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Neural Networks-Based Adaptive Exponential Quasi-Passification and Output Tracking Control for Uncertain Switched Nonlinear Systems

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ABSTRACT In this paper, we mainly study the adaptive exponential quasi-passivity and adaptive tracking control of lower triangular uncertain switched nonlinear systems, even though the adaptive output tracking control problem of individual subsystem is unsolvable. First, the exponential quasipassivity concept is proposed to describe the energy changing of the overall switched nonlinear systems without the exponential quasi-passivity property of all the subsystems. Then, for switched nonlinear systems, the semiglobally uniformly ultimate boundedness is achieved by using exponential quasipassivity. Second, this result is applied to solve adaptive tracking control problem uncertain switched nonlinear systems in lower-triangular form. A new adaptive tracking control technique is developed by combining quasi-passification methods with adaptive backstepping techniques. The unknown nonlinear functions are approximated by the radial basis function neural networks. In contrast to the existing results, the multiple storage functions method reduces the conservativeness caused by a common Lyapunov function for all subsystems. Finally, the effectiveness of the proposed method is verified by an example.

INDEX TERMS Switched nonlinear systems, neural networks, output tracking control, exponential quasipassification.

I. INTRODUCTION

The notion of passivity originated in electrical network theory was first proposed by Willems [1]. In general, passive system dissipates no more energy than the external supply. Passivity theory was widely applied in analysis and synthesis of nonlinear systems [2], [3]. This motivates the researchers [4] to establish necessary and sufficient conditions for feedback passification of nonlinear systems. In practical systems, In practical systems, structural uncertainty and other uncertainties are ubiquitous. These effects of uncertainties on plants can be eliminated with the aid of robust control and adaptive control [5], [6]. To deal with with structural uncertainties, [5] adopted robust control method to solve passfication and stabilization problems. In [6], adaptive passification method was developed for nonlinear systems with

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parameter uncertainties. When the controlled system is considered unknown, neural networks passivity was investigated in [7], [8]. Nevertheless, the aforementioned feedback passification method is its limitation to systems with relative degree one and being minimum-phase. To overcome these restrictive conditions, [5], [9] combined backstepping technique with passification method to solve the stabilization problem of uncertain nonlinear systems. Because of the existence of the large uncertainties, it was often hard to realize exact feedback passification. In [10], [11], only passivity except for a compact region was achieved. Similar practical passivity property was investigated in [12]–[14]. [12] proposed a concept of quasi-passivity. Compared with passive systems, quasipassive systems can produce energy itself. In [13], [14], semipassivity was studied and applied to solve synchronization in networks of neuronal oscillators. This practical passivity was also used to solve nonlinear control problem of uncertain nonlinear systems in lower-triangular form [11], [15]. In [11],

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exponential quasipassivity and backstepping technique were adopted to anaylize stability of power system. In [15], based on the set passivity theory, adaptive fuzzy tracking problem was solved by backstepping technique for a class of unknown nonlinear systems with saturation input nonlinearity.

On the other hand, switched systems have received extensive research attention in recent years [16]-[23]. Many real-world systems can be modeled as switched systems [16]-[19]. Several methods were often used to investigate switched nonlinear systems [18]-[23]. Passivity is an important system property of switched systems as nonswitched systems. So far, many scholars have studied the passivity of switched systems [24]-[31]. [24] defined passivity of switched systems using a common storage function method. But it is often hard to obtain a common storage function even not exist. Therefore, it is natural to study passivity of switched systems by the adoption of multiple storage functions [25], [26]. For switched nonlinear systems with structural uncertainties, robust passification and stabilization were investigated in [27]. In [28], H_{∞} control problem was solved for uncertain switched nonlinear systems with passive and non-passive subsystems. For switched nonlinear systems with parameter uncertainties, the adaptive control technique and passification method were combined to solve the ouput tracking and stabilization problems in [29], [30]. Nevertheless, there are few results on passvity of the switched system with unknown nonlinear functions. [31] investigated passivity analysis and feedback passification for a class of switched T-S fuzzy systems with the sampled-data-dependent switching strategy and controllers. As nonswitched systems, quasipassivity is useful for practical switched systems. Although exponential quasi-passivity property for non-switched systems has been studied thoroughly in [13]-[17], a switched system does not necessarily inherit the properties of its subsystems. Hence, [27] studied expnential quasi-dissipativity and boundedness of switched nonlinear systems without considering uncertainties. In [28], practical stability for uncertain switched nonlinear systems using exponential quasi-passivity property of subsystems was obtained via the average dwell time method. Nevertheless, at least a subsystem was required to be feedback quasipassive, which is a conservative condition, because non-quasi-passive system is commonly encountered in the real world. Subsequently, in [29], robust semipassivity of switched nonlinear systems with structural uncertainties was studied by the design of a state dependent switching law, even if each subsystem was not semipassive. Nevertheless, a switched system with unknown nonlinear functions has not been studied in [29]. Moreover, to solve the feedback quasi-passification problem, the relative degree of each subsystem was required to be one. How to remove this restriction is worth studying.

Motivated by the above analysis, this paper will solve adaptive exponential quasi-passification and adaptive tracking control for uncertain switched nonlinear systems in lower-triangular form. The contributions are in three aspects. First, an exponential quasipassivity is proposed. It is a generalization of exponential passivity. Second, the semiglobally uniformly ultimate boundedness is obtained using the quasi-passivity concept. This result was firstly applied to solve adaptive tracking control for uncertain switched nonlinear systems in lower-triangular form. In contrast to conventional backstepping, a class of nonlinear adaptive controllers with new control inputs are designed constructively. The new control inputs can be redesigned to solve the tracking and stabilization problem and so on. This method can remove the major obstacle to feedback quasipassification that requires the relative degree of each subsystem is one [29]. Finally, a more general switching law which allows the storage function increase at each switching point is designed constructively to achieve output tracking control, while the dwell time and the common Lyapunov method adopted in [20]-[22] are infeasible for the problem under study.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. PRELIMINARIES

Consider a switched nonlinear system described by

$$\dot{x} = f_{\sigma} (x, u_{\sigma}) , y = h_{\sigma} (x) ,$$
 (1)

where $x \in \mathbb{R}^n$ is the state, $\sigma : [0, \infty) \to I = \{1, 2, \dots, M\}$, called a switching signal, is a piecewise constant function. $u_i \in \mathbb{R}^m$ and $y \in \mathbb{R}^m$ are the input and output vectors of the *i*-th subsystem, respectively. $f_i(\cdot)$ and $h_i(\cdot)$ are assumed to be smooth with $f_i(0, 0) = 0$ and $h_i(0) = 0$. The switching signal can be characterized by

$$\sum = \{x_0; (i_0, t_0), (i_1, t_1), \dots, (i_k, t_k), \dots | i_k \in I, k \in Z_+\},$$
(2)

in which x_0 denotes the initial state at the initial time t_0 and Z_+ stands for the set of non-negative integers. When $\sigma(t) = i_k$, the i_k -th subsystem is active during $[t_k, t_{k+1})$. In addition, we assume that the state of the system (1) does not jump at the switching instants.

The definitions of class \mathcal{K}_{∞} functions, class \mathcal{GK} function are introduced as follows.

Definition 1 [35]: A function $\gamma : \mathbb{R}^+ \to \mathbb{R}^+$ is called a class \mathcal{K}_{∞} function if it is continuous, positive definite, strictly increasing and $\gamma(r) \to \infty$ as $r \to \infty$.

Definition 2 [19]: A function $\alpha : R_+ \to R_+$ is called a class \mathcal{GK} function if it is increasing and right continuous at the origin with α (0) = 0.

Next, we will give an exponential quasi-passivity definition of system (1).

Definition 3: System (1) is said to be exponentially quasipassive, if, for a given switching signal σ , there is a nonnegative function $V(\sigma(t), x) : I \times \mathbb{R}^n \to \mathbb{R}^+$, called storage function, constants $\lambda > 0$, $c_i \ge 0$, class \mathcal{GK} function α such that

$$e^{\lambda t} V\left(\sigma\left(t\right), x\left(t\right)\right) - e^{\lambda t_{0}} V\left(\sigma\left(t_{0}\right), x\left(t_{0}\right)\right)$$

$$\leq \int_{t_{0}}^{t} e^{\lambda \tau} u_{\sigma\left(\tau\right)}\left(\tau\right), y\left(\tau\right) + c_{\sigma\left(\tau\right)}\right) \mathrm{d}\tau + e^{\lambda t_{0}} \alpha\left(\|x\left(t_{0}\right)\|\right)$$
(3)

holds for all the initial state $x(t_0)$.

Remark 1: Obviously, system (1) is quasi-passive by letting $\lambda = 0$. c_i is seen as internal supply rates, which means system can produce energy by itself. If $c_i = 0$, Definition 2 degenerates into passivity of switched system in [27]. By (3), the energy of switched system may increase at any switching time and is bounded by $e^{\lambda t_0} \alpha$ ($||x(t_0)||$).

We will investigate ultimately boundedness of system (1) using exponential quasi-passivity.

Definition 4: System (1) is semiglobally uniformly ultimately bounded, find, if possible, feedback controllers u_i , $i \in I$ and a switching signal $\sigma(t)$ such that for given any initial compact set Ω , the closed-loop system possesses the following properties.

(a) (Uniform boundedness) there exists compact set Ω_{ε} such that for all $t_0 \ge 0$, $x_0 \in \Omega$ implies $x(t) \in \Omega_{\varepsilon}$, $\forall t \ge t_0$.

(b) (convergence) for all initial condition $x_0 \in \Omega$, there exists T > 0 such that states x(t) eventually converge to the compact set Ω_s , i.e. $x(t) \in \Omega_s$, $t \ge T$.

We will investigate ultimately boundedness of system (1) using exponential quasi-passivity.

Lemma 1: Suppose that system (1) is exponentially quasipassive with a storage function $V(\sigma(t), x) = V_{\sigma(t)}(x)$. If there exist class \mathcal{K}_{∞} functions ϕ_1, ϕ_2 satisfying $\phi_1(||x||) \le V_i(x) \le \phi_2(||x||)$. Then, the closed-loop system (1) with the output feedback controllers $u_i = -\Psi_i(y)$ is semiglobally uniformly ultimately bounded, where $\Psi_i(y), i \in I$ are continuous functions satisfying $\psi_i(0) = 0$ and $y^T \psi_i(y) \ge 0$.

Proof: Since system (1) is exponentially quasi-passive, for $t \in [t_k, t_{k+1})$, substituting $u_i = -\Psi_i(y)$ into (3) gives

$$e^{\lambda t} V (\sigma (t), x (t)) - e^{\lambda t_0} V (\sigma (t_0), x (t_0)) = e^{\lambda t} V_{i_k} (x (t)) - e^{\lambda t_0} V_{i_0} (x (t_0)) \leq \int_{t_0}^t e^{\lambda \tau} c_{\sigma(\tau)} d\tau + e^{\lambda t_0} \alpha (\|x (t_0)\|).$$
(4)

It follows from $\phi_1(||x||) \leq V_i(x) \leq \phi_2(||x||)$ that

$$\begin{aligned} \phi_{1} (\|x(t)\|) &\leq V_{i_{k}} (x(t)) \\ &\leq e^{-\lambda(t-t_{0})} V_{i_{0}} (x(t_{0})) + e^{-\lambda(t-t_{0})} \alpha (\|x(t_{0})\|) \\ &\quad + \frac{c}{\lambda} \left(1 - e^{-\lambda(t-t_{0})} \right) \\ &\leq e^{-\lambda(t-t_{0})} \left(V_{i_{0}} (x(t_{0})) - \frac{c}{\lambda} + \alpha (\|x(t_{0})\|) \right) + \frac{c}{\lambda} \quad (5) \\ &\leq \overline{\alpha} (\|x(t_{0})\|) + \frac{c}{\lambda}, \end{aligned}$$

where $c = \max_{i \in I} \{c_i\}, \overline{\alpha} = \alpha + \phi_2$. From (6), for any bounded initial conditions $x_0 \in \Omega$, there exists a compact

set $\Omega_{\varepsilon} = \left\{ x \mid ||x|| \le \phi_1^{-1} \left(\overline{\alpha} \left(||x(t_0)|| \right) + \frac{c}{\lambda} \right) \right\}$ such that for all $t_0 \ge 0, x(t) \in \Omega_{\varepsilon}$.

(1) When $V_{i_0}(x(t_0)) - \frac{c}{\lambda} + \alpha(\|x(t_0)\|) = 0$, for any bounded initial conditions $x_0 \in \Omega$, (5) implies that $\lim_{t\to\infty} \|x(t)\| = \phi_1^{-1}\left(\frac{c}{\lambda}\right)$, i.e. there is T > 0 such that state $x(t) \in \Omega_s = \left\{ x \| \|x\| \le \phi_1^{-1}\left(\frac{c}{\lambda}\right) \right\}, t \ge T$. (2) When $V_{i_0}(x(t_0)) - \frac{c}{\lambda} + \alpha(\|x(t_0)\|) \ne 0$, for any $x_0 \in \Omega$, we have $\lim_{t\to\infty} \|x(t)\| = \phi_1^{-1}\left(\frac{c}{\lambda}\right)$.

Therefore, the closed-loop system is semiglobally uniformly ultimately bounded.

B. PROBLEM FORMULATION

1) SYSTEM DESCRIPTION

Consider system (1) in the special form

$$\begin{cases} \dot{z} = f_{0\sigma} (z, x_1), \\ \dot{x}_1 = x_2 + f_{1\sigma} (z, \bar{x}_1), \\ \dot{x}_2 = x_3 + f_{2\sigma} (z, \bar{x}_2), \\ \dots \\ \dot{x}_{n-1} = x_n + f_{(n-1)\sigma} (z, \bar{x}_{n-1}), \\ \dot{x}_n = u_{\sigma} + f_{n\sigma} (z, \bar{x}_n), \\ y = x_1, \end{cases}$$
(7)

where $z \in \mathbb{R}^p$, $x = (x_1, x_2 \dots x_n)^T$ are the system states with $\bar{x}_k = (x_1, \dots x_k)^T$, $k = 1, 2 \dots n$. $f_{0i}(.)$, $f_{ki}(.)$, $k = 1, 2 \dots n$, $i \in I$ are unknown smooth functions and $f_{0i}(0) = 0$, $f_{ki}(0) = 0$, $k = 1, 2 \dots n$, $i \in I$. The switching sequences can be characterized by

$$\Sigma = \{ \left(z_0^T, x_0^T \right); (i_0, t_0), (i_1, t_1), \dots, (i_l, t_l), \dots | i_l \in I , l \in N \}$$
(8)

with the initial states $(z_0^T, x_0^T)^T$

In this paper, for a given reference signal $y_d(t)$, the main control objective is to design adaptive NNs-based controller for each subsystem of system (7) and an appropriate switching law such that

(a) all the signals of the resulting closed-loop system are semiglobally uniformly ultimately bounded.,

(b) the output of the system can follow the reference signal $y_d(t)$ to a small compact.

To this end, we make the following assumptions.

Assumption 1: There exists a positive definite and radially unbounded smooth function $W_i(z)$, $\beta_{ij}(z) \le 0$, and smooth functions $\mu_{ij}(z)$ with $\mu_{ij}(0) = 0$ and $\mu_{ii}(z) = 0$, constants $c_i \ge 0$, $\lambda_i > 0$ for $i, j \in I$ such that

$$L_{f_{i}^{0}(z,y_{d})}W_{i}(z) + \lambda_{i}W_{i}(z) + \sum_{k=1}^{M} \beta_{ik}(z)(W_{i}(z) - W_{k}(z) + \mu_{ik}(z)) \leq c_{i}, \quad (9)$$

$$L_{f_{i}^{0}(z,x_{1})}\mu_{ik}(z) + \lambda\mu_{ik}(z) \le 0,$$
(10)

$$\mu_{ij}(z) + \mu_{jk}(z) \le \min\{0, \mu_{ik}(z)\}$$
(11)

hold for $i, j \in I$.

Assumption 2: The reference signal $y_d(t)$ and its time derivatives $\dot{y}_d(t)$, $\ddot{y}_d(t)$... $y_d^{(n)}(t)$ are continuous and bounded.

Remark 2: Under Assumption 1, the tracking control problem for each subsystem is not required to be solvable. Therefore, Assumption 1 is much weaker than the conditions provided in [23], [26], [30]. Assumption 1 can degenerate into [30] by letting $c_i = 0$.

2) FUNCTION APPROXIMATION USING RBF NNs

RBF neural network is usually used to model nonlinear functions. In [35], an unknown continuous function $f(X) : \mathbb{R}^n \to \mathbb{R}$ can be expressed as $f(X) = \hat{W}S(X) + \varepsilon(X)$, where $X = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$ is the input vector of NN, $\hat{W} \in \mathbb{R}^l$ is a weight vector of the NN, l > 1 is the number of the NN nodes. $\varepsilon(X)$ is the approximation error which satisfies $|\varepsilon(X)| \leq |\varepsilon|$ and ε is an unknown bounded parameter, $S(X) = [s_1(X), \ldots, s_l(X)]^T \in \mathbb{R}^l$ is the basis function vector, where $s_i(X)$ can be chosen as the Gaussian functions in th form

$$s_i(X) = \exp\left[\frac{-(X-\mu_i)^T(X-\mu_i)}{\xi_i^2}\right], \quad i = 1, 2, \dots l,$$
(12)

where $\mu_i = [\mu_{i1}, \mu_{i2}, \dots, \mu_{in}]^T$ is the center of the receptive field and ξ_i is the width of the Gaussian function.

As mentioned in [36], for sufficiently large nodes number *l*, the RBF neural networks $\theta^*S(X)$ can approximate any continuous function f(X) over a compact set Ω , that is, for $\forall \varepsilon > 0$, there exists the RBF neural networks $W^*S(X)$ such that

$$f(X) = \theta^* S(X) + \varepsilon^*(X), \quad \forall X \in \Omega \subset \mathbb{R}^n, \quad (13)$$

where W^* is the optimal weight and $|\varepsilon^*(X)| \le \varepsilon$ is the smallest approximation error.

III. MAIN RESULTS

This section will solve the output tracking control problem of system (7) using the exponential quasi-passivity concept.

A. NEURAL NETWORKS-BASED ADAPTIVE EXPONENTIAL QUASI-PASSIFICATION CONTROLLER AND SWITCHING LAW DESIGN

In the following, a set of adaptive controllers and a statedependent switching law will be designed constructively to solve the exponential quasi-passification problem of system (7).

Step 1: Consider the *i*-th (z, x_1) subsystem. Construct the output $e_1 = x_1 - y_d$. Thus, the *i*-th (z, x_1) subsystem is described by

$$\dot{z} = f_{0i} (z, e_1 + y_d), \dot{e}_1 = x_2 + f_{1i} (z, e_1 + y_d) - \dot{y}_d,$$
(14)

where x_2 is a virtual input of (14). Thus, the zero dynamics of system (14) is $\dot{z} = f_{0i}(z, y_d)$. Using the mean value theorem

gives:

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$$f_{0i}(z, e_1 + y_d) = f_{0i}(z, y_d) + e_1^T \tilde{f}_{0i}(z, e_1 + y_d), \quad (15)$$

where $\tilde{f}_{0i}(z, e_1 + y_d) = \int_0^1 \frac{\partial f_{0i}(z, \zeta + y_d)}{\partial \zeta} \bigg|_{\zeta} = se_1 ds$ are unknown smooth functions. Let

$$R_{1i}(X_1) = f_{1i}(z, x_1) + \frac{1}{l} \frac{\partial W_i(z)}{\partial z} \tilde{f}_{0i}(z, x_1) - \dot{y}_d, X_1 = (z, x_1, y_d, \dot{y}_d).$$

Using an RBF neural network $\theta_{i1}^{T*}S_1(X_1) + \varepsilon_{i1}^*$ to approximate $R_{1i}(X_1)$ gives

$$R_{1i}(X_1) = \theta_{i1}^{T*} S_1(X_1) + \varepsilon_{i1}^{\bullet}(X_1), \left|\varepsilon_{i1}^{\bullet}\right| \le \overline{\varepsilon}_1$$

with the ideal constant weights θ_{i1}^* , and the approximation error ε_{i1}^* . Define an unknown constant $\Theta_1 = \max_{i \in I} \{ \|\theta_{i1}\|^2 \}$. A storage function for system (14) is constructed as $V_{1i}\left(z, e_1, \tilde{\Theta}_1\right) = \frac{1}{l} W_i(z) + \frac{e_1^2}{2} + \frac{1}{2l} \tilde{\Theta}_1^2$ with the estimation error $\tilde{\Theta}_1 = \Theta_1 - \hat{\Theta}_1$, where $\hat{\Theta}_1$ is the estimation of $\Theta_1, l > 0$ is a design parameter.

Differentiating V_{1i} together with (9) gives

$$V_{1i}$$

$$= e_1 \left(x_2 + \frac{1}{l} \frac{\partial W_i}{\partial z} \tilde{f}_{0i} \left(z, e_1 + y_d \right) + f_{1i} \left(z, e_1 + y_d \right) - \dot{y}_d \right)$$

$$+ \frac{1}{l} \frac{\partial W_i}{\partial z} f_{0i} \left(z, y_d \right) - \frac{1}{l} \tilde{\Theta}_1 \hat{\Theta}_1$$

$$= e_1 \left(x_2 + \theta_{i1}^{T*} S_1 \left(X_1 \right) + \varepsilon_{i1}^{\bullet} \left(X_1 \right) \right)$$

$$+ \frac{1}{l} \frac{\partial W_i}{\partial z} f_{0i} \left(z, y_d \right) - \frac{1}{l} \tilde{\Theta}_1 \hat{\Theta}_1 \qquad (16)$$

Applying the Young's inequality gives

$$e_1\theta_{i1}^{T*}S_1(X_1) \le \frac{e_1^2}{2a_1^2}\Theta_1S_1^TS_1 + \frac{a_1^2}{2}, e_1\varepsilon_{i1}^{\bullet} \le \frac{e_1^2}{2} + \frac{\bar{\varepsilon}_1^2}{2}, \quad (17)$$

where $a_1 > 0$ is a design parameter. Therefore,

$$\dot{V}_{1i} \leq e_1 \left(x_2 + \frac{e_1}{2a_1^2} \Theta_1 S_1^T S_1 + \frac{e_1}{2} \right) + \frac{a_1^2}{2} + \frac{\bar{\varepsilon}_1^2}{2} + \frac{1}{l} \frac{\partial W_i}{\partial z} f_{0i}(z, y_d) - \frac{1}{l} \tilde{\Theta}_1 \hat{\Theta}_1.$$
(18)

Design the virtual common controller as:

$$x_{2} = \alpha_{1} \left(\hat{\Theta}_{1}, e_{1}, X_{1} \right) + e_{2},$$

$$\alpha_{1} = -\frac{e_{1}}{2a_{1}^{2}} \hat{\Theta}_{1} S_{1}^{T} S_{1} - \left(\frac{\lambda}{2} + \frac{1}{2} \right) e_{1} + \dot{y}_{d},$$

$$\dot{\hat{\Theta}}_{1} = l \frac{e_{1}^{2}}{2a_{1}^{2}} S_{1}^{T} S_{1} - \lambda \hat{\Theta}_{1},$$
(19)

where $\lambda = \min_{i \in I} \{\lambda_i\}$. From (9) and (19), we have

$$\dot{V}_{1i} \leq e_1 e_2 - \lambda \frac{e_1^2}{2} + \frac{a_1^2}{2} + \frac{\bar{\varepsilon}_{i1}^2}{2} + \frac{1}{2} \frac{\partial W_i}{\partial z} f_{0i}(z, y_d) + \frac{\lambda}{l} \tilde{\Theta}_1 \hat{\Theta}_1$$

$$\leq e_{1}e_{2} - \frac{\lambda}{l}W_{i}(z) - \lambda\frac{e_{1}^{2}}{2} - \frac{\lambda}{l}\frac{\Theta_{1}^{2}}{2} + \frac{a_{1}^{2}}{2} + \frac{\bar{\varepsilon}_{1}^{2}}{2} + \frac{\lambda}{l}\frac{\Theta_{1}^{2}}{2} \\ + \sum_{k=1}^{M}\beta_{ik}(z)\left(\frac{W_{i}(z)}{l} - \frac{W_{k}(z)}{l} + \frac{\mu_{ik}(z)}{l}\right) + \frac{c_{i}}{l} \\ \leq e_{1}e_{2} - \lambda V_{1i} + \frac{a_{1}^{2}}{2} + \frac{\bar{\varepsilon}_{1}^{2}}{2} + \frac{\lambda}{l}\frac{\Theta_{1}^{2}}{2} + \frac{c_{i}}{l} \\ + \sum_{k=1}^{M}\beta_{ik}(z)(V_{1i}(z,e_{1}) - V_{k}(z,e_{1}) + \tilde{\mu}_{ik}(z,e_{1})),$$

where $\tilde{\mu}_{ik}(z,e_1) = \frac{1}{T}\mu_{ik}(z)$.

Step $j(1 \le j \le n-1)$: Let $X_k = (z, \bar{x}_k, \hat{\Theta}_1, \dots, \hat{\Theta}_{k-1}, y_d,$ $(\dots, y_d^{(k)})^T, Z_j = (z, e_1, \dots, e_j), \vartheta_j = (\tilde{\Theta}_1, \dots, \tilde{\Theta}_j).$ Repeating this process, we can obtain, at the *j*-th step, that the virtual output, virtual input and the storage functions of $\begin{pmatrix} z, \bar{x}_{j-1}, \hat{\Theta}_1, \dots, \hat{\Theta}_{j-2} \end{pmatrix}$ subsystem: $e_{j-1} = x_{j-1} - \alpha_j$ and $x_j = \alpha_{j-1} (X_j) + e_j, V_{j-1,i} (Z_{j-1}, \vartheta_{j-1}) = \frac{1}{l} W_i (z) + \sum_{k=1}^{j-1} \frac{e_k^2}{2} + \frac{\hat{\Theta}_k^2}{2l}$ satisfying

$$\dot{V}_{j-1,i} \leq e_{j-1}e_j - \lambda V_{j-1i} + \sum_{k=1}^{j-1} \left(\frac{a_k^2}{2} + \frac{\bar{\varepsilon}_k^2}{2} + \frac{\lambda}{l}\frac{\Theta_k^2}{2}\right) + \frac{c_i}{l} + \sum_{\substack{k=1\\k=1}}^{M} \beta_{ik}\left(z\right)\left(V_{j-1,i}\left(Z_{j-1},\vartheta_{j-1}\right) - V_{j-1,k}\left(Z_{j-1},\vartheta_{j-1}\right) + \tilde{\mu}_{ik}\left(Z_{j-1},\vartheta_{j-1}\right)\right).$$
(20)

where $\tilde{\mu}_{ik} = \frac{1}{T} \mu_{ik} (z)$.

Consider the *i*-th $(z, \bar{x}_j, \hat{\Theta}_1, \dots, \hat{\Theta}_{j-1})$ subsystem. Let the output $e_j = x_j - \alpha_{j-1} \cdot x_{j+1}$ is a virtual input of *i* th $\left(z, \bar{x}_j, \hat{\Theta}_1, \dots, \hat{\Theta}_{j-1}\right)$ subsystem. Let $R_{j,i}\left(X_j\right) = f_{j,i}\left(z, \bar{x}_j\right) - f_{j,i}\left(z, \bar{x}_j\right)$ $\dot{\alpha}_{j-1}$, which are the unknown functions. Employing an RBF neural network $\theta_{ij}^{T*}S_j(X_j)$ to approximate $R_{j,i}(X_j)$ gives

$$R_{j,i}(X_j) = \theta_{ij}^{T*} S_j(X_j) + \varepsilon_{ij}^*(X_j), \left|\varepsilon_{ij}^*(X_j)\right| \le \bar{\varepsilon}_j, \quad (21)$$

where θ_{ij}^{T*} denotes the ideal constant weights, ε_{ij}^{*} is the approximation error with constant $\bar{\varepsilon}_{j} > 0$. Define $\Theta_{j} =$ $\max_{i \in I} \left\{ \left\| \theta_{ij} \right\|^2 \right\}.$ Choose the storage function as

$$V_{ji}\left(Z_{j},\vartheta_{j}\right)=V_{j-1,i}\left(Z_{j-1},\vartheta_{j-1}\right)+\frac{e_{j}^{2}}{2}+\frac{\tilde{\Theta}_{j}^{2}}{2l},$$

where $\hat{\Theta}_{j} = \Theta_{j} - \hat{\Theta}_{j}$, $\hat{\Theta}_{j}$ is the estimation of Θ_{j} .

The derivative of V_{ii} along the trajectory of the *i*-th (z, \bar{x}_i) subsystem together with (20) and (21) is

$$\begin{split} \dot{V}_{ji} &\leq \left(e_{j-1} + x_{j+1} + f_{ji}\left(z, \bar{x}_{j}\right) - \dot{\alpha}_{j}\right)e_{j} - \lambda V_{j-1,i} \\ &+ \sum_{k=1}^{j-1} \left(\frac{a_{k}^{2}}{2} + \frac{\bar{\varepsilon}_{k}^{2}}{2} + \frac{\lambda}{l}\frac{\Theta_{k}^{2}}{2}\right) + \frac{c_{i}}{l} - \frac{\tilde{\Theta}_{j}}{l}\hat{\Theta}_{j} \\ &+ \sum_{k=1}^{M} \beta_{ik}\left(z\right)\left(V_{ji}\left(Z_{j}, \vartheta_{j}\right) - V_{jk}\left(Z_{j}, \vartheta_{j}\right) + \tilde{\mu}_{ik}\right) \end{split}$$

$$\leq \left(e_{j-1} + x_{j+1} + \theta_{ij}^{T*}S_{j}\left(X_{j}\right) + \varepsilon_{ij}^{*}\right)e_{j} - \lambda V_{j-1,i}$$

$$+ \sum_{k=1}^{j-1}\left(\frac{a_{k}^{2}}{2} + \frac{\bar{\varepsilon}_{k}^{2}}{2} + \frac{\lambda}{l}\frac{\Theta_{k}^{2}}{2}\right) + \frac{c_{i}}{l} - \frac{\tilde{\Theta}_{j+1}}{l}\hat{\Theta}_{j+1}$$

$$+ \sum_{k=1}^{M}\beta_{ik}\left(z\right)\left(V_{ji}\left(Z_{j},\vartheta_{j}\right) - V_{jk}\left(Z_{j},\vartheta_{j}\right) + \tilde{\mu}_{ik}\right). \quad (22)$$

Applying the Young's inequality gives

$$e_{j}\theta_{ij}^{T*}S_{j}\left(X_{j}\right) \leq \frac{e_{j}^{2}}{2a_{j}^{2}}\Theta_{j}S_{j}^{T}S_{j} + \frac{a_{j}^{2}}{2}, e_{j}\varepsilon_{i,j}^{\bullet} \leq \frac{e_{j}^{2}}{2} + \frac{\bar{\varepsilon}_{j}^{2}}{2}, \quad (23)$$

where $a_{i+1} > 0$ is a design parameter.

Design the virtual common controller as:

$$x_{j+1} = \alpha_j + e_{j+1},$$

$$\alpha_j = -\frac{e_j}{2a_j^2} \hat{\Theta}_j S_j^T S_j - e_{j-1} - \frac{\lambda}{2} e_j, \, \dot{\hat{\Theta}}_j = l \frac{e_j^2}{2a_j^2} S_j^T S_j - \lambda \hat{\Theta}_j.$$
(24)

Substituting (23), (24) into (22) gives

$$\dot{V}_{j,i} \leq e_{j}e_{j+1} - \lambda V_{j,i} + \sum_{k=1}^{j} \left(\frac{a_{k}^{2}}{2} + \frac{\bar{\varepsilon}_{k}^{2}}{2} + \frac{\lambda}{l}\frac{\Theta_{k}^{2}}{2}\right) + c_{i} + \sum_{k=1}^{M} \beta_{ik}\left(z\right)\left(V_{j,i}\left(Z_{j},\vartheta_{j}\right) - V_{j,k}\left(Z_{j},\vartheta_{j}\right) + \mu_{ik}\left(z\right)\right).$$
(25)

Step n: According to the above analysis, we have

$$\begin{split} \dot{V}_{n-1,i} &\leq e_{n-1}e_n - \lambda V_{n-1,i} + \sum_{k=1}^{n-1} \left(\frac{a_k^2}{2} + \frac{\bar{\varepsilon}_k^2}{2} + \frac{\lambda}{l} \frac{\Theta_k^2}{2}\right) + \frac{c_i}{l} \\ &+ \sum_{k=1}^M \beta_{ik} \left(z\right) \left(V_{n-1,i} \left(Z_{n-1}, \vartheta_{n-1}\right) - V_{n-1,k} \left(Z_{n-1}, \vartheta_{n-1}\right) + \tilde{\mu}_{ik}\right). \end{split}$$

Consider the *i*-th $(z, \bar{x}_n, \hat{\Theta}_1, \dots, \hat{\Theta}_{n-1})$ subsystem. Define the output $e_n = x_n - \alpha_{n-1}$. Use an RBF neural network $\theta_{in}^{T*}S_n(X_n)$ to approximate $f_{n,i}(z, \bar{x}_n) - \dot{\alpha}_{n-1}$.

Let $\Theta_{j+1} = \max_{i \in I} \left\{ \left\| \theta_{ij+1} \right\|^2 \right\}$. Choose the storage function

$$V_{ni}(Z_n,\vartheta_n) = V_{n-1,i}(Z_{n-1},\vartheta_{n-1}) + \frac{e_n^2}{2} + \frac{\tilde{\Theta}_n^2}{2l},$$

where $\tilde{\Theta}_n = \Theta_n - \hat{\Theta}_n$ and $\hat{\Theta}_n$ is the estimation of Θ_n . Design the controllers as:

$$u_{i} = \alpha_{n} + v_{i},$$

$$\alpha_{n} = -\frac{e_{n}}{2a_{n}^{2}}\hat{\Theta}_{n}S_{n}^{T}S_{n} - e_{n-1} - \frac{\lambda}{2}e_{n},$$

$$\dot{\hat{\Theta}}_{n} = l\frac{e_{n}^{2}}{2a_{n}^{2}}S_{n}^{T}S_{n} - \lambda\hat{\Theta}_{n},$$
(26)

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The derivative of V_{ni} is

$$\dot{V}_{ni} \leq e_n v_i - \lambda V_{ni} + \sum_{k=1}^n \left(\frac{a_k^2}{2} + \frac{\bar{\varepsilon}_k^2}{2} + \frac{\lambda}{l} \frac{\Theta_k^2}{2}\right) + \frac{c_i}{l} + \sum_{k=1}^M \tilde{\beta}_{ik} \left(Z_n, \vartheta_n\right) \left(V_{ni} \left(Z_n, \vartheta_n\right) - V_{nk} \left(Z_n, \vartheta_n\right) + \tilde{\mu}_{ik} \left(Z_n, \vartheta_n\right)\right),$$
(27)

where $\tilde{\beta}_{ik}(Z_n, \vartheta_n) = \beta_{ik}(z)$, $\tilde{\mu}(Z_n, \vartheta_n) = \frac{1}{l}\mu_{ik}(z)$. Let

$$\Omega_{i} = \{ (Z_{n}, \vartheta_{n}) | V_{ni} (Z_{n}, \vartheta_{n}) \\ -V_{nk} (Z_{n}, \vartheta_{n}) + \tilde{\mu}_{ik} (Z_{n}, \vartheta_{n}) \le 0, j \in I \}$$

and

$$\begin{split} \tilde{\Omega}_{ij} &= \{ \left(Z_n, \vartheta_n \right) | V_{ni} \left(Z_n, \vartheta_n \right) \\ &- V_{nk} \left(Z_n, \vartheta_n \right) + \tilde{\mu}_{ik} \left(Z_n, \vartheta_n \right) = 0, \, i \neq k \} \,. \end{split}$$

According to [22], the set { $\Omega_i | i \in I$ } is a partition of R^{2n+p} . When $V_{ni}(Z_n, \vartheta_n) - V_{nk}(Z_n, \vartheta_n) + \tilde{\mu}_{ik}(Z_n, \vartheta_n) \leq 0$, we have

$$\dot{V}_{ni} \le e_n v_i - \lambda V_{ni} + \sum_{k=1}^n \left(\frac{a_k^2}{2} + \frac{\bar{\varepsilon}_k^2}{2} + \frac{\lambda}{l} \frac{\Theta_k^2}{2}\right) + \frac{c_i}{l}.$$
 (28)

Multiplying both sides of (28) by $e^{\lambda t}$, respectively, yields:

$$\frac{d}{dt}\left(e^{\lambda t}V_{ni}\right) \le e^{\lambda t}\left(e_nv_i + C_i\right),\tag{29}$$

where $C_i = \sum_{k=1}^{n} \left(\frac{a_k^2}{2} + \frac{\overline{\epsilon}_k^2}{2} + \frac{\lambda}{l} \frac{\Theta_k^2}{2}\right) + \frac{c_i}{l}$. Integrating (29) over [s, t] for $\forall t > s \ge t_0$ gives:

$$e^{\lambda t} V_{ni}(t) - e^{\lambda s} V_{ni}(s) \le \int_{s}^{t} e^{\lambda \tau} (e_{n}(\tau) v_{i}(\tau) + C_{i}) d\tau .$$
(30)

Design the switching law as:

$$\sigma(t) = i \text{ if } \sigma(t^{-}) = i \text{ and } x(t) \in \Omega_i,$$

$$\sigma(t) = \min\left\{ j \left| x(t) \in \tilde{\Omega}_{ij} \right\} \text{ if } \sigma(t^{-}) = i \text{ and } x(t) \in \tilde{\Omega}_{ij}.$$
(31)

From (31), we have

$$V_{ni} (Z_n (t_l), \vartheta_n (t_l)) - V_{nk} (Z_n (t_l), \vartheta_n (t_l)) = \tilde{\mu}_{ik} (Z_n (t_l), \vartheta_n (t_l)), \quad l = 0, 1, \dots \quad (32)$$

and $e^{\lambda t} \tilde{\mu}_{i_k j} (Z_n(t), \vartheta(t))$ are decreasing on $[t_l, t_{l+1})$.

The storage function of system (7) is defined as $V(\sigma(t), Z_n, \vartheta_n) = V_{\sigma(t)}(Z_n, \vartheta_n)$. For $t \ge t_0 > 0$ and $t \in [t_k, t_{k+1})$.

From (30), (32), and (21), we have:

$$\begin{split} e^{\lambda t} V \left(\sigma \left(t \right), Z_{n} \left(t \right), \vartheta_{n} \left(t \right) \right) &- e^{\lambda t_{0}} V \left(\sigma \left(t_{0} \right), Z_{n} \left(t_{0} \right), \vartheta_{n} \left(t_{0} \right) \right) \\ &= e^{\lambda t} V_{i_{k}} \left(Z_{n} \left(t \right), \vartheta_{n} \left(t \right) \right) - e^{\lambda t_{k}} V_{i_{k}} \left(Z_{n} \left(t_{k} \right), \vartheta_{n} \left(t_{k} \right) \right) \\ &+ \sum_{p=0}^{k-1} \left(e^{\lambda t_{p+1}} V_{i_{p}} \left(Z_{n} \left(t_{p+1} \right), \vartheta_{n} \left(t_{p+1} \right) \right) \right) \end{split}$$

$$-e^{\lambda t_{p}}V_{i_{p}}(Z_{n}(t_{p}), \vartheta_{n}(t_{p})))$$

$$+\sum_{p=1}^{k}e^{\lambda t_{p}}\left(V_{i_{p}}(Z_{n}(t_{p}), \vartheta_{n}(t_{p}))\right)$$

$$-V_{i_{p-1}}\left((Z_{n}(t_{p}), \vartheta_{n}(t_{p}))\right))$$

$$\leq \int_{t_{0}}^{t}e^{\lambda \tau}\left(e_{n}(\tau)v_{\sigma}(\tau)+C_{\sigma}\right)d\tau$$

$$+\sum_{p=1}^{k}e^{\lambda t_{p}}\tilde{\mu}_{i_{p-1}i_{p}}(Z_{n}(t_{p}), \vartheta_{n}(t_{p}))$$

$$\leq \begin{cases}\int_{t_{0}}^{t}e^{\lambda \tau}\left(e_{n}(\tau)v_{\sigma}(\tau)+C_{\sigma}\right)d\tau & \text{if k is even}\\\int_{t_{0}}^{t}e^{\lambda \tau}\left(e_{n}(\tau)v_{\sigma}(\tau)+C_{\sigma}\right)d\tau & \text{if k is odd}\end{cases}$$

$$\leq \int_{t_{0}}^{t}e^{\lambda \tau}\left(e_{n}(\tau)v_{\sigma}(\tau)+C_{\sigma}\right)d\tau$$

$$+e^{\lambda t_{0}}\tilde{\mu}_{i_{0}i_{1}}(Z_{n}(t_{0}), \vartheta_{n}(t_{0})) & \text{if k is odd}\end{cases}$$

$$\leq \int_{t_{0}}^{t}e^{\lambda \tau}\left(e_{n}(\tau)v_{\sigma}(\tau)+C_{\sigma}\right)d\tau$$

$$+e^{\lambda t_{0}}\alpha(\|(Z_{n}(t_{0}), \vartheta_{n}(t_{0}))\|)$$
(33)

where $\alpha(s) = \max_{\|(Z_n,\vartheta_n)\| \le s} \{ |\mu_{ik}(Z_n,\vartheta_n)| | i, k \in I \}$ is a class \mathcal{GK} function. Therefore, system (7) is exponentially quasi-passive.

Remark 3: Because of the existence of uncertainty, quasipassification was only achieved instead of exact feedback passification.

B. STABILITY ANALYSIS

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Based on the above quasipassive system, the output tracking problem will be solved by output feedback controllers.

Theorem 1: Consider system (7). For the given reference signal $y_d(t)$, suppose Assumptions 1,2 hold. Then, there exists the switching law (31) and adaptive controllers (26) such that closed- loop system is exponentially quasipassive from the inputs v_i to the output $e_n = x_n - \alpha_n$. Furthermore, if the new control input is redesigned as $v_i = -\Psi_i(e_n)$ with the continuous function $\phi_i(\cdot)$ satisfying $e_n^T \Psi_i(e_n) \ge 0$, the output tracking control problem is solvable.

Proof: By Lemma 1, all the signals of the resulting closedloop system are semiglobally uniformly ultimately bounded. Thus, all the signals $(z, e_1, \ldots e_n, \tilde{\Theta}_1, \ldots \tilde{\Theta}_n)$ in the closedloop system are bounded. Because $\Theta_1, \Theta_2 \ldots \Theta_n$ are constants, $\hat{\Theta}_1, \ldots \hat{\Theta}_n$ are bounded. By Assumption 2, *z*, *x*₁, ... *x*_n are bounded. The states and parameter estimation errors converge to compact sets whose sizes can be reduced by choosing appropriate design parameters *l* and *a_i*.On the other hand, for all initial condition, *e*₁ eventually converge to the compact set. Therefore, the output can follow the reference signal *y_d*(*t*) in a small compact.

Remark 4: In contrast to conventional backstepping, a class of nonlinear adaptive controllers with new control inputs are designed constructively. The new control inputs can be redesigned to solve the tracking and stabilization problem and so on. The switching law (31) can degenerate into the well-known "min-switching" law in [23] by setting $\mu_{ij} \equiv 0$.

IV. EXAMPLE

In this section, a numerical example is given to demonstrate the effectiveness of the results.

Consider the following system

$$\begin{cases} \dot{z} = f_{0\sigma} (z, x_1), \\ \dot{x}_1 = x_2 + f_{1\sigma} (z, \bar{x}_1), \\ \dot{x}_2 = u_{\sigma} + f_{2\sigma} (z, \bar{x}_2), \\ y = x_1, \end{cases}$$
(34)

where $\sigma(t): [0, +\infty) \to I = \{1, 2\} z = [z_1, z_2]^T \in \mathbb{R}^2, x = [x_1, x_2]^T \in \mathbb{R}^2, f_{01} = [-9z_1 + 3x_1, 2z_2 + \cos(z_1 z_2)]^T, f_{11} = z_1 x_1 + x_1^2, f_{21} = z_2^2 + e^{x_1^2}, f_{12} = x_1^2 z_2 + \cos x_1, f_{02} = [2z_1 + \cos(1.2z_2), -16z_2 + 2x_1]^T, f_{22} = z_1^2 x_2^3$. The reference signal is $y_d(t) = \sin t$.

First, let $W_1 = \frac{1}{2}z_1^2 + z_2^2$, $W_2 = z_1^2 + \frac{1}{2}z_2^2$. It is easy to verify that the conditions in Assumption 1 are satisfied with $\lambda_1 = 1, \lambda_2 = 6, c_1 = 2, c_2 = 2, \mu_{12} = \mu_{21} = 0$ and $\beta_{12} = -12, \beta_{21} = -22$.

Similar to the proof of Theorem 1, the adaptive NN controllers and update laws as:

$$x_2 = \cos t - \frac{e_1}{2a_1^2} \hat{\Theta}_1 S_1^T S_1 - \left(\frac{\lambda}{2} + \frac{1}{2}\right) e_1 + e_2, \quad (35)$$

$$u_1 = -\frac{e_2}{2a_2^2} \hat{\Theta}_2 S_2^T S_2 - e_1 - \frac{\lambda_1}{2} e_2 + v_1,$$
(36)

$$u_2 = -\frac{e_2}{2a_2^2}\hat{\Theta}_2 S_2^T S_2 - e_1 - \frac{\lambda_2}{2}e_2 + v_1, \qquad (37)$$

$$\dot{\hat{\Theta}}_1 = l \frac{e_1^2}{2a_1^2} S_1^T S_1 - 2\hat{\Theta}_1,$$
(38)

$$\dot{\hat{\Theta}}_2 = l \frac{e_2^2}{2a_2^2} S_2^T S_2 - 6 \hat{\Theta}_2,$$
(39)

where $\lambda = \min \{\lambda_i | i = 1, 2\} = 1$. Then

 $\begin{aligned} \dot{V}_{21} &\leq 12 \left(V_{21} \left(z, e_2 \right) - V_{22} \left(z, e_2 \right) \right) + e_2 v_1 - V_{21} + d_{21}, \\ \dot{V}_{22} &\leq 22 \left(V_{22} \left(z, \bar{e}_2 \right) - V_{21} \left(z, \bar{e}_2 \right) \right) + e_2 v_2 - V_{22} + d_{22}, \end{aligned}$

where

$$d_{2i} = \sum_{k=1}^{2} \left(\frac{a_k^2}{2} + \frac{\bar{\varepsilon}_k^2}{2} + \frac{\lambda}{l} \frac{\Theta_k^2}{2}\right) + \frac{c_i}{l},$$

$$\bar{\varepsilon}_1 = \bar{\varepsilon}_2 = 2, i = 1, 2.$$

$$V_{2i}(z, \bar{e}_2) = V_{1i} + \frac{e_2^2}{2} + \frac{1}{2l} \tilde{\Theta}_2^2,$$

$$V_{1i}(z, e_1) = \frac{1}{l} W_i(z) + \frac{e_1^2}{2} + \frac{1}{2l} \tilde{\Theta}_1^2, \quad i = 1, 2.$$

Design the switching law as follows:

$$\sigma (t) = 1, \text{ when } V_{21}(z, \bar{e}_2) - V_{22}(z, \bar{e}_2) \le 0;$$

$$\sigma (t) = 2, \text{ when } V_{22}(z, \bar{e}_2) - V_{21}(z, \bar{e}_2) \le 0; \quad (40)$$

By Theorem 1, system (33) is exponentially quasi-passive from v_i to the output e_2 . Therefore, the output tracking problem for closed-loop system (33,36-39) is solvable by $v_1 = -e_2$, $v_2 = -2e_2$ under the switching law (40).



FIGURE 1. State responses of the switched system.



FIGURE 2. Output and reference signal.



FIGURE 3. Tracking error.

Let $a_1 = 0.24, a_2 = 0.2, l = 20$.

The input vectors of RBFNNs are $X_1 = (z_1, z_2, x_1, y_d, \dot{y}_d)$ and $X_2 = (z_1, z_2, \bar{x}_k, \hat{\Theta}_1, \dots, \hat{\Theta}_{k-1}, y_d, \dots, y_d^{(k)})^T$. The centers and widths are chosen on a regular lattice in the respective compact sets. The centers and widths of RBFNNs evenly spaced on [-2, 2], [-2, 2]. The widths of RBFNNs are $\xi_1 = \xi_2 = 2$. The simulation was performed with the initial state (z(0), x(0)) = (-1.5, 0.6, -0.2, -0.05), $\hat{\Theta}_1(0) = 0, \hat{\Theta}_2(0) = 0.$

The simulation results are shown in Figs. 1-4. See from Figure 1, the states are bounded under the switching law described in Fig. 4. Figs. 2 and 3 imply the outputs of the system (33) can track the reference signal under the controllers



FIGURE 4. Switching law (39).



FIGURE 5. Control input.

as shown in Fig. 5. Therefore, the adaptive tracking control problem of the resulting closed-loop system (33) - (39) is solvable. The simulation results well illustrate the effective-ness of the proposed approach.

V. CONCLUSION

This paper has studied adaptive exponential quasipassification and adaptive tracking control for uncertain switched nonlinear systems in lower-triangular form. A more general switching law and adaptive controller are designed constructively. There are relevant problems that need to be investigated. One of such problems is how to solve finite time tracking problems using finite-time quasi-passivity for switched nonlinear systems.

REFERENCES

- J. C. Willems, "Dissipative dynamical systems part I: General theory," Arch. Rational Mech. Anal., vol. 45, no. 5, pp. 321–351, Jan. 1972.
- [2] J.-H. Ryu, C. Preusche, B. Hannaford, and G. Hirzinger, "Time domain passivity control with reference energy following," *IEEE Trans. Control Syst. Technol.*, vol. 13, no. 5, pp. 737–742, Sep. 2005.
- [3] H. Wang, C. C. Cheah, W. Ren, and Y. Xie, "Passive separation approach to adaptive visual tracking for robotic systems," *IEEE Trans. Control Syst. Technol.*, vol. 26, no. 6, pp. 2232–2241, Nov. 2018.
- [4] C. I. Byrnes, A. Isidori, and J. C. Willems, "Passivity, feedback equivalence, and the global stabilization of minimum phase nonlinear systems," *IEEE Trans. Autom. Control*, vol. 36, no. 11, pp. 1228–1240, Dec. 1991.
- [5] W. Lin and T. Shen, "Robust passivity and feedback design for minimumphase nonlinear systems with structureal uncertainty," *Automatica*, vol. 35, no. 1, pp. 35–47, Jan. 1999.

[6] M. M. Seron, D. J. Hill, and A. L. Fradkov, "Nonlinear adaptive control of feedback passive systems," *Automatica*, vol. 31, no. 7, pp. 1053–1060, Jul. 1995.

IEEEAccess

- [7] W. Yu, "Passivity analysis for dynamic multilayer neuro identifier," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 50, no. 1, pp. 173–178, Jan. 2003.
- [8] W. Yu, "Neural feedback passivity of unknown nonlinear systems via sliding mode technique," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 26, no. 7, pp. 1560–1566, Jul. 2015.
- [9] R. Sepulchre, M. Jankovic, and P. Kokotovic, *Constructive Nonlinear Control*. London, U.K.: Springer, 2017, doi: 10.1007/978-1-4471-0967-9.
- [10] A. K. Kostarigka and G. A. Rovithakis, "Approximate adaptive output feedback stabilization via passivation of MIMO uncertain systems using neural networks," *IEEE Trans. Syst. Man, Cybern. B, Cybern.*, vol. 39, no. 5, pp. 1180–1191, Oct. 2009.
- [11] H. E. Psillakis and A. T. Alexandridis, "A simple nonlinear adaptive fuzzy passivity-based control of power systems," *Nonlinear Dyn. Syst. Theory*, vol. 7, no. 1, pp. 51–67, Mar. 2007.
- [12] I. G. Polushin and J. H. Marquez, "Boundedness properties of nonlinear quasi-dissipative systems," *IEEE Trans. Autom. Control*, vol. 49, no. 12, pp. 2257–2261, Jan. 2004.
- [13] A. Pogromsky and H. Nijmeijer, "Cooperative oscillatory behavior of mutually coupled dynamical systems," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 48, no. 2, pp. 152–162, Mar. 2001.
- [14] E. Steur, I. Tyukin, and H. Nijmeijer, "Semi-passivity and synchronization of diffusively coupled neuronal oscillators," *Phys. D, Nonlinear Phenomena*, vol. 238, no. 21, pp. 2119–2128, Jun. 2009.
- [15] A. Molavi, A. Jalali, and M. Ghasemi Naraghi, "Adaptive fuzzy control of a class of nonaffine nonlinear system with input saturation based on passivity theorem," *ISA Trans.*, vol. 69, pp. 202–213, Jul. 2017.
- [16] R. Ma, Y. Liu, S. Zhao, M. Wang, and G. Zong, "Nonlinear adaptive control for power integrator triangular systems by switching linear controllers," *Int. J. Robust Nonlinear Control*, vol. 25, no. 14, pp. 2443–2460, Sep. 2015.
- [17] J. Fu, R. Ma, and T. Chai, "Adaptive finite-time stabilization of a class of uncertain nonlinear systems via logic-based switchings," *IEEE Trans. Autom. Control*, vol. 62, no. 11, pp. 5998–6003, Nov. 2017.
- [18] D. Yang and J. Zhao, "H_∞ output tracking control for a class of switched LPV systems and its application to an aero-engine model," *Int. J. Robust Nonlinear Control*, vol. 27, no. 12, pp. 2102–2120, Aug. 2017.
- [19] J. Zhao and D. J. Hill, "On stability, L2-gain and control for switched systems," *Automatica*, vol. 44, no. 5, pp. 1220–1232, Jan. 2009.
- [20] L. Liu, Y.-J. Liu, and S. Tong, "Neural networks-based adaptive finitetime fault-tolerant control for a class of strict-feedback switched nonlinear systems," *IEEE Trans. Cybern.*, vol. 49, no. 7, pp. 2536–2545, Jul. 2019.
- [21] B. Niu and L. Li, "Adaptive backstepping-based neural tracking control for MIMO nonlinear switched systems subject to input delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 6, pp. 2638–2644, Jun. 2018.
- [22] Y. Li, S. Tong, L. Liu, and G. Feng, "Adaptive output-feedback control design with prescribed performance for switched nonlinear systems," *Automatica*, vol. 80, pp. 225–231, Jun. 2017.
- [23] L. Long and J. Zhao, "Adaptive fuzzy tracking control of switched uncertain nonlinear systems with unstable subsystems," *Fuzzy Sets Syst.*, vol. 273, pp. 49–67, Aug. 2015.
- [24] W. Chen and M. Saif, "Passivity and passivity based controller design of a class of switched control systems," in *Proc. 19th IFAC World Congr.*, Prague, CZ, USA, 2005, pp. 143–147.
- [25] J. Zhao and D. J. Hill, "Dissipativity theory for switched systems," *IEEE Trans. Autom. Control*, vol. 53, no. 4, pp. 941–953, May 2008.
- [26] H. Wang and J. Zhao, "Finite-time passivity of switched non-linear systems," *IET Control Theory Appl.*, vol. 12, no. 3, pp. 338–345, Feb. 2018.
- [27] H. Pang and J. Zhao, "Robust passivity, feedback passification and global robust stabilisation," *IET Control Theory Appl.*, vol. 9, no. 11, pp. 1723–1739, Jul. 2015.
- [28] C. Li and J. Zhao, "Robust passivity-based H_∞ control for uncertain switched nonlinear systems," *Int. J. Robust Nonlinear Control*, vol. 26, no. 14, pp. 3186–3206, Sep. 2016.
- [29] H. Pang and J. Zhao, "Adaptive passification and stabilization for switched nonlinearly parameterized systems," *Int. J. Robust Nonlinear Control*, vol. 27, no. 7, pp. 1147–1170, May 2017.
- [30] Y. Sun and J. Zhao, "Passivity-based adaptive output tracking control for switched nonlinear systems with uncertain parameters," *Int. J. Adapt. Control Signal Process.*, vol. 32, no. 1, pp. 170–184, Jan. 2018.

- [31] H. Sang and J. Zhao, "Passivity and passification for switched T–S fuzzy systems with sampled-data implementation," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 7, pp. 1219–1229, Jul. 2020.
- [32] S. Liu and H. Pang, "Exponential quasi (Q,S,R) dissipativity and practical stability for switched nonlinear systems," *Asian J. Control*, vol. 22, no. 5, pp. 2029–2040, Sep. 2020.
- [33] H. Pang and J. Zhao, "Quasi-passivity-based adaptive stabilization for switched nonlinearly parameterized systems," *Int. J. Adapt. Control Signal Process.*, vol. 31, no. 8, pp. 1111–1125, Aug. 2017.
- [34] S. Liu and H. Pang, "Robust semipassivity and practical stability for uncertain switched nonlinear systems," *IEEE Access*, vol. 8, pp. 12096–12104, 2020.
- [35] H. K. Khalil, Nonlinear Systems, 3rd ed. Uppen Saddle River, NJ, USA: Prentice-Hall, Jan. 2002.
- [36] W. Lin and C. Qian, "Adaptive control of nonlinearly parameterized systems: The smooth feedback case," *IEEE Trans. Autom. Control*, vol. 47, no. 8, pp. 1249–1266, Aug. 2002.
- [37] S. S. Ge, C. C. Hang, T. H. Lee, and T. Zhang, *Stable Adaptive Neural Network Control*. Boston, MA, USA: Kluwer, 2001.
- [38] R. M. Sanner and J.-J.-E. Slotine, "Gaussian networks for direct adaptive control," *IEEE Trans. Neural Netw.*, vol. 3, no. 6, pp. 837–863, Nov. 1992.



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