IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received November 9, 2020, accepted November 22, 2020, date of publication November 25, 2020, date of current version December 9, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3040480

Maximum and Optimal Number of Activated Links for D2D-Aided Underlaying Cellular Networks

JIAN SUN[1], ZHIGUANG LI[2], ZHONGSHAN ZHANG[3], (Senior Member, IEEE), AND CHENGWEN XING[3]

[1]School of Computer Science and Technology, Shandong University of Technology, Zibo 255000, China
[2]Bohai Drilling Engineering Company Ltd., CNPC, Tianjin 300457, China
[3]Beijing Institute of Technology, Beijing 100081, China

Corresponding author: Zhongshan Zhang (zhangzs@bit.edu.cn)

This work was supported in part by the key project of the National Natural Science Foundation of China under Grant 61431001, in part by the Beijing Institute of Technology Research Fund Program for Young Scholars, and in part by the Beijing Natural Science Foundation under Grant L172026.

ABSTRACT With the rapid popularity of smart terminals, the device-to-device(D2D) communication is recognized as one of the most promising techniques in the fifth-generation(5G) communications networks due to its capabilities of substantially improving the spectral efficiency, relieving the traffic burden in the base stations(BSs), and reducing the terminals' power consumptions, etc. By properly activating some D2D links(DLs), the sum data rate of the D2D-aided cellular networks(CNs) can be substantially improved. However, constrained by the severe interference imposed on the conventional cellular users (CUs) by the activating D2D users (DUs), the sum data rate of the D2D-aided CNs cannot be unlimitedly improved in a crude way of simply increasing the number/density of DLs. In other words, there must exist a maximum/optimal number of activating DLs in terms of the sum data rate, as revealed in this paper. By identifying the maximum/optimal number of activating DLs, the closed-form expressions for sum data rates of both cellular links(CLs) and DLs can be obtained. Numerical results show that the optimum number of activating DLs, as a function of several critical parameters such as DU's transmit power, signal-to-interference-plus-noise ratio threshold and outage probability of CLs/DLs, etc, can be determined by implementing the proposed algorithms.

INDEX TERMS Underlaying cellular networks, device-to-device, maximum D2D number, optimal D2D number.

I. INTRODUCTION

With the rapid popularity of smart terminals, more and more advanced applications such as pilotless automobile, telemedicine and smart home, etc, have emerged [1]–[4]. Accordingly, both the number of connected devices and the volume of mobile data traffic have undergone an exponential growth in the past decade, highly demanding a substantial improvement in the capacity of mobile networks [5], [6]. However, the harsh reality of "spectrum resources are becoming scarce" has posed a severe challenge to both operators and vendors. All the above-mentioned conditions have greatly stimulated the demand for emerging of new techniques such as the fifth-generation (5G) wireless communications and Internet of Things (IoT) [7]–[12].

The associate editor coordinating the review of this manuscript and approving it for publication was Zihuai Lin.

At present, many new spectral-efficient technologies have been proposed by either academia or industry. Among them, the Device-to-device (D2D) technique, which enables the D2D peers to communicate directly without relying on the involvement of base stations (BSs), has attracted a wide concern. In the past few years, D2D technique has exhibited several promising advantages, such as the capabilities of substantially improving the spectral efficiency of wireless networks, significantly relieving the heavy traffic burden of the BSs, enhancing the connectivity of mobile devices, and saving power of mobile terminals, etc [13]–[23].

Basically, the benefits brought about by employing D2D technique comes from the fact that "the D2D links (DLs) are allowed to reuse the licensed spectrum that was supposed to be exclusively occupied by the conventional cellular links (CLs)". The spectrum utilization can thus be substantially improved by implementing D2D transmissions [24].

Even so, there are still many core problems that should be broken through. For instance, the activated D2D transmitters (DTs) may impose a severe interference on their co-spectrum CLs [25]–[27], thus causing an intolerable bit error rate (BER) on the latter [28], [29]. In other words, it is impossible for us to attain an unlimitedly increasing sum data rate in the D2D-aided underlaying cellular networks (CNs) in a crude way of simply enhancing the density of DLs.

Until now, the relationship between the system's performance and the number of DLs in D2D-aided underlaying CNs has been widely investigated [30]–[34]. For instance:

- In [30], the authors investigated the maximum-multiplexing problem in D2D-aided underlaying CNs by considering two multi-carrier modulations, i.e., Orthogonal Frequency Division Multiplex (OFDM) and Filter Bank Multi-Carrier (FBMC) modulation. It was illustrated that the average data rate can be greatly improved by multiplexing licensed spectrum with DLs. However, the optimal number of DLs was not evaluated in [30].
- In [31], the sum data rate of the D2D-aided CNs was shown to be improved by increasing the number of DLs. However, the authors only considered the scenarios of fixed CLs/DLs, without giving out the closed-form expressions for the sum data rate under variant number of DLs.
- The authors in [32] pointed out that the number of potential DLs plays an important role in impacting the system's capacity. The analytic expressions for the average coverage probability of the conventional cellular users (CUs) under variant number of potential DLs were computed relying on mathematical tools such as stochastic geometry and Poisson point process (PPP). However, neither the closed-form expression for the DLs' coverage probability nor that for the sum data rate was given out in [32].
- In [33], the authors proposed an adaptive group-head-selection algorithm for maximizing the number of connections in machine-type communication (MTC). This algorithm can be categorized as a joint-signaling-and-data-resource-optimization model that is constrained by both network resources and data rate. It was shown that the number of connections is mainly impacted by two parameters, namely the ratio of the number of MTC users to that of the conventional CUs and the ratio of communication-target-threshold of MTC users to that of the CUs. However, by adopting the proposed group-based resource allocation algorithm, only the number of connections can be improved (i.e., neither the maximum nor the optimal number of MTC users was investigated).
- In [34], the impact of DLs' number on the sum data rate was investigated by considering the scenario in which an individual spectrum was reused by multiple CLs/DLs. By fixing the total number of users, it was shown that the smaller the proportion of CUs, the higher the sum data rate. Furthermore, it was revealed that an optimal number of DLs under the fixed-CUs condition always exists and can be found. However, the maximum number of DLs supported by a single sub-channel was still unknown.
- In [35], based on Lagrangian duality theory, energy efficiency algorithms are proposed to solve an optimal power and rate control problem, whicih is used to achieve proportional fairness between DUs and CUs.

From the above-mentioned discussions [31]–[33], [36], the sum data rate of the D2D-aided underlaying CNs can be improved to some extent by increasing the density of DLs. However, without giving out the closed-form expressions of the sum data rate (i.e., neither the optimal nor the maximum number of DLs is calculated), it would be really hard to intuitively illustrate the capacity's changing trend in the D2D-aided CNs. To our best knowledge, there exist very few literatures concerning the optimal/maximum tolerable number of activating DLs in D2D-aided CNs. Although the authors in [34] attained the closed-form expressions for the sum throughput, and the maximum value is obtained by balancing the scale factor between the DUs and the total users (i.e. accounting both CUs and DUs). However, the maximum/optimal number of DLs were neglected.[1] Furthermore, with the rapid development of IoTs, the maximum number of DLs that can be accommodated by a single channel has been regarded as one of the most important performance indicators (as well as major problems) in D2D-aided underlaying CNs due to the depleting spectral resources.

In this paper, both the optimal value of DLs in terms of the maximum throughput and the maximum number of connected devices in the D2D-aided underlaying CNs are investigated. The main contributions of this paper are reflected in the following aspects:

1) Deriving the closed-form expression of the sum data rate in the D2D-aided underlaying CNs, in which an individual sub-channel can be employed for supporting one CL and K DLs simultaneously;
2) Attaining the optimal number of activated DLs in terms of the maximum throughout;
3) Obtaining the maximum number of connected devices that can be supported in the D2D-aided underlaying CNs.

The remainder of this paper is organized as follows. Section (II) introduces the system model of the D2D-aided underlaying CNs. In Section (III), the closed-form

[1] In theory, the optimal activated link indicates the number of DLs corresponding to the maximum capacity of the proposed system. In other words, if the number of DLs is beyond this optimal value, the excessive interference imposed by these additional DLs will erode the sum data rate. At that time, the SINR received at BS is still greater than the threshold, and the number of D2D links can be further increased. However, the interference imposed by increasing the activated DLs exceeds the increment of signal, thus decreasing the system capacity. When the received SINR becomes lower than the threshold due to the interference induced by the activated D2D links, the corresponding number of DLs is called "maximum". In this sense, the concept "the maximum number" should be understood as "the upper-bound for the number of DLs carried in the same sub-channel under the premise of satisfying the CUs' QoS requirements".

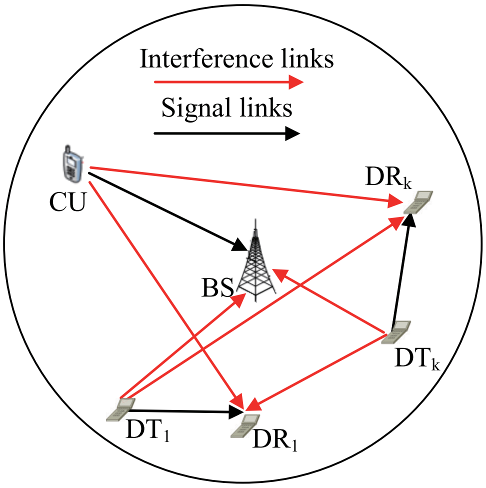

FIGURE 1. System model for the proposed D2D-aided CNs with $1 \leq k \leq K$.

expressions of coverage probabilities for both CLs and DLs are derived, followed by solving the optimal/maximum number of DLs in Section (IV). Furthermore, numerical results are provided in Section (V). Finally, Section (VI) concludes the paper.

II. SYSTEM MODEL

In this paper, the spectrum-sharing scheme in D2D-aided underlaying CNs will be investigated. Without loss of generality, we assume that M activated CUs occupy a total of M sub-channels based on some resource-allocation techniques (e.g. orthogonal frequency division multiplexing (OFDM)). In this case, there would exist no available spectrum resource for the DUs. To address this problem, a new scheme can be implemented by allowing the reuse of an individual sub-channel between one (and only one) CU and K DLs. In brief, the licensed uplink spectrum, which was supposed to be exclusively occupied by the conventional CUs, can be reused by the DUs in the proposed model, as depicted in Fig.1. Furthermore, we assume that a single cell is capable of providing services to a number of mobile customers (i.e., comprising M CUs and K D2D pairs) simultaneously. In addition, both CLs and DLs have their own minimum Quality of Service (QoS) requirements in terms of a tolerable signal-to-interference-plus-noise ratio (SINR), provided that the channel state information (CSI) information concerning each link is available at the BS. Finally, all the K co-channel DTs are assumed to be distributed within the cell coverage following a homogeneous PPP Φ_1 model with density λ_d [32].

In the following, we assume that (if necessary) the CUs can always establish their connections to the serving BS. Without loss of generality, the inter-CU-interference is assumed to be negligible. Meanwhile, the geographically close-by D2D peers are allowed to establish DLs between them, as depicted in Fig.1. By and large, there exist three primary interference sources between DLs and the conventional CLs, namely the inter-DU interference, the DL-to-BS interference and the CL-to-D2D receiver (DR) interference.

In the following, we employ subscripts b and c to stand for the BS and the CU, respectively, and use subscripts i and j to specify the i-th DT and the j-th DR, respectively. In particular, $d_{i,i}$ is used to denote the distance between i-th DT and its peer DR. Furthermore, we assume that the distance $d_{i,i}$ is statistically time invariant. Following the above settings, the received signals at the BS can be expressed:

$$y_b = \sqrt{d_{c,b}^{-\alpha} P_c} \cdot h_{c,b} \cdot s_c + \sum_{i=1}^{K} \sqrt{d_{i,b}^{-\alpha} P_i} \cdot h_{i,b} \cdot s_i + z, \quad (1)$$

where P_i and $d_{i,b}$ denote the transmit powers of the i-th DT and the distance between the i-th DT and the BS, respectively. Furthermore, $h_{i,b} \sim \mathcal{CN}(0, 1)$ denotes the coefficient of the channel attenuation between the i-th DT and the BS. Meanwhile, α and s_i are used to denote the path-loss exponent and the transmitted signals of the i-th DT, respectively. In addition, we assume that $E\left\{|s|^2\right\} = 1$ always holds. Finally, $z \sim \mathcal{CN}(0, \sigma^2)$ stands for the additive white Gaussian noise (AWGN).

Similarly, the received signals at the j-DR can be represented:

$$y_j = \sqrt{d_{c,j}^{-\alpha} P_c} \cdot h_{c,j} \cdot s_c + \sum_{\substack{i=1 \\ i \neq j}}^{K} \sqrt{d_{i,j}^{-\alpha} P_i} \cdot h_{i,j} \cdot s_i + z. \quad (2)$$

III. COVERAGE PROBABILITY ANALYSIS

In this section, the closed-form expressions for the coverage probabilities of both CLs and DLs will be given out.

A. COVERAGE PROBABILITY OF CONVENTIONAL CELLULAR LINKS

For a given realization of the PPP Φ_1, under the above-mentioned assumptions by taking the k-th CL as an example, the SINR observed at the BS side (i.e., due to the interference imposed by the DLs) can be computed:

$$\mathrm{SINR}_b^k = \frac{P_c d_{c,b}^{-\alpha} \left|h_{c,b}\right|^2}{\sum\limits_{i \in \Phi_1} P_i d_{i,b}^{-\alpha} \left|h_{i,b}\right|^2 + \sigma^2}. \quad (3)$$

Given a pre-defined SINR threshold (i.e., β_c),[2] the averaged uplink coverage probability can be defined:

$$P_{\mathrm{cov}}^{\mathrm{C}}\left(\beta_c, \lambda_d, \alpha\right) = \mathbb{E}\left[\mathbb{P}\left\{\mathrm{SINR}_b^k > \beta_c\right\}\right], \quad (4)$$

where

$$\begin{aligned} \mathbb{P}\left\{\mathrm{SINR}_b^k > \beta_c\right\} &= \mathbb{P}\left\{\frac{P_c d_{c,b}^{-\alpha} \left|h_{c,b}\right|^2}{I_d + \sigma^2} > \beta_c\right\} \\ &= \mathbb{P}\left\{\left|h_{c,b}\right|^2 > \frac{\beta_c d_{c,b}^{\alpha}}{P_c}\left(I_d + \sigma^2\right)\right\} \\ &= \exp\left(-\frac{\beta_c d_{c,b}^{\alpha} \sigma^2}{P_c}\right) \mathcal{L}_{I_d}(s) \quad (5) \end{aligned}$$

[2]The received power at the BSs can be regarded as high enough to meet the QoS requirement of CLs only if it is beyond the threshold.

with $I_d = \sum_{i\in\Phi_1} P_i d_{i,b}^{-\alpha} \left|h_{i,b}\right|^2$ and $s = \frac{\beta_c d_{c,b}^{\alpha}}{P_c}$. Furthermore, $\mathcal{L}_{I_d}(s)$ is used to stand for the Laplace transform of random variables I_d evaluated at s, as expressed:

$$\begin{aligned}\mathcal{L}_{I_d}(s) &= \mathbb{E}\left[\exp\left(-s\sum_{i\in\Phi_1} P_i d_{i,b}^{-\alpha}\left|h_{i,b}\right|^2\right)\right]\\ &= \exp\left[-\frac{2\pi\lambda_d}{\alpha}\left(\frac{\beta_c P_i}{P_c}\right)^{\frac{2}{\alpha}} \mathrm{B}\left(1-\frac{2}{\alpha},\frac{2}{\alpha}\right) d_{c,b}^2\right], \quad (6)\end{aligned}$$

where $\mathrm{B}(P,Q)$ denotes the Beta function.[3] The detailed derivation of the above-mentioned expression is given by Appendix A.

By substituting (6) into (5) and utilizing the Euler's Reflection Formula $\Gamma(1-x)\Gamma(x) = \frac{\pi}{\sin \pi x}$, the probability density function (PDF) $f_r(r)$ can be simplified as $2r/R^2$, provided that the CUs are randomly distributed within the radius-R coverage of a cell. Therefore, the coverage probability of CLs can be calculated:

$$P_{\text{cov}}^{\text{C}}(\beta_c,\lambda_d,\alpha) = \int_0^R \mathrm{e}^{-ar^{\alpha}-br^2} f_r(r)dr, \quad (7)$$

where $a = \frac{\beta_c\sigma^2}{P_c}$, $b = \frac{2\pi^2\lambda_d}{\alpha\sin(2\pi/\alpha)}\left(\frac{\beta_c P_i}{P_c}\right)^{\frac{2}{\alpha}}$ and $r = d_{c,b}$.

Here we must emphasize that it would be pretty hard to derive the exact closed-form expression of the coverage probability, if not impossible. Fortunately, by picking out some specific α values, we can always attain the simplified closed-form expression. In the following, like in [31], [32], [38], we give out the coverage probability of CLs by considering the special case of $\alpha = 4$:

$$P_{\text{cov}}^{\text{C}}(\beta_c,\lambda_d,4) = \frac{1}{R^2}\sqrt{\frac{\pi}{4a}}\exp\left(\frac{b^2}{4a}\right)[\Phi(\iota)-\Phi(\kappa)] \quad (8)$$

where $\iota = b\sqrt{\frac{1}{4a}} + R^2\sqrt{a}$, $\kappa = b\sqrt{\frac{1}{4a}}$, and $\Phi(x) = \frac{1}{\sqrt{\pi}}\int_0^{x^2}\frac{e^{-t}}{\sqrt{t}}dt$.

Basically, the impact of noise can be neglected in calculating the coverage probability, provided that the noise power is low enough. By neglecting the impact of noise and substituting both $\sigma^2 = 0$ and $\alpha = 4$ into (8), the closed-form expression of CL's coverage probability (i.e. $P_{\text{cov}}^{\text{C}}$) can be simplified:

$$P_{\text{cov}}^{\text{C}} = \frac{1-\exp\left(-\frac{\pi^2}{2}\lambda_d\sqrt{\frac{\beta_c P_i}{P_c}}R^2\right)}{\frac{\pi^2}{2}\lambda_d\sqrt{\frac{\beta_c P_i}{P_c}}R^2}. \quad (9)$$

Evidently, the coverage probability of CLs (by neglecting the impact of noise) will be impacted mainly by the following three factors, i.e., the number of the co-spectrum DLs (i.e., $\lambda_d\pi R^2$), the ratio P_i/P_c and the SINR threshold of CLs (i.e., β_c).

[3] In particular, we have $\mathrm{B}\left(1-\frac{2}{\alpha},\frac{2}{\alpha}\right) = \Gamma\left(\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)$, where $\Gamma(s) = \int_0^{\infty} t^{s-1}e^{-t}dt$ is defined as the Gamma function, refered to (8.350) in [37].

B. COVERAGE PROBABILITY OF D2D LINKS

For a given D2D pair (i.e., the j-th DL), the interference imposed on the j-th DR mainly comes from the co-spectrum CL as well as its neighboring DLs ($i \neq j$). In this case, the SINR observed at the j-th DR can be expressed:

$$\text{SINR}_j^k = \frac{P_j d_{j,j}^{-\alpha}\left|h_{j,j}\right|^2}{P_c d_{c,j}^{-\alpha}\left|h_{c,j}\right|^2 + \sum_{i\in\Phi_1\setminus\{j\}}^{K} P_i d_{i,j}^{-\alpha}\left|h_{i,j}\right|^2 + \sigma^2}. \quad (10)$$

Denoting by β_d the SINR threshold at the j-th DR, the coverage probability averaged over a given planar area can be expressed:

$$P_{\text{cov}}^{\text{D2D}}(\beta_d,\lambda_d,\alpha) = \mathbb{E}\left[\mathbb{P}\left\{\text{SINR}_j^k > \beta_d\right\}\right], \quad (11)$$

where

$$\begin{aligned}\mathbb{P}\left\{\text{SINR}_j^k > \beta_d\right\} &= \mathbb{P}\left\{\frac{P_j d_{j,j}^{-\alpha}\left|h_{j,j}\right|^2}{I_c' + I_d' + \sigma^2} > \beta_d\right\}\\ &= \mathbb{P}\left\{\left|h_{j,j}\right|^2 > \frac{\beta_d d_{j,j}^{\alpha}}{P_j}\left(I_c' + I_d' + \sigma^2\right)\right\}\\ &= \exp\left(-\frac{\beta_d d_{j,j}^{\alpha}\sigma^2}{P_j}\right)\mathcal{L}_{I_c'}\left(s'\right)\mathcal{L}_{I_d'}\left(s'\right),\end{aligned} \quad (12)$$

with $I_c' = P_c d_{c,j}^{-\alpha}\left|h_{c,j}\right|^2$, $I_d' = \sum_{i\in\Phi_1\setminus\{j\}}^{K} P_i d_{i,j}^{-\alpha}\left|h_{i,j}\right|^2$ and $s' = \frac{\beta_d d_{j,j}^{\alpha}}{P_j}$.

Similar to (6), $\mathcal{L}_{I_d'}(s')$ can be expressed:

$$\mathcal{L}_{I_d'}\left(s'\right) = \exp\left[-\frac{2\pi\lambda_d}{\alpha}\left(\frac{\beta_d d_{j,j}^{\alpha} P_i}{P_j}\right)^{\frac{2}{\alpha}}\mathrm{B}\left(1-\frac{2}{\alpha},\frac{2}{\alpha}\right)\right], \quad (13)$$

where P_j denotes the transmit power of the j-th DT.

Next, let us validate the above-mentioned expression by taking Rayleigh-fading model as an example. In particular, for Rayleigh-fading channels with $\left|h_{c,j}\right|^2 \to \exp(1)$, the Laplace transform $\mathcal{L}_{I_c'}\left(s'\right)$ can be calculated:

$$\begin{aligned}\mathcal{L}_{I_c'}\left(s'\right) &= \mathbb{E}\left[e^{-\left(s' P_c d_{c,j}^{-\alpha}\left|h_{c,j}\right|^2\right)}\right]\\ &= \frac{1}{1+s' P_c \mathbb{E}\left[d_{c,j}\right]^{-\alpha}}\end{aligned} \quad (14)$$

where the equality $\mathbb{E}\left[e^{-\delta A}\right] = \frac{1}{1+A}$ when $\delta \to \exp(1)$ is used. Like in [39], an approximation $\mathbb{E}\left[d_{c,j}\right] \approx \frac{128R}{45\pi}$ can be obtained.

By substituting (13) and (14) into (11), the coverage probability of the j-th DL can be obtained:

$$P_{\text{cov}}^{\text{D2D}}(\beta_d,\lambda_d,\alpha) = \exp\left(-a' d_{j,j}^{\alpha} - b' d_{j,j}^2\right)\frac{1}{1+\frac{\beta_d P_c d_{j,j}^{\alpha}}{P_j \mathbb{E}[d_{c,j}]^{\alpha}}} \quad (15)$$

where $a' = \frac{\beta_d\sigma^2}{P_j}$ and $b' = \frac{2\pi^2\lambda_d}{\alpha\sin(2\pi/\alpha)}\left(\frac{\beta_d P_i}{P_j}\right)^{\frac{2}{\alpha}}$.

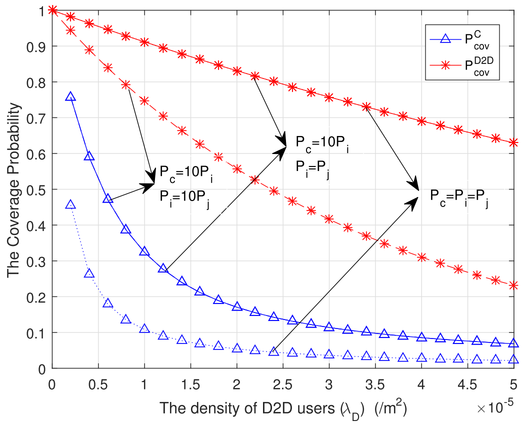

FIGURE 2. **Coverage probability as a function of DL density (i.e., λ_d) with variant power ratios.**

Next, let us substitute $\alpha = 4$ into (15) for deriving the closed-form expression of the coverage probability of DLs. By ignoring the impact of noise, we get:

$$P_{\rm cov}^{\rm D2D} = \exp\left(-\frac{\pi^2\lambda_d}{2}\sqrt{\frac{\beta_d P_i}{P_j}}d_{j,j}^2\right)\frac{1}{1+\frac{\beta_d P_c d_{j,j}^4}{P_j \mathbb{E}[d_{c,j}]^4}}. \quad (16)$$

Evidently, the coverage probability of the j-th DL will mainly be dominated by the following four factors, including the density of the co-spectrum DLs (i.e., λ_d), the ratios P_i/P_j and P_c/P_j, the SINR threshold of DUs (i.e. β_d) and the distance between the j-th DT and its peer DR (i.e., $d_{j,j}$).

In Fig.2, the relationship between the coverage probability and the density of DUs under variant power ratios is revealed. The coverage probabilities of both CLs and DLs are shown to gradually decrease as the DUs' density increases. This observation matches the theoretical analysis well. Evidently, the coverage probability $P_{\rm cov}^{\rm C}$ can be improved by increasing the ratio P_c/P_i. Unlike $P_{\rm cov}^{\rm C}$, it would be beneficial to improving the coverage probability $P_{\rm cov}^{\rm D2D}$ by decreasing the power ratio P_i/P_j or P_c/P_j.

IV. OPTIMAL AND MAXIMUM NUMBER OF D2D LINKS

In this section, the optimal/maximum numbers of activated DLs will be identified relying on the analysis of ergodic data rate. We assume that each licensed spectrum that has already been allocated to a CU is allowed to be reused by multiple DLs in the proposed underlaying CNs. The optimal number of DLs indicates the number of DLs corresponding to the maximum capacity of the underlaying CNs. In other words, if the number of DLs is beyond this optimal value, an excessive interference imposed by these additional DLs will deteriorate the sum data rate. Although the number of DLs is still allowed to continually increase in this case (because the SINR received at the serving BS is still beyond the threshold), the excessive interference imposed by increasing the activated DLs will exceed the useful signal's increment, thus deteriorating the system's capacity. If in a time the received SINR is becoming less than the threshold if an additional DL is activated, the instantaneous number of activated DLs is defined as "the maximum number" of DLs. In this sense, the concept "the maximum number" should be understood as "the upper-bound of the number of DLs carried in the same sub-channel under the premise of satisfying the CUs' QoS requirements".

A. SUM DATA RATE OF D2D-AIDED UNDERLAYING NETWORKS

From (29) in [31], the ergodic data rate of D2D-aided underlaying CNs can be expressed:

$$\begin{aligned}\bar{R} &= \int_0^\infty \log_2(1+\beta)\mathbb{P}\left\{{\rm SINR}_j^k > \beta_d\right\}d\beta \\ &= \frac{1}{\ln 2}\int_0^\infty \ln(1+\beta)d(-P_{\rm cov}) \\ &= \frac{\ln(1+\beta)}{\ln 2}(-P_{\rm cov})\Big|_0^\infty + \frac{1}{\ln 2}\int_0^\infty \frac{P_{\rm cov}}{1+\beta}d\beta, \end{aligned} \quad (17)$$

where we have $\frac{\ln(1+\beta)}{\ln 2}(-P_{\rm cov})\Big|_0^\infty \to 0$.

By substituting (9) into (17), the ergodic data rate of each CL (i.e., $\bar{R}^{\rm C}$) can be calculated:

$$\begin{aligned}\bar{R}^{\rm C} &= \frac{1}{\ln 2}\int_0^\infty \frac{1-\exp\left(-A\sqrt{\beta_c}\right)}{A\left(1+\beta_c\right)\sqrt{\beta_c}}d\beta_c \\ &= \frac{2}{A\ln 2}\left[\frac{\pi}{2} - {\rm Ci}\left(A\right)\sin A + {\rm si}\left(A\right)\cos A\right], \end{aligned} \quad (18)$$

where $A = \frac{\pi^2 R^2 \lambda_d}{2}\sqrt{\frac{P_i}{P_c}}$ and ${\rm si}(x) = -\int_x^\infty \frac{\sin t}{t}dt$ and ${\rm Ci}(x) = -\int_x^\infty \frac{\cos t}{t}dt$, as defined in [37].

Similar to (18), the ergodic data rate of a DL (i.e., $\bar{R}^{\rm D2D}$) can be computed by substituting (16) into (17):

$$\begin{aligned}\bar{R}^{\rm D2D} &= \frac{1}{\ln 2}\int_0^\infty \frac{\exp\left(-C\sqrt{\beta_d}\right)}{\left(1+\beta_d\right)\left(1+B\beta_d\right)}d\beta_d \\ &= \frac{2}{(B-1)\ln 2}\left[\cos C\,{\rm Ci}\left(C\right) - \cos\left(\frac{C}{\sqrt{B}}\right){\rm Ci}\left(\frac{C}{\sqrt{B}}\right)\right. \\ &\qquad \left. + \sin C\,{\rm si}(C) - \sin\left(\frac{C}{\sqrt{B}}\right){\rm si}\left(\frac{C}{\sqrt{B}}\right)\right], \end{aligned} \quad (19)$$

where $B = \frac{P_c d_{j,j}^4}{P_j \mathbb{E}[d_{c,j}]^4}$ and $C = \frac{\pi^2 d_{j,j}^2 \lambda_d}{2}\sqrt{\frac{P_i}{P_j}}$.

By combining (18) and (19), the sum data rate of the proposed underlaying CNs can be obtained:

$$R^{\rm sum} = \bar{R}^{\rm C} + \pi R^2 \lambda_d \bar{R}^{\rm D2D}. \quad (20)$$

B. MAXIMUM NUMBER OF D2D LINKS

Following the above-mentioned analysis, the following conclusion can obviously be drawn: the larger the number of DLs, the severer the inter-link interference. In other words, we cannot unlimitedly improve the sum data rate in a crude manner of "continuously increasing the number of activated DLs". Evidently, there must exist a maximum[4] (tolerable)

[4]Here we must emphasize that the maximum (tolerable) number of activated DLs may not necessarily correspond to the maximum sum data rate. As compared to this marginal condition, adopting an appropriately lower density of DLs may be more beneficial to optimizing the sum data rate, as proven in the next subsection.

number of activated DLs. If the maximum (tolerable) number of activated DLs in terms of link's quality is met in the underlaying CNs, further increasing the activated DLs (i.e., make it be beyond this maximum number) will definitely erode the sum data rate, because the received SINRs at both BS and DRs may no longer satisfy the minimum QoS requirements of CL and DLs, respectively.

In order to achieve the above-mentioned goals, we must implement some appropriate schemes such as resource allocation and power control. Before doing that, let us first denote by N_c and N_d the CLs and DLs of interest, respectively. To maximize the number of connections while simultaneously satisfying the SINR threshold of both links, we may formulate the optimization problem as:

$$\max_{P_s,\lambda_d} N_c + N_d$$
$$s.t. \begin{cases} C1 : 1-\gamma_c < P_{\mathrm{cov}}^{\mathrm{C}} < 1; \\ C2 : 1-\gamma_d < P_{\mathrm{cov}}^{\mathrm{D2D}} < 1. \\ C3 : 0 \le P_s \le P_{\max} \end{cases} \tag{21}$$

where γ_c and γ_d denote the thresholds of outage probabilities of CLs and DLs, respectively, and P_s may denote either P_c or P_d [40]. Here, the expression $(1-\gamma_c)$ represents the minimum probability of establishing the link that is accepted by each user. In other words, if the coverage probability is lower than $(1-\gamma_c)$, the link is not allowed to be established.

The goal of the above-mentioned optimization problem is to maximizing the number of connections by both choosing an appropriate number of activated DLs and adjusting the transmit power of each user. Furthermore, the constraints $C1$ and $C2$ must be met simultaneously for satisfying the probability threshold in terms of the successful creation of DLs. In addition, constraint $C3$ gives out the allowed transmit power of DUs in the proposed formulation.

Under constraints $C1$ and $C2$, we can give out the upper-bound of λ_d:

$$\begin{cases} \lambda_d \le \dfrac{1+(1-\gamma_c)\,\mathcal{P}_{\mathcal{L}}[-\frac{e^{-1/(1-\gamma_c)}}{1-\gamma_c}]}{A'\sqrt{\beta_c}\,(1-\gamma_c)} = \lambda_1; \\ \lambda_d \le -\dfrac{\ln[(1-\gamma_d)(1+B\beta_d)]}{C'\sqrt{\beta_d}} = \lambda_2; \end{cases} \tag{22}$$

where $A' = \frac{\pi^2 R^2}{2}\sqrt{\frac{P_i}{P_c}}$, $C' = \frac{\pi^2}{2} d_{j,j}^2$. In addition, $\mathcal{P}_{\mathcal{L}}$, whose integration interval is $\left[\frac{-1}{e}, +\infty\right]$, denotes the inverse function of $f(x) = xe^x$. Since $\mathcal{P}_{\mathcal{L}}$ is not a Liouville integrable function, we cannot obtain its closed-form solution relying on any elementary method. However, we can still conclude that the upper bounds for the density of total users must satisfy $\lambda_d^{up} \le \min[\lambda_1, \lambda_2]$.

In light of the fact that only one CU is accommodated by an individual sub-channel, the above-mentioned optimization problem can be transformed into the problem of "maximizing the number of activated DLs (i.e., $N_d = \pi R^2 \lambda_d P_{\mathrm{cov}}^{\mathrm{D2D}}$)". By substituting (16) into N_d, the objective function can be rewritten as:

Algorithm 1 Algorithm for Calculating the Maximum Number of DLs

Steps for Calculating the Maximum Number of DLs

Initialize: $\lambda_0 = 0.001, R, P, d, \varepsilon$;
  Computer: λ_2, ξ; then
    Substitute λ_2 into $P_{\mathrm{cov}}^{\mathrm{C}}$
  if $1-\gamma_c < P_{\mathrm{cov}}^{\mathrm{C}} < 1$; then
    $\lambda_d^{\mathrm{up}} = \lambda_2$;
  else
    $\lambda_d^{\mathrm{up}} = (\lambda_0 + \lambda_2)/2$
    Substitute λ_d^{up} into $P_{\mathrm{cov}}^{\mathrm{C}}$
    if $1-\gamma_c < P_{\mathrm{cov}}^{\mathrm{C}} < 1$; then
      λ_2 is the Maximum Number of DLs, i.e., $\lambda_d^{\mathrm{up}} = \lambda_2$;
    else
      while ($|\lambda_2 - \lambda_0| > \varepsilon$)
        if $1-\gamma_c < P_{\mathrm{cov}}^{\mathrm{C}} < 1$; then
          $\lambda_0 = \lambda_d^{\mathrm{up}}$;
        else
          $\lambda_2 = \lambda_d^{\mathrm{up}}$;
        endif
        $\lambda_d^{\mathrm{up}} = (\lambda_0 + \lambda_2)/2$;
    endif
  endif
  Maximum Number of DLs $\lambda_d^{max} = \min\left[\frac{1}{\xi}, \lambda_d^{\mathrm{up}}\right]$

$$\max_{\lambda_d} N_d = \lambda_d \zeta \exp(-\xi\lambda_d), \tag{23}$$

where $\xi = \frac{\pi^2 d_{j,j}^2}{2}\sqrt{\frac{\beta_d P_i}{P_j}}$ and $\zeta = \frac{\pi R^2}{1+\frac{\beta_d P_c d_{j,j}^{\alpha}}{P_j \mathbb{E}[d_{c,j}]^{\alpha}}}$. Note that the constraints of this optimization is similar to that of (21).

Following (23), we can identify the optimal value of λ_d by taking the first derivative to the cost function with respect to λ_d, provided that the objective function has an unique optimum point (although this function is not concave). We can thus obtain:

$$\frac{\partial N_d}{\partial \lambda_d} = \zeta \exp(-\xi\lambda_d)(1-\xi\lambda_d). \tag{24}$$

Evidently, $\lambda_d = \frac{1}{\xi}$ is the solution of the above equation. Considering the constraints of the proposed formulation, the maximum connections of (simultaneously activating) DLs can be expressed:

$$\lambda_d^{max} = \min\left[\frac{1}{\xi}, \lambda_d^{\mathrm{up}}\right]. \tag{25}$$

C. OPTIMAL NUMBER OF D2D LINKS

As emphasized in the former subsection, adopting the maximum number of DLs may not necessarily correspond to the maximum sum data rate. In fact, the system working with the maximum number of DLs usually does not maximize the sum data rate due to the impact of a severer interference imposed by these excessively activating DLs. Evidently, there should exist an "optimal" number of DLs corresponding to the maximum sum data rate, as investigated in the following.

Let us analyze the performance of the proposed D2D-aided underlaying CNs in a scenario that allows us to continuously increase the density of DLs. We start at the condition that the density of activated DLs is zero (i.e., all the activated users are CUs). Obviously, by gradually increasing the number of activated DLs, the sum data rate of this underlaying system can be improved in the low-DL-density scenarios. Note that the interference imposed by these on-activating DLs will also increase accordingly. This DL-induced interference will become the main constraint factor for preventing the growth of the sum data rate. The sum data rate will continuously increase until an "optimal" number of activated DLs is touched.

As shown in (20), both the distance $d_{j,j}$ and the transmit power (i.e. P_c and P_d) are assumed to be kept fixed for a given cellular radius R, in which case the sum data rate will be determined solely by the DUs' density λ_d. To obtain the optimal number of activated DLs in terms of the maximum sum data rate, the objective function of the proposed optimization task can be formulated:

$$\max_{\lambda_d} \frac{2}{A\ln 2}\left(\frac{\pi}{2}-f(A)\right) + \frac{2\pi R^2\lambda_d}{(B-1)\ln 2}\left(g(C)-g\left(\frac{C}{\sqrt{B}}\right)\right)$$
$$s.t. \begin{cases} C1: 1-\gamma_c < P_{\text{cov}}^{\text{C}} < 1; \\ C2: 1-\gamma_d < P_{\text{cov}}^{\text{D2D}} < 1; \\ C3: 0 \le P \le P_{\max}. \end{cases} \quad (26)$$

where $f(x) = \text{Ci}(x)\sin x - \text{si}(x)\cos x$ and $g(x) = \text{Ci}(x)\cos x + \text{si}(x)\sin x$, with their first-order derivatives:

$$\begin{cases} f'(x) = g(x) \\ g'(x) = \dfrac{1}{x} - f(x) \end{cases} \quad (27)$$

Proof: Please refer to Appendix B for details.

To maximize the sum data rate by identifying the optimal number of activated DLs, we may take the first and second-order derivatives, respectively, to R_{sum} in terms of λ_d, thus leading to:

$$\frac{\partial R^{\text{sum}}}{\partial \lambda_d} = \frac{2}{\ln 2}\left[-\frac{1}{A'\lambda_d^2}\left(\frac{\pi}{2}-f(A)\right)-\frac{g(A)}{\lambda_d} + \frac{\pi R^2}{(B-1)}\left(g(C)-g\left(\frac{C}{\sqrt{B}}\right)+\frac{Cf\left(\frac{C}{\sqrt{B}}\right)}{\sqrt{B}}-Cf(C)\right)\right] \quad (28)$$

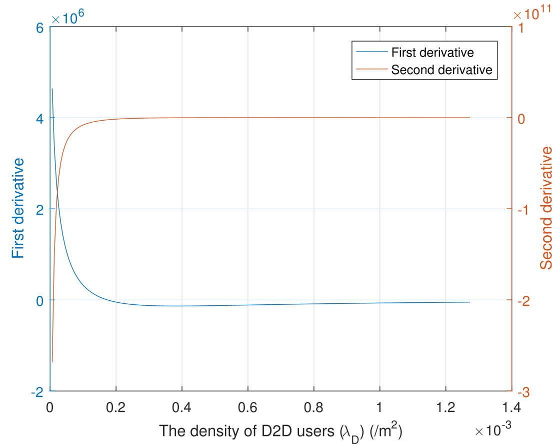

FIGURE 3. The first and second-order derivatives as functions of DUs' density.

and

$$\frac{\partial^2 R^{\text{sum}}}{\partial \lambda_d^2} = \frac{2}{\ln 2}\left[\frac{\pi}{A'\lambda_d^3}-\frac{2f(A)}{A'\lambda_d^3}+\frac{2g(A)}{\lambda_d^2}-\frac{1}{\lambda_d^2}+\frac{A'f(A)}{\lambda_d} + \frac{\pi R^2 C''}{(B-1)}\mathcal{T}(B,C)\right], \quad (29)$$

respectively, where $\mathcal{T}(B,C) = -2f(C)+\frac{2f\left(\frac{C}{\sqrt{B}}\right)}{\sqrt{B}}+\frac{Cg\left(\frac{C}{\sqrt{B}}\right)}{B}-Cg(C)$, and $C'' = \frac{\pi^2 d_{j,j}^2}{2}\sqrt{\frac{P_i}{P_j}}$.

According to (28), it would be very hard to give out the exact closed-form solution of $\frac{\partial R^{\text{sum}}}{\partial \lambda_d} = 0$, if not impossible. Fortunately, we may employ the graphic rather than algebraic method to address this issue. As illustrated in Fig.3, the dotted curve corresponds to the first derivative to R^{sum} in terms of λ_d.[5] The first derivative of R^{sum} is shown to have only null point (i.e., the density of DLs - λ_d^{null} - is attainable), which guarantees that the specific value to be identified can be calculated by using dichotomy. As revealed in the solid curve in Fig.3, on the other hand, R^{sum} is shown to be a monotonically increasing function of density of DLs (i.e., the value of this function cannot always be negative). However, by taking the second-order derivative to R^{sum}, the result will be negative at the point that the first-order derivative of this function is zero. To prove it, we can first identify the value of $\frac{\partial R^{\text{sum}}}{\partial \lambda_d} = 0$ by using dichotomy, followed by substituting this specific value into (29) to determine the sign of the cost function. In other words, there must exist an optimal number of users (i.e., comprising both CUs and DUs) in terms of sum data rate. The steps for determining this optimal value are described as follows:

Based on the above-mentioned analysis, the optimal density of activated DLs can be calculated:

$$\lambda_d^{\text{opt}} = \min[\lambda_d^{\text{null}}, \lambda_d^{\max}]. \quad (30)$$

In summary, as the density of activated DLs increases, the radio environment of the proposed underlaying CNs will

[5] Since this function is continuous and smooth, it is derivable.

Algorithm 2 Algorithm for Calculating the Optimal Number of DLs

Steps for Calculating the Optimal Number of DLs

Initialize: $a = 0.001$, $b = 1000^1$, ε;
 if $\frac{\partial R^{\text{sum}}}{\partial \lambda_d}|_{\lambda_d=a} \cdot \frac{\partial R^{\text{sum}}}{\partial \lambda_d}|_{\lambda_d=b} < 0$; then
  Substitute $c = (a+b)/2$ into $\frac{\partial R^{\text{sum}}}{\partial \lambda_d}|_{\lambda_d=c}$
  if $\frac{\partial R^{\text{sum}}}{\partial \lambda_d}|_{\lambda_d=c} = 0$; then
   c is the desired zero point, i.e., $\lambda_d^{\text{null}} = c$;
  else
   while ($|a-b| > \varepsilon$)
    if $\frac{\partial R^{\text{sum}}}{\partial \lambda_d}|_{\lambda_d=a} \cdot \frac{\partial R^{\text{sum}}}{\partial \lambda_d}|_{\lambda_d=c} < 0$; then
     $b = c$;
    else
     $a = c$;
    endif
   $\lambda_d^{\text{null}} = (a+b)/2$;
  endif
 endif
 Substitute λ_d^{null} into $\frac{\partial^2 R^{\text{sum}}}{\partial^2 \lambda_d}|_{\lambda_d=c}$;
 if λ_d^{null} satisfies $\frac{\partial^2 R^{\text{sum}}}{\partial^2 \lambda_d}|_{\lambda_d=c} < 0$
  λ_d^{null} is the optimal value of DLs.
 endif

[1] Note that the numbers a and b denote the upper and lower boundaries of λ_d's interval, respectively. Here, we employ $a = 0.001$ to approximately represent the lower-bound value that tends to 0, while employ $b = 1000$ to approximately represent the upper-bound value that tends to ∞.

change accordingly. On the one hand, the system's capacity becomes greater by activating more DLs (provided that the optimal number of DLs has not been touched); on the other hand, the DL-induced interference will become severer. However, beyond the optimal number of DLs, the system's capacity will decrease, i.e., $R_{K+1}^{\text{SUM}} < R_K^{\text{SUM}}$, where K represents the optimal number of activated DLs. Furthermore, both the constraints $\text{SINR}_o > \beta_c$ and $\text{SINR}_i > \beta_d$ must be satisfied[6] for guaranteeing the required SINRs of both CLs and DLs. In practical scenarios, we can attain the optimal solution of problems (26) and (23) by implementing the algorithm below:

V. NUMERICAL ANALYSIS

In this section, we assume that each sub-channel is allowed to be reused by multiple users, comprising one CU as well as N_d (i.e., $\lambda_d \pi R^2$) D2D pairs. Next, let us evaluate the performance of the proposed algorithms via simulation. The simulation parameters are elaborated on in Table 1.

In Fig.4, the sum data rate as a function of DUs' density is evaluated by considering variant power ratios (i.e., $k = P_i/P_j$). It is shown that both the maximum number of activated DLs reusing the same sub-channel and the optimal number of activated DLs in terms of the sum data rate exist

[6] Of course, the number of activated DLs in this case corresponds to the maximum rather than optimal number of activated DLs that reuse the same spectrum. In particular, if the required QoS in either CL or DLs cannot be satisfied, the maximum instead of optimal number of DLs can be employed.

Algorithm 3 Simulation Algorithm for Calculating the Optimal/Maximum Numbers of DLs

Steps of Simulation Algorithm

Initialize: M=1; L=1000[1]
 for K=1: L
  for j=1: K
   if $\text{SINR}_o > \beta_c$ and $\text{SINR}_j > \beta_d$, then
    $N^{max} = K$;
    if $R_K^{\text{SUM}} > R_{K-1}^{\text{SUM}}$, then
    $N^{\text{opt}} = K$;
    endif
   elseif break;
  endfor
 endfor

[1] Note that L signifies the number of cycles for finding the maximum number of activated DLs. Here, L can be set to be infinity. The algorithm is forced to exit whenever this infinity condition is met. If the (calculated) maximum number of activated DLs is beyond L, we can increase L to suit for the needs of this algorithm.

TABLE 1. Simulation parameters for the proposed analysis.

Symbol	Parameters	Value
R	Cell radius	500m
P_c	Transmit power of CU	100mW
P_j	Transmit power of DT	10mW
P_i	Transmit power of DT	$P_i = k \times P_j$
$d_{j,j}$	Distance of DT-to-DR	50m
α	Pathloss exponential	4
σ^2	AWGN noise power	-174dBm/Hz

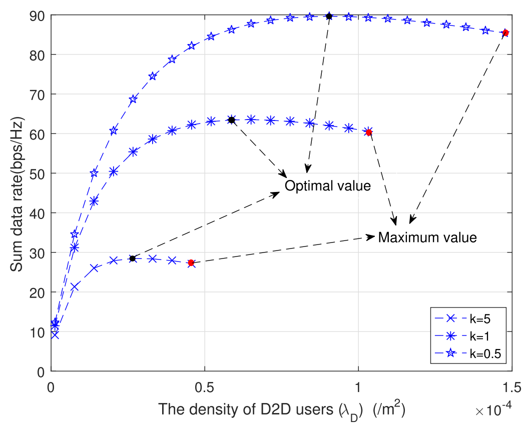

FIGURE 4. Sum data rate as a function of the DLs' density (i.e., λ_d) by considering variant power ratios k, where we assumed that $\gamma = 0.8$ and $\beta = -2.5$db.

and can be found. As k increases, either the maximum or the optimal number of activated DLs decreases. In other words, we may adaptively adjust the DUs' transmit power for achieving the optimal/maximum number of activated DLs.

The number of activated DLs as a function of SINR threshold is described in Fig.5, where we assumed that $k = 0.5; 1; 5$. As the SINR threshold increases, the number of maximum/optimal DLs decreases. We can explain it as follows: without changing the received signal at the DR, increasing the SINR threshold corresponds to decreasing the DUs' interference-withstanding ability. In other words, a higher

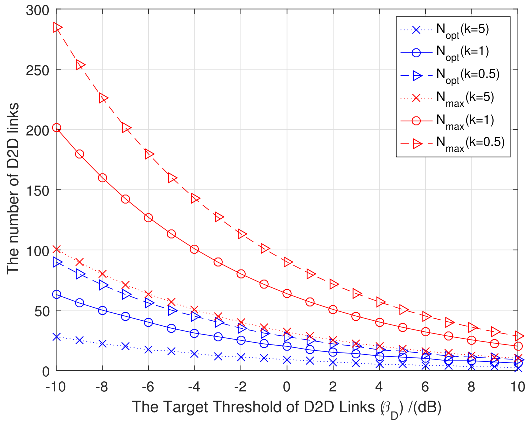

FIGURE 5. **Relationship between the number of activated DLs and SINR threshold with variant power ratio k.**

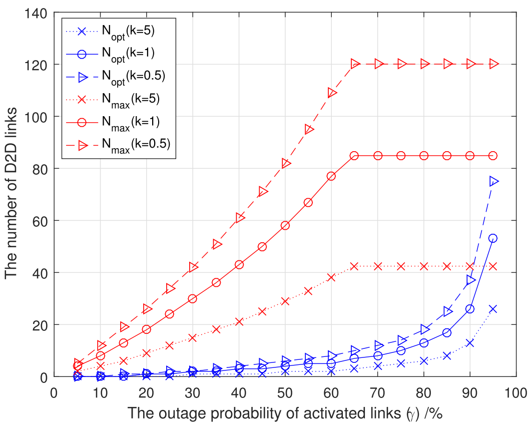

FIGURE 6. **Relationship between the number of activated DLs and outage probability with variant power ratio k.**

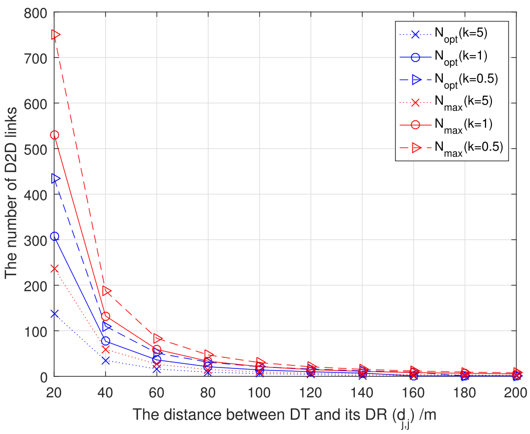

FIGURE 7. **Relationship between the number of activated DLs and the DT-to-DR distance $d_{j,j}$, in which we assumed that $\gamma = 0.8$ and $\beta = -2.5$db.**

SINR threshold usually corresponds to a lower anti-jamming capability of DLs. Furthermore, like in Fig.4, the number of maximum/optimal activated DLs is also shown to decrease as k increases.

To more effectively optimize both the maximum and optimal numbers of activated DLs, the total number of (supportable) DLs as a function of outage probability in these links

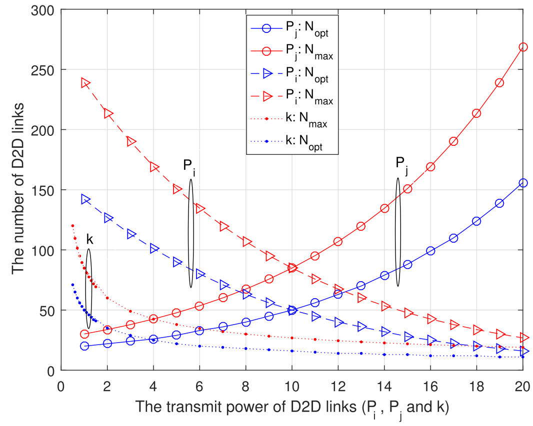

FIGURE 8. **Relationship between number of activated DLs and the transmit power P_j, P_i and $k = P_j/P_i$, in which we assumed that $\gamma = 0.8$ and $\beta = -2.5$db.**

(i.e., γ) must be computed. For a given γ, as illustrated in Fig.6, there always exists an optimum/maximum number of activated DLs in terms of sum data rate. Obviously, increasing γ corresponds to improving the number of maximum/optimal activated DLs. It is worth noting that the maximum number of activated DLs tends to approach a fixed value, because, beyond 65%, further increasing γ will contribute a negligible performance under the condition $\frac{1}{a} > \lambda_d^{up}$.

In Fig.7, the relationship between the number of maximum/optimal activated DLs and the DT-to-DR distance (say, $d_{j,j}$) is revealed. Obviously, the greater $d_{j,j}$, the lower the former, because increasing $d_{j,j}$ may erode the useful signal at the receiver.

In Fig.8, the number of maximum/optimal activated DLs as a function of the transmit power (including P_j, P_i and $k = P_j/P_i$) is illustrated. Taking the j-th DL as the reference DL, it is shown that a greater transmit power ratio k corresponds to a lower number of maximum/optimal activated DLs, because $k = P_j/P_i$ is a function of both powers. Furthermore, the solid and the dotted curves denote the changing trends of the maximum/optimal numbers with powers P_j and P_i, respectively. Of course, increasing P_j for the j-th DL will definitely improve the received power at the receiver, but this operation will as well increase the interference imposed on the neighboring DLs. Consequently, the increased interference will definitely erode the benefit that was brought about by increasing the transmit power.

VI. CONCLUSION

In this paper, we investigated the maximum/optimal number of activated DLs in terms of the sum data rate of the proposed D2D-aided underlaying CNs. The optimal and maximum numbers of activated DLs correspond to the maximum capacity and the minimum QoS requirements of the users, respectively, were identified. It was shown that there always exist the maximum/optimum number of activated DLs on terms of sum data rate. Furthermore, the closed-form expressions of the coverage probabilities of activated DLs were derived, followed by analyzing the sum data rate of the proposed underlaying CNs. By employing algorithms that can adaptively adjust some critical parameters, including the DUs' transmit power, the SINR threshold and the outage probability of CLs/DLs, etc, we can always obtain the maximum/optimum number of activated DLs in practical systems.

APPENDIX A
THE LAPLACE TRANSFORM OF RANDOM VARIABLES I_d

We may calculate $\mathcal{L}_{I_d}(s)$ as follows:

$$\begin{aligned}\mathcal{L}_{I_d}(s) &\triangleq \mathbb{E}_{I_d}\left[\exp\left(-s\sum_{i\in\Phi_1} P_i d_{i,b}^{-\alpha}\left|h_{i,b}\right|^2\right)\right]\\ &= \mathbb{E}_{\Phi_1}\left[\prod_{i\in\Phi_1}\mathbb{E}_{d_{i,b}}\left[\exp\left(-sP_i d_{i,b}^{-\alpha}\left|h_{i,b}\right|^2\right)\right]\right]\\ &\underline{\underline{(a)}}\ \exp\left[-\lambda_d\int_{\mathbb{R}^2}\left(1-\mathbb{E}\left[e^{-sP_i t^{-\alpha}\left|h_{i,b}\right|^2}\right]\right)dt\right]\\ &\underline{\underline{(b)}}\ \exp\left[-2\pi\lambda_d\int_0^\infty\left(1-\mathbb{E}\left[e^{-sP_i t^{-\alpha}\left|h_{i,b}\right|^2}\right]\right)tdt\right]\\ &\underline{\underline{(c)}}\ \exp\left[-2\pi\lambda_d\int_0^\infty\left(\frac{sP_i t^{-\alpha+1}}{1+sP_i t^{-\alpha}}\right)dt\right], \qquad (31)\end{aligned}$$

where $t = d_{i,b}$. Note that we have used the following equations:

(a) $\mathbb{E}_\Phi\left[\prod_{i=\Phi} f(x)\right] = \exp\left[-\lambda\int_{\mathbb{R}^2}(1-f(x))dx\right]$

(b) $\int_{\mathbb{R}^2} f(x)dx = 2\pi\int_0^\infty xf(x)dx$,

(c) $\delta \to \exp(1) \Longrightarrow \mathbb{E}\left[e^{-\delta A}\right] = \frac{1}{1+A}$,

Relying on [37, eqn.3.241.4], the result in (6) can be obtained.

APPENDIX B
DERIVATIVE RELATIONSHIP BETWEEN $f(x)$ AND $g(x)$ FUNCTIONS

The functions $f(x)$ and $g(x)$ can be expressed:

$$\begin{cases} f(x) = \mathrm{Ci}(x)\sin x - \mathrm{si}(x)\cos x \\ g(x) = \mathrm{Ci}(x)\cos x + \mathrm{si}(x)\sin x \end{cases} \qquad (32)$$

We can take the first-order derivative to $f(x)$ and $g(x)$ in terms of x, showing that:

$$\begin{aligned} f'(x) &= \mathrm{Ci}(x)\cos x + \frac{\cos x}{x}\sin x \\ &\quad + \mathrm{si}(x)\sin x - \frac{\sin x}{x}\cos x \\ &= \mathrm{Ci}(x)\cos x + \mathrm{si}(x)\sin x \\ &= g(x). \qquad (33)\end{aligned}$$

Similarly, by taking the first-order derivative to $g(x)$ in terms of x, we get:

$$\begin{aligned} g'(x) &= \frac{\cos x}{x}\cos x - \mathrm{Ci}(x)\sin x + \frac{\sin x}{x}\sin x + \mathrm{si}(x)\cos x \\ &= \frac{1}{x} - \mathrm{Ci}(x)\sin x + \mathrm{si}(x)\cos x \\ &= \frac{1}{x} - f(x). \qquad (34)\end{aligned}$$

The desired result (27) can thus be obtained.

REFERENCES

[1] X. Cheng, L. Yang, and X. Shen, "D2D for intelligent transportation systems: A feasibility study," *IEEE Trans. Intell. Transp. Syst.*, vol. 16, no. 4, pp. 1784–1793, Aug. 2015.

[2] A. Lo, Y. W. Law, M. Jacobsson, and M. Kucharzak, "Enhanced LTE-advanced random-access mechanism for massive machine-to-machine (M2M) communications," in *Proc. 27th World Wireless Research Forum (WWRF) Meeting WWRF27-WG4-08*, 2011, pp. 1–5.

[3] R. Zhang, X. Cheng, and L. Yang, "Cooperation via spectrum sharing for physical layer security in device-to-device communications underlaying cellular networks," *IEEE Trans. Wireless Commun.*, vol. 15, no. 8, pp. 5651–5663, Aug. 2016.

[4] C. Wu, S. Ohzahata, Y. Ji, and T. Kato, "Joint fuzzy relays and network-coding-based forwarding for multihop broadcasting in VANETs," *IEEE Trans. Intell. Transp. Syst.*, vol. 16, no. 3, pp. 1415–1427, Jun. 2015.

[5] G. Zhang, K. Yang, P. Liu, and J. Wei, "Power allocation for full-duplex relaying-based D2D communication underlaying cellular networks," *IEEE Trans. Veh. Technol.*, vol. 64, no. 10, pp. 4911–4916, Oct. 2015.

[6] Y. Zhou, H. Liu, Z. Pan, L. Tian, J. Shi, and G. Yang, "Two-stage cooperative multicast transmission with optimized power consumption and guaranteed coverage," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 2, pp. 274–284, Feb. 2014.

[7] Z. Zhang, K. Long, A. V. Vasilakos, and L. Hanzo, "Full-duplex wireless communications: Challenges, solutions, and future research directions," *Proc. IEEE*, vol. 104, no. 7, pp. 1369–1409, Jul. 2016.

[8] Z. Zhang, K. Long, and J. Wang, "Self-organization paradigms and optimization approaches for cognitive radio technologies: A survey," *IEEE Wireless Commun.*, vol. 20, no. 2, pp. 36–42, Apr. 2013.

[9] Z. Chen, L. Tang, X. Zhang, and Y. Bai, "Resource allocation in multiple-relay systems exploiting opportunistic energy harvesting," *Sci. China Inf. Sci.*, vol. 61, no. 2, Feb. 2018, Art. no. 029302.

[10] B. Zhong and Z. Zhang, "Opportunistic two-way full-duplex relay selection in underlay cognitive networks," *IEEE Syst. J.*, vol. 12, no. 1, pp. 725–734, Mar. 2018.

[11] J. An, K. Yang, J. Wu, N. Ye, S. Guo, and Z. Liao, "Achieve sustainable ultra-dense heterogeneous networks for 5G," *IEEE Commun. Mag.*, vol. 55, no. 12, pp. 84–90, Dec. 2017.

[12] B. Zhong and Z. Zhang, "Secure full-duplex two-way relaying networks with optimal relay selection," *IEEE Commun. Lett.*, vol. 21, no. 5, pp. 1123–1126, May 2017.

[13] R. Zhang, Y. Li, C.-X. Wang, Y. Ruan, and H. Zhang, "Energy efficient power allocation for underlaying mobile D2D communications with peak/average interference constraints," *Sci. China Inf. Sci.*, vol. 61, no. 8, Mar. 2018, Art. no. 089301.

[14] S. Andreev, A. Pyattaev, K. Johnsson, O. Galinina, and Y. Koucheryavy, "Cellular traffic offloading onto network-assisted device-to-device connections," *IEEE Commun. Mag.*, vol. 52, no. 4, pp. 20–31, Apr. 2014.

[15] J. Sun, Z. Zhang, H. Xiao, and C. Xing, "Uplink interference coordination management with power control for D2D underlaying cellular networks: Modeling, algorithms, and analysis," *IEEE Trans. Veh. Technol.*, vol. 67, no. 9, pp. 8582–8594, Sep. 2018.

[16] K. Doppler, M. Rinne, C. Wijting, C. Ribeiro, and K. Hugl, "Device-to-device communication as an underlay to LTE-advanced networks," *IEEE Commun. Mag.*, vol. 47, no. 12, pp. 42–49, Dec. 2009.

[17] C. Xu, P. Zeng, W. Liang, and H. Yu, "Secure resource allocation for green and cognitive device-to-device communication," *Sci. China Inf. Sci.*, vol. 61, no. 2, Feb. 2018, Art. no. 029305.

[18] G. Wang, Q. Liu, R. He, F. Gao, and C. Tellambura, "Acquisition of channel state information in heterogeneous cloud radio access networks: Challenges and research directions," *IEEE Wireless Commun.*, vol. 22, no. 3, pp. 100–107, Jun. 2015.

[19] X. Lin, J. Andrews, A. Ghosh, and R. Ratasuk, "An overview of 3GPP device-to-device proximity services," *IEEE Commun. Mag.*, vol. 52, no. 4, pp. 40–48, Apr. 2014.

[20] J. Sun, Z. Zhang, C. Xing, and H. Xiao, "Uplink resource allocation for relay-aided device-to-device communication," *IEEE Trans. Intell. Transp. Syst.*, vol. 19, no. 12, pp. 3883–3892, Dec. 2018.

[21] H. Min, W. Seo, J. Lee, S. Park, and D. Hong, "Reliability improvement using receive mode selection in the device-to-device uplink period underlaying cellular networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 2, pp. 413–418, Feb. 2011.

[22] Z. Zhang, K. Long, J. Wang, and F. Dressler, "On swarm intelligence inspired self-organized networking: Its bionic mechanisms, designing principles and optimization approaches," *IEEE Commun. Surveys Tuts.*, vol. 16, no. 1, pp. 513–537, 1st Quart., 2014.

[23] S. W. H. Shah, M. M. U. Rahman, A. N. Mian, A. Imran, S. Mumtaz, and O. A. Dobre, "On the impact of mode selection on effective capacity of device-to-device communication," *IEEE Wireless Commun. Lett.*, vol. 8, no. 3, pp. 945–948, Jun. 2019.

[24] H. Xiao, Y. Hu, K. Yan, and S. Ouyang, "Power allocation and relay selection for multisource multirelay cooperative vehicular networks," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 11, pp. 3297–3305, Nov. 2016.

[25] H. Min, J. Lee, S. Park, and D. Hong, "Capacity enhancement using an interference limited area for device-to-device uplink underlaying cellular networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 12, pp. 3995–4000, Dec. 2011.

[26] R. Yin, G. Yu, H. Zhang, Z. Zhang, and G. Y. Li, "Pricing-based interference coordination for D2D communications in cellular networks," *IEEE Trans. Wireless Commun.*, vol. 14, no. 3, pp. 1519–1532, Mar. 2015.

[27] C. Ma, J. Liu, X. Tian, H. Yu, Y. Cui, and X. Wang, "Interference exploitation in D2D-enabled cellular networks: A secrecy perspective," *IEEE Trans. Commun.*, vol. 63, no. 1, pp. 229–242, Jan. 2015.

[28] J. Liu, Y. Kawamoto, H. Nishiyama, N. Kato, and N. Kadowaki, "Device-to-device communications achieve efficient load balancing in LTE-advanced networks," *IEEE Wireless Commun.*, vol. 21, no. 2, pp. 57–65, Apr. 2014.

[29] C.-Y. Oh, D. Hwang, and T.-J. Lee, "Joint access control and resource allocation for concurrent and massive access of M2M devices," *IEEE Trans. Wireless Commun.*, vol. 14, no. 8, pp. 4182–4192, Aug. 2015.

[30] M. Pischella, R. Zakaria, and D. Le Ruyet, "On maximum D2D multiplexing in asynchronous communications," in *Proc. IEEE 87th Veh. Technol. Conf. (VTC Spring)*, Jun. 2018, pp. 1–5.

[31] N. Lee, X. Lin, J. G. Andrews, and R. W. Heath, "Power control for D2D underlaid cellular networks: Modeling, algorithms, and analysis," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 1, pp. 1–13, Jan. 2015.

[32] H. A. Mustafa, M. Z. Shakir, M. A. Imran, A. Imran, and R. Tafazolli, "Coverage gain and device-to-device user density: Stochastic geometry modeling and analysis," *IEEE Commun. Lett.*, vol. 19, no. 10, pp. 1742–1745, Oct. 2015.

[33] X. Zhang, Y. Wang, R. Wang, Y. Hou, and D. Wang, "Group-based joint signaling and data resource allocation in MTC-underlaid cellular networks," *Sci. China Inf. Sci.*, vol. 60, no. 10, Oct. 2017, Art. no. 100304.

[34] J. Sun, T. Liu, X. Wang, C. Xing, H. Xiao, A. V. Vasilakos, and Z. Zhang, "Optimal mode selection with uplink data rate maximization for D2D-aided underlaying cellular networks," *IEEE Access*, vol. 4, pp. 8844–8856, 2016.

[35] S. Mumtaz, K. M. Saidul Huq, J. Rodriguez, and V. Frascolla, "Energy-efficient interference management in LTE-D2D communication," *IET Signal Process.*, vol. 10, no. 3, pp. 197–202, May 2016.

[36] P. Sun, K. G. Shin, H. Zhang, and L. He, "Transmit power control for D2D-underlaid cellular networks based on statistical features," *IEEE Trans. Veh. Technol.*, vol. 66, no. 5, pp. 4110–4119, May 2017.

[37] A. Jeffrey and D. Zwillinger, *Table of Integrals, Series, and Products*, 4th ed. New York, NY, USA: Academic, 2007.

[38] J. G. Andrews, F. Baccelli, and R. K. Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Trans. Commun.*, vol. 59, no. 11, pp. 3122–3134, Nov. 2011.

[39] D. Moltchanov, "Distance distributions in random networks," *Ad Hoc Netw.*, vol. 10, no. 6, pp. 1146–1166, Aug. 2012.

[40] R. F. Guiazon, K.-K. Wong, and M. Fitch, "Coverage probability of cellular networks using interference alignment under imperfect CSI," *Digit. Commun. Netw.*, vol. 2, no. 4, pp. 162–166, Nov. 2016.

JIAN SUN received the B.S. and M.S. degrees in engineering from Northeast Petroleum University, in 2004 and 2011, respectively. He is currently pursuing the D.S. degree with the University of Science and Technology Beijing (USTB). His research interests are in the theory of wireless communication and in the modeling and performance evaluation of communication systems. His main research interests include the interference coordination management and power control for D2D communication systems.

ZHIGUANG LI received the B.E. degree in electronic engineering from Northeast Petroleum University, in 2004, and the M.E. degree in geological engineering from Yangtze University, in 2015. He joined CNPC Bohai Drilling Engineering Company Limited, since 2004, as a Research and Development Engineer, and was promoted to be a Senior Engineer, in 2016. His main research interests include wireless communication in underground environment, logging while drilling (LWD), measurement while drilling (MWD), and digital signal processing.

ZHONGSHAN ZHANG (Senior Member, IEEE) received the B.E. and M.S. degrees in computer science from the Beijing University of Posts and Telecommunications (BUPT) in 1998 and 2001, respectively, and the Ph.D. degree in electrical engineering from BUPT, in 2004. Since August 2004, he joined DoCoMo Beijing Laboratories as an Associate Researcher, and was promoted to be a Researcher, in December 2005. Since February 2006, he joined the University of Alberta, Edmonton, AB, Canada, as a Postdoctoral Fellow. Since April 2009, he joined the Department of Research and Innovation (R&I), Alcatel-Lucent, Shanghai, as a Research Scientist. Since August 2010 to July 2011, he worked in NEC China Laboratories, as a Senior Researcher. He served or is serving as a Guest Editor and/or an editor for several technical journals, such as the *IEEE Communications Magazine and KSII Transactions on Internet and Information Systems*. He is currently a Professor with the School of Information and Electronics, Beijing Institute of Technology (BIT). His main research interests include statistical signal processing, full-duplex communications, self-organized networking, cognitive radio, and cooperative communications.

CHENGWEN XING received the B.Eng. degree from Xidian University, Xi'an, China, in 2005, and the Ph.D. degree from the Department of Electrical and Electronic Engineering, The University of Hong Kong, Hong Kong, in 2010. Since September 2010, he has been with the School of Information and Electronics, Beijing Institute of Technology (BIT), Beijing, China, where he is currently an Associate Professor. His current research interests include statistical signal processing, convex optimization, multivariate statistics, combinatorial optimization, and cooperative communication systems.