A Degenerate Spatial ARMA Process of External Sources Impinging Upon an Azimuth-Only ULA and the Estimation of Doas and Noise Variances

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ABSTRACT Array processing is an interdisciplinary topic of both physics and signal processing. Physical basis of array processing is the orderly-setup of sensors in space that induces regular change of the phase of external sources incident upon sensor array. In the current paper the direction-of-arrival (DoA) estimation of an azimuth-only uniform linear array (ULA) is investigated in theory and method from the fused perspective of signal and physical properties of the incident sources. By organically fusing the stationary assumption on source signals and noises with difference operation on the phases, a distinctive system of linear equations satisfied by the incident sources is theoretically derived for the azimuth-only ULA with the Hankel-block-matrix of signal correlations as coefficient matrix and the elementary power-sum symmetric functions of the propagators of incident sources as the unknowns. Based on the derived system of linear equations, signal model of the incident sources is first proved as a degenerate spatial *ARMA* process subject to the identical autoregressive and moving average parameters and simultaneously obeying the dimensional homogeneity principle (DHP) in physics. The explicit root-finding polynomial is proposed with the unknowns of the system of linear equations as polynomial coefficients and the propagators as the roots. No extraneous roots and conjugate symmetry constraint on polynomial coefficients are involved. The DoAs and noise variances can be separately estimated under the backgrounds of spatially white and colored noises, which are numerically analyzed with the different coherent lengths of noises. A simple sound experiment is designed and performed to verify the proposed DoA estimation method. It is promising to investigate the DoA estimation of the ULA model, particularly of the ULAs of the multi-dimensional array from the fused perspective of physical and signal properties of the incident sources.

INDEX TERMS Array signal processing, direction-of-arrival estimation, phase-difference, the azimuth-only ULA, the degenerate spatial ARMA process, the modified Yule-Walker (MYW) system of linear equations, the root-finding polynomial.

I. INTRODUCTION

Array processing has been extensively applied to the diverse engineering applications including radar, sonar, wireless communication and seismic prospecting and so forth. The target of array processing is to extract the desired parameters or properties of the incident sources from sensor data. The DoA (Direction-of-arrival) estimation has been a representative and central problem in array processing.

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Array processing is physically model-based. The sensors are artificially and orderly setup in space and comprise various array geometries such as the one-dimensional (1-D) ULA (Uniform linear array), 2-D rectangular and 3-D cuboid arrays and so on. The orderly setup of sensors has induced regular change of the phase and phase-difference of the external sources incident upon the array, which is a common physical phenomenon. More specifically, for the extensively-studied azimuth-only ULA, under the narrow-band far-field condition, phase-difference of the incident source is the constant and simplified as an explicit function of azimuth angle, sensor interspacing, and carrier frequency after sensor

calibration. The phase is proportional to the corresponding phase-difference. In contrast, for the multi-dimensional (M-D) array or near-field source, function form of the phase becomes highly complicated due to the introduction of azimuth, elevation angles and propagation distance.

Meanwhile, array processing is signal model-based, which is reflected by the statistical property of source signals and sensor noises. In practice, the well-known concepts such as non-stationary/stationary, deterministic/stachastic, independent/correlated/coherent, narrow-band and wide-band and so forth have been frequently introduced to reflect signal properties of source and noise and impacted on array processing including the DoA estimation in theory and method.

Physical and signal properties have formed a pair to fully describe the incident sources. In this sense, array processing can be taken as a problem of both physics and signal processing. It is beneficial and meaningful to study array processing from the fused perspective of physical and signal properties of the incident sources.

In past, various popular DoA estimation methods had been proposed from signal perspective of the incident sources and noises and mainly classified into beamforming technique [1]–[3], linear prediction (LP) [4]–[6], the subspace-based methods such as the conventional MUSIC (Multiple signal classification) and ESPRIT (Estimation of signal parameters via rotational invariance techniques) [7]–[12], Maximum likelihood (ML) [13] and their variants and combinations. The reviews could be found in [14]–[16].

It was noted that a well-known concept was the so-called subspace in the DoA estimation, in particularly the signal subspace that stemmed from the partition of data covariance matrix. Signal subspace was proved to be spanned both by the implicit steering vectors of the ULA representation and by signal eigenvectors after eigenvalue decomposition (EVD) on data covariance matrix, which showed signal subspace in effect had played a role of bridge connecting steering vector and the ULA data. After considering the one-to-one map from steering vector to azimuth angle, azimuth angles could be estimated from the ULA data via signal subspace in theory. The DoA estimation of the azimuth-only ULA had been elegantly accomplished by the MUSIC and ESPRIT methods, which also resulted in the popularity of the subspace-based methods in array processing.

In the recent years, the sparsity of the incident source signals in the spatial domain has drawn growing attention, spare signal processing techniques based on CS (Compressed sensing) theory [17] have been extensively introduced and exploited for the DoA estimation of array processing. Under the CS frame, the DoA estimation had been formulated as a sparse signal representation problem after discretizing angular area and formulating an overcomplete dictionary and then performed by the convex optimization problems of the l_1-norm penalty [18] or l_p-norm penalty with $p \leq 1$ [19], [20] and the sparse Bayesian Learning (SBL) [21]–[23]. The CS-based DoA estimation methods

exhibited some advantages including the increased resolution, improved robustness to noise and limitations in data quantity, as well as not requiring an accurate initialization. On the other hand, the accuracy and performance of the above methods relied on the exact-alignment requirement of the true DoAs of the incident sources with the discretized grids and the tuning of regularization parameter balancing the sparsity of the spectrum and the residual norm, which have been the focuses in study and also resulted in the promising off-grid models based on the Taylor series expansion method and the linear interpolation method.

Physical properties of the incident sources induced by the orderly-setup of sensors has been more-or-less exploited in some estimation methods. The noted one was the rotational-invariance structure of array manifold developed in the ESPRIT. The propagators of all the sources were wholly extracted out from array manifolds of the displaced or partitioned subarrays and explicitly formed a diagonal unitary matrix. Owing to avoiding array calibration and reducing computational complexity, the ESPRIT had been the other representative subspace method, the successive methods such as MPM (Matrix pencil method) [24], [25], PM (Propagator method) [26]–[28], and SUMWE (The subspace-based method without eigen-decomposition) method [29] as well as CODE (The cross-correlation based 2-D DOA estimation) method [30] had borrowed ESPRIT idea. The rotational transformation was a whole operation acting on the displaced or partitioned subarrays. Analogous to the MUSIC, the estimation of propagators still required to make the connection with the ULA data, which was completed by exploiting the rotational invariance of the underlying signal subspace.

Array processing is an interdisciplinary problem of both physics and signal processing. As a physical parameter containing the unknown DoAs, phase-difference can be treated as a result of difference operation on phases of the same source impinging upon the adjacent sensors. Once such a connection is made, except for the rotational-invariance structure of array manifold, by making use of difference operation on phases, phase-difference also can be individually extracted out from the ULA representation and explicitly expressed for the subsequent estimation. By fusing difference operation on phases with the appropriate signal assumption on source and noise, here we take the azimuth-only ULA as an object and probe the DoA estimation in theory and method.

Indeed, the more deep and significant insight into the DoA estimation benefits from the fused perspective. It is from the fused perspective that the MYW system of linear equations and the root-finding polynomial are first derived for the azimuth-only ULA. Signal model of the incident sources is revealed as a degenerate spatial *ARMA* process (Autoregressive moving-average process), neither *AR* (Autoregressive) nor *MA* (Moving-average) models. Furthermore, the *ARMA* process is not arbitrary, but explicit in function forms of model coefficients. The degenerate *ARMA* process constrains the DoA estimation to possess its own distinctive method.

By contrast, in past signal model had been uncertain in principle, which led various AR processes or LP equations to be introduced into the DoA estimation [4]–[6], [8].

The paper is structured as follows: The extraction manners of phase-difference are described in Section II. The systems of linear equations and the root-finding polynomial are derived for the azimuth-only ULA, respectively, with the forward and backward difference operations (FDO and BDO) in Section III. A degenerate spatial *ARMA* process of the incident sources is developed and further analyzed from the standpoints of physics and signal processing in Section IV. The properties of the MYW system of linear equations are discussed in Section V. The implementation of the DoA estimation is described in Section VI. Numerical analysis and experimental verification are performed in Section VII and VIII.

II. THE EXTRACTION OF PHASE-DIFFERENCE UNDER THE AZIMUTH-ONLY ULA

A. THE AZIMUTH-ONLY ULA MODEL

Consider a typical one-dimensional azimuth-only ULA comprising the M identical and omnidirectional sensors, the K far-field narrow-band sources $\{s_k(t)\}_{k=1}^K$ with azimuth angles $\{\theta_k\}_{k=1}^K$ are impinging upon the array. The azimuth angle θ_k is defined as the one between the incident direction of the k-th source and the normal of the ULA. The representation of the azimuth-only ULA is expressed in matrix notation as [14]

$$X(t) = A(\theta)S(t) + W(t), \quad (1)$$

where the column vectors of sensor data, source signals and sensor noises are $X(t) = [x_1(t), x_2(t), \cdots, x_M(t)]^T$, $S(t) = [s_1(t), s_2(t), \cdots, s_K(t)]^T$ and $W(t) = [w_1(t), w_2(t), \cdots, w_M(t)]^T$ with t as time variable, $(\cdot)^T$ denotes transpose operator. The statistical property of source signal is reflected via $s_k(t)$. $A(\theta)$ stands for array manifold or array steering matrix with $\theta = [\theta_1, \theta_2, \cdots, \theta_K]^T$.

Array manifold $A(\theta)$ of the azimuth-only ULA is a Vandermonde matrix given by

$$A(\theta) = [a(\theta_1), \cdots, a(\theta_k), \cdots, a(\theta_K)], \quad (2)$$

where steering vector $a(\theta_k) = [e^{j\varphi_1(\theta_k)}, \ldots, e^{j\varphi_m(\theta_k)}, \cdots, e^{j\varphi_M(\theta_k)}]^T$ with j as imaginary unit.

Steering vector $a(\theta_k)$ of array manifold $A(\theta)$ in (2) contains the two physical parameters, one is the phase $\varphi(\theta)$, the other is phase-difference $\Delta\varphi(\theta)$. Suppose $\varphi_m(\theta_k)$ in $a(\theta_k)$ is the phase of the k-th source impinging upon the m-th sensor after the first sensor is calibrated as the zero-phase sensor, then

$$\varphi_m(\theta_k) = (m-1)\Delta\varphi(\theta_k), \quad m = 0, \cdots, M-1, \quad (3)$$

where $\Delta\varphi(\theta_k)$ is the corresponding k-th phase-difference.

For the far-field narrow-band source, the k-th phase-difference $\Delta\varphi(\theta_k)$ is simplified as

$$\Delta\varphi(\theta_k) = 2\pi f d \sin\theta_k/c, \quad (4)$$

In (4) the k-th phase-difference $\Delta\varphi(\theta_k)$ is a function of azimuth angle θ_k, sensor interspacing d, carrier frequency f

and propagation velocity c. The corresponding propagator v_k is defined as

$$v_k = e^{j\Delta\varphi(\theta_k)}, \quad (5)$$

It is shown in (3) that the phase $\varphi_m(\theta)$ is the product of phase-difference $\Delta\varphi(\theta)$ and the index $(m-1)$ after sensor calibration (To simplify the notation, here we omit the index k of $\varphi_m(\theta_k)$ and $\Delta\varphi(\theta_k)$; we will reinstate the index k later on, when needed.). For the azimuth-only ULA, only one phase-difference $\Delta\varphi(\theta)$ appearing in the phase $\varphi_m(\theta)$ guarantees the one-to-one map from the propagator v to azimuth angle θ, while the index $(m-1)$ in (3) describes sensor position and is linearly-varied. The function form of the phase $\varphi_m(\theta)$ is induced by the artificial and orderly setup of sensors.

In contrast to the phase $\varphi_m(\theta)$, owing to independent of position index of sensor and also containing the DoAs, phase-difference $\Delta\varphi(\theta)$ is the more essential parameter in physics. The DoA estimation of the azimuth-only ULA in effect has been viewed as the estimation of propagator or phase-difference from the noise-contaminated ULA data.

B. THE EXTRACTION OF PHASE-DIFFERENCE WITH DIFFERENCE OPERATION

Except for the rotational-invariance structure of array manifold proposed by the ESPRIT which was used to wholly extract the propagators of the incident sources, in accordance with the above definition, here we provide the other manner of explicit extraction of phase-difference.

The proportional relation between the phase and phase-difference in (3) yields

$$\Delta\varphi(\theta) = \varphi_m(\theta) - \varphi_{m-1}(\theta), \quad (6)$$

and also

$$\Delta\varphi(\theta) = [-\varphi_{m-1}(\theta)] - [-\varphi_m(\theta)], \quad (7)$$

For comparison, with positive phase $\varphi_m(\theta)$, phase-difference $\Delta\varphi(\theta)$ is extracted by using the forward difference operation (FDO) in (6), while with negative phase $-\varphi_m(\theta)$, $\Delta\varphi(\theta)$ obtained by using the backward difference operation (BDO) in (7). The BDO can be implemented after conjugate preprocessing of the ULA data, which will be discussed next.

Here we probe the DoA estimation of the azimuth-only ULA in theory and method via implementing difference operation on phases and explicitly extracting phase-difference. Apparently the implementation of difference operation requires the appropriate condition on source signals and noises since the phases $\{\varphi_m(\theta)\}_{m=1}^M$ and the signals $\{s_k(t)\}_{k=1}^K$ and the noises $\{w_m(t)\}_{m=1}^M$ coexist into the representation of the azimuth-only ULA in (1).

III. THE SYSTEMS OF LINEAR EQUATIONS AND THE ROOT-FINDING POLYNOMIAL WITH THE FDO AND BDO

A. THE CONSTRUCTION OF CORRELATION SEQUENCE OF THE ULA DATA

In order to study the DoA estimation via performing the FDO, a raw sequence S_0^f composed of correlation functions

of sensor data $\{x_m(t)\}_{m=1}^M$ is first constructed and given by

$$S_0^f = (R_{x_1 x_n}, R_{x_2 x_n}, \cdots, R_{x_m x_n}, \cdots, R_{x_M x_n}), \quad (8)$$

where the m-th element $R_{x_m x_n} = E\{x_m(t) x_n^H(t)\}$. $E\{\cdot\}$ and $(\cdot)^H$ denote the expectation operation and conjugate transpose, respectively. For the sake of convenience, the data $x_n(t)$ used in $R_{x_m x_n}$ is called reference signal and can be chosen from sensor data as needed.

After the first sensor is calibrated as the zero-phase sensor, the m-th element $R_{x_m x_n}$ is expanded as

$$R_{x_m x_n} = v_1^{m-1} R_{s_1 x_n} + \cdots + v_K^{m-1} R_{s_K x_n} + R_{w_m x_n}, \quad (9)$$

where $R_{s_k x_n} = E\{s_k(t) x_n^H(t)\}$ denotes the correlation function between the k-th source signal $s_k(t)$ and $x_n(t)$, $R_{w_m x_n} = E\{w_m(t) x_n^H(t)\}$ is the one between the m-th noise $w_m(t)$ and $x_n(t)$.

The property of $R_{s_k x_n}$ and $R_{w_m x_n}$ in $R_{x_m x_n}$ are usually unknown. However, under the stationary assumption on source signals and noises, $R_{s_k x_n}$ and $R_{w_m x_n}$ become the time-independent constants.

B. THE SYSTEM OF LINEAR EQUATIONS WITH THE FDO

For the first element $R_{x_1 x_n}$ in S_0^f, we have

$$R_{s_1 x_n} = R_{x_1 x_n} - R_{s_2 x_n} - \cdots - R_{s_K x_n} - R_{w_1 x_n}, \quad (10)$$

Similarly, for the second element $R_{x_2 x_n}$,

$$v_1 R_{s_1 x_n} = R_{x_2 x_n} - v_2 R_{s_2 x_n} - \cdots - v_K R_{s_K x_n} - R_{w_2 x_n}, \quad (11)$$

Dividing the right-side and left-side hands of (11) by the corresponding sides of (10), we preform the FDO on the phases of $s_1(t)$ incident on the first and second sensors. To guarantee the implementation of the FDO, the involved correlation function $R_{s_1 x_n}$ is required to be a time-independent constant, which implies that source signals and noises are stationary. After the FDO, we get

$$R_{x_1 x_n}^1 = R_{s_2 x_n}^1 + \cdots + R_{s_k x_n}^1 + \cdots + R_{s_K x_n}^1 + R_{w_1 x_n}^1, \quad (12)$$

where $R_{x_1 x_n}^1 = R_{x_2 x_n} - v_1 R_{x_1 x_n}$, $R_{w_1 x_n}^1 = R_{w_2 x_n} - v_1 R_{w_1 x_n}$. $R_{s_k x_n}^1 = (v_k - v_1) R_{s_k x_n}$ $(k \geq 2)$.

It is shown in (12) that the unknown correlation function $R_{s_1 x_n}$ appearing in (10) and (11) is eliminated by the FDO under the stationary assumption on source signals and noises.

For the m-th and $(m+1)$-th sensors, it follows (12) that

$$R_{x_m x_n}^1 = v_2^{m-1} R_{s_2 x_n}^1 + \cdots + v_k^{m-1} R_{s_k x_n}^1$$
$$+ \cdots + v_K^{m-1} R_{s_K x_n}^1 + R_{w_m x_n}^1, \quad (13)$$

where $R_{x_m x_n}^1 = R_{x_{m+1} x_n} - v_1 R_{x_m x_n}$, $R_{w_m x_n}^1 = R_{w_{m+1} x_n} - v_1 R_{w_m x_n}$.

For the convenience next, $R_{x_m x_n}^1$ $(m = 1, \cdots, M-1)$ in (13) is named as the compound correlation function in respect to the index m after the first FDO.

After the first FDO, it is noted that the original sequence S_0^f in (8) changes as $S_1^f = (R_{x_1 x_n}^1, R_{x_2 x_n}^1, \cdots, R_{x_{M-1} x_n}^1)$.

The compound correlation function $R_{x_m x_n}^1$ in (13) remains the similar function structure as $R_{x_m x_n}$ in (9), which shows that

the FDO can be performed on the phases of $R_{s_2 x_n}^1$ in $R_{x_{m-1} x_n}^1$ and $R_{x_m x_n}^1$.

After the k-th FDO, by derivation, the correlation functions $R_{s_1 x_n}, R_{s_2 x_n}, \cdots, R_{s_k x_n}$ are shown to be orderly eliminated under the stationary assumption on source signals and sensor noises, the compound correlation function $R_{x_m x_n}^k$ is derived as

$$R_{x_m x_n}^k = v_{k+1}^{m-1} R_{s_{k+1} x_n}^k + \cdots + v_K^{m-1} R_{s_K x_n}^k + R_{w_m x_n}^k, \quad (14)$$

where the involved recurrence formulas satisfy

$$R_{x_i x_n}^k = R_{x_{i+1} x_n}^{k-1} - v_k R_{x_i x_n}^{k-1}, \quad (15)$$
$$R_{s_{k'} x_n}^k = (v_{k'} - v_k) R_{s_{k'} x_n}^{k-1}, \ (k' \geq k+1), \quad (16)$$
$$R_{w_i x_n}^k = R_{w_{i+1} x_n}^{k-1} - v_k R_{w_i x_n}^{k-1}, \quad (17)$$

The sequence after the k-th FDO is given by $S_k^f = (R_{x_1 x_n}^k, R_{x_2 x_n}^k, \cdots, R_{x_{M-k} x_n}^k)$, which similarly shows that the FDO can continue to be performed for the elements in S_k^f.

After the K-th FDO, all the K correlation functions $\{R_{s_k x_n}\}_{k=1}^K$ are eliminated in derivation, the compound correlation function $R_{x_m x_n}^K$ in respect to the index m contains no signal correlations $\{R_{s_k x_n}\}_{k=1}^K$ and satisfies

$$R_{x_m x_n}^K = R_{w_m x_n}^K, \quad m = 1, \cdots, M-K, \quad (18)$$

The sequence after the K-th FDO is $S_K^f = (R_{x_1 x_n}^K, R_{x_2 x_n}^K, \cdots, R_{x_{M-K} x_n}^K)$.

Employing recurrence formulas in (15), (16), and (17), Equation (18) in respect to the index m is expanded as

$$-(R_{x_{m+K} x_n} - R_{w_{m+K} x_n}) = b_1(R_{x_{m+K-1} x_n} - R_{w_{m+K-1} x_n})$$
$$+ b_2(R_{x_{m+K-2} x_n} - R_{w_{m+K-2} x_n})$$
$$+ \cdots$$
$$+ b_K(R_{x_m x_n} - R_{w_m x_n}), \quad (19)$$

where the unknown coefficients $\{b_k\}_{k=1}^K$ denote the elementary power-sum symmetric functions of the propagators and have the following distinctive forms

$$\begin{cases} b_1 = (-1)^1 (v_1 + v_2 + \cdots + v_K), \\ b_2 = (-1)^2 (v_1 v_2 + \cdots + v_1 v_K + v_2 v_3 + \cdots + v_i v_j \\ \quad + \cdots + v_{K-1} v_K), \ (i \neq j), \\ \vdots \\ b_K = (-1)^K v_1 v_2 \cdots v_K, \end{cases} \quad (20)$$

Equation (19) is a difference equation satisfied by the propagators of the K sources impinging upon the azimuth-only ULA under the stationary assumption on source signals and noises.

Writing (19) for the indices $m = 1, \cdots, M-K$ gives the following system of linear equations in matrix notation

$$H_f b = r_f, \quad (21)$$

where $\boldsymbol{H}_f$, $\boldsymbol{b}$ and $\boldsymbol{r}_f$ denote coefficient matrix, the vectors of the unknowns and constant terms and are given by

$$\begin{cases} \boldsymbol{H}_f = [\boldsymbol{h}_{f1}, \ \boldsymbol{h}_{f2}, \ \cdots, \ \boldsymbol{h}_{fm}, \ \cdots, \ \boldsymbol{h}_{f(M-K)}]^T, \\ \boldsymbol{b} = [b_K, \ b_{K-1}, \ \cdots, \ b_1]^T, \\ \boldsymbol{r}_f = -[\boldsymbol{h}_{f(K+1)}, \ \boldsymbol{h}_{f(K+2)}, \ \cdots, \ \boldsymbol{h}_{fM}]^T, \end{cases} \quad (22)$$

where the $1 \times K$ vector $\boldsymbol{h}_{fm} = [h_{fm}, \ \cdots, \ h_{f(m+K-1)}]$. The element h_{fl} ($l = 1, \cdots, M$) in $\boldsymbol{H}_f$ and $\boldsymbol{r}_f$ denotes $h_{fl} = R_{x_l x_n} - R_{w_l x_n}$. It is shown that coefficient matrix $\boldsymbol{H}_f$ is a standard block-Hankel matrix.

In order to solve the K unknowns $\{b_k\}_{k=1}^K$, in accordance with the concrete form of (22), the relation between M and K is required to satisfy $K \le \frac{M}{2}$. As $K = \frac{M}{2}$, the system of the K equations in K unknowns is obtained and all the sensors of the ULA are employed for estimation.

Although the azimuth-only ULA has the highly complicated representation and array manifold $\boldsymbol{A}(\boldsymbol{\theta})$ contains no explicit propagator and phase-difference, by fusing the FDO with the stationary assumption on source signals and noises, it is proven by theoretical derivation that the azimuth-only ULA model has a system of linear equations with the block-Hankel matrix as coefficient matrix and the elementary power-sum symmetric functions of the propagators as the unknowns.

Matrix equation (21) and the corresponding derivation manner are based on the equal-interspacing azimuth-only ULA model. Since any ULA is always one-dimensional, the linearly-combined relation between the phase and its component phase-difference still holds, no matter how complicated function forms of the phase and phase-difference are, therefore the derivation manner for the azimuth-only ULA including difference operation can be extended to any ULA of the M-D array and further used to investigate the complicated 2-D DoA estimation problem.

If $\{h_{fm}\}_{m=1}^M$ are known, the unknowns $\{b_k\}_{k=1}^K$ can be estimated and explicitly visualized by solving the system of linear equations in (21).

C. THE SYSTEM OF LINEAR EQUATIONS WITH THE BDO
As shown in (7), phase-differences $\{\Delta\varphi(\theta_k)\}_{k=1}^K$ also can be explicitly extracted out by making using of the BDO on the negative phase. To generate the negative phase, here correlation function $R_{x_n x_m}$ between reference signal $x_n(t)$ and $x_m(t)$ ($m = 1, \cdots, M$) are proposed to constitute the other sequence S_0^b.

$$S_0^b = (R_{x_n x_1}, R_{x_n x_2}, \cdots, R_{x_n x_m}, \cdots, R_{x_n x_M}), \quad (23)$$

where the m-th correlation function $R_{x_n x_m} = E\{x_n(t)x_m^H(t)\}$.

The correlation function $R_{x_n x_m}$ is expanded as

$$R_{x_n x_m} = v_1^{-(m-1)} R_{x_n s_1} + \cdots + v_K^{-(m-1)} R_{x_n s_K} + R_{x_n w_m}, \quad (24)$$

where $R_{x_n s_k} = E\{x_n(t)s_k^H(t)\}$ ($k = 1, \cdots, K$) and $R_{x_n w_m} = E\{x_n(t)w_m^H(t)\}$.

In (24) the phase $\varphi_m(\theta_k)$ of the k-th signal $s_k(t)$ incident on the m-th sensor is negative and equal to $-(m-1)\Delta\varphi(\theta_k)$.

Owing to the introduction of negative phase, the BDO can be performed on the elements of the sequence S_0^b. Under the stationary assumption on source signals and noises, by making use of the similar derivation manner as used for the system of linear equations in (21), the difference equation in respect to m ($m = M, \cdots, K+1$) is given by

$$\begin{aligned} -(R_{x_n x_{m-K}} - R_{x_n w_{m-K}}) &= b_1(R_{x_n x_{m-K+1}} - R_{x_n w_{m-K+1}}) \\ &+ b_2(R_{x_n x_{m-K+2}} - R_{x_n w_{m-K+2}}) \\ &+ \cdots \\ &+ b_K(R_{x_n x_m} - R_{x_n w_m}), \quad (25) \end{aligned}$$

Equation (25) is the other difference equation satisfied by the propagator of the incident source with the BDO on negative phase. Similarly, writing (25) for the indices $m = M, \cdots, K+1$ gives the other system of linear equations

$$\boldsymbol{H}_b \boldsymbol{b} = \boldsymbol{r}_b, \quad (26)$$

$$\begin{cases} \boldsymbol{H}_b = [\boldsymbol{h}_{bM}, \ \boldsymbol{h}_{b(M-1)}, \ \cdots, \ \boldsymbol{h}_{bm}, \ \cdots, \ \boldsymbol{h}_{b(K+1)}]^T, \\ \boldsymbol{b} = [b_K, \ b_{K-1}, \ \cdots, \ b_1]^T, \\ \boldsymbol{r}_b = -[\boldsymbol{h}_{b(M-K)}, \ \boldsymbol{h}_{b(M-K+1)}, \ \cdots, \ \boldsymbol{h}_{b1}]^T, \end{cases} \quad (27)$$

where the vector $\boldsymbol{h}_{bm} = [h_{bm}, \ h_{b(m-1)}, \ \cdots, \ h_{b(m-K+1)}]$. The element h_{bl} ($l = 1, \cdots, M$) in $\boldsymbol{H}_b$ and $\boldsymbol{r}_b$ denotes $h_{bl} = R_{x_n x_l} - R_{x_n w_l}$.

D. THE ROOT-FINDING POLYNOMIAL OF THE AZIMUTH-ONLY ULA
After $\{b_k\}_{k=1}^K$ are estimated and known by solving (21) or (26), there still requires one step to obtain the propagators $\{v_k\}_{k=1}^K$ from the known $\{b_k\}_{k=1}^K$. In accordance with the distinctive function forms of $\{b_k\}_{k=1}^K$ in (20), we construct the following K-degree monic polynomial in propagator variable v as

$$f(v) = v^K + b_1 v^{K-1} + \cdots + b_k v^{K-k} + \cdots + b_K, \quad (28)$$

It is not difficult to show that $f(v)$ is factored as follows:

$$f(v) = (v - v_1)(v - v_2) \cdots (v - v_K), \quad (29)$$

Thus the propagators $\{v_k\}_{k=1}^K$ are proven as the roots of the polynomial $f(v)$. With the known propagators $\{v_k\}_{k=1}^K$, the individual azimuth angle θ_k is straightforward calculated via

$$\theta_k = \arcsin\left[\frac{c \ln v_k}{j 2\pi f d}\right], k = 1, \cdots, K \quad (30)$$

Hence the DoA estimation of the azimuth-only ULA is taken as a root-finding problem of the higher-order monic polynomial with $\{b_k\}_{k=1}^K$ as the explicit polynomial coefficients and the propagators $\{v_k\}_{k=1}^K$ as the roots.

As a simple example, let the number of sources $K = 2$, from (20), the coefficients b_1 and b_2 have the following simple and explicit forms as

$$b_1 = -(v_1 + v_2), \quad b_2 = v_1 v_2,$$

After b_1 and b_2 are estimated from the ULA data, the 2-degree monic polynomial $f(v)$ is constructed as

$$f(v) = v^2 + b_1 v + b_2,$$

The closed-form solutions of two propagators are

$$v_{1,2} = \frac{-b_1 \pm \sqrt{b_1^2 - 4b_2}}{2}.$$

IV. THE DEGENERATE SPATIAL ARMA PROCESS OF THE INCIDENT SOURCES

A. DIMENSIONAL ANALYSIS ON DIFFERENCE EQUATIONS (19) AND (25)

Difference equations (19) and (25) can be analyzed from the standpoint of physics.

An evident difference in function forms of (19) and (25) lies in the arrangements of $\{h_{fm}\}_{m=1}^M$ and $\{h_{bm}\}_{m=1}^M$. The elements $\{h_{fi}\}_{i=m}^{(m+K)}$ in (19) are paired and multiplied by the corresponding coefficients $b_1, b_2, \cdots, b_K$ in the descending order from the index $m+K-1$ to m. By contrast, $\{h_{bi}\}_{i=m-K}^m$ in (25) are arranged in the ascending order from $m-K+1$ to m. The reason for the above difference is simple to explain from the dimensional analysis on two difference equations.

In effect, after noise correlation $R_{w_m x_n}$ is subtracted from $R_{x_m x_n}$, the m-th element h_{fm} is given by

$$h_{fm} = R_{x_m x_n} - R_{w_m x_n} = v_1^{m-1} R_{s_1 x_n} + \cdots + v_K^{m-1} R_{s_K x_n}, \quad (31)$$

Then h_{fm} is contributed only by source correlations $\{R_{s_k x_n}\}_{k=1}^K$, similarly, h_{bm} contributed by $\{R_{x_n s_k}\}_{k=1}^K$.

Difference equations (19) and (25) should physically obey the dimensional homogeneity principle (DHP) in which the propagator in each term of the same equation remains identical in dimension.

The dimension of propagator is reflected by the corresponding power. For example, the dimension of b_k ($k = 1, \cdots, K$) is equal to k. Similarly, for h_{fm} in (31), without considering $x_n(t)$ appearing in all the K terms, the dimension of propagator in each term is equal to $(m-1)$, while for h_{bm}, is equal to $-(m-1)$.

Now we perform the dimensional analysis on difference equation (19). The dimension of propagator in the left-hand term $h_{f(m+K)}$ is equal to $m + K - 1$, each of the right-hand terms is the product of b_i and $h_{f(m+K-i)}$ ($i = 1, \cdots, K$), then the dimension of propagator in each of right-hand terms is the sum of i and $K + m - i - 1$ and equal to $m + K - 1$. Hence difference equation (19) physically obeys the DHP with the arrangement of $\{h_{fi}\}_{i=m}^{(m+K)}$ in the descending order. Meanwhile, difference equation (25) obeys the DHP with the dimensions of the K terms all equal to $-(m + K - 1)$.

The dimension of propagator reflects the equation-independent property. By the similar analysis, all the equations in (21) and (26) obey the DHP with the different dimensions of propagator and are the independent equations.

B. SIGNAL MODEL OF THE INCIDENT SOURCES OF THE AZIMUTH-ONLY ULA

Difference equations (19) and (25) also can be analyzed from the standpoint of signal processing.

Let $c_{m+K} = R_{x_{m+K} x_n}$ and $e_{m+K} = R_{w_{m+K} x_n}$, $c_{m+K-k} = R_{x_{m+K-k} x_n}$ and $e_{m+K-k} = R_{w_{m+K-k} x_n}$, then (19) is rewritten as

$$c_{m+K} + \sum_{k=1}^K b_k c_{m+K-k} = e_{m+K} + \sum_{k=1}^K b_k e_{m+K-k}$$
$$(m = 1, \cdots, M - K), \quad (32)$$

Similarly, difference equation (25) is rewritten as

$$c_{m-K}^* + \sum_{k=1}^K b_k c_{m-K+k}^* = e_m^* + \sum_{k=1}^K b_k e_{m-K+k}^*,$$
$$(m = M, \cdots, K + 1), \quad (33)$$

As shown in (32) and (33), under the stationary assumption, via the FDO or BDO, signal model of the external sources impinging upon the azimuth-only ULA is definitely revealed as the so-called degenerate spatial $ARMA(K, K)$ process subject to the identical autoregressive and moving average parameters. The definition of degenerate $ARMA(K, K)$ process can be found in [31].

As the stationary assumption is narrowed as the conventional one in which source signals and noises are uncorrelated and the noises are supposed as the GWN (Gaussian white noise) with zero mean and identical variance σ^2, then (32) is further simplified by reasonably choosing reference signal. For example, let reference index $n = m + K$, then only $e_{m+K} = \sigma^2$, other $e_{m+K-1}, \cdots, e_m$ are equal to zero, from (32) we get

$$c_{m+K} + \sum_{k=1}^K b_k c_{m+K-k} = e_{m+K}, \quad (34)$$

The degenerate $ARMA(K, K)$ process is shown as the $AR(K)$ process with the following power spectral density

$$P(\omega) = \frac{\sigma^2}{|v^K + b_1 v^{K-1} + \cdots + b_k v^{K-k} + \cdots + b_K|^2}, \quad (35)$$

In contrast to (35), indeed, the MUSIC was the spatial pseudo-spectral method.

In general, signal model of the incident sources is the degenerate spatial $ARMA$ process under stationary assumption on source signals and noises. Meanwhile, it should be stressed that such an $ARMA$ process is not a common one. Model coefficients $\{b_k\}_{k=1}^K$ are required to remain the explicit and distinctive function forms and satisfy the DHP.

For convenience next, Equations (21) is called the modified Yule-Walker (MYW) system of linear equations with the FDO.

C. THE DISCUSSION OF THE ROOT-FINDING POLYNOMIALS IN THE DoA ESTIMATION

The various root-finding polynomials for the DoA estimation had been developed in the Root-Music [11], Min-norm

method [8], IQML (Iterative quadratic maximum likelihood) algorithm [32], [33], Root-WSF (Weighted subspace fitting) and MODE (Method of direction estimation) [13], [34]. The Root-WSF algorithm had been recognized as 'a strong candidate for the best method for ULAs' [14].

It was an ordinary operation to prespecify the propagators as the roots of the polynomial. However, a key problem was how to construct the reasonable root-finding polynomial. There mainly existed two constructing manners.

In the Root-MUSIC and Min-norm method, the root-finding polynomials stemmed from noise eigenvectors, the propagators and noise zeroes became the roots of the polynomial in accordance with the orthogonality between steering vectors and noise eigenvectors. Therefore the polynomials in the Root-MUSIC and Min-norm method were not the pure one in propagator. In estimation, the roots distributed in and on the unit circle, it was required to separate the propagators from noise zeroes. Otherwise, when noise zeroes got closer to the unit circle, the spurious DoA estimates arose.

In the IQML, Root-WSF and MODE, the ML criterions in terms of angles were parameterised in terms of another parameter vector whose elements were set as polynomial coefficients, which was the other constructing manner. To guarantee the roots on the unit circle, polynomial coefficients were imposed to satisfy the conjugate symmetry constraint.

The diverse constructing manners, nonuniform polynomial forms and the attached constraints all showed that the root-finding polynomial still deserved further study, even for the simple azimuth-only ULA. In addition, from the standpoint of physics, the propagator is a physical parameter, one source corresponds to one propagator. Once the K propagators are prespecified as the roots of polynomial, the dimensions of propagator in all the terms of the root-finding polynomial are required to be equal in accordance with the DHP, which is a sound way of checking the validity of the root-finding polynomial.

Different from the root-finding polynomials mentioned above, the proposed polynomial shown in (23) is derived from the fused perspective of physics and signal processing. From the signal perspective, polynomial coefficients are explicit and distinctive in form. The propagators are clearly proved as the roots of the proposed polynomial. No other extraneous roots are involved and no conjugate symmetry constraint is imposed on polynomial coefficients. Meanwhile, the propagators are both the poles and zeroes of the so-called degenerate spatial $ARMA(K, K)$ process that are only distributed on the unit circle. From the standpoint of physics, it is shown that the proposed polynomial obeys the DHP in which the dimensions of all the terms of $f(v)$ are equal to the number K of sources.

In addition, for the simple azimuth-only ULA modle, the reason on the different function forms of the root-finding polynomials induced by difference operation and ML parameterization is an interesting problem.

V. THE PROPERTIES OF THE MYW SYSTEM OF LINEAR EQUATIONS

A. THE DoA ESTIMATION UNDER MULTIPATH SCENARIO
Multipath propagation due to various reflections was frequently encountered in the DoA estimation. Similar to the unknown DoAs, the uncorrelated/coherent properties of source signals reflecting multipath propagation are also unknown to the processor.

Rank deficiency of source covariance matrix induced by the presence of multipath propagation was main reason for performance degradation and the failure of the subspace-based methods. Considerable efforts had been spent to handle the DoA estimation of the highly correlated and coherent sources.

The spatial smoothing operation had been recognized as an effective preprocessing technique in which the total array was partitioned into the subarrays and generated the average of covariance matrices of subarray data, however, which still depended on the uncorrelated condition between source signals and noises.

By contrast, the proposed difference operation on phases can protect the applicability of the DoA estimation to multipath propagation under the more looser stationary condition on source signals and noises.

As an illustration, suppose the k-th source signal $s_k(t)$ is rewritten as $s_k(t) = \alpha_{ki}s_i(t)$, where the coefficient α_{ki} denotes the complex-valued attenuation of the k-th $s_k(t)$ with respect to the i-th signal $s_i(t), (i, k = 1, \cdots, K)$, then correlation function $R_{s_k x_n}$ changes as $R_{s_k x_n} = \alpha_{ki}E\{s_i(t)x_n^H(t)\}$, under the stationary assumption on source signals and noises, $R_{s_k x_n}$ containing α_{ki} is still a time-independent constant and can be eliminated by difference operation as we use above. Hence Multipath propagation do not pose a problem and is unnecessary to be considered in estimation.

B. THE MAXIMUM NUMBER OF DETECTABLE SOURCES WITH THE COMBINED SYSTEM OF LINEAR EQUATIONS
To solve the MYW system of linear equations in (21), coefficient matrix H_f and the vector r_f are required to be known. However, owing to the presence of the unknown noise correlation $R_{w_m x_n}$ $(m = 1, \cdots, M)$ in the corresponding element h_{fm}, H_f and r_f comprising $\{h_{fm}\}_{m=1}^{M}$ in effect are unknown, the MYW system of linear equations in effect is unsolvable.

The more concrete signal property of sources and noises is desirable or needed. When the stationary source signals and noises are further assumed to be uncorrelated and suppose the noises are the spatially GWNs with zero mean and unequal variances $\{\sigma_m^2\}_{m=1}^{M}$, the DoA estimation is readily implemented by constructing and solving the eligible MYW system of linear equations without the presence of noise variance (PNV).

1) THE MAXIMUM NUMBER OF DETECTABLE SOURCES WITH THE MYW SYSTEM OF LINEAR EQUATIONS
Under the above assumption, the construction of the eligible MYW system of linear equations depends on the chosen

reference signal. When the first sensor is chosen as reference one, namely $n = 1$, the unknown $R_{w_1 x_1}$ is equal to the nonzero σ_1^2. Except for the first element h_{f1} containing $R_{w_1 x_1}$, other $M - 1$ elements $\{h_{fm}\}_{m=2}^{M}$ without PNV can be employed to construct the eligible MYW system of linear equations. Similarly, when $n = M$, the elements without PNV are $\{h_{fm}\}_{m=1}^{M-1}$. For other sensors chosen as reference sensor, the number of the elements without PNV is less than $M - 1$.

By analysis, the number n_c of the elements without PNV, the number n_e of the equations without PNV and the number K satisfy

$$n_e = n_c - K \qquad (36)$$

As reference index $n = 1$ or $n = M$, $n_c = M - 1$.

With the single FDO, as $n = 1$, owing to noise variance σ_1^2 only appearing into h_{f1}, in the system of linear equations in (21), only the first equation containing h_{f1} is not eligible. However, the first equation can be replaced by the counterpart with the M-th sensor chosen as reference sensor. Thus there have a total of $n_e + 1$ equations without PNV. As $n_e + 1 = K$ and $n_c = M - 1$, the maximum number K_{max} of detectable sources is reached, then

$$(n_c - K) + 1 = K, \qquad (37)$$

After $n_c = M - 1$ is substituted into (37), the maximum number K_{max} of detectable sources is obtained and equal to $int(M)/2$. The same K_{max} can be reached with the BDO.

2) THE MAXIMUM NUMBER OF DETECTABLE SOURCES WITH THE COMBINED SYSTEM OF LINEAR EQUATIONS

Each equation of the systems of linear equations in (21) and (26) has the different dimension of propagator, then all the equations without PNV are independent and readily combined to increase the number of detectable sources. The eligible combined system of linear equations is given by

$$\boldsymbol{H}\boldsymbol{b} = \boldsymbol{r}, \qquad (38)$$

where $\boldsymbol{H}$ and $\boldsymbol{r}$ stand for the combined coefficient matrix and the vector of constant terms without PNV.

As an example, the first sensor is chosen as reference sensor to investigate the maximum number of detectable sources with the combined system of linear equations. The number n_c of the elements without PNV is equal to $2(M - 1)$. In accordance to the construction of (21) and (26), the $2(M-1)-2K$ equations without PNV are constructed by the $2(M-1)$ eligible elements, namely, $n_e = 2(M-1)-2K$. After including the two counterpart equations with $n = M$ to replace the first equations of (21) and (26) with $n = 1$, respectively, the number of the equations without PNV becomes $n_e + 2$. To estimate the K sources requires $n_e + 2 \geq K$, then

$$2(M - 1) - 2K + 2 \geq K, \qquad (39)$$

The maximum number K_{max} of detectable sources is given by

$$K_{max} = floor[\frac{2M}{3}], \qquad (40)$$

It is shown that the maximum number K_{max} is equal to that by making use of the subspace-based method based on the FSBB, however, is completed by the portion of the elements of data covariance matrix in the proposed method.

C. THE CALCULATION OF NOISE VARIANCES

Signal model of the external sources impinging upon the azimuth-only ULA is revealed as the degenerate spatial $ARMA(K, K)$ process. As source signals and noises are assumed to be uncorrelated and suppose the noises are the spatially GWNs with unequal variances $\{\sigma_m^2\}_{m=1}^{M}$, by reasonably choosing reference sensors, the DoA estimation and the calculation of noise variances are performed separately. Noise variances of all the sensors are calculated from the ULA data with the known $\{b_k\}_{k=1}^{K}$.

When the first sensor $x_1(t)$ is chosen as reference one, the unknown noise variance σ_1^2 only appears in the first equation of the MYW system of linear equations in (21) and given by

$$-R_{x_{K+1} x_1} = b_1 R_{x_K x_1} + b_2 R_{x_{K-1} x_1} + \cdots + b_K (R_{x_1 x_1} - \sigma_1^2), \qquad (41)$$

Once $\{b_k\}_{k=1}^{K}$ are determined, the variance σ_1^2 is calculated as

$$\sigma_1^2 = \frac{R_{x_{K+1} x_1} + b_1 R_{x_K x_1} + b_2 R_{x_{K-1} x_1} + \cdots + b_K R_{x_1 x_1}}{b_K}, \qquad (42)$$

Similarly, as the M-th sensor is chosen as reference one, the variance σ_M^2 can be extracted from the last equation and is given by

$$\sigma_M^2 = R_{x_M x_M} + b_1 R_{x_{M-1} x_M} + \cdots + b_K R_{x_{M-K} x_M}, \qquad (43)$$

With the similar manner, we choose other sensors as reference sensor, respectively, and calculate the corresponding noise variance from the corresponding equation. Each of noise variances has the different calculation formula.

It is a distinct advantage of the proposed method to extract more parameters including the DoAs and noise variances from the ULA data.

VI. THE IMPLEMENTATION OF THE DoA ESTIMATION OF THE AZIMUTH-ONLY ULA

A. THE TLS ESTIMATION FOR THE COMBINED SYSTEM OF LINEAR EQUATIONS

In practice, the true data correlations $\{R_{x_m x_n}\}_{m=1}^{M}$ are usually unavailable and replaced by the finite sampling ones $\{\hat{R}_{x_m x_n}\}_{m=1}^{M}$ given by

$$\hat{R}_{x_m x_n} = \frac{1}{P} \sum_{p=1}^{P} x_m(p) x_n^*(p), \qquad (44)$$

where P is the number of snapshots, $(\cdot)^*$ denotes conjugate operation. $x_m(p)$ and $x_n(p)$ are the sampling data with p as the discreted time index.

After noise correlation is removed from the corresponding data correlation, the combined system of linear equations

in (38) is still subject to various unknown 'residual' errors. Hence the determination of $\{b_k\}_{k=1}^K$ is a classical total least squares (TLS) problem. Central to the TLS is stated as perturbing both $\boldsymbol{H}$ and $\boldsymbol{r}$ in a minimal fashion to obtain a consistent matrix equation.

$$(\boldsymbol{H} + \boldsymbol{E})\boldsymbol{b} = (\boldsymbol{r} + \boldsymbol{\delta}), \quad (45)$$

where $\boldsymbol{E}$ and $\boldsymbol{\delta}$ are the minimal TLS perturbations. Equation (48) is rewritten as

$$(\boldsymbol{B} + \boldsymbol{D})\boldsymbol{z} = 0, \quad (46)$$

where augmented matrix $\boldsymbol{B} = [-\boldsymbol{r}, \boldsymbol{H}]$, perturbation matrix $\boldsymbol{D} = [-\boldsymbol{\delta}, \boldsymbol{E}]$ and $\boldsymbol{z} = \begin{bmatrix} 1 \\ \boldsymbol{b} \end{bmatrix}$.

The TLS aims to solve the following constrained minimization problem

$$\min_{\boldsymbol{D},x} \|\boldsymbol{D}\|_F^2$$
$$subject \quad to \quad (\boldsymbol{r} + \boldsymbol{\delta}) \in range(\boldsymbol{H} + \boldsymbol{E}) \quad (47)$$

where $\|\cdot\|_F$ denotes the Frobenius norm.

The TLS property does not depend on the distribution of 'residual' errors. The TLS estimates can be obtained from the SVD of augmented matrix, which was discussed in [31], [35]–[37].

B. THE IMPLEMENTATION OF THE DoA ESTIMATION AND THE CALCULATION OF NOISE VARIANCES

Here the uncorrelated assumption on source signals and noises is considered and the noises are preset as the GWNs with zero mean and unequal variances $\{\sigma_m^2\}_{m=1}^M$. As the degenerate spatial $ARMA(K, K)$ process, the construction of the combined system of linear equations without PNV is readily realized by flexibly choosing reference signals, especially for the ULA with the large number of sensors.

To implement the DoA estimation, it is first assumed that the number K of incident sources is known or estimated by the existing enumeration techniques in advance.

The step-by-step description of the proposed method is summarized as follows:

Step 1. Construct and solve the sampling combined system of linear equations without PNV to obtain the estimates of $\{\hat{b}_k\}_{k=1}^K$.

Step 2. Substitute the estimates $\{\hat{b}_k\}_{k=1}^K$ into the equation containing the single σ_m^2 ($m = 1, \cdots, M$), obtain the estimated $\hat{\sigma}_m^2$ by direct calculation.

Step 3. Construct the root-finding polynomial with the known $\{\hat{b}_k\}_{k=1}^K$, obtain the estimates of $\{\hat{v}_k\}_{k=1}^K$ and $\{\hat{\theta}_k\}_{k=1}^K$ via the formulas of roots of polynomial or the numerical root-finding algorithm.

If only the DoAs need to be estimated, **Step 2** can be cancelled.

VII. NUMERICAL ANALYSIS

The validity and performance of the proposed method are first evaluated by several numerical examples. We consider four far-field narrow-band sources ($K = 4$) impinging upon an azimuth-only ULA comprising seven sensors ($M = 7$) with sensor separation of half the wavelength of signal. Azimuth angles $[\theta_1, \theta_2, \theta_3, \theta_4]$ are preset as $[-60^0, -40^0, 20^0, 35^0]$. The ULA are corrupted by the so-called spatially colored noise. For simplicity, here the sources are supposed to have zero mean and unitary power, while sensor noises have the unequal powers $\{\sigma_m^2\}_{m=1}^7$. In simulation, source signals and noises all are generated by the complex-valued zero-mean Gaussian white random process. The number of snapshots is equal to 1000.

For the seven-sensor ULA, only three DoAs can be estimated by the single system of linear equations with the FDO or BDO. However, in accordance with (40), the number of the detectable DoAs can be up to four with the combined system of linear equations.

In simulation, following **Step 1**, by means of the first sensor as the reference sensor, a total of six correlation elements $\{h_{fm}^1\}_{m=2}^7$ and the corresponding conjugate counterparts $\{h_{bm}^1\}_{m=2}^7$ without PNV in data covariance matrix are chosen to construct the four independent equations in which the superscript 1 is used to describe the first sensor data as reference data. The four independent equations are enough to form the eligible combined system of linear equations to estimate the unknowns $\{b_k\}_{k=1}^4$. So the equations with the seven-th sensor as reference sensor are not considered.

The concrete form of the used combined system of linear equations in simulation is constructed as

$$\begin{bmatrix} h_{f2}^1 & h_{f3}^1 & h_{f4}^1 & h_{f5}^1 \\ h_{f3}^1 & h_{f4}^1 & h_{f5}^1 & h_{f6}^1 \\ h_{b6}^1 & h_{b5}^1 & h_{b4}^1 & h_{b3}^1 \\ h_{b7}^1 & h_{b6}^1 & h_{b5}^1 & h_{b4}^1 \end{bmatrix} \begin{bmatrix} b_4 \\ b_3 \\ b_2 \\ b_1 \end{bmatrix} = - \begin{bmatrix} h_{f6}^1 \\ h_{f7}^1 \\ h_{b2}^1 \\ h_{b3}^1 \end{bmatrix}, \quad (48)$$

where $h_{fm}^1 = R_{x_m x_1} - R_{w_m x_1}$ and $h_{bm}^1 = R_{x_1 x_m} - R_{x_1 w_m}$ with the first sensor data $x_1(t)$ as reference data. According to (20), the unknowns $\{b_k\}_{k=1}^4$ is given by

$$\begin{cases} b_1 = -(v_1 + v_2 + v_3 + v_4), \\ b_2 = v_1 v_2 + v_1 v_3 + v_1 v_4 + v_2 v_3 + v_2 v_4 \\ \qquad + v_3 v_4, \\ b_3 = -(v_1 v_2 v_3 + v_1 v_2 v_4 + v_1 v_3 v_4 + v_2 v_3 v_4), \\ b_4 = v_1 v_2 v_3 v_4, \end{cases} \quad (49)$$

Under the uncorrelated assumption between source signals and noises and further suppose the noises are the GWNs, theoretically, $R_{w_m x_1}$ and $R_{x_1 w_m}$ are all equal to zero as $m \neq 1$, then the used elements $\{h_{fm}^1\}_{m=2}^7$ and $\{h_{bm}^1\}_{m=2}^7$ in (48) contain no noise variances so that the unknowns $\{b_k\}_{k=1}^4$ is separately solved from (48) without considering the unknowns noise variances $\{\sigma_m^2\}_{m=1}^7$.

Once the estimates $\{\hat{b}_k\}_{k=1}^4$ are obtained, following **Step 2**, constructing the corresponding equations with PNV yields the estimates $\{\hat{\sigma}_m^2\}_{m=1}^M$ by the direction calculation.

Azimuth angles are obtained from the 4-order root-finding polynomial proposed in **Step 3**. For example, in one Monte
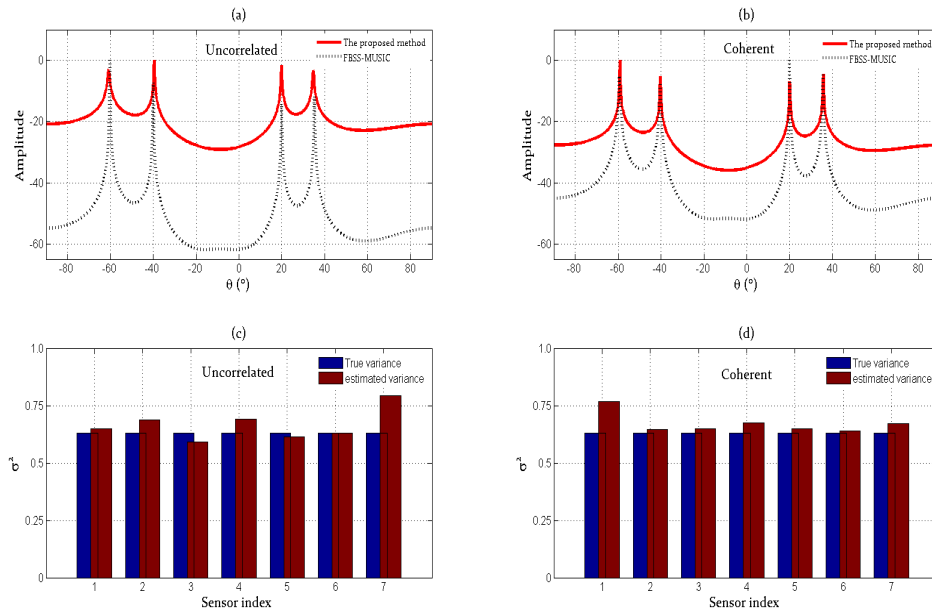
FIGURE 1. The azimuth-searching results of the uncorrelated and coherent sources and the calculation of noise variances under the background of the spatially-homogeneous white noise. (a) and (b) the azimuth-searching results of the uncorrelated and coherent sources; (c) and (d) the calculated noise variances of the uncorrelated and coherent sources with the proposed method.
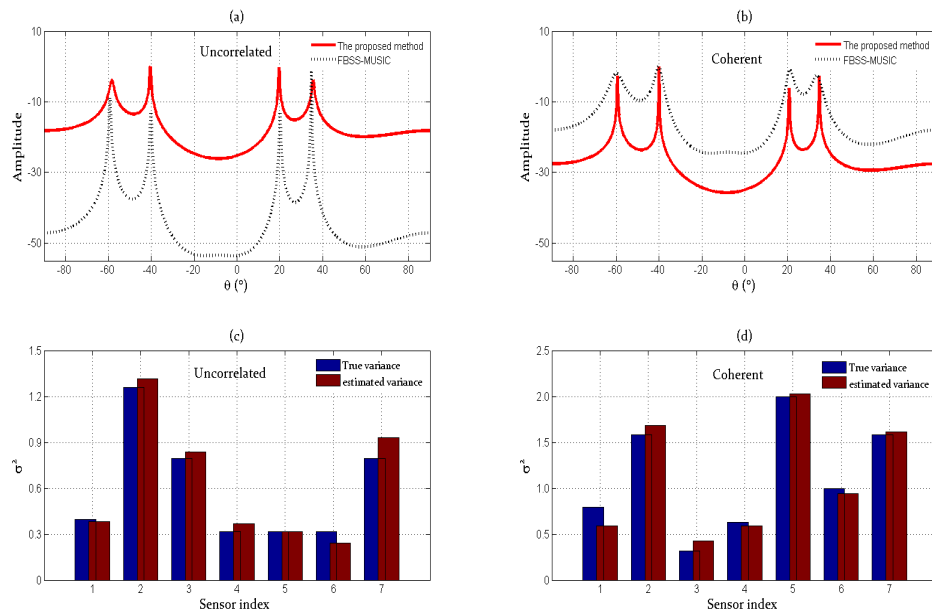

FIGURE 2. The azimuth-searching results of the uncorrelated and coherent sources and the calculation values of noise variances under the unequal SNRs. (a) and (b) the azimuth-searching results of the uncorrelated and coherent sources; (c) and (d) the calculated noise variances of the uncorrelated and coherent sources with the proposed method.

Carlo trial, with the known $\{\hat{b}_k\}_{k=1}^4$, the root-finding polynomial is given by

$$
\begin{aligned}
f(v) &= v^4 + \hat{b}_1 v^3 + \hat{b}_2 v^2 + \hat{b}_3 v + \hat{b}_4 \\
&= v^4 + (1.09 + 0.57j)v^3 + (1.09 + 1.58j)v^2 \\
&\quad + (0.15 + 1.22j)v + (-0.35 + 0.93j).
\end{aligned}
$$

Here numerical approach in which the root-finding problem of the complex-valued propagator is converted into the real-valued angle-searching operation is proposed. We construct the cost function $h(v) = 1/\log_{10}[|f(v)|]$ and plot its amplitude for points versus θ in the interval of $[-\pi/2, \pi/2]$ and pick peak positions as the estimated angles. In angle-searching process, in order to obtain more accurate estimates,
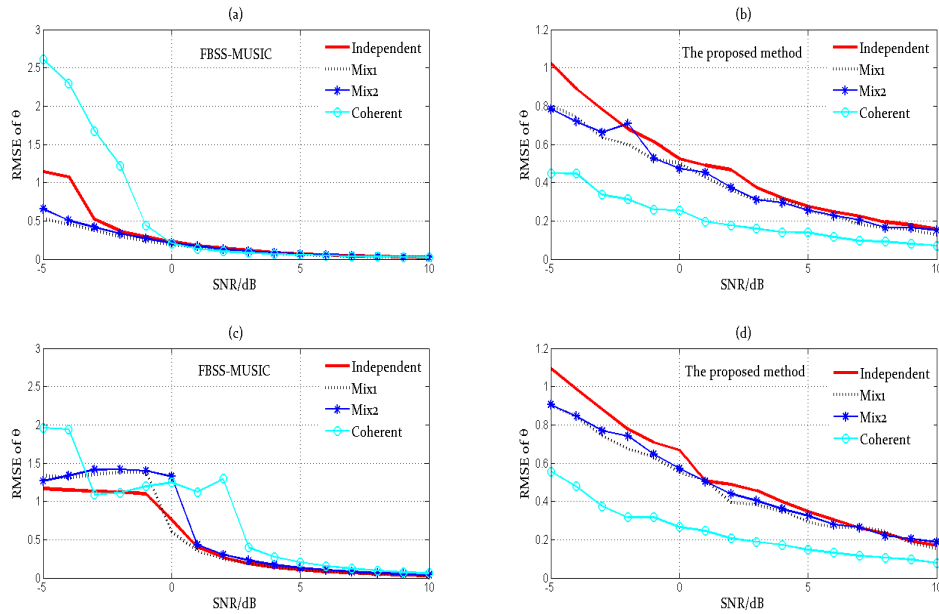
FIGURE 3. The RMSEs of $\hat{\theta}$ for the four combined cases of the incident sources versus the SNR with two coherent lengths of noises under the equal SNR. (a) and (b) the FBSS-MUSIC and the proposed method under the coherent length of noises equal to 3 with the last three noises being coherent. (c) and (d) the FBSS-MUSIC and the proposed method under the coherent length of noises equal to 4 with the last four noises being coherent. 'Uncorrelated': Four sources are uncorrelated; 'Mix1': The first two sources are coherent, others uncorrelated; 'Mix2': The first three sources are coherent; 'Coherent': Four sources are coherent.
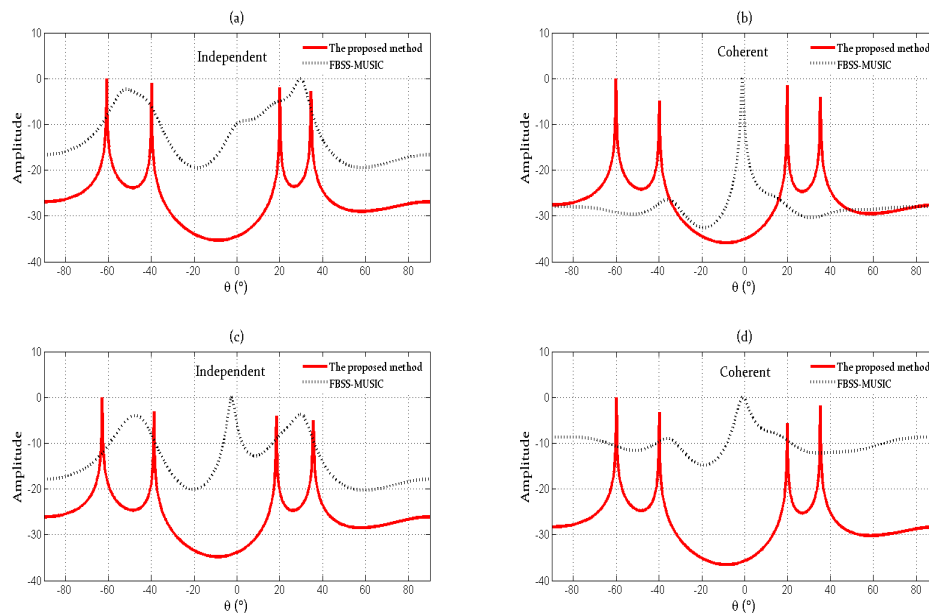

FIGURE 4. The azimuth-searching results of the uncorrelated and coherent sources under the coherent length of six noises. All the sensors but the first are coherent. (a) and (b) uncorrelated and coherent cases under the condition of equal SNR that is same as in Fig.1. (c) and (d) are uncorrelated and coherent cases under unequal SNRs preset as same as in Fig.2.

the effective off-gird searching algorithms are highly recommended [38], [39].

A. THE UNCORRELATED CASES OF SENSOR NOISES
Example 1: We first assume that sensor noises are uncorrelated with one another and have an identical noise variance,

which is termed the so-called spatially white. In simulation, noise variance is preset via the unitary signal power and $SNR = 2dB$. Since the FBSS (Forward and backward spatial smoothing) had been recognized as an effective preprocessing technique to combat multipath scenario, the comparison is focused on the proposed method and the MUSIC based on

the FBSS (FBSS-MUSIC). The DoAs of the uncorrelated and coherent sources can be well estimated by two methods in Fig.1(a) and (b). Besides the estimated DoAs, $\{\hat{\sigma}_m^2\}_{m=1}^7$ also can be directly calculated by the proposed method and shown in Fig.1(c) and (d).

Example 2: Sensor noises are still assumed to be uncorrelated but with unequal variances $\{\sigma_m^2\}_{m=1}^M$, which is the simple spatially-colored noise background. In simulation, noise variances are preset via the unitary power of signal and the SNR vector randomly generated by computer program within a prespecified range $[-5dB, 5dB]$. The azimuth-searching results of the uncorrelated and coherent sources are plotted in Fig.2(a) and (b), the calculated noise variances with the proposed method are in Fig.2(c) and (d).

As shown in Fig.1 and 2, the estimates of azimuth angles, regardless of under the equal or unequal SNRs, all can be obtained by the proposed method and FBSS-MUSIC, which shows that the spatially homogeneous or colored noises in effect have the less impact on the DoA estimation when the noises are uncorrelated with one another. The extraction of noise variances from the ULA data is an advantage of the proposed method.

B. THE COHERENT CASES OF SENSOR NOISES

Example 3: The proposed method can be applied to the background in which the noises are correlated/coherent with a 'correlated/coherent length' [40], [41] when source signals and noises are still uncorrelated. In simulation, sensor noises are assumed to have an identical variance under equal SNR with two coherent lengths. Owing to employing the different elements and amounts of data, the RMSEs of $\hat{\boldsymbol{\theta}}$ are separately plotted to show the behaviors of two methods. As shown in Fig.4(a) and (c), as the coherent length increases from 3 to 4, estimation performance gradually degrades, which spreads from the low to moderate SNR. By contrast, the proposed method performs more stable.

Example 4: The azimuth-searching results with the coherent length of six noises under equal SNR and unequal SNRs are obtained and plotted in Fig.4. As source signals are uncorrelated or coherent, the FBSS-MUSIC all breaks down, regardless of equal or unequal SNRs. It is shown that the coherent length of noises has a significant impact on the FBSS-MUSIC. By contrast, the proposed method still provides the correct DoA estimation even with one independent noises. Hence the proposed method also can be applicable to the so-called partly calibrated arrays in spatially correlated noise fields in which the noise in the calibrated sensors is uncorrelated with the noise in the other sensors [42].

VIII. EXPERIMENTAL VERIFICATION OF THE PROPOSED DoA ESTIMATION METHOD

The proposed DoA estimation method is verified by a simple sound experiment. Fig.5 shows the schematic diagram of the experimental set-up comprising three microphones and one sound source. Cellphone 2 is taken as one source emitting a sinusoidal waveform. The Cellphone 1 is added to align with
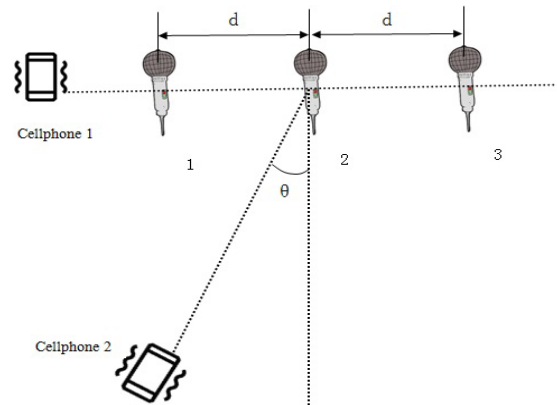

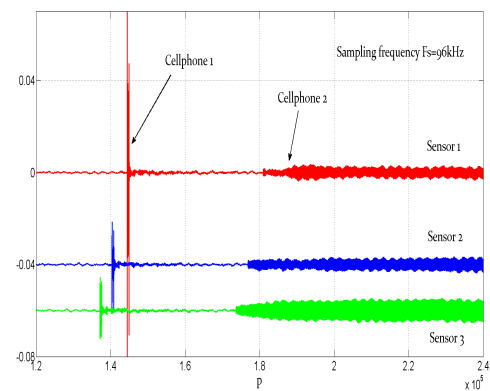**FIGURE 5.** The schematic diagram of the experimental set-up.

FIGURE 6. The signals received by three microphones.

three microphones for time synchronization. The frequency of sinusoidal waveform $f = 1.0kHz$ and microphone interspacing $d = 0.17m$. With time synchronization, the DoA is determined by the proposed method after the received waveforms of the microphones are transformed into the complex data with the Hilbert transform. Meanwhile, the differences between time-of-arrivals (DToAs) of three received waveforms also can be readily marked to calculate the DoA. The two types of the DoA results are compared to analyze and verify the proposed method.

Fig.6 shows the signals received by three microphones in one experiment in which the signals emitted by Cellphone 1 is employed for time synchronization. After time synchronization, the used waveforms and the corresponding results of three experiments are displayed in Fig.7. The source has the different incident angle in each experiment. For the first experiment, the starting waveforms are intentionally chosen and plotted in Fig.7(a). For other experiments, the non-starting waveforms are plotted in Fig.7(c) and (e), respectively.

We take Fig.7(a) for an example to show the DToAs method, the time t_i $(i = 1, 2, 3)$ of the first peak of the waveform received by the i-th sensor is marked as the corresponding time-of-arrival, so $\Delta t_{im} = t_i - t_m$ $(i > m)$ denotes time-difference between the i-th and m-th sensors, then the angle θ_{im}^t is determined via Δt_{im} from t_i and t_m. Each concrete
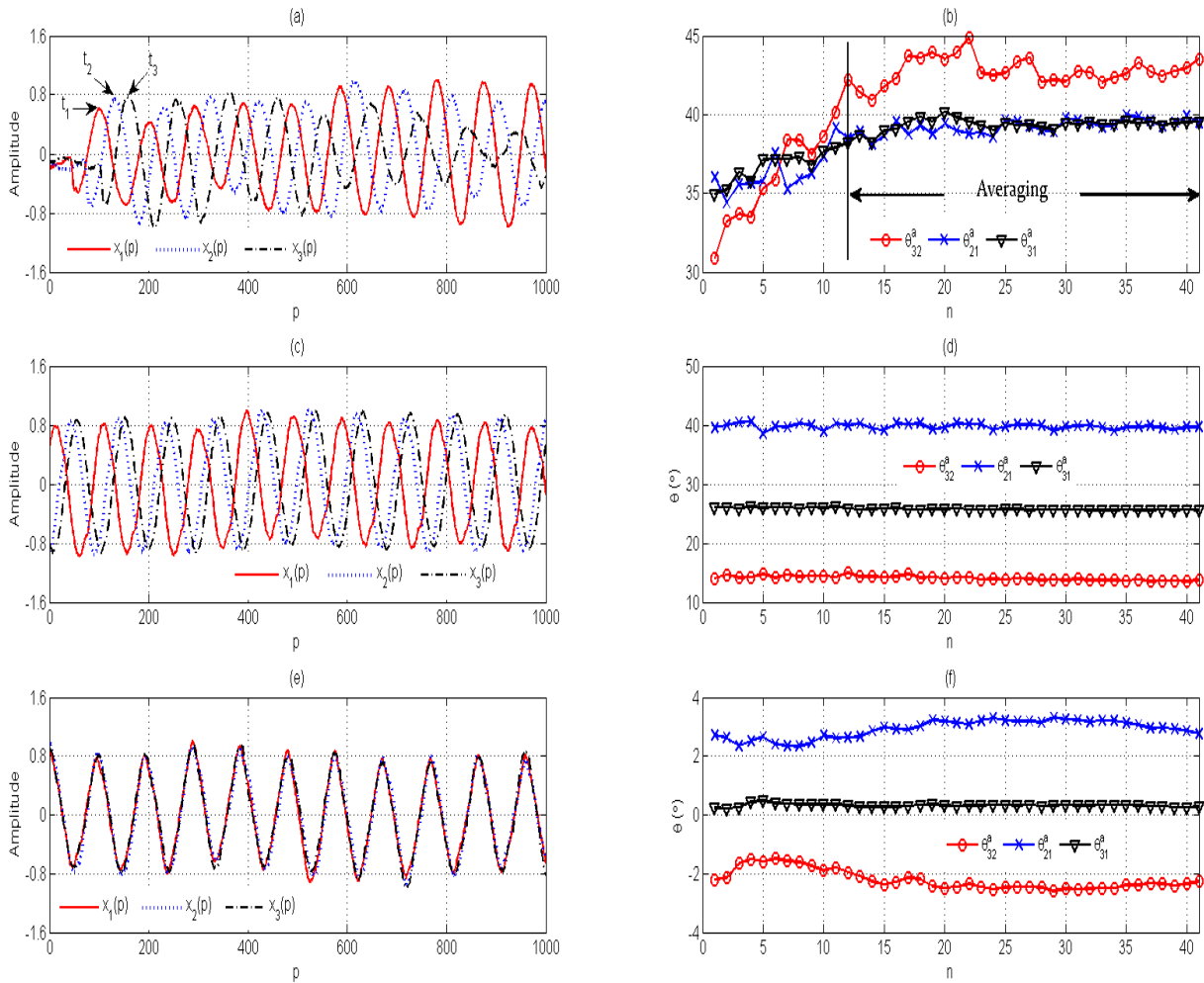
FIGURE 7. The used waveforms and the corresponding results of three different incident angles.

θ_{im}^t is calculated as described above. Here the averaging value $\bar{\theta}^t$ of the corresponding θ_{32}^t, θ_{21}^t and θ_{31}^t is used to describe the DoA estimate with the DToAs.

In the proposed method, data covariance matrix is first constructed by windowing three received waveform. the lengths of the windowed waveform increase from 200 points to 1000 points with the increment of 20 points and the zero starting point. Employing (21) and (22), each windowing operation provides one result for each concrete angle θ_{im}^a with the following expression

$$\theta_{im}^a = Re\{\frac{c\ln\frac{\hat{R}_{x_i^h x_n^h}}{\hat{R}_{x_m^h x_n^h}}}{j2\pi f(i-m)d}\}, \quad i > m, \qquad (50)$$

where $x_i^h(p)$ denotes the Hilbert transform of the i-th sampling data $x_i(p)$, the correlation function $\hat{R}_{x_i^h x_n^h}$ is obtained by making use of (44) with $x_n^h(p)$ as the reference signal and $n \neq i$, $n \neq m$. $Re\{.\}$ stands for real part.

The results of θ_{32}^a, θ_{21}^a and θ_{31}^a of each incident angle versus the windowing times are plotted in Fig.7(b), (d) and (f),

TABLE 1. The angle results with DToAs and the proposed method for three experiments.

Experiment	θ_{32}^t	θ_{21}^t	θ_{31}^t	$\bar{\theta}^t$	$\bar{\theta}_{32}^a$	$\bar{\theta}_{21}^a$	$\bar{\theta}_{31}^a$	$\bar{\theta}^a$
1	34.2^o	41.8^o	39.7^o	38.6^o	42.8^o	39.2^o	39.3^o	40.4^o
2	25.9^o	22.0^o	23.9^o	23.9^o	14.0^o	39.7^o	25.7^o	26.5^o
3	7.2^o	0°	3.6^o	3.6^o	-2.2^o	2.9^o	0.3^o	0.3^o

respectively. Because the non-signals before the received waveforms in Fig.7(a) are included in the windowing operations and result in the inaccurate and unstable results in calculation, only as the windowing times is larger than 12, the impact of the non-signals disappears in Fig.7(b). It is shown that all the results of each concrete angle θ_{im}^a are almost equal and nearly form the straight line with the averaging value $\bar{\theta}_{im}^a$. Similarly, the total averaging value $\bar{\theta}^a$ of the corresponding $\bar{\theta}_{32}^a$, $\bar{\theta}_{21}^a$ and $\bar{\theta}_{31}^a$ is taken as the DoA estimate with the proposed method. The angle results with the DToAs and the proposed method are listed in TABLE 1. It is shown that

each concrete $\bar{\theta}^a$ is well consistent with the corresponding $\bar{\theta}^t$, which verifies the effectiveness of the proposed method.

As shown in TABLE 1, an interesting phenomenon is observed for Source 2, the large value difference arises between θ_{32}^t and $\bar{\theta}_{32}^a$, and also between θ_{21}^t and $\bar{\theta}_{21}^a$. However, qualitatively and quantitatively, the values of $\bar{\theta}_{32}^a$ and $\bar{\theta}_{21}^a$ in Fig.7(d) are observed to be more consistent with the corresponding time-of-arrivals of the received waveforms shown in Fig.7(c). The main reason lies in that the time-of-arrival of the waveform of sensor 2 is slightly varied in propagation in the practical experiment and become more closer to that of sensor 3 and away from that of sensor 1 in Fig.7(c), which is well reflected by $\bar{\theta}_{32}^a$ and $\bar{\theta}_{21}^a$ with the proposed method.

IX. CONCLUSION

The DoA estimation of the azimuth-only ULA is investigated in theory and method from the fused perspective of physical and signal properties of the incident sources induced by the orderly-setup of array sensors.

Under the mild stationary assumption on source signals and noises, it is theoretically proved that signal model of the external sources incident on the azimuth-only ULA is a degenerate spatial *ARMA* process subject to the identical autoregressive and moving average parameters. The azimuth-only ULA has its own distinctive MYW system of linear equations and the root-finding polynomial. The DoAs of the azimuth-only ULA are estimated directly from the ULA data via the chain comprising the MYW system of linear equations and the root-finding polynomial. The EVD is not involved.

The proposed method can be applied to multipath scenario. The DoAs estimation and the calculation of noise variances are separately performed under the backgrounds of spatially white and colored noises. The maximum number of the detectable sources is equal to $floor[\frac{2M}{3}]$ by making use of the combined system of linear equations with the FDO and BDO, where M is the number of sensors.

The MYW system of linear equations and the root-finding polynomial are analyzed to obey the DHP, which manifests that the conclusion of the degenerate spatial *ARMA* process is self-consistent both in physics and signal processing.

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