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Impulsive Synchronization of Delayed Chaotic Neural Networks With Actuator Saturation

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ABSTRACT This article focuses on the impulsive synchronization of chaotic neural networks (CNNs) with time-varying delays subject to actuator saturation. By constructing discontinuous Lyapunov function and employing linear matrix inequality (LMI) approach, some sufficient conditions are derived to guarantee the synchronization object of the delayed chaotic neural networks. In addition, the control methods in this article have no strict requirements on the size of time delay and the actuator saturation domain, which is more flexible and practical in real system. Finally, a numerical example is given to verify the effectiveness of the proposed method.

INDEX TERMS Impulsive control, synchronization, actuator saturation, chaotic neural networks.

I. INTRODUCTION

Over that last few decades, the research on chaos synchronization has become research hotspots from foreign and domestic scholars in various perspectives, such as information technique, secure communication, biological science, and so forth [1]. The concept of chaos synchronization was explored in [2] firstly, and a large number of control schemes were emerged subsequently to achieve the chaos synchronization scheme, such as state feedback approach [3], [4], sliding mode approach [5], [6], adaptive approach [7], [8], event-triggered approach [9]–[11], anti-disturbance control [12], [13], fuzzy control [14], [15], impulsive approach [16], [17], etc. In some cases, the common continuous control methods will be invalid, and the system state cannot change instantaneously. As a discrete control method, impulsive control can provide an effective solution [18]–[26]. Furthermore, in the process of impulsive synchronization (only at the impulsive instants), the response (slave) system is controlled by obtaining the state information of the drive (master) system. Obviously, the information transmission loads will be alleviated enormously, which obtains low control cost and strong robustness in actual applications.

Note that the influence of time delay is a nonnegligible factor in practical applications because of the limited switching speed and transmitting signals [27]–[29]. Facts proved that the delayed system is very common in the case of signal transmission and manual control. Moreover, the response trend of the system state depends on its current value and past one concurrently. Thus, it is meaningful and significant to discuss the problems of delayed impulsive control system. There are some meaningful results in synchronization of delayed chaotic systems with impulsive control strategy [30]–[35]. In [30], the novel impulsive synchronization control criteria of delayed chaotic neural networks (DCNNs) are established to handle with the scheme with uncertain nonlinear coupling function. In [31], the synchronization of delayed fractional order systems with impulsive control approach was discussed, and the scheme with same structure and different

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structure was investigated simultaneously. In [32], the impulsive synchronization scheme of DCNNs with distributed and time-varying delays was discussed. In [33], the power-rate synchronization of delayed (i.e., proportional delay) CNNs with impulsive control approach was studied. The lag synchronization of DCNNs was studied, and the sampled-data and impulsive control (so-called hybrid control) was designed in [34]. In [35], the synchronization of coupled delayed multistable neural networks with directed topology was investigated, and two new concepts (i.e., DMS and SMS) were firstly proposed to describe the two novel kinds of synchronization manifolds.

Furthermore, actuator saturation is a very common and important nonlinearity in practical control systems because the actuators cannot produce unrestricted amplitude signals in real systems. It is common knowledge that actuator fault and saturation may give rise to performance deterioration of the system and even make the stable closed-loop system unstable for external perturbations. Recently, many meaningful research results on actuator saturation are investigated because of the significance and importance of actuator saturation [36]–[42]. For instance, in [36], based on the master-slave synchronization concept, the synchronization with input time-delay and input saturation was discussed. In [37], the synchronization of fractional order chaotic systems with impulsive control approach was discussed, and both control gain error and actuator saturation were considered simultaneously. The master-slave synchronization with input saturation, model mismatches and external perturbations was addressed in [38]. In [39], the synchronization of the uncertain coupled memristive NNs with switching topology and actuator saturation was discussed, and the non-fragile reliable controller was designed to realize the synchronization asymptotically under directed topology. Considering the control advantage of impulsive control method, it is very important and significant to investigate the impulsive synchronization with actuator saturation. In [40], the impulsive synchronization of coupled DNNs with actuator saturation via sector nonlinearity model method was investigated, and the derived results were verified in image encryption. In [41], the time-delayed impulsive control for discrete-time dynamical systems with actuator saturation was discussed, some new sufficient criteria were derived by impulsive differential inequality techniques and convex analysis method. Several fault-tolerant control laws for singularly perturbed systems with actuator faults and disturbances were discussed in [42].

Based on the above discussions, the impulsive synchronization for delayed (i.e., time-varying delays) CNNs with actuator saturation is explored in this works, which is discussed firstly in the literatures to the authors' knowledge. By Lyapunov analysis method, linear matrix inequality (LMI) and impulsive control system theory, some new sufficient criteria on impulsive synchronization or stabilization of DCNNs with actuator saturation are derived, which is more effective and rigorous in actual control systems. It is particularly important to note that the stabilization and synchronization conditions in this article of DCNNs also apply to the impulsive synchronization or stabilization of the delayed nonlinear systems with similar system models. Moreover, the relation between the control parameters and the control performance is discussed intensively, which can bring design guidance for obtaining better control performance.

The organization of this article is outlined below. Section 2 provides the problem formulation and some necessary preliminaries. Main results for the impulsive synchronization and stabilization of DCNNs with actuator saturation are given in Section 3. In Section 4, one numerical simulation is given to show the correctness of the obtained main results. Finally, in Section 5, some brief conclusions are included.

Notations: I_N, $\otimes$, $\lambda_{\max}(\cdot)$, $\mathbb{R}$, $\mathbb{R}^+$, $\mathbb{R}^n$, $\mathbb{R}^{m\times n}$ refer to the identity matrix with N dimensions, the Kronecker product, the maximal eigenvalue, the real numbers, the positive real numbers, the Euclidean space with n dimensions, the $m \times n$ real matrices respectively. $\mathbb{N} = \{1, 2, \ldots\}$, $\mathrm{diag}\{d_1, \ldots, d_N\}$ is the diagonal matrix.

II. PROBLEM DESCRIPTION

The master system is considered as

$$\dot{x}(t) = -Bx(t) + A_1 f(x(t)) + A_2 f(x(t-\tau(t))), \quad (1)$$

where $x(t) = [x_1(t), \cdots, x_n(t)]^T \in \mathbb{R}^n$, $A_1 \in \mathbb{R}^{n\times n}$, $A_2 \in \mathbb{R}^{n\times n}$, $B = \mathrm{diag}\{b_1, \cdots, b_n\}$, $b_i > 0$, the state time delay satisfies $0 \le \tau(t) \le \tau$, $f(x(t)) = [f_1(x_1(t)), \ldots, f_n(x_n(t))]^T \in \mathbb{R}^n$ is the activation function respectively.

Assumption 1: The nonlinear function $f_i : \mathbb{R} \to \mathbb{R}$ satisfies

$$\|f_i(\omega_1) - f_i(\omega_2)\| \le \sigma_i \|\omega_1 - \omega_2\|, \sigma_i > 0, \quad \forall \omega_1, \omega_2 \in \mathbb{R}.$$

It follows from the master–slave chaos synchronization viewpoint that the slave one is described as

$$\dot{y}(t) = -By(t) + A_1 f(y(t)) + A_2 f(y(t-\tau(t))) + u(t), \quad (2)$$

where $y(t) = [y_1(t), \cdots, y_n(t)]^T \in \mathbb{R}^n$. Note that system (1) and (2) have the same system model. The controller $u(t) \in \mathbb{R}^n$ in (2) is chosen as

$$u(t) = \mathrm{sat}(\Gamma_k e(t_k))\delta(t - t_k), \quad (3)$$

where $\mathrm{sat}(\Gamma_k e(t_k)) \doteq (\mathrm{sat}(\gamma_{1k} e_1(t_k)), \ldots, \mathrm{sat}(\gamma_{nk} e_n(t_k)))^T$ with $\mathrm{sat}(\vartheta) = \mathrm{sign}(\vartheta)\min\{\Delta, |\vartheta|\}$, $\vartheta \in \mathbb{R}$, $\Delta \in \mathbb{R}^+$ is the saturation level. $e(t) = y(t) - x(t) = [e_1(t), \ldots, e_n(t)]^T$ denotes the synchronization error vector. $\Gamma_k = \mathrm{diag}\{\gamma_{1k}, \ldots, \gamma_{nk}\}$ is the gain matrix of impulsive control. The discrete time sequence $\{t_k\}$ satisfies $0 \le t_0 < t_1 < t_2 < \cdots < t_{k-1} < t_k < \cdots$, $x(t_k^+) = \lim_{s\to 0^+} x(t_k + s)$, $x(t_k^-) = x(t_k) = \lim_{s\to 0^+} x(t_k - s)$, $\delta(t)$ is the Dirac delta function.

Subtract system (1) from (2), the synchronization error system is given as

$$\begin{cases} \dot{e}(t) = -Be(t) + A_1\varphi(e(t)) + A_2\varphi(e(t-\tau(t))), t \ne t_k, \\ \Delta e(t_k) = e(t_k^+) - e(t_k^-) = sat(\Gamma_k e(t_k)), k \in \mathbb{N}, \end{cases} \quad (4)$$

where

$$\varphi(e(t)) \doteq [(f_1(y_1(t)) - f_1(x_1(t)), \ldots, f_n(y_n(t)) - f_n(x_n(t))]^T,$$
$$\varphi(e(t-\tau(t))) \doteq [(f_1(y_1(t-\tau(t))) - f_1(x_1(t-\tau(t))), \ldots, f_n(y_n(t-\tau(t))) - f(x_n(t-\tau(t)))]^T.$$

To understand the characteristics of actuator saturation well, we define a time-varying function $h_i(t_k) \in \mathbb{R}^+$ as

$$h_i(t_k) = \begin{cases} \dfrac{\Delta}{|\gamma_{ik} e_i(t_k)|} & |\gamma_{ik} e_i(t_k)| > \Delta, \\ 1 & |\gamma_{ik} e_i(t_k)| \le \Delta. \end{cases} \tag{5}$$

It is easy to check that $h_i(t_k) \in (0, 1]$ and then the actuator saturation in (3) is described by

$$\begin{aligned} \text{sat}(\Gamma_k e(t_k)) &= (\text{sat}(\gamma_{1k} e_1(t_k)), \text{sat}(\gamma_{2k} e_2(t_k)), \\ &\quad \ldots, \text{sat}(\gamma_{nk} e_n(t_k)))^T \\ &= (\gamma_{1k} h_1(t_k) e_1(t_k), \gamma_{2k} h_2(t_k) e_2(t_k), \\ &\quad \ldots, \gamma_{nk} h_n(t_k) e_n(t_k))^T \\ &= \Gamma_k H(t_k) e(t_k), \end{aligned} \tag{6}$$

where $H(t_k) = \text{diag}\{h_1(t_k), \ldots, h_n(t_k)\} \in \mathbb{R}^{n\times n}$.

In this article, the control goal is to achieve the asymptotic synchronization of system (1) and (2) via impulsive control, i.e., $\lim_{t\to\infty} e(t) = 0$.

In the following sections, all time-varying parameters will be simplified as $x \doteq x(t)$ or $x_\tau \doteq x(t-\tau(t))$ for convenience.

III. MAIN RESULTS

In the section that follows, the main synchronization conditions are studied to accomplish impulsive synchronization of DCNNs subject to actuator saturation.

Before giving the main theorems, one necessary lemma is introduced, which will help to obtain the main results.

Lemma 1 [43]: For any real matrices S_1, S_2 and $Y > 0$, scalar $\varepsilon > 0$, it has

$$S_1^T S_2 + S_2^T S_1 \le \varepsilon S_1^T Y S + \varepsilon^{-1} S_2^T Y^{-1} S_2.$$

Theorem 1: The synchronization of systems (1) and (2) is realized with impulsive controller (3) if the following inequalities holds:

$$\begin{bmatrix} \Xi_i & \Sigma \\ * & -Z_i \end{bmatrix} < 0, \quad i = 1, 2, \tag{7}$$
$$(\Gamma_k H(t_k) + I_n)^T (\Gamma_k H(t_k) + I_n) \le \eta_k I_n, \tag{8}$$
$$(\alpha + \frac{\beta}{\eta_k})(t_{k+1} - t_k) + \ln \eta_k < \ln \xi, \tag{9}$$

where $\Xi_1 = -2B + A_1 Z_1 A_1^T + A_2 Z_2 A_2^T - \alpha I_n$, $\Xi_2 = -\beta I_n$, $\Sigma = \text{diag}\{\sigma_1, \ldots, \sigma_n\}$. Z_1 and Z_2 are positive diagonal matrices, and $\alpha, \beta, \eta_k < \xi < 1 (k \in \mathbb{N})$ are positive constants.

Proof: Consider the Lyapunov functions as

$$V(t) = e^T e. \tag{10}$$

and its derivative is given as

$$D^+ V(t) = 2e^T(-Be + A_1\varphi(e) + A_2\varphi(e_\tau)). \tag{11}$$

From Lemma 1, one gets

$$\begin{aligned} 2eA_1\varphi(e) &\le e^T A_1 Z_1 A_1^T e + \varphi^T(e) Z_1^{-1} \varphi(e) \\ &\le e^T(A_1 Z_1 A_1^T + \Sigma Z_1^{-1} \Sigma) e. \end{aligned} \tag{12}$$

and

$$\begin{aligned} 2eA_2\varphi(e_\tau) &\le e^T A_2 Z_2 A_2^T e + \varphi^T(e_\tau) Z_2^{-1} \varphi(e_\tau) \\ &\le e^T A_2 Z_2 A_2^T e + e_\tau^T \Sigma Z_2^{-1} \Sigma e_\tau. \end{aligned} \tag{13}$$

Thus, from (12) and (13), one has

$$\begin{aligned} &D^+ V(t) \\ &\le e^T(-2B + A_1 Z_1 A_1^T + \Sigma Z_1^{-1}\Sigma + A_2 Z_2 A_2^T) e \\ &\quad + e_\tau^T \Sigma Z_2^{-1} \Sigma e_\tau \\ &= e^T(-2B + A_1 Z_1 A_1^T + \Sigma Z_1^{-1}\Sigma + A_2 Z_2 A_2^T - \alpha I_n) e \\ &\quad + e_\tau^T(\Sigma Z_2^{-1}\Sigma - \beta I_n) e_\tau + \alpha e^T e + \beta e_\tau^T e_\tau \\ &= e^T \Pi_1 e + e_\tau^T \Pi_2 e_\tau + \alpha e^T e + \beta e_\tau^T e_\tau, \end{aligned} \tag{14}$$

where

$$\Pi_1 = -2B + A_1 Z_1 A_1^T + \Sigma Z_1^{-1}\Sigma + A_2 Z_2 A_2^T - \alpha I_n$$
$$\Pi_2 = \Sigma Z_2^{-1}\Sigma - \beta I_n.$$

Note that the condition (7) implies inequality $\Pi_i < 0$ $(i = 1, 2)$, therefore, it gets

$$D^+ V(t) \le \alpha e^T e + \beta e_\tau^T e_\tau = \alpha V(t) + \beta V(t - \tau(t)). \tag{15}$$

For $t = t_k$, from (3) and (6), it yields

$$e(t_k^+) = (\Gamma_k H(t_k) + I_n) e(t_k), \tag{16}$$

then one can get

$$\begin{aligned} V(t_k^+) &= e^T(t_k^+) e(t_k^+) \\ &= e^T(t_k)((\Gamma_k H(t_k) + I_n)^T (\Gamma_k H(t_k) + I_n)) e(t_k) \\ &\le \eta_k V(t_k) \\ &\le \eta_k \tilde{V}(t_k), \end{aligned} \tag{17}$$

where $\tilde{V}(t) \doteq \sup_{s\in[t-\tau,t]} V(s)$.

Next, for $t \in (t_k, t_{k+1}]$, $k \in \mathbb{N}$, we are going to obtain

$$V(t) < \xi \tilde{V}(t_k), 0 < \eta_k < \xi < 1. \tag{18}$$

If (18) is not correct, from the continuity of $V(t)$ and $V(t_k^+) \le \eta_k \tilde{V}(t_k)$ for $t \in (t_k, t_{k+1}]$, there must exist a $\tilde{t}_k \in (t_k, t_{k+1}]$ such that

$$V(\tilde{t}_k) = \xi \tilde{V}(t_k), \tag{19}$$

and $V(t) < \xi \tilde{V}(t_k)$, for $t \in (t_k, \tilde{t}_k)$.

From (19) and $V(t_k^+) \le \eta_k \tilde{V}(t_k)$, there exists $\hat{t}_k \in (t_k, \tilde{t}_k)$ such that

$$V(\hat{t}_k) = \eta_k \tilde{V}(t_k), \tag{20}$$

and $\eta_k \tilde{V}(t_k) \le V(t) \le \xi \tilde{V}(t_k), fort \in [\hat{t}_k, \tilde{t}_k]$, where $\hat{t}_k = \sup\{t \in (t_k, \tilde{t}_k), V(t) \le \eta_k \tilde{V}(t_k)\}$.

For $t \in [\hat{t}_k, \tilde{t}_k]$, it yields

$$V(t+s) \leq \tilde{V}(t_k), \quad \text{for } s \in [-\tau, 0]. \tag{21}$$

From (20) and (21), it yields

$$\eta_k V(t+s) \leq \eta_k \tilde{V}(t_k) \leq V(t), \quad \text{for } s \in [-\tau, 0), t \in [\hat{t}_k, \tilde{t}_k].$$

Thus, for $t \in [\hat{t}_k, \tilde{t}_k]$, (15) yields

$$D^+V(t) \leq \alpha V(t) + \beta(V(t-\tau(t)) \leq (\alpha + \frac{\beta}{\eta_k})V(t). \tag{22}$$

Integrating the both sides of (22) from $\hat{t}_k$ to $\tilde{t}_k$, where $t_k < \hat{t}_k < \tilde{t}_k < t_{k+1}$, it yields

$$\begin{aligned}\ln(V(\tilde{t}_k)) - \ln(V(\hat{t}_k)) &\leq (\alpha + \frac{\beta}{\eta_k})(\tilde{t}_k - \hat{t}_k) \\ &\leq (\alpha + \frac{\beta}{\eta_k})(t_{k+1} - t_k).\end{aligned} \tag{23}$$

Moreover, from (9), (19) and (20), one has

$$\begin{aligned}\ln(V(\tilde{t}_k)) - \ln(V(\hat{t}_k)) &= \ln(\xi V(\tilde{t}_k)) - \ln(\eta_k V(\tilde{t}_k)) \\ &= \ln\xi - \ln\eta_k > (\alpha + \frac{\beta}{\eta_k})(t_{k+1} - t_k),\end{aligned} \tag{24}$$

which contradicts (23), and therefore (18) holds.

Next, we will prove the result $\tilde{V}(t_k) \leq \tilde{V}(t_{k-1})$, $k \in \mathbb{N}$

1) If $t_k - \tau \geq t_{k-1}$, then it can get

$$\tilde{V}(t_k) \leq \sup_{s\in[t_k-\tau, t_k)} V(s) \leq \xi\tilde{V}(t_{k-1}). \tag{25}$$

2) If $t_k - \tau < t_{k-1}$, it gets

$$\begin{aligned}\tilde{V}(t_k) &\leq \sup_{s\in[t_k-\tau, t_k)} V(s) \\ &\leq \max\{\tilde{V}(t_{k-1}), \xi\tilde{V}(t_{k-1})\} \\ &= \tilde{V}(t_{k-1}).\end{aligned} \tag{26}$$

From (25) and (26), one gets

$$\tilde{V}(t_k) \leq \tilde{V}(t_{k-1}), \quad k \in \mathbb{N}. \tag{27}$$

In general, there exist $1 < k_1 < k_2 < \cdots < k_{l-1} < k_l < k_{l+1} < \cdots (k_l \in \mathbb{N}, l \in \mathbb{N})$ such that

$$\begin{cases} t_{k_1} - \tau \in (t_1, t_2], \\ t_{k_2} - \tau \in (t_{k_1}, t_{k_1+1}], \\ \vdots \\ t_{k_{l-1}} - \tau \in (t_{k_{l-2}}, t_{k_{l-2}+1}], \\ t_{k_l} - \tau \in (t_{k_{l-1}}, t_{k_{l-1}+1}], \\ t_{k_{l+1}} - \tau \in (t_{k_l}, t_{k_l+1}], \\ \vdots \end{cases} \tag{28}$$

Thus, for $t \in (t_{k_l}, t_{k_{l+1}}]$, from (18), (27) and (28), it yields

$$V(t) \leq \xi\tilde{V}(t_{k_l}). \tag{29}$$

Moreover, it gets $t_{k_l+1} - \tau > t_{k_l}$, which yields

$$\tilde{V}(t_{k_l+1}) \leq \xi\tilde{V}(t_{k_l}). \tag{30}$$

Thus, from (30), (29) can further yield that

$$\begin{aligned}V(t) &\leq \xi\tilde{V}(t_{k_l}) \\ &\leq \xi^2\tilde{V}(t_{k_l-1}) \\ &\leq \xi^3\tilde{V}(t_{k_l-2}) \\ &\leq \cdots \\ &\leq \xi^l\tilde{V}(t_{k_1}) \\ &\leq \xi^{l+1}\tilde{V}(t_1).\end{aligned} \tag{31}$$

From (27) and (31), one has

$$V(t) \leq \xi^{l+1}\tilde{V}(t_1) \leq \xi^{l+1}\tilde{V}(t_0),$$

which further implies that

$$\|e(t)\|^2 \leq \xi^{l+1}\tilde{V}(t_0).$$

Note that $\xi^{l+1} \to 0$ as $l \to \infty$ (i.e., $t \to \infty$). Obviously, the error vector $e(t)$ converge to zero asymptotically. This completes the proof. □

Remark 1: It follows from (8) and $h_i(t_k) \in (0, 1]$ that $\eta_k \in (0, 1)$ satisfies for $\gamma_{ik} \in \Theta \doteq (-2, -1) \cup (-1, 0)$. The master-slave synchronization goal can be obtained for choosing suitable control gain $\gamma_{ik} \in \Theta$ and impulsive interval $\tau_k = t_{k+1} - t_k$.

Remark 2: It follows from (5) that $\text{sat}(\gamma_{ik}e_i(t_k)) = \text{sign}(e_i(t_k))\Delta$ if $|\gamma_{ik}e_i(t_k)| > \Delta$. In this situation, the strength of the impulsive control will be weakened to some extent.

Note that the main results in Theorem 1 is also applicable to the stabilization of single delayed system (1), and the controlled impulsive system can be further described as

$$\begin{cases} \dot{x}(t) = -Bx(t) + A_1 f(x(t)) + A_2 f(x(t-\tau(t))), & t \neq t_k, \\ \Delta x(t_k) = x(t_k^+) - x(t_k^-) = \text{sat}(\Gamma_k x(t_k)), & k \in \mathbb{N}. \end{cases} \tag{32}$$

The stabilization conditions can be derived in the following corollary, which has similar format with Theorem 1.

Corollary 1: The stabilization of DCNNs (32) can be realized if the following inequalities holds:

$$\begin{bmatrix} \Xi_i & \Sigma \\ * & -Z_i \end{bmatrix} < 0, \quad i = 1, 2, \tag{33}$$

$$(\Gamma_k H(t_k) + I_n)^T(\Gamma_k H(t_k) + I_n) \leq \eta_k I_n, \tag{34}$$

$$(\alpha + \frac{\beta}{\eta_k})(t_{k+1} - t_k) + \ln\eta_k < \ln\xi, \tag{35}$$

where $\Xi_1 = -2B + A_1Z_1A_1^T + A_2Z_2A_2^T - \alpha I_n$, $\Xi_2 = -\beta I_n$, $\Sigma = \text{diag}\{\sigma_1, \ldots, \sigma_n\}$. Z_1 and Z_2 are positive diagonal matrices, and α, β, $\eta_k < \xi < 1 (k \in \mathbb{N})$ are some positive constants.

Proof: Compared with the controlled synchronization error system (4) and the controlled DCNNs (32), they have similar structure and control goal (i.e., error vector and state vector asymptotically convergence to zero). Thus, the detailed proof of Corollary 1 can be referred to Theorem 1, and it is omitted here for brevity. □

Remark 3: Some literatures have studied the impulsive stabilization and synchronization of master-slave (chaotic)

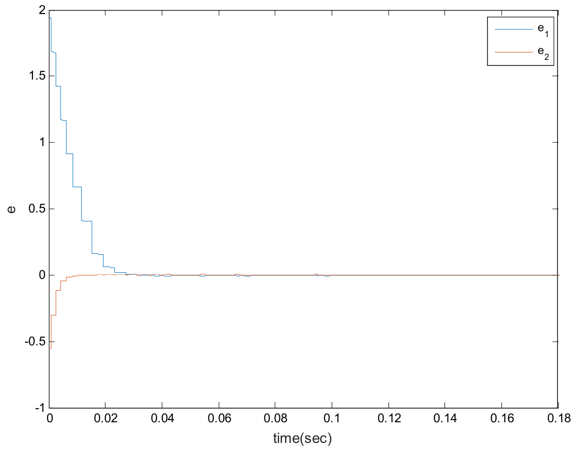

FIGURE 1. Synchronization error trajectories for Theorem 1.

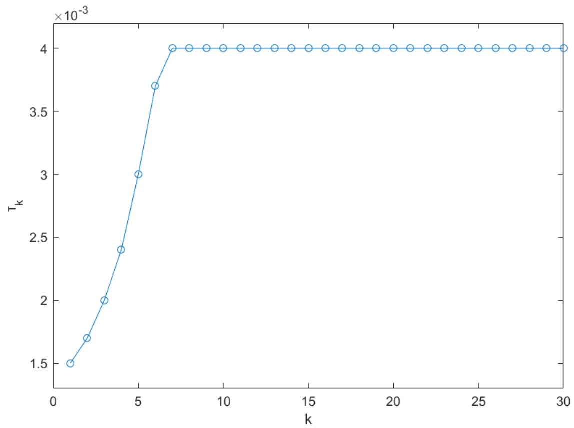

FIGURE 2. The impulsive interval $\tau_k = t_{k+1} - t_k$ vs k.

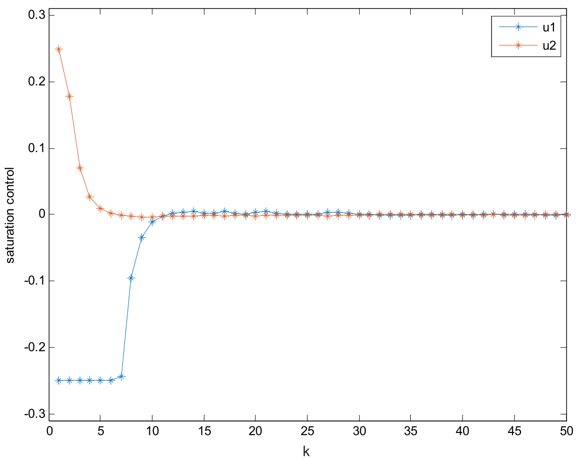

FIGURE 3. The impulsive controller $u(t_k)$ vs k.

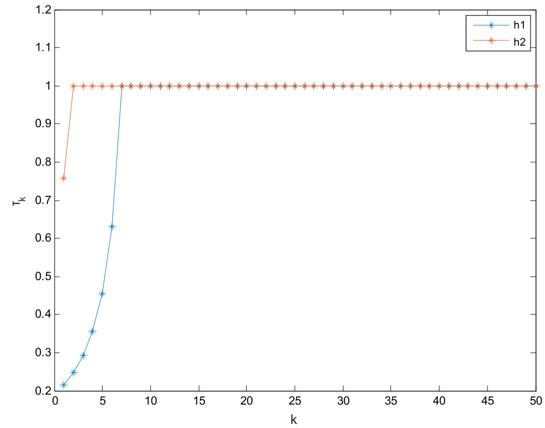

FIGURE 4. The time-varying function $h_i(t_k)$ vs k.

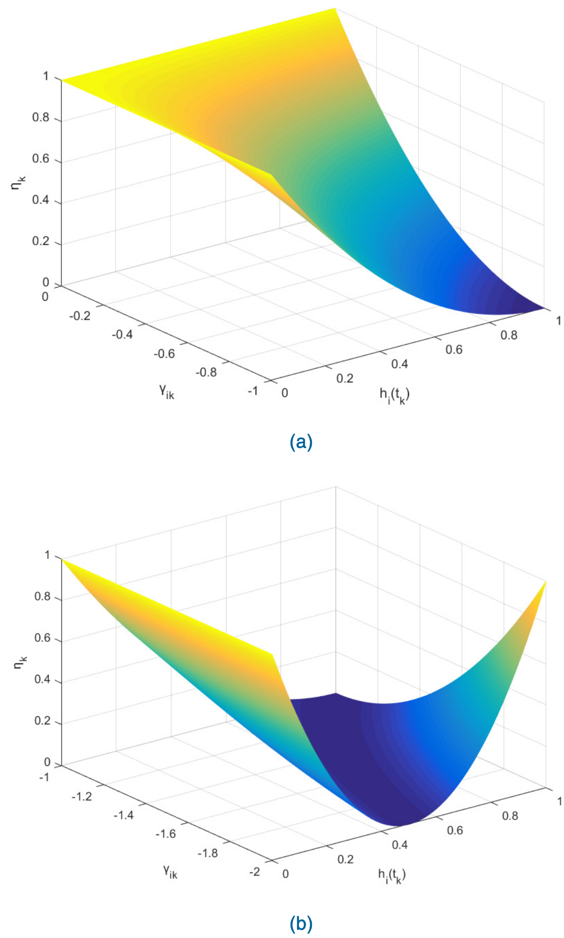

FIGURE 5. Parametric comparison diagram for $h_i(t_k)$, γ_{ik} and η_k. (a) the case for $\gamma_{ik} \in (-1, 0)$ (b) the case for $\gamma_{ik} \in (-2, -1)$.

systems simultaneously [44]–[46]. Note that the criteria of stabilization and synchronization in [44]–[46] are also similar because of their similar controlled systems. For example, Theorem 1 and Theorem 2 in [44], Theorem 1 and Theorem 3 in [45], Theorem 3 and Theorem 4 in [46].

IV. SIMULATION RESULTS

An example is given to verify the feasibility and correctness of the synchronization conditions, and the control performance under the given control parameters is discussed. The models of system (1) and (2) are considered as

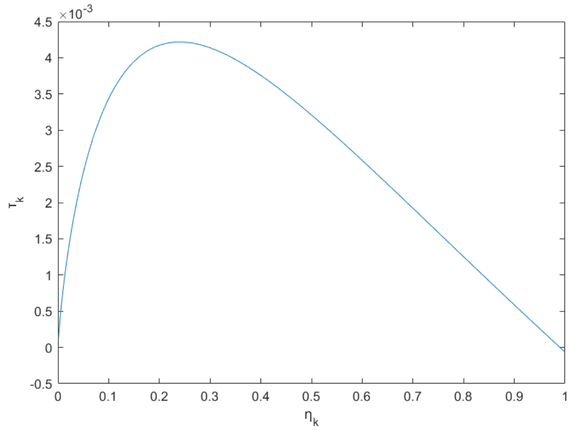

FIGURE 6. The relation between η_k and τ_k.

follows:

$$\dot{x} = -Bx + A_1 f(x) + A_2 f(x_\tau),$$

$$\begin{cases} \dot{y} = -By + A_1 f(y) + A_2 f(y_\tau), & t \neq t_k, \\ \Delta y(t_k) = \mathrm{sat}(\Gamma_k e(t_k)), & k \in \mathbb{N}, \end{cases}$$

where $B = I_2$, $f(x) = \frac{|x+1|-|x-1|}{2}$, $\tau = 0.1s A_1 = \begin{bmatrix} 1+\pi/4 & 20 \\ 0.1 & 1+\pi/4 \end{bmatrix}, A_2 = \begin{bmatrix} -1.3\sqrt{2}\pi/4 & 0.1 \\ 0.1 & -1.3\sqrt{2}\pi/4 \end{bmatrix}$ It follows from Assumption 1 that $\Sigma = I_2$. Let $\alpha = 99.2227$, $\beta = 56.8160$, $\gamma_{1k} = \gamma_{2k} = -0.6$, $\xi = 0.99$ such that the inequalities (7)∼(9) holds. The saturation level is chosen as $\Delta = 0.25$.

The curves in Fig. 1 show that the impulsive synchronization is achieved less than 0.11s, which presents the feasibility of the proposed theoretical approach. The curves of impulsive interval τ_k, impulsive controller $u(t_k)$ and function $h_i(t_k)$ are shown in Figs. 2∼4 respectively, which reflects the response trend of the acceptable impulsive interval and control intensity. With the decrease of the error magnitude, the controlled system is not under the influence of the input saturation (note that the function matrix $H(t_k) = I_2$ in this case), and the acceptable impulsive interval keep unchanged. For further analyze the parametric relationship in inequalities (8) and (9), the parametric comparison diagram for $h_i(t_k)$, γ_{ik} and η_k is shown in Fig. 5, and the relation of η_k and τ_k is shown in Fig. 6, which provides design guidance to obtain better synchronization performance. In this section, the initial state are chosen as $x = [0.05, 0.05]^T$ and $y = [2, -0.5]^T$ respectively.

V. CONCLUSION

This article investigates the impulsive synchronization of delayed chaotic neural networks subject to actuator saturation. By using Lyapunov analysis method and some helpful inequality approaches, the asymptotic convergence of the error system is studied via the designed impulsive controller. Accordingly, this article gives some necessary discussions of control parameters and synchronization performance. The numerical simulation results can prove the correctness of the proposed protocol.

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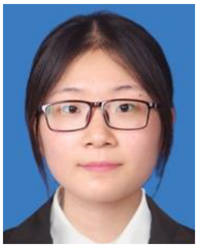

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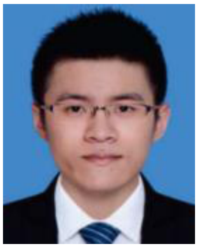

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