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Peer-to-Peer Energy Transaction Mechanisms Considering Fairness in Smart Energy Communities

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ABSTRACT This study presents peer-to-peer (P2P) energy transaction mechanisms to maximize social welfare considering the uncertainty and profit fairness of the players. The P2P energy transaction problem is formulated as a P2P energy transaction pair matching and the determination of the P2P transaction price. To solve the problem, the optimal condition to maximize social welfare is determined using stochastic P2P energy transaction performance analysis based on the uncertainty characteristics. The analysis results show that social welfare is maximized to match the producer and consumer pairs that have similar demand characteristics; the P2P transaction price balances the profit fairness between the pair. Using these results, two centralized P2P energy transaction mechanisms are proposed by modifying the optimization problem. Moreover, a decentralized P2P energy transaction mechanism that operates in a distributed manner is suggested with the operational signal flow for the implementation of the mechanism. The simulation results show that the centralized and decentralized mechanisms have near optimal performance, with less than a 0.5% and 1% optimal gap compared to the optimal solution that requires perfect information including uncertainty, respectively. However, the decentralized mechanism is less computationally complex and uses less information than the centralized mechanisms; consequently, it can alleviate the operational burden and security and privacy problems. In addition, the results show that the performance of P2P energy transaction is related to the relative demand ratio between the producer and consumer. The optimal condition and results suggest a guide to the design of the P2P energy transaction.

INDEX TERMS Demand-side management, distributed energy transaction, distributed generation, energy community, energy trading, fairness, peer-to-peer, prosumer, uncertainty.

I. INTRODUCTION

A. MOTIVATION

Driven by the development of advanced grid technologies—such as the smart grid—and environmental issues to reduce CO_2 emissions, the energy system framework has experienced significant changes in recent years [1]. Distributed energy generators (DEGs) have stimulated increasing interest in addressing these issues. DEGs provide economic benefits to the grid in terms of transmission and distribution savings [2] as well as low carbon renewable energy sources [3]. On the customer side, small-scale DEGs, such as solar photovoltaics (PVs), introduce prosumers who work as producers to sell domestically generated energy, as well as consumers who buy energy, from the grid and/or other prosumers [4].

However, the ever-increasing number of DEGs and prosumers reduces the reliability of energy systems and increases their management complexity [5]. The result requires the reorganization of the management of the energy system. Consequently, the smart energy community (SEC) framework has been proposed. The role of the SEC is the effective

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management of self-produced renewable energy, DEGs, and the development of localized energy transactions [6]. The participants of the SEC expect an energy cost reduction of energy vector procurement, the use of local resources and utilities that will reduce the complexity of energy system management, and improve the reliability and quality of supply [7].

B. PRIOR WORKS

In an SEC, the peer-to-peer (P2P) energy transaction between energy producers and consumers is an important problem [8].

Various studies have been conducted under an optimization problem-based approach. Liu *et al.* formulated the bi-level optimal problem (day-ahead and real-time markets) to minimize the inconvenience cost and solve the distributed iteration process (real-time pricing) [9]. A dynamic internal pricing model was proposed for the operation of an energy sharing zone, which was defined based on the supply and demand ratio of shared PV energy. Morstyn and McCulloch formulated a multi-class energy management problem to minimize the total cost considering power flow and prosumer constraints. They proposed a distributed price-directed optimization mechanism using an alternating direction method of multipliers [10]. The proposed distributed mechanism converged with less than 300 iterations of shorter operating times compared to the centralized mechanism. Oh and Son presented an energy matching problem for minimizing the social welfare with fixed transaction prices [11]. They suggested an energy trading rule with a guaranteed minimum quantity to manage the distributed P2P market risk from the unpredictability of resources and proposed centralized and distributed P2P matching algorithms. Jogunola *et al.* suggested a slime-mold inspired optimization method for addressing the path cost problem for energy routing and the capacity constraints of distribution lines for congestion control [12]. The paper demonstrated that the cost of energy had a direct correlation with the distance between the energy producer and consumer, and, using that, a path-optimized system was developed for energy routing. Paudel *et al.* formulated a P2P energy trading problem for social welfare maximization considering power loss and network fees [13]. The problem was decoupled using Lagrangian multiplier methods and a decentralized market clearing algorithm was proposed for the P2P energy trading considering the privacy of the agents, power losses, and the utilization fees. Zhang *et al.* presented a P2P trading market as a centralized optimization problem to maximize the total revenue and proposed a P2P trading market mechanism including energy and uncertainty trading according to flexibility characteristics [14]. Sorin *et al.* formulated a P2P energy market as a multi-bilateral economic dispatch problem and proposed a relaxed consensus and innovation approach to solve the problem in a decentralized manner [15]. Morstyn *et al.* presented P2P energy trading using bilateral contract networks including an upstream–downstream energy balance and forward market uncertainty [16]. Leeuwen *et al.* proposed an integrated blockchain-based energy management platform that optimized energy flows, implemented a bilateral trading mechanism, and presented a smart contract as a virtual aggregator to solve the problem in a distributed manner [17]. Khorasany *et al.* proposed a primal-dual gradient method based on bilateral P2P energy trading considering line flow constraints to avoid overloaded or congested lines in the system [18]. These works showed that the P2P energy transaction problem could be used by centralized and decentralized mechanisms to solve the optimization problem. However, to apply the solution, the centralized mechanism requires a central controller and is computationally complex, whereas the decentralized mechanisms require relaxation parameters such as Lagrangian multipliers and an iteration process.

Tushar *et al.* discussed various game- and auction-based theoretical approaches to provide an overview of the use of game-theory approaches to P2P energy trading as a feasible and effective means of energy management [19]. They also proposed a canonical coalition game approach for P2P energy trading, in which a set of participating prosumers formed a coalition group to trade their energy with one another and utilized a mid-market rate as a pricing mechanism for the proposed P2P trading [20], [21]. Liu *et al.* designed a Stackelberg game-based energy sharing method for PV prosumers using a feed-in-tariff where the utility acts as the leader and all participating prosumers are considered to be the followers [22]. Paudel *et al.* modeled price competition among sellers for P2P energy trading as a noncooperative game and proposed an M-leader and N-follower—based on Stackelberg's game-based approach—using direct interactions between buyers and sellers taking into consideration the demand response capability and privacy of prosumers [23]. Anoh *et al.* formulated P2P energy trading as a Stackelberg game with producers as leaders and consumers as followers and optimized both the cost for consumers and the utility for producers [24]. Chen modeled P2P energy sharing as a generalized Nash-demand bidding game and showed that Nash equilibrium occurs [25]. Cadre *et al.* characterized a P2P energy market solution as a variational equilibrium and proved that the set of variational equilibria coincides with the set of social welfare optimal solutions of market design [26]. Wang *et al.* proposed a real-time double auction market with a continuous bidding mechanism for a distributed P2P energy transaction so that the coordination and complementarity of energy among prosumers could be achieved [27]. It is clear that these game-theory approaches can solve P2P energy transaction problems. However, an iterative process to converge the solution is required.

Although many studies have been conducted in this P2P energy transaction area, there are still several problems to be resolved, including the operational burden. Firstly, further research needs to be carried out on the effect of supply and demand uncertainty. Prosumers' resources are generally based on renewable energy with high fluctuation and predictive uncertainty. Few works touch on these uncertainty issues. In [11], the trading rule considering the risk of uncertainty has been suggested for P2P energy transactions. A P2P energy trading mechanism has been proposed considering forecast power and its uncertainty [14]. These studies show that the uncertainty of resources affect the systems' performance including aspects such as the economic benefits. Therefore, it is necessary to study P2P energy transactions considering uncertainty and analyze its effect on performance. Secondly, the consideration of profit fairness is required. Most of the discussed research has aimed to maximize social welfare defined as the overall prosumer profit. A few studies have introduced fairness during the P2P energy transaction using the Shapley value [28] and Nash-type non-cooperative game-theory approach between two prosumers [29]. It is important to guarantee the fairness of energy sharing; the result of fairness may affect the participation willingness of prosumers [9]. Therefore, research into P2P energy transactions considering profit fairness among prosumers through profit balancing is required. To fill this void in the research gap, this study deals with the P2P energy transaction problem including uncertainty and fairness problems and uses the proposed centralized and decentralized P2P energy transaction mechanisms to solve the problem. The centralized mechanism has near optimal performance, and the decentralized mechanism reduces the operational burden using a marginal performance gap.

C. CONTRIBUTION

This study focuses on the P2P energy transaction mechanism while considering uncertainty and profit fairness. To do this, we first analyze the stochastic performance of the P2P energy transaction and propose P2P energy transaction mechanisms. Our contribution is summarized below:

- *Stochastic performance analysis*: Net profit, an increase in the profit in comparison to trading with the grid, is used as a P2P energy transaction performance metric. The net profit is related to the characteristics of the players such as the demand and its uncertainty, and the transaction parameters, such as the P2P transaction energy and transaction price between the transaction pairs. In this study, we analyzed the expected pair profit using the stochastic properties of the players and determined the optimal P2P transaction energy to maximize it. The main contribution of the analysis is the determination of the optimal condition to maximize the social welfare—that is, the total profit of players using the P2P energy transaction. The P2P energy transaction performance is analyzed as the profit bound due to the imbalance between production and consumption and the profit loss due to uncertainty. The analysis shows that the social welfare is maximized when the prosumer and consumer with similar stochastic properties is matched. This is because the P2P transaction energy is maximized in this condition. The P2P transaction price balances the profit between the transaction pair. Using these results, the P2P energy transaction mechanisms are proposed. Moreover, the analysis suggests the theoretical bound that each player can obtain. The bound provides a criterion for examining the performance of the P2P energy transaction.
- *P2P energy transaction mechanisms*: A P2P energy transaction is a problem that needs to be solved to determine the P2P transaction energy and the price of each transaction pair. The problem becomes a non-convex optimization problem that requires a central controller with high computational complexity so that it can be optimally solved. From the stochastic performance analysis, the problem decomposes into two parts: pair matching to maximize the profit that is related to the P2P transaction energy and profit balancing between the matched pair using the P2P transaction price. Under this observation, we propose two central P2P energy transaction mechanisms by modifying the problem. The proposed mechanisms require less computational complexity, and they use the stochastic properties of the players alone. Nevertheless, the performance gap of the proposed central mechanisms is less than 0.5% in comparison to the optimal solution that is required for perfect information, which includes future information. Moreover, a decentralized mechanism is proposed without a central controller. The proposed decentralized mechanism is designed as a one-by-one method in which one or more transaction pairs are matched in a single process. The method has a linear computational complexity based on the player size. The proposed decentralized mechanism is less computationally complex and requires less information in comparison to the central mechanisms. Therefore, the operational burden and security and privacy problems are alleviated in the decentralized mechanism. The performance gap of the proposed decentralized mechanism is also less than 1% of the optimal solution.

The rest of this paper is organized as follows. Section II describes the system models and the problem formulation of the P2P energy transaction. Section III analyzes the stochastic performance of the P2P energy transaction. Section IV discusses how to design the proposed P2P energy transaction mechanisms. Section V and VI demonstrate the measurement studies that are applied to the proposed mechanisms in comparison to the optimal method. Finally, Section VII concludes the paper.

II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

A. SMART ENERGY COMMUNITY MODEL

The SEC studied in this investigation is shown in Figure 1, and it includes three parts: the community member, the smart energy service provider (SESP), and the grid utility.

The community member consists of the consumer and producer, such as renewable generators and a prosumer with a PV system or an electric vehicle (EV). Information for the P2P energy transaction is exchanged between the members. A P2P energy transaction is determined based on a predetermined mechanism as suggested in this study, and an energy

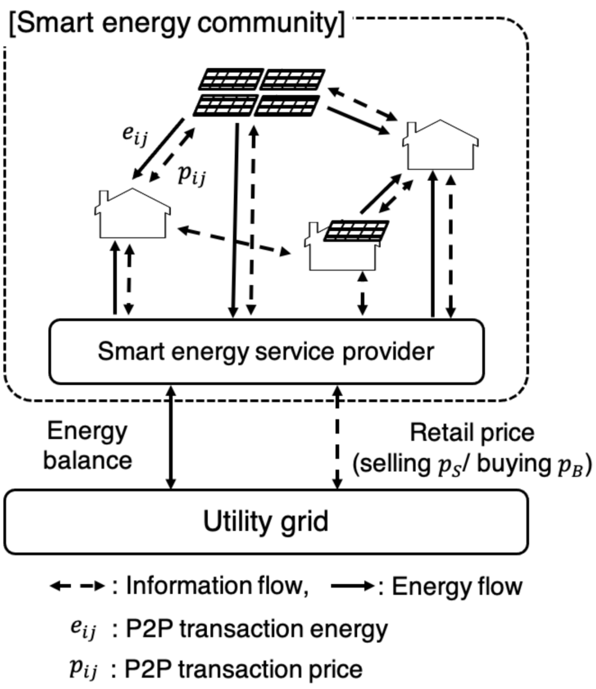

FIGURE 1. Smart energy community model.

transaction is performed from the seller to the buyer during a specified period, such as one day.

The SESP is introduced to deal with the energy transactions between the members and the utility grid. When one fails to generate or consume the required amount of energy, it can buy or sell energy through the SESP for energy balance. If the P2P energy transaction is operated in a centralized manner, the SESP can function as a central controller.

The utility announces the retail price—i.e., selling to the grid p_S and buying from the grid p_B—and transmits the energy for the energy balance of the SEC. The retail price can be used to control SEC access to the grid. However, this study focuses on the P2P energy transaction mechanism; hence, it is assumed that the retail price is fixed.

B. PROSUMER MODEL

Even as prosumers, players operate as either producers or consumers in the P2P energy transaction decision-making process. Therefore, players are defined as producers and consumers. Accordingly, there are sets of producers ($\mathfrak{I}$) and consumers ($\mathfrak{J}$). Let $i \in \mathfrak{I}$ be the index of the ith producer and $j \in \mathfrak{J}$ be the index of the jth consumer.

The generation and demand of the players are predicted using forecasting methods, such as renewable generation forecasting methods [30], [31] and customer baseline load calculation methods [32], [33]. Forecasting values include the uncertainty of the forecasting error. Therefore, the actual generation of the ith producer, g_i, and the demand of the jth consumer, d_j, are written as:

$$\begin{aligned} g_i &= \hat{g}_i + \varepsilon_i, \quad \forall i \in \mathfrak{I} \\ d_j &= \hat{d}_j + \varepsilon_j, \quad \forall j \in \mathfrak{J} \end{aligned} \tag{1}$$

where ε_i and ε_j are the uncertainty of the forecasting error. The uncertainty is modeled as a random value in a set of distributions in the exponential family, such as normal and exponential distributions [11], [34]. In this study, it is assumed that the uncertainty has the Laplace distribution that is a double-side exponential distribution [35]:

$$\varepsilon_k \sim \mathcal{L}(\mu_k, \sqrt{2}/\lambda_k), \quad \forall k \in \mathfrak{I} \cup \mathfrak{J} \tag{2}$$

C. PROFIT MODEL

When the P2P transaction energy e_{ij} and price p_{ij} between the ith producer and the jth consumer are determined through the P2P energy transaction mechanism, the profit of the producer and the consumer are measured, as follows.

1) PROFIT OF THE PRODUCER

The generated energy by the producer is traded with the consumer. However, due to the uncertainty of the generation, additional or insufficient energy is generated that is mismatched with the transaction energy. The energy is sold or purchased on the grid through the SESP. Therefore, the profit of the producer i when trading with the consumer j, B_{ij}, is expressed as:

$$B_{ij} = \underbrace{e_{ij}p_{ij}}_{BP1} + \underbrace{[g_i - e_{ij}]^+ p_S}_{BP2} + \underbrace{[g_i - e_{ij}]^- p_B}_{BP3} \tag{3}$$

where $[a]^+ = \max\{0, a\}$ and $[a]^- = \min\{0, a\}$. The profit in (3) consists of three parts:

- $BP1$: Profit as a result of the P2P energy transaction,
- $BP2$: Profits from selling extra energy to the grid,
- $BP3$: Penalty from buying insufficient energy from the grid.

Moreover, the net profit of the producer i in comparison to trading with the grid becomes:

$$B_i^{net} = B_{ij} - g_i p_S \tag{4}$$

2) PROFIT OF THE CONSUMER

Similarly to the producer, the consumer is also affected by mismatching due to the uncertainty of the consumption. When this is considered the cost of the consumer j when trading with the producer i, C_{ji} is obtained as:

$$C_{ji} = \underbrace{e_{ij}p_{ij}}_{BC1} + \underbrace{[d_j - e_{ji}]^+ p_B}_{BC2} + \underbrace{[d_j - e_{ji}]^- p_S}_{BC3} \tag{5}$$

where $e_{ji} = e_{ij}$ and $p_{ji} = p_{ij}$. Each term to the right of (5) expresses:

- $BC1$: Cost incurred as a result of the P2P energy transaction,
- $BC2$: Cost incurred from buying insufficient energy from the grid,
- $BC3$: Profits obtained from selling extra energy to the grid.

The net profit of consumer j from the P2P energy transaction becomes:

$$B_j^{net} = d_j p_B - C_{ji} \tag{6}$$

Note that this study assumes that the P2P energy transaction consists of one producer and one consumer matching case, i.e., a one-to-one model. However, the model can be easily extended to a multi-to-multi matching case that replaces g_i and d_j with g_{ij} and d_{ji},

$$B_i^{net} = \sum\nolimits_{j\in\mathcal{J}} \left(B_{ij} - g_{ij}p_S\right)$$
$$B_j^{net} = \sum\nolimits_{i\in\mathfrak{I}} \left(d_{ji}p_B - C_{ji}\right) \tag{7}$$

and the constraints:

$$\sum\nolimits_{j\in\mathcal{J}} g_{ij} = g_i$$
$$\sum\nolimits_{i\in\mathfrak{I}} d_{ji} = d_i \tag{8}$$

The notation used in this study is summarized in Table 1.

TABLE 1. Notation summary.

Symbol	Description
	System symbol
$i \in \mathfrak{I}$	Producer index
$j \in \mathcal{J}$	Consumer index
$\hat{g}_i$	Generation forecasting of producer i, [kWh]
$\hat{d}_j$	Demand forecasting of consumer j, [kWh]
g_i	Actual generation of producer i, [kWh]
d_j	Actual demand of consumer j, [kWh]
ε_k	Uncertainty of the generation and demand $k \in \mathfrak{I} \cup \mathcal{J}$
	P2P transaction
$e_{ij} = e_{ji}$	Transaction energy between i and j, [kWh]
$p_{ij} = p_{ji}$	Transaction price between i and j, [\$/kWh]
p_B	Buying price from the grid, [\$/kWh]
p_S	Selling price to the grid, [\$/kWh]
B_{ij}	Profit of producer i to consumer j, [\$]
C_{ji}	Cost of consumer j to producer i, [\$]
B_k^{net}	Net profit of producer and consumer $k \in \mathfrak{I} \cup \mathcal{J}$, [\$]

D. PROBLEM FORMULATION

The goal of the P2P energy transaction in this study is to maximize the social welfare that aggregates the total profit of the players while considering profit fairness. The problem can be formulated as follows:

$$\mathbf{P0}: \max_{e_{ij},p_{ij},\nu} \sum\nolimits_{k\in\mathfrak{I}\cup\mathcal{J}} B_k^{net}$$
$$\text{subject to } B_k^{net} \geq \nu, \quad \forall k \in \mathfrak{I}\cup\mathcal{J}$$
$$\nu \geq 0 \tag{9}$$

where ν is the fairness constraint. When considering the problem of (9), for the P2P energy transaction, the P2P transaction energy e_{ij} and price p_{ij} between each transaction pair are determined to achieve this purpose. Through the first constraint, the minimum net profit is guaranteed for all players. This is a restriction to ensure max-min fairness [36]. The max-min method increases the level of fairness to reduce the variance of values. The second constraint expresses that all players achieve a positive net profit through the P2P energy transaction. This is a condition for the players to participate in the P2P energy transactions. When the fairness constraint ν is zero, it has the same meaning as individual rationality that each player should achieve a non-negative profit to participate in the P2P energy transaction.

The net profit that is the objective function of the problem in (9) is a non-convex function due to the $[a]^+$ and $[a]^-$ operations. Moreover, for finding the optimum set of the P2P transaction pairs, theoretically high computational complexity is required among $|\mathfrak{I}| \times |\mathcal{J}|$ [37], and a central controller that requires information from all players is also needed. These difficulties create a burden on the implementation of the P2P energy transaction in a distributed P2P energy market with low accountability. In this study, effective P2P energy transaction mechanisms are suggested to deal with these difficulties.

III. STOCHASTIC PERFORMANCE OF THE P2P ENERGY TRANSACTION

To design effective P2P energy transaction mechanisms, the stochastic performance of the players using the P2P energy transaction is analyzed while considering their uncertainty. This analysis suggests the optimal condition and the expected profit bound to solve the problem of (9); it is used to design the P2P energy transaction mechanisms.

A. EXPECT PROFIT FROM THE PLAYERS

When a P2P energy transaction between producer i and consumer j proceeds with e_{ij} and p_{ij}, the profits of the players are predicted using the stochastic properties of the uncertainty.

1) PRODUCER'S PROFIT

From (3), the profit of producer i by trading to consumer j is projected as:

$$\begin{aligned}\hat{B}_{ij} &= \mathbb{E}\left\{e_{ij}p_{ij} + [g_i - e_{ij}]^+p_S + [g_i - e_{ij}]^-p_B\right\}\\ &= \mathbb{E}\left\{e_{ij}p_{ij} + [\hat{g}_i + \varepsilon_i - e_{ij}]^+p_S + [\hat{g}_i + \varepsilon_i - e_{ij}]^-p_B\right\}\\ &= e_{ij}p_{ij} + \mathbb{E}\left\{[\varepsilon_i + l_{ij}]^+\right\}p_S + \mathbb{E}\left\{[\varepsilon_i + l_{ij}]^-\right\}p_B \end{aligned} \tag{10}$$

where $l_{ij} = \hat{g}_i - e_{ij}$. In (10), p_S and p_B are announced from the grid utilty, e_{ij} and p_{ij} are determined thorough the P2P energy transaction mechanism, and $\hat{g}_i$ is measured by the generation forecasting of producer i.

Therefore, if the uncertainty ε_i has a zero mean with λ_i, the expected profit of producer i, $\hat{B}_{ij}$, can be measured as:

$$\hat{B}_{ij} = e_{ij}p_{ij} + [l_{ij}]^+p_S + [l_{ij}]^-p_B - (p_B - p_S)\frac{1}{2\lambda_i}e^{-\lambda_i|l_{ij}|} \tag{11}$$

and the expected net profit of producer i, $\hat{B}_i^{net}$, is expressed as:

$$\hat{B}_i^{net} = \hat{B}_{ij} - \hat{g}_i p_S \tag{12}$$

The detailed calculations are described in Appendix A.

2) CONSUMER'S PROFIT

The cost of consumer j by trading to producer i in (5) is predicted as:

$$\begin{aligned}\hat{C}_{ji} &= \mathbb{E}\left\{e_{ji}p_{ji} + [d_j - e_{ji}]^+ p_B + [d_j - e_{ji}]^- p_S\right\} \\ &= \mathbb{E}\left\{e_{ji}p_{ji} + [\hat{d}_j + \varepsilon_j - e_{ji}]^+ p_B + [\hat{d}_j + \varepsilon_j - e_{ji}]^- p_S\right\} \\ &= e_{ji}p_{ji} + \mathbb{E}\left\{[\varepsilon_j + l_{ji}]^+\right\} p_B + \mathbb{E}\left\{[\varepsilon_j + l_{ji}]^-\right\} p_S \end{aligned} \tag{13}$$

where $l_{ji} = \hat{d}_j - e_{ji}$.

Similar to the case of the producer, assuming that the uncertainty has a zero mean with λ_j, the expected cost of consumer j, $\hat{C}_{ji}$, can be measured as:

$$\hat{C}_{ji} = e_{ji}p_{ji} + [l_{ji}]^+ p_B + [l_{ji}]^- p_S + (p_B - p_S)\frac{1}{2\lambda_j}e^{-\lambda_j|l_{ji}|} \tag{14}$$

and the expected net profit of consumer j, $\hat{B}_j^{net}$, is shown as:

$$\hat{B}_j^{net} = \hat{d}_j p_B - \hat{C}_{ji} \tag{15}$$

The detailed calculations are shown in Appendix B.

B. EXPECTED PAIR PROFIT

The objective in (9) is to maximize the social welfare of the overall players. It can be converted to the sum of the profits of all transaction pairs.

From (12) and (15), the expected pair profit of the transaction pair (i, j), $\hat{B}_{(i,j)}$, is measured as:

$$\begin{aligned}\hat{B}_{(i,j)} &= \hat{B}_i^{net} + \hat{B}_j^{net} \\ &= \hat{d}_j p_B - \hat{g}_i p_S \\ &\quad + [l_{ij}]^+ p_S + [l_{ij}]^- p_B - (p_B - p_S)\frac{1}{2\lambda_i}e^{-\lambda_i|l_{ij}|} \\ &\quad - [l_{ji}]^+ p_B - [l_{ji}]^- p_S - (p_B - p_S)\frac{1}{2\lambda_j}e^{-\lambda_j|l_{ji}|}\end{aligned} \tag{16}$$

The expected pair profit is the sum of the prosumer's profit and the consumer's cost saving. The prosumer's profit consists of profit from the P2P energy transaction (BP1) and the extra profit or penalty from the uncertainty of the prosumer (BP2 and BP3) as shown in (3). Likewise, the cost of the consumer consists of the cost incurred by the P2P energy transaction (BC1) and the penalty or the extra profit from the uncertainty of the consumer (BC2 and BC3) as shown in (5). The P2P transaction price affects the profit and the cost of the P2P energy transaction (BP1 and BC1). In (16), due to the mutual P2P energy transaction profit and the cost offset between the producer and consumer, the expected pair profit is isolated from the P2P transaction price p_{ij} and is decided based on the P2P transaction energy e_{ij}.

As described in detail in Appendix C, the optimal P2P transaction energy e_{ij}^* to maximize the expected pair profit $\hat{B}_{(i,j)}$ is calculated through the derivative of (16), as follows:

$$e_{ij}^* = \frac{\lambda_i}{\lambda_i + \lambda_j}\hat{g}_i + \frac{\lambda_j}{\lambda_i + \lambda_j}\hat{d}_j \tag{17}$$

This value is a risk balancing point for the producer and consumer according to the uncertainty that satisfies the following condition:

$$\frac{1}{\lambda_i}e^{-\lambda_i|l_{ij}|} = \frac{1}{\lambda_j}e^{-\lambda_j|l_{ji}|} \tag{18}$$

Substituting (17) into (16), the expected maximum pair profit between producer i and consumer j through the P2P energy transaction, $\hat{B}_{(i,j)}^*$, is measured as:

$$\hat{B}_{(i,j)}^* = \frac{p_B - p_S}{2}\left\{2\min(\hat{g}_i, \hat{d}_j) - \frac{\lambda_i + \lambda_j}{\lambda_i\lambda_j}e^{-\frac{\lambda_i\lambda_j}{\lambda_i+\lambda_j}|\hat{g}_i - \hat{d}_j|}\right\} \tag{19}$$

In (19), the coefficient $(p_B - p_S)/2$ is a systematic parameter that controls the participation of the P2P energy transaction. The first term becomes the profit bound due to the imbalance between production and consumption, and the second term expresses the profit loss due to uncertainty.

IV. P2P ENERGY TRANSACTION MECHANISM

A. DESIGN RATIONALE

The problem of the P2P energy transaction to maximize social welfare while considering profit fairness becomes the decision of the P2P transaction energy e_{ij} and price p_{ij} between the pairs and the fairness constraint ν that is formulated in (9).

However, when the social welfare, which is the total profit of all players, is converted into total pair profit, the effect of the P2P transaction price on the social welfare is isolated due to the P2P energy transaction profit and the cost offset between the producer and consumer—as described in the performance analysis of the previous section. The P2P transaction price only affects the profit balance for each producer and consumer pair.

As a result, the social welfare maximization problem is simplified as a pair matching problem that determines the P2P transaction energy among the players. The profit fairness constraint is considered as profit balancing between the pair by determining the P2P transaction price.

B. CENTRALIZED P2P ENERGY TRANSACTION MECHANISM

Herein, two centralized P2P energy transaction mechanisms are suggested. For the central mechanism, the SESP is operated as a central controller and all players exchange information with the controller. As described earlier in the design rationale, the mechanisms consist of two parts: pair matching and profit balancing.

1) PAIR MATCHING

When the producer and consumer match, the expected maximum pair profit is determined by (19) through the optimum P2P transaction energy of (17). Therefore, the pair matching problem to maximize the social welfare is expressed as:

$$\mathbf{P1}: \max_{(i,j)} \sum\nolimits_{(i,j)\in(\mathcal{I},\mathcal{J})} \hat{B}_{(i,j)}^* \tag{20}$$

This problem is a mixed-integer problem with 0-1 binary decision variables, so it can be solved through iterative algorithms by relaxing variables such as branch-and-bound procedures [38].

Moreover, to reduce the complexity, the expected maximum pair profit, which is the objective function in (20), is approximated as:

$$\hat{B}^{*}_{(i,j)} \leq (p_B - p_S)\min(\hat{g}_i, \hat{d}_j) \propto \min\left(\hat{g}_i, \hat{d}_j\right) = \tilde{B}_{(i,j)} \tag{21}$$

caused by $\frac{\lambda_i+\lambda_j}{\lambda_i\lambda_j} e^{-\frac{\lambda_i\lambda_j}{\lambda_i+\lambda_j}|\hat{g}_i-\hat{d}_j|} \geq 0$. Therefore, the second pair matching problem is presented as:

$$\mathbf{P2}: \max_{(i,j)} \sum\nolimits_{(i,j)\in(\mathfrak{I},\mathfrak{J})} \tilde{B}_{(i,j)} \tag{22}$$

The problem in (22) is solved in a similar fashion to (20). However, it only uses the generation and demand forecasting information without its uncertainty characteristics. Therefore, it can reduce not only the information exchange between the controller and the players but also the computational complexity.

2) PROFIT BALANCING

When the pairs match, the profits between the pairs are balanced by controlling the P2P transaction price while considering profit fairness. To achieve profit fairness, the profit between the producer and consumer pair is equally balanced, as follows:

$$\hat{B}^{net}_i = \hat{B}^{net}_j = 0.5\hat{B}^{*}_{(i,j)} \tag{23}$$

In this study, the max-min fairness is considered to be the maximized value of the minimum player. This is a weak fairness constraint that guarantees the worst profit player. However, without the loss of social welfare, the profit balancing simply satisfies the profit fairness between the producer and consumer pair.

In the proposed mechanisms, using the analysis discussed in Section III, the producer and consumer pair is matched by those who have similar demand and generation characteristics for maximizing the pair benefit. The transaction price is determined to balance the profit between the pair. If the number of players is sufficient to match the pair with the same characteristics, the transaction price is decided by the uniform price, that is, the median price between the price to sell to the grid of the producer and to buy from the grid of the consumer.

C. DECENTRALIZED P2P ENERGY TRANSACTION MECHANISM

Although the problem is simplified by the approximation of the objective function, a set search in $|\mathfrak{I}| \times |\mathfrak{J}|$ is needed to operate the centralized P2P energy transaction mechanisms. Moreover, the central controller requests information from all the players. Information, especially on demand, can create security and privacy concerns.

By exchanging information between the players without a central controller, the decentralized P2P energy transaction mechanism can be designed as follows.

Decentralized P2P Energy Transaction Mechanism

Initialization

1. Set the player set $\mathfrak{I}$ and $\mathfrak{J}$.
2. Broadcast $(\hat{g}_i, \lambda_i)$ from producer $i \in \mathfrak{I}$.

P2P energy transaction matching

3. ***do while*** $\mathfrak{I} \neq \emptyset$ and $\mathfrak{J} \neq \emptyset$
4. Each consumer $j \in \mathfrak{J}$
5. Select $k \leftarrow \mathrm{argmax}_{k\in\mathfrak{I}} \hat{B}^{*}_{(k,j)}$.
6. Send the request to transaction (RTT) with $\hat{B}^{*}_{(k,j)}$ to producer k.
7. Each producer $i \in \mathfrak{I}$
8. Check the RTT and select $l \leftarrow \mathrm{argmax}_{l\in\mathfrak{J}_i} \hat{B}^{*}_{(i,l)}$ where $\mathfrak{J}_i$ is the consumer set sent from RTT to i.
9. Send the confirmation to the transaction (CTT) to consumer l.
10. Update the player sets
11. $\mathfrak{I} \leftarrow \mathfrak{I} - \{k\}$ and $\mathfrak{J} \leftarrow \mathfrak{J} - \{l\}$.
12. ***end do***

In this decentralized P2P energy transaction mechanism, each consumer requests the P2P energy transaction pairing to the prosumer to maximize the expected pair profit using the broadcasting information of prosumers and its own information. The prosumer confirms the pair matching to the consumer for whom the expected pair profit is the greatest among the consumers that sent the request. Therefore, the mechanism is a one-by-one method in which one or more transaction pairs are matched in one process. This mechanism has a linear computational burden of $\max(|\mathfrak{I}|, |\mathfrak{J}|)$. In this decentralized mechanism, the expected pair profit is exchanged between the players without detailed information. Therefore, the security and privacy problems are alleviated. For a more efficient implementation of the mechanism, a clear timer can be used as the lifetime of the RTT/CTT signal, which is similar to the information exchange procedures such as IEEE 802.11 RTS/CTS [39]. The player set update as presented in line 10 and 11 is also replaced by a determined market time when the mechanism is implemented in the real world.

The signal flow of the decentralized P2P energy transaction mechanism is expressed in Figure 2.

V. NUMERICAL RESULTS

The uncertainty of the generation in the producer and the demand in the consumer causes a loss of profit as shown in (19). Therefore, the ideal case without uncertainty such as $1/\lambda_k = 0, \forall k \in \mathfrak{I} \cup \mathfrak{J}$ is the maximum performance bound of the pair profit. Considering the profit balancing between the producer and consumer, the profit bound of the players becomes:

$$\hat{B}^{*net}_i = \hat{B}^{*net}_j \leq \frac{(p_B - p_S)}{2}\min(\hat{g}_i, \hat{d}_j) \tag{24}$$

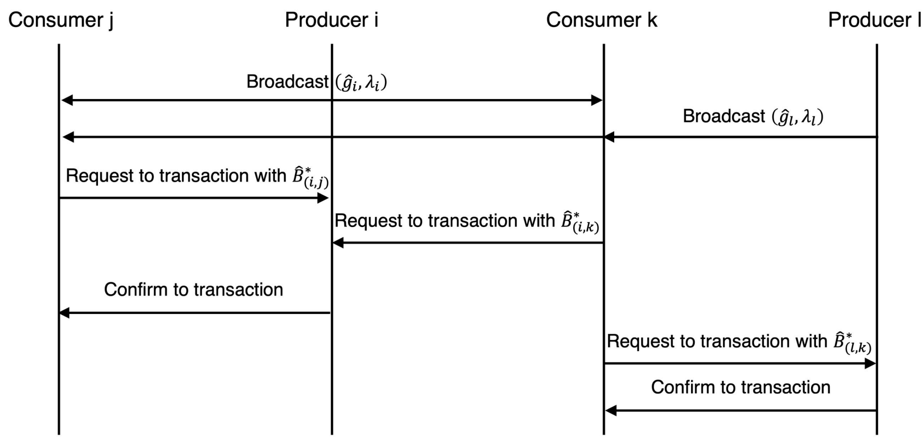

FIGURE 2. Signal flow of the decentralized P2P energy transaction mechanism.

Assuming that the player set is large enough, all players asymptotically have the unit profit bound (UPB), as follows:

$$\text{Unit Profit Bound} = \frac{(p_B - p_S)}{2} [\$/\text{kWh}] \tag{25}$$

The characteristics that affect the profit are the generation and demand forecasting and its uncertainty, as shown in (19). Therefore, the profit is expressed as a relative value. The relative demand is the demand ratio over generation forecasting, i.e. $\hat{d}_j/\hat{g}_i$. The relative λ_i and λ_j, i.e. $\tilde{\lambda}_i$ and $\tilde{\lambda}_j$, are the values that make the standard deviation of the uncertainty of the generation and demand forecasting to be x% of the forecasting values, respectively. It expresses the value of using the mean absolute percentage error (MAPE) as a forecasting performance indicator [30].

Figure 3 shows the achievement ratio of the UPB according to the change of the player's characteristics with $p_B = 0.15$ and $p_S = 0.10$. Figures 3(a) and 3(b) illustrate the UPB achievement ratio when $\tilde{\lambda}_i$ is set as 5% and 15%, respectively. As shown in the figures, the maximum achievement ratio is obtained when the generation and demand are balanced, such as the relative demand when it becomes 1. Moreover, the result shows that the profit is highly related to matching the size of the generation and demand rather than their uncertainty. As the mismatching between generation and demand increases, the influence of the uncertainty decreases by the exponential term in the second part of (19). When generation and demand is balanced, the uncertainty is mainly affected. In this case, the uncertainty affects the UPB achievement. For example, to obtain the 90% UPB achievement ratio, the prosumers with $\tilde{\lambda}_i = 5\%$ and $\tilde{\lambda}_i = 15\%$ should match the consumers with $\tilde{\lambda}_j = 23\%$ and $\tilde{\lambda}_j = 13\%$ in Figures 3(a) and 3(b), respectively. Finally, when the relative demand is less than 1, the UPB achievement decreases more in comparison to the case when the relative demand is greater than 1. The first part of (19) is responsible for this effect. Therefore, the players can selfishly choose those with forecasting values that are greater than theirs for a stable profit.

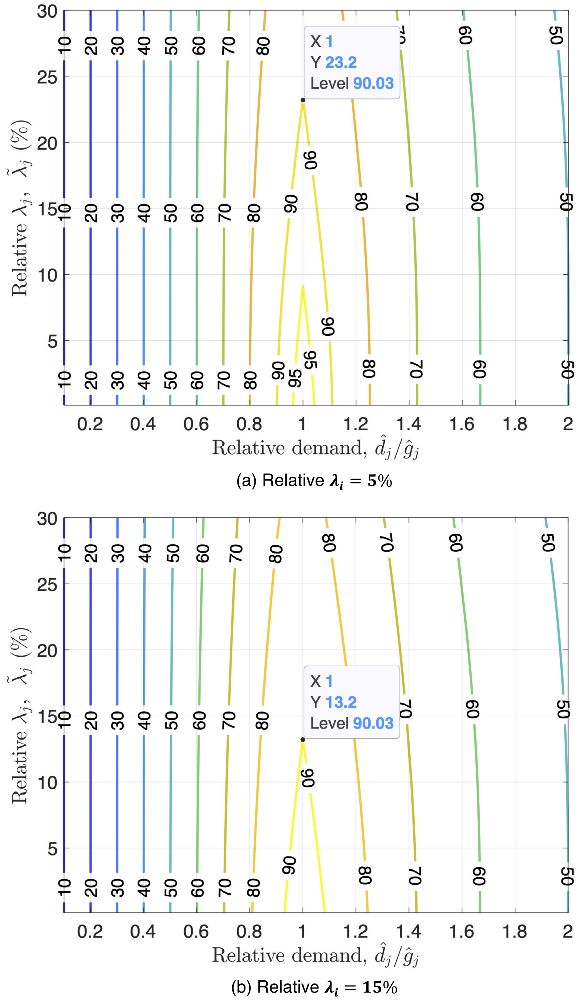

FIGURE 3. Unit profit bound achievement ratio according to the change of the player's characteristics. The relative λ_i and λ_j, i.e., $\tilde{\lambda}_i$ and $\tilde{\lambda}_j$, are the values that create the mean absolute percentage error (MAPE) of the generation and demand forecasting uncertainty, which is x%, respectively.

VI. CASE STUDY

To verify the effectiveness of the proposed mechanisms, the performance of the P2P energy transaction mechanisms is measured and the effect of the characteristics of the players is discussed. The results are compared to the optimal performance by solving the problem **P0** in (9) that requires perfect information, including future information.

This study considered the experimental environment presented in [11]. The daily energy transaction is considered to be a player trading energy within the same pair for a day. The transaction is measured every half hour. The buying and selling price to the grid is assumed to be $p_B = 0.12$ $/kWh and $p_S = 0.08$ $/kWh, which are the average electricity prices in USD [40]. The average daily demand and the uncertainty of the players is uniformly distributed in a specific range depending on the case. For the player sets, eight producers and consumers were considered, but the case for varying the set size is also described. The performances are calculated by averaging the results over 1,000 days.

A. PERFORMANCE

Table 2 shows the performance of the average unit profit when applying the proposed P2P energy transaction mechanism, **P1**, **P2**, and the decentralized method, as well as the optimal solution of the problem **P0**. Based on statistics from the U.S. Energy Information Administration [40] and the accuracy of the forecasting methods [30]–[33], the average daily demand and relative λ_j of the reference case were set to 30 kWh and 15%, respectively. It was assumed that the producer had the same characteristics as the consumer, except for the demand imbalance case.

TABLE 2. Average Unit Profit [¢/kWh].

	P0	P1	P2	Decent. method
[Ref. case] Daily demand: (20, 40) [kWh], Relative λ_j: (0,30) [%]				
Profit of a player	1.762	1.757	1.756	1.747
UPB achievement (%)	88.1	87.9	87.8	87.3
Optimal gap (%)	-	0.25	0.33	0.92
[High uncertainty] Ref. case when the relative λ_j is (20, 30) [%]				
Profit of a player	1.762	1.757	1.756	1.747
UPB achievement (%)	88.1	87.9	87.8	87.3
Optimal gap (%)	-	0.25	0.33	0.92
[Demand imbalance] Ref. case when the relative demand is 1.5.				
Profit of a player	1.762	1.757	1.756	1.747
UPB achievement (%)	88.1	87.9	87.8	87.3
Optimal gap (%)	-	0.25	0.33	0.92

In the reference case, the results applying the proposed mechanisms achieved more than 87% of the UPB, which was 2 $/kWh, and they had less than a 1% performance gap in comparison to the optimal solution of **P0**. By increasing the uncertainty that was presented in the high uncertainty case, the UPB achievement was reduced to approximately 81%. However, the performance gap of the proposed mechanisms was still less than 1%. In the demand imbalance case, the UPB achievements of the proposed mechanisms were approximately 64%. However, the numerical UPB achievement bound in this case was 65.3% (Figure 3(b)). This verified that the proposed mechanisms were well designed, and that they worked effectively.

By comparing the results of applying **P1** and **P2**, an identical performance was observed. As discussed in the previous numerical results section, this showed that the profit was highly related to the matched energy quantity rather than its uncertainty. The decentralized mechanism had a lower performance than the centralized mechanisms of **P1** and **P2**. This was because the decentralized mechanism could select a local maximum solution. The characteristics of the results are discussed in the next section.

Even with a slight performance degradation, the decentralized mechanism benefits in terms of computational complexity. The centralized methods of **P1** and **P2** are solutions of a mixed-integer problem with 0-1 binary decision variables. Therefore, the centralized methods require a computational complexity of $\mathcal{O}(|\mathcal{I}| \times |\mathcal{J}|)$ for archiving the solution. However, the decentralized method needs a computational complexity of $\mathcal{O}(\max(|\mathcal{I}|, |\mathcal{J}|))$ to solve the problem because the method matches the prosumer and consumer using a one-by-one method. Therefore, in the case of the centralized mechanisms, the computational complexity increased in the form of a square based on the size of the player sets, but in the case of the decentralized mechanism, it increases linearly.

Figure 4 shows the profit achievement related to the size of the player sets for the reference case. The black line is the solution of the optimal problem **P0** using the perfect information, including future information. The blue line with a circle and the red line with a diamond are the results obtained when applying the proposed centralized mechanisms of **P1** and **P2**, and the dashed purple line with a square is the solution of the proposed decentralized mechanism. Random matching represented by the dash-dot lines is a result of performing the pair matching randomly. The optimal exchange is the case where the optimal value of (17) is used as the P2P transaction energy, and the equal exchange is the case where the average value of the generation and demand forecasting is used.

By increasing the size of the player sets, the UPB achievement ratios that apply the optimal and proposed mechanisms also increased, as shown in Figure 4(a). This is because as the size increases, the multi-user diversity gain is enhanced, and the performance approaches the UPB [41]. However, in the case of random matching, since the multi-user diversity gain cannot be obtained, the performance converges even if the size increases. Similarly, the decentralized mechanism has less multi-user diversity gain; therefore, the optimal gap slightly increases as the size increases as shown

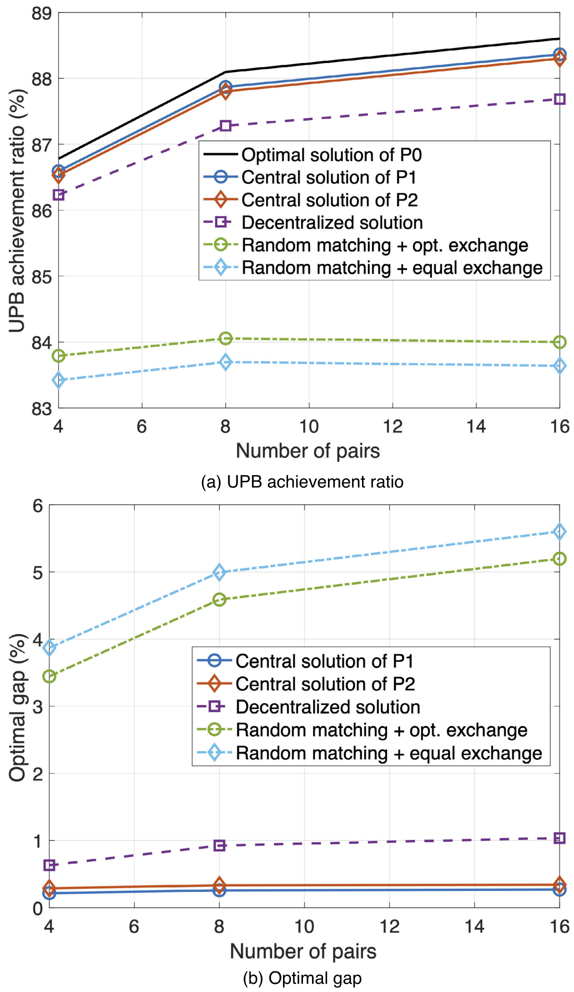

FIGURE 4. Comparison of the profit achievement with a varying player set size.

in Figure 4(b). However, the optimal gap of the decentralized mechanism is maintained when it is less than 1%. In addition, the UPB achievement gap between the proposed mechanisms and the method using random matching was approximately 3%, and it was approximately 0.5% between the optimal exchange case and the equal exchange case. This was because the P2P transaction energy was coarsely determined through pair matching and it was finely adjusted in the actual exchange.

To satisfy the fairness constraint, the Jain's fairness index is measured [42]:

$$F = \frac{(\mathbb{E}\left\{B_i^{net}\right\})^2}{\mathbb{E}\left\{(B_i^{net})^2\right\}}$$

In all cases that apply the proposed P2P energy transaction mechanisms, the fairness index was 0.97, which was the same as the optimal case. Since the Jain's fairness index is 1 for the best fairness, this indicates a very high degree of fairness. When using the uniform price as the median price between the selling price to the grid and the buying price from the grid, the fairness index increased from 0.93 to 0.95 with an increasing number of pairs—from 4 to 16—in the reference case. In the reference case, the players had similar characteristics; hence, the pair was well matched. Therefore, even if using the uniform price, the profit was allocated fairly. It can also be seen that the fairness index improved as the number of pairs increased. However, in the imbalance case, the fairness index was reduced to 0.74 when using the uniform price. This suggests that profit balancing is required by controlling the transaction price.

B. CHARACTERISTICS

As described in the previous numerical results section, the performance of the P2P energy transaction is related to the relative characteristics between the players. This section discusses how the relative characteristics are determined through the P2P energy transaction mechanisms. In Figures 5 and 6, the black dashed line, the blue line with a circle, and the red line with a square express the results using the optimal solution, the proposed central solution, and the proposed decentralized solution, respectively. Because the two central solutions using **P1** and **P2** have similar results, the result from **P1** is shown as the central solution.

Figure 5 shows the characteristics of the players using the P2P energy transaction as the reference case. The reference case is a demand balanced environment, in which the relative demand becomes 1 on average. Therefore, as shown in Figure 5(a), in the case of the optimal and central solutions, players are matched with the highest probability that the relative demand is close to 1 through the P2P energy transaction mechanisms. In particular, the central solution matches with a higher probability than the optimal solution, in which the relative demand of the transaction pair is 1. The optimal solution matches the transaction pairs while considering the perfect information—which includes the future information—however, the central solution only has stochastic information. Therefore, the central solution is more biased to demand balancing, which is a dominant factor to determine the profit. In the case of a decentralized solution, the highest probability point matched the transaction pairs, and the relative demand was less than 1. In the proposed decentralized P2P energy transaction, the customers first chose the producers. Therefore, the customers selfishly selected the producers, which generated a value that was larger than their demand to mitigate the risk using the uncertainty. However, the consumer who was not selected by the producer was matched to a high relative demand, as shown by the probability, which had a 1.4–1.6 relative demand range. Figure 5(b) shows the effect of the uncertainty through the P2P energy transaction mechanisms. At an average value of 30%, most of the transaction pairs were constructed, and there was little difference between the results using the mechanisms in the reference case.

Figure 6 shows the characteristics of the players using the P2P energy transaction for the demand imbalance case. In this case, the relative demand was 1.5 on average. Therefore, as shown in Figure 6(a), the optimal solution

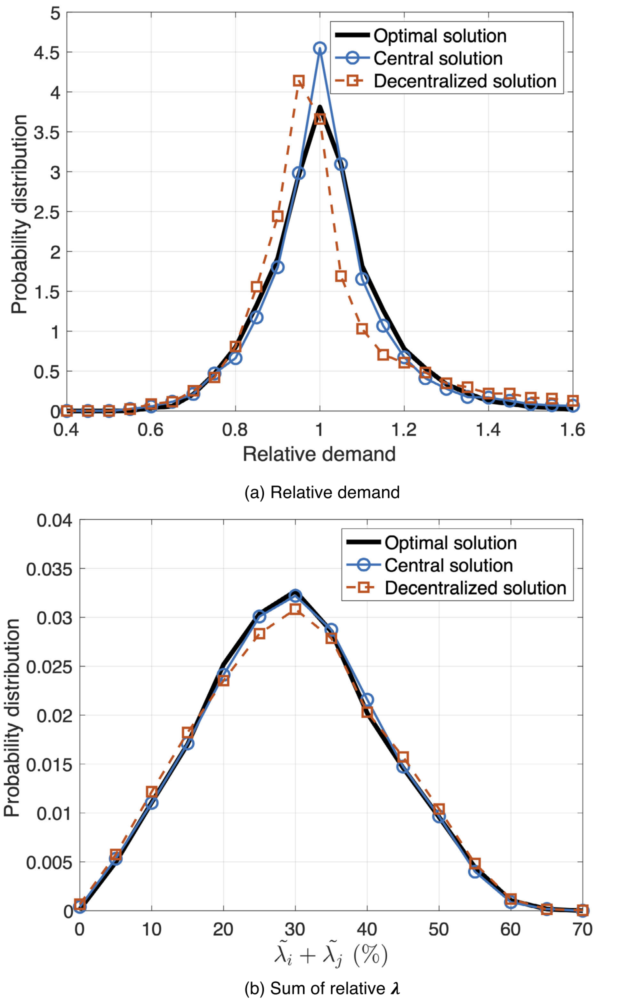

FIGURE 5. **Characteristics of the players using the P2P energy transaction as the reference case.**

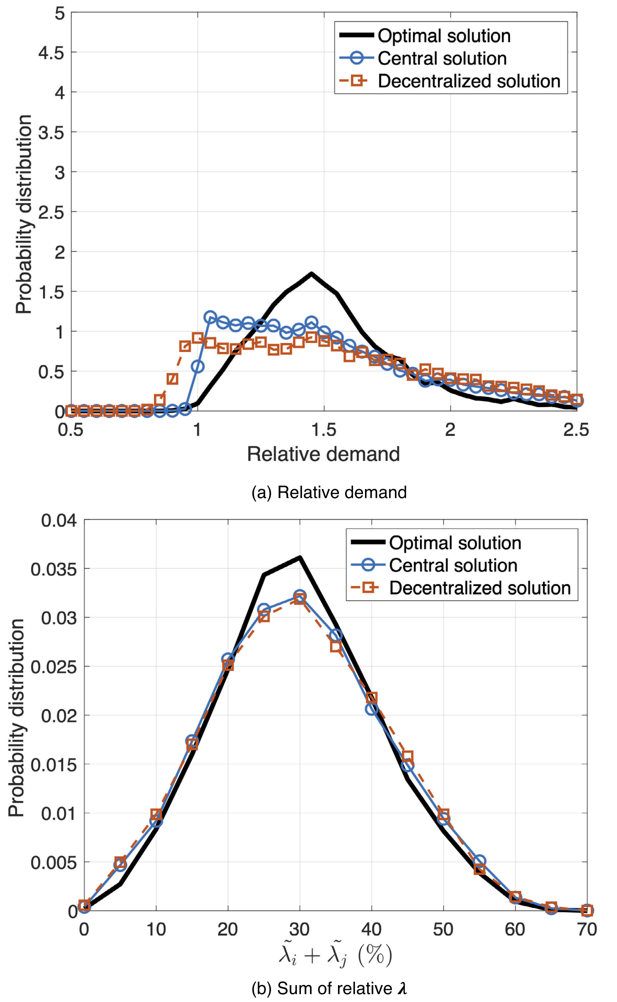

FIGURE 6. **Characteristics of the players using the P2P energy transaction as the demand imbalance case.**

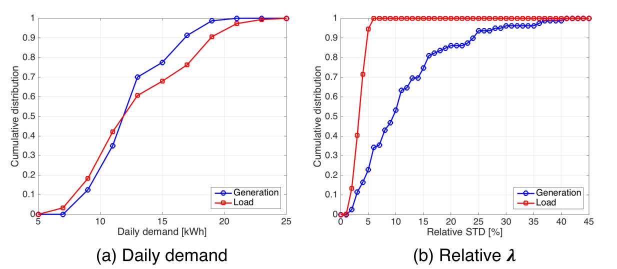

FIGURE 7. **Real demand properties.**

matched the transaction pairs, which was close to 1.5. However, the proposed central and decentralized solutions matched the transaction pairs with a similar probability via a relative demand range from 1–1.5. The proposed P2P energy transaction mechanisms used imperfect information; thus, the results were biased toward demand balancing to achieve the maximum profit. Moreover, the optimal solution in Figure 6(b) reduced the variance of the transaction pair uncertainty in comparison to the results shown in Figure 5(b). However, the proposed central and decentralized solutions shown in Figure 6(b) exhibited similar results to Figure 5(b) since the relative λ is the same in the reference case and the demand imbalance case. These differences in the characteristic analysis created an optimal gap between the optimal solution and the proposed solutions. The results suggest that the proposed P2P energy transaction mechanisms was less adaptive than the optimal method. However, the proposed mechanisms demonstrated a similar trend to the optimal method in most cases and exhibited only a slight difference in performance.

C. PRACTICAL CASE

For more practical results, we ran simulations using the sample data sets from demand profiles of wind generation and building loads presented in [11]. The generation data were collected from the Bonneville Power Administration (BPA), United States Department of Energy [43], and the building load data were recorded as part of the Korea Micro Grid Energy Project (K-MEG) [44]. The statistical properties of demand are shown in Figure 7. The performance was averaged based on the results over 30 days with eight producer and consumer sets.

TABLE 3. Average Unit Profit [¢/kWh].

	P0	P1	P2	Decent. method
Profit of a player	1.695	1.677	1.676	1.667
UPB achievement (%)	84.8	83.9	83.8	83.4
Optimal gap (%)	-	1.06	1.12	1.65

In Table 3, the performance of the average unit profit using real data when applying the proposed P2P energy transaction mechanisms, **P1**, **P2**, and the decentralized method, as well as the optimal solution of the problem **P0,** is shown. It exhibits the same trend as that of the ideal case. The optimal gap was slightly increased because the characteristics of demand and load were imbalanced. However, the gap was less than 2%. This confirms that the proposed mechanisms work well.

VII. CONCLUSION

In this study, we focused on the P2P energy transaction problem for social welfare maximization while considering the profit fairness between transaction pairs. First, we analyzed the stochastic performance of the P2P energy transaction. The key finding of the stochastic performance analysis was that the social welfare using the P2P energy transaction was maximized when a prosumer and consumer with similar characteristics were matched. The analysis also showed that the P2P energy transaction problem can be decomposed into two parts: pair matching to maximize the pair profit and profit balancing between the matched pair. When converting the P2P energy transaction problem to the pair matching problem, the P2P transaction price was decoupled from the matching problem due to the P2P energy transaction profit and cost offset between the producer and consumer. Theoretically, the optimal P2P transaction energy to maximize the pair profit and P2P transaction price to satisfy the profit fairness between the matched pair are suggested. Using these findings, we proposed two centralized mechanisms and one decentralized mechanism for the P2P energy transaction. We showed, empirically, that the proposed mechanisms performed closely to the optimal solution that was required for perfect information, i.e., within 0.5% of the optimal gap for the centralized mechanisms and within 1% of the optimal gap for the decentralized mechanisms, respectively. In addition, we discussed the relationship between the performance of the P2P energy transaction and the characteristics of the matched pairs and demonstrated that it was primarily related to the relative demand between the producer and the consumer. The theoretical performance analysis of the P2P energy transaction and the discussion of the relationship suggest a guide to design the P2P energy transaction.

This paper proposed not only the P2P energy transaction mechanisms but also the operational signal. However, for the practical implementation of the proposed mechanisms, additional considerations are required. With regard to the utility, grid constraints such as transaction capacity and congestion could be considered. With regard to the P2P energy transaction system, additional attention could be given to settlement of the service fee.

APPENDIX

A. EXPECTED PROFIT OF THE PRODUCER

The expected profit of producer i to consumer j by the P2P energy transaction with e_{ij} and p_{ij} in (10) is rewritten as:

$$\begin{aligned}\hat{B}_{ij} &= \mathbb{E}\left\{e_{ij}p_{ij} + [g_i - e_{ij}]^+ p_S + [g_i - e_{ij}]^- p_B\right\}\\ &= \mathbb{E}\left\{e_{ij}p_{ij} + [\hat{g}_i + \varepsilon_i - e_{ij}]^+ p_S + [\hat{g}_i + \varepsilon_i - e_{ij}]^- p_B\right\}\\ &= e_{ij}p_{ij} + \mathbb{E}\left\{[\varepsilon_i + l_{ij}]^+\right\} p_S + \mathbb{E}\left\{[\varepsilon_i + l_{ij}]^-\right\} p_B,\\ &= e_{ij}p_{ij} + p_S\int_0^\infty (x+l_{ij})f_{\varepsilon_i}(x)dx + p_B\int_{-\infty}^0 (x+l_{ij})f_{\varepsilon_i}(x)dx,\end{aligned} \tag{26}$$

where $l_{ij} = \hat{g}_i - e_{ij}$ and $f_{\varepsilon_i}(x)$ is the probability distribution function (PDF) of the uncertainty ε_i. In this study, the uncertainty is assumed to be a random variable that has a Laplace distribution with a zero mean and λ_i. Therefore, the PDF of ε_i becomes:

$$f_{\varepsilon_i}(x) = \frac{\lambda_i}{2}e^{-\lambda_i|x|}. \tag{27}$$

Applying the PDF of (27), each term of (26) is calculated as:

$$p_S\int_0^\infty (x + l_{ij})f_{\varepsilon_i}(x)dx = \begin{cases} p_S l_{ij} + p_S\dfrac{1}{2\lambda_i}e^{-\lambda_i l_{ij}}, & for\ l_{ij} \geq 0,\\ p_S\dfrac{1}{2\lambda_i}e^{\lambda_i l_{ij}}, & for\ l_{ij} < 0,\end{cases} \tag{28}$$

and:

$$p_B\int_{-\infty}^0 (x+l_{ij})f_{\varepsilon_i}(x)dx = \begin{cases} -p_B\dfrac{1}{2\lambda_i}e^{-\lambda_i l_{ij}}, & for\ l_{ij} \geq 0,\\ p_B l_{ij} - p_B\dfrac{1}{2\lambda_i}e^{\lambda_i l_{ij}}, & for\ l_{ij} < 0.\end{cases} \tag{29}$$

As a result, the expected profit of producer i is summarized as:

$$\hat{B}_{ij} = e_{ij}p_{ij} + [l_{ij}]^+ p_S + [l_{ij}]^- p_B - (p_B - p_S)\frac{1}{2\lambda_i}e^{-\lambda_i|l_{ij}|}, \tag{30}$$

and the expected net profit of producer i is calculated as:

$$\begin{aligned}\hat{B}_i^{net} &= \hat{B}_{ij} - \mathbb{E}\{g_i p_S\},\\ &= \hat{B}_{ij} - \hat{g}_i p_S.\end{aligned} \tag{31}$$

B. EXPECTED PROFIT OF THE CONSUMER

The expected cost of consumer j by trading to producer i in (13) is predicted as:

$$\begin{aligned}\hat{C}_{ji} &= \mathbb{E}\left\{e_{ji}p_{ji} + [d_j - e_{ji}]^{+}p_B + [d_j - e_{ji}]^{-}p_S\right\}\\ &= \mathbb{E}\left\{e_{ji}p_{ji} + [\hat{d}_j + \varepsilon_j - e_{ji}]^{+}p_B + [\hat{d}_j + \varepsilon_j - e_{ji}]^{-}p_S\right\}\\ &= e_{ji}p_{ji} + \mathbb{E}\left\{[\varepsilon_j + l_{ji}]^{+}\right\}p_B + \mathbb{E}\left\{[\varepsilon_j + l_{ji}]^{-}\right\}p_S\\ &= e_{ji}p_{ji} + p_B\int_0^{\infty}(x+l_{ji})f_{\varepsilon_j}(x)dx + p_S\int_{-\infty}^{0}(x+l_{ji})f_{\varepsilon_j}(x)dx,\end{aligned} \tag{32}$$

where $l_{ji} = \hat{d}_j - e_{ji}$ and $f_{\varepsilon_j}(x)$ is the PDF of the uncertainty ε_j with a zero mean and λ_j.

Similarly to the producer case, each term in (32) is calculated as:

$$p_B\int_0^{\infty}(x + l_{ij})f_{\varepsilon_j}(x)dx = [l_{ji}]^{+}p_B + p_B\frac{1}{2\lambda_j}e^{-\lambda_j|l_{ji}|}, \tag{33}$$

and:

$$p_S\int_{-\infty}^{0}(x + l_{ji})f_{\varepsilon_j}(x)dx = [l_{ji}]^{-}p_S - p_S\frac{1}{2\lambda_j}e^{-\lambda_j|l_{ji}|}. \tag{34}$$

Therefore, the expected cost of consumer j is summarized as:

$$\hat{C}_{ji} = e_{ji}p_{ji} + [l_{ji}]^{+}p_B + [l_{ji}]^{-}p_S + (p_B - p_S)\frac{1}{2\lambda_j}e^{-\lambda_j|l_{ji}|}, \tag{35}$$

and the expected net profit of consumer j is presented as:

$$\begin{aligned}\hat{B}_j^{net} &= \mathbb{E}\left\{d_jp_B\right\} - \hat{C}_{ji},\\ &= \hat{d}_jp_B - \hat{C}_{ji}.\end{aligned} \tag{36}$$

C. THE OPTIMUM e_{ij}^{*} TO MAXIMIZE THE EXPECTED PAIR PROFIT

The expected pair profit (16) is expressed as:

$$\begin{aligned}\hat{B}_{(i,j)} &= \hat{B}_i^{net} + \hat{B}_j^{net}\\ &= \hat{d}_jp_B - \hat{g}_ip_S\\ &\quad + \left([l_{ij}]^{+} - [l_{ji}]^{-}\right)p_S + \left([l_{ij}]^{-} - [l_{ji}]^{+}\right)p_B\\ &\quad - (p_B - p_S)\left\{\frac{1}{2\lambda_i}e^{-\lambda_i|l_{ij}|} + \frac{1}{2\lambda_j}e^{-\lambda_j|l_{ji}|}\right\}.\end{aligned} \tag{37}$$

To obtain the optimum P2P transaction energy e_{ij}, it is differentiated as follows:

$$\frac{\partial\hat{B}_{(i,j)}}{\partial e_{ij}} = \begin{cases}(p_B-p_S)-\dfrac{(p_B-p_S)}{2}\left\{e^{-\lambda_il_{ij}}+e^{-\lambda_jl_{ji}}\right\}, & \text{for } l_{ij}\geq 0,\ l_{ji}\geq 0, \quad \text{(38a)}\\ -(p_B-p_S)+\dfrac{(p_B-p_S)}{2}\left\{e^{\lambda_il_{ij}}+e^{\lambda_jl_{ji}}\right\}, & \text{for } l_{ij}\leq 0,\ l_{ji}\leq 0, \quad \text{(38b)}\\ -\dfrac{(p_B-p_S)}{2}\left\{e^{-\lambda_il_{ij}} - e^{\lambda_jl_{ji}}\right\}, & \text{for } l_{ij}\geq 0,\ l_{ji}\leq 0, \quad \text{(38c)}\\ \dfrac{(p_B-p_S)}{2}\left\{e^{\lambda_il_{ij}} - e^{-\lambda_jl_{ji}}\right\}, & \text{for } l_{ij}\leq 0,\ l_{ji}\geq 0. \quad \text{(38c)}\end{cases}$$

For the cases of (38a) and (38b), the optimum value should satisfy the condition:

$$e^{-\lambda_i|l_{ij}|} + e^{-\lambda_j|l_{ji}|} = 2. \tag{39}$$

The condition is only satisfied when:

$$l_{ij} = l_{ji} = 0, \tag{40}$$

because $e^{-\lambda_i|l_{ij}|}, e^{-\lambda_j|l_{ji}|} \leq 1, \forall i \in I, j \in J$.

For the cases of (38c) and (38d), the condition to achieve the optimum value is expressed as:

$$e^{-\lambda_i|l_{ij}|} = e^{-\lambda_j|l_{ji}|}, \tag{41}$$

and it is rewritten as:

$$\lambda_i|l_{ij}| = \lambda_j|l_{ji}|. \tag{42}$$

The condition in (42) includes the condition of (40).

As a result, using the condition of (42), the optimal transaction energy e_{ij}^{*} is measured as:

$$e_{ij}^{*} = \frac{\lambda_i}{\lambda_i + \lambda_j}\hat{g}_i + \frac{\lambda_j}{\lambda_i + \lambda_j}\hat{d}_j. \tag{43}$$

The solution is from the condition of (41) that consists of two strictly decreasing functions bounded by (0,1). Therefore, the optimal solution of (43) is the unique solution to maximize the expected pair profit in (37) [45].

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