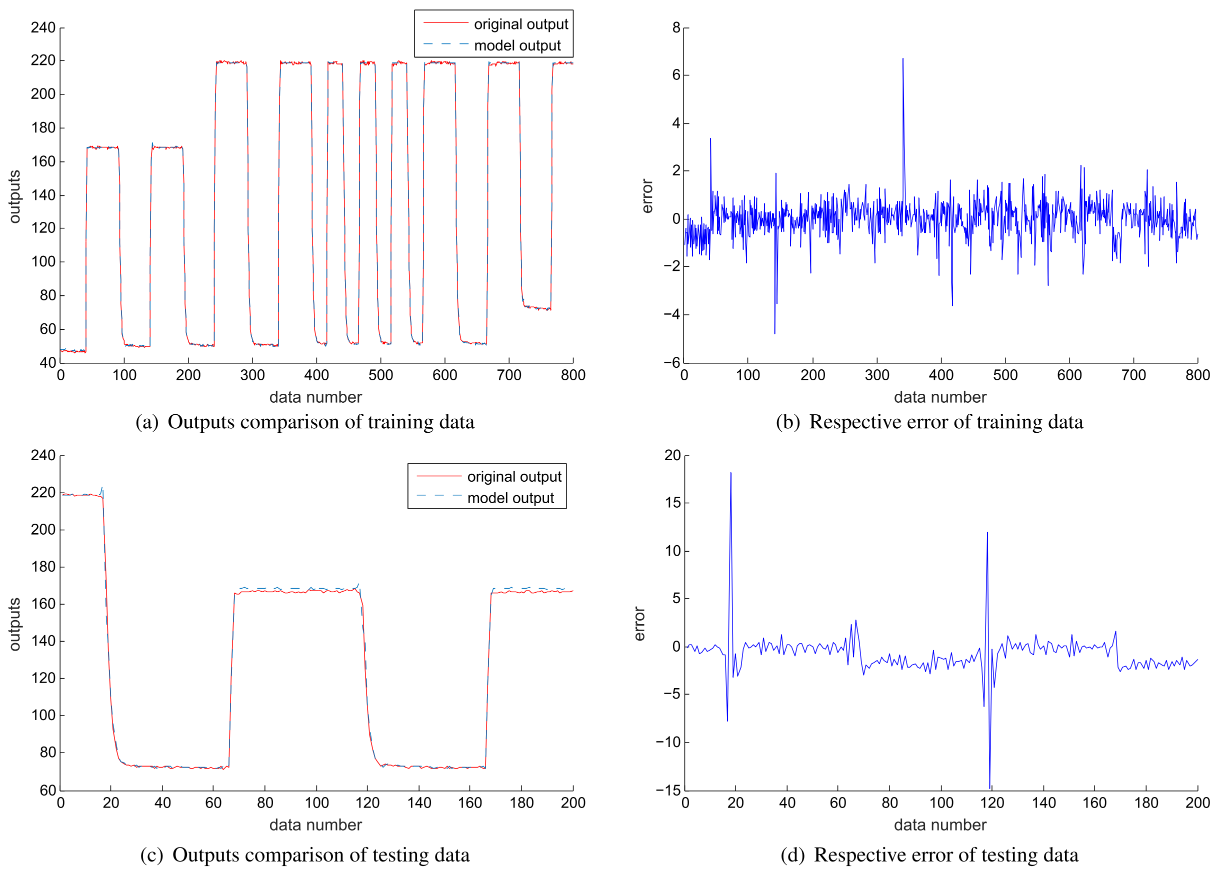
state and collects the input and output data of the system. The sampling period is 0.1s, the sampling time is 100s, and 1000 sample points $[u(k), y(k)]$ are obtained. Then, training

TABLE 9. List of performance index for air pressure loading system.

x_i	x_{12}	x_9	x_8	x_{10}	x_{13}	x_7
input	$y(k-1)$	$u(k-2)$	$u(k-3)$	$u(k-1)$	$y(k-2)$	$u(k-4)$
P_i	0.0212	0.0262	0.0375	0.0577	0.0598	0.0702

the collected data according to the TSFC method as mentioned above, we can sort the input variables based on their importance (see Table 9). The following variables: $u(k-1)$, $u(k-2)$, $u(k-3)$, $u(k-4)$, $y(k-1)$ and $y(k-2)$ are selected as the candidate input variables of the model, and $y(k)$ as the output variable. The number of fuzzy rules is set as 3, the parameters of antecedents are determined by FCM algorithm.

Fig. 7 shows the off-line modeling process curve of the variable load pneumatic loading system based on the selection of important input variables, where Fig. 7 (a) shows the outputs of the fuzzy model comparing with that of the real system, and Fig. 7 (b) shows the errors between the outputs of the fuzzy model and the outputs of the real system. The performance of the fuzzy model based on IVS-FCM-PSO comparing with no IVS is shown in Table 10. It can be seen from Table 10, if the TSFC method is used to select important input variables, when the variables: $u(k-1)$, $u(k-2)$, $u(k-3)$ and $y(k-1)$ are chosen as inputs, the training MSE



(a) Outputs comparison of training data

(b) Respective error of training data

(c) Outputs comparison of testing data

(d) Respective error of testing data

FIGURE 7. Square wave loading test results for the variable load pneumatic loading system.

TABLE 10. Comparison of the results of selecting input variables for Box-Jenkins system(Case2).

Model	Input variables	No. of rules	MSE1 (Training)	MSE2 (Testing)
FCM-PSO	$u(k-1), u(k), y(k-1), y(k-2)$	3	12.0288	36.8988
FCM-PSO with IVS	$u(k-1), u(k-2), u(k-3), y(k-1)$	3	0.6954	10.5019
FCM-PSO	$u(k), u(k-1), u(k-2), y(k-1), y(k-2), y(k-3)$	3	0.7136	10.9543
FCM-PSO with IVS	$u(k-1), u(k-2), u(k-3), u(k-4), y(k-1), y(k-2)$	3	0.6376	5.7597

and the testing MSE are 0.6954 and 10.5019 respectively. When all of the 6 variables in Table 9 are chosen as inputs, the training MSE of our model is 0.6376 and the testing MSE is 5.7597. If the selection of important input variables is not considered, when the number of input variables is 4, the training errors and test errors are 12.0288 and 36.8988; when the number of input variables is 6, the training and testing errors are 0.7136 and 10.9543.

The experimental results show that the proposed fuzzy modeling method based on the selection of important input variables has a better performance index and generalization ability, which can better approximate the output of the actual pneumatic loading system. It can effectively reduce the influence of time delay on the system, which make it easier to control the variable load pneumatic loading system and realize the rapid response and accurate tracking of the system. And the proposed fuzzy modeling method has good adaptive ability. The practical application shows that the fuzzy identification method of nonlinear dynamic system based on TSFC method is of great significance to the modeling of practical dynamic system.

V. CONCLUSION

In this paper, the influence of the important input variables selection on the accuracy of fuzzy model identification has been analyzed and studied in detail. The fuzzy premise parameters have been optimized by intelligent optimization algorithm. The proposed identification method has been applied to two international standard examples and a practical variable load pneumatic loading system. Compared with the previous fuzzy identification methods, this method firstly preprocesses the selection of important input variables offline, which not only improves the identification accuracy, but also reduces the complexity of the model and saves the cost of calculation. The experimental results in this paper show that the important input variable selection method can accurately select the most suitable output for the system. Therefore, for some unknown nonlinear systems, this paper can accurately select the system inputs and model them.

With the continuous development and maturity of intelligent optimization algorithms, more and more excellent optimization algorithms have emerged, such as hybrid frog leaping algorithm, firefly algorithm and cockroach algorithm and so on. For a specific fuzzy identification problem, it is a more practical research direction to combine the appropriate important input variable selection algorithm with the intelligent optimization algorithm for parameter identification, to explore a fuzzy identification method with faster convergence speed and higher accuracy, and to better and more successfully apply it to the actual fuzzy system identification.

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