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A Joint Inventory and Transport Capacity Problem With Carbon Emissions

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ABSTRACT This paper considers an inventory system consisting of one supplier and one retailer, and studies a joint inventory policy and transport capacity decision for this inventory system with carbon emissions. The joint decision simultaneously determines the reorder interval of the retailer and the number of the vehicles used to transport the product from the supplier to the retailer while minimizing the inventory replenishment related cost and the carbon emission cost. This paper considers the carbon emissions from holding and replenishing inventory at the retailer, and calculates the carbon emissions based on the reorder interval of the retailer and the number of the vehicles used to transport the product. This problem is formulated as a nonlinear mixed integer programming, and an algorithm is designed to solve the nonlinear mixed integer programming to optimality. The computational results show that the integrated model proposed in this paper can reduce the system-wide cost and the carbon emissions by 5.6% and 14.42% in average, respectively. For the cases that the product with low fixed ordering cost and the vehicle with high fuel consumption, the superiority of the integrated model is more prominent. Besides, the computational results also provide some other management insights.

INDEX TERMS Inventory management, transport capacity, carbon emissions.

I. INTRODUCTION

Inventory decision is to determine when to place an order, and how much to order when an order is placed, i.e., inventory policy decision. When the inventory policy is determined, the vehicles should be arranged to transport the product for replenishing inventory. One of the main problem associated with the vehicles for replenishing inventory is to determine how many vehicles should be used to transport the product, which is called the transport capacity decision. The transport capacity decision can be made by use of the ordering quantities of the product according to the inventory policy. In practice, the inventory decision and the transport capacity decision are always made sequentially and separately, which might lead to suboptimal decision results, especially in the case that carbon emissions are considered. This paper studies a joint decision problem for the inventory replenishment policy and the transport capacity decision with carbon emissions, which simultaneously determines the inventory policy and the number of the vehicles used to transport the product for the inventory system with considering the cost of carbon emissions.

The problem studied in this paper is motivated by the following reasons. Firstly, in order to calculate the carbon emissions from the inventory replenishment activities accurately and explore the impacts of carbon emissions on the inventory policy, the number of the vehicles used to transport the product in the inventory replenishment should be known in advance. This requires that when making inventory decision with carbon emissions, the transport capacity decision should be integrated into the inventory decision. Secondly, from the point of view of the firm, the legislation for curbing carbon emissions has been enacted in many countries, and the firm needs to optimize the number of the vehicles used to transport the product in the inventory replenishment for reducing the carbon emissions from the transportation. Finally, from the actuality of the logistics in China, the statistics show that the empty return ratio of the vehicles for delivering the product in the logistics in China is up to 40%, and it is urgent for the firm to conduct the transport capacity decision in the inventory replenishment to reduce the effects of the high empty return ratio.

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A. LITERATURE REVIEW

The studies in this paper are mainly related to two aspects of research, namely, inventory management with carbon emissions and transport capacity decisions. Inventory management with carbon emissions has been extensively studied, in which the studies are carried out by integrating carbon emissions into the classic inventory models, such as Economic Order Quantity (EOQ) ([5], [7], [10], [15], [16], [25], and [26]), dynamic lot-sizing ([1], [2], [3], [13], [22], [21], and [29]), Economic Production Quantity (EPQ) ([17]), and newsvendor model ([28]). Additionally, for this research stream, there are some studies conducted in the framework of supply chain. For example, Cachon studies the retail supply chain network design with considering the carbon emission cost under the carbon tax and cap-and-trade regulations ([8]); Sarkar *et al.* study a three-echelon supply chain model with single-setup-multiple-delivery policy and variable carbon emission costs ([23]); Peng *et al.* and Sun *et al.* study the contract designs for the supply chain under the carbon cap-and-trade regulation ([20], and [24]); Zhao *et al.* study the carbon emission reduction related issues in the forestry-pulp and paper supply chain ([30]). In the above studies, the transport capacities (the number of the vehicles) employed for delivering the product are always assumed to be sufficient and given in advance. This paper relaxes the assumption on the transport capacities, and studies the joint inventory and transport capacity decision with carbon emissions.

Similarly, the topic of transport capacity decisions to determine the number of the vehicles in the fleet in transportation networks has also received extensive attentions ([4], [9], [12], [19], and [27]). However, the studies on the transport capacity decision in the framework of inventory management are few. Chen studies a purchase quantity decision problem, in which he needs to determine the optimal purchase quantities and the number of the hired vehicles used to transport the product while minimizing the inventory cost, transport cost, and purchase cost ([10]). This paper studies the transport capacity decision in the inventory replenishment with carbon emissions, and the purposes of the research in this paper is to curb the carbon emissions in the inventory replenishment, and explore the impacts of the carbon emissions on the inventory policies.

Table 1 shows the comparisons of the research in this paper with the literature, and the characteristics of the research in this paper. In summary, differing with the above studies, this paper considers a simple inventory system consisting of one supplier and one retailer, and studies the inventory policy decision for this inventory system with carbon emissions, in which the number of the vehicles used to transport the product from the supplier to the retailer is not given in advance. This paper needs to determine the reorder interval of the retailer and the number of the vehicles used to transport the product from the supplier to the retailer simultaneously. This paper considers the carbon emissions from holding and replenishing inventory at the retailer, and calculates the carbon emissions based on the reorder interval of the retailer, and the number of the vehicles used to transport the product from the supplier to the retailer. This problem is formulated as a nonlinear mixed integer programming problem, and an algorithm is designed to solve this nonlinear mixed integer programming to optimality. Finally, the computational experiments are conducted to explore the advantages of the joint decision for the inventory replenishment and the transport capacity with carbon emissions, and the impacts of some key parameters in the model on the computational results.

TABLE 1. Characteristics of the research in this paper.

	Transport capacity decisions	Inventory and supply chain	Carbon emissions
Absi et al. (2013)		√	√
Absi et al. (2016)		√	√
Beaujon and Turnquist (1991)	√		
Benjaafar et al. (2013)		√	√
Bouchery et al. (2012)		√	√
Bozorgi et al. (2014)		√	√
Cachon (2014)		√	√
Carbajal et al. (2013)	√	√	
Chen (2013)	√	√	
Chen et al. (2013)		√	√
Du and Hall (1997)	√	√	
Ghosh et al. (2017)		√	√
Hua et al. (2011)		√	√
Konur (2014)		√	√
Mishra et al. (2020)		√	√
Papier and Thonemann (2008)	√		
Peng et al. (2018)		√	√
Purohit et al. (2016)		√	√
Rete Helmrich et al. (2015)		√	√
Sarkar et al. (2016)		√	√
Sun et al. (2018)		√	√
Taleizadeh et al. (2018)		√	√
Toptal et al. (2014)		√	√
Tsao et al. (2017)		√	√
Turnquist and Jordan (1986)	√	√	
Zhang et al. (2016)		√	√
Zhao et al. (2019)		√	√
This paper	√	√	√

B. CONTRIBUTIONS AND PAPER STRUCTURE

The main contributions of this paper are as follows:

- This paper studies an inventory management problem with carbon emissions, in which the transport capacity is endogenous. This means that the inventory policy and the number of the vehicles used to transport the product are determined simultaneously in the inventory management with considering carbon emissions. The main reason for this is to curb the carbon emissions from the inventory replenishment by optimizing the inventory policy and the number of the vehicles used to transport the product, and meanwhile, to explore the impacts of carbon emissions on the inventory policy.
- The computational experiments show that the integrated model proposed in this paper is more superior than the traditional sequenced decision model. For example, by optimizing the number of the vehicles used to transport the product in the inventory management, the system-wide cost is decreased by 5.6% in average, and the carbon emissions are decreased by 14.42% in average. And furthermore, for the product with low fixed ordering cost and for the vehicle with high fuel consumption, the superiority of the integrated model is

more prominent. The computational experiments also provide some other management insights.

The remainder of this paper is organized as follows. Section II formulates this problem as a nonlinear mixed integer programming, Section III introduces a solution approach to solve this nonlinear mixed integer programming, Section IV presents the computational results, and Section V concludes the paper.

II. MODEL FORMULATION

This paper considers an inventory system with one supplier and one retailer, and assumes that the inventory system is a centralized system. The retailer replenishes its inventory from the outside supplier via EOQ model. Whenever the retailer places an order, a fixed ordering cost K is incurred, and meanwhile, holding inventory at the retailer incurs the inventory holding cost, where the inventory holding cost per unit per time is denoted by h. This paper also assumes that the retailer's demand is deterministic, and sets the demand rate at the retailer as λ. The objective for this inventory system is to determine the order quantities or the reorder interval for the retailer. The reorder interval of the retailer is denoted by T, and then the average inventory cost can be formulated as:

$$Z_1(T) = \frac{K}{T} + \frac{1}{2}\lambda h T.$$

Note that this paper assumes the replenishment leading time is deterministic, and without loss of generality, sets the leading time as zero. In fact, the leading time has no effect on the results of the model in this paper, and the model can be easily extended to the case that the leading time is nonzero ([18]).

In the process of inventory control for the retailer, this paper wants to explore how the carbon emissions from holding and replenishing inventory activities affect the order quantity or the reorder interval. In order to do this, this paper presents a method to calculate the carbon emissions in this inventory system, which characters the impacts of the order frequency on the carbon emissions. Obviously, the major energy for the retailer to hold inventory at the warehouse is electricity. Then let the amount of energy needed to hold per unit product per time as H_e (*kWh*), and let the carbon emissions from one unit of energy for holding the product as E_h (*kg/kWh*). The average carbon emissions from holding inventory at the warehouse for the retailer are $\frac{1}{2} \times \lambda \times H_e \times E_h \times T$.

Next this paper calculates the carbon emissions from replenishing inventory from the supplier to the retailer. This paper assumes only one type of the vehicles used to transport the product, and the capacities of the vehicles are finite. After transporting the product from the supplier to the retailer, the vehicles must be return to the supplier and no cargo is transported by the vehicles on the way back. Then the carbon emissions for transporting the product from the supplier to the retailer are determined by the weight of the shipment, the fuel consumption of the vehicles, the fuel emissions, and the transport distance ([14]). The transport distance is the distance that the vehicle travels. The notations for calculating the carbon emissions from replenishing inventory are depicted in the following:

- M_L: the maximum load capacity of the vehicle; D_{0r}: the transport distance between the supplier and the retailer (*km*);
- FC_e: the fuel consumption of the vehicle per unit distance when it is empty (*l/km*);
- FC_f: the fuel consumption of the vehicle per unit distance when it is full (*l/km*);
- FE: the carbon emissions released by the consumption of one unit of fuel (*kg*);

For every order of the retailer, this paper firstly needs to determine the number of the vehicles used to transport the product from the supplier to the retailer. The number of the vehicles is denoted by N, and then

$$N = \lceil \frac{T\lambda}{M_L} \rceil,$$

where $T\lambda$ is the demand of the retailer in the period of one reorder interval T, and $\lceil x \rceil$ represents the smallest integer that is greater than or equal to x.

Let LF_i be the load factor of the ith vehicle, where the load factor means the actual load transported by the vehicle versus the maximum load of the vehicle, $i = 1, 2, \cdots, N$. Without loss of generality, this paper considers the first $(N - 1)$ vehicles are full load, i.e., $LF_i = 1,\ i = 1, 2, \cdots, N - 1$, and the load factor of the Nth vehicle can be formulated as

$$LF_N = \frac{T\lambda}{M_L} - (N - 1).$$

The fuel consumption for transporting the product from the supplier to the retailer per unit of distance is $\sum_{i=1}^{N}[FC_e + (FC_f - FC_e) \times LF_i]$, and then the carbon emissions for transporting the product from the supplier to the retailer in the period of one reorder interval T, denoted by E_{0r}, can be figured out by the following formulas:

$$\begin{aligned} E_{0r} &= FE \times D_{0r} \times \sum_{i=1}^{N}[FC_e + (FC_f - FC_e) \times LF_i] \\ &\quad + N \times FE \times D_{0r} \times FC_e && (1) \\ &= FE \times D_{0r} \times [N \times FC_e + (FC_f - FC_e) \\ &\quad \times \frac{T\lambda}{M_L}] + N \times FE \times D_{0r} \times FC_e && (2) \\ &= FE \times D_{0r} \times (FC_f - FC_e) \times \frac{T\lambda}{M_L} \\ &\quad + 2 \times N \times FE \times D_{0r} \times FC_e && (3) \end{aligned}$$

where the first term of (1) is the carbon emissions for transporting the product from the supplier to the retailer, and the second term of (1) is the carbon emissions associated with the empty vehicle returns from the retailer to the supplier.

Let φ be the cost of carbon emissions per unit released, i.e., the carbon tax (*1/kg*). Therefore, the average cost of carbon emissions per cycle, denoted by $Z_2(T,N)$, can be written as follows:

$$Z_2(T,N) = \frac{\varphi \times E_{0r}}{T} + \varphi \times \frac{1}{2} \times \lambda \times H_e \times E_h \times T$$
$$= \frac{NC_1}{T} + \frac{1}{2}C_2 T + C_3\lambda, \quad (4)$$

where $C_1 \equiv 2 \times \varphi \times FE \times D_{0r} \times FC_e$, $C_2 \equiv \varphi \times \lambda \times H_e \times E_h$, and $C_3 \equiv \varphi \times FE \times D_{0r} \times (FC_f - FC_e) \times \frac{1}{M_L}$.

Adding the average cost of carbon emissions into the EOQ model, the following objective function is got:

$$Z(T,N) = Z_1(T) + Z_2(T,N)$$
$$= \frac{K}{T} + \frac{1}{2}h\lambda T + \frac{NC_1}{T} + \frac{1}{2}C_2T + C_3\lambda$$
$$= \frac{K+NC_1}{T} + \frac{1}{2}(h\lambda + C_2)T + C_3\lambda. \quad (5)$$

Proposition 1: The objective function $Z(T,N)$ is nonconvex and nonconcave on $T > 0, N > 0$.

Proof: Let $Z(T,N) = f_1(T) + f_2(T,N)$, where $f_1(T) = \frac{K}{T} + \frac{1}{2}(h\lambda + C_2)T + C_3\lambda$, and $f_2(T,N) = \frac{NC_1}{T}$. Since $f_1''(T) = \frac{2K}{T^3} > 0$, then $f_1(T)$ is convex on $T > 0$. The following only needs to prove $f_2(T,N)$ is nonconvex and nonconcave on $T > 0$ and $N > 0$.

The Hessian matrix of $f_2(T,N)$ is

$$\begin{pmatrix} \frac{2NC_1}{T^3} & -\frac{C_1}{T^2} \\ 0 & -\frac{C_1}{T^2} \end{pmatrix}, \quad (6)$$

and obviously, the matrix (6) is nonpositive semidefinite and nonnegative semidefinite. Thus $f_2(T,N)$ is nonconvex and nonconcave on $T > 0$ and $N > 0$. ■

This paper wants to minimize $Z(T,N)$ over T and N subject to some constraints, and obtains the following optimization problem:

$$\mathcal{P}: \ \min Z(T,N) = \frac{K+NC_1}{T} + \frac{1}{2}(h\lambda + C_2)T + C_3\lambda \quad (7)$$
$$s.t.\ (N-1)\cdot M_L < T\lambda \le N \cdot M_L, \quad (8)$$
$$0 < N \le N_{MAX}, \text{ and integer}, \quad (9)$$
$$T > 0, \quad (10)$$

where N_{MAX} represents the maximum number of the vehicles available for transporting the products, i.e., the maximum transport capacity employed for transporting the products, and constraint (8) states that the number of the vehicles employed for replenishing inventory at the retailer r in the period of one reorder interval T must be suitable. Since the objective funciton $Z(T,N)$ is nonconvex and nonconcave, and the constraint (9) requires integer, obtaining the optimal solution of the optimization problem $\mathcal{P}$ directly is more difficult. In the next section, this paper will introduce a procedure to solve the optimization $\mathcal{P}$ to optimality quickly.

Note that this paper ignores the transport cost, as do in most of the inventory management literature ([18]). For the fixed and variable transport cost structure that is adopted by most of the literature ([9]), considering transport cost in the problem studied in this paper does not increase the complexity of the problem, which is equivalent to increase the values of C_1 and C_3 in the optimization problem $\mathcal{P}$.

III. SOLUTION APPROACH

This section shows how to solve the optimization problem $\mathcal{P}$. The optimization problem $\mathcal{P}$ is a nonlinear mixed integer programming, and the objective function of $\mathcal{P}$ is nonconvex and nonconcave. Then, it is hard to solve the optimization problem $\mathcal{P}$ to optimality directly. Therefore, this paper solves the optimization problem $\mathcal{P}$ in the following ways.

For each $N = 1, 2, \cdots, N_{MAX}$, this paper first focusses on the following subproblem:

$$\mathcal{P}'_N: \ \min Z'_N(T) = \frac{K+NC_1}{T} + \frac{1}{2}(h\lambda + C_2)T + C_3\lambda$$
$$s.t.\ T\lambda \le N \cdot M_L, \quad (11)$$
$$T > 0,$$

where constraint (11) limits the order quantities of the retailer. In the subproblem $\mathcal{P}'_N$, the number of the vehicles, N, is fixed, and then $Z'_N(T)$ is convex on constraint (11) and $T > 0$. Thus the subproblem $\mathcal{P}'_N$ is a convex optimization, and the Karush-Kuhn-Tucker (KKT) conditions provide necessary and sufficient conditions for optimality for the subproblem $\mathcal{P}'_N$ ([6]).

This paper obtains the optimal solution of the subproblem $\mathcal{P}'_N$ by solving the corresponding KKT conditions, which are given in the following:

$$-\frac{K+NC_1}{T^2} + \frac{1}{2}(h\lambda + C_2) + \mu\lambda = 0, \quad (12)$$
$$\mu(T\lambda - N\cdot M_L) = 0, \quad (13)$$
$$T\lambda - N \cdot M_L \le 0, \quad (14)$$
$$\mu \ge 0, \quad (15)$$

where the parameter μ is the lagrange multiplier associated with the constraint (11). By analyzing the KKT conditions, this paper first considers $\mu = 0$, and by the condition (12), this paper gets

$$T_1^* = \sqrt{\frac{2(K+NC_1)}{h\lambda + C_2}}.$$

If T_1^* satisfies the condition (14), i.e., $T_1^*\lambda - N\cdot M_L \le 0$, then T_1^* is the optimal solution of the subproblem $\mathcal{P}'_N$. Otherwise, this paper obtains that $\mu > 0$, and by the condition (13), this paper gets

$$T_2^* = \frac{N \cdot M_L}{\lambda}.$$

Then T_2^* is the optimal solution of the subproblem $\mathcal{P}'_N$.

Based on the subproblem $\mathcal{P}'_N$, $N = 1, 2, \cdots, N_{MAX}$, this paper has the following theorem.

Theorem 1: For the subproblem $\mathcal{P}'_N$, let $\hat{T}_N$ denote the optimal solution of the subproblem $\mathcal{P}'_N$, $N = 1, 2, \cdots, N_{MAX}$, and $\mathcal{P}'_{\hat{N}}$ denote the subproblem with minimum objective function value, i.e., $Z'_{\hat{N}}(\hat{T}_{\hat{N}}) = \min\{Z'_1(\hat{T}_1), Z'_2(\hat{T}_2), \cdots, Z'_{N_{MAX}}(\hat{T}_{N_{MAX}})\}, 0 < \hat{N} \leq N_{MAX}$. Then $(\hat{T}_{\hat{N}}, \hat{N})$ is the optimal solution of the optimization problem $\mathcal{P}$.

Proof: Since $\hat{T}_{\hat{N}}$ is the optimal solution of the subproblem $\mathcal{P}'_{\hat{N}}$, then $\hat{T}_{\hat{N}}\lambda \leq \hat{N} \cdot M_L$. In order to prove that $(\hat{T}_{\hat{N}}, \hat{N})$ is the optimal solution of the optimization problem $\mathcal{P}$, this paper first proves that $(\hat{T}_{\hat{N}}, \hat{N})$ is a feasible solution of the optimization problem $\mathcal{P}$, i.e.,

$$\hat{T}_{\hat{N}}\lambda \leq \hat{N} \cdot M_L \Longrightarrow (\hat{N} - 1) \cdot M_L < \hat{T}_{\hat{N}}\lambda \leq \hat{N} \cdot M_L.$$

This paper needs only to prove $(\hat{N} - 1) \cdot M_L < \hat{T}_{\hat{N}}\lambda$. If $\hat{T}_{\hat{N}}\lambda \leq (\hat{N} - 1) \cdot M_L$, then $\hat{T}_{\hat{N}}$ is feasible for the subproblem $\mathcal{P}'_{\hat{N}-1}$, and then $Z'_{\hat{N}}(\hat{T}_{\hat{N}}) > Z'_{\hat{N}-1}(\hat{T}_{\hat{N}}) \geq Z'_{\hat{N}-1}(\hat{T}_{\hat{N}-1})$, which contradicts with the condition that $Z'_{\hat{N}}(\hat{T}_{\hat{N}})$ is the minimum objective function value. Thus $(\hat{N} - 1) \cdot M_L < \hat{T}_{\hat{N}}\lambda \leq \hat{N} \cdot M_L$.

Next this paper proves that $(\hat{T}_{\hat{N}}, \hat{N})$ is optimal for the optimization problem $\mathcal{P}$. For any feasible solution of the optimization problem $\mathcal{P}$, denoted by (T^0, N^0), this paper has $(N^0 - 1) \cdot M_L < T^0\lambda \leq N^0 \cdot M_L$. Then this paper has $T^0\lambda \leq N^0 \cdot M_L$, and T^0 is feasible for the subproblem $\mathcal{P}'_{N^0}$. Thus this paper obtains that $Z(T^0, N^0) = Z'_{N^0}(T^0) \geq Z'_{N^0}(T^0_{N^0}) \geq Z'_{\hat{N}}(\hat{T}_{\hat{N}}) = Z(\hat{T}_{\hat{N}}, \hat{N})$. ■

In order to solve the optimization problem $\mathcal{P}$ and based on **Theorem 1**, this paper firstly needs to solve the N_{MAX} subproblems, and then to select the one with minimum objective function value. The reorder interval of the retailer and the number of the vehicles used to transport the product corresponding to the subproblem with minimum objective function value is the optimal solution of the optimization problem $\mathcal{P}$. In practice, the maximum number of the vehicles available for transporting the product in the distribution network is finite, and usually a small integer, i.e., N_{MAX} is usually a small integer. Thus, this paper can solve the optimization problem $\mathcal{P}$ quickly and efficiently by making use of the procedure described above.

IV. COMPUTATIONAL RESULTS

This section conducts the computational experiments to explore the advantages of the proposed model in reducing the system-wide cost and the carbon emissions, and meanwhile to analyze the impacts of the parameters of the optimization problem $\mathcal{P}$ on the computational results, such as the fixed ordering cost and the carbon tax. For ease of expression, the model proposed in this paper is called as the *integrated decision model*. For comparison purpose, this paper defines a *sequenced decision model*, in which this paper firstly obtains the order interval of the retailer via the EOQ model with the constraint $T\lambda \leq N_{MAX} \cdot M_L$, and then determines the number of the vehicles used to transport the product and the carbon emission cost by the approach proposed in the Section II. In practice, many firms adopt the sequenced decision model to determine the reorder interval at the retailer, and the number of the vehicles used to transport the product in the inventory replenishment.

TABLE 2. Values of the parameters associated with the vehicles.

Parameter	FE	FC_e	FC_f	M_L
Value	2.25	0.1	0.2	1000

A. TEST ENVIRONMENT

Before presentation of the computational results, this paper firstly describes the test environment of the computational experiments in detail. This paper considers the demand of the retailer, λ, as 100, 200, 600, 800, 1000, and 2000, respectively, and for each demand, this paper randomly generates the transport distance from the supplier to the retailer, D_{0r}, in [100, 1000]. This paper sets the fixed ordering cost, K, as 1500, the holding cost per unit per time, h, as 1, and the carbon tax, i.e., the cost of carbon emissions per unit, φ, as 2, 4, 6, 8, and 10, respectively. By combination of the demand of the retailer and the carbon tax, this paper generates 30 $(6 \times 5 = 30)$ basic problem instances. For different objectives of the computational experiments, this paper will generate more test problem instances based on the 30 basic problem instances.

This paper sets the amount of energy needed to hold per unit product per time, H_e, as 0.01, and the carbon emissions from one unit of energy for holding the product, E_h, as 0.55 ([8]). The values of the parameters associated with the vehicles are summarized in Table 2.[1] Without loss of generality, this paper sets the maximum number of the vehicles available for replenishing inventory, N_{MAX}, as 10.

B. INTEGRATED MODEL VS. SEQUENCED MODEL

In this subsection, this paper explores the changes of the reorder interval of the retailer and the number of the vehicles used to transport the product by considering the carbon emissions, and more important, explores the reductions in the carbon emissions and the system-wide cost by adopting the integrated model proposed in this paper. This paper solves the 30 basic problem instances, and reports the computational results in the following.

Figure 1 presents the impacts of the carbon emissions on the reorder interval of the retailer, and Figure 2 presents the changes of the number of the vehicles used to transport the product by considering the carbon emissions. Obviously, this paper finds that with considering the carbon emissions in the inventory replenishment activities, the reorder interval of the retailer may increase, and may decrease as well, and the number of the vehicles used to transport the product is non-increasing. But this paper further finds that the number of the vehicles used to transport the product per unit time, i.e., N/T, decreases with considering the carbon emissions, which means the frequency of transporting the product from the supplier to the

[1] https://en.wikipedia.org/wiki/Fuel_economy_in_automobiles.

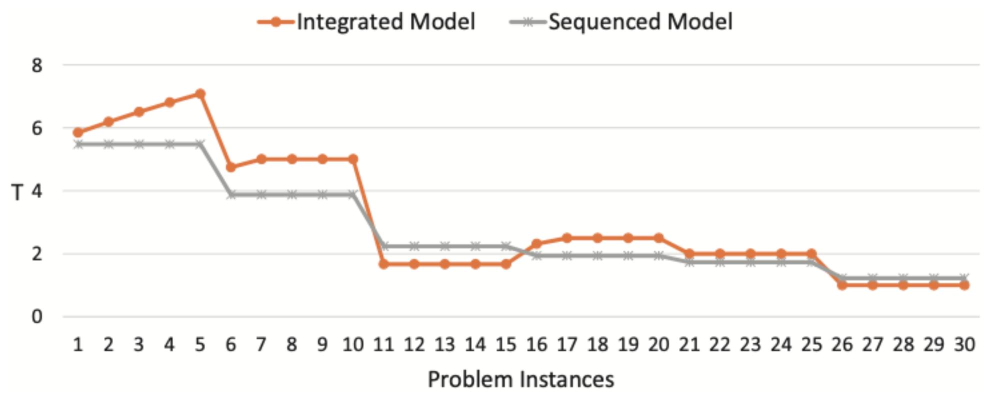

FIGURE 1. The reorder interval of the retailer *r*.

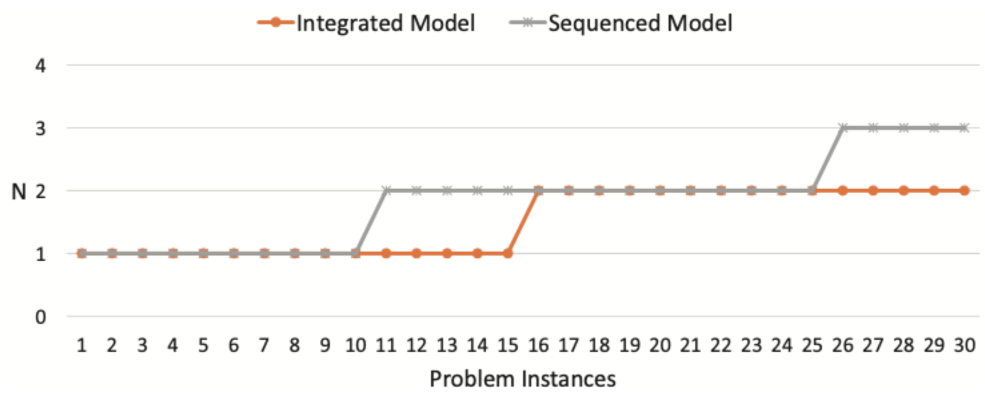

FIGURE 2. The number of the vehicles.

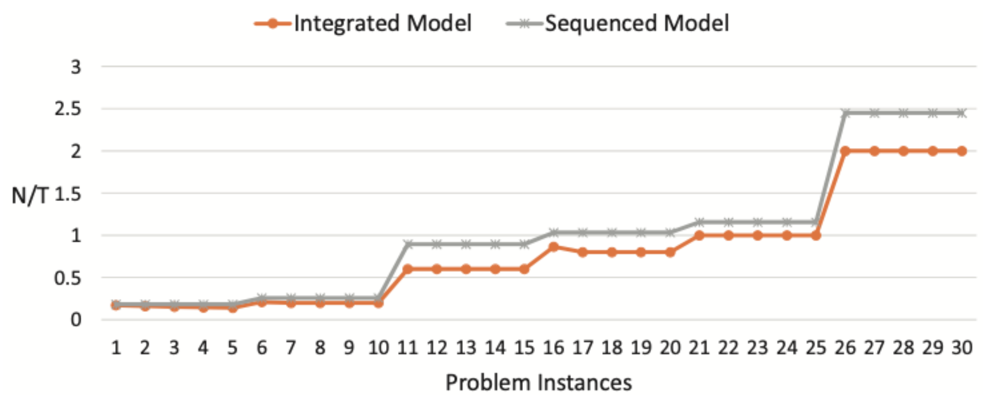

FIGURE 3. The ratio of the number of the vehicles to the reorder interval of the retailer.

retailer decreases for reducing the carbon emissions from the inventory replenishment. Figure 3 presents the changes of N/T between the integrated model and the sequenced model.

Next, this paper compares the integrated model and the sequenced model in terms of the system-wide cost and the carbon emissions. Note that the system-wide cost for the integrated model and sequenced model is equal to the carbon emission cost plus the inventory cost. In order to do that, this paper firstly defines Δ_1, and Δ_2 in the following:

$$\Delta_1 = \frac{\text{System-wide Cost(SM) - System-wide Cost(IM)}}{\text{System-wide Cost(SM)}} \times 100,$$

and

$$\Delta_2 = \frac{\text{Carbon Emission(SM) - Carbon Emission(IM)}}{\text{Carbon Emission(SM)}} \times 100,$$

where IM represents the integrated model, and SM represents the sequenced model.

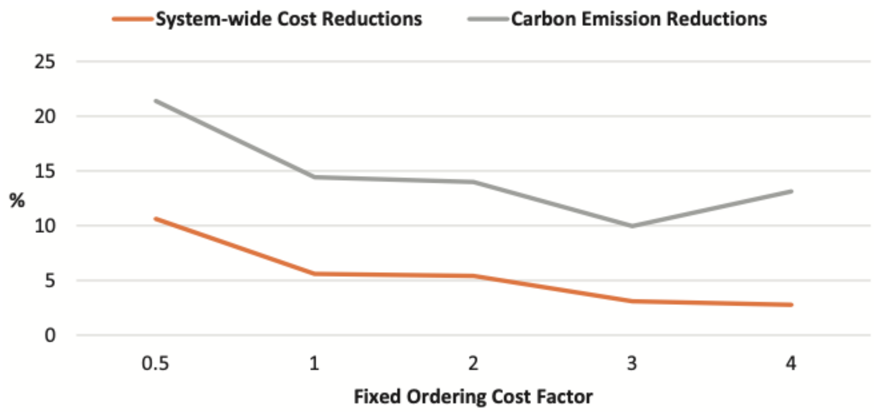

FIGURE 4. Impacts of the fixed ordering cost on the computational results.

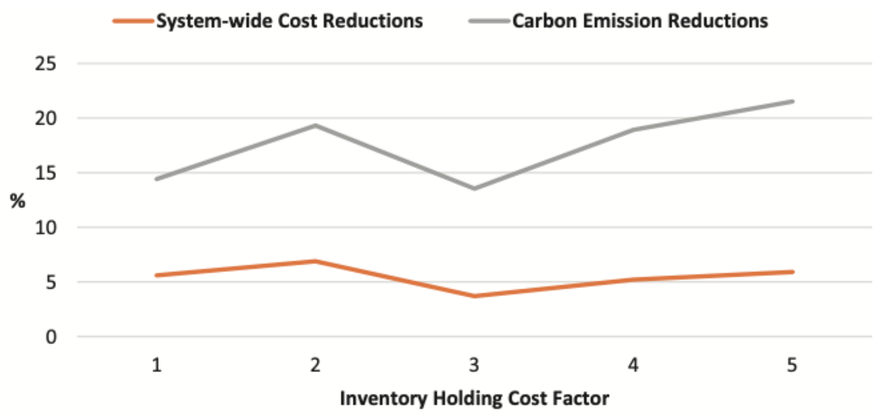

FIGURE 5. Impacts of the inventory holding cost on the computational results.

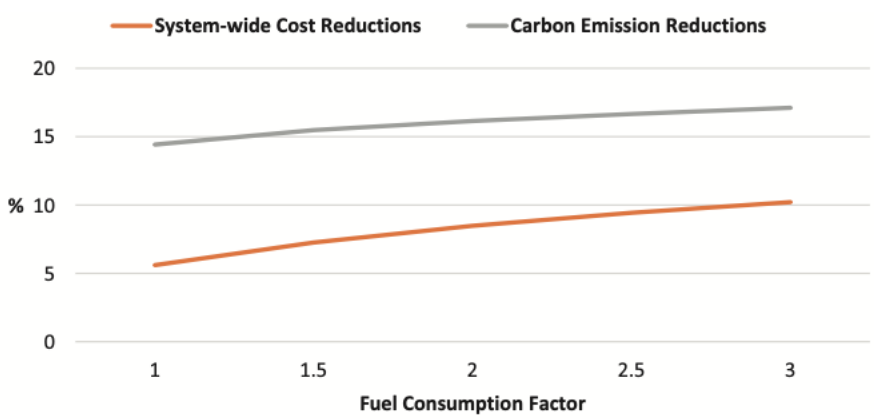

FIGURE 6. Impacts of the fuel consumption of the vehicles on the computational results.

Table 3 reports the computational results on average across the carbon tax. From Table 3, this paper finds that by adopting the integrated model under different demands, the average reductions in the system-wide cost are from 1.49% to 11.74%, and the average reductions in the carbon emissions are from 8.92% to 24.66%. For all the test problem instances, the average reductions in the system-wide cost are 5.60%, and in the carbon emissions are 14.42%. Then this paper concludes that the integrated model has more superiorities in reducing the system-wide cost and the carbon emissions comparing with the sequenced model. From the computational results, this paper also obtains that the reductions in the system-wide

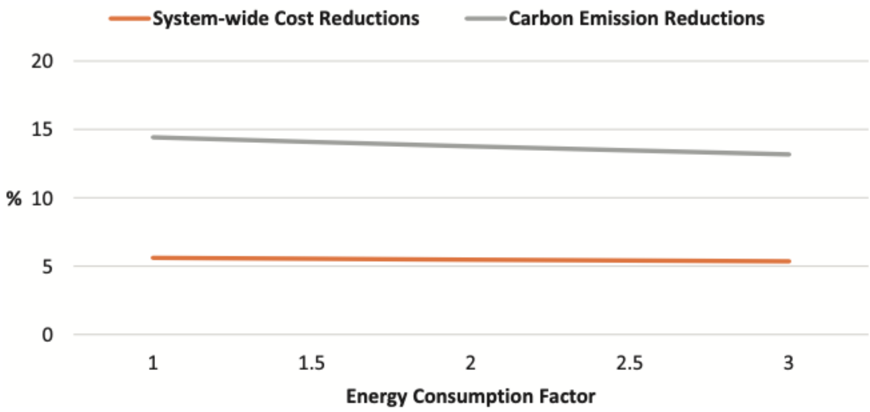

FIGURE 7. Impacts of the energy consumption of holding the product on the computational results.

TABLE 3. Reductions in the system-wide cost and the carbon emissions.

Demand	$\Delta_1(\%)$	$\Delta_2(\%)$
100	1.49	10.27
200	5.96	14.98
600	11.74	24.66
800	5.31	14.61
1000	3.52	8.92
2000	5.58	13.11

cost and the carbon emissions increase with increasing of the carbon tax for the integrated model, and therefore, this paper suggests that the carbon tax can be as an incentive for the firms to conduct the green strategies in their inventory replenishment and transportation activities.

C. IMPACT OF THE PARAMETER

In this subsection, this paper studies how the parameters of the model proposed in this paper affect the computational results. this paper firstly use $\alpha_1\ K$, $\alpha_2\ h$, $\alpha_3\ FC_e$, $\alpha_3\ FC_f$, and $\alpha_4\ H_e$ to replace K, h, FC_e, FC_f, and H_e in the optimization problem $\mathcal{P}$, respectively, where this paper calls α_1 the fixed ordering cost factor, α_2 the inventory holding cost factor, α_3 the fuel consumption factor for the vehicles, and α_4 the energy consumption factor for holding the product. Then this paper changes these factors, and shows the impacts of the corresponding parameters on the computational results, specifically on the system-wide cost and the carbon emissions. For each value of α_1, α_2, α_3, and α_4, this paper solves the 30 basic problem instances, and reports the computational results on average in the Figures 4-7.

From Figures 4 and 5, this paper finds that for different settings of the fixed ordering cost and the inventory holding cost, the integrated model has the same advantages on reducing the system-wide cost and the carbon emissions, and that the reductions in the system-wide cost and the carbon emissions for the integrated model are more sensitive to the fixed ordering cost and the inventory holding cost. This paper also finds that the reductions in the system-wide cost decrease with increasing of the fixed ordering cost, which means the integrated model can make more profit for the kind of products with small fixed ordering cost than the kind of products with large fixed ordering cost.

From Figure 6, this paper finds that with increasing of the fuel consumption of the vehicles, the reductions in the system-wide cost and the reductions in the carbon emissions both increase smoothly. This is to say, for the vehicles with high fuel consumption, the more profit can be obtained by the integrated model in reducing the system-wide cost and the carbon emissions. From Figure 7, this paper finds that the impacts of the energy consumption of holding the product on the computational results are slight, particularly on the reductions in the system-wide cost. The reason is obvious. For the non-perishable product, the carbon emissions from holding the product within the warehouse at the retailer are far less than those from transporting the product from the supplier to the retailer in the inventory replenishment.

V. CONCLUSION

In order to reduce the carbon emissions from the process of inventory replenishment, this paper considers an inventory system consisting of one supplier and one retailer, and studies a joint decision problem for the inventory policy and the transport capacity for this inventory system with carbon emissions, which simultaneously determines the reorder interval of the retailer, and the number of the vehicles used to transport the product from the supplier to the retailer while minimizing the inventory ordering and holding cost, and the carbon emission cost. This paper considers the carbon emissions from holding inventory and replenishing inventory at the retailer. The carbon emissions from replenishing inventory are calculated based on the reorder interval of the retailer, and the number of the vehicles used to transport the product from the supplier to the retailer. This problem is formulated as a nonlinear mixed integer programming, in which the objective function of the nonlinear mixed integer programming is nonconvex and nonconcave, and an algorithm is introduced to solve the nonlinear mixed integer programming optimally. Finally, this paper conducts the computational experiments to explore the advantages of the joint decision for the inventory policy and the transport capacity, i.e., the integrated model, comparing with the sequenced decision model. The computational results show that the integrated model is more profitable than the sequenced decision model in reducing the carbon emissions and the system-wide cost. Specifically, for all the test problem instances, the average reductions in the carbon emissions are 14.42%, and in the system-wide cost are 5.60%. From the computational results, this paper also obtains that the integrated decision model is more sensitive to the fixed ordering cost and the per unit inventory holding cost than the other parameters, such as fuel consumption factor, and energy consumption factor.

This paper studies a joint decision problem for the inventory replenishment and the transport capacity with carbon emissions, and only considers the carbon tax regulation. In practice, there are other regulations on curbing carbon emissions, such as carbon caps, carbon cap-and-offset, and carbon cap-and-trade ([3]), and thus, studies on the joint decision problem under those regulations are also interesting research problems.

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