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A Data-Driven Distributed Adaptive Control Approach for Nonlinear Multi-Agent Systems

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ABSTRACT In this paper the distributed leader-follower consensus tracking problem is investigated for unknown nonlinear non-affine discrete-time multi-agent systems. Via a dynamic linearization method both for the agent system and the local ideal distributed controller, a distributed adaptive control scheme is proposed in this paper using the Newton-type optimization method. The proposed approach is data-driven since only the local measurement information among neighboring agents is utilized in the control system design. The consensus tracking stabilities of the proposed approach are rigorously guaranteed in the cases of fixed and switching communication topologies. The simulations are conducted to verify the effectiveness of the proposed approach.

INDEX TERMS Dynamic linearization, data-driven control, adaptive control, multi-agent systems, consensus tracking.

I. INTRODUCTION

The recent two decades have witnessed a burgeoning research direction in the automatic control of interconnected systems [1], [2]. Such systems are the well-known multi-agent systems (MASs). Cooperative control of MASs is aimed to exploit the local interactive control protocols among networked agents for achieving a global objective that is difficult to be accomplished by a single agent. Due to its powerful potential applications [3]–[5], a considerable attention has been attracted for different cooperating tasks, such as consensus, formation, coverage control, flocking and containment control. Among these research topics, consensus control is an important and fundamental problem. Remarkable results on consensus have been investigated from different perspectives, and readers are referred to [6]–[8] and references therein.

Because of the pioneering works [9], [10] on consensus, many scholars have extensively investigated different consensus problems. For instance, in [11]–[13], the distributed consensus of linear continuous-time and discrete-time homogeneous systems was discussed. Moreover, some works, such as [14], [15], were extended to heterogeneous network systems. Since almost all the physical dynamics

of controlled systems in practice are inherently nonlinear, the aforementioned control schemes cannot be directly applied to nonlinear systems. Recently, the adaptive control schemes were developed for nonlinear MASs [6], [7]. However, the aforementioned works are usually based on availability of the dynamic models or structural information of the controlled MAS. This means that the first principle or an identification method is required for these distributed control schemes, which have the problems of unmodeled dynamics and model/controller reduction [16].

For the control problems of unknown systems, data-driven control methodologies are a concerned research topic, in which the model free adaptive control (MFAC), proposed by Hou in [17] and further developed in [18], is valuable. The MFAC has been extended from the original single-input single-output systems [19], [20] and multi-input multi-output systems [21], [22] to nonlinear MASs [23]–[25]. Besides, the dynamic linearization based control methods have been successfully applied to many practical applications, such as servo motor systems [26] and exoskeleton robotic systems [27]. The detail of the dynamic linearization based control methods can be found in [16].

However, some challenging issues still have not been developed for unknown nonlinear discrete-time MASs on data-driven consensus control. One issue is to how to

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design a distributed consensus controller structure through a systematic approach. The local controller structures, such as the distributed proportional and proportional-integral control laws in [7], [28], are usually determined a priori by experience, which leads to the difficulty in determining their appropriation or effectiveness in applications. Another issue is to how to design a distributed control gain updating algorithm on condition that the local measurements are only applicable. The control gains in existing distributed control schemes are usually calibrated heuristically and chosen as fixed values if the physical dynamics of the controlled MAS are unknown. The dynamic linearization based control methods motivate us to explore a novel data-driven distributed consensus tracking approach for addressing these issues.

Comparing to existing distributed control methods, the main contributions of this paper are as follows.

- Provide a systematic way of directly designing the distributed controller structure for unknown MASs on the consensus tracking, and the designed distributed controller is independent of the controlled MAS.
- Propose a data-driven distributed adaptive control approach, where the local control law and control gain updating algorithm are designed only using the local information among neighboring agents.
- Establish the consensus tracking stability properties of the proposed approach under fixed and switching topologies.

The rest of this paper is organized as follows. Section II formulates the consensus tracking problem. Section III concludes the main results, including the designed control law, distributed control gain updating algorithm, adaptive control approach and its convergence properties. Section IV conducts some simulations. Section V provides some concluding remarks. In this paper, $\|\cdot\|$ denotes any generic vector or matrix norm.

II. PROBLEM FORMULATION

The communication topology of a leader-follower system including the leader is represented by the graph \mathcal{G} , where the topology among agents is fixed and directional. The leader's command is only accessible to a subset of the follower agents with unidirectional paths from the leader to the follower agents. Each follower agent exchanges local measurement information only with its neighboring follower agents under a directional graph. It is assumed that the topology among the follower agents is a fixed strongly connected graph and at least one follower agent is communicated to the leader.

We consider a set of N heterogeneous nonlinear non-affine discrete-time follower agents, where the physical model of follower agent q , $q = 1, 2, \dots, N$, is described by

$$y_q(k+1) = f_q(y_q(k), \dots, y_q(k-n_y), u_q(k), \dots, u_q(k-n_u)), \quad (1)$$

where $y_q(k) \in \mathbb{R}^1$ and $u_q(k) \in \mathbb{R}^1$ are the system output and control input of agent q at the time instant k , respectively,

and $k = 1, 2, \dots, n_y \in \mathbb{Z}_+$ and $n_u \in \mathbb{Z}_+$ are unknown orders for the system outputs and control inputs of agent q ; $f_q(\cdot) : \mathbb{R}^{n_y+n_u+2} \mapsto \mathbb{R}^1$ is an unknown nonlinear function.

Following [19], the agent system (1) can be transformed into the following equivalent dynamic linearization data model:

$$\delta y_q(k+1) = \psi_q(k) \delta u_q(k), \quad (2)$$

where $\delta y_q(k+1) = y_q(k+1) - y_q(k)$; $\delta u_q(k) = u_q(k) - u_q(k-1)$; the unknown time-varying parameter $\psi_q(k)$ is called the pseudo partial derivative (PPD) of the agent system (1), satisfying $|\psi_q(k)| \leq b_p$, $b_p > 0$ is a known constant.

Assumption 1: The sign of $\psi_q(k)$, remains invariant, satisfying $\psi_q(k) > 0$ or $\psi_q(k) < 0$ for all $k = 1, 2, \dots$. Without loss of generality, we consider $0 < \psi_q(k) \leq b_p$ in this paper.

Remark 1: The considered condition $0 < \psi_q(k) \leq b_p$ in Assumption 1 implies that the control direction is known and positive. This condition is reasonable since many practical systems, such as autonomous underwater vehicles, unmanned aerial vehicles and mobile robots, feature this property.

Note that the time-varying parameter $\psi_q(k)$ is only a concept in the sense of mathematics, and its existence is rigorously guaranteed by the theorem in [19]. While the time-invariant parameters usually introduced for traditional adaptive control methods indicate the variables of dynamics of a controlled plant. It can be seen from the theorem in [19] that $\psi_q(k)$ is obviously time-varying even if the controlled plant (1) is linear time-invariant since $\psi_q(k)$ is only related to the system outputs and control inputs by the time k . Moreover, all of the possible properties of the controlled system (1), such as the nonlinearity and time-varying parameters or structures, are involved into $\psi_q(k)$, which may lead to its complicated characteristic, but the simple numerical behavior that can be easily estimated. This implies that $\psi_q(k)$ is capable of managing these properties that are difficult to be handled in the framework of traditional adaptive control due to its possible insensitivity to these properties. More detail on the parameter can be referred to [29].

Equation (2) is only a data model that is equivalent to (1) in the sense of mathematics and it has no physical meanings. This equivalent transformation is achieved by using the compact form dynamic linearization method, and the detailed derivations can be referred to [19] and [29]. The data model (2) is only related to the system outputs and control inputs of the controlled plant, and it not explicitly or implicitly includes the parametric and structural information of physical dynamics of a controlled plant. In addition, the data model (2) is only purposed to the control system design, and it is not suitable for other objectives, such as monitoring and diagnosis.

The consensus tracking problem of the leader-follower MAS described by (1) under the graph \mathcal{G} is summarized as follows.

The leader's command at the time instant k is denoted as $y_d(k)$. The global control objective is to develop a data-driven distributed adaptive control approach that drives the system

output $y_q(k)$ to $y_d(k)$ when the time instant k tends to infinity; that is, the tracking error $\lim_{k \rightarrow \infty} e_q(k) = y_d(k) - y_q(k) = 0$, although the local information among neighboring agents is only used and the leader's command is only accessible to a subset of the follower agents. To describe the local measurement information of agent q with its neighbors, we define the following distributed tracking error $\xi_q(k)$ under \mathcal{G} as:

$$\xi_q(k) = \sum_{p \in N_q} a_{q,p} (y_p(k) - y_q(k)) + d_q (y_d(k) - y_q(k)), \quad (3)$$

where N_q denotes the set of neighbors of agent q ; $a_{q,p} = 1$ if agent q can receive information from its neighboring agent p , otherwise $a_{q,p} = 0$, specially $a_{q,q} = 0$; $d_q = 1$ if agent q receives the leader's command $y_d(k)$, otherwise $d_q = 0$.

The first issue for developing the distributed control approach is the structure design of a distributed control law. Since the physical model of the MAS described by (1) is completely unknown, so far there is not a systematic way to determine the distributed controller structure. The second issue is related to the design of the distributed control gain updating algorithm using only the local measurement information. The last important issue is the stability properties of the developed data-driven distributed control approach. In next section, these issues are discussed in detail.

III. MAIN RESULTS

A. DISTRIBUTED CONTROL LAW

This subsection considers the design of the distributed controller structure through only known local information. Assume there exists an ideal distributed consensus controller in theory that can guarantee the system output of agent q equal to $y_d(k+1)$ in one step-ahead. The ideal distributed controller can be written in the following mathematical form:

$$u_q(k) = C_q(\xi_q(k+1), \dots, \xi_q(k-n_e+2), \xi_q(k-1), \dots, u_q(k-n_c)), \quad (4)$$

where $C_q(\cdot) : \mathbb{R}^{n_e+n_c} \mapsto \mathbb{R}^1$ is an unknown nonlinear function; $n_e \in \mathbb{Z}_+$ and $n_c \in \mathbb{Z}_+$ are the unknown orders of ideal distributed controller (4) on the distributed tracking errors and control inputs, respectively.

The assumption on (4) implies that it is reachable and the detailed discussions are referred to [30]. In practice, the controller (4) is difficult to derive. Thus the key task is to transform it into a practical distributed controller, while keeping it equivalent to (4) in the input-output data sense. In achieving this, the following two assumptions are required.

Assumption 2: The partial derivative of $C_q(\cdot)$ with respect to the distributed tracking error $\xi_q(k+1)$ is continuous.

Assumption 3: $C_q(\cdot)$ satisfies the generalized Lipschitz condition, that is, if $|\delta\xi_q(k+1)| \neq 0$, then there exists an unknown constant $\beta > 0$ such that

$$|\delta u_q(k)| \leq \beta |\delta\xi_q(k+1)|, \quad (5)$$

where $\delta\xi_q(k+1) = \xi_q(k+1) - \xi_q(k)$.

Assumption 2 is common since many controllers, such as the distributed proportional controller [31] and the distributed adaptive controller [32], generally satisfy this condition. Assumption 3 implies that the ideal distributed controller (4) is required to be stable [16].

Lemma 1: The controller (4) satisfies Assumptions 2 and 3. If $|\delta\xi_q(k+1)| \neq 0$, then there exists an unknown controller parameter $\theta_q(k)$, such that (4) can be transformed into the following equivalent distributed control law using the compact form dynamic linearization (CFDL) method:

$$\delta u_q(k) = \theta_q(k) \delta\xi_q(k+1), \quad (6)$$

where $|\theta_q(k)| \leq b_t$, and $b_t > 0$ is an unknown constant.

Proof: Lemma 1 can be proved by utilizing the differential mean value theorem through Assumptions 2 and 3. The derivative detail is similar to the results in [30], [33] and thus the derivations are omitted.

For simplicity, we label the obtained distributed control law (6) as CFDLc controller (CFDLc).

Remark 2: The CFDLc (6), with a time-varying linearization structure, is equivalent to the ideal distributed controller (4), which implies two points. One point is that the structure complexity of (6) does not increase even though the agent system (1) is highly nonlinear. Another point is that the CFDLc (6) can be considered as a candidate consensus controller for unknown nonlinear MASs as described by (1) since (6) can drive $e_q(k+1) = 0$ in one-step. In other words, the issue of designing a distributed controller structure is addressed through the CFDL method, while the existing distributed controller structures are usually given in an ad hoc way.

Remark 3: Lemma 1 indicates that the CFDLc (6) is independent of the agent system (1), and $\theta_q(k)$ can be obtained through only the local information using some data analytical approaches when the dynamic model of agent q is unavailable. It can also be obtained via the model based optimization method through submitting (6) into the agent system (1) when its dynamic model is known. This paper just considers the issue of obtaining $\theta_q(k)$ using only the local information communicated with agent q .

The CFDLc (6) cannot be implemented in practice due to the presence of the noncausal term $\xi_q(k+1)$ in (6). Similar to [33], the following practical CFDLc is obtained:

$$\delta u_q(k) = -\theta_q(k) \xi_q(k), \quad (7)$$

which means that $u_q(k)$ can be computed directly according to the measured $\xi_q(k)$ at current time instant k . Note that (7) is not an approximation to (6), but a direct derivation from the observation that (6) can drive $e_q(k+1) = 0$.

B. DISTRIBUTED CONTROL GAIN UPDATING ALGORITHM

This subsection considers the second issue on tuning $\theta_q(k)$ in the CFDLc (7) using only the local information communicated with agent q via the data model (2).

We first consider the following control objective function:

$$J_q = \frac{1}{2} \sum_{p \in N_q} a_{q,p} (y_p(k+1) - y_q(k+1))^2 + \frac{1}{2} d_q (y_d(k+1) - y_q(k+1))^2 + \frac{1}{2} \lambda \delta u_q^2(k), \quad (8)$$

where $\lambda > 0$ is a weight factor used as penalty for $\delta u_q(k)$.

In order to obtain the optimal control gain $\theta_q(k)$ under the control objective function (8), the relationship between $y_q(k+1)$ and $u_q(k)$ for agent q is required. In achieving this, the data model (2) is applied and we rewrite it as

$$y_q(k+1) = y_q(k) + \psi_q(k) \delta u_q(k). \quad (9)$$

Taking the CFDLc (7) and data model (9) into the control objective function (8), we obtain

$$J_q = \frac{1}{2} \sum_{p \in N_q} a_{q,p} (y_p(k+1) - y_q(k) + \psi_q(k) \theta_q(k) \xi_q(k))^2 + \frac{1}{2} d_q (y_d(k+1) - y_q(k) + \psi_q(k) \theta_q(k) \xi_q(k))^2 + \frac{1}{2} \lambda (\theta_q(k) \xi_q(k))^2. \quad (10)$$

Equation (10) indicates that the control objective function (8) is transformed into an identification function of $\theta_q(k)$. Then the tuning of $\theta_q(k)$ is achieved by applying the following Newton-type optimization method:

$$\begin{aligned} \theta_q(k+1) &= \theta_q(k) - \gamma \left(\frac{\partial^2 J_q}{\partial \theta_q^2(k)} \right)^{-1} \frac{\partial J_q}{\partial \theta_q(k)} \\ &= \theta_q(k) - \gamma \frac{\psi_q(k) \xi_q(k+1) + \lambda \theta_q(k) \xi_q(k)}{(\lambda + \psi_q^2(k)) \xi_q(k)}, \end{aligned} \quad (11)$$

with the given resetting mechanism

$$\begin{aligned} \theta_q(k+1) &= -b_l \quad \text{if } \theta_q(k+1) < -b_l, \\ \text{or } \theta_q(k+1) &= 0 \quad \text{if } \theta_q(k+1) > 0, \end{aligned} \quad (12)$$

where $\gamma \in (0, 1]$ is the step size of $\theta_q(k)$.

Note that the control gain $\theta_q(k)$ is not required to be updated if the distributed tracking error $\xi_q(k) = 0$ since $y_d(k) = y_q(k)$ in this case; that is, a perfect tracking for agent q is achieved.

However the control gain updating algorithm (11) is not realizable since $\psi_q(k)$ is unknown and $\xi_q(k+1)$ is noncausal. For simplicity, we consider the updating algorithm given in [23] to estimate $\psi_q(k)$:

$$\hat{\psi}_q(k) = \hat{\psi}_q(k-1) + \frac{\eta \epsilon_q(k) \delta u_q(k-1)}{\mu + \delta u^2(k-1)}, \quad (13)$$

with the resetting mechanism

$$\hat{\psi}_q(k) = \hat{\psi}_q(k-1) \quad \text{if } \hat{\psi}_q(k) < \sigma \text{ or } \hat{\psi}_q(k) > b_p, \quad (14)$$

where

$$\epsilon_q(k) = \delta y_q(k) - \hat{\psi}_q(k-1) \delta u_q(k-1),$$

$\mu > 0$ is a weight factor, $\eta \in (0, 1]$ is the step size of $\psi_q(k)$, and σ is a tiny positive constant.

Based on the estimation given in (13) and (14), the estimation of the noncausal term $\xi_q(k+1)$ is given by

$$\begin{aligned} \hat{\xi}_q(k+1) &= \sum_{p \in N_q} a_{q,p} (\hat{y}_p(k+1) - \hat{y}_q(k+1)) \\ &\quad + d_q (y_d(k+1) - \hat{y}_q(k+1)), \end{aligned} \quad (15)$$

where

$$\hat{y}_p(k+1) = y_p(k) - \hat{\psi}_p(k) \delta u_p(k), \quad (16)$$

$$\hat{y}_q(k+1) = y_q(k) - \hat{\psi}_q(k) \delta u_q(k). \quad (17)$$

C. SUMMARIZED DISTRIBUTED ADAPTIVE CONTROL APPROACH

The CFDLc (7), the two updating algorithms (11) and (13) with the resetting mechanisms (12) and (14), and the distributed tracking error estimation (15), formulate the distributed consensus tracking approach. The detailed steps are as follows.

Step 1: Set $k = 1$, initialize the local measurement data and the $\hat{\psi}_q(1)$ satisfying $\sigma \leq \hat{\psi}_q(1) \leq b_p$, and randomly set $\theta_q(1)$ satisfying $-b_l \leq \theta_q(1) < 0$.

Step 2: Compute the control input

$$u_q(k) = u_q(k-1) - \theta_q(k) \xi_q(k), \quad (18)$$

rewritten from the CFDLc (7), apply it to agent q , and collect $y_q(k+1)$ and $u_q(k)$.

Step 3: Update the PPD estimation value $\hat{\psi}_q(k)$ using (13) with the resetting mechanism (14).

Step 4: Compute $\hat{\xi}_q(k+1)$ by (15) with (16) and (17).

Step 5: Update the control gain

$$\theta_q(k+1) = \theta_q(k) - \gamma \frac{\hat{\psi}_q(k) \hat{\xi}_q(k+1) + \lambda \theta_q(k) \xi_q(k)}{(\lambda + \hat{\psi}_q^2(k)) \xi_q(k)}, \quad (19)$$

with the resetting mechanism (12).

Step 6: Set $k = k + 1$, and go back to Step 2.

For convenient descriptions, we label the proposed approach as CFDL based distributed adaptive control (CFDL-DAC).

Remark 4: The proposed CFDL-DAC illustrates that no physical dynamics of the controlled MAS are involved into the distributed controller design. The parameter updating algorithm (13) is based on only the input-output data of each agent. The distributed tracking error estimation (15), the distributed control law (18) and the distributed control gain updating algorithm (19) are designed using only the local information communicated to agent q . Further, the design for the distributed control law (18) is an independent process of the dynamics of agent q . Hence, the proposed CFDL-DAC is a pure data-driven distributed control approach.

Note that the proposed CFDL-DAC can be extended to deal with the leaderless control problems [34], [35] since it is applicable as long as the local measurement information can be described, which is demonstrated by (3). However,

the results in [34], [35] are obtained for continues-time multi-agent systems with unknown control directions, and this paper considers discrete-time multi-agent systems where the control directions are known. Therefore, the two obstacles are required to be tackled before utilizing the proposed approach.

D. CONVERGENCE ANALYSES

The lemma following [36] is applied to facilitate the convergence analyses.

Lemma 2: $\mathbf{H}(k) \in \mathbb{R}^{N \times N}$ is an irreducible stochastic matrix with positive diagonal entries and \mathcal{H} is the set of all possible $\mathbf{H}(k)$. The multiplication of Q matrixes satisfies

$$\|\mathbf{H}(Q)\mathbf{H}(Q-1)\cdots\mathbf{H}(1)\| \leq \iota, \tag{20}$$

where $\{\mathbf{H}(r)|r = 1, 2, \dots, Q, Q \in \mathbb{Z}_+\}$ is the subset of the Q matrixes arbitrarily selected from \mathcal{H} , and $0 < \iota < 1$.

Theorem 1: Let the MAS described by (1) under the communication graph \mathcal{G} satisfying Assumptions 1–3, be controlled by the proposed CFDL-DAC. The leader’s command $y_d(k)$ is assumed to be time-invariant, namely, $y_d(k) \equiv c$, c is a constant. Then $e_q(k)$ converges to zero as $k \rightarrow \infty$ for all $q = 1, 2, \dots, N$, if the following condition is satisfied:

$$b_l < \frac{1}{b_p(\max_{q=1,\dots,N} \sum_{p=1}^N a_{q,p} + d_q)}. \tag{21}$$

Proof: We let

$$\begin{aligned} \mathbf{y}(k) &= [y_1(k), y_2(k), \dots, y_N(k)]^T \in \mathbb{R}^N, \\ \mathbf{u}(k) &= [u_1(k), u_2(k), \dots, u_N(k)]^T \in \mathbb{R}^N, \\ \mathbf{e}(k) &= [e_1(k), e_2(k), \dots, e_N(k)]^T \in \mathbb{R}^N, \\ \boldsymbol{\xi}(k) &= [\xi_1(k), \xi_2(k), \dots, \xi_N(k)]^T \in \mathbb{R}^N, \end{aligned}$$

and rewrite equations (2) and (3) respectively as the following form based on $y_d(k) \equiv c$:

$$e_q(k+1) = e_q(k) - \psi_q(k)\delta u_q(k), \tag{22}$$

$$\xi_q(k) = \sum_{p \in N_q} a_{q,p}(e_q(k) - e_p(k)) + d_q e_q(k), \tag{23}$$

Then equations (22) and (23) can be respectively expressed by the following vector forms:

$$\mathbf{e}(k+1) = \mathbf{e}(k) - \boldsymbol{\Psi}(k)\delta\mathbf{u}(k), \tag{24}$$

$$\boldsymbol{\xi}(k) = (\mathbf{L} + \mathbf{D})\mathbf{e}(k), \tag{25}$$

where

$$\begin{aligned} \boldsymbol{\Psi}(k) &= \text{diag}(\psi_1(k), \psi_2(k), \dots, \psi_N(k)) \in \mathbb{R}^{N \times N}, \\ \delta\mathbf{u}(k) &= \mathbf{u}(k) - \mathbf{u}(k-1), \\ \mathbf{D} &= \text{diag}(d_1, d_2, \dots, d_N) \in \mathbb{R}^{N \times N}, \end{aligned}$$

and $\mathbf{L} \in \mathbb{R}^{N \times N}$ is the Laplacian matrix of the follower agents under the communication graph \mathcal{G} .

Similarly, we rewrite (7) as the following vector form:

$$\delta\mathbf{u}(k) = -\boldsymbol{\Theta}(k)\boldsymbol{\xi}(k), \tag{26}$$

where $\boldsymbol{\Theta}(k) = \text{diag}(\theta_1(k), \theta_2(k), \dots, \theta_N(k)) \in \mathbb{R}^{N \times N}$.

Substituting (26) into (24) yields the following closed-loop error dynamics:

$$\mathbf{e}(k+1) = \mathbf{e}(k) + \boldsymbol{\Psi}(k)\boldsymbol{\Theta}(k)\boldsymbol{\xi}(k). \tag{27}$$

Then based on equation (25), it has

$$\begin{aligned} \mathbf{e}(k+1) &= \mathbf{e}(k) + \boldsymbol{\Psi}(k)\boldsymbol{\Theta}(k)(\mathbf{L} + \mathbf{D})\mathbf{e}(k) \\ &= (\mathbf{I} + \boldsymbol{\Theta}(k)\boldsymbol{\Psi}(k)(\mathbf{L} + \mathbf{D}))\mathbf{e}(k). \end{aligned} \tag{28}$$

From Assumption 1, we have that $0 < \psi_q(k) \leq b_p$. Besides, $\mathbf{I} + \boldsymbol{\Theta}(k)\boldsymbol{\Psi}(k)(\mathbf{L} + \mathbf{D})$ must be an irreducible matrix since the communication graph \mathcal{G} is assume to be strongly connected. Based on the resetting mechanism (12), for the matrix $\mathbf{I} + \boldsymbol{\Theta}(k)\boldsymbol{\Psi}(k)(\mathbf{L} + \mathbf{D})$, if the condition (21) is satisfied, then there is at least one row sum of the matrix strictly less than one, which means that it is an irreducible stochastic matrix with positive diagonal entries.

With equation (28), we can conclude that

$$\mathbf{e}(k+1) = \mathbf{G}(k, 1)\mathbf{e}(1), \tag{29}$$

where

$$\mathbf{G}(k, 1) = \prod_{j=1}^k (\mathbf{I} + \boldsymbol{\Theta}(k-j+1)\boldsymbol{\Psi}(k-j+1)(\mathbf{L} + \mathbf{D})).$$

Taking norms on both sides of equation (29) yields

$$\|\mathbf{e}(k+1)\| \leq \|\mathbf{G}(k, 1)\|\|\mathbf{e}(1)\|, \tag{30}$$

By grouping all Q matrixes together for $\mathbf{G}(k, 1)$ in (30), and applying Lemma 2, we have

$$\|\mathbf{e}(k+1)\| \leq \iota^{\lfloor \frac{k}{Q} \rfloor} \|\mathbf{e}(1)\|, \tag{31}$$

where $\lfloor \cdot \rfloor$ indicates the smaller but nearest integer to the real number k/Q . Then it is obtained that $\lim_{k \rightarrow \infty} \|\mathbf{e}(k+1)\| = 0$; that is, the tracking error $e_q(k)$ converges to zero as $k \rightarrow \infty$ for all $q = 1, 2, \dots, N$. This proof is completed.

Next the communication graph for the MAS described by (1) is extended to switching topologies, where each communication graph is strongly connected and at least one agent is communicated to the leader’s command at each time instant k for each graph. To facilitate the description of the switching topologies, we denote $\mathcal{G}(k)$ as a time-varying graph at time instant k , then we can have matrixes $\mathbf{L}(k)$ and $\mathbf{D}(k)$ with the same denotation as aforementioned. Furthermore, we denote $\mathcal{G}_t = \{\mathcal{G}_1, \mathcal{G}_2, \dots, \mathcal{G}_P\}$ as the set of all possible directed graphs, describing the switching communication topologies, where $P \in \mathbb{Z}_+$ is the total number of possible communication topologies. In this case, the stability of the CFDL-DAC is summarized as follows.

Corollary 1: Let the MAS described by (1) satisfying Assumptions 1–3, be controlled by the proposed CFDL-DAC, where the communication topology is the switching graphs $\mathcal{G}_t = \{\mathcal{G}_1, \mathcal{G}_2, \dots, \mathcal{G}_P\}$, each graph is strongly connected, and the leader’s command $y_d(k)$ is assumed to be time-invariant, namely, $y_d(k) \equiv c$. Then the tracking error $e_q(k)$ converges

to zero as $k \rightarrow \infty$ for all $q = 1, 2, \dots, N$, if we select b_t satisfying the following condition:

$$b_t < \frac{1}{b_p \left(\max_{\substack{q=1, \dots, N \\ m=1, \dots, P}} \sum_{p=1}^N a_{q,p}(m) + d_q(m) \right)}, \quad (32)$$

where $(a_{q,p}(m))$ is the weighted adjacent matrix of \mathcal{G}_m , $d_q(m)$ is the entries of $D(m) = \text{diag}(d_1(m), \dots, d_N(m))$ under \mathcal{G}_m , \mathcal{G}_m is the element of set \mathcal{G}_t , and $m = 1, 2, \dots, P$.

Proof: In this case, equation (25) is rewritten as

$$\xi(k) = (\mathbf{L}(k) + \mathbf{D}(k))\mathbf{e}(k), \quad (33)$$

then based on equations (24) and (26), we have

$$\mathbf{e}(k+1) = (\mathbf{I} + \Theta(k)\Psi(k)(\mathbf{L}(k) + \mathbf{D}(k)))\mathbf{e}(k). \quad (34)$$

Since all the possible communication topologies are strongly connected, $\mathbf{I} + \Theta(k)\Psi(k)(\mathbf{L}(k) + \mathbf{D}(k))$ is still an irreducible matrix. It is noted that the set $\{\mathbf{L}_1 + \mathbf{D}_1, \mathbf{L}_2 + \mathbf{D}_2, \dots, \mathbf{L}_P + \mathbf{D}_P\}$ includes all the possible matrices of $\mathbf{L}(k) + \mathbf{D}(k)$. If the condition (32) is satisfied, then the greatest diagonal entry of $\mathbf{I} + \Theta(k)\Psi(k)(\mathbf{L}(k) + \mathbf{D}(k))$ is less than 1; that is, the matrix $\mathbf{I} + \Theta(k)\Psi(k)(\mathbf{L}(k) + \mathbf{D}(k))$ is irreducibly stochastic with positive diagonal entries. Similar to the proof of Theorem 1, it then can be obtained that the tracking error $e_q(k)$ converges to zero as $k \rightarrow \infty$ for all $q = 1, 2, \dots, N$. This completes the proof.

Remark 5: The results of Theorem 1 and Corollary 1 are based on the time-invariant leader's command, and the convergence conditions (21) and (32) require a global communication topology to determine b_t . The limitation in determining b_t probably can be avoided by introducing the stability analysis methods given in [32]. However, the results in [32] are based on the availability of physical model of a controlled plant. The agent system considered in this paper is unknown. Therefore, it may need to integrate other analysis methods in addressing this problem. In future work the case of time-varying leader's command will be investigated for generalizing the proposed approach given in this paper.

The conditions (21) and (32) given in Theorem 1 and Corollary 1 seem from their mathematical forms that they can be ensured by simply choosing $-b_t \geq \theta_q(k) < 0$. However, it should be noted that $\theta_q(k)$ is designed to achieve its automatic tuning using only the local information among neighboring agents. This is different from the most existing distributed control schemes where the control gains are usually calibrated heuristically and chosen as fixed values if the physical dynamics of a controlled plant are unknown. As presented by the control gain updating algorithm (11), this automation helps search and approximate the optimal value of the control gain in the sense of minimizing the control objective function (8). Moreover, b_t is purposed to be chosen a value as large as possible under the conditions of (21) and (32), so that $\theta_q(k)$ can be searched in a larger space in order to better approximate the optimal value.

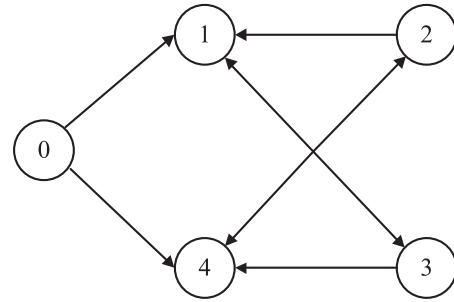


FIGURE 1. Fixed communication topology.

IV. SIMULATION RESULTS

To illustrate the effectiveness of the proposed CFDL-DAC, three examples are simulated in this paper. The three examples consider the same nonlinear heterogeneous discrete-time MAS, where the first two examples are conducted under the fixed and switching communication topologies, respectively, with time-invariant leader's command, and the third example is proceeded with time-varying leader's command.

The nonlinear heterogeneous discrete-time MAS consists of four follower agents, where the follower agent models are governed by

$$\begin{cases} y_1(k+1) = \frac{y_1(k-1)y_1(k)}{1 + y_1^2(k-1) + y_1^2(k)} + 3u_1(k), \\ y_2(k+1) = \frac{y_2(k)}{1 + y_2^4(k)} + u_2^3(k), \\ y_3(k+1) = \frac{y_3(k-1)y_3(k)u_3(k-1) + u_3(k)}{1 + y_3^2(k-1) + y_3^2(k)} \\ \quad + u_3^3(k), \\ y_4(k+1) = \frac{y_4(k)u_4(k)}{1 + y_4^6(k)} + 2u_4(k), \end{cases} \quad (35)$$

and the initial system outputs of the four follower agents are set as $y_1(1) = y_2(1) = y_3(1) = y_4(1) = 0$. Furthermore, we would like to point out that the dynamic models of the simulated MAS are only for generating the input-output data, and are not involved in the control system design.

A. EXAMPLE 1: FIXED COMMUNICATION TOPOLOGY

Fig. 1 shows the communication topology, where the leader is described by vertex 0, and only agents 1 and 4 receive the leader's command. The communication among neighboring agents is depicted by solid arrows. We use 0 and 1 as weights for the information communicated between two adjacent follower agents, therefore the Laplacian matrix among the follower agents is given by

$$\mathbf{L} = \begin{pmatrix} 2 & -1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 2 \end{pmatrix},$$

and $\mathbf{D} = \text{diag}(1, 0, 0, 1)$.

It is seen that the communication topology is strongly connected. The bound of $\theta_q(k)$ is set as $b_t = 0.2$, and the

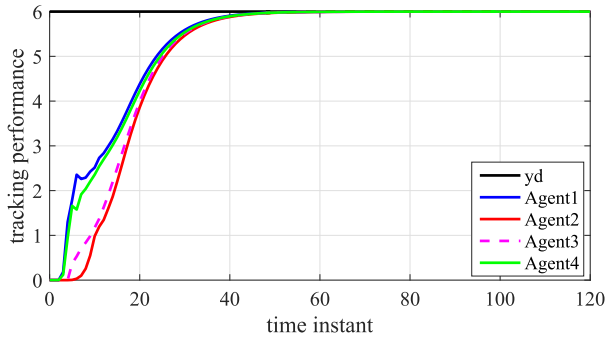


FIGURE 2. Tracking performance (Example 1).

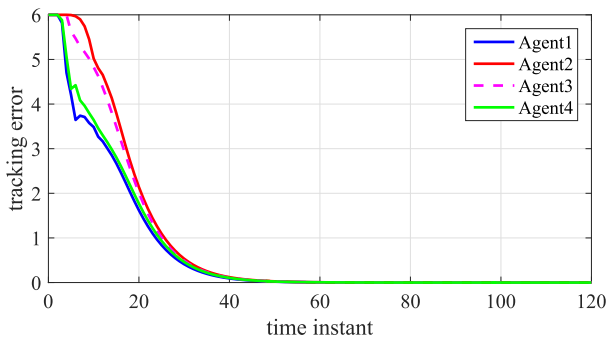


FIGURE 3. Tracking error (Example 1).

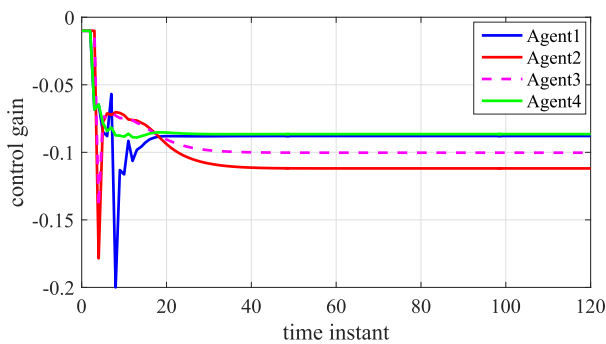


FIGURE 4. Control gains (Example 1).

bound of $\psi_q(k)$ is given as $b_p = 1$. Thus it can be obtained:

$$\begin{aligned} \{0.2, 0.15\} &< \frac{1}{1 \times \left(\max_{q=1, \dots, 4} \sum_{p=1}^4 a_{q,p} + d_q \right)} \\ &= \frac{1}{1 \times 3} \approx 0.3, \end{aligned} \quad (36)$$

which indicates that the convergence condition (21) of Theorem 1 is satisfied.

We set the leader's command as $y_d(k) = 6$, and the simulation is executed with 120 time instants. The simulation results are shown in Figs. 2–4. Fig. 2 and Fig. 3 are the tracking performances and tracking errors of the four follower agents, respectively. Fig. 4 shows the updating values of the control gains for the four follower agents.

It is obvious that the system outputs of the four follower agents have large deviations from the leader's command at

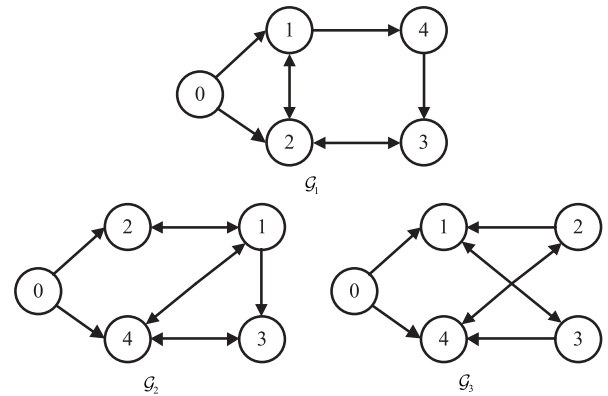


FIGURE 5. Switching communication topologies.

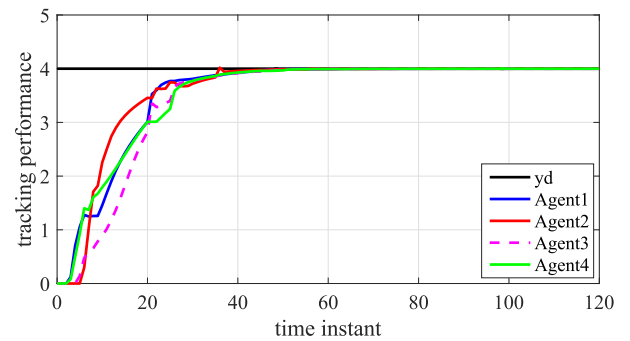


FIGURE 6. Tracking performance (Example 2).

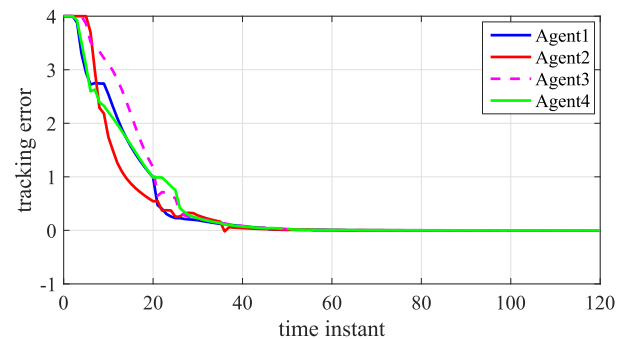


FIGURE 7. Tracking error (Example 2).

the primary time instants, but all the tracking errors of the four agents gradually decrease and the consensus tracking is basically achieved after 60 time instants. Furthermore, we can conclude from Fig. 4 that the proposed CFDL-DAC keeps automatically tuning and updating the control gains for the four follower agents to search the optimal values before the consensus tracking is achieved.

B. EXAMPLE 2: SWITCHING COMMUNICATION TOPOLOGIES

In this subsection, we represent that the proposed CFDL-DAC also works well under switching communication topologies. The communication topologies switch randomly among three graphs, which are described by the set $\mathcal{G}_t = \{\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3\}$, as shown in Fig. 5. Fig. 5 shows that each graph

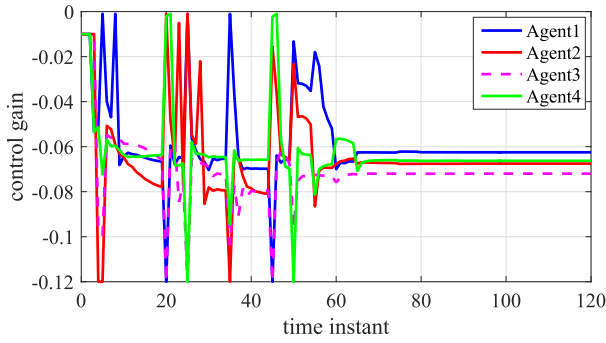


FIGURE 8. Control gain (Example 2).

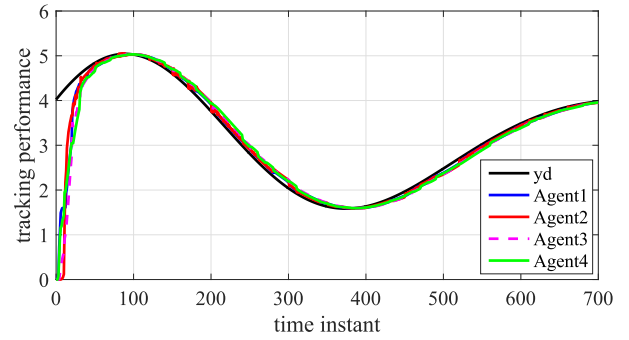


FIGURE 11. Tracking performance for switching topologies (Example 3).

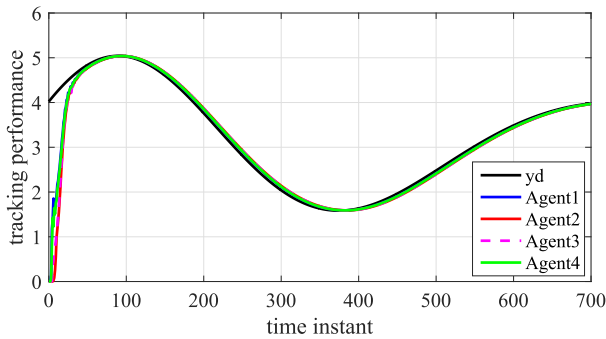


FIGURE 9. Tracking performance for fixed topology (Example 3).

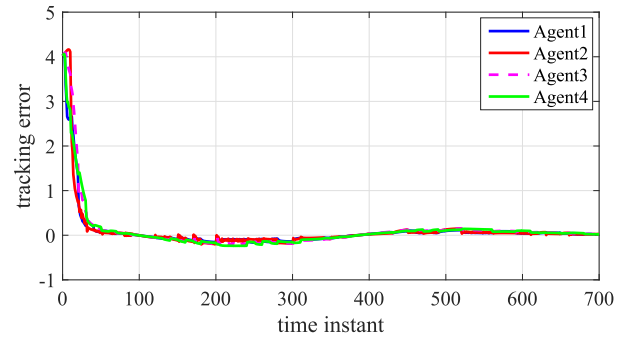


FIGURE 12. Tracking error for switching topologies (Example 3).

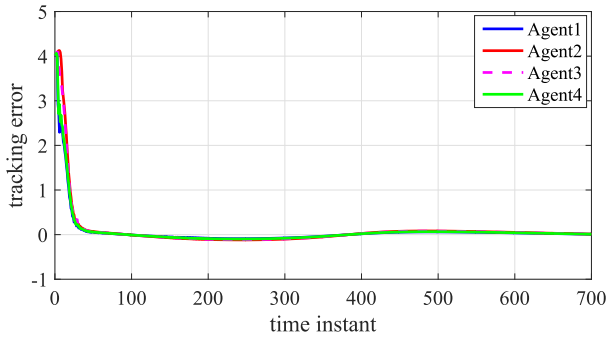


FIGURE 10. Tracking error for fixed topology (Example 3).

of the three communication topologies is strongly connected. The bound of $\theta_q(k)$ and $\psi_q(k)$ are respectively set as $b_t = 0.12$ and $b_p = 2$, therefore it can be obtained that

$$0.12 < \frac{1}{2 \times \left(\max_{q=1, \dots, 4} \sum_{p=1}^4 a_{q,p} + d_q \right)} = \frac{1}{2 \times 3} \approx 0.17, \quad (37)$$

which indicates that the convergence condition (32) for Corollary 1 is satisfied.

We set the leader's command as $y_d(k) = 4$, and the simulation results are shown in Figs. 6–8. It is observed that the consensus tracking is achieved, and the tracking errors of all the follower agents converge to zero after 120 time instants which verifies the result of Corollary 1. The automatic tuning of the control gain, as shown in Fig. 8, contributes to the ability of tracking the leader's command for the proposed CFDL-DAC even under the switching communication topologies.

C. EXAMPLE 3: TIME-VARYING LEADER'S COMMAND

To further demonstrate the effectiveness of the proposed approach, in this subsection we consider the time-varying leader's command described by

$$y_d(k) = 3 + 2 \sin(0.9k\pi/260) + \cos(k\pi/240), \quad (38)$$

and the simulation is executed with 700 time instants. In this simulation, the fixed graph as shown in Fig. 1 and the switching topologies as depicted in Fig. 5 are considered.

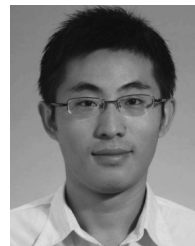
The simulation results are presented in Figs. 9-12. These results show that the system outputs of the four follower agents rapidly approximate to the neighborhood of the leader's command from a large deviation at the initial time instant. Although the tracking errors do not converge to zero, they reduce to a small bound.

V. CONCLUSION

This paper investigated a distributed leader-follower consensus tracking approach for a class of unknown nonlinear non-affine discrete-time MASs. A data-driven distributed adaptive control scheme was designed using only the local measurements exchanged among neighboring agents via the dynamic linearization method applied to the controlled MAS and the ideal distributed controller. The stabilities of the proposed distributed adaptive control approach were rigorously guaranteed under both the fixed and switching communication topologies. In future, investigating a more general distributed adaptive control scheme and analyzing its stability properties for a time-varying leader's command are interesting topics.

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