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Delay-Dependent H_∞ Control for Singular Markovian Jump Systems With Generally Uncertain Transition Rates

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ABSTRACT This article is devoted to the problem of H_∞ control for a class of singular Markovian jump systems with time-varying delay and generally uncertain transition rates, which means each transition rate is completely unknown or only its estimated value is known. By using Lyapunov stability theory, a new delay-dependent H_∞ admissible criterion in terms of strict linear matrix inequalities is obtained, which guarantees that the singular Markovian jump system with known transitions rates is regular, impulse-free and stochastically stable with a prescribed H_∞ disturbance attenuation level γ. Based on this obtained criterion, some suitable state feedback controllers are designed such that the closed-loop delayed singular Markovian jump system with generally uncertain transition rates is H_∞ stochastically admissible. Finally, numerical examples are included to illustrate the effectiveness and the less conservativeness of the proposed method.

INDEX TERMS H_∞ control, singular Markovian jump systems, time-varying delays, generally uncertain transition Rates.

I. INTRODUCTION

Singular systems, also known as generalized systems, descriptor systems, differential-algebraic systems or implicit systems, can provide comprehensive and natural representations in the description of many physical systems, such as electrical circuits, economic systems, robotic manipulator systems, chemical systems and other practical systems [1]–[5]. And Markovian jump systems have been extensively studied in the past decades [6]–[14] due to the better describing dynamic systems subject to abrupt variations including abrupt environment disturbances, changing subsystem interconnections and random component failures or repairs [15]–[17]. When the singular systems experience the aforementioned random abrupt changes in their parameters or structures, they can be effectively modeled as singular Markovian jump systems (SMJSs) [18]. For example, in networked control systems, network delay or packet dropouts often occur randomly. An effective method is to use Markov process to describe such phenomena and model the system as a SMJSs, and then a controller is designed to eliminate the influence of these random factors. In recent years, considerable attention has been paid to the stability analysis, controller synthesis, and filtering problems of SMJSs, see [19]–[35], and the references therein.

On the other hand, time-delays are necessary to be considered since they can lead to serious performance degradation or system instability [36]. There are two types of literature on the study of the systems with time-delay: delay-independent results and delay-dependent results. In recent years, more and more attention has been paid to the delay-dependent results [6], [22], [23], [30]–[32], [38]. For example, in a recent article [38], a new approach to analyze delay-dependent stability of linear impulsive delay systems is proposed based on the delay-partitioning method, the time-dependent Lyapunov functional method, and the looped-functional method. It is well known that the delay-dependent results are less conservative than the delay-independent ones especially when the time-delay is small. Recently, numerous articles analyze the admissibility and stabilization of SMJSs with or without time delays. In [29], the authors dealt with the admissibility analysis of stochastic SMJSs with time delays and presented a more general condition for the existence and uniqueness of the impulse-free solution to delayed

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SMJSs. The problems of mean-square exponential stability for discrete-time nonlinear SMJSs were investigated in [30], and based on the free-weighting matrix method, the authors presented a delay-dependent sufficient condition which guarantees the considered systems to be mean-square exponentially stable. In [31], on the basis of the delay subinterval decomposition approach, the authors proposed a strict linear matrix inequality (LMI) sufficient condition to guarantee SMJSs stochastically admissible with H_∞ performance γ. In [35], the problems of optimal H_∞ filtering problem for SMJSs were considered, and a necessary and sufficient condition in terms of strict LMIs for optimal H_∞ filtering was derived.

What should be noticed is that the results in the above-mentioned literature are under the assumption of completely known transition rates (TRs). However, in practice, the TRs in some jumping processes are difficult to precisely acquire due to the limitations of equipment and the influence of uncertain complex factors. For example, as a result of the existence of the packet dropout and channel delays in networked control systems, the TRs cannot be measured precisely, and even if it is measured, the cost may be high [23] . At present, there are three types of descriptions of uncertain TRs. The first one is bounded uncertain transition rates (BUTRs), where the exact value of each TR may be unknown, but its boundaries (upper and lower bounds) are known [14]. The second one is partly unknown transition rates (PUTRs), where each TR is either completely known or completely unknown, see [20]–[21]. However, in practice, either the aforementioned two cases are too difficult to be satisfied or the cost is too much. For example, working modes of vertical takeoff and landing aircraft need to be determined according to the wind speed, but the accurate value of the transition probability matrix of wind speed changes is very difficult to obtain. Generally, only a certain value range can be obtained, sometimes even completely unknown [37]. Based on the above factors, some researchers proposed a new type of TRs, namely general unknown transition rates (GUTRs), where each TR can be completely unknown or only its estimated value is available [13]. Compared with the first two cases BUTRs and PUTRs, the case of GUTRs is more general (in fact, BUTRs and PUTRs are the special cases of GUTRs), and has a wider scope of application [13], [22]–[28].

It is worth noting that although scholars have done extensive researches on SMJSs in recent years, there are few researches on the control problem of SMJSs under GUTRs, especially considering the time-varying delays and H_∞ performance simultaneously. Currently, H_∞ control for a class of nonlinear stochastic SMJSs with GUTRs has been investigated in [25] using adaptive control, but the sufficient condition in [25] includes non-strict matrix inequality which cannot be solved directly by standard LMI solvers. Therefore, finding a more general strict LMI sufficient condition for stochastic admissibility of SMJSs with time-varying delay and GUTRs and seeking more effective techniques to design a controller that can ensure the closed-loop system is H_∞ stochastically admissible are of great significance.

Motivated by the aforementioned discussion, this article is concerned with H_∞ control for a class of SMJSs with time-varying delays and GUTRs. The main contributions are as follows: (i) a new delay-dependent sufficient condition for H_∞ stochastically admissible of SMJSs is derived, which is less conservative than some existing methods; (ii) sufficient condition is obtained to make the SMJSs with GUTRs and time-varying delays stochastically admissible with a prescribed H_∞ performance index γ; (iii) the desired H_∞ state feedback controller is designed by solving a set of strict LMIs.

Notation: Throughout this article, $\mathbb{R}^{m\times n}$ denotes the set of all $m \times n$ real matrices, and $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathbb{N}^+$ represents the set of positive integers. $\|\cdot\|$ represents for the Euclidean norm for a vector. The symbol $Sym\{\cdot\}$ denotes $Sym\{M\} = M + M^T$ for any square matrix M, $diag(\cdot)$ represents a block diagonal matrix. $\mathcal{L}_2[0,\infty)$ stands for the space of square integrable functions on $[0,\infty)$. $(\Omega, \mathcal{F}, \mathcal{P})$ is a probability space, Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $\varepsilon\{\cdot\}$ denotes the expectation operator with respect to some probability measure $\mathcal{P}$. The superscript 'T' and '−1' represent the transpose and the inverse of a matrix, respectively, and '$*$' denotes the term that is induced by symmetry.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following continuous-time SMJSs defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ as follows

$$\begin{cases} E\dot{x}(t) = A(r_t)x(t) + A_d(r_t)x(t-d(t)) + B(r_t)u(t) \\ \qquad\quad + B_\omega(r_t)\omega(t) \\ z(t) = C(r_t)x(t) + D(r_t)u(t) \\ x(t) = \phi(t), \quad t \in [-\bar{d}, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $z(t) \in \mathbb{R}^q$ is the controlled output, $\omega(t) \in \mathbb{R}^p$ is the external disturbance input which belongs to $\mathcal{L}_2[0,\infty)$. $\phi(t)$ is a compatible vector valued initial function. The matrix $E \in \mathbb{R}^{n\times n}$ may be singular with rank $(E) = r \le n$.$A(r_t), A_d(r_t), B(r_t), B_\omega(r_t), C(r_t)$, and $D(r_t)$ are known real constant matrices with appropriate dimensions for each $r_t \in \mathcal{S}$. $d(t)$ is the time-varying delay satisfying

$$0 < d(t) \le \bar{d}, \quad \dot{d}(t) \le \mu \tag{2}$$

where $\bar{d}$ is the time delay upper bound, and μ is the upper bound of time delay variation rate.

In system (1), the mode jumping process $\{r_t, t \ge 0\}$ is a right-continuous Markov process taking values in a finite set $\mathcal{S} = \{1, 2, \cdots, s\}$ with the mode transition probabilities

$$\mathbb{P}_r\{r_{t+h} = j \mid r_t = i\} = \begin{cases} \pi_{ij}h + o(h), & i \ne j, \\ 1 + \pi_{ij}h + o(h), & i = j. \end{cases}$$

where $h > 0$, $\lim_{h\to 0} \frac{o(h)}{h} = 0$, and $\pi_{ij} \ge 0$, for $i \ne j$, is the transition rate from mode i at time t to mode j at time $t+h$, and $\pi_{ii} = -\sum_{j=1,j\neq i}^{s}\pi_{ij}$. The transition rate matrix $\Pi \triangleq \{\pi_{ij}\}$, is considered to be generally uncertain, which can be expressed as

$$\begin{bmatrix} \hat{\pi}_{11}+\Delta_{11} & ? & \hat{\pi}_{13}+\Delta_{13} & \cdots & ? \\ ? & ? & \hat{\pi}_{23}+\Delta_{23} & \cdots & \hat{\pi}_{2s}+\Delta_{2s} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ ? & \hat{\pi}_{s2}+\Delta_{s2} & ? & \cdots & \hat{\pi}_{ss}+\Delta_{ss} \end{bmatrix} \quad (3)$$

where $\hat{\pi}_{ij}$ and $\Delta_{ij} \in [-\delta_{ij}, \delta_{ij}]$ $(\delta_{ij} \geq 0)$ are the estimated value of the uncertain transition rate π_{ij} and the estimated error bound, respectively. And both $\hat{\pi}_{ij}$ and δ_{ij} are known. "?" represents the completely unknown transition rates, which means its estimate value $\hat{\pi}_{ij}$ and estimate error bound δ_{ij} are the completely unknown. In a Markovian jump system, if the system is in mode i and the transition rate to mode j at the next moment cannot be measured, then the position of ith row and jth column in the transition rate matrix Π is "?".

For all $i \in \mathcal{S}$, denote the set U^i by $U^i = U_k^i \cup U_{Uk}^i$ with $U_k^i \triangleq \{j$: The estimated value of π_{ij} is known for $j \in \mathcal{S}\}$ and $U_{Uk}^i \triangleq \{j$: The estimated value of π_{ij} is unknown for $j \in \mathcal{S}\}$. Moreover, if $U_k^i \neq \emptyset$, it is further described as $U_k^i = \{k_1^i, k_2^i, \cdots, k_m^i\}$, where $k_m^i \in \mathbb{N}^+$ represents the mth bound-known element with the index k_m^i in the ith row of the transition rate matrix Π. And then, the following three assumptions can be defined reasonable, since they can be directly derived from the features of transition rates $\pi_{ij} \geq 0$, for $i \neq j$ and $\pi_{ii} = -\sum_{j=1,j\neq i}^{s}\pi_{ij}$.

Assumption 1: If $U_k^i = \mathcal{S}$, then $\hat{\pi}_{ij} - \delta_{ij} \geq 0, (\forall j \in \mathcal{S}, j \neq i), \hat{\pi}_{ii} = -\sum_{j=1,i\neq j}^{s}\hat{\pi}_{ij} \leq 0$, and $\delta_{ii} = \sum_{j=1,i\neq j}^{s}\delta_{ij} > 0$;

Assumption 2: If $U_k^i \neq \mathcal{S}$ and $i \in U_k^i$, then $\hat{\pi}_{ij} - \delta_{ij} \geq 0, (\forall j \in U_k^i, j \neq i), \hat{\pi}_{ii} + \delta_{ii} \leq 0$, and $\sum_{j\in U_k^i}\hat{\pi}_{ij} \leq 0$;

Assumption 3: If $U_k^i \neq \mathcal{S}$ and $i \notin U_k^i$, then $\hat{\pi}_{ij} - \delta_{ij} \geq 0, (\forall j \in U_k^i)$.

For simplicity, in this article, when $r(t) = i, i \in \mathcal{S}$, a matrix $M(r_t)$ will be denoted by M_i; for example, $A(r_t)$ is denoted by A_i, $A_d(r_t)$ is denoted by A_{di} and so on.

Remark 1: The BUTRs model [14] is

$$\begin{bmatrix} \hat{\pi}_{11}+\Delta_{11} & \hat{\pi}_{12}+\Delta_{12} & \cdots & \hat{\pi}_{1s}+\Delta_{1s} \\ \hat{\pi}_{21}+\Delta_{21} & \hat{\pi}_{22}+\Delta_{22} & \cdots & \hat{\pi}_{2s}+\Delta_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\pi}_{s1}+\Delta_{s1} & \hat{\pi}_{s2}+\Delta_{s2} & \cdots & \hat{\pi}_{ss}+\Delta_{ss} \end{bmatrix}$$

with $\hat{\pi}_{ij} - \delta_{ij}\,(\forall j \in \mathcal{S}, j \neq i), \hat{\pi}_{ii} = -\sum_{j=1,i\neq j}^{s}\hat{\pi}_{ij}$, and $\delta_{ii} = \sum_{j=1,i\neq j}^{s}\delta_{ij}$. And the PUTRs model [20]–[21] is

$$\begin{bmatrix} \pi_{11} & ? & \cdots & \pi_{s1} \\ ? & ? & \cdots & ? \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{s1} & ? & \cdots & \pi_{ss} \end{bmatrix}$$

Obviously, if $U_k^i = \mathcal{S}$, the GUTRs model (3) is reduced to BUTRs model, and if $\delta_{ij}= 0, \forall i \in \mathcal{S}, \forall j \in U_k^i$, the GUTRs model (3) is reduced to PUTRs model. Therefore, the GUTRs model considered in this article is more general than BUTRs and PUTRs. Furthermore, the systems considered in this article are more universal.

The system (1) with $u(t) = 0$ can be described as follows

$$\begin{cases} E\dot{x}(t) = A_i x(t) + A_{di}(t - d(t)) + B_{\omega i}\omega(t) \\ z(t) = C_i x(t) \\ x(t) = \phi(t), \quad t \in [-\bar{d}, 0] \end{cases} \quad (4)$$

Definition 1 [17]: 1) The system (4) is said to be regular and impulse-free, if the pairs (E, A_i) and $(E, A_i + A_{di})$ are regular and impulse-free for every $i \in \mathcal{S}$.

2) The system (4) is said to be stochastically stable, if for any initial mode r_0 and any initial condition $x(t) = \phi(t)$, $t \in [-\bar{d}, 0]$, there exists a scalar $M(r_0, \phi(\cdot))$ such that

$$\lim_{T-\infty} \varepsilon\left\{\int_0^T x^T(t)\,x(t)\,dt|\phi(t), r_0\right\} \leq M(r_0, \phi(\cdot)).$$

3) The system (4) is said to be stochastically admissible, if it is regular, impulse-free and stochastically stable.

Definition 2: The singular Markovian jump time-delay system (4) is said to be stochastically admissible with a given H_∞ performance index γ, if it is stochastically admissible, and under zero initial condition, for any external disturbance $\omega(t) \in \mathcal{L}_2[0, \infty)$,

$$\varepsilon\left\{\int_0^\infty z(t)^T z(t)\,dt\right\} \leq \gamma^2 \int_0^\infty \omega(t)^T\omega(t)\,dt. \quad (5)$$

Consider the following state feedback controller

$$u(t) = K_i x(t) \quad (6)$$

where $K_i (i = 1, 2, \cdots, s)$ are the controller gain matrices to be determined.

Substituting the controller (6) into system (1), we can obtain the close-loop system

$$\begin{cases} E\dot{x}(t) = (A_i + B_i K_i)\,x(t) + A_{di}x(t - d(t)) \\ \qquad\qquad +B_{\omega i}\omega(t) \\ z(t) = (C_i + D_i K_i)\,x(t) \\ x(t) = \phi(t), \quad t \in [-\bar{d}, 0] \end{cases} \quad (7)$$

The objective of this article is to design state feedback controller (6) for system (1) with GUTRs such that the closed-loop system (7) is stochastically admissible while satisfying a prescribed H_∞ performance γ.

Lemma 1 [31]: The singular Markovian jump system $E\dot{x}(t) = A_i x(t)$ is stochastically admissible if and only if there exist symmetric positive-definite matrices P_i and matrices S_i such that for each $i \in \mathcal{S}$,

$$\sum_{j=1}^{s}\pi_{ij}E^T P_j E + E^T P_i A_i + S_i R^T A_i + A_i^T P_i E + A_i^T R S_i^T < 0$$

where $R \in \mathbb{R}^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E^T R = 0$.

Lemma 2 [5]: For any constant matrices $N_1 \in \mathbb{R}^{n\times n}$, $N_2 \in \mathbb{R}^{n\times n}$, $W \in \mathbb{R}^{n\times n}$, a positive definitive symmetric matrix $Z \in \mathbb{R}^{n\times n}$, and the time-varying delay $d(t)$, we have

$$-\int_{t-d(t)}^{t} \dot{x}^T(s) E^T Z E \dot{x}(s) \leq \xi^T(t) \left\{ \mathcal{H} + d(t) Y^T Z^{-1} Y \right\}$$

$\xi(t)$ where $\xi(t) = [x^T(t)\, x^T(t-d(t)) \omega^T(t))]^T$ and

$$\mathcal{H} = \begin{bmatrix} N_1^T E + E^T N_1 & E^T N_2 - N_1^T E & E^T W \\ * & -N_2^T E - E^T N_2 & -E^T W \\ * & * & 0 \end{bmatrix},$$

$$Y = \begin{bmatrix} N_1 & N_2 & W \end{bmatrix}.$$

Lemma 3 [1]: Given any scalar α and matrix $\mathcal{Q}$, and any matrix $T > 0$ of appropriate dimension, the following matrix inequality holds

$$\alpha\left(\mathcal{Q}+\mathcal{Q}^T\right) \leq \alpha^2 T + \mathcal{Q} T^{-1} \mathcal{Q}^T.$$

III. MAIN RESULTS

In this section, the H_∞ state feedback control problem will be investigated for the SMJSs (1) with time-varying delay and GUTRs. To begin with, considering the systems (4) with a special situation of completely known transition rates, a new delay-dependent condition is proposed such that the SMJSs (4) is stochastically admissible with H_∞ performance γ.

Theorem 1: Given scalars $\bar{d} > 0$, $\gamma > 0$, and μ, the system (4) with completely known transition rates is stochastically admissible with H_∞ performance γ for any time-varying delay $d(t)$ satisfying (2), if there exist the symmetric positive-definite matrices P_i, Q_1, Q_2, Z and the matrices S_i, S_{di}, N_1, N_2, W such that for each $i \in \mathcal{S}$,

$$\begin{bmatrix} \Gamma_{11i} & \Gamma_{12i} & \Gamma_{13i} & \Gamma_{14i} & \bar{d}N_1^T & \bar{d}A_i^T Z & C_i^T \\ * & \Gamma_{22i} & \Gamma_{23i} & \Gamma_{24i} & \bar{d}N_2^T & \bar{d}A_{di}^T Z & 0 \\ * & * & \Gamma_{33i} & \Gamma_{34i} & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & \bar{d}W^T & \bar{d}B_{\omega i}^T Z & 0 \\ * & * & * & * & -\bar{d}Z & 0 & 0 \\ * & * & * & * & * & -\bar{d}Z & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{8}$$

where $R \in \mathbb{R}^{n\times(n-r)}$ is any matrix with full column satisfying $E^T R = 0$, and

$$\Gamma_{11i} = Sym\left\{E^T P_i A_i + A_i^T R S_i^T + N_1^T E\right\} + Q_1 + Q_2 + \sum_{j=1}^{s} \pi_{ij} E^T P_j E,$$

$$\Gamma_{22i} = -(1-\mu) Q_1 + Sym\left\{A_{di}^T R S_{di}^T - N_2^T E\right\},$$

$$\Gamma_{33i} = -Q_2,$$

$$\Gamma_{12i} = E^T P_i A_{di} + E^T N_2 - N_1^T E + A_i^T R S_{di}^T + S_i R^T A_{di},$$

$$\Gamma_{13i} = -A_i^T R S_{di}^T,$$

$$\Gamma_{14i} = E^T W + E^T P_i B_{\omega i} + S_i R^T B_{\omega i},$$

$$\Gamma_{23i} = -A_{di}^T R S_{di}^T,$$

$$\Gamma_{24i} = -E^T W + S_{di} R^T B_{\omega i},$$

$$\Gamma_{34i} = -S_{di} R^T B_{\omega i}.$$

Proof: The regularity and impulse-free of SMJSs (4) will be proved firstly.

Since rank $(E) = r \leq n$, there must exist two non-singular matrices G and $H \in \mathbb{R}^{n\times n}$ such that $GEH = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ where I_r is an identity matrix with r-dimension.

Denote

$$GA_iH = \begin{bmatrix} \bar{A}_{i1} & \bar{A}_{i2} \\ \bar{A}_{i3} & \bar{A}_{i4} \end{bmatrix}, \quad G^{-T} P_i G^{-1} = \begin{bmatrix} \bar{P}_{i1} & \bar{P}_{i2} \\ \bar{P}_{i3} & \bar{P}_{i4} \end{bmatrix},$$

$$G^{-T} N_1 H = \begin{bmatrix} \bar{N}_1 & \bar{N}_2 \\ \bar{N}_3 & \bar{N}_4 \end{bmatrix}, \quad H^T S_i = \begin{bmatrix} \bar{S}_{i1} \\ \bar{S}_{i2} \end{bmatrix}.$$

And it follows from $E^T R = 0$ that the matrix R can be parameterized as

$$R = G^T \begin{bmatrix} 0 \\ \bar{R} \end{bmatrix}$$

where $\bar{R} \in \mathbb{R}^{(n-r)\times(n-r)}$ is any non-singular matrix.

Pre- and post-multiplying Γ_{11i} by H^T and H, respectively, we have

$$Sym\left\{\bar{S}_{i2} \bar{R}^T \bar{A}_{i4}\right\} < 0$$

which implies $\bar{A}_{i4}$ are non-singular for each $i \in \mathcal{S}$, thus the pair (E, A_i) is regular and impulse-free for each $i \in \mathcal{S}$. From (8), it is easy to see that

$$\begin{bmatrix} \Gamma_{11i} & \Gamma_{12i} & \Gamma_{13i} \\ * & \Gamma_{22i} & \Gamma_{23i} \\ * & * & \Gamma_{33i} \end{bmatrix} < 0 \tag{9}$$

Pre- and post-multiplying on both sides of (9) by $\begin{bmatrix} I & I & I \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$ and its transpose, respectively, we have

$$\Gamma_{11i} + \Gamma_{22i} + \Gamma_{33i} + Sym\left\{\Gamma_{12i} + \Gamma_{13i} + \Gamma_{23i}\right\} < 0$$

Hence,

$$\sum_{j=1}^{s} \pi_{ij} E^T P_j E + Sym\left\{\left(E^T P_i + S_i R^T\right)(A_i + A_{di})\right\} < 0 \tag{10}$$

From Lemma 1, (10) implies that the pair $(E, A_i + A_{di})$ is regular and impulse-free for each $i \in \mathcal{S}$. Then according to Definition 1, system (4) is regular and impulse-free.

Define the following stochastic Lyapunov functional candidate for system (4)

$$V(x, r_t, t) = \sum_{m=1}^{4} V_m(x, r_t, t) \tag{11}$$

where

$$V_1(x, r_t, t) = x^T(t) E^T P(r_t) Ex(t) = x^T(t) E^T P_i Ex(t),$$

$$V_2(x, r_t, t) = \int_{t-d(t)}^{t} x^T(s) Q_1 x(s) ds,$$

$$V_3(x, r_t, t) = \int_{-\bar{d}}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) E^T Z E \dot{x}(s) ds d\theta,$$

$$V_4(x, r_t, t) = \int_{t-\bar{d}}^{t} x^T(s) Q_2 x(s) ds.$$

and $P(r_t) = P_i, Q_1, Q_2$, and Z are symmetric positive definite matrices. Noting that $E^T R = 0$, we have

$$\begin{aligned}
&\dot{x}^T(t) E^T R \left(S_i^T x(t) + S_{di}^T x(t-d(t))\right) \\
&= \left(x^T(t) S_i + x^T(t-d(t)) S_{di}\right) R^T E \dot{x}(t) \\
&= \dot{x}^T(t) E^T R S_{di}^T x(t-\bar{d}) = x^T(t-\bar{d}) S_{di} R^T E \dot{x}(t) = 0
\end{aligned}$$

Let $\mathcal{L}$ be the weak infinitesimal generator of the random process $\{(x, r_t), t \geq 0\}$, and then

$$\begin{aligned}
&\mathcal{LV}(x, r_t, t) \\
&\leq Sym\left\{\dot{x}^T(t) E^T P_i^T E x(t)\right\} + x^T(t)\left(\sum_{j=1}^{s} \pi_{ij} E^T P_j E\right) x(t) \\
&\quad + x^T(t) Q_1 x(t) - (1-\mu) x^T(t-d(t)) Q_1 x(t-d(t)) \\
&\quad + \bar{d}\dot{x}^T(t) E^T Z E \dot{x}(t) - \int_{t-d(t)}^{t} \dot{x}^T(s) E^T Z E \dot{x}(s) ds \\
&\quad + x^T(t) Q_2 x(t) - x^T(t-\bar{d}) Q_2 x(t-\bar{d}) \\
&\quad + Sym\left\{\left(x^T(t) S_i + x^T(t-d(t)) S_{di}\right) R^T E \dot{x}(t)\right\} \\
&\quad - Sym\left\{x^T(t-\bar{d}) S_{di} R^T E \dot{x}(t)\right\}.
\end{aligned}$$

It follows from Lemma 2 that

$$-\int_{t-d(t)}^{t} \dot{x}^T(s) E^T Z E \dot{x}(s) ds \leq \xi^T(t)\left\{\mathcal{H} + d(t) Y^T Z^{-1} Y\right\}\xi(t)$$

where $\xi^T(t) = \left[x^T(t)\ x^T(t-d(t))\ 0\right]$, $\mathcal{H}$ and Y are the same as that defined in Lemma 2.

Then,

$$\mathcal{LV}(x_t, r_t, t) \leq \eta^T(t) \mho_i \eta(t)$$

where $\eta^T = \left[x^T(t)\ x^T(t-d(t))\ x^T(t-\bar{d})\right]$, and

$$\mho_i = \begin{bmatrix} \nabla_{11i} & \nabla_{12i} & \Gamma_{13i} \\ * & \nabla_{22i} & \Gamma_{23i} \\ * & * & \Gamma_{33i} \end{bmatrix}$$

with

$$\begin{aligned}
\nabla_{11i} &= Sym\left\{E^T P_i A_i + A_i^T R S_i^T + N_1^T E\right\} + Q_1 + Q_2 \\
&\quad + \sum_{j=1}^{s} \pi_{ij} E^T P_j E + \bar{d} N_1^T Z^{-1} N_1 + \bar{d} A_i^T Z A_i, \\
\nabla_{22i} &= -(1-\mu) Q_1 + \bar{d} N_2^T Z^{-1} N_2 + \bar{d} A_{di}^T Z A_{di} \\
&\quad + Sym\left\{A_{di}^T R S_{di}^T - N_2^T E\right\} \\
\nabla_{12i} &= E^T P_i A_{di} + E^T N_2 - N_1^T E + A_i^T R S_{di}^T + S_i R^T A_{di} \\
&\quad + \bar{d} N_1^T Z^{-1} N_2 + \bar{d} A_i^T Z A_{di}.
\end{aligned}$$

From Schur complement formula, if the inequality (8) holds, which implies $\mho_i < 0$, thus $\mathcal{LV}(x, r_t, t) \leq \eta^T(t) \mho_i \eta(t) < 0$, moreover, there exists a scalar $\lambda > 0$ such that

$$\mathcal{LV}(x_t, r_t, t) \leq -\lambda \|x(t)\|^2$$

Hence, for any $t \geq 0$,

$$\varepsilon\{V(x_t, r_t, t)\} - \varepsilon\{V(x_0, r_0, 0)\} \leq -\lambda\varepsilon\left\{\int_0^t \|x(s)\|^2 ds\right\}$$

which yields

$$\varepsilon\left\{\int_0^t \|x(s)\|^2 ds\right\} \leq \lambda^{-1}\varepsilon\{V(x_0, r_0, 0)\}$$

According to Definition 1, the inequality (8) can guarantee the system (4) with $\omega(t) \equiv 0$ is stochastically stable.

Next, consider the following performance

$$J_{z\omega} = \varepsilon\left\{\int_0^\infty \left[z(s)^T z(s) - \gamma^2 \omega(s)^T \omega(s)\right] ds\right\} \quad (12)$$

Under zero initial condition, it follows from (8) and Schur complement formula that

$$\begin{aligned}
J_{z\omega} &= \varepsilon\left\{\int_0^\infty \left[z(s)^T z(s) - \gamma^2 \omega(s)^T \omega(s) + \mathcal{L}V(x, i, s)\right] ds\right\} \\
&= \varepsilon\left\{\int_0^\infty \left[x(s)^T C_i^T C_i x(s)\right.\right. \\
&\quad \left.\left. - \gamma^2 \omega(s)^T \omega(s) + \mathcal{L}V(x, i, s)\right] ds\right\} \\
&\leq \varepsilon\left\{\int_0^\infty \varsigma^T(s) \Theta_i \varsigma(s) ds\right\}
\end{aligned}$$

where

$$\varsigma^T(s) = \left[x^T(s)\ x^T(t-d(s))\ x^T(t-\bar{d})\ \omega^T(s)\right]$$

$$\Theta_i = \begin{bmatrix} \nabla_{11i} + C_i^T C_i & \nabla_{12i} & \Gamma_{13i} & \nabla_{14i} \\ * & \nabla_{22i} & \Gamma_{23i} & \nabla_{24i} \\ * & * & \Gamma_{33i} & \Gamma_{34i} \\ * & * & * & \nabla_{44i} \end{bmatrix}$$

with

$$\begin{aligned}
\nabla_{14i} &= E^T W + E^T P_i B_{\omega i} + S_i R^T B_{\omega i} + \bar{d} N_1^T Z^{-1} W \\
&\quad + \bar{d} A_i^T Z B_{\omega i}, \\
\nabla_{24i} &= -E^T W + S_{di} R^T B_{\omega i} + \bar{d} N_2^T Z^{-1} W + \bar{d} A_{di}^T Z B_{\omega i}, \\
\nabla_{44i} &= -\gamma^2 I + \bar{d} W^T Z^{-1} W + \bar{d} B_{\omega i}^T Z B_{\omega i}.
\end{aligned}$$

Hence, $J_{z\omega} < 0$, and the inequality (5) holds. Thus, from Definition 2, the SMJSs (4) with completely known transition rates is stochastically admissible with H_∞ performance γ. Thus, this completes the proof. □

Remark 2: Currently, many solvability conditions in terms of the filtering or the control for SMJSs are non-strict LMIs, such as the theorems given in references [19], [22], [23], and [25] containing semi-definite matrix inequalities $E^T P_i = P_i^T E \geq 0$. Notably, those semi-definite matrix inequalities fail to meet the standard LMI and thus could not be solved by it directly. Due to the round-off errors in digital computation, it will result in further problems of checking the inequality conditions numerically. Nevertheless, the Theorem 1 in this article is a strict LMI and it does not have the trouble mentioned above. Thereby, the theorem in this article is more general than those referred in the articles above.

Theorem 1 solves the H_∞ stochastically admissible problem of SMJSs (4) with completely known transition probabilities. In the sequel, based on the obtained Theorem 1, the H_∞ stochastically admissible problem for SMJSs (4) with GUTRs is further investigated and the following Theorem 2 is immediate.

Theorem 2: Given scalars $\bar{d} > 0$, $\gamma > 0$ and μ, system (4) with GUTRs is stochastically admissible with H_∞ performance γ for any time-varying delay $d(t)$ satisfying (2), if there exist symmetric positive-definite matrices P_i, Q_1, Q_2, Z and the matrices S_i, S_{di}, N_1, N_2, W such that the following inequalities hold for each $i \in \mathbb{S}$.

Case I: If $i \notin U_k^i$ and $U_k^i = \{k_1^i, k_2^i, \cdots, k_m^i\}$, existing a set of positive definite matrices $T_{ij} \in \mathbb{R}^{n\times n}$ $(i \notin U_k^i, j \in U_k^i)$ such that

$$\begin{bmatrix} \tilde{\Gamma}_{11i} & \Gamma_{2i} & \tilde{\Gamma}_{3i} \\ * & \Gamma_{4i} & 0 \\ * & * & \tilde{\Gamma}_{5i} \end{bmatrix} < 0 \tag{13a}$$

with $E^T(P_j - P_i)E \geq 0, \forall j \in U_k^i$.
where

$$\tilde{\Gamma}_{11i} = Sym\left\{E^T P_i A_i + A_i^T R S_i^T + N_1^T E\right\} + Q_1 + Q_2 + \sum_{j\in U_k^i} \hat{\pi}_{ij} E^T(P_j - P_i)E + \sum_{j\in U_k^i} \frac{\delta_{ij}^2}{4} T_{ij},$$

$$\Gamma_{2i} = \left[\Gamma_{12i}\ \Gamma_{13i}\ \Gamma_{14i}\ \bar{d}N_1^T\ \bar{d}A_i^T Z\ C_i^T\right],$$

$$\tilde{\Gamma}_{3i} = \left[E^T(P_{k_1^i} - P_i)E\ E^T(P_{k_2^i} - p_i)E \cdots E^T(P_{k_m^i} - P_i)E\right],$$

$$\Gamma_{4i} = \begin{bmatrix} \Gamma_{22i} & \Gamma_{23i} & \Gamma_{24i} & \bar{d}N_2^T & \bar{d}A_{di}^T Z & 0 \\ * & \Gamma_{33i} & \Gamma_{34i} & 0 & 0 & 0 \\ * & * & -\gamma^2 I & \bar{d}W^T & \bar{d}B_{\omega i}^T Z & 0 \\ * & * & * & -\bar{d}Z & 0 & 0 \\ * & * & * & * & -\bar{d}Z & 0 \\ * & * & * & * & * & -I \end{bmatrix},$$

$$\tilde{\Gamma}_{5i} = diag\left\{-T_{ik_1^i}, -T_{ik_2^i}, \cdots, -T_{ik_m^i}\right\},$$

and other notations are defined as in Theorem 1.

Case II: If $i \in U_k^i$, $U_{Uk}^i \neq \emptyset$ and $U_k^i = \{k_1^i, k_2^i, \cdots, k_m^i\}$, there exist a set of positive definite matrices $V_{ijl} \in \mathbb{R}^{n\times n}(i, j \in U_k^i, l \in U_{Uk}^i)$ such that

$$\begin{bmatrix} \bar{\Gamma}_{11i} & \Gamma_{2i} & \bar{\Gamma}_{3i} \\ * & \Gamma_{4i} & 0 \\ * & * & \bar{\Gamma}_{5i} \end{bmatrix} < 0 \tag{13b}$$

where

$$\bar{\Gamma}_{11i} = Sym\left\{E^T P_i A_i + A_i^T R S_i^T + N_1^T E\right\} + Q_1 + Q_2 + \sum_{j\in U_k^i} \hat{\pi}_{ij} E^T(P_j - P_l)E + \sum_{j\in U_k^i} \frac{\delta_{ij}^2}{4} V_{ijl},$$

$$\bar{\Gamma}_{3i} = \left[E^T(P_{k_1^i} - P_l)E\ E^T(P_{k_2^i} - p_l)E \cdots E^T(P_{k_m^i} - P_l)E\right],$$

$$\hat{\Gamma}_{5i} = diag\left\{-R_{i1}, \cdots, -R_{i(i-1)}, -R_{i(i+1)}, \cdots, -R_{is}\right\},$$

and other notations are defined as in Case I and Theorem 1.

Case III: If $i \in U_k^i$, $U_{Uk}^i = \emptyset$, there exist a set of positive definite matrices $R_{ij} \in \mathbb{R}^{n\times n}(i, j \in U_k^i)$ such that

$$\begin{bmatrix} \hat{\Gamma}_{11i} & \Gamma_{2i} & \hat{\Gamma}_{3i} \\ * & \Gamma_{4i} & 0 \\ * & * & \hat{\Gamma}_{5i} \end{bmatrix} < 0 \tag{13c}$$

Where

$$\hat{\Gamma}_{11i} = Sym\left\{E^T P_i A i + A_i^T R S_i^T + N1^T E\right\} + Q_1 + Q_2 + \sum_{j\in\mathbb{S}, j\neq i} \left[\hat{\pi}_{ij} E^T(P_j - P_i)E + \frac{\delta_{ij}^2}{4} R_{ij}\right],$$

$$\hat{\Gamma}_{3i} = \left[E^T(P_1 - P_i)E \cdots E^T(P_{i-1} - P_i)E \times E^T(P_{i+1} - P_i)E \cdots E^T(P_s - P_i)E\right]$$

$$\hat{\Gamma}_{5i} = diag\left\{-R_{i1}, \cdots, -R_{i(i-1)}, -R_{i(i+1)}, \cdots, -R_{is}\right\},$$

and other notations are defined as in Case I and Theorem 1.

Proof: Firstly, we prove that (8) can be guaranteed from inequalities (13a), (13b), and (13c) in three different cases respectively. And then based on Theorem 1, we can derive that system (4) with GUTRs is stochastically admissible.

Similar to Theorem 1, pre- and post-multiplying $\tilde{\Gamma}_{11i} < 0$, $\bar{\Gamma}_{11i} < 0$ and $\hat{\Gamma}_{11i} < 0$ by H^T and H, respectively. In all the above three cases, we have

$$Sym\left\{\bar{S}_{i2}\bar{R}^T\bar{A}_{i4}\right\} < 0$$

which implies $\bar{A}_{i4}$ are non-singular for every $i \in \mathbb{S}$ and thus the pairs (E, A_i) are regular and impulse-free for each $i \in \mathbb{S}$. By (13a), (13b), and (13c), it is easy to see that

Case I: $i \notin U_k^i$ and $U_k^i = \{k_1^i, k_2^i, \cdots, k_m^i\}$, according to Schur complement and inequality (13a), we have

$$\begin{bmatrix} \tilde{\tilde{\Gamma}}_{11i} & \Gamma_{2i} \\ * & \Gamma_{4i} \end{bmatrix} < 0 \tag{14}$$

where

$$\tilde{\tilde{\Gamma}}_{11i} = \sum_{j\in U_k^i} \left[\frac{\delta_{ij}^2}{4} T_{ij} + E^T(P_j - P_i)ET_{ij}^{-1}E^T(P_j - P_i)E\right] + Sym\left\{E^T P_i A_i + A_i^T R S_i^T + N_1^T E\right\} + Q_1 + Q_2 + \sum_{j\in U_k^i} \hat{\pi}_{ij} E^T(P_j - P_i)E$$

It follows from (13a) that

$$\begin{bmatrix} \tilde{\tilde{\Gamma}}_{11i} & \Gamma_{12i} & \Gamma_{13i} \\ * & \Gamma_{22i} & \Gamma_{23i} \\ * & * & \Gamma_{33i} \end{bmatrix} < 0 \tag{15}$$

Pre- and post-multiplying (15) by $\begin{bmatrix} I & I & I \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$ and its transpose respectively, yields

$$\tilde{\tilde{\Gamma}}_{11i} + \Gamma_{22i} + \Gamma_{33i} + Sym\{\Gamma_{12i} + \Gamma_{13i} + \Gamma_{23i}\} < 0$$

Hence,

$$\sum\nolimits_{j\in U_k^i}\left[\frac{\delta_{ij}^2}{4}T_{ij}+E^T\left(P_j-P_i\right)ET_{ij}^{-1}E^T\left(P_j-P_i\right)E\right]$$
$$+Sym\left\{\left(E^TP_i+S_iR^T\right)(A_i+A_{di})\right\}$$
$$+\sum\nolimits_{j\in U_k^i}\hat{\pi}_{ij}E^T\left(P_j-P_i\right)E<0 \tag{16}$$

Since $E^T\left(P_j-P_i\right)E\geq 0, \forall j\in U_k^i$, and note that in this case, $\sum_{j\in U_k^i i\neq j}\pi_ij=-\pi_ii-\sum_{j\in U_ki}\pi_{ij}$ and $\pi_{ij}\geq 0$, we have

$$\sum_{j=1}^{s}\pi_{ij}E^TP_jE=\pi_{ii}E^TP_iE+\sum_{j\in U_k^i}\pi_{ij}E^TP_jE$$
$$+\sum_{j\in U_{Uk}^i,j\neq i}\pi_{ij}E^TP_jE\leq\sum_{j\in U_k^i}\pi_{ij}E^TP_jE+\pi_{ii}E^TP_iE$$
$$+\left(-\pi_{ii}E^TP_iE-\sum_{j\in U_k^i}\pi_{ij}\right)E^TP_iE$$
$$=\sum_{j\in U_k^i}\left(\hat{\pi}_{ij}+\Delta_{ij}\right)E^T\left(P_j-P_i\right)E$$
$$=\sum_{j\in U_k^i}\left[\frac{\Delta_{ij}}{2}E^T\left(P_j-P_i\right)E+\frac{\Delta_{ij}}{2}E^T\left(P_j-P_i\right)E\right]$$
$$+\sum_{j\in U_k^i}\hat{\pi}_{ij}E^T\left(P_j-P_i\right)E$$

By using Lemma 3, we have

$$\sum_{j=1}^{s}\pi_{ij}E^TP_jE\leq\sum_{j\in U_k^i}\hat{\pi}_{ij}E^T\left(P_j-P_i\right)E$$
$$+\sum_{j\in U_k^i}\left[\frac{\delta_{ij}^2}{4}T_{ij}+E^T\left(P_j-P_i\right)ET_{ij}^{-1}E^T\left(P_j-P_i\right)E\right] \tag{17}$$

According to Lemma 1, the formula (16) and (17) imply that the pairs (E,A_i+A_{di}) are regular and impulse-free for each $i\in\mathbb{S}$. Thus, the system (4) with GUTRs is regular and impulse-free. On the other hand, from inequalities (13a) and (14), we have $\Gamma_{11i}\leq\tilde{\tilde{\Gamma}}_{11i}<0$, therefore, the inequality (8) holds.

Case II: $i\in U_k^i$, $U_{Uk}^i\neq\emptyset$ and $U_k^i=\left\{k_1^i,k_2^i,\cdots,k_m^i\right\}$, according to Schur complement and inequality (13b), we have

$$\begin{bmatrix}\bar{\bar{\Gamma}}_{11i} & \Gamma_{2i}\\ * & \Gamma_{4i}\end{bmatrix}<0 \tag{18}$$

where

$$\bar{\bar{\Gamma}}_{11i}\sum_{j\in U_k^i}\left[\frac{\delta_{ij}^2}{4}V_{ijl}+E^T\left(P_j-P_l\right)EV_{ijl}^{-1}E^T\left(P_j-P_l\right)E\right]$$
$$+Sym\left\{E^TP_iA_i+A_i^TRS_i^T+N_1^TE\right\}+Q_1+Q_2$$
$$+\sum_{j\in U_k^i}\hat{\pi}_{ij}E^T\left(Pj-P_l\right)E$$

Similar to Case I, the following formula holds

$$\sum_{j\in U_k^i}\left[\frac{\delta_{ij}^2}{4}V_{ijl}+E^T\left(P_j-P_l\right)EV_{ijl}^{-1}E^T\left(P_j-P_l\right)E\right]$$
$$+Sym\left\{\left(E^TP_i+S_iR^T\right)(A_i+A_{di})\right\}$$
$$+\sum\nolimits_{j\in U_k^i}\hat{\pi}_{ij}E^T\left(P_j-P_l\right)E<0 \tag{19}$$

Because of $U_k^i=\left\{k_1^i,k_2^i,\cdots,k_m^i\right\}$, there must exist $l\in U_{Uk}^i$ so that $E^TP_lE\geq EP_jE^T\left(\forall j\in U_{Uk}^i\right)$. In this case, since $\pi_{ii}=-\sum_{j=1,j\neq i}^{s}\pi_{ij}\leq 0$ and $\pi_{ii}\in U_k^i$, then $\sum_{j\in U_{Uk}^i}\pi_{ij}=-\sum_{j\in U_k^i}\pi_{ij}$. And we have

$$\sum\nolimits_{j=1}^{s}\pi_{ij}E^TP_jE$$
$$=\sum\nolimits_{j\in U_k^i}\pi_{ij}E^TP_jE+\sum\nolimits_{j\in U_{Uk}^i}\pi_{ij}E^TP_jE$$
$$\leq\sum\nolimits_{j\in U_k^i}\pi_{ij}E^TP_jE+\sum\nolimits_{j\in U_{Uk}^i}\pi_{ij}E^TP_lE$$
$$=\sum\nolimits_{j\in U_k^i}\pi_{ij}E^TP_jE-\sum\nolimits_{j\in U_k^i}\pi_{ij}E^TP_lE$$
$$=\sum\nolimits_{j\in U_k^i}\hat{\pi}_{ij}E^T\left(P_j-P_l\right)E+\sum\nolimits_{j\in U_k^i}\Delta_{ij}E^T\left(P_j-P_l\right)E$$
$$=\sum\nolimits_{j\in U_k^i}\left[\frac{\Delta_{ij}}{2}E^T\left(P_j-P_l\right)E+\frac{\Delta_{ij}}{2}E^T\left(P_j-P_l\right)E\right]$$
$$+\sum\nolimits_{j\in U_k^i}\hat{\pi}_{ij}E^T\left(P_j-P_l\right)E$$

It follows from Lemma 3 that,

$$\sum\nolimits_{j=1}^{s}\pi_{ij}E^TP_jE\leq\sum\nolimits_{j\in U_k^i}\hat{\pi}_{ij}E^T\left(P_j-P_l\right)E$$
$$+\sum_{j\in U_k^i}\left[\frac{\delta_{ij}^2}{4}V_{ijl}+E^T\left(P_j-P_l\right)EV_{ijl}^{-1}E^T\left(P_j-P_l\right)E\right] \tag{20}$$

According to Lemma 1, the formula (19) and (20) imply that the pairs (E,A_i+A_{di}) are regular and impulse-free for each $i\in\mathbb{S}$. Thus, the system (4) with GUTRs is regular and impulse-free. On the other hand, from inequalities (13b) and (18), we have $\Gamma_{11i}\leq\bar{\bar{\Gamma}}_{11i}<0$. Therefore, the inequality (8) holds.

The proof of Case III is similar to that of Case I, and it is omitted here. Therefore, if the inequalities (13a), (13b) and (13c) hold, then according to Definition 1 and Theorem 1, we conclude that the system (4) with GUTRs is stochastically admissible with a given H_∞ performance γ for any time-varying delay $d(t)$ satisfying (2), thus, it completes the proof. □

Remark 3: In many engineering areas, there is a need to model the dynamics of a control system in partial functional differential equations [39]–[40]. It should be noted that by using the methods mentioned in this article, it is easy to extend the results of this article to SMJSs with reaction-diffusion terms under GUTRs.

In the following theorem, we aim at designing the controller (6) such that the closed-loop SMJSs (7) with GUTRs is stochastically admissible with H_∞ performance γ.

Theorem 3: Given scalars $\bar{d} > 0, \gamma > 0$, and μ, for the system (1) with GUTRs, there exist a state feedback controller (6) such that the closed loop systems (7) are stochastically admissible with H_∞ performance γ for any time-varying delay $d(t)$ satisfying (2), if there exist positive definite matrices P_i, Q_1, Q_2, Z and the matrices S_i, N_1, N_2, W, X_i, H_i such that the following linear matrix inequalities hold. And the controller gain matrices can be obtained as $K_i = H_i X_i^{-1}$.

Case I: If $i \notin U_k^i$ and $U_k^i = \{k_1^i, k_2^i, \cdots, k_m^i\}$, there exist a set of positive definite matrices $T_{ij} \in \mathbb{R}^{n\times n}\ (i \notin U_k^i, j \in U_k^i)$ such that

$$\begin{bmatrix} \tilde{\Psi}_{11i} & \Psi_{2i} & \tilde{\Psi}_{3i} \\ * & \Psi_{4i} & 0 \\ * & * & \tilde{\Psi}_{5i} \end{bmatrix} < 0 \tag{21a}$$

with $E^T(P_j - P_i)E \geq 0, \forall j \in U_k^i$.
where

$$\tilde{\Psi}_{11i} = \sum_{j\in U_k^i} \hat{\pi}_{ij} E(P_j - P_i)E^T + \sum_{j\in U_k^i} \frac{\delta_{ij}^2}{4} T_{ij} + Sym\{A_iX_i + B_iH_i + EN_1\} + Q_1 + Q_2,$$

$$\Psi_{2i} = [\Psi_{12i}\ \Psi_{13i} \Psi_{14i}\ \Psi_{15i}\ \bar{d}N_1^T\ 0\ B_{\omega i}],$$

$$\tilde{\Psi}_{3i} = \left[E^T(P_{k_1^i} - P_i)E\ E^T(P_{k_2^i} - p_i)E \cdots E^T(P_{k_m^i} - P_i)E^T\right],$$

$$\Psi_{4i} = \begin{bmatrix} \Psi_{22i} & \Psi_{23i} & \Psi_{24i} & \Psi_{25i} & 0 & -\bar{d}Z & 0 \\ * & \Psi_{33i} & \Psi_{34i} & \Psi_{35i} & \bar{d}N_2^T & 0 & 0 \\ * & * & -Q_2 & \Psi_{45i} & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & \bar{d}W^T & 0 & 0 \\ * & * & * & * & -\bar{d}Z & 0 & 0 \\ * & * & * & * & * & -\bar{d}Z & 0 \\ * & * & * & * & * & * & -I \end{bmatrix},$$

$$\tilde{\Psi}_{5i} = diag\left\{-T_{ik_1^i}, -T_{ik_2^i}, \cdots, -T_{ik_m^i}\right\},$$

$$\Psi_{22i} = -X_i - X_i^T,$$

$$\Psi_{33i} = -(1-\mu)Q_1 + Sym\left\{A_{di}X_iS_{di}^T - EN_2^T\right\},$$

$$\Psi_{12i} = EP_i + A_iX_i + B_iH_i + S_iR^T - X_i^T,$$

$$\Psi_{13i} = EN_2 - N_1^TE^T + X_i^TA_{di}^T + A_iX_iS_{di}^T + B_iH_iS_{di}^T,$$

$$\Psi_{14i} = -A_iX_iS_{di}^T - B_iH_iS_{di}^T,$$

$$\Psi_{15i} = EW + X_i^TC_i^T + H_i^TD_i^T,$$

$$\Psi_{23i} = X_i^TA_{di}^T - X_iS_{di}^T,$$

$$\Psi_{24i} = -X_iS_{di}^T,$$

$$\Psi_{25i} = X_i^TC_i^T + H_i^TD_i^T,$$

$$\Psi_{34i} = -A_{di}X_iS_{di}^T,$$

$$\Psi_{35i} = -EW + S_{di}X_i^TC_i^T + S_{di}H_i^TD_i^T,$$

$$\Psi_{45i} = -S_{di}X_i^TC_i^T - S_{di}H_i^TD_i^T,$$

and $R \in \mathbb{R}^{n\times(n-r)}$ is any matrix with full column satisfying $E^T\mathbb{R} = 0$;S_{di} is any matrix with appropriate dimensions; $X_i \in \mathbb{R}^{n\times n}$ is any non-singular matrix.

Case II: If $i \in U_k^i$, $U_{Uk}^i \neq \emptyset$ and $U_k^i = \{k_1^i, k_2^i, \cdots, k_m^i\}$, there are a set of positive definite matrices $V_{ijl} \in \mathbb{R}^{n\times n}(i, j \in U_k^i, l \in U_{Uk}^i)$ such that

$$\begin{bmatrix} \bar{\Psi}_{11i} & \Psi_{2i} & \bar{\Psi}_{3i} \\ * & \Psi_{4i} & 0 \\ * & * & \bar{\Psi}_{5i} \end{bmatrix} < 0 \tag{21b}$$

where

$$\bar{\Psi}_{11i} = \sum_{j\in U_k^i} \hat{\pi}_{ij} E(P_j - P_l)E^T + \sum_{j\in U_k^i} \frac{\delta_{ij}^2}{4} V_{ijl} + Sym\{A_iX_i + B_iH_i + EN_1\} + Q_1 + Q_2,$$

$$\bar{\Psi}_{3i} = \left[E(P_{k_1^i} - P_l)E^T\ E(P_{k_2^i} - p_l)E \cdots E^T(P_{k_m^i} - P_i)E^T\right],$$

$$\bar{\Psi}_{3i} = \left[E(P_{k_1^i} - P_l)E^T\ E(P_{k_2^i} - p_l)E \cdots E^T(P_{k_m^i} - P_i)E^T\right],$$

and other notations are defined as in Case I.

Case III: If $i \in U_k^i$, $U_{Uk}^i = \emptyset$, there exist a set of positive definite matrices $R_{ij} \in \mathbb{R}^{n\times n}(i, j \in U_k^i)$ such that

$$\begin{bmatrix} \hat{\Psi}_{11i} & \Psi_{2i} & \hat{\Psi}_{3i} \\ * & \Psi_{4i} & 0 \\ * & * & \hat{\Psi}_{5i} \end{bmatrix} < 0 \tag{21c}$$

where

$$\hat{\Psi}_{11i} = \sum_{j\in \mathbb{S}, j\neq i} \left[\hat{\pi}_{ij} E(P_j - P_i)E^T + \frac{\delta_{ij}^2}{4} R_{ij}\right] + Sym\{A_iX_i + B_iH_i + EN_1\} + Q_1 + Q_2,$$

$$\hat{\Psi}_{3i} = \Big[E(P_1 - P_i)E^T \cdots E(P_{i-1} - P_i)E^T\ , \times E(P_1 - P_i)E^T \cdots E^T(P_{k_m^i} - P_i)E^T\Big]$$

$$\hat{\Psi}_{5i} = diag\{-R_{i1}, \cdots, -R_{i(i-1)}, -R_{i(i+1)}, \cdots, -R_{is}\},$$

and other notations are defined as in Case I.

Proof: Rewrite the close-loop system (7) with GUTRs in the following form

$$\begin{cases} \bar{E}\dot{x}(t) = \bar{A}_i\bar{x}(t) + \bar{A}_{di}\bar{x}(t - d(t)) + \bar{B}_{\omega i}\omega(t) \\ z(t) = \bar{C}_i x(t) \\ x(t) = \phi(t), t \in [-\bar{d}, 0] \end{cases} \tag{22}$$

where

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ E\dot{x}(t) \end{bmatrix}, \quad \bar{E} = \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix},$$

$$\bar{A}_i = \begin{bmatrix} 0 & I \\ A_i + B_i & K_i - I \end{bmatrix}, \quad \bar{A}_{di} = \begin{bmatrix} 0 & 0 \\ A_{di} & 0 \end{bmatrix},$$

$$\bar{B}_{\omega i} = \begin{bmatrix} 0 \\ B_{\omega i} \end{bmatrix}, \quad \bar{C}_i = \begin{bmatrix} C_i + D_iK_i & 0 \end{bmatrix}.$$

It follows from Theorem 2 that the closed-loop system (22) with GUTRs is stochastically admissible with H_∞ performance γ, if there exist positive definite matrices $\bar{P}_i, \bar{Q}_1, \bar{Q}_2, \bar{Z}$ and matrices $\bar{S}_i, \bar{S}_{di}, \bar{R}$ such that the inequalities (13a), (13b), and (13c) hold.

As a particular case, we set

$$\bar{P}_i = \begin{bmatrix} P_i & 0 \\ 0 & \varepsilon I \end{bmatrix}, \quad \bar{Q}_1 = \begin{bmatrix} Q_1 & 0 \\ 0 & \varepsilon I \end{bmatrix}, \quad \bar{Q}_2 = \begin{bmatrix} Q_2 & 0 \\ 0 & \varepsilon I \end{bmatrix},$$

TABLE 1. Minimum allowed H_∞ performance γ.

	[9]	[11]	[34]	Theorem 1
γ	3.7698	3.7207	3.7070	2.0576

$$\bar{Z}=\begin{bmatrix}Z & 0\\ 0 & \varepsilon I\end{bmatrix},\quad \bar{S}_i=\begin{bmatrix}S_i & I\\ 0 & I\end{bmatrix},\quad \bar{R}=\begin{bmatrix}R & 0\\ 0 & X_i\end{bmatrix},$$
$$\bar{S}_{di}=\begin{bmatrix}0 & S_{di}\\ \varepsilon I & 0\end{bmatrix},\quad \bar{T}_{ij}=\begin{bmatrix}T_{ij} & 0\\ 0 & \varepsilon I\end{bmatrix},\quad \bar{V}_{ijl}=\begin{bmatrix}V_{ijl} & 0\\ 0 & \varepsilon I\end{bmatrix},$$
$$\bar{R}_{ij}=\begin{bmatrix}R_{ij} & 0\\ 0 & \varepsilon I\end{bmatrix}. \tag{23}$$

where $R \in \mathbb{R}^{n\times(n-r)}$ is any matrix with full column satisfying $E^T R = 0$;S_{di} is any matrix with appropriate dimensions, and $X_i \in \mathbb{R}^{n\times n}$ is any non-singular matrix, $\varepsilon > 0$. It is obvious that $\bar{E}^T\bar{R} = 0$ and $\bar{R} \in \mathbb{R}^{2n\times(2n-r)}$ is of full column rank.

Replace matrices $P_i, Q_1, Q_2, Z, S_i, S_{di}, R, T_{ij}, V_{ijl}, R_{ij}, E, A_i, A_{di}, B_{\omega i}$, and C_i in Theorem 2 with matrices $\bar{P}_i, \bar{Q}_1, \bar{Q}_2, \bar{Z}, \bar{S}_i, \bar{S}_{di}, \bar{R}, \bar{T}_{ij}, \bar{V}_{ijl}, \bar{R}_{ij}, \bar{E}, \bar{A}_i, \bar{A}_{di}, \bar{B}_{\omega i}$, and $\bar{C}_i$ respectively, similar to the proof of Theorem 2 [31], the results in Theorem 3 (21a), (21b), and (21c) can be easily derived, the detail is omitted here for the sake of brevity and readability. It completes the proof. □

IV. NUMERICAL EXAMPLES

To show the effectiveness of the proposed method for the SMJSs with time-varying delays, some numerical examples are presented in this section.

Example 1: Consider the time-delay SMJSs (4) with two modes and the following parameters [34]:

$$E=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad \begin{bmatrix}\pi_{11} & \pi_{12}\\ \pi_{21} & \pi_{22}\end{bmatrix}=\begin{bmatrix}-0.5 & 0.5\\ 0.5 & -0.5\end{bmatrix},$$
$$A_1=\begin{bmatrix}-2 & 1\\ 1 & -2\end{bmatrix},\quad A_{d1}=\begin{bmatrix}0.5 & -0.2\\ 0.2 & 0.3\end{bmatrix},\quad B_{\omega 1}=\begin{bmatrix}0.1\\ 0.2\end{bmatrix},$$
$$A_2=\begin{bmatrix}-2 & 1\\ 1 & -2\end{bmatrix},\quad A_{d2}=\begin{bmatrix}0.3 & 0.5\\ 0.4 & 0.5\end{bmatrix},\quad B_{\omega 2}=\begin{bmatrix}0.2\\ 0.3\end{bmatrix},$$
$$C_1=\begin{bmatrix}0.1\\ 0.2\end{bmatrix}^T,\quad C_2=\begin{bmatrix}-0.1\\ 0.2\end{bmatrix}^T. \tag{24}$$

Choose the known scalars $\bar{d} = 2$ and $\mu = 0.37$. Table 1 presents the comparison results on minimum allowed H_∞ performance γ, which shows that Theorem 1 in this article is better than that in [9], [11], [34].

Next, consider the time-delay SMJSs (4) with $E = I$, two modes and the following parameters [32]:

$$\Pi=\begin{bmatrix}\pi_{11} & \pi_{12}\\ \pi_{21} & \pi_{22}\end{bmatrix},$$
$$A_1=\begin{bmatrix}-3.4888 & 0.8057\\ -0.6451 & -3.2684\end{bmatrix},$$
$$A_{d1}=\begin{bmatrix}-0.8620 & -1.2919\\ -0.6841 & -2.0729\end{bmatrix},$$
$$A_2=\begin{bmatrix}-2.4898 & 0.2895\\ 1.3396 & -0.0211],\end{bmatrix},$$
$$A_{d2}=\begin{bmatrix}-2.8306 & 0.4978\\ -0.8436 & -1.0115\end{bmatrix}.$$

TABLE 2. Allowable upper bound $\bar{d}$ for different π_{11}.

π_{11}	-0.1	-0.3	-0.5	-0.7	-0.9
[32]	0.4252	0.4250	0.4248	0.4246	0.4244
Theorem 1	0.4542	0.4530	0.4519	0.4510	0.4504

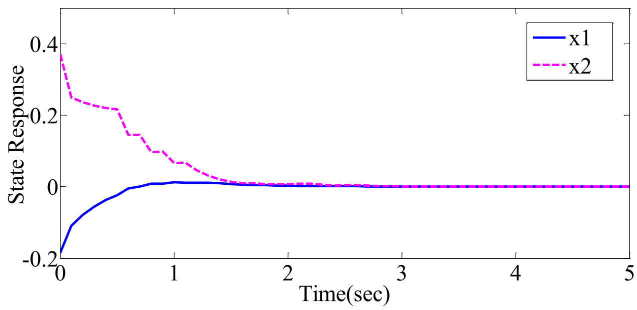

FIGURE 1. State trajectories of system (4) when $\pi_{22}= -0.5$ and $\bar{d}= 0.4519$.

Similar to [32], let $\pi_{22} = -0.8$ and $\mu = 0.9$, compute upper bound $\bar{d}$ with various π_{11}, and Table 2 presents the comparison results. It is clear that the result of Theorem 1 is less conservative than that in [32]. In special, when $\pi_{22} = -0.5$ and $\bar{d}= 0.4519$, Fig. 1 shows the state trajectories of the open-loop system (4). It is obvious that the system (4) is asymptotically stochastic admissible, which illustrates the accuracy and benefits of Theorem 1.

Remark 4: It can be seen from the above example that the admissible H_∞ performance index γ in Theorem 1 of this article is smaller than that in [9], [11], [34], and the upper bound of the time delay allowed is larger than that in [32]. This means that the theorem in this article is less conservative. In addition, it should be pointed out that the transition rates considered in the above literature is completely known, but when the transition probabilities are not completely known, the approach in the above literature cannot be available, Theorem 2 and Theorem 3 proposed in this article are effective.

Example 2: This example shows the effectiveness of Theorem 3 when the considered SMJSs meet three different cases of the TRs (BUTRs, PUTRs, and GUTRs).

Case 1: Consider the SMJSs (7) with two modes, that is, $S = \{1, 2\}$ and the upper and lower bounds of each transition rate are known, that is, the type of TRs is BUTRs. The system parameters and transition rates are described as follows [14]:

$$E=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad A_1=\begin{bmatrix}0 & 0.1\\ 0 & 1\end{bmatrix},\quad A_2=\begin{bmatrix}-1 & 0.1\\ 0 & -1\end{bmatrix},$$
$$B_1=\begin{bmatrix}0.9\\ -1\end{bmatrix},\quad B_2=\begin{bmatrix}0.1\\ 1\end{bmatrix},\quad B_{\omega 1}=\begin{bmatrix}1\\ 0\end{bmatrix},\quad B_{\omega 2}=\begin{bmatrix}0.1\\ 1\end{bmatrix},$$
$$C_1=\begin{bmatrix}1 & -0.1\\ 0 & 1\end{bmatrix},\quad C_2=\begin{bmatrix}1 & 0.1\\ 0 & 1\end{bmatrix},\quad D_1=\begin{bmatrix}0\\ 0\end{bmatrix},$$
$$D_2=\begin{bmatrix}0\\ 0\end{bmatrix}.$$

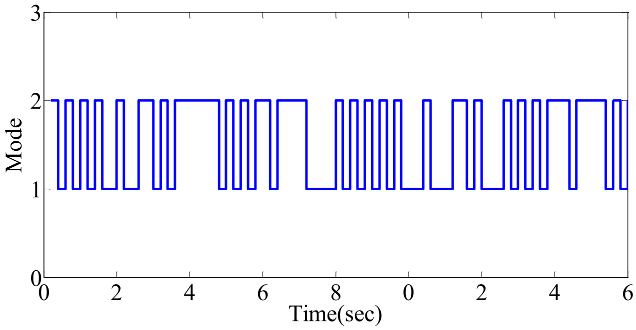

FIGURE 2. System jumping modes of SMJSs with BUTRs.

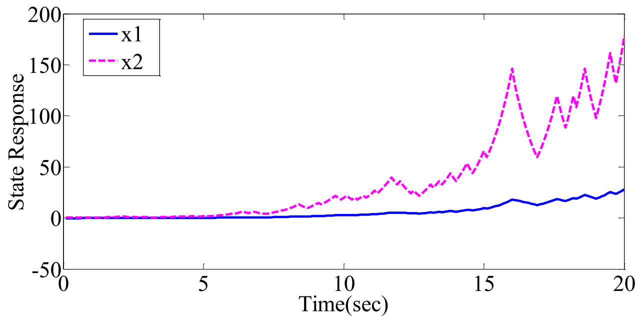

FIGURE 3. State trajectories of open-loop SMJSs with BUTRs.

It can be seen from [14], when $i = 1$, the lower bound of transition probability $\underline{\lambda}_1 = -3.7$, the upper bound of transition probability $\overline{\lambda}_1 = 3.7$, and when $i = 2$, the lower bound of transition probability $\underline{\lambda}_2 = -18$, the upper bound of transition probability $\overline{\lambda}_2 = 18$. We can assume that the estimate of the real transition probability is as follows: $\hat{\pi}_{11} = -2, \hat{\pi}_{12} = 2, \hat{\pi}_{21} = 8, \hat{\pi}_{22} = -8$. According to the lower and upper bounds, we can assume that $\Delta_{11}, \Delta_{12} \in [-1.7, 1.7], \Delta_{21}, \Delta_{22} \in [-10, 10]$. And as defined above, for all $i \in \mathcal{S}$, we denote the set U^i by $U^i = U_k^i \cup U_{Uk}^i$ with $U_k^1 = \{1, 2\}, U_k^2 = \{1, 2\}$.

Using Theorem 3 and LMI toolbox in MATLAB, the closed-loop systems (7) are stochastically admissible and the desired state feedback controller gains are obtained as follows:

$$K_1 = \begin{bmatrix} 6.3416 & 13.4877 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -1.0382 & -1.5392 \end{bmatrix}.$$

Fig. 2 shows the Markovian process system switches between mode 1 to mode 2 and Fig. 3 indicates the state trajectories of open-loop SMJSs with BUTRs, respectively. Fig.4 shows the state trajectories of closed-loop system obtained by the above controller gains. It can be known from Fig.4 that the stochastically admissible problem of SMJSs with BUTRs can be solved by Theorem 3 in this article.

Case 2: Consider the SMJSs (1) with three modes, that is, $\mathcal{S} = \{1, 2, 3\}$. And the type of TRs is PUTRs. The system parameters and transition rates are described as follows [21]:

$$\begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix} = \begin{bmatrix} -1.3 & ? & ? \\ ? & ? & 1.1 \\ 0.2 & 0.3 & -0.5 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

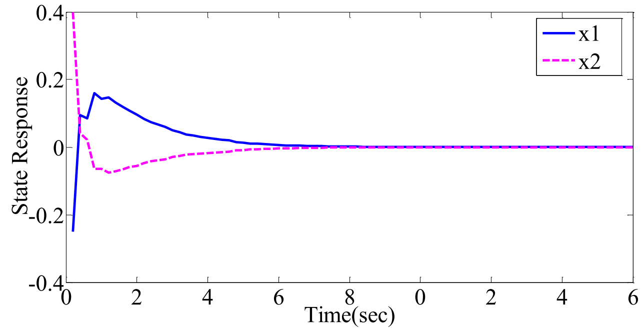

FIGURE 4. State trajectories of closed-loop SMJSs with BUTRs.

$$A_1 = \begin{bmatrix} 0.5 & -0.75 \\ 1 & 2 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.3 & -0.5 \\ 0.2 & -0.4 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.5 \\ 1.0 \end{bmatrix}, B_{\omega 1} = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}^T, \quad D_1 = 2,$$
$$A_2 = \begin{bmatrix} 3.4 & -2 \\ 1 & -3 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0.8 & 1.5 \\ -0.4 & -0.3 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix},$$
$$B_{\omega 2} = \begin{bmatrix} 0.05 \\ 0.1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}^T, \quad D_2 = 3,$$
$$A_3 = \begin{bmatrix} 0.2 & 1 \\ 1 & -0.5 \end{bmatrix}, \quad A_{d3} = \begin{bmatrix} -0.8 & -1 \\ -0.9 & -1.5 \end{bmatrix},$$
$$B_3 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad B_{\omega 3} = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}^T, \quad D_3 = 1.5.$$

According to the description of GUTRs matrix Π in (3), we have

$$\begin{aligned} \Pi &= \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix} \\ &= \begin{bmatrix} \hat{\pi}_{11} + \Delta_{11} & ? & ? \\ ? & ? & \pi_{23} + \Delta_{23} \\ \pi_{31} + \Delta_{31} & \pi_{32} + \Delta_{32} & \pi_{33} + \Delta_{33} \end{bmatrix} \\ &= \begin{bmatrix} -1.3 & ? & ? \\ ? & ? & 1.1 \\ 0.2 & 0.3 & -0.5 \end{bmatrix} \end{aligned} \tag{25}$$

where $\hat{\pi}_{11} = -1.3, \hat{\pi}_{23} = 1.1, \hat{\pi}_{31} = 0.2, \hat{\pi}_{32} = 0.3, \hat{\pi}_{33} = -0.5$, $\Delta_{11} = \Delta_{23} = \Delta_{31} = \Delta_{32} = \Delta_{33} = 0$. And we know $U_k^1 = \{1\}, U_k^2 = \{3\}, U_k^3 = \{1, 2, 3\}$.

From Theorem 3 and LMI toolbox in MATLAB, the SMJSs (7) are stochastically admissible with the following state feedback controller gains:

$$\begin{aligned} K_1 &= \begin{bmatrix} 0.3994 & -4.6430, \end{bmatrix}, \\ K_2 &= \begin{bmatrix} -2.9448 & -0.5971 \end{bmatrix}, \\ K_3 &= \begin{bmatrix} -2.0343 & -2.7217 \end{bmatrix}. \end{aligned}$$

Fig. 5 shows the Markovian process system switches between mode 1 to mode 3 and Fig 6 indicates the state trajectories of open-loop SMJSs with PUTRs, respectively. Fig. 7 shows the state trajectories corresponding to the obtained control gains. It can be seen from Fig. 7 that the stochastically

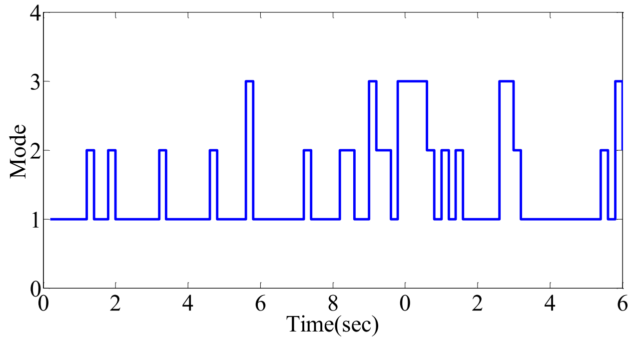

FIGURE 5. System jumping modes of SMJSs with PUTRs.

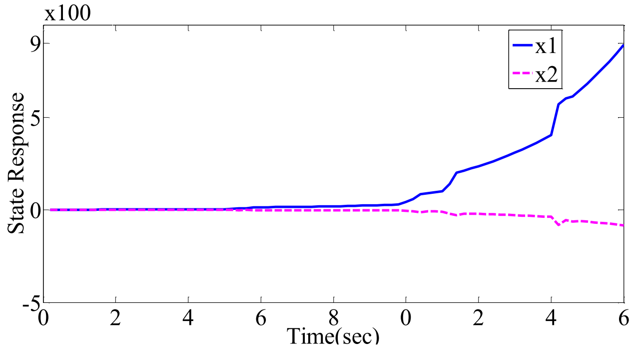

FIGURE 6. State trajectories of open-loop SMJSs with PUTRs.

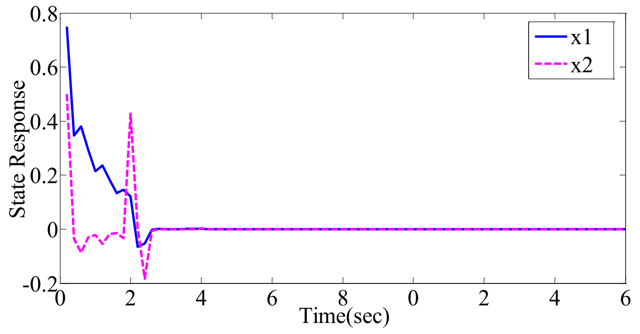

FIGURE 7. State trajectories of closed-loop SMJSs with PUTRs.

admissible problem of SMJSs with PUTRs can also be solved by Theorem 3 in this article.

Case 3: Consider the SMJSs (7) with three modes, that is, $\mathcal{S} = \{1, 2, 3\}$ and each transition probability is completely unknown or only its estimate is known. The type of TRs is GUTRs. We use the same method in [26], and the system parameters are described as follows [26]:

$$A_1 = \begin{bmatrix} -1.0344 & -0.8755 & 2.0747 & 0.1249 \\ 0.4124 & 1.4002 & 0.1 & 0.8501 \\ -1.1249 & 1.3997 & 0.4499 & 0.7499 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.1688 & -3.4001 & -1.3907 & -0.9748 \\ 0.3375 & -1.3996 & -0.1989 & -1.9500 \\ 0 & -0.2501 & 2.3743 & -1.8747 \\ 0.2249 & 2.4989 & 11.2475 & -1.9495 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -1.0126 & -1.7748 & -1.0521 & -1.5377 \\ -0.2248 & -2.1091 & -6.6669 & -0.2697 \\ -0.75 & 0.3003 & -1.6343 & -0.775 \\ 1.3498 & 2.0995 & 1.2299 & 2.2496 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -0.2751 & 4.5008 \\ 0.4701 & 6.7507 \\ 0.0499 & -0.2506 \\ 0.7999 & 4.9988 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.8002 & 0.05 \\ 1.5201 & -0.26 \\ 0.1499 & 0.2 \\ 1.2997 & 0.6 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} -0.225 & 0.2749 \\ 0 & 0.4001 \\ -0.05 & 0.7499 \\ 0.1 & 0.6998 \end{bmatrix}.$$

The mode switching is governed by the rate matrix

$$\Pi = \begin{bmatrix} -3.2 + \Delta_{11} & ? & ? \\ ? & ? & 2 + \Delta_{23} \\ 1.5 + \Delta_{31} & 2.1 + \Delta_{32} & -3.6 + \Delta_{33} \end{bmatrix}$$

where $\hat{\pi}_{11} = -3.2, \hat{\pi}_{23} = 2, \hat{\pi}_{31} = 1.5, \hat{\pi}_{32} = 2.1, \hat{\pi}_{33} = -3.6, \Delta_{11}, \Delta_{32} \in [-0.1, 0.1], \Delta_{23}, \Delta_{31}, \Delta_{33} \in [-0.2, 0.2]$. And we know $\pi_{11} \in [-3.3, -3.1], \pi_{12}=?, \pi_{13}=?, \pi_{21}=?, \pi_{22}=?, \pi_{23} \in [1.8, 2.2], \pi_{31} \in [1.3, 1.7], \pi_{32} \in [2.0, 2.2], \pi_{33} \in [-3.8, -3.4]$, and $U_k^1 = \{1\}, U_k^2 = \{3\}, U_k^3 = \{1, 2, 3\}$.

Without losing generality, we can assume that $\pi_{11} = -3.0, \pi_{12} = 2.0, \pi_{13} = 1.0, \pi_{21} = 1.7, \pi_{22} = -3.6, \pi_{23} = 1.9, \pi_{31} = 1.4, \pi_{32} = 2.2$, and $\pi_{33} = -3.6$.

In this case, we can assume that

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad x(0) = \begin{bmatrix} 0.5 \\ -1 \\ -1.5 \\ 1.2 \end{bmatrix}, \quad R = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

$$S_{d1} = S_{d2} = S_{d3} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

According to Theorem 3 and LMI toolbox in MATLAB, the SMJSs (7) is stochastically admissible with the following state feedback controller gains:

$$K_1 = \begin{bmatrix} 5.2040 & -9.7542 & -10.9004 & -6.6986 \\ -0.3067 & 0.2164 & 0.8476 & 0.3304 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 0.8033 & -2.6911 & -2.1159 & -0.6558 \\ -0.8234 & 0.6351 & -12.1663 & -0.0656 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} -3.8502 & -8.6180 & 7.1047 & -14.9156 \\ -3.6980 & -2.3098 & -1.2993 & -4.2519 \end{bmatrix}.$$

Fig. 8 shows that the Markovian process system switches between mode 1 to mode 3. Fig. 9 shows that the open-loop system trajectories of x_1, x_2, x_3, and x_4 under above switching modes are not stochastically stable. And Fig. 10 shows the closed-loop system state trajectories obtained by the above controller gains. The simulation results demonstrate the above controller gains is effective and that the system is stochastically admissible. It should be pointed out that [26]

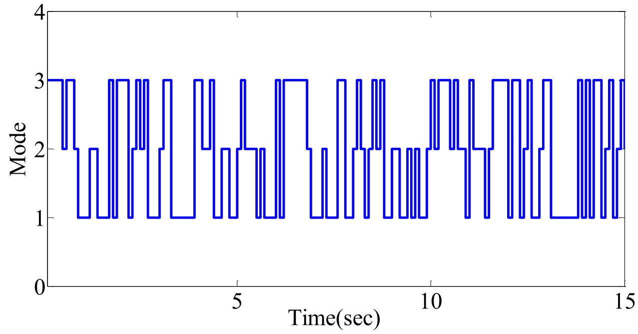

FIGURE 8. System jumping modes of SMJSs with GUTRs.

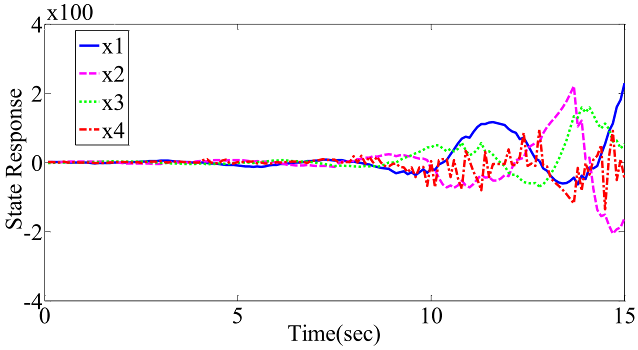

FIGURE 9. State trajectories of open-loop SMJSs with GUTRs.

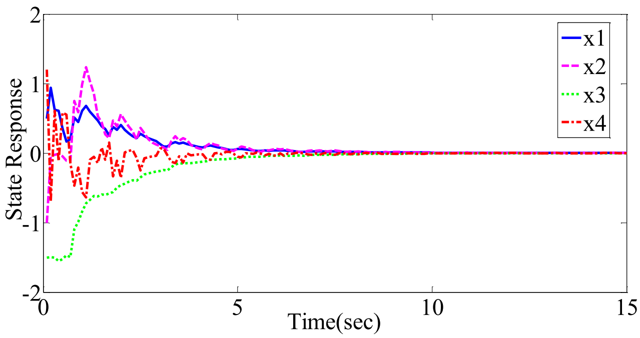

FIGURE 10. State trajectories of closed-loop SMJSs with GUTRs.

is concerned with SMJSs without disturbance and time-delay, while the SMJSs we consider is the one with external disturbance $\omega(t)$ and time-varying delay $d(t)$. Therefore, the theorems in this article are more general than those in [26].

Remark 5: Example 2 shows that whether the transition probability model of SMJSs is BUTRs or PUTRs, the state feedback controller can be designed by Theorem 3 in this article, which makes the system stochastically admissible with H_∞ performance γ. However, the theorems in [14] and [21] can only be used to solve the specific transition probability model. That is to say, [14] can only solve the case where the lower and upper bounds of each transition probability is known, and [21] can only solve the case where the transition probability is partly unknown.

V. CONCLUSION

In this article, by using the Lyapunov theory and LMIs approach, the problem of H_∞ control for continuous-time singular Markovian jump time-varying delay system with general uncertain transition rates has been analyzed. Firstly, under the assumption that the transition probability is completely known, a new delay-dependent strict LMI sufficient condition is obtained, which ensures that the system is stochastically admissible and that a given H_∞ performance index γ is satisfied. Then, some strict LMIs criteria on stochastically admissible for singular Markovian jump time-varying delay systems with general uncertain transition rates are proposed. Furthermore, a H_∞ state feedback controller is designed to guarantee that the closed-loop system is stochastically admissible and the gain of the controller can be obtained by solving a set of strict LMIs. Finally, numerical examples demonstrate the advantage and effectiveness of our proposed method.

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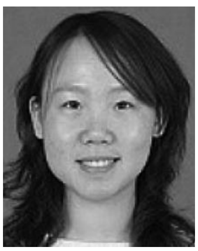

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