IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received October 21, 2020, accepted November 16, 2020, date of publication November 27, 2020,
date of current version December 14, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3041030

Robust I&I Adaptive Control for a Class of Quadrotors With Disturbances

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This work was supported by the National Natural Science Foundation of China under Grant 61673050.

ABSTRACT This article presents a robust adaptive control based on Immersion and Invariance (I&I) method for a class of quadrotors subject to disturbances. The nonlinear model of the considered quadrotor including position subsystem and attitude subsystem is established, which has the characteristics of high nonlinearity, underactuation and strong coupling between the subsystems. To achieve satisfying position tracking performance, an I&I adaptive control approach is proposed for the uncertain position subsystem. To enhance the robustness of the attitude subsystem subject to disturbances, a robust adaptive control law based on disturbance observer (DO) is developed to stabilize the subsystem. The DO for the attitude subsystem is constructed to accommodate unknown external disturbances and a robust adaptive bounding law is designed to dominate the modelling errors. The ultimate boundedness of all the signals in the closed-loop system is proved by Lyapunov-based stability analysis. Experimental results performed on an actual indoor micro quadrotor indicate a better performance of the proposed controller compared with the nominal controller without robust and adaptive parts.

INDEX TERMS Quadrotor UAV, robust adaptive control, immersion and invariance, disturbance observer.

I. INTRODUCTION

Quadrotor, as one type of unmanned aerial vehicles (UAV) which consisting of four rotors with a crossing arrangement, has been increasingly used as a preferred UAV platform for various field applications. The outperformed features of quadrotor compared to other UAV are its maneuverability, vertical takeoff/landing (VTOL) ability, low cost and low maintenance [1]. It is not an easy task to synthesis a control law for a quadrotor due to its structural problems such as strongly nonlinearity, coupling characteristics and underactuation of the system.

Based on accurate model, many studies have been explored on the design of attitude and position controllers for different types of quadrotors. Backstepping controller is designed in [2] by decomposing the model into translation part and rotation part. This decomposition is widely adopted by many researchers in quadrotor controller design. A full state backstepping control for quadrotor is presented in [3] from a different perspective, in which the quadrotor model is composed of three interconnected subsystems: under-actuated subsystem, fully-actuated subsystem and rotors subsystem. In [4], to compensate the Coriolis and gyroscopic torques in attitude subsystem, a quaternion-based feedback control scheme is proposed. The compensation of nonlinearities is especially effective in the case of large-angle maneuvers.

The aforementioned control schemes are developed based on accurate model information. However, quadrotor dynamic systems practically have internal uncertainties and external disturbances as well. This fact requests researchers to find effective ways to address these unwanted terms. As a kind of robust control methods, sliding mode control (SMC) has been widely used to dominate uncertainties and disturbances in quadrotor control problem, where control input signals are switched to enforce system trajectories onto the pre-defined sliding surface. In [2], a SMC combined with backstepping is designed and verified both in simulations and experiments. In [5], a SMC is proposed to stabilize a class of underactuated systems and applied to simplified quadrotor model. A second order SMC is proposed for quadrotors in [6], where the coefficients of sliding manifolds are determined through Hurwitz stability analysis. To address chattering problem, a chattering-free SMC is presented in [7] for the quadrotor subject to known bounded disturbances. To counteract the

The associate editor coordinating the review of this manuscript and approving it for publication was Bin Xu.

mismatched disturbance, DO in conjunction with SMC is introduced in [8] for a class of systems. In [9], the result in [8] is extended to deal with the system subject to a large class of disturbances without any particular form. In [10], this DO based SMC method is successfully applied to control the quadrotor with disturbance, and the effectiveness is verified through simulations as well as real flight tests. From a compensator point of view, separating the quadrotor attitude model into nominal part and lumped uncertain part, robust attitude control methods are designed in [11] and [12] for different models to weaken the influence of lumped disturbances on system performance, and the robust properties of the closed-loop system are highlighted.

Meanwhile, adaptive control attracts great attention for the researchers to solve control problem of parametric uncertainties in quadrotor systems. In [13], adaptive backstepping techniques are utilized to compensate for the uncertain mass, however, over-parametrization is introduced as well. In [14], a novel adaptive control scheme is presented for quadrotors subject to external disturbances with varying center of mass (CoM). To solve the drawback in transient behaviour of classical adaptive control, I&I adaptive methodology provides a freedom via tuning function to shape the attractive estimation error manifold to improve the transient performance of the error system. In [15] and [16], I&I estimators are applied to deal with unknown aerodynamic damping coefficients in the position loop of a quadrotor, and the asymptotic tracking performance of the closed-loop system is proved. In [17], a saturated backstepping control strategy is developed in the position loop, where an I&I estimator is introduced to recover the mass of quadrotor.

In view of the respective advantages of robust and adaptive methodologies, a robust adaptive controller is developed for a class of quadrotors subject to disturbances in this article. To achieve Cartesian position trajectory tracking, integral action of tracking error is employed in position control to eliminate the possible steady-state error. To compensate for the parametric uncertainties in the position dynamics, including unknown aerodynamic damping coefficients, mass and thrust factor, an I&I adaptive method is introduced to recover them with guaranteed satisfying performance via shaping the converging dynamics of parameter estimation. To cope with disturbances in the attitude loop, inspired by [9], the DO is designed in control law to partially compensate a large class of disturbances without structure information. Noticing the fact that there exists uncertainties in the attitude dynamics, a robust adaptive bounding estimate law is developed to dominate the modelling errors between nominal parameters and real parameters. The contribution of this article are given as follows,

1) Considering unknown thrust factor in the actuator dynamics of quadrotors, an I&I adaptation law is designed to estimate it.
2) Under Persistence of Excitation (PE) condition, the designed I&I adaptation laws guarantees that the estimate of unknown control gain in the height subsystem converges to its true value.
3) Except for the use of nominal system parameters in control law, a robust adaptive bounding estimation law combined with the DO is developed to enhance the robustness of the attitude subsystem.
4) Experimental results performed in an actual indoor micro quadrotor are carried out in different cases, and the effectiveness of the proposed controller is verified.

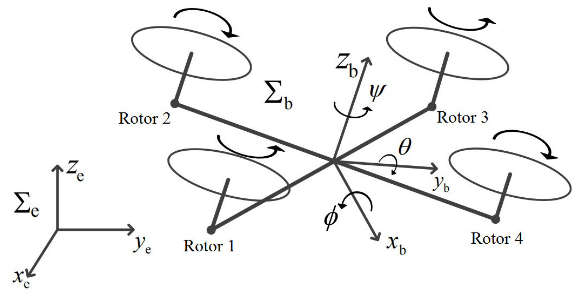

FIGURE 1. A quadrotor sketch.

The remainder of this article is organized as follows: The description of the quadrotor model and the control objective are given in Section II. The control algorithm is proposed in Section III. Section IV gives the stability analysis of overall system with several notable remarks. Experimental results are presented in Section V. Finally, the conclusions of this article and future works are given in Section VI.

Notations: Let $I_{N\times N} \in \mathbb{R}^{N\times N}$ denote a unit matrix, $0_{N\times N} \in \mathbb{R}^{N\times N}$ a zero matrix, $diag\{\cdot\}$ denote a diagonal matrix, and $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote respectively the minimum and maximum eigenvalue of a matrix.

II. QUADROTOR MODELLING AND PROBLEM FORMULATION

X-type quadrotor (whose heading direction is between the two body arms) is considered in this article, whose configuration is depicted in Fig 1. Based on right-hand rule, two reference frames are defined to describe the motion of the 6-degree of freedom rigid body: earth inertial reference frame and body-fixed reference frame denoted by $\sum_e = \{x_e, y_e, z_e\}$ and $\sum_b = \{x_b, y_b, z_b\}$, respectively.

A. QUADROTOR KINEMATICS

To describe motion information, let $\xi = [x, y, z]^T$ and $\Theta = [\phi, \theta, \psi]^T$ be the variables of the position and the attitude of the quadrotor in $\sum_e$, where the Euler angles ϕ, θ and ψ denote roll angle, pitch angle, and yaw angle, respectively. The translational kinematic of the quadrotor can be written as,

$$\dot{\xi} = R\nu \tag{1}$$

where $\nu = [\nu_x, \nu_y, \nu_z]^T$ is the velocity vector in $\sum_b$, and the rotation matrix R can be obtained through three successive rotations around the three axes of $\sum_b$ as

$$R = \begin{bmatrix} C_\psi C_\theta & -S_\psi C_\phi + C_\psi S_\theta S_\phi & S_\psi S_\phi + C_\psi S_\theta C_\phi \\ S_\psi C_\theta & C_\psi C_\phi + S_\psi S_\theta S_\phi & -C_\psi S_\phi + S_\psi S_\theta C_\phi \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{bmatrix}$$

where $C_\cdot \triangleq \cos(\cdot)$ and $S_\cdot \triangleq \sin(\cdot)$.

Let $\Omega = [\Omega_x, \Omega_y, \Omega_z]^T$ be the angular velocity of the body in $\sum_b$, and the rotational kinematic of a quadrotor describing the relationship between Θ and Ω is expressed as

$$\dot{\Theta} = T\Omega \tag{2}$$

where T is the transfer matrix defined as

$$T = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix}.$$

Assumption 1: The angle ϕ and θ satisfy $|\phi| < \pi/2$, $|\theta| < \pi/2$.

B. QUADROTOR DYNAMICS

Based on Newton-Euler formulation, the quadrotor dynamics is described by

$$F_{total} = m\dot{v} + \Omega \times mv \tag{3}$$

$$\tau_{total} = J\dot{\Omega} + \Omega \times J\Omega \tag{4}$$

where m denotes the total mass, $J = diag\{J_x, J_y, J_z\} > 0$ is the inertia matrix in $\sum_b$. F_{total} and τ_{total} include all the external forces and torques applied to the mass center of a quadrotor in $\sum_b$ as follows,

$$F_{total} = F - F_g - F_a \tag{5}$$

$$\tau_{total} = \tau - \tau_g - \tau_a + d(t) \tag{6}$$

where $F_g = mgRe_3$ with $e_3 = [0, 0, 1]^T$ is a unit vector, g is the gravitational acceleration; $F_a = K_\xi v$ and $\tau_a = K_\Theta \Omega$ are the aerodynamic friction force and torque vectors, respectively, and $K_\xi = diag\{K_{\xi 1}, K_{\xi 2}, K_{\xi 3}\}$ and $K_\Theta = diag\{K_{\Theta 1}, K_{\Theta 2}, K_{\Theta 3}\}$ are two positive diagonal aerodynamic damping matrices; $\tau_g = \sum_{i=1}^{4} J_r(\Omega \times e_3)(-1)^{i+1}\omega_i$ represents the force vector due to gyroscopic effect, in which J_r and ω_i denote the inertia of the rotor and the angular speed of rotor i, respectively; $d(t) = [d_1, d_2, d_3]^T$ is the unknown external disturbance; F and τ produced by the propellers are defined as

$$F = [0, 0, b\sum_{i=1}^{4}\omega_i^2]^T$$

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} bl(-\omega_1^2 - \omega_2^2 + \omega_3^2 + \omega_4^2) \\ \frac{\sqrt{2}}{2} bl(-\omega_1^2 + \omega_2^2 + \omega_3^2 - \omega_4^2) \\ c_d \sum_{i=1}^{4}(-1)^{i+1}\omega_i^2 \end{bmatrix} \tag{7}$$

where b is the thrust coefficient, l denotes the length from each rotor to the center of the mass and c_d is the drag coefficient.

Remark 1: To model $d(t)$, Dryden wind gust model is adopted which can be formulated as a summation of distinct sinusoidal excitations with bias [18], [19],

$$d_i(t) = d_{i0} + \sum_{k=1}^{n} a_{i,k}\sin(\varpi_{i,k}t + q_{i,k}), \quad i = 1, 2, 3 \tag{8}$$

where n is the number of sinusoidal sinusoids, $a_{i,k}$, $\varpi_{i,k}$ and $q_{i,k}$ are the amplitude, frequency and phase shift of the corresponding sinusoid, respectively, and bias d_{i0} stands for the static part of the wind gust. It can be observed that the disturbance $d_i(t)$ is continuous and bounded up to its jth derivatives, $j = 0, 1, \ldots, \infty$.

Remark 2: In this article, the nominal values of system parameters, denoted by the subscript N, are available for controller design.

Assumption 2: The desired position of the quadrotor $\xi_d = [\xi_{d1}, \xi_{d2}, \xi_{d3}]^T = [x_d, y_d, z_d]^T$, the desired yaw angle ψ_d and their jth derivatives $\xi_d^{(j)}$ and $\psi_d^{(j)}$, $j = 0, 1, 2, 3$ are bounded.

The control objective of this article is to design controller for the quadrotor subject to uncertainties and disturbances to track the desired position trajectory x_d, y_d, z_d and the desired yaw angle ψ_d.

III. ROBUST ADAPTIVE CONTROLLER DESIGN FOR THE QUADROTOR

A. POSITION CONTROLLER DESIGN

To derive the position dynamics of the considered quadrotor, the derivative of (1) is calculated as

$$\ddot{\xi} = R\dot{v} + \dot{R}v = R\dot{v} + RS(\Omega)v = R(\dot{v} + \Omega \times v) \tag{9}$$

where the skew-symmetric matrix $S(\Omega)$ is given by:

$$S(\Omega) = \begin{bmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{bmatrix}.$$

By virtue of (3), (5) and (9), the equation of the translational dynamics of a quadrotor with parameterized uncertainties in $\sum_e$ is written as

$$\ddot{\xi} = \frac{b}{m}\bar{f}Re_3 - \frac{K_\xi}{m}\dot{\xi} - ge_3 \tag{10}$$

where $\bar{f} = \sum_{i=1}^{4}\omega_i^2$ is the equivalent control input. (10) can be rewritten as

$$\ddot{\xi} = u\vartheta_2 + \varphi_0\vartheta_1 - G \tag{11}$$

where $\varphi_0 = diag\{\varphi_{01}, \varphi_{02}, \varphi_{03}\} = -diag\{\dot{x}, \dot{y}, \dot{z}\}$ is the regressor matrix, $G = [G_1, G_2, G_3]^T = [0, 0, g]^T$ is the gravity vector, $\vartheta_1 = [\vartheta_{11}, \vartheta_{12}, \vartheta_{13}]^T = \frac{1}{m}\left[K_x, K_y, K_z\right]^T$ and $\vartheta_2 = [\vartheta_{21}, \vartheta_{22}, \vartheta_{23}]^T = \frac{1}{m}[b, b, b]^T$ are unknown parameter vectors and $u = [u_1, u_2, u_3]^T = \bar{f}\left[u_x, u_y, C_\phi C_\theta\right]^T$ is the equivalent control input. The virtual control input u_x and u_y are defined as

$$u_x = \sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi,$$
$$u_y = -\cos\psi\sin\phi + \sin\psi\sin\theta\cos\phi \tag{12}$$

which are used to obtain desired roll angle ϕ_d and pitch angle θ_d of attitude system as follows,

$$\phi_d = \arcsin(u_x\sin\psi_d - u_y\cos\psi_d),$$
$$\theta_d = \arcsin\left(\frac{u_x - \sin\phi_d\sin\psi_d}{\cos\phi_d\cos\psi_d}\right). \tag{13}$$

Define the position error vector $e_\xi = \left[e_I^T, e_P^T, e_D^T\right]^T \in \mathbb{R}^9$, where $e_I = \int_0^t e_P dt \in \mathbb{R}^3$, $e_P = \xi - \xi_d \in \mathbb{R}^3$, $e_D = \dot{e}_P \in \mathbb{R}^3$, then their derivatives with respect to time are given as

$$\dot{e}_I = e_P,$$
$$\dot{e}_P = e_D,$$
$$\dot{e}_D = u\vartheta_2 + \varphi_0\vartheta_1 - G - \ddot{\xi}_d. \tag{14}$$

Due to unknown parameters ϑ_1 and ϑ_2 in (14), I&I adaptive method will be introduced to recover them.

Different from the classical adaptive method, I&I adaptive control adds a tuning function to shape estimation error convergence dynamics [20]. To begin with, define the off-the-manifold coordinate $\zeta = \left[\zeta_1^T, \zeta_2^T\right]^T \in \mathbb{R}^6$ in which $\zeta_1 \in \mathbb{R}^3$ and $\zeta_2 \in \mathbb{R}^3$ are defined as

$$\zeta_1 = \hat{\vartheta}_1 - \vartheta_1 + \beta_1(e_\xi), \quad \zeta_2 = \hat{\vartheta}_2 - \bar{\vartheta}_2 + \beta_2(e_\xi, \hat{\vartheta}_1, \ddot{\xi}_d) \tag{15}$$

where $\hat{\vartheta}_1 \in \mathbb{R}^3$ and $\hat{\vartheta}_2 \in \mathbb{R}^3$ are the estimates of ϑ_1 and $\bar{\vartheta}_2 = \left[\bar{\vartheta}_{21}, \bar{\vartheta}_{22}, \bar{\vartheta}_{23}\right]^T = \frac{1}{b}[m, m, m]^T$, respectively, $\beta_1 = [\beta_{11}, \beta_{12}, \beta_{13}]^T$ and $\beta_2 = [\beta_{21}, \beta_{22}, \beta_{23}]^T$ are tuning functions to be designed. Taking time derivative of (15) gives

$$\begin{aligned}
\dot{\zeta}_1 &= \dot{\hat{\vartheta}}_1 + \frac{\partial\beta_1}{\partial e_I^T}e_P + \frac{\partial\beta_1}{\partial e_P^T}e_D \\
&\quad + \frac{\partial\beta_1}{\partial e_D^T}(u\vartheta_2 + \varphi_0\vartheta_1 - G - \ddot{\xi}_d), \\
\dot{\zeta}_2 &= \dot{\hat{\vartheta}}_2 + \frac{\partial\beta_2}{\partial e_I^T}e_P + \frac{\partial\beta_2}{\partial e_P^T}e_D \\
&\quad + \frac{\partial\beta_2}{\partial e_D^T}(u\vartheta_2 + \varphi_0\vartheta_1 - G - \ddot{\xi}_d) \\
&\quad + \frac{\partial\beta_2}{\partial\hat{\vartheta}_1^T}\dot{\hat{\vartheta}}_1 + \frac{\partial\beta_2}{\partial\ddot{\xi}_d^T}\dddot{\xi}_d.
\end{aligned} \tag{16}$$

The control input u is proposed as follows for $i = 1, 2, 3$,

$$\begin{aligned}
u_i = (\hat{\vartheta}_{2i} + \beta_{2i})(&-k_{Pi}e_{Pi} - k_{Ii}e_{Ii} - k_{Di}e_{Di} \\
&-(\hat{\vartheta}_{1i} + \beta_{1i})\varphi_{0i} + G_i + \ddot{\xi}_{di})
\end{aligned} \tag{17}$$

where $k_{Pi} > 0$, $k_{Ii} > 0$ and $k_{Di} > 0$, $i = 1, 2, 3$ are positive control gains chosen by users.

For (16), the adaptation laws for $\hat{\vartheta}_1$ and $\hat{\vartheta}_2$ are designed as,

$$\begin{aligned}
\dot{\hat{\vartheta}}_1 &= -\frac{\partial\beta_1}{\partial e_I^T}e_P - \frac{\partial\beta_1}{\partial e_P^T}e_D - \frac{\partial\beta_1}{\partial e_D^T}(-k_Pe_P - k_Ie_I \\
&\quad -k_De_D) - \gamma_1\sigma_{\xi1}(\hat{\vartheta}_1 + \beta_1) \\
\dot{\hat{\vartheta}}_2 &= -\frac{\partial\beta_2}{\partial e_I^T}e_P - \frac{\partial\beta_2}{\partial e_P^T}e_D - \frac{\partial\beta_2}{\partial e_D^T}(-k_Pe_P - k_Ie_I \\
&\quad -k_De_D) - \frac{\partial\beta_2}{\partial\hat{\vartheta}_1^T}\dot{\hat{\vartheta}}_1 - \frac{\partial\beta_2}{\partial\ddot{\xi}_d^T}\dddot{\xi}_d \\
&\quad -\gamma_2\sigma_{\xi2}(\hat{\vartheta}_2 + \beta_2)
\end{aligned} \tag{18}$$

with $k_P = diag\{k_{P1}, k_{P2}, k_{P3}\}$, $k_I = diag\{k_{I1}, k_{I2}, k_{I3}\}$, $k_D = diag\{k_{D1}, k_{D2}, k_{D3}\}$, $\gamma_1 = diag\{\gamma_{11}, \gamma_{12}, \gamma_{13}\}$, $\gamma_2 = diag\{\gamma_{21}, \gamma_{22}, \gamma_{23}\}$, $\sigma_{\xi1} = diag\{\sigma_{\xi11}, \sigma_{\xi12}, \sigma_{\xi13}\}$ and $\sigma_{\xi2} = diag\{\sigma_{\xi21}, \sigma_{\xi22}, \sigma_{\xi23}\}$ positive gain matrices, and the tuning functions β_1 and β_2 are chosen as

$$\begin{aligned}
\beta_{1i} &= \gamma_{1i}\int_0^{e_{Di}}\varphi_{0i}(e_\xi, \chi)d\chi, \\
\beta_{2i} &= \gamma_{2i}(k_{Pi}e_{Pi}e_{Di} + k_{Ii}e_{Ii}e_{Di} \\
&\quad + \frac{1}{2}k_{Di}e_{Di}^2 - G_ie_{Di} - \ddot{\xi}_{di}e_{Di} \\
&\quad + \int_0^{e_{Di}}\varphi_{0i}(e_\xi, \chi)(\hat{\vartheta}_{1i} + \beta_{1i}(e_\xi, \chi))d\chi)
\end{aligned} \tag{19}$$

with $i = 1, 2, 3$. By choosing appropriate gains K_P, K_I, K_D, γ_1, γ_2 and σ_ξ, the position tracking error e_ξ converges to an adjustable size around zero, which is illustrated by the following theorem in detail.

Theorem 1: For the position error system (14) *and the off-the-manifold dynamics* (16)*, given the control law* (17)*, the adaptation law* (18) *as well as the tuning function* (19)*, all the signals in the closed-loop system are ultimately bounded and the position tracking error e_ξ is ultimately bounded by*

$$\|e_\xi\| \le \sqrt{\mu_\xi/\|P\|} \tag{20}$$

where $\mu_\xi > 0$ is a positive constant and $P > 0$ is a symmetric positive-definite matrix which will be given later.

Proof: Noting that (19), one can obtain that

$$\frac{\partial\beta_1}{\partial e_D^T} = \gamma_1\varphi_0, \quad \frac{\partial\beta_2}{\partial e_D^T} = -\gamma_2 r \tag{21}$$

where $r = diag\{r_1, r_2, r_3\}$ with $r_i = -k_{Pi}e_{Pi} - k_{Ii}e_{Ii} - k_{Di}e_{Di} - (\hat{\vartheta}_{1i} + \beta_{1i})\varphi_{0i} + G_i + \ddot{\xi}_{di}$, $i = 1, 2, 3$. Substituting (17), (18) and (21) into (16) yields

$$\begin{aligned}
\dot{\zeta}_1 &= \gamma_1\varphi_0(\vartheta_2'r\zeta_2 - \varphi_0\zeta_1) - \gamma_1\sigma_{\xi1}(\hat{\vartheta}_1 + \beta_1), \\
\dot{\zeta}_2 &= -\gamma_2 r(\vartheta_2'r\zeta_2 - \varphi_0\zeta_1) - \gamma_2\sigma_{\xi2}(\hat{\vartheta}_2 + \beta_2)
\end{aligned} \tag{22}$$

where $\vartheta_2' = diag\{\vartheta_{21}, \vartheta_{22}, \vartheta_{23}\}$. Moreover, (22) can be further rewritten as

$$\dot{\zeta} = -\Gamma\Phi\Phi^T\zeta - \Gamma\gamma'\sigma_\xi(\hat{\vartheta} + \beta) \tag{23}$$

where $\Gamma = diag\{\gamma_1, \gamma_2\vartheta_2'^{-1}\}$, $\Phi = \left[\varphi_0, -r\vartheta_2'\right]^T$, $\gamma' = diag\{I_{3\times3}, \vartheta_2'\}$, $\sigma_\xi = diag\{\sigma_{\xi1}, \sigma_{\xi2}\}$, $\hat{\vartheta} = \left[\hat{\vartheta}_1^T, \hat{\vartheta}_2^T\right]^T$ and

$\beta = \left[\beta_1^T, \beta_2^T\right]^T$. Then, the position error dynamics (14) can be rewritten as

$$\dot{e}_\xi = Ae_\xi + B(-\Phi^T\zeta) \tag{24}$$

where

$$A = \begin{bmatrix} 0_{3\times3} & I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & I_{3\times3} \\ -k_I & -k_P & -k_D \end{bmatrix}, \quad B = \begin{bmatrix} 0_{3\times3} \\ 0_{3\times3} \\ I_{3\times3} \end{bmatrix}.$$

Define a composite Lyapunov function $V_\xi = e_\xi^T P e_\xi + \|PB\|^2 \zeta^T \Gamma^{-1} \zeta$ with P a positive-definite matrix of the equation $A^T P + PA = -2I$, and its derivative is calculated as

$$\begin{aligned} \dot{V}_\xi &= -2e_\xi^T e_\xi - 2e_\xi^T PB\Phi^T\zeta - 2\|PB\|^2\zeta^T\Phi\Phi^T\zeta \\ &\quad -2\|PB\|^2\zeta^T\gamma'\sigma_\xi(\zeta+\vartheta) \\ &\le -2\|e_\xi\|^2 + 2\|PB\|\|e_\xi\|\|\Phi^T\zeta\| \\ &\quad -2\|PB\|^2\|\Phi^T\zeta\|^2 - 2\|PB\|^2\|\gamma'\sigma_\xi\|\|\zeta\|^2 \\ &\quad +2\|PB\|^2\|\gamma'\sigma_\xi\|\|\vartheta\|\|\zeta\| \end{aligned} \tag{25}$$

where $\vartheta = \left[\vartheta_1^T, \bar{\vartheta}_2^T\right]^T$. Using Young's inequality on the above cross terms, we have

$$\begin{aligned} \|e_\xi\|\|\Phi^T\zeta\| &\le \frac{1}{2\|PB\|}\|e_\xi\|^2 + \frac{\|PB\|}{2}\|\Phi^T\zeta\|^2, \\ \|\zeta\|\|\vartheta\| &\le \frac{1}{2}\|\zeta\|^2 + \frac{1}{2}\|\vartheta\|^2, \end{aligned}$$

then

$$\begin{aligned} \dot{V}_\xi &\le -\|e_\xi\|^2 - \|PB\|^2\|\gamma'\sigma_\xi\|\|\zeta\|^2 \\ &\quad +\|PB\|^2\|\gamma'\sigma_\xi\|\|\vartheta\|^2 \\ &\le -\kappa_\xi V_\xi + M_\xi \end{aligned} \tag{26}$$

where $\kappa_\xi = \min\{\lambda_{\min}(P^{-1}), \frac{\lambda_{\min}(\gamma')\lambda_{\min}(\sigma_\xi)}{\lambda_{\max}(\Gamma^{-1})}\}$ and $M_\xi = \|PB\|^2\|\gamma'\sigma_\xi\|\|\vartheta\|^2$. Solving (26), one has

$$V_\xi(t) \le \left(V_\xi(0) - \frac{M_\xi}{\kappa_\xi}\right)\exp(-\kappa_\xi t) + \frac{M_\xi}{\kappa_\xi}. \tag{27}$$

Therefore, $V_\xi(t)$ is ultimately bounded, then e_ξ and ζ are ultimately bounded and exponentially converge to the bound M_ξ/κ_ξ. From the definition of e_ξ, it can be obtained that ξ, $\dot{\xi}$ and $\int_0^t \xi dt$ are also bounded. The boundedness of ζ ensures the boundedness of $\hat{\vartheta} + \beta$ noting that (15). Consequently, the control law u is bounded as well. Thus, all the signals in the closed-loop system are ultimately bounded. Furthermore, denote $\mu_\xi = M_\xi/\kappa_\xi$, and from the definition of V_ξ and inequality (27), one can obtain the result in (20). Therefore, by choosing parameters K_P, K_I, K_D, γ_1, γ_2, $\sigma_{\xi 1}$ and $\sigma_{\xi 2}$, the tracking error e_ξ converges to an adjustable residual set $\sqrt{\mu_\xi/\|P\|}$. □

B. ATTITUDE CONTROLLER DESIGN

Let us define attitude angle error and angular rate error as $e_\Theta = \Theta - \Theta_d$ and $e_\Omega = \Omega - \Omega_d$, then the attitude error dynamics can be derived by virtue of (2), (4) and (6),

$$\begin{aligned} \dot{e}_\Theta &= Te_\Omega, \\ \dot{e}_\Omega &= A_1\varphi_1 + A_2\varphi_2 + A_3\varphi_3 + B\bar{\tau} + D - \dot{\Omega}_d \end{aligned} \tag{28}$$

in which $A_1 = diag\{A_{11}, A_{12}, A_{13}\} = diag\{(J_y - J_z)/J_x, (J_z - J_x)/J_y, (J_x - J_y)/J_z\}$, $A_2 = diag\{A_{21}, A_{22}, A_{23}\} = diag\{J_r/J_x, J_r/J_y, 0\}$, $A_3 = diag\{A_{31}, A_{32}, A_{33}\} = diag\{K_{\Theta 1}/J_x, K_{\Theta 2}/J_y, K_{\Theta 3}/J_z\}$, $\varphi_1 = [\varphi_{11}, \varphi_{12}, \varphi_{13}]^T = \left[\Omega_y\Omega_z, \Omega_x\Omega_z, \Omega_x\Omega_y\right]^T$, $\varphi_2 = [\varphi_{21}, \varphi_{22}, \varphi_{23}]^T = [-\Omega_y, \Omega_x, 0]^T$, $\varphi_3 = [\varphi_{31}, \varphi_{32}, \varphi_{33}]^T = \left[-\Omega_x, -\Omega_y, -\Omega_z\right]^T$, $B = diag\{B_1, B_2, B_3\} = diag\{bl/J_x, bl/J_y, c_d/J_z\}$, $\bar{\tau} = [\bar{\tau}_1, \bar{\tau}_2, \bar{\tau}_3]^T$ with $\bar{\tau}_1 = -\omega_1^2 - \omega_2^2 + \omega_3^2 + \omega_4^2$, $\bar{\tau}_2 = -\omega_1^2 + \omega_2^2 + \omega_3^2 - \omega_4^2$ and $\bar{\tau}_3 = \sum_{i=1}^{4}(-1)^{i+1}\omega_i^2$ and $D = [D_1, D_2, D_3]^T = J^{-1}d$. Define a manifold $s = [s_1, s_2, s_3]^T$ as

$$s = \Lambda e_\Theta + e_\Omega \tag{29}$$

where $\Lambda \in \mathbb{R}^{3\times3}$ is a positive-definite gain matrix, and the motion of (28) is governed by the manifold (29). Taking the derivative of s, we have

$$\begin{aligned} \dot{s} &= \Lambda\dot{e}_\Theta + \dot{e}_\Omega, \\ &= \Lambda Te_\Omega + A_1\varphi_1 + A_2\varphi_2 + A_3\varphi_3 + B\bar{\tau} + D - \dot{\Omega}_d, \end{aligned} \tag{30}$$

and then the control law should be designed such that all trajectories reach the manifold s and stay on it for all future time. The control law $\bar{\tau}$ consists of two terms as

$$\bar{\tau} = \bar{\tau}_N + \bar{\tau}_{RA} \tag{31}$$

where $\bar{\tau}_N$ is the nominal part designed to stabilize the system and $\bar{\tau}_{RA}$ for coping with uncertainties and disturbances. Considering the nominal part of the system, we design

$$\bar{\tau}_N = B_N^{-1}(-k_s s - \Lambda Te_\Omega - A_{1N}\varphi_1 - A_{2N}\varphi_2 - A_{3N}\varphi_3 + \dot{\Omega}_d) \tag{32}$$

where $A_{1N} = diag\{A_{11N}, A_{12N}, A_{13N}\} = diag\{(J_{yN} - J_{zN})/J_{xN}, (J_{zN} - J_{xN})/J_{yN}, (J_{xN} - J_{yN})/J_{zN}\}$, $A_{2N} = diag\{A_{21N}, A_{22N}, A_{23N}\} = diag\{J_{rN}/J_{xN}, J_{rN}/J_{yN}, 0\}$, $A_{3N} = diag\{A_{31N}, A_{32N}, A_{33N}\} = diag\{K_{\Theta 1\,N}/J_{xN}, K_{\Theta 2\,N}/J_{yN}, K_{\Theta 3\,N}/J_{zN}\}$ and $B_N = diag\{B_{1N}, B_{2N}, B_{3N}\} = diag\{b_N l_N/J_{xN}, b_N l_N/J_{yN}, c_{dN}/J_{zN}\}$ are known matrices with nominal parameters, and $k_s = diag\{k_{s1}, k_{s2}, k_{s3}\}$ is a designed positive-definite gain matrix. Actually, there exist deviations between the nominal values and the true ones. Since the system states and the control input do not go infinite generally, we assume that there exists a positive constant bound vector $\eta = [\eta_1, \eta_2, \eta_3]^T$ such that

$$|\tilde{A}_{1i}\varphi_{1i} + \tilde{A}_{2i}\varphi_{2i} + \tilde{A}_{3i}\varphi_{3i} + \frac{\tilde{B}_i}{B_{Ni}}\bar{\tau}_i| \le \eta_i, i = 1, 2, 3 \tag{33}$$

where $\tilde{A}_{1i} = A_{1i} - A_{1iN}$, $\tilde{A}_{2i} = A_{2i} - A_{2iN}$, $\tilde{A}_{3i} = A_{3i} - A_{3iN}$ and $\tilde{B}_i = B_i - B_{iN}$. Since η is unavailable, the adaptation law of $\hat{\eta}$ for η is designed as follows,

$$\dot{\hat{\eta}} = \gamma_\eta \tanh\left(\frac{s}{\varepsilon}\right)s - \gamma_\eta\sigma_\eta\hat{\eta} \tag{34}$$

where $\tanh(\frac{s}{\varepsilon}) \triangleq diag\{\tanh(\frac{s_1}{\varepsilon}), \tanh(\frac{s_2}{\varepsilon}), \tanh(\frac{s_3}{\varepsilon})\}$, $\gamma_\eta = diag\{\gamma_{\eta 1}, \gamma_{\eta 2}, \gamma_{\eta 3}\}$ and $\sigma_\eta = diag\{\sigma_{\eta 1}, \sigma_{\eta 2}, \sigma_{\eta 3}\}$ are positive-definite gain matrices, and $\varepsilon > 0$ is a positive constant. The DO for D is proposed as

$$\begin{aligned}\hat{D} &= p + l_\Omega e_\Omega \\ \dot{p} &= -l_\Omega \left(-k_s s - \Lambda T e_\Omega - \tanh\left(\frac{s}{\varepsilon}\right)\hat{\eta}\right)\end{aligned} \tag{35}$$

where $l_\Omega = diag\{l_{\Omega 1}, l_{\Omega 2}, l_{\Omega 3}\}$ is a positive-definite control gain matrix.

The term $\bar{\tau}_{RA}$ now can be derived as follows

$$\bar{\tau}_{RA} = B_N^{-1}\left(-\tanh\left(\frac{s}{\varepsilon}\right)\hat{\eta} - \hat{D}\right). \tag{36}$$

Theorem 2: Consider the attitude error system (28). *Given the control law* (31), (32) *and* (36), *the adaptive bounding law* (34) *as well as the DO* (35), *one has the following results: all the signals in the closed-loop system are ultimately bounded and the error signals e_Θ and e_Ω ultimately converges to a small neighborhood around its origin.*

Proof: By substituting the control law (31), (32) and (36) into (30), we have

$$\dot{s} = -k_s s - \tanh\left(\frac{s}{\varepsilon}\right)\hat{\eta} + \tilde{A}_1\varphi_1 + \tilde{A}_2\varphi_2 + \tilde{A}_3\varphi_3 + \frac{\tilde{B}}{B_N}\bar{\tau} + \tilde{D} \tag{37}$$

where $\tilde{D} = D - \hat{D}$ denotes the estimation error of DO, whose dynamics can be derived from (35) as follows,

$$\begin{aligned}\dot{\tilde{D}} &= \dot{D} - (\dot{p} + l_\Omega \dot{e}_\Omega) \\ &= \dot{D} - l_\Omega\left(\tilde{A}_1\varphi_1 + \tilde{A}_2\varphi_2 + \tilde{A}_3\varphi_3 + \frac{\tilde{B}}{B_N}\bar{\tau} + \tilde{D}\right)\end{aligned} \tag{38}$$

Define a candidate Lyapunov function $V_\Theta = s^T s + \tilde{D}^T k_s^{-1} l_\Omega^{-1}\tilde{D} + (\hat{\eta} - \eta)^T \gamma_\eta^{-1}(\hat{\eta} - \eta)$, whose derivative is

$$\begin{aligned}\dot{V}_\Theta = {} & 2s^T\left(-k_s s - \tanh\left(\frac{s}{\varepsilon}\right)\hat{\eta} + \tilde{A}_1\varphi_1 + \tilde{A}_2\varphi_2 \right. \\ & \left. + \tilde{A}_3\varphi_3 + \frac{\tilde{B}}{B_N}\bar{\tau} + \tilde{D}\right) + 2\tilde{D}^T k_s^{-1} l_\Omega^{-1}\Big(\dot{D} \\ & - l_\Omega\left(\tilde{A}_1\varphi_1 + \tilde{A}_2\varphi_2 + \tilde{A}_3\varphi_3 + \frac{\tilde{B}}{B_N}\bar{\tau} + \tilde{D}\right)\Big) \\ & + 2(\hat{\eta} - \eta)^T\left(\tanh\left(\frac{s}{\varepsilon}\right)s - \sigma_\eta\hat{\eta}\right).\end{aligned} \tag{39}$$

Noting that (33), (39) is rewritten as

$$\begin{aligned}\dot{V}_\Theta \le {} & 2s^T\left(-k_s s - \tanh\left(\frac{s}{\varepsilon}\right)\hat{\eta} + \eta + \tilde{D}\right) \\ & + 2\tilde{D}^T k_s^{-1} l_\Omega^{-1}(\dot{D} - l_\Omega(-\eta + \tilde{D})) \\ & + 2(\hat{\eta} - \eta)^T\left(\tanh\left(\frac{s}{\varepsilon}\right)s - \sigma_\eta\hat{\eta}\right) \\ \le {} & -2s^T k_s s + 2\sum_{i=1}^{3}\eta_i\left(|s_i| - s_i\tanh\left(\frac{s_i}{\varepsilon}\right)\right) \\ & + 2s^T\tilde{D} - 2\tilde{D}^T k_s^{-1}\tilde{D} + 2\tilde{D}^T k_s^{-1}\eta \\ & + 2\tilde{D}^T k_s^{-1} l_\Omega^{-1}\dot{D} - 2(\hat{\eta} - \eta)^T\sigma_\eta\hat{\eta}.\end{aligned} \tag{40}$$

Note that function $\tanh(\cdot)$ has the following property [21],

$$0 \le |\varsigma| - \varsigma\tanh\left(\frac{\varsigma}{\varepsilon}\right) \le \varrho\varepsilon, \quad \varrho = 0.2785$$

for any $\varepsilon > 0$ and $\varsigma \in \mathbb{R}$, which yields

$$\begin{aligned}\dot{V}_\Theta \le {} & -2s^T k_s s - 2\tilde{D}^T k_s^{-1}\tilde{D} + 2s^T\tilde{D} + 2\tilde{D}^T k_s^{-1}\eta \\ & + 2\tilde{D}^T k_s^{-1} l_\Omega^{-1}\dot{D} - 2(\hat{\eta} - \eta)^T\sigma_\eta\hat{\eta} \\ & + 2\sum_{i=1}^{3}\varrho\varepsilon\eta_i.\end{aligned} \tag{41}$$

Using Young's inequality on the above cross terms, we have

$$\begin{aligned} s^T\tilde{D} &\le \sum_{i=1}^{3}\frac{k_{si}}{2}s_i^2 + \sum_{i=1}^{3}\frac{1}{2k_{si}}\tilde{D}_i^2, \\ \tilde{D}^T\eta &\le \sum_{i=1}^{3}\frac{1}{4}\tilde{D}_i^2 + \sum_{i=1}^{3}\eta_i^2, \\ \tilde{D}^T\dot{D} &\le \sum_{i=1}^{3}\frac{l_{\Omega i}}{8}\tilde{D}_i^2 + \sum_{i=1}^{3}\frac{2}{l_{\Omega i}}\dot{D}_i^2, \\ -(\hat{\eta} - \eta)^T\hat{\eta} &\le -\sum_{i=1}^{3}\frac{1}{2}(\hat{\eta}_i - \eta_i)^2 + \sum_{i=1}^{3}\frac{1}{2}\eta_i^2,\end{aligned}$$

then (41) can be rewritten as

$$\begin{aligned}\dot{V}_\Theta \le {} & -\sum_{i=1}^{3}k_{si}s_i^2 - 0.25\sum_{i=1}^{3}k_{si}^{-1}\tilde{D}_i^2 \\ & - \sum_{i=1}^{3}\sigma_{\eta i}(\hat{\eta}_i - \eta_i)^2 + 4\sum_{i=1}^{3}k_{si}^{-1}l_{\Omega i}^{-1}\dot{D}_i^2 \\ & + 2\sum_{i=1}^{3}(k_{si}^{-1} + \sigma_{\eta i})\eta_i^2 + 2\sum_{i=1}^{3}\varrho\varepsilon\eta_i \\ \le {} & -\kappa_\Theta V_\Theta + M_\Theta\end{aligned} \tag{42}$$

in which $\kappa_\Theta = \min\{\lambda_{\min}(k_s), 0.25, \frac{\lambda_{\min}(\sigma_\eta)}{\lambda_{\max}(\gamma_\eta^{-1})}\}$ and $M_\Theta = 4\|k_s^{-1}l_\Omega^{-1}\|\|\dot{D}\|^2 + 2\|k_s^{-1} + \sigma_\eta\|\|\eta\|^2 + 2\varrho\varepsilon\|\eta\|$. Note that $\dot{D}$ is bounded according to Remark 1. Thus, following a similar procedure from the proof of Theorem 1, it is concluded that the attitude error e_Θ and e_Ω, the surface s, the bounding estimate $\hat{\eta}$, the variable p, the DO $\hat{D}$ and the control input $\bar{\tau}$ are ultimately bounded. Also, by the definition of V_Θ, the ultimate bound of s can be found as $\|s\| \le \sqrt{M_\Theta/\kappa_\Theta}$. By the definition of s, the size of e_Θ and e_Ω can be tuned through the selection of parameters Λ, k_s, l_Ω, γ_η, σ_η and ε. □

IV. STABILITY AND PERFORMANCE ANALYSIS

This section presents the stability analysis of the whole system with some essential remarks. By virtue of Theorem 1 and Theorem 2, it is very straightforward for us to obtain the stability for the whole closed-loop system via the following theorem.

Theorem 3: For the whole error system of the quadrotor (14), (28), *it is closed with the control law* (17), (31), (32) *and* (36), *I&I adaptive law* (18), *the tuning function* (19), *the adaptive bounding law* (34) *as well as the DO* (35) *such that: 1) all the signals in the closed-loop system are uniformly* *and ultimately bounded; 2) overall closed-loop tracking error e_ξ and e_Θ ultimately converges to a small neighborhood around the origin.*

Proof: It is straightforward to complete the proof. Define an extended Lyapunov candidate as $V = V_\xi + V_\Theta$, it time derivative can be simply obtained by (26) and (42):

$$\dot{V} \leq -\kappa V + M \tag{43}$$

where

$$\begin{aligned}\kappa &= \min\{\lambda_{\min}(P^{-1}), \frac{\lambda_{\min}(\gamma')\lambda_{\min}(\sigma_\xi)}{\lambda_{\max}(\Gamma^{-1})}, \\ &\quad \times \lambda_{\min}(k_s), 0.25, \frac{\lambda_{\min}(\sigma_\eta)}{\lambda_{\max}(\gamma_\eta^{-1})}\} \\ M &= \|PB\|^2\|\gamma'\sigma_\xi\|\|\vartheta\|^2 + 4\|k_s^{-1}l_\Omega^{-1}\|\|\dot{D}\|^2 \\ &\quad +2\|k_s^{-1} + \sigma_\eta\|\|\eta\|^2 + 2\varrho\varepsilon\|\eta\|.\end{aligned}$$

Taken a similar procedure previously as the proof of Theorem 1 and 2, all the signals in the closed-loop system are uniformly and ultimately bounded, and the sizes of position and attitude errors e_ξ and e_Θ are tunable by choosing appropriate control parameters. However, it is a process of trade-off for users to choose these control parameters. From a practical point of view, it is not desirable for us to increase the control gains arbitrarily, since unmodeled high frequency dynamics of the system might be excited and can even lead to instability. □

Remark 3: Although we do not assume external disturbance appearing in the position dynamics, the disturbance in the attitude loop may enlarge the steady-state error of position. For instance, a hovering quadrotor, when being affected by sustained external wind gusts in the horizontal direction, would be apart from its original position on x–y plane. In this situation, the existence of integral action is able to drive the quadrotor back to its initial position by increasing desired roll angle or pitch angle (since horizontal motion is driven by roll and pitch actions).

Some issues about adaptation laws are discussed here. Neither the traditional Lyapunov-based adaptive control nor the I&I adaptive control can be said to guarantee that the estimates can converge to their true values, since PE condition of the system is not always satisfied, which is discussed by the following lemma [22]:

Lemma 1: The regressor vector $\rho \in \mathbb{R}^k, k > 0$ is PE if there exist $\delta > 0$ such that

$$\int_t^{t+T} \rho(\chi)\rho(\chi)^T d\chi \geq \delta I \tag{44}$$

for some $T > 0$ and $\forall t \geq 0$. The estimation error globally exponentially converges to zero, if and only if the regressor ρ is persistently-excited.

Theorem 4: Consider the off-the-manifold dynamics (23)*. The last term ζ_{23} of the vector ζ is exponentially stable, i.e., the estimate $\hat{\vartheta}_{23} + \beta_{23}$ in the height control converges to its actual value exponentially if the parameter $\sigma_{\xi 23}$ in σ-modification is set to zero.*

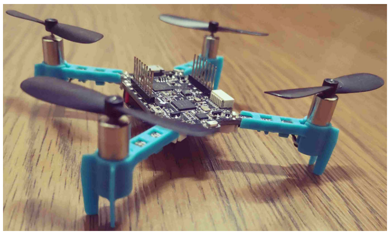

FIGURE 2. Micro quadrotor experimental platform.

Proof: For the dynamics of ζ_{23} of (23), we can obtain

$$\dot{\zeta}_{23} = -\gamma_{23}\vartheta_{23}'^{-1}r_3^2\zeta_{23} - \gamma_{23}\sigma_{\xi 23}(\hat{\vartheta}_{23} + \beta_{23}) \tag{45}$$

where r_3 contains term g (for compensating the gravity effect), which makes r_3 always larger than zero, i.e., $r_3 > 0, \forall t \geq 0$. By virtue of Lemma 1, the integral of r_3^2 is greater than zero over any time interval, thus, the signal r_3 is PE. By setting $\sigma_{\xi 23} = 0$, we have

$$\begin{aligned}\dot{\zeta}_{23}(t) &= -\gamma_{23}\vartheta_{23}'^{-1}r_3^2(t)\zeta_{23}(t) \leq -\lambda\zeta_{23}(t) \\ &\Rightarrow \zeta_{23}(t) = \zeta_{23}(0)\exp(-\lambda t)\end{aligned} \tag{46}$$

where $\lambda = \inf_{t\geq 0}\{\gamma_{23}\vartheta_{23}'^{-1}r_3^2(t)\} > 0$. Thus, we can draw the conclusion that the estimation error ζ_{23} exponentially decays to zero, and $\lim_{t\to\infty}(\hat{\vartheta}_{23} + \beta_{23}) = \vartheta_{23}$ is achieved. □

From a practical perspective, it is also rather intuitive that a quadrotor flies and hovers in the air by resisting the effect of gravity, and the thrust force generated by propellers always points upward with respect to its body frame, thus the corresponding *regressor* of unknown control gain of height control is guaranteed to be positive. However, other estimates in this article may not satisfy condition (44).

Remark 4: It can be observed that in the estimation of unknown control gains of position control, over-parametrization is employed where each element of the vector $\hat{\vartheta}_2 + \beta_2$ is designed to estimate the same unknown parameter $\frac{m}{b}$. However, consider the discussion above, it is possible to replace the estimates of horizontal ones $\hat{\vartheta}_{21} + \beta_{21}$ and $\hat{\vartheta}_{22} + \beta_{22}$ with the PE-guaranteed $\hat{\vartheta}_{23} + \beta_{23}$. Therefore, it is able for us to remove the undesired feature of over-parametrization. This skill will be used in the experiments next section.

V. EXPERIMENTAL RESULTS

In this section, experimental results are carried out to illustrate the effectiveness of the proposed control scheme applied to an indoor micro quadrotor shown in Fig.2. The onboard electronic system of the quadrotor consists of a flight control computer based on STM32F411 and a sensor system. The sensor system includes an optical flow sensor, an air pressure sensor, a 3-axis gyroscope and a 3-axis accelerometer, and the motion data of quadrotor such as its position, attitude and their velocities can be available. Four coreless motors with the rated speed of 50000 rad/min is equipped on the quadrotor. All onboard components are powered by one 3.7V battery. The nominal parameters of the UAV are shown in Table 1.

TABLE 1. Quadrotor nominal physical parameters.

Parameter	Description	Nominal value
m	Quadrotor mass	0.0337kg
l	Quadrotor arm length	0.05m
g	Gravity acceleration	$9.8m/s^2$
J_x	Roll inertia	$1.5 \times 10^{-5} kg \cdot m^2$
J_y	Pitch inertia	$1.5 \times 10^{-5} kg \cdot m^2$
J_z	Yaw inertia	$2.7 \times 10^{-5} kg \cdot m^2$
J_r	Rotor inertia	$1 \times 10^{-7} kg \cdot m^2$
b	Thrust factor	$2.5 \times 10^{-7} N \cdot s^2/rad^2$
c_d	Drag factor	$5{\times}10^{-10} Nm{\cdot}s^2/rad^2$
K_ξ, K_Θ	Aerodynamic factor	0 (not measured)

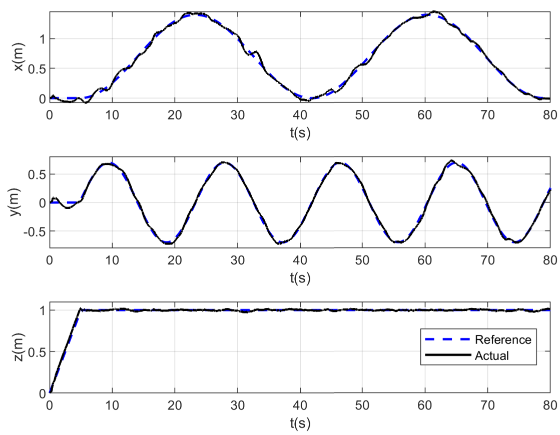

FIGURE 3. Position tracking in case 1.

To highlight the performance of the proposed controller, two cases of experiments are implemented. In the first case, the quadrotor is required to track a time-varying trajectory. The second case is a set-point test subject to wind disturbance compared with a nominal controller. The initial values of system states and adaptation laws are all set as 0, except for $\hat{\vartheta}_{23}(0) = 1.155 \times 10^5$. The control parameters are listed as: 1) for position controller: $K_P = diag\{4, 4, 50\}$, $K_I = diag\{2, 2, 30\}$, $K_D = diag\{4, 4, 20\}$, $\gamma_1 = diag\{1, 1, 1\}$, $\gamma_{23} = 100$ (note that Remark 4, $\hat{\vartheta}_{21} + \beta_{21}$ and $\hat{\vartheta}_{22} + \beta_{22}$ are not put into use), $\sigma_{\xi 1} = diag\{0.01, 0.01, 0.01\}$ and $\sigma_{\xi 23} = 0$; 2) for attitude controller: $\Lambda = diag\{5, 5, 5\}$, $k_s = diag\{80, 80, 80\}$, $l_\Omega = diag\{2, 2, 5\}$, $\gamma_\eta = diag\{1, 1, 1\}$, $\sigma_\eta = diag\{0.1, 0.1, 0.1\}$ and $\varepsilon = 0.1$.

A. CASE 1: PATH FOLLOWING

In this case, the quadrotor is required to track a time-varying path. The references for horizontal motion are two sinusoidal signals of distinct frequencies with the magnitude of 0.7m. The desired height is 1m and the desired yaw angle is 0. Experiment results of this case are shown in Fig.3-7.

In Fig.3 and Fig.4, the position tracking errors are converged to a small range, where the tracking errors of x and y direction are kept about 0.1m, and the height error does not exceed 0.03m. Fig.5 and Fig.6 illustrates the attitude tracking performance. Furthermore, the time evolution of $\hat{\vartheta}_{23} + \beta_{23}$ is depicted in Fig.7, where the estimated value is kept tightly

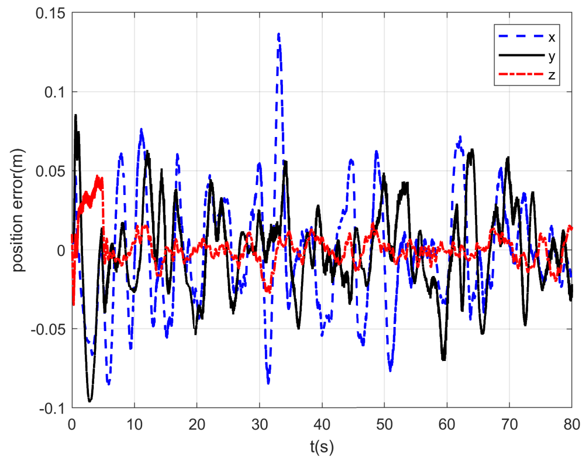

FIGURE 4. Position tracking error in case 1.

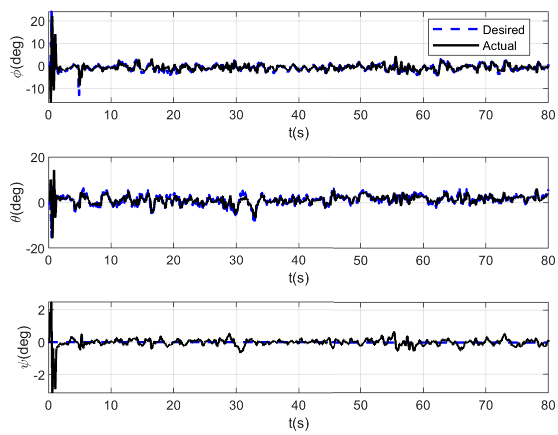

FIGURE 5. Attitude tracking in case 1.

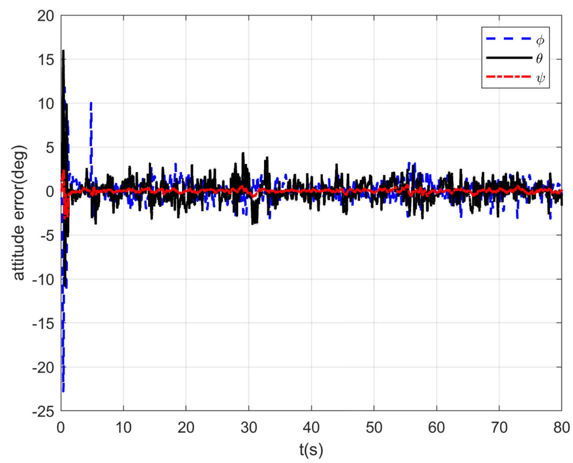

FIGURE 6. Attitude tracking error in case 1.

around its nominal value with zero σ-modification parameter, which endorses the claim of Theorem 4.

B. CASE 2: PERFORMANCE COMPARISON

In this case, the control objective is to regulate the quadrotor to point $[x_d, y_d, z_d]^T = [1, 1, 1]^T$ and $\psi_d = 0$ with zero initial conditions. Note that in order to avoid large maneuver, slope signals are added in position reference in the first 5s. For

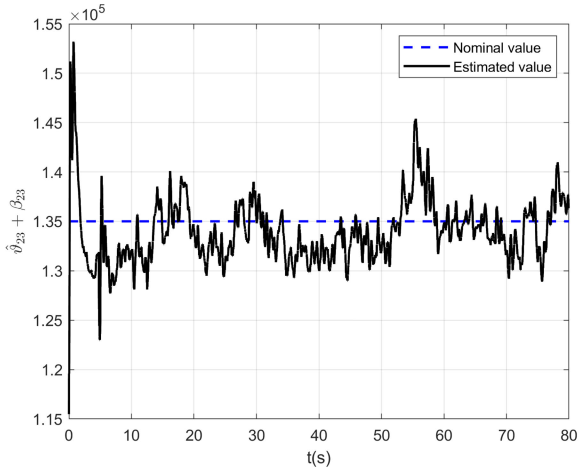

FIGURE 7. Estimate of $\bar{\vartheta}_{23}$ in case 1.

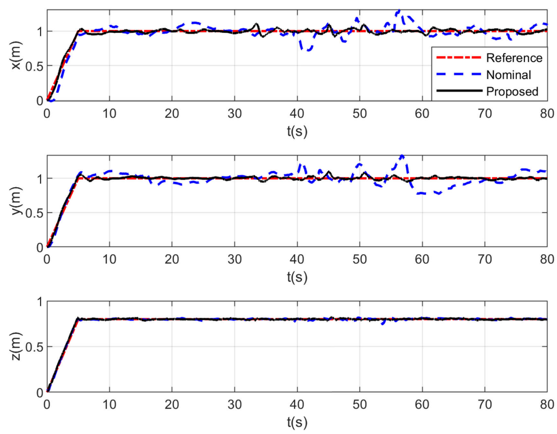

FIGURE 8. Position tracking in case 2.

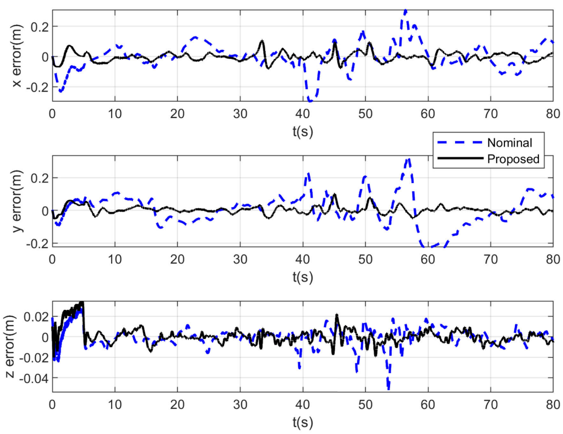

FIGURE 9. Position tracking error in case 2.

the sake of performance comparison, a nominal information based, non-robust adaptive controller is also implemented in the quadrotor platform, whose position part is given by

$$u_N = m_N(-k_P e_P - k_I e_I - k_D e_D + G + \ddot{\xi}_d)/b_N \quad (47)$$

with the same control parameters as our proposed controller. The attitude controller is the same as $\bar{\tau}_N$ in (32) as well. Also, to test the robustness of the two kinds of controllers,

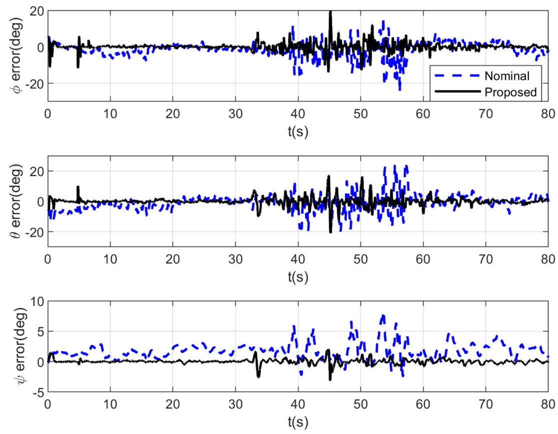

FIGURE 10. Attitude tracking error in case 2.

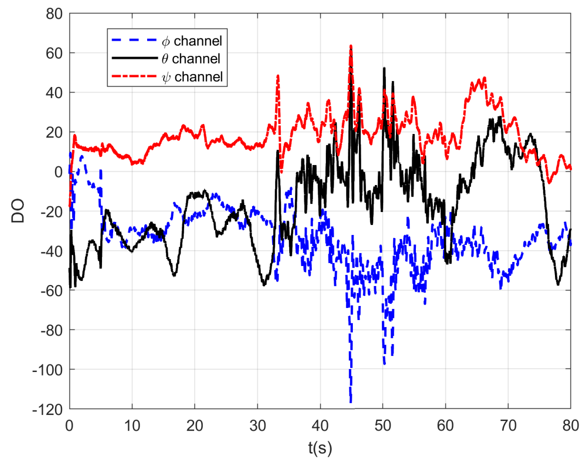

FIGURE 11. DO in case 2.

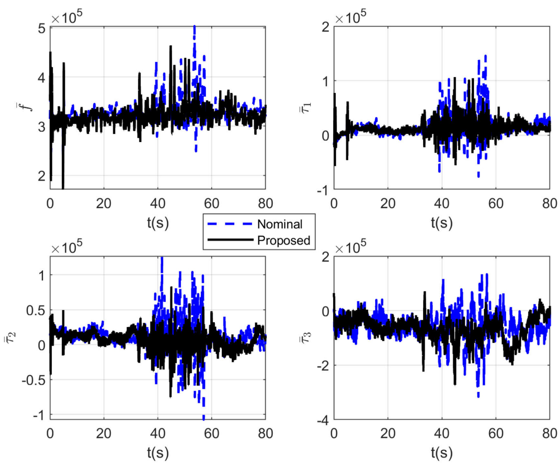

FIGURE 12. Control inputs in case 2.

an electrical fan is used to generate wind disturbance acting on the quadrotor body at a fixed distance. The electrical fan is switched on at approximate 33s and switched off at approximate 60s. Comparison results of this case are presented in Fig.8-12.

In Fig.8-10, it can be observed that when the wind disturbance is actuated, both the position and attitude tracking errors of the nominal controller increase significantly while the proposed one is less effected. In addition to that, when there is no disturbance, the proposed controller also remarkably outperforms the nominal one. The estimated values of DO in three angle channels are shown in Fig.11, and it can be seen that wind disturbances are effectively detected by DO. Finally, Fig.12 presents the time profile of control inputs of the two controllers.

VI. CONCLUSION

A robust adaptive full control for a class of quadrotor UAVs is presented in this article. To compensate for the parameters of the uncertainties in position subsystem such as aerodynamic damping coefficient, mass and thrust factor, an I&I based adaptive control approach is proposed. To stabilize the attitude subsystem, a DO based robust adaptive control is designed, where the DO in the control law is to accommodate unknown external disturbances and a robust adaptive bounding law to dominate the modelling errors. By Lyapunov-based stability analysis, the ultimate boundedness of all the signals in the closed-loop system is proved. In order to verify the effectiveness of the proposed controller, experimental results in two different cases are carried out to illustrate its good performance comparing to the controller without robust adaptive technique. Future work will involve the implementation of this control strategy in the formation control of multiple quadrotor UAVs.

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