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Polynomials of Degree-Based Indices for Swapped Networks Modeled by Optical Transpose Interconnection System

ALI AHMAD[1], ROSLAN HASNI[2], KASHIF ELAHI[2,3], AND MUHAMMAD AHSAN ASIM[1]
[1]College of Computer Science and Information Technology, Jazan University, Jazan 45142, Saudi Arabia
[2]School of Informatics and Applied Mathematics, Universiti Malaysia Terengganu, Kuala Terengganu 21030, Malaysia
[3]Deanship of E-learning and Information Technology, Jazan University, Jazan 45142, Saudi Arabia

Corresponding author: Roslan Hasni (hroslan@umt.edu.my)

ABSTRACT The Optical Transpose Interconnection System (OTIS) has applications in parallel processing, distributed processing, routing, and networks. It is used for efficient usage of multiple parallel algorithms or parallel systems, with different global interconnections in a network as it is an optoelectronic (combination of light signals and electronics). In chemical graph theory, topological indices are used to study characteristics of the chemical structures or biological activities. Topological indices are sometimes studied with the assistance to their polynomial. In this article, polynomials of degree-based topological indices for OTIS and swapped networks have been studied. Results can be used to compute any degree-based topological polynomials for OTIS swapped network.

INDEX TERMS Topological polynomials, degree-based index, optical transpose interconnection system.

I. INTRODUCTION AND PRELIMINARY RESULTS

Graph theory has been proved as a vast field by solving problems in multiple fields, like chemistry, physics, computer science, statistics, robotics, networks and routing. Graph can be represented numerically in different forms of data container like matrices, vectors and polynomials. Different real life problems can be represented with the help of a graph. Graph theory provides further operations to find the solution of a problem. Chemical graph theory has further branches quantitative structure-property relationship (QSPR) and quantitative structure-activity relationship (QSAR), which are essential part in studying the characteristics or chemical properties of molecules and atoms [1]–[4].

Topological index is a single value used to represent the characteristics of the graph. It is invariant under graph automorphism. Topological indices have played an important part in the study of chemical properties under the branches of graph theory QSPR and QSAR. They are used to correlate biological activity or other properties of molecules with their chemical structure. Weiner Index was the first topological index, introduced by Harry Weiner in 1947. Topological indices can be categorized on the bases of their calculation mechanism. Degree based topological indices involves degree of vertices of graph in the calculation. Randić index, Zagreb index, harmonic index, atom bond connectivity and geometric-arithmetic index are some known degree-based topological indices and polynomials [5]–[17].

OTIS is an optoelectronic. In a network, it provides competency in both optical and electronic technologies. Multiple groups are connected efficiently, electronic connections are used within the same group while optical links are used to communicate between different groups. Multiple algorithms are used for routing, image processing, parallel processing, matrix multiplication (hybrid network model), sorting, selecting and fourier transform (sound and signals) [18]–[20]. A network can be represented graphically. Servers and processors can be represented by vertices while the connections between them can be represented by edges. The number of links on servers or processors are degree of vertices. The maximum distance between two network heads is grid diameter [21]–[23].

II. DEGREE-BASED INDICES AND THEIR POLYNOMIALS

Let $\mathcal{G}$ be a graph with the vertex set $V(\mathcal{G})$ and the edge set $E(\mathcal{G})$. The degree d_υ of a vertex $\upsilon \in V(\mathcal{G})$ is the number of neighbours of υ. The most general indices based on degrees

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are the general Randić index of a graph $\mathcal{G}$,

$$R_\alpha(\mathcal{G}) = \sum_{\upsilon\nu\in E(\mathcal{G})} (d_\upsilon d_\nu)^\alpha, \tag{1}$$

the general sum-connectivity index

$$\chi_\alpha(\mathcal{G}) = \sum_{\upsilon\nu\in E(\mathcal{G})} (d_\upsilon + d_\nu)^\alpha, \tag{2}$$

and the generalized Zagreb index

$$GZ_{\alpha,\beta}(\mathcal{G}) = \sum_{\upsilon\nu\in E(\mathcal{G})} d_\upsilon^\alpha d_\nu^\beta + d_\nu^\alpha d_\upsilon^\beta, \tag{3}$$

Note that the third redefined Zagreb index is defined as

$$ReZ(\mathcal{G}) = \sum_{\upsilon\nu\in E(\mathcal{G})} d_\upsilon d_\nu (d_\upsilon + d_\nu), \tag{4}$$

the harmonic index is defined as

$$H(\mathcal{G}) = \sum_{\upsilon\nu\in E(\mathcal{G})} \frac{2}{d_\upsilon + d_\nu}. \tag{5}$$

the third Zagreb index

$$M_3(\mathcal{G}) = \sum_{\upsilon\nu\in E(\mathcal{G})} (d_\upsilon - d_\nu) \tag{6}$$

the fourth Zagreb index

$$M_4(\mathcal{G}) = \sum_{\upsilon\nu\in E(\mathcal{G})} d_\upsilon (d_\upsilon + d_\nu) \tag{7}$$

and the fifth Zagreb index

$$M_5(\mathcal{G}) = \sum_{\upsilon\nu\in E(\mathcal{G})} d_\nu (d_\upsilon + d_\nu) \tag{8}$$

Let us introduce a general invariant for polynomials of above mentioned topological indices.

$$P(\mathcal{G}, x) = \sum_{u\nu\in E(\mathcal{G})} x^{\varphi(d_\upsilon, d_\nu)}, \tag{9}$$

where $\varphi(d_\upsilon, d_\nu)$ is a function of d_υ and d_ν such that $\varphi(d_\upsilon, d_\nu) = \varphi(d_\nu, d_\upsilon)$.

- If $\varphi(d_\upsilon, d_\nu) = (d_\upsilon d_\nu)^\alpha$, where α is a positive integer, then $P(\mathcal{G}, x)$ is the general Randić polynomial of $\mathcal{G}$. Moreover, $P(\mathcal{G}, x)$ is the second Zagreb polynomial if $\alpha = 1$.
- If $\varphi(d_\upsilon, d_\nu) = (d_\upsilon + d_\nu)^\alpha$, where α is a positive integer, then $P(\mathcal{G}, x)$ is the general sum-connectivity polynomial of G. Furthermore, $P(\mathcal{G}, x)$ is the first Zagreb polynomial for $\alpha = 1$ and the hyper-Zagreb polynomial for $\alpha = 2$.
- If $\varphi(d_\upsilon, d_\nu) = d_\upsilon^\alpha d_\nu^\beta + d_\nu^\alpha d_\upsilon^\beta$, where α is a positive integer and β is a non-negative integer, then $P(\mathcal{G}, x)$ is the generalized Zagreb polynomial of $\mathcal{G}$. Moreover, $P(\mathcal{G}, x)$ is the forgotten polynomial if $\alpha = 2$ and $\beta = 0$.
- If $\varphi(d_\upsilon, d_\nu) = d_\upsilon d_\nu (d_\upsilon + d_\nu)$, then $P(\mathcal{G}, x)$ is the third redefined Zagreb polynomial of $\mathcal{G}$.
- If $\varphi(d_\upsilon, d_\nu) = d_\upsilon + d_\nu - 1$, then $P(\mathcal{G}, x)$ is one half of the harmonic polynomial $H(\mathcal{G}, x)$ of $\mathcal{G}$. Note that the harmonic polynomial is defined differently from the other polynomials.
- If $\varphi(d_\upsilon, d_\nu) = |d_\upsilon - d_\nu|$, then $P(\mathcal{G}, x)$ is the third Zagreb polynomial of $\mathcal{G}$.
- If $\varphi(d_\upsilon, d_\nu) = d_\upsilon(d_\upsilon + d_\nu)$, then $P(\mathcal{G}, x)$ is the fourth Zagreb polynomial of $\mathcal{G}$.
- If $\varphi(d_\upsilon, d_\nu) = d_\nu(d_\upsilon + d_\nu)$, then $P(\mathcal{G}, x)$ is the fifth Zagreb polynomial of $\mathcal{G}$.

So the general Randić polynomial of any graph $\mathcal{G}$ is defined as

$$R_\alpha(G, x) = \sum_{\upsilon\nu\in E(G)} x^{(d_\upsilon d_\nu)^\alpha}, \tag{10}$$

the general sum-connectivity polynomial is

$$\chi_\alpha(\mathcal{G}, x) = \sum_{\upsilon\nu\in E(\mathcal{G})} x^{(d_\upsilon + d_\nu)^\alpha}, \tag{11}$$

the generalized Zagreb polynomial of any graph $\mathcal{G}$,

$$GZ_{\alpha,\beta}(\mathcal{G}, x) = \sum_{\upsilon\nu\in E(\mathcal{G})} x^{d_\upsilon^\alpha d_\nu^\beta + d_\nu^\alpha d_\upsilon^\beta}, \tag{12}$$

the third redefined Zagreb polynomial is defined as

$$ReZ(\mathcal{G}, x) = \sum_{\upsilon\nu\in E(\mathcal{G})} x^{d_\upsilon d_\nu (d_\upsilon + d_\nu)}, \tag{13}$$

the harmonic polynomial is

$$H(\mathcal{G}, x) = 2 \sum_{\upsilon\nu\in E(\mathcal{G})} x^{d_\upsilon + d_\nu - 1}. \tag{14}$$

the third Zagreb polynomial

$$M_3(\mathcal{G}) = \sum_{\upsilon\nu\in E(\mathcal{G})} x^{|d_\upsilon - d_\nu|} \tag{15}$$

the fourth Zagreb polynomial

$$M_4(\mathcal{G}) = \sum_{\upsilon\nu\in E(\mathcal{G})} x^{d_\upsilon (d_\upsilon + d_\nu)} \tag{16}$$

and the fifth Zagreb polynomial

$$M_5(\mathcal{G}) = \sum_{\upsilon\nu\in E(\mathcal{G})} x^{d_\nu (d_\upsilon + d_\nu)} \tag{17}$$

III. OPTICAL TRANSPOSE INTERCONNECTION SYSTEM SWAPPED NETWORK

Considered a graph G having vertex set $V(G)$ and edge set $E(G)$, OITS swapped network O_G can be defined as below: $V(O_G) = \{(x, y)|x, y \in V(G)\}$ $E(O_G) = \{(x, y_1), (x, y_2)|x \in V(G), (y_1, y_2) \in E(G)\} U\{(x, y), (y, x)|x, y \in E(G), x \neq y\}$ [24].

For the mutual OTIS network O_G, the graph G is called the factor of the graph or grid. If there are m primary network G. So, O_G consists of a separate subnet from the m node groups are called, and they are similar to G [24]. The node name (x, y) in O_G select the y node handle in the group [18]–[20], [25]–[27]. Next some some degree based topological polynomials of swapped networks are calculated. For a given path P_m on m vertices and O_{P_m} as its OTIS swapped network with basis network P_m is shown in Figure 1.

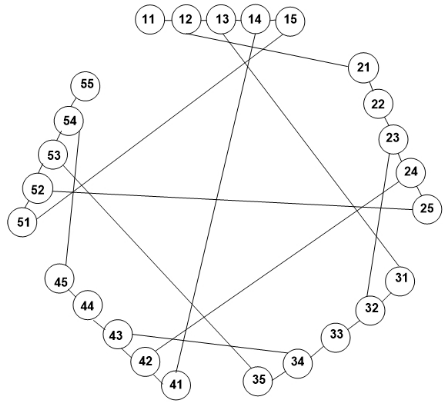

FIGURE 1. OTIS swapped network O_{P_5}.

IV. RESULTS FOR OTIS SWAPPED NETWORK O_{P_m}

We present results, which can be used to compute any degree-based topological polynomials. Our results generalize known results in the area. We give exact values of the most well-known degree-based polynomials for optical transpose interconnection system O_{P_m}. Vetrík [28] introduced a new method to calculate the topological indices and also in [29], we follow the same technique in this paper. Let us give a formula, which can be used to obtain any polynomial of indices based on degrees for optical transpose interconnection system O_{P_m}.

Lemma 1: Let O_{P_m} be a optical transpose interconnection system . Then $P(O_{P_m},x) = \frac{3m^2}{2}x^{\lambda(3,3)} + m\left\{6x^{\lambda(2,3)} - \frac{15}{2}x^{\lambda(3,3)}\right\} + 2x^{\lambda(1,3)} + 3x^{\lambda(2,2)} - 14x^{\lambda(2,3)} + 9x^{\lambda(3,3)}$.

Proof 1: The graph O_{P_m} contains m^2 vertices and $\frac{3m^2-3m}{2}$ edges. Each vertex of O_{P_m} has degree 1,2 or 3, vertices of O_{P_m} can be partitioned according to their degrees. Let

$$V_j = \{\varepsilon \in V(O_{P_m}) : d_\varepsilon = j\}.$$

This means that the set V_j contains the vertices of degree j. The set of vertices with respect to their degrees are as follows:

$$V_1 = \{\varepsilon \in V(O_{P_m}) : d_\varepsilon = 1\}$$
$$V_2 = \{\varepsilon \in V(O_{P_m}) : d_\varepsilon = 2\}$$
$$V_3 = \{\varepsilon \in V(O_{P_m}) : d_\varepsilon = 3\}$$

Since, $|V_1| = 2$, $|V_2| = 3m-4$ and $|V_3| = |V(O_{P_m})| - |V_1| - |V_2| = m^2-2-(3m-4) = m^2-3m+2$. Let us divide the edges of O_{P_m} into partition sets according to the degree of its end vertices. Let

$$\Xi_{1,3} = \{\varepsilon\nu \in E(O_{P_m}) : d_\varepsilon = 1, d_\nu = 3\}$$
$$\Xi_{2,2} = \{\varepsilon\nu \in E(O_{P_m}) : d_\varepsilon = 2, d_\nu = 2\}$$
$$\Xi_{2,3} = \{\varepsilon\nu \in E(O_{P_m}) : d_\varepsilon = 2, d_\nu = 3\}$$
$$\Xi_{3,3} = \{\varepsilon\nu \in E(O_{P_m}) : d_\varepsilon = 3, d_\nu = 3\}.$$

Note that $E(O_{P_m}) = \Xi_{1,3} \sqcup \Xi_{2,2} \sqcup \Xi_{2,3} \sqcup \Xi_{3,3}$. The number of edges incident to one vertex of degree 1 and another vertex of degree 3 is 2, so $|\Xi_{1,3}| = 2$. The number of edges incident to two vertices of degree 2 is 3, so $|\Xi_{2,2}| = 3$. The number of edges incident to one vertex of degree 2 and other vertex of degree 3 are $6m-14$, so $|\Xi_{2,3}| = 6m-14$. Now, the remaining number of edges are those edges which are incident to two vertices of degree 3, i.e $\Xi_{3,3} = |E(O_{P_m})| - \Xi_{1,3} - \Xi_{2,2} - \Xi_{2,3} = \frac{3m^2-3m}{2} - 2 - 3 - (6m-14) = \frac{3m^2-15m+18}{2}$.

Hence, $P(O_{P_m},x) = \sum_{\varepsilon\nu\in E(O_{P_m})} x^{\lambda(d_\varepsilon,d_\nu)} = \sum_{\varepsilon\nu\in\Xi_{1,3}} x^{\lambda(1,3)} + \sum_{\varepsilon\nu\in\Xi_{2,2}} x^{\lambda(2,2)} + \sum_{\varepsilon\nu\in\Xi_{2,3}} x^{\lambda(2,3)} + \sum_{\varepsilon\nu\in\Xi_{3,3}} x^{\lambda(3,3)}$. After Simplification, we get $P(O_{P_m},x) = \frac{3m^2}{2}x^{\lambda(3,3)} + m\left\{6x^{\lambda(2,3)} - \frac{15}{2}x^{\lambda(3,3)}\right\} + 2x^{\lambda(1,3)} + 3x^{\lambda(2,2)} - 14x^{\lambda(2,3)} + 9x^{\lambda(3,3)}$.

Now we present polynomials of the best-known degree based polynomials of optical transpose interconnection system in the following theorem.

Theorem 1: For the optical transpose interconnection system O_{P_m}, we have the general Randić polynomial of O_{P_m},

$R_\alpha(O_{P_m},x) = \frac{3}{2}m^2x^{9^\alpha} + m(6x^{6^\alpha} - \frac{15}{2}x^{9^\alpha}) + 2x^{3^\alpha} + 3x^{4^\alpha} - 14x^{6^\alpha} + 9x^{9^\alpha}$

the second Zagreb polynomial of O_{P_m},

$R_1(O_{P_m},x) = \frac{3}{2}m^2x^9 + m(6x^6 - \frac{15}{2}x^9) + 2x^3 + 3x^4 - 14x^6 + 9x^9$.

Proof 2: For $R_\alpha(O_{P_m},x)$ which is the general Randić polynomial of O_{P_m}, we have $\lambda(d_\varepsilon,d_\nu) = (d_\varepsilon d_\nu)^\alpha$, therefore $\lambda(1,3) = (3)^\alpha$, $\lambda(2,2) = (4)^\alpha$, $\lambda(2,3) = (6)^\alpha$ and $\lambda(3,3) = (9)^\alpha$. Thus by Lemma 1,

$R_\alpha(O_{P_m},x) = \frac{3}{2}m^2x^{9^\alpha} + m(6x^{6^\alpha} - \frac{15}{2}x^{9^\alpha}) + 2x^{3^\alpha} + 3x^{4^\alpha} - 14x^{6^\alpha} + 9x^{9^\alpha}$.

For $\alpha = 1$, the second Zagreb polynomial is

$R_1(O_{P_m},x) = \frac{3}{2}m^2x^9 + m(6x^6 - \frac{15}{2}x^9) + 2x^3 + 3x^4 - 14x^6 + 9x^9$.

In the next theorem, we determined general sum-connectivity polynomial, first Zagreb polynomial and hyper-Zagreb polynomial of the optical transpose interconnection system O_{P_m}.

Theorem 2: For the optical transpose interconnection system O_{P_m}, we have

the general sum-connectivity polynomial of O_{P_m},

$\chi_\alpha(O_{P_m},x) = \frac{3}{2}m^2x^{6^\alpha} + m(6x^{5^\alpha} - \frac{15}{2}x^{6^\alpha}) + 5x^{4^\alpha} - 14x^{5^\alpha} + 9x^{6^\alpha}$,

the first Zagreb polynomial of O_{P_m},

$\chi_1(O_{P_m},x) = \frac{3}{2}m^2x^6 + m(6x^5 - \frac{15}{2}x^6) + 5x^4 - 14x^5 + 9x^6$.

the hyper-Zagreb polynomial of O_{P_m},

$\chi_2(O_{P_m},x) = \frac{3}{2}m^2x^{36} + m(6x^{25} - \frac{15}{2}x^{36}) + 5x^{16} - 14x^{25} + 9x^{36}$.

Proof 3: For $\chi_\alpha(O_{P_m},x)$ which is the general sum-connectivity polynomial of O_{P_m}, we have $\lambda(d_\varepsilon,d_\nu) = (d_\varepsilon + d_\nu)^\alpha$, therefore $\lambda(1,3) = (4)^\alpha$, $\lambda(2,2) = (4)^\alpha$, $\lambda(2,3) = (5)^\alpha$ and $\lambda(3,3) = (6)^\alpha$. Thus by Lemma 1,

$\chi_\alpha(O_{P_m},x) = \frac{3}{2}m^2x^{6^\alpha} + m(6x^{5^\alpha} - \frac{15}{2}x^{6^\alpha}) + 5x^{4^\alpha} - 14x^{5^\alpha} + 9x^{6^\alpha}$

For $\alpha = 1$, the first Zagreb polynomial is $\chi_1(O_{P_m}, x) = \frac{3}{2} m^2 x^6 + m(6x^5 - \frac{15}{2} x^6) + 5x^4 - 14x^5 + 9x^6$,
For $\alpha = 2$, the hyper-Zagreb polynomial is
$\chi_2(O_{P_m}, x) = \frac{3}{2} m^2 x^{36} + m(6x^{25} - \frac{15}{2} x^{36}) + 5x^{16} - 14x^{25} + 9x^{36}$.

In the following theorem, we determined generalized Zagreb polynomial and forgotten polynomial of the optical transpose interconnection system O_{P_m}.

Theorem 3: For the optical transpose interconnection system O_{P_m}, we have

the generalized Zagreb polynomial of O_{P_m},
$GZ_{\alpha,\beta}(O_{P_m}, x) = \frac{3}{2} m^2 x^{(3^\alpha 3^\beta + 3^\alpha 3^\beta)} + m(6x^{(2^\alpha 3^\beta + 3^\alpha 2^\beta)} - \frac{15}{2} x^{(3^\alpha 3^\beta + 3^\alpha 3^\beta)}) + 2x^{(1^\alpha 3^\beta + 3^\alpha 1^\beta)} + 3x^{(2^\alpha 2^\beta + 2^\alpha 2^\beta)} - 14x^{(2^\alpha 3^\beta + 3^\alpha 2^\beta)} + 9x^{(3^\alpha 3^\beta + 3^\alpha 3^\beta)}$,
the forgotten polynomial of O_{P_m},
$GZ_{2,0}(O_{P_m}, x) = \frac{3}{2} m^2 x^{18} + m(6x^{13} - \frac{15}{2} x^{18}) + 2x^{10} + 3x^4 - 14x^8 + 9x^{18}$.

Proof 4: For $GZ_{\alpha,\beta}(O_{P_m}, x)$ which is the generalized Zagreb polynomial of O_{P_m}, we have $\lambda(d_\varepsilon, d_\nu) = (d_\varepsilon + d_\nu)^\alpha$, therefore $\lambda(1, 3) = (4)^\alpha, \lambda(2, 2) = (4)^\alpha, \lambda(2, 3) = (5)^\alpha$ and $\lambda(3, 3) = (6)^\alpha$. Thus by Lemma 1,
$GZ_{\alpha,\beta}(O_{P_m}, x) = \frac{3}{2} m^2 x^{(3^\alpha 3^\beta + 3^\alpha 3^\beta)} + m(6x^{(2^\alpha 3^\beta + 3^\alpha 2^\beta)} - \frac{15}{2} x^{(3^\alpha 3^\beta + 3^\alpha 3^\beta)}) + 2x^{(1^\alpha 3^\beta + 3^\alpha 1^\beta)} + 3x^{(2^\alpha 2^\beta + 2^\alpha 2^\beta)} - 14x^{(2^\alpha 3^\beta + 3^\alpha 2^\beta)} + 9x^{(3^\alpha 3^\beta + 3^\alpha 3^\beta)}$.
For $\alpha = 2, \beta = 0$, the forgotten polynomial is
$GZ_{2,0}(O_{P_m}, x) = \frac{3}{2} m^2 x^{18} + m(6x^{13} - \frac{15}{2} x^{18}) + 2x^{10} + 3x^4 - 14x^8 + 9x^{18}$.

In the following theorem, we determined the third redefined Zagreb polynomial and harmonic polynomial of the optical transpose interconnection system O_{P_m}.

Theorem 4: For the optical transpose interconnection system O_{P_m}, we have

the third redefined Zagreb polynomial of O_{P_m},
$ReZ(O_{P_m}, x) = \frac{3}{2} m^2 x^{54} + m(6x^{30} - \frac{15}{2} x^{54}) + 2x + 3x^{16} - 14x^{30} + 9x^{54}$. and the harmonic polynomial of O_{P_m},
$H(O_{P_m}, x) = \frac{3}{2} m^2 x^5 + m(6x^4 - \frac{15}{2} x^5) + 5x^3 - 14x^4 + 9x^5$.

Proof 5: For $ReZ(O_{P_m}, x)$ which is the third redefined Zagreb polynomial of O_{P_m}, we have $\lambda(d_\varepsilon, d_\nu) = d_\varepsilon d_\nu(d_\varepsilon + d_\nu)$, therefore $\lambda(1, 3) = (12)^\alpha, \lambda(2, 2) = (8)^\alpha, \lambda(2, 3) = (30)^\alpha$ and $\lambda(3, 3) = (54)^\alpha$. Thus by Lemma 1,
$ReZ(O_{P_m}, x) = \frac{3}{2} m^2 x^{54} + m(6x^{30} - \frac{15}{2} x^{54}) + 2x + 3x^{16} - 14x^{30} + 9x^{54}$.

For $H(O_{P_m}, x)$ which is the harmonic polynomial of O_{P_m}, we have $\lambda(d_\varepsilon, d_\nu) = d_\varepsilon + d_\nu - 1$, therefore $\lambda(1, 3) = (3)^\alpha, \lambda(2, 2) = (3)^\alpha, \lambda(2, 3) = (4)^\alpha$ and $\lambda(3, 3) = (5)^\alpha$. Thus by Lemma 1,
$H(O_{P_m}, x) = \frac{3}{2} m^2 x^{5)} + m(6x^4 - \frac{15}{2} x^5) + 5x^3 - 14x^4 + 9x^5$.

In the following theorem, we determined the third Zagreb polynomial, fourth Zagreb polynomial and fifth Zagreb polynomial of the optical transpose interconnection system O_{P_m}.

Theorem 5: For the optical transpose interconnection system O_{P_m}, we have

the third Zagreb polynomial of O_{P_m},
$M_3(O_{P_m}, x) = \frac{3}{2} m^2 + m(6x - \frac{15}{2}) + 2x^2 - 14x + 12$.
the fourth Zagreb polynomial of O_{P_m},
$M_4(O_{P_m}, x) = \frac{3}{2} m^2 x^{18} + m(6x^{10} - \frac{15}{2} x^{18}) + 2x^4 + 3x^8 - 14x^{10} + 9x^{18}$.
the fifth Zagreb polynomial of O_{P_m},
$M_5(O_{P_m}, x) = \frac{3}{2} m^2 x^{18} + m(6x^{15} - \frac{15}{2} x^{18}) + 2x^{12} + 3x^8 - 14x^{15} + 9x^{18}$.

Proof 6: For $M_3(O_{P_m}, x)$ which is the third Zagreb polynomial of O_{P_m}, we have $\lambda(d_\varepsilon, d_\nu) = |d_\varepsilon - d_\nu|$, therefore $\lambda(1, 3) = (2)^\alpha, \lambda(2, 2) = (0)^\alpha, \lambda(2, 3) = (1)^\alpha$ and $\lambda(3, 3) = (0)^\alpha$. Thus by Lemma 1,
$M_3(O_{P_m}, x) = \frac{3}{2} m^2 + m(6x - \frac{15}{2}) + 2x^2 - 14x + 12$.

For $M_4(O_{P_m}, x)$ which is the fourth Zagreb polynomial of O_{P_m}, we have $\lambda(d_\varepsilon, d_\nu) = d_\varepsilon(d_\varepsilon + d_\nu)$, therefore $\lambda(1, 3) = (4)^\alpha, \lambda(2, 2) = (8)^\alpha, \lambda(2, 3) = (10)^\alpha$ and $\lambda(3, 3) = (18)^\alpha$. Thus by Lemma 1,
$M_4(O_{P_m}, x) = \frac{3}{2} m^2 x^{18} + m(6x^{10} - \frac{15}{2} x^{18}) + 2x^4 + 3x^8 - 14x^{10} + 9x^{18}$.

For $M_5(O_{P_m}, x)$ which is the fifth Zagreb polynomial of O_{P_m}, we have $\lambda(d_\varepsilon, d_\nu) = d_\nu(d_\varepsilon + d_\nu)$, therefore $\lambda(1, 3) = (12)^\alpha, \lambda(2, 2) = (8)^\alpha, \lambda(2, 3) = (15)^\alpha$ and $\lambda(3, 3) = (18)^\alpha$. Thus by Lemma 1,
$M_5(O_{P_m}, x) = \frac{3}{2} m^2 x^{18} + m(6x^{15} - \frac{15}{2} x^{18}) + 2x^{12} + 3x^8 - 14x^{15} + 9x^{18}$.

V. OTIS SWAPPED NETWORK O_{K_m}

The complete graph denoted by K_m with m vertices and O_{K_m} be the OTIS swapped network for O_K4 as example shown in Figure 2.

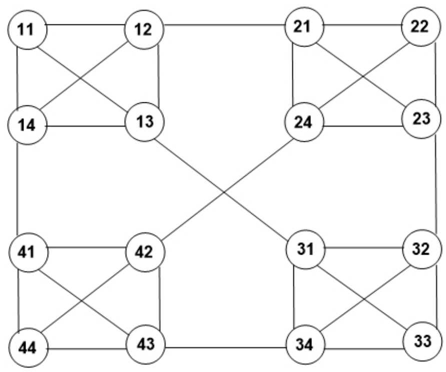

FIGURE 2. OTIS swapped network O_{K5}.

Lemma 2: Let O_{K_m} be a. Then $P(O_{K_m}, x) = \frac{m^3}{2} x^{\lambda(m,m)} + m^2\{x^{\lambda(m,m-1)} - x^{\lambda(m,m)}\} + m\{\frac{x^{\lambda(m,m)}}{2} - x^{\lambda(m,m-1)}\}$.

Proof 7: The graph O_{K_m} contains m^2 vertices and $\frac{m^3-m}{2}$ edges. Each vertex of O_{K_m} has degree $m-1$ or m, vertices of O_{K_m} can be partitioned according to their degrees. Let

$$V_j = \{\varepsilon \in V(O_{K_m}) : d_\varepsilon = j\}.$$

This means that the set V_j contains the vertices of degree j. The set of vertices with respect to their degrees are as follows:

$$V_{m-1} = \{\varepsilon \in V(O_{K_m}) : d_\varepsilon = m-1\}$$
$$V_m = \{\varepsilon \in V(O_{K_m}) : d_\varepsilon = m\}.$$

Since, $|V_{m-1}| = m$ and $|V_m| = |V(O_{K_m})| - |V_{m-1}| = m^2 - m$. Let us divide the edges of O_{K_m} into partition sets according to the degree of its end vertices. Let

$$\Xi_{m,m-1} = \{\varepsilon\nu \in E(O_{K_m}) : d_\varepsilon = m, d_\nu = m-1\}$$
$$\Xi_{m,m} = \{\varepsilon\nu \in E(O_{K_m}) : d_\varepsilon = m, d_\nu = m\}.$$

Note that $E(O_{K_m}) = \Xi_{m,m-1} \sqcup \Xi_{m,m}$. The number of edges incident to one vertex of degree $m-1$ and other vertex of degree m is $m^2 - m$, so $|\Xi_{m,m-1}| = m^2 - m$. Now, the remaining number of edges are those edges which are incident to two vertices of degree m, i.e $\Xi_{m,m} = |E(O_{K_m})| - \Xi_{m,m-1} = \frac{m^3-m}{2} - (m^2-m) = \frac{m^3-2m^2+m}{2} = \frac{m(m-1)^2}{2}$.

Hence,
$P(O_{K_m}, x) = \sum_{\varepsilon\nu \in E(O_{K_m})} x^{\lambda(d_\varepsilon, d_\nu)} = \sum_{\varepsilon\nu \in \Xi_{m,m-1}} x^{\lambda(m,m-1)} + \sum_{\varepsilon\nu \in \Xi_{m,m}} x^{\lambda(m,m)}$. After Simplification, we get
$P(O_{K_m}, x) = \frac{m^3}{2} x^{\lambda(m,m)} + m^2\{x^{\lambda(m,m-1)} - x^{\lambda(m,m)}\} + m\{\frac{x^{\lambda(m,m)}}{2} - x^{\lambda(m,m-1)}\}$.

Theorem 6: For the optical transpose interconnection system swapped network O_{K_m}, we have

the general Randić polynomial of O_{K_m},
$R_\alpha(O_{K_m}, x) = \frac{m^3}{2} x^{(m^2)^\alpha} + m^2\{x^{(m^2-m)^\alpha} - x^{(m^2)^\alpha}\} + m\{\frac{x^{(m^2)^\alpha}}{2} - x^{(m^2-m)^\alpha}\}$

the second Zagreb polynomial of O_{K_m},
$R_1(O_{K_m}, x) = \frac{m^3}{2} x^{(m^2)} + m^2\{x^{(m^2-m)} - x^{(m^2)}\} + m\{\frac{x^{(m^2)}}{2} - x^{(m^2-m)}\}$.

Proof 8: For $R_\alpha(O_{K_m}, x)$ which is the general Randić polynomial of O_{K_m}, we have $\lambda(d_\varepsilon, d_\nu) = (d_\varepsilon d_\nu)^\alpha$, therefore $\lambda(m, m-1) = (m^2-m)^\alpha$ and $\lambda(m,m) = (m^2)^\alpha$. Thus by Lemma 2,
$R_\alpha(O_{K_m}, x) = \frac{m^3}{2} x^{(m^2)^\alpha} + m^2\{x^{(m^2-m)^\alpha} - x^{(m^2)^\alpha}\} + m\{\frac{x^{(m^2)^\alpha}}{2} - x^{(m^2-m)^\alpha}\}$.

For $\alpha = 1$, the second Zagreb polynomial is
$R_1(O_{K_m}, x) = \frac{m^3}{2} x^{(m^2)} + m^2\{x^{(m^2-m)} - x^{(m^2)}\} + m\{\frac{x^{(m^2)}}{2} - x^{(m^2-m)}\}$.

In the next theorem, we determined general sum-connectivity polynomial, first Zagreb polynomial and hyper-Zagreb polynomial of the optical transpose interconnection system swapped network O_{K_m}.

Theorem 7: For the optical transpose interconnection system swapped network O_{K_m}, we have

the general sum-connectivity polynomial of O_{K_m},
$\chi_\alpha(O_{K_m}, x) = \frac{m^3}{2} x^{(2m)^\alpha} + m^2\{x^{(2m-1)^\alpha} - x^{(2m)^\alpha}\} + m\{\frac{x^{(2m)^\alpha}}{2} - x^{(2m-1)^\alpha}\}$,

the first Zagreb polynomial of O_{K_m},
$\chi_1(O_{K_m}, x) = \frac{m^3}{2} x^{(2m)} + m^2\{x^{(2m-1)} - x^{(2m)}\} + m\{\frac{x^{(2m)}}{2} - x^{(2m-1)}\}$.

the hyper-Zagreb polynomial of O_{K_m},
$\chi_2(O_{K_m}, x) = \frac{m^3}{2} x^{(2m)^2} + m^2\{x^{(2m-1)^2} - x^{(2m)^2}\} + m\{\frac{x^{(2m)^2}}{2} - x^{(2m-1)^2}\}$.

Proof 9: For $\chi_\alpha(O_{K_m}, x)$ which is the general sum-connectivity polynomial of O_{K_m}, we have $\lambda(d_\varepsilon, d_\nu) = (d_\varepsilon + d_\nu)^\alpha$, therefore $\lambda(m, m-1) = (2m-1)^\alpha$ and $\lambda(m,m) = (m^2)^\alpha$. Thus by Lemma 2,
$\chi_\alpha(O_{K_m}, x) = \frac{m^3}{2} x^{(2m)^\alpha} + m^2\{x^{(2m-1)^\alpha} - x^{(2m)^\alpha}\} + m\{\frac{x^{(2m)^\alpha}}{2} - x^{(2m-1)^\alpha}\}$

For $\alpha = 1$, the first Zagreb polynomial is
$\chi_1(O_{K_m}, x) = \frac{m^3}{2} x^{(2m)} + m^2\{x^{(2m-1)} - x^{(2m)}\} + m\{\frac{x^{(2m)}}{2} - x^{(2m-1)}\}$,

For $\alpha = 2$, the hyper-Zagreb polynomial is
$\chi_2(O_{K_m}, x) = \frac{m^3}{2} x^{(2m)^2} + m^2\{x^{(2m-1)^2} - x^{(2m)^2}\} + m\{\frac{x^{(2m)^2}}{2} - x^{(2m-1)^2}\}$.

In the following theorem, we determined generalized Zagreb polynomial and forgotten polynomial of the optical transpose interconnection system swapped network O_{K_m}.

Theorem 8: For the optical transpose interconnection system swapped network O_{K_m}, we have

the generalized Zagreb polynomial of O_{K_m},
$GZ_{\alpha,\beta}(O_{K_m}, x) = \frac{m^3}{2} x^{(2m^\alpha m^\beta)} + m^2\{x^{((m^\alpha(m-1)^\beta + m^\beta(m-1)^\alpha)} - x^{(2m^\alpha m^\beta)}\} + m\{\frac{x^{(2m^\alpha m^\beta)}}{2} - x^{((m^\alpha(m-1)^\beta + m^\beta(m-1)^\alpha)}\}$,

the forgotten polynomial of O_{K_m},
$GZ_{2,0}(O_{K_m}, x) = \frac{m^3}{2} x^{(2m^2)} + m^2\{x^{(m^2+(m-1)^2)} - x^{(2m^2)}\} + m\{\frac{x^{(2m^2)}}{2} - x^{(m^2+(m-1)^2)}\}$.

Proof 10: For $GZ_{\alpha,\beta}(O_{K_m}, x)$ which is the generalized Zagreb polynomial of O_{K_m}, we have $\lambda(d_\varepsilon, d_\nu) = (d_\varepsilon^\alpha d_\nu^\beta + d_\varepsilon^\beta d_\nu^\alpha)^\alpha$, therefore $\lambda(m, m-1) = m^\alpha(m-1)^\beta + m^\beta(m-1)^\alpha$ and $\lambda(m,m) = 2m^\alpha m^\beta$. Thus by Lemma 2,
$GZ_{\alpha,\beta}(O_{K_m}, x) = \frac{m^3}{2} x^{(2m^\alpha m^\beta)} + m^2\{x^{((m^\alpha(m-1)^\beta + m^\beta(m-1)^\alpha)} - x^{(2m^\alpha m^\beta)}\} + m\{\frac{x^{(2m^\alpha m^\beta)}}{2} - x^{((m^\alpha(m-1)^\beta + m^\beta(m-1)^\alpha)}\}$.

For $\alpha = 2$, $\beta = 0$, the forgotten polynomial is
$GZ_{2,0}(O_{K_m}, x) = \frac{m^3}{2} x^{(2m^2)} + m^2\{x^{(m^2+(m-1)^2)} - x^{(2m^2)}\} + m\{\frac{x^{(2m^2)}}{2} - x^{(m^2+(m-1)^2)}\}$.

In the following theorem, we determined the third redefined Zagreb polynomial and harmonic polynomial of the optical transpose interconnection system swapped network O_{K_m}.

Theorem 9: For the optical transpose interconnection system swapped network O_{K_m}, we have

the third redefined Zagreb polynomial of O_{K_m},
$ReZ(O_{K_m}, x) = \frac{m^3}{2} x^{(2m^3)^\alpha} + m^2\{x^{((2m-1)(m^2-m))^\alpha} - x^{(2m^3)^\alpha}\} + m\{\frac{x^{(2m^3)^\alpha}}{2} - x^{((2m-1)(m^2-m))^\alpha}\}$. and the harmonic polynomial of O_{K_m},
$H(O_{K_m}, x) = \frac{m^3}{2} x^{(m^2-1)^\alpha} + m^2\{x^{(m^2-1)^\alpha} - x^{(2m-1)^\alpha}\} + m\{\frac{x^{(m^2-1)^\alpha}}{2} - x^{(2m-1)^\alpha}\}$

Proof 11: For $ReZ(O_{K_m}, x)$ which is the third redefined Zagreb polynomial of O_{K_m}, we have $\lambda(d_\varepsilon, d_\nu) = d_\varepsilon d_\nu(d_\varepsilon + d_\nu)$, therefore $\lambda(m, m-1) = x^{((2m-1)(m^2-m))^\alpha}$ and $\lambda(m,m) = x^{(2m^3)^\alpha}$. Thus by Lemma 2,
$ReZ(O_{K_m}, x) = \frac{m^3}{2} x^{(2m^3)^\alpha} + m^2\{x^{((2m-1)(m^2-m))^\alpha} - x^{(2m^3)^\alpha}\} + m\{\frac{x^{(2m^3)^\alpha}}{2} - x^{((2m-1)(m^2-m))^\alpha}$.

For $H(O_{K_m}, x)$ which is the harmonic polynomial of O_{K_m}, we have $\lambda(d_\varepsilon, d_\nu) = d_\varepsilon + d_\nu - 1$, therefore $\lambda(m, m-1) = x^{(2m-1)^\alpha}$ and $\lambda(m, m) = x^{(m^2-1)^\alpha}$. Thus by Lemma 2,

$H(O_{K_m}, x) = \frac{m^3}{2} x^{(m^2-1)^\alpha} + m^2\{x^{(m^2-1)^\alpha} - x^{(2m-1)^\alpha}\} + m\{\frac{x^{(m^2-1)^\alpha}}{2} - x^{(2m-1)^\alpha}\}$.

In the following theorem, we determined the third Zagreb polynomial, fourth Zagreb polynomial and fifth Zagreb polynomial of the optical transpose interconnection system swapped network O_{K_m}.

Theorem 10: For the optical transpose interconnection system swapped network O_{K_m}, we have

the third Zagreb polynomial of O_{K_m},
$M_3(O_{K_m}, x) = \frac{m^3}{2} + m^2\{x - 1\} + m\{\frac{1}{2} - x\}$.
the fourth Zagreb polynomial of O_{K_m},
$M_4(O_{K_m}, x) = \frac{m^3}{2} x^{(2m^2)^\alpha} + m^2\{x^{(m(2m+1))^\alpha} - x^{(2m^2)^\alpha}\} + m\{\frac{x^{(2m^2)^\alpha}}{2} - x^{(m(2m-1))^\alpha}\}$.
the fifth Zagreb polynomial of O_{K_m},
$M_5(O_{K_m}, x) = \frac{m^3}{2} x^{(2m^2)^\alpha} + m^2\{x^{((m-1)(2m-1))^\alpha} - x^{(2m^2)^\alpha}\} + m\{\frac{x^{(2m^2)^\alpha}}{2} - x^{((m-1)(2m-1))^\alpha}\}$.

Proof 12: For $M_3(O_{K_m}, x)$ which is the third Zagreb polynomial of O_{K_m}, we have $\lambda(d_\varepsilon, d_\nu) = |d_\varepsilon - d_\nu|$, therefore $\lambda(m, m-1) = x$ and $\lambda(m, m) = 1$. Thus by Lemma 2,

$M_3(O_{K_m}, x) = \frac{m^3}{2} + m^2\{x - 1\} + m\{\frac{1}{2} - x\}$.

For $M_4(O_{K_m}, x)$ which is the fourth Zagreb polynomial of O_{K_m}, we have $\lambda(d_\varepsilon, d_\nu) = d_\varepsilon(d_\varepsilon + d_\nu)$, therefore $\lambda(m, m-1) = x^{(m(2m+1))^\alpha}$ and $\lambda(m, m) = x(2m^2)^\alpha$. Thus by Lemma 2,

$M_4(O_{K_m}, x) = \frac{m^3}{2} x^{(2m^2)^\alpha} + m^2\{x^{(m(2m+1))^\alpha} - x^{(2m^2)^\alpha}\} + m\{\frac{x^{(2m^2)^\alpha}}{2} - x^{(m(2m-1))^\alpha}\}$.

For $M_5(O_{K_m}, x)$ which is the fifth Zagreb polynomial of O_{K_m}, we have $\lambda(d_\varepsilon, d_\nu) = d_\nu(d_\varepsilon + d_\nu)$, therefore $\lambda(m, m-1) = x^{(m-1)(2m-1))^\alpha}$ and $\lambda(m, m) = x(2m^2)^\alpha$. Thus by Lemma 2,

$M_5(O_{K_m}, x) = \frac{m^3}{2} x^{(2m^2)^\alpha} + m^2\{x^{((m-1)(2m-1))^\alpha} - x^{(2m^2)^\alpha}\} + m\{\frac{x^{(2m^2)^\alpha}}{2} - x^{((m-1)(2m-1))^\alpha}\}$.

VI. CONCLUSION

Optical Transpose Interconnection Systems (OTIS) swapped networks are optoelectronic and have been used in efficient parallel processing and services in large global networks. Topological indices are often studied with the assistance of their polynomials. Formulae for degree-based topological polynomials for Optical Transpose Interconnection Systems swapped network have been derived. Results can be used to compute any degree-based topological polynomials for OTIS swapped network. These results will help in the future research of networks, mechanics, computer science, and chemistry.

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ALI AHMAD received the M.Sc. degree in mathematics from Punjab University, Lahore, Pakistan, in 2000, the M.Phil. degree in mathematics from Bahauddin Zakariya University, Multan, Pakistan, in 2005, and the Ph.D. degree in mathematics from the Abdus Salam School of Mathematical Sciences, GC University, Lahore, in 2010. He is currently an Assistant Professor with the College of Computer Science and Information Technology, Jazan University, Saudi Arabia. His research interests include graph labeling, metric dimension, minimal doubly resolving sets, distances in graphs, and topological indices of graphs.

ROSLAN HASNI received the M.Sc. degree from Universiti Kebangsaan Malaysia (UKM), Selangor, Malaysia, and the Ph.D. degree in pure mathematics (graph theory) from Universiti Putra Malaysia (UPM), Selangor, in 2005. He was formerly attached with the School of Mathematical Sciences, Universiti Sains Malaysia (USM) Penang, Malaysia, from 2005 to 2012. He is currently an Associate Professor with Universiti Malaysia Terengganu (UMT), Kuala Terengganu, Malaysia. His research interests include chromaticity in graphs, domination graph theory, graph labeling, and chemical graph theory.

KASHIF ELAHI received the master's degree in information and operational management from the University of the Punjab, Lahore, Pakistan, in 2005. He is currently working as a Lecturer with Jazan University, Jazan, Saudi Arabia. His research interests include algorithms, graph theory, distance in graph, software engineering, and topological indices.

MUHAMMAD AHSAN ASIM received the master's degree from the Blekinge Institute of Technology (BTH), Sweden, and the Ph.D. degree in mathematical sciences from University Malaysia Terengganu (UMT), Malaysia, in 2018. He is currently working as a Lecturer with the Department of Information Technology and Security, Jazan University, Saudi Arabia. His research interests include graph theory, graph algorithms, and experimental mathematics.