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SVR-ND Method for Online Aerodynamic Parameter Estimation

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ABSTRACT Online aerodynamic parameter estimation plays an important role in compensating control system of aircraft under parameter uncertainties and unknown disturbance. In this paper, stability and control derivatives of aircraft are estimated online via support vector regression-numerical differential(SVR-ND) method. Small-sample real-time flight data reflecting real-time aerodynamic characteristics of aircraft is processed as training samples. For the small-size training samples, SVR technique is used for aerodynamic modeling. To pursue good performance in both computation efficiency and estimation accuracy, offline parameter estimation simulations are performed to select training sample size. It is observed that parameter estimation accuracy is related to the number of training samples and the noise level of samples. After that, an empirical formula is proposed to select training sample size according to results of simulations. To adapt the variation of samples, empirical formulas to tune hyper-parameters of SVR are presented based on the estimation of noise variance of samples. Finally, aerodynamic parameters are obtained by numerical differential in real-time. In a simulated maneuver, the proposed method is applied to online aerodynamic parameter estimation, and a Monte Carlo simulation is carried out to validate the robustness of SVR-ND method. Results indicate that the proposed method could realize accurate and robust estimation of aerodynamic parameters online.

INDEX TERMS Online aerodynamic parameter estimation, SVR-ND method, online model tuning.

NOMENCLATURE

V	= airspeed, m/s
θ	= inclination angle, deg or rad
α	= angle of attack, deg or rad
φ	= pitch angle, deg or rad
ω_z	= pitch rate, deg/s or rad/s
q	= dynamic pressure, N/m^2
m	= mass, kg
g	= acceleration of gravity, m/s^2
S	= reference area, m^2
J_z	= mass moment of inertia about z-axis of body reference, kg·m^2
L	= chord length of aircraft, m
P	= body-axis engine thrust, N
δ_e	= elevator input, deg or rad
d_{in}	= dimension of input vector
n_c	= number of samples for training
C_D	= drag coefficient
C_L	= lift coefficient
C_m	= pitch moment coefficient
C_{D0}	= zero-lift drag coefficient
C_D^α	= derivative of drag coefficient with respect to angle of attack
C_D^δ	= derivative of drag coefficient with respect to elevator
C_L^α	= derivative of lift coefficient with respect to angle of attack
C_L^δ	= derivative of lift coefficient with respect to elevator
$C_{m\alpha}$	= derivative of pitch moment coefficient with respect to angle of attack
$C_{m\delta_e}$	= derivative of pitch moment coefficient with respect to elevator
$C_{m\omega_z}$	= derivative of pitch moment coefficient with respect to pitch rate

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$\boldsymbol{x}$ = input vector of samples
y = target value
$\boldsymbol{y}$ = vector of target value
$\boldsymbol{R}$ = feature space
$\boldsymbol{R}^l$ = input space of $l$dimensional, $l \in Z$
$\boldsymbol{\varphi}(\cdot)$ = operator of a mapping from input space $\boldsymbol{R}^l$ to feature space
$\boldsymbol{w}$ = weighting vector
b = bias term
C = penalty factor
ε = insensitive factor
ξ_i^*, ξ_i = slack variables
$K(\cdot)$ = operator of kernel function
$f(\cdot)$ = operator of regression function
n = number of samples
k = current moment of sampling
N_s = number of sampling
$f_D(\cdot)$ = operator of regression function for modeling C_D
$f_L(\cdot)$ = operator of regression function for modeling C_L
$f_m(\cdot)$ = operator of regression function for modeling C_m
n_x = body axial overload
n_y = body normal overload
t = time, s
d = degree of freedom of a high-complexity estimator
σ = standard deviation of noise of target value of training samples
$\tilde{\sigma}$ = estimated value of σ
C_X, C_Z = body forces coefficients
Y = measurement data sequence for a variable
Up = upper bound of normalization
Low = lower bound of normalization
y_{mean} = mean of the target values of training samples
σ_y = standard deviation of the target values of training samples
e = prediction error, or noise of training sample (they are equivalent in the paper)
λ_1, λ_2 = critical value to construct confidence interval
a = confidence coefficient
μ = mean of Gaussian distribution
λ = standard deviation of Gaussian distribution
$\lambda_{a/2}$ = critical value of Gaussian distribution
μ_p = mean of total distribution of one aerodynamic parameter
μ_s = mean of estimates of one aerodynamic parameter
λ_p = standard deviation of total distribution of one aerodynamic parameter
λ_s = standard deviation of estimates of one aerodynamic parameter
n_s = total runs of estimation (number of results of one aerodynamic parameter)

SUPERSCRIPT

$\cdot$ = time derivatives
$-$ = normalize
$\wedge$ = predict
$\sim$ = estimate

I. INTRODUCTION

A. MOTIVATION

Acquiring accurate aerodynamic characteristic of an aircraft is crucial for high-performance control system design and expanding flight envelope [1], which helps verify overall performance of aircraft via flight simulations and reduces the risk in flight tests. Three approaches have been developed to obtain accurate aerodynamic characteristic of an aircraft, including wind tunnel tests, computational method and parameter estimation method. Wind tunnel tests and computational method are always performed before parameter estimation to establish an aerodynamic model and obtain aerodynamic parameters. For aircrafts with simple aerodynamic characteristics, such as axisymmetric aircraft at low speed, wind tunnel test or computational method can generally obtain accurate aerodynamic parameters. However, when the flight condition becomes complex, the parameters obtained by wind tunnel tests will be deviated from truth value a lot because wind tunnel tests could not reproduce complex flight environment with current technology. Moreover, limited wind tunnel tests can not cover all flight envelopes of aircraft. Due to the deficiency of aerodynamic methodology, computational method is prone to lose its efficiency under complicated environmental flow, too. As a result, real flight tests involving aerodynamic parameter estimation are often carried out for verification [2]. Extracting the stability and control derivatives from flight data is defined as aerodynamic parameter estimation, which has become a routine step in aircraft design and system performance evaluation [3]. Moreover, it helps enhance the confidence in the estimates of the first two approaches. Up to now, a variety of aerodynamic parameter estimation methods have been developed, which can be divided into online estimation methods and offline estimation methods. Offline estimation methods use historical flight data sampled from flight trials. However, an extremely limited number of trials are performed for one-shot aircrafts such as missiles in order to reduce cost. Therefore, it seems to be impractical to cover all flight conditions in limited number of flight tests. The challenge becomes even more serious for supersonic aircrafts due to a larger velocity range, great uncertainty from aero-elastic coupling, and transition/turbulence in the hypersonic flow. To this end, online estimation techniques may be an ideal approach to solve the problem. Online estimation methods utilize real-time flight data so that aerodynamic parameters can be obtained and applied to control system design in real-time. It is pressing to develop an effective approach to estimate aerodynamic parameters online for aircrafts with complex aerodynamic characteristics.

B. LITERATURE REVIEW

Conventional offline estimation methods include Equation Error Method (EEM) [4], Output Error Method (OEM) [5], and various Kalman-type Filter Methods(KFM) [6], such as unscented Kalman filter (UKF) [7]–[9], cubature Kalman filter (CKF) [10], [11], and extended Kalman filter (EKF) [12]. However, they usually need a priori and accurate knowledge of dynamic model or initial values of the parameters. In the past few years, Artificial Neural Network(ANN) serves a black-box method for aircraft system estimation with perfect fault tolerant ability. The trained ANN often acts as the dynamic model or the aerodynamic model. ANN algorithms have been applied to aerodynamic parameter estimation to avoid the deficiencies of traditional offline estimation techniques. Delta method [13], [14], Zero method [15] and Neural Partial Differential(NPD) [16] method are typical methods to extract aircraft aerodynamic parameters from past flight data, in which ANN is employed to represent the aerodynamic model. However, training process of ANN is somewhat time-consuming and the generalization of ANN is usually poor. Hence, parameter estimation methods based on ANN are insufficient to realize online aerodynamic parameter estimation. With the improvement of flight capability, the flight envelope of aircraft is becoming larger and the environment of flight gets more complex. In this situation, high performance autopilot depends more on accurate and real-time aerodynamic parameters and the support of online estimation techniques is increasingly needed.

A method [17] in the frequency domain based on finite Fourier transform has been proposed to estimate aerodynamic parameters in real-time. However, the frequency-domain method requires empirical selection of size of time window and single-size time window may not produce satisfactory coherence over the whole frequency range. Least Squares (LS) [18] and EKF are the most widely used time-domain online aerodynamic parameter estimation methods, but both of them require a prior dynamic model [19], [20]. Moreover, LS only can be utilized to estimate parameters of linear aerodynamic model, and it is difficult for EKF to obtain a Jacobian matrix for system with complex nonlinear traits. Machine learning technique is an effective approach to deal with complex nonlinear system, two representatives of which are ANN and Support Vector Machine(SVR) [21]. However, as aforementioned, ANN has deficiencies in online modeling of aerodynamics. Instead, SVM has a great potential to fulfill the goal.

C. LIMITATIONS AND CONTRIBUTIONS

SVM has been widely used in the field of classification and regression [22], [23]. Similar to ANN, SVR is able to realize nonlinear mapping from input space to output space, and shows a superior generalization performance over ANN especially for small size sample [24]. SVR model is established by solving quadratic programming and therefore its solution is single and global minimum, while solution of ANN may be trapped into local minimum [25]. In addition, SVR can process high-dimension data and overcome the ''curse of dimensionality'' compared to ANN. The real-time aerodynamic characteristics of aircraft are hidden in the real-time flight data in short time interval, in other words, current aerodynamic characteristics are reflected in a small sample of real-time flight data. ε- SVR provides a good option for real-time aerodynamic modeling based on small samples. However, the hyper-parameters of SVR have a great impact on model generalization performance and it is vital to tune hyper-parameters when using ε- SVR online, there is no systematic method to provide a guide for selection of hyper-parameters of ε- SVR.

In the work, a method christened SVR-ND based on ε- SVR for online aerodynamic parameter estimation is developed. The contributions of this paper are as follows:

1. there is no need for any knowledge of aircraft dynamic model or aerodynamic model. SVR-ND achieves real-time aerodynamic modeling by ε- SVR by taking advantage of the excellent learning performance of SVR on small size samples. The SVR model has satisfactory generalization capability and can be utilized for aerodynamic prediction. Then numerical differential is adopted to extract aerodynamic derivatives in real-time from the SVR model.

2. To ensure both estimation accuracy and computational efficiency, empirical formulation in terms of dimension of input and noise level is developed to select training sample size online. Considering that hyper-parameters of ε- SVR are difficult to be optimized online, empirical formulas to tune model parameters are proposed to realize an excellent generalization performance of ε- SVR model. Based on the asymptotic estimation of noise variance of training samples, hyper-parameters of ε- SVR are selected online.

3. The proposed method can estimate aerodynamic parameter online accurately and robustly with a low time overhead. It also has a good scalability and can be used not only as an offline method, but also an online method to provide support for online flight ability prediction based on aerodynamic prediction capability of SVR model.

D. PAPER ORGANIZATION

The paper is organized as follows: Section II presents aircraft dynamic model and aerodynamic parameters. Methodology of ε- SVR is given in Section III. Section IV presents SVR-ND method and describes the process of aerodynamic modeling using ε- SVR, including offline parameter estimation simulation analysis. In Section V, online aerodynamic parameter estimation is implemented using proposed method and a Monte Carlo test is carried out to examine robustness of the method, with results and discussions given. Finally, the last part draws a conclusion of the study.

II. DYNAMIC MODEL AND POSTULATED AERODYNAMIC MODEL

To describe the motion of aircraft and generate flight data, a longitudinal motion of aircraft is considered in this work. The longitudinal model with 3 degree of freedom(DOF) is given as follows:

$$\begin{aligned} \dot{V} &= (P\cos\alpha - qSC_D - mg\sin\theta)/m \\ \dot{\theta} &= P\sin\alpha/mV + qSC_L/mV - mg\cos\theta/V \\ \dot{\omega}_z &= C_m qSL/J_z \\ \dot{\varphi} &= \omega_z \end{aligned} \tag{1}$$

For the convenience of evaluating the performance of the proposed method, assuming that the longitudinal aerodynamic model is linear. The postulated aerodynamic model is given as:

$$\begin{aligned} C_D &= C_{D0} + C_D^{\alpha}|\alpha| + C_D^{\delta}\delta_e \\ C_L &= C_L^{\alpha}\alpha + C_L^{\delta}\delta_e \\ C_m &= C_{m\alpha}\alpha + C_{m\delta_e}\delta_e + C_{m\omega_z}\omega_z \end{aligned} \tag{2}$$

The unknown parameters to be estimated include C_{D0}, C_D^{α}, C_D^{δ}, C_L^{α}, C_L^{δ}, $C_{m\alpha}$, $C_{m\delta_e}$, $C_{m\omega_z}$.

III. METHODOLOGY OF SVR

In the paper, ε- SVR is used to model aerodynamics of the aircraft in real-time. A brief introduction of the methodology of ε- SVR is described as below.

For a set of n training samples

$$(\boldsymbol{x}_1, y_1), (\boldsymbol{x}_2, y_2), \ldots, (\boldsymbol{x}_i, y_i), \ldots, (\boldsymbol{x}_n, y_n) \quad \boldsymbol{x} \in \boldsymbol{R}^l,\ y \in \boldsymbol{R}$$

the goal of SVR is to find a regression function

$$f(\boldsymbol{x}) = \boldsymbol{w}^{\mathrm{T}}\boldsymbol{\varphi}(\boldsymbol{x}) + b \tag{3}$$

such that the training data can be accurately fitted, where $\boldsymbol{\varphi}(\boldsymbol{x})$ represents a mapping from input space $\boldsymbol{R}^l$ to feature space, $\boldsymbol{w}$ and b are weighting vector and bias term, respectively.

According to the principle of structural risk minimization, SVR yields the following optimization goal:

$$\begin{aligned} &\min \frac{1}{2}\|\omega\|^2 + C\sum_{i=1}^{n}\left(\xi_i + \xi_i^*\right) \\ &\text{subject to} \begin{cases} y_i - f(\boldsymbol{x}_i, \omega) - b \le \varepsilon + \xi_i^* \\ f(\boldsymbol{x}_i, \omega) + b - y_i \le \varepsilon + \xi_i \\ \xi_i, \quad \xi_i^* \ge 0,\ i = 1, 2 \ldots, n \end{cases} \end{aligned} \tag{4}$$

where C is a positive constant called penalty factor, ε is insensitive factor, ξ_i^* and ξ_i are slack variables which guarantee that there exists a solution under the constraint.

The above optimization problem can be transformed into a dual problem [26]. In this paper, the sequence minimum optimization (SMO) method is used to solve the dual problem, and the solution can be rewritten as

$$f(\boldsymbol{x}) = \sum_{i=1}^{n}(\beta_i - \beta_i^*)K(\boldsymbol{x}_i, \boldsymbol{x}) + b \tag{5}$$

where $K(\boldsymbol{x}_i, \boldsymbol{x}) = (\varphi(\boldsymbol{x}_i) \cdot \varphi(\boldsymbol{x}))$ is the kernel function, such as Linear kernel, Polynomial kernel and Gaussian kernel; β_i and β_i^* are the solution for the dual problem, which are called Lagrange multipliers and subjected to constraints $0 \le \beta_i, \beta_i^* \le C$.

The generalization performance of SVR depends on the choice of hyper-parameters C, ε and the kernel parameters. C adjusts the ratio between confidence interval and the empirical risk, i.e. determining the trade-off between the model flatness and the training error. ε determines the width of the ε- insensitive zone, thus affecting the number of support vectors and fitting precision. The bigger the value of ε, the more complex the model. The choice of kernel parameters can be neglected, as the linear kernel is selected in the present work. To minimize the structural risk, the paper mainly focuses on the tuning of hyper-parameters C and ε.

There are a number of optimization approaches to choose C and ε, which can mainly be categorized into unintelligent optimization method and intelligent method. The typical unintelligent methods include Grid Search method [27], gradient descent algorithm [28] and Cross-validation method. Intelligent optimization method comprises Particle Swarm Optimization algorithm [29], Artificial Bee Colony algorithm [30], Sine Cosine algorithm [31], etc. However, no systematic methodology has been developed for tuning hyper-parameters of SVR up to now.

The following empirical formula [32] has been proposed to determine C, ε.

$$\begin{aligned} C &= \max\left(\left|y_{mean} + 3\sigma_y\right|, \left|y_{mean} - 3\sigma_y\right|\right) \\ \varepsilon &= 3\sigma\sqrt{\frac{\ln n}{n}} \end{aligned} \tag{6}$$

The variance of noise is estimated according to the following formula [26]

$$\tilde{\sigma}^2 = \frac{n}{n-d} \cdot \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2 \tag{7}$$

where d is the degree of freedom of a high-complexity estimator such as polynomial regression. The method has a simple selection mechanism but may not achieve optimal choice of C and ε.

IV. THE SVR-ND METHOD

For the purpose of online aerodynamic parameter estimation, the SVR-ND method combines nonlinear modeling capability of SVR with numerical differential principles of Delta method. SVR avoids the deficiency of overfitting in Delta method and improves generalization of aerodynamic model. SVR-ND method begins with real-time flight data processing to obtain training samples. Then ε- SVR machine is employed to construct aerodynamic model online based on training samples.

Given the following real-time flight data $[\alpha(i), \delta_e(i), \omega_z(i), C_D(i), C_L(i), C_m(i)]\ i = k, k-1 \ldots, k-1-N_s$ where k represents the current moment of sampling and N_s is the number of sampling.

The objective of aerodynamic modeling is to establish the relationship between target variables (C_D, C_L, C_m) and state variables (α, δ_e, ω_z). Specifically, three multi-input single-output(MISO) aerodynamic models will be separately constructed using ε- SVR, namely

$$\begin{aligned} C_D &= f_D(\alpha, \delta_e) \\ C_L &= f_L(\alpha, \delta_e) \\ C_m &= f_m(\alpha, \delta_e, \omega_z) \end{aligned} \tag{8}$$

The size of samples will be separately selected to model C_D, C_L, C_m and the selection of sample size will be discussed in subsection IV.B.

Once the aerodynamic modeling is completed, numerical differential is performed to obtain stability and control derivatives in real-time. The detailed process is as follows: Perturb one of the input variables by adding a small variable and keep all other variables constant. The corresponding derivative equals the output variation divided by the input variation.

To ensure the robustness of the method, numerical differentiation is performed at each sample point. And the derivative to be estimated is equal to the mean value of all estimates of such derivative at different sample points. For instance, C_L^α is obtained by

$$C_L^\alpha = \frac{1}{n}\sum_{i=1}^{n}\left(\bar{f}_L(\bar{\alpha}_i + \Delta\bar{\alpha}, \bar{\delta}_{ei}) - \bar{f}_L(\bar{\alpha}_i, \bar{\delta}_{ei})\right)/\Delta\alpha \tag{9}$$

Eq.(9) can be easily derived by Min–Max normalization. The normalization of training samples will be discussed later.

However, there exists a special case, where a constant term C_{D0} in the expression of C_D in Eq.(2) can be obtained by the following formula

$$C_{D0} = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - C_L^\alpha \alpha_i - C_D^\delta \delta_{ei}\right) \tag{10}$$

Notate that the SVR-ND method could also be applied to offline estimation, where the data to be processed in the procedure of estimation is historical flight data.

A. DATA PROCESSING

Training samples is an essential precondition for aerodynamic modeling. The remainder of the subsection describes how to process data to acquire training samples.

The measurable variables include angle of attack α, the elevator input δ_e, the pitch rate ω_z, the body axial overload n_x, the body normal overload n_y and the dynamic pressure q.

Force coefficients and moment coefficients cannot be measured directly, so following simplified formulas are needed to acquire them

$$\begin{aligned} C_D &= -C_X\cos(\alpha) - C_Z\sin(\alpha) \\ C_L &= C_X\sin(\alpha) - C_Z\cos(\alpha) \\ C_m &= [J_z\dot{\omega}_z]/qSL \end{aligned} \tag{11}$$

where $\dot{\omega}_z$ is calculated based on measurement value of the pitch rate ω_z using numerical differential method, the body forces coefficients (C_X, C_Z) are expressed as:

$$\begin{aligned} C_X &= mn_x/qS \\ C_Z &= mn_y/qS \end{aligned} \tag{12}$$

Before training, there is an another problem that needs to be considered. In present application, different original variables in sample are obtained in different scale or in various range. Thus, normalization is supposed to be performed on training samples. Studies have proved that normalization helps improve the generalization performance and the learning speed [33]–[35].

An alternative and simple choice is the Min–Max normalization. The form of the Min–Max normalization is as follows:

$$\bar{Y}(i) = \frac{Y(i) - \min(Y)}{\max(Y) - \min(Y)} \cdot (Up - Low) + Low \tag{13}$$

Here we take $Low = -1$, $Up = 1$, thus each variable in samples is separately normalized into $[-1, 1]$.

B. SELECTION OF TRAINING SAMPLE SIZE

Since aerodynamic derivatives are obtained by perturbing input variable at sample points, the generalization performance of SVR model has a great influence on the estimation accuracy of parameters. Exactly speaking, the generalization performance of the model is affected by noise level and data sparsity.

On one hand, the generalization performance of model depends on the sparseness of training data. As the input dimension increases, the available data becomes sparse relatively. An accurate and reliable modeling requires more training samples for machine learning approaches such as SVR. Since input dimension of model of C_m is larger, modeling for C_m requires more training samples compared with modeling for C_D or C_L.

On the other hand, the noise level is related to the generalization performance of model. Suppose an effective SVR model has been constructed based on training samples. For truth target value y and prediction target value $\hat{y}$ at any input point $\boldsymbol{x}$, there is $y = \hat{y} + e$, where e is error which can be considered as the noise of training samples. Assuming that e obeys Gaussian distribution with zero mean and variance σ^2, written in the form $e \sim N(0, \sigma^2)$. It follows that $y \sim N\left(\hat{y}, \sigma^2\right)$. Then the following relation can be derived: $\frac{y-\hat{y}}{\sigma} \sim N(0, 1)$. Define a function $Z\left(\hat{y}\right)$

$$Z\left(\hat{y}\right) = \frac{y - \hat{y}}{\sigma} \sim N(0, 1) \tag{14}$$

where the distribution of $Z\left(\hat{y}\right)$ is independent of the distribution of $\hat{y}$. For a given confidence coefficient a, there exists critical value λ_1 and λ_2 such that

$$P\left(\lambda_1 \leq Z \leq \lambda_2\right) = 1 - a \tag{15}$$

where λ_1 and λ_2 could be obtained by looking up tables of Gaussian distribution. From Eq.(15), confidence interval of $\hat{y}$

in terms of a can be solved. For $a = 0.05$, 95% confidence interval of $\hat{y}$ is $\left[\hat{y} - 1.96\sigma, \hat{y} + 1.96\sigma\right]$.

Obviously, with the increase of σ, the width of the confidence interval of $\hat{y}$ increases and the uncertainty of the predicted value $\hat{y}$ becomes greater accordingly. In practice, the noise variance σ^2 is estimated from the sum of square of fitting error of training samples as Eq.(9). For small-size samples, the estimate of the variance σ^2 may be deviated from truth value. In consequence, for reducing the estimation risk of noise variance caused by small samples, variance estimation should depend on large samples. In addition, the time cost of SVR-ND method should be considered. Online parameter estimation using training samples with different sizes will cost different time. Hence, the relationship between the cost of estimated time and the sample size needs to be explored.

According to later simulation results, with the increase of the number of samples, time cost of aerodynamic parameter estimation increases. And when a maximum training sample size (about 250 samples) is adopted, the SVR-ND takes no more than 180ms running on a computer with a main frequency of 2.5 GHz, which can satisfy the needs of online parameter estimation. Hence, we no longer consider the influence of sample size on computational efficiency and the influence of the training sample size on accuracy of aerodynamic parameter estimation will be discussed in detail.

To demonstrate the relationship between accuracy of estimation and noise level and training sample size, the SVR-ND method was firstly applied to off-line aerodynamic estimation and the Grid search method [27] was used to determine the hyper-parameters C, ε. The dynamic model Eq.(1) and aerodynamic model Eq.(2) were used to generate simulated historical flight data. Relevant parameters of aircraft are listed in TABLE 1.

TABLE 1. **Parameters of aircraft.**

variables	Value
Mass $m(kg)$	100
Reference area $S(m^2)$	0.01327
Chord length $L(m)$	1.5
Gravity acceleration $g(m/s^2)$	9.8066
Trust $P(N)$	0
Moment of inertia $J_z(kg \cdot m^2)$	6.279

To excite the mode of motion of the aircraft, a longitudinal maneuver was carried out by applying sine elevator input with a duration of 4 s. The sine elevator input is expressed as

$$\delta_e(t) = 2\sin(\pi t)\,(\deg) \qquad t \in [0, 4]$$

The historical flight data was recorded with a sampling time of 0.01 s. Assuming that there was no wind effect on aircraft.

The time histories of $\alpha, \delta_e, w_z, n_x, n_y, q$ are shown in Fig.1. ε- SVR was used for aerodynamic modeling. In Fig.2, the coefficients of force and moment C_D, C_L, C_m through calculation and SVR modeling based on the first 150 samples are given. It can be seen that the SVR model outputs in the first 1.5 seconds (corresponding to the first 150 samples) and the last 2.5 seconds (corresponding to the last 250 samples) fit well with the calculated force and moment coefficients. It is shown that the SVR aerodynamic models have excellent generalization performance.

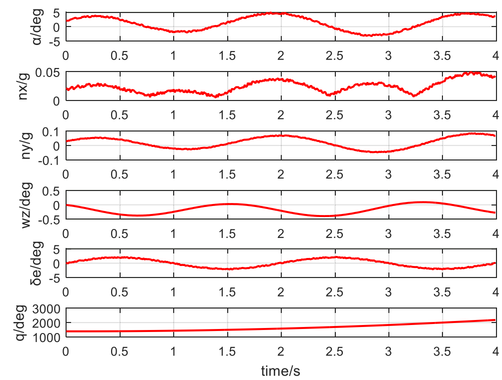

FIGURE 1. **Simulated flight data(3% noise).**

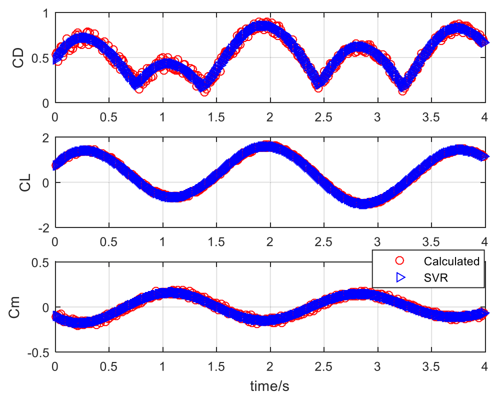

FIGURE 2. **The coefficients of force and moment by calculating and SVR modeling(3% noise, sample size n=150).**

Then stability and control derivatives were estimated through perturbing each input variable individually, namely, by adding disturbance variable with amplitude of 0.05 to each input variable. The results of parameter estimation are listed in TABLE 2, where results in parenthesis represent the absolute value of relative deviation of estimated value with respect to truth value of parameters, given as

$$|RD| = \left|\frac{estimated\ value - truth\ value}{truth\ value}\right| \times 100\% \quad (16)$$

Under 3% noise, the estimated value is close to truth value, which verifies the effectiveness of the proposed method. To further explore the influence of noise on identification results, Gaussian white measurement noise with zero mean and intensities of 5% and 7% were added to all motion

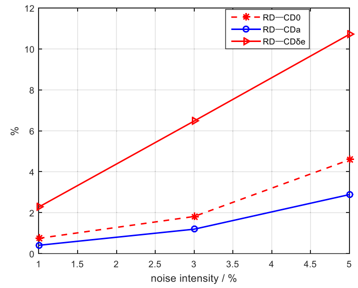

FIGURE 3. |RD| of parameters of drag force for noise of varying intensity.

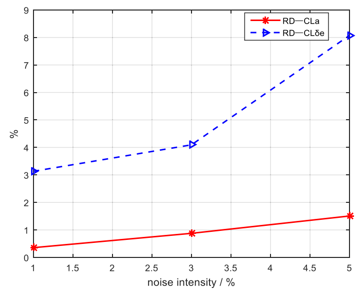

FIGURE 4. |RD| of parameters of lift force for noise of varying intensity.

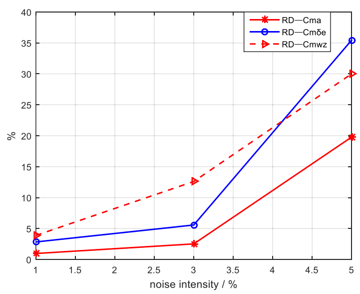

FIGURE 5. |RD| of parameters of pitch moment for noise of varying intensity.

variables. Parameter estimation results with noise of varying intensity are shown in TABLE 2. The absolute value of RD of estimated parameter are shown in Figs.3-5. It is observed that the estimation accuracy decreases with the increase of noise level. Nevertheless, as the number of samples increases, the adverse effect of noise on estimation accuracy is decreased.

Specifically, to examine the sensitivity of estimation results to the number of training samples via experiments, samples of different size were used to construct aerodynamic model with 3% noise. The results of offline parameter estimation are given in TABLE 3. Figs.6-8 show the absolute value of relative deviation of estimated parameter based on various number of training samples. It can be clearly seen that the larger the sample size, the higher the estimation accuracy.

From the above simulation results, it is concluded that parameter estimation accuracy has a negative correlation with noise level and has a positive correlation with the number of training samples or sparsity of data. Selection of the number of samples to achieve the goal of estimate aerodynamic parameters accurately (satisfy certain accuracy) under certain noise level is concerned. And a large number of offline trials were carried out to find a proper training sample size for

TABLE 2. Offline parameter estimation results with noise of varying intensity.

Parameters	Truth value	3% Noise	5% Noise	7% Noise
C_{D0}	0.1789	0.18036 (0.74%)	0.18214 (1.81%)	0.18709 (4.58%)
C_D^α	0.144	0.14287 (0.40%)	0.14228 (1.19%)	0.13984 (2.88%)
C_D^δ	0.0085	0.01006 (2.27%)	0.00794 (6.47%)	0.00758 (10.71%)
C_L^α	0.3417	0.34462 (0.86%)	0.34298 (3.71%)	0.34167 (7.86%)
C_L^δ	0.0984	0.10148 (3.13%)	0.09437 (4.09%)	0.09044 (8.07%)
$C_{m\alpha}$	-0.0450	-0.04543 (0.96%)	-0.04613 (2.52%)	-0.0539 (19.78%)
$C_{m\delta_e}$	-0.0432	-0.04442 (2.83%)	-0.04079 (5.57%)	-0.02788 (35.4%)
C_{mw_z}	-0.30	-0.31166 (3.89%)	-0.35582 (18.6%)	-0.30090 (0.3%)

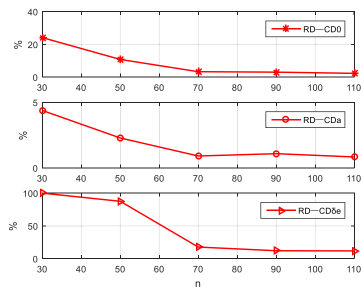

FIGURE 6. |RD| of parameters of drag force for training samples of different size.

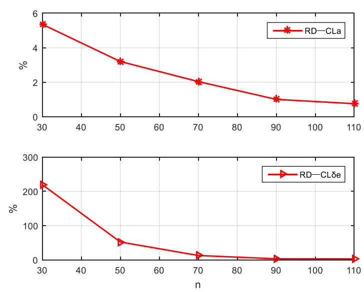

FIGURE 7. |RD| of parameters of lift force for training samples of different size.

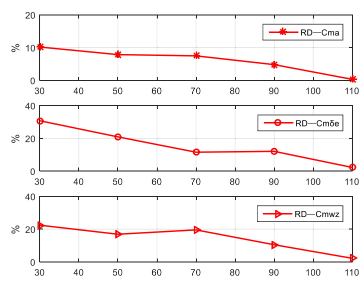

FIGURE 8. |RD| of parameters of pitch moment for training samples of different size.

aerodynamic modeling. The strategy in the paper is to select the minimum number of samples on the premise of obtaining accurate aerodynamic derivative ($|RD|$ of estimation is less than 10%). Then, the satisfied number of samples in various simulation conditions can be obtained by changing the noise level and input dimension. Finally, the relationship between number of samples and the noise level and input dimension could be obtained by a fitting formula. The following empirical formula in terms of dimension of input and noise level is presented to choose the number of samples for online modeling.

$$n_c = 40d_{in}(1 + \tau\sigma) \tag{17}$$

where τ is a constant taken as 15~20. In online use, σ would be estimated based on real-time flight data and input dimen-sion d_{in} are known. Once σ and d_{in} are given, the number of samples to be used for aerodynamic modeling could be determined.

C. ONLINE MODEL SELECTION

Flight is a dynamic process, where the characteristics of flight data and training samples change constantly, so it is necessary to tune hyper-parameters online to adapt the variation of samples. There has been no completely proper method to optimize hyper-parameters C and ε online. To this end, the empirical formula Eq.(6) provides a potential approach to fulfill the goal.

Due to normalization of the samples, the distribution characteristic of data is changed relative to the original data. Eq.(6) needs to be modified according to normalized parameters. Define mean value of sample errors

$$\upsilon = \frac{1}{n}\sum_{i=1}^{n}\hat{y}_i - y_i \tag{18}$$

The choice of ε depends on the variance of υ [32]. Given the following theorem of the Gaussian distribution: for a variable X, if $X \sim N\left(\mu, \lambda^2\right)$, then $cX + d \sim N\left(c\mu + d, (c\sigma)^2\right)$ exists for real numbers c and d. According to the above theorem, $\upsilon \sim N\left(0, \bar{\sigma}^2\right)$ is could be derived, and $\bar{\sigma}$ can be obtained by

$$\bar{\sigma} = K\sigma \tag{19}$$

where $K = \frac{(Up-Low)}{\max(\boldsymbol{y})-\min(\boldsymbol{y})}$, $\boldsymbol{y}$ is the vector of target value.

Combining Eqs.(6)(17)(19) yields the following equation

$$\varepsilon = 3K\bar{\sigma}\sqrt{\frac{\ln\left(40d_{in}(1+\tau\bar{\sigma})\right)}{40d_{in}(1+\tau\bar{\sigma})}} \tag{20}$$

In order to reduce the overhead of online computation, SVR aerodynamic model (estimator) is used to estimate standard deviation σ instead of selecting other estimators. In each estimation, σ is estimated based on new training samples by Eq.(7). The estimated σ is employed to yield new estimates of standard deviation, which are then applied to the determination of ε in the later estimations in the following way

$$\tilde{\sigma}_{j-1} = \frac{1}{j-2}\sum_{p=1}^{j-2}\tilde{\sigma}_p, \quad j > 1 \tag{21}$$

where $\tilde{\sigma}_{j-1}$ represents the $(j-1)^{\text{th}}$ estimated value of σ. As the j increased, more and more samples are employed to estimate σ, which makes $\tilde{\sigma}$ approximate its truth value gradually.

Parameter C is given by

$$C = \max\left(\left|\bar{y}_{mean} + 3\bar{\sigma}_y\right|, \left|\bar{y}_{mean} - 3\bar{\sigma}_y\right|\right) \tag{22}$$

In general, the following formulas are applied to online selection of hyper-parameters in the j^{th} estimation:

$$\begin{aligned} C_j &= \max\left(\left|\bar{y}_{meanj} + 3\bar{\sigma}_{yj}\right|, \left|\bar{y}_{meanj} - 3\bar{\sigma}_{yj}\right|\right) \\ \varepsilon_j &= 3K_j\tilde{\sigma}_{j-1}\sqrt{\frac{\ln\left(40d_{in}\left(1+\tau\tilde{\sigma}_{j-1}\right)\right)}{40d_{in}\left(1+\tau\tilde{\sigma}_{j-1}\right)}} \end{aligned} \tag{23}$$

In online use, once the samples for modeling are determined, the mean value of samples $\bar{y}_{mean}$ and the standard deviation $\bar{\sigma}_y$ are then determined. K could also be determined by nor-malized parameters. Then parameter C is determined. Input dimension d_{in} are known. The standard deviation of sample noise σ could be estimated by Eqs. (7) (17). Then ε could be determined.

V. ONLINE PARAMETER ESTIMATION RESULTS AND DISCUSSION

Online aerodynamic parameter estimation was implemented during a longitudinal maneuver. Real-time flight data were generated using dynamic model Eq.(1) and aerodynamic model Eq.(2). Sine elevator input with a duration of 10s was applied to the longitudinal maneuver. The expression of sine elevator input is adopted as

$$\delta_e(t) = \begin{cases} 2\sin(\pi t)\,(\text{deg}) & t \in [0, 6) \\ 6\sin(\pi t)\,(\text{deg}) & t \in [6, 10] \end{cases}$$

Taking measured noise with intensity of 5% into account, real-time flight data of α, δ_e, w_z, n_x, n_y, q is shown in Fig. 9. Coefficients of force and moment C_D, C_L, C_m are computed online. Fig. 10 shows the variation of the force and moment coefficients during the maneuver.

During the maneuver, the SVR-ND method was applied to real-time estimation of aerodynamic derivatives. Figs.11-13 show the online parameter estimation results. To ensure the estimation accuracy, parameter estimation should

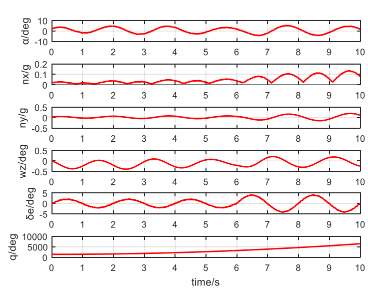

FIGURE 9. Simulated flight data.

TABLE 3. Offline parameter estimation for training samples of different size (3% Noise).

Parameters	Truth value	n=30	n=50	n=70	n=90	n=110
C_{D0}	0.1789	0.13573 (24.12%)	0.15957 (10.80%)	0.17294 (3.32%)	0.17343 (3.05%)	0.17474 (2.32%)
C_D^α	0. 144	0.15031 (4.38%)	0.14728 (2.28%)	0.14530 (0.90%)	0.14556 (1.08%)	0.14520 (0.83%)
C_D^δ	0. 0085	0.04635 (100%)	0.01593 (87.51%)	0.01001 (17.73%)	0.00953 (12.12%)	0.00950 (11.84%)
C_L^α	0. 3417	0.32339 (5.35%)	0.35264 (3.20%)	0.348670 (2.04%)	0.34514 (1.01%)	0.34429 (0.75%)
C_L^δ	0. 0984	0.31425 (219.36%)	0.14981 (52.24%)	0.110954 (12.75%)	0.10148 (3.13%)	0.10123 (2.88%)
$C_{m\alpha}$	−0. 0450	-0.04958 (10.18%)	-0.04854 (7.87%)	-0.04837 (7.48%)	-0.04716 (4.8%)	-0.04485 (0.32%)
$C_{m\delta_e}$	−0. 0432	-0.05648 (30.76%)	-0.05193 (20.92%)	-0.03819 (11.58%)	-0.03797 (12.09%)	-0.04418 (2.28%)
C_{mw_z}	−0. 30	-0.23275 (22.41%)	-0.24927 (16.91%)	-0.24134 (19.55%)	-0.26849 (10.50%)	-0.30663 (2.20%)

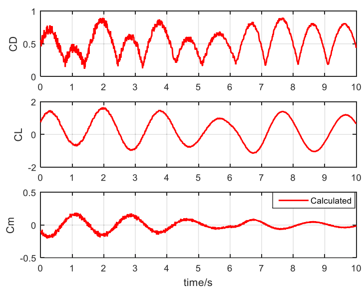

FIGURE 10. The calculated coefficient of force and moment.

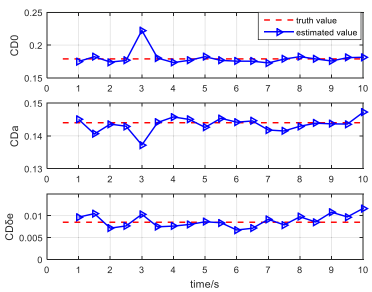

FIGURE 11. Online parameter estimates for derivatives of CD.

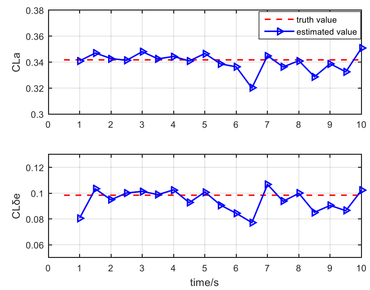

FIGURE 12. Online parameter estimates for derivatives of CL.

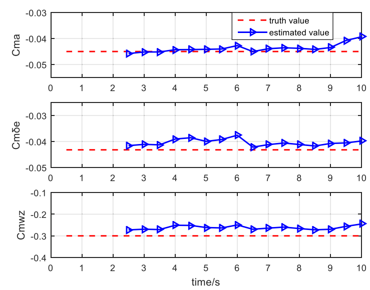

FIGURE 13. Online parameter estimates for derivatives of Cm.

be carried out after a certain number of samples are accumulated. Therefore, one second after maneuver started, estimation of aerodynamic parameters for drag force coefficient and lift force coefficient was carried out for the first time. As described in subsection 4.2, more samples are required to model coefficient of pitch moment. Hence, for derivatives of pitch moment, the first estimation was carried out at 2.5 seconds during the maneuver. Each subsequent estimation was performed every 0.5s for all aerodynamic parameters as there were enough samples in the database. It can be seen that the results of each estimation are different and scattered around the truth value.

The estimated value of σ and variation of training samples size n_c for modeling C_D, C_L and C_m are shown in Figs.14-16. It can be seen that the size of training sample for pitch moment modeling is greater than that for lift and drag force modeling due to larger input dimension and larger noise variance of target value. In addition, the number of samples n_c varies with the noise level. To be specific, training sample size increases with increase of the noise variance. This adaptation to noise and input dimensions makes sense. Generally speaking, a larger sample size means a higher computational overhead, while too small samples will lead to a decrease in estimation accuracy. The online selection of training sample size avoids the contradiction.

In each estimation, the aerodynamic model was first constructed using ε- SVR. The insensitive factor ε and penalty factor C were given as $\varepsilon = 0.01, C = 1.0$ in the first identification. Except for the first estimation, the size of training sample and model hyper-parameters were selected online based on the estimation of noise variance, as expressed

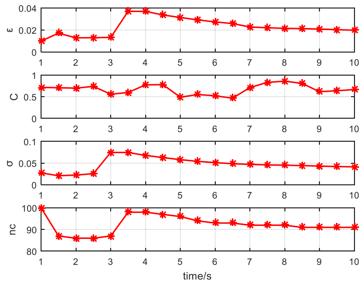

FIGURE 14. **Online tuning results of model parameters for modeling CD.**

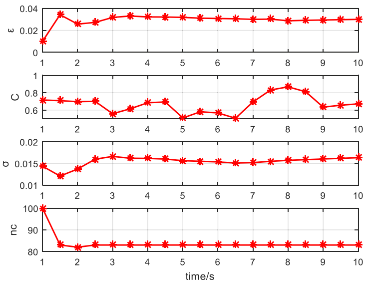

FIGURE 15. **Online tuning results of model parameters for modeling CL.**

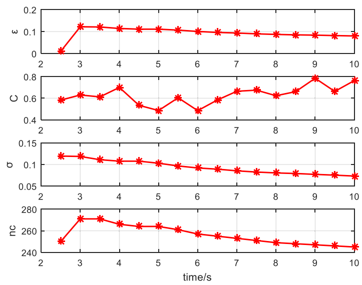

FIGURE 16. **Online tuning results of SVR model parameters for modeling Cm.**

in Eq.(17) and Eq.(23). Figs.14-16 show online tuning of hyper-parameters of ε- SVR for modeling C_D, C_L and C_m, respectively. The variation process of ε, σ and n_c indicates that a larger σ corresponds to a larger ε and a larger n_c. Adjusting hyper-parameters online avoids possible problems caused by fixed hyper-parameters, such as poor generalization performance and low fitting accuracy.

A. MONTE CARLO TEST

To validate the noise robustness of the proposed method, a Monte Carlo test was carried out by applying Gaussian white noise with intensity of 5% on measurable variables. Considering that the attitude of the aircraft may be disturbed by parameter uncertainties or unknown state input, online estimation should be performed as soon as possible. The Monte Carlo test was carried out particularly for the first estimation during the maneuver. The first estimation was repeated 200 times. SVR-ND method was used to estimate aerodynamic parameters at each run. Fig.17 shows histograms of estimation results of C_{D0}, C_D^α, C_L^α for 200 runs, where the red line represents truth value of aerodynamic parameters.

Since it is difficult to get certain information about the total distribution of aerodynamic parameter estimates, Central-Limit Theorem (CLT) is adopted to perform an interval estimation. When the number of estimates for each aerodynamic parameter is large enough, according to CLT there exists

$$\frac{\mu_s - \mu_p}{\lambda_p}\sqrt{n_s} \sim N(0, 1) \tag{24}$$

And for a given confidence coefficient a, there exists a critical value $\lambda_{a/2}$ such that

$$P\left(-\lambda_{a/2} \leq \frac{\mu_s - \mu_p}{\lambda_p}\sqrt{n_s} \leq \lambda_{a/2}\right) = 1 - a \tag{25}$$

where $\lambda_{a/2}$ could be obtained by looking up tables of Gaussian distribution. By solving Eq.(26), confidence interval of μ_p about confidence $1 - a$ can be derived as

$$\left(\mu_s - \lambda_{a/2}\frac{\lambda_p}{\sqrt{n_s}}, \mu_s + \lambda_{a/2}\frac{\lambda_p}{\sqrt{n_s}}\right) \tag{26}$$

Since mean of total distribution of aerodynamic parameter is unknown, λ_s could be used in place of λ_p. Then confidence interval of μ_p about confidence $1 - a$ is

$$\left(\mu_s - \lambda_{a/2}\frac{\lambda_s}{\sqrt{n_s}}, \mu_s + \lambda_{a/2}\frac{\lambda_s}{\sqrt{n_s}}\right) \tag{27}$$

TABLE 4 shows the results of interval estimation of parameter estimates for 200 runs, where a is taken as 5% ($\lambda_{a/2}$ equals 0.5199). It is obviously that the mean of parameter estimates is close to the truth value and the scattering magnitude of almost all parameter estimates is small, except for C_D^δ and $C_{m\omega_z}$. The situation is acceptable because C_{D0} and $C_D^\alpha\alpha$ account for a large proportion of C_D and $C_{m\omega_z}\omega_z$ accounts for a small proportion of the C_m. As shown in TABLE 4, for all aerodynamic parameters, width of 95% confidence interval of is small and critical value of confidence interval is close to their truth values. The results of interval estimation confirm the effectiveness and robustness of the proposed method in obtaining real-time parameter estimates, as far as estimation accuracy and reliability are concerned.

Consequently, it is reasonable to use mean value of parameter estimates when a great number of flight tests are performed.

B. COMPUTATIONAL EFFICIENCY

To further evaluate the computational efficiency of the method, 10 runs of aerodynamic parameter estimation using SVR-ND method were carried out for entire maneuver process (10 s) on a computer with a main frequency of 2.5 GHz. Other simulation conditions are the same as Monte Carlo test. The average time overhead of some estimations is demonstrated in TABLE 5.

The results in Table 5 suggest that with the increase of the number of samples, time cost of aerodynamic parameter estimation increases. And when a maximum training sample size (264 samples) is adopted, the SVR-ND takes no more than 181ms, which can satisfy the needs of online

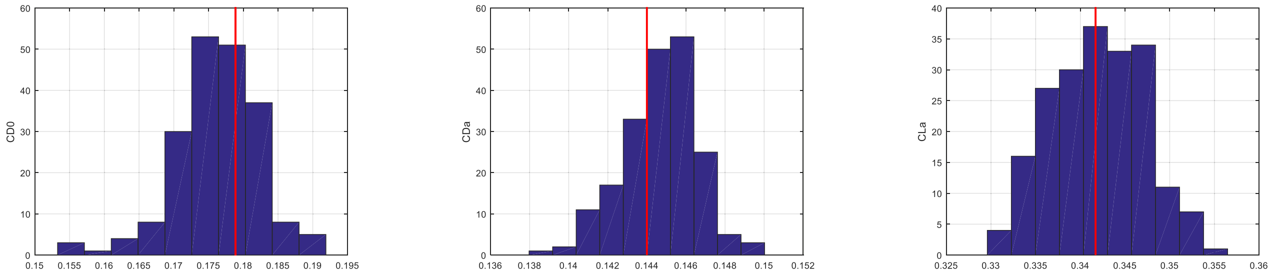

FIGURE 17. Online parameter estimation results of Monte Carlo Simulation for 200 Runs.

TABLE 4. Online parameter estimation results using SVR-ND method.

Parameter	truth value	95% Confidence interval of the parameter	$\mu_s \pm \lambda_s$
C_{D0}	0.1789	[0.1699,0.1704]	0.1702±0.0061
C_D^α	0.144	[0.1446, 0.1448]	0.1447±0.0019
C_D^δ	0.0085	[0.0086,0.0087]	0.0087±0.0032
C_L^α	0.3417	[0.340,0.3403]	0.3402±0.0052
C_L^δ	0.0984	[0.1006,0.1012]	0.1009±0.0094
$C_{m\alpha}$	-0.0450	[-0.0450,-0.0449]	-0.0451±0.0005
$C_{m\delta_e}$	-0.0432	[-0.0411, -0.0410]	-0.0411±0.0025
C_{mw_z}	-0.30	[-0.281,-0.279]	-0.280±0.0268

TABLE 5. Estimated time of 10 runs for different sample size.

Coefficient	Order	Average sample size	Average time(ms)
C_D	1st	100	95.0
	5th	86	81.6
	10th	95	89.2
C_L	1st	100	95
	5th	83	79.8
	10th	82	79.5
C_m	1st	250	176.4
	5th	264	180.1
	10th	252	176.5

aerodynamic parameter estimation (performed per 0.5s). And when less samples are used, the estimated time will decrease accordingly. Therefore, the proposed method is efficient to estimate aerodynamic parameters in real-time.

VI. CONCLUSION

In this work, SVR-ND method is exploited to estimate stability and control derivatives of aircraft online. Real-time flight data is employed to model the aerodynamic force and moment coefficients by applying ε- SVR technique. Numerical differential is used to estimate aerodynamic parameters. To demonstrate the relationship between estimation accuracy and noise level or training sample size, the proposed method is first applied to offline parameter estimation based on simulated historical data. Simulations show that the estimation accuracy is negatively correlated with noise level but positively correlated with the number of samples. In a simulated maneuver, SVR-ND method is applied to online parameter estimation. And the Monte Carlo simulation results indicate that SVR-ND algorithm is able to accurately estimate stability and control derivatives and shows excellent robustness to unknown measure noises. The aerodynamic model generated by SVR-ND method is proved to have a good fitting precision and an excellent generalization performance. In the process of online aerodynamic modeling, variance noise of flight data is estimated, and empirical formulas are proposed to tune training sample size and hyper-parameters of model according to noise level.

Based on the intelligent nonlinear modeling capability of SVR, the SVR-ND method is a model-independent online estimation algorithm. Even if a linear aerodynamic model is adopted in the research, the proposed method based on SVR can be extended to aerodynamic modeling and parameter estimation for nonlinear system. For an aircraft with large flight envelope, such as hypersonic vehicle, flight tests cannot cover all real flight envelopes. Under this condition, online aerodynamic estimation method based on real-time flight data of small sample is required to realize accurate disturbance observation and precise compensation for control system of aircraft. The method proposed in this paper provides a promising way to solve the problem. In addition, the proposed method has a potential to solve other estimation problems in engineering, such as estimation of dynamic coefficient of ship [36], [37].

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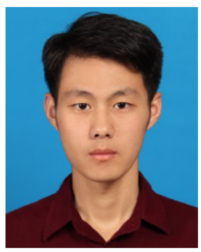

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