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An Efficient Codebook Design for Uplink SCMA

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ABSTRACT Sparse code multiple access (SCMA) is one of the most intensively investigated non-orthogonal multiple access (NOMA) techniques for the fifth generation communication (5G) system. In this paper, a new multi-stage optimization approach for codebook design is proposed for the uplink SCMA system. The proposed schemes includes two parts: i) multi-dimensional mother constellation design ii) mapping matrix design. Firstly, the multi-dimensional complex mother constellation design is implemented by a series of two-dimensional real constellations which can be obtained by the lattice theory and the symbol switch algorithm (SSA). The Secondly, we construct user-specific mapping matrix by optimizing the phase rotation of the user constellation to effectively reduce the interference between the multi-user codewords superimposed on every single time-domain resource elements (REs). Finally, the multi-user codebooks are generated by combining the multi-dimensional mother constellation and the user-specific mapping matrix. The simulation results demonstrate that the proposed codebooks can greatly decrease bit error rate (BER) value compared to the existing codebooks under different channels, different codebook sizes and different overloading ratios, and the performance is also improved for the large-scale SCMA codebook. More importantly, in order to verify the proposed codebook applicability, two different message passing algorithm (MPA) are used to simulate the SCMA system with different codebook sizes, both of which obtain faster convergence speed and better BER performance than other existing codebooks.

INDEX TERMS SCMA, codebook design, mapping matrix, BER, message passing algorithm.

I. INTRODUCTION

With the rapid development of wireless communication, there are tighter requirements such as throughput, spectrum utilization, very low latency, and massive device connectivity for the 5G system [1]–[3]. Due to the limitations of frequency-domain and time-domain resource elements (REs), the conventional orthogonal multiple access (OMA)-based is hard to meet increasing application demands. Different from conversional OMA technologies, non-orthogonal multiple access (NOMA) can accommodate much more users via resource allocation, so as to achieve massive connectivity, low latency and high spectral efficiency, but its system performance still be affected by interference and receiver complexity [4], [5]. The several NOMA techniques can be categorized into three main classes: the power-domain NOMA (PD-NOMA) [6], [7], code-domain NOMA (CD-NOMA) [8], [9], and power-domain sparse code multiple access (PSMA) [10], [11].

The low-density signature (LDS) [8] technique is a special approach of CD-NOMA spreading sequence with a few nonzero elements within a large signature length. Like the low-density parity-check coding (LDPC), widely used message passing algorithm (MPA) can be used for multi-user detection at the receiver due to sparsity of LDS [12]–[14]. The sparse code multiple access (SCMA) is a novel CD-NOMA technique which is an enhanced variation of LDS. At the transmitter, an SCMA encoder can combine modulation mapping and spreading so that information bits can be directly mapped to multi-dimensional complex domain sparse codewords selected from predefined SCMA codebooks. Compared to LDS, SCMA benefits from shaping gains which are brought by multi-dimensional codewords [15]–[17]. Due to the SCMA codewords being sparse, MPA also can be used for multi-user detection at the receiver, which can sharply reduce the complexity of multi-user detection. The performance of MPA detector is sensitive to the distance between

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codewords superimposed on the same RE. In order to reduce the interference between the codewords and achieve good system performance, SCMA codebook need to be designed delicately by increasing power diversity and Euclidean distance between interfering codewords. Both designing high efficient multi-user codebooks and a low complexity detection algorithm are critical to SCMA system.

A. RELATED WORKS AND MOTIVATION

The codebook design is one of the key factors for the performance of SCMA system which has generated significant efforts among research community since the inception of SCMA was proposed, but designing an efficient SCMA codebook is still a challenge problem. Nikopour *et al.* [9], [18] first proposed a multi-stage suboptimal SCMA codebook design method based on lattice constellation for SCMA systems. The multi-dimensional mother constellation can be obtained by the Cartesian product of quadrature amplitude modulation (QAM), and then different constellation operators can be applied on the mother constellation to build multi-user codebooks. In [19], a codebook design scheme based on the star-quadrature amplitude modulation (star-QAM) signaling constellations is proposed under Gaussian channel, which had better BER performance and lower complexity than those in [18]. However, the scheme exhibits poor BER performance as codebook size increases. Cai *et al.* showed a multi-dimensional SCMA codebook design method based on constellation rotation and interleaving under downlink Rayleigh fading channel [20]. This method can be used to design different codebooks for the aim of spectral efficiency or power efficiency, but the minimum Euclidean distance (MED) between mother codebook did not reach its expectation under the large codebook size. The codebook design based on constellation rotation for downlink SCMA system is presented [21]. This scheme is able to achieve a good shaping gain by constructing proper sub-constellation and Latin matrix. However, the MED between codewords superimposed on sub-constellation is not enough large, thus this scheme shows poor BER performance under large-scale codebook. Most of their codebook schemes exhibit better BER performance under the downlink SCMA system when SCMA codebook is small-scale codebook, but their performance degrades significantly in uplink SCMA system when codebook size increases. Mheich *et al*. proposed to utilize golden angle modulation (GAM) points to generate codebooks for uplink and downlink SCMA systems [22]. This design method can achieve better BER performance and lower PAPR than star-QAM codebooks [19] and the codebook [20], but the BER performance is not enough good for the large-scale codebook. In [23], A codebook design method is proposed, which constructs mother constellation with large coding gain and shaping gain. This codebook has an advantage for the large-scale codebook while it is hard to ensure BER performance of the small-scale codebook for downlink SCMA system. To the best of our knowledge, there is no other efficient design method proposed in present literatures to design SCMA codebooks suitable for both Gaussian channel and uplink Rayleigh channel.

B. CONTRIBUTIONS

An efficient codebook design algorithm for uplink SCMA system is proposed with multi-stage optimization. We use the subset of the two-dimensional lattice constellation and the SSA algorithm to construct the multi-dimensional complex mother constellation. Then the mother constellation combined with the user-specific mapping matrix to generate a new SCMA codebook for good BER performance. The main contributions of this paper are summarized as follows.

- The detailed codebook design criterions which are MED and minimum product distance (MPD) of the mother constellation are established for Gaussian channel and uplink Rayleigh fading channel.
- The optimization problem of the multi-dimensional complex mother constellation is constructed, which can be decomposed into N two-dimensional real constellations optimization. Thus the two-dimensional real constellation with large coding gain is obtained by lattice theory. Then the first dimension of the mother constellation is generated by a subset of the two-dimensional real constellation, the others are obtained by using the SSA algorithm for the subset. Therefore, our mother constellation has larger the MED and MPD than others, and constellation points of that are not overlapped for the large-scale codebook.
- We propose the mapping matrix based on user-specific constellation rotation. Therefore, it greatly reduces the collision of several user information on the same RE. The proposed codebook is generated by combining mother constellation and mapping matrix. The codebook we proposed shows better improvements in BER performance than existing codebook schemes under different channels, different codebook sizes and different overloading ratios.

We use bold upper case letters to denote matrices, bold lower case letters to denote vectors and normal letters to denote scalars. $\mathbb{B}, \mathbb{C}, \mathbb{R}, \mathbb{Z}$ represent the fields of binary, complex, real and integer, respectively. $\mathbf{I}_k$ denotes an identity matrix of size $K \times K$. Also, diag($\mathbf{V}$) is a diagonal matrix. We use the operations $(\cdot)^T$ and $\|\cdot\|$ to represent transpose and Euclidean norm, respectively.

The remainder of this paper is organized as follows: Section II demonstrates uplink SCMA system model. The detailed design scheme and criterions of our proposed SCMA codebook are discussed in III section. The codebook design is considered from two perspectives: i) multi-dimensional mother constellation design ii) mapping matrix design. Computer simulation results of BER performance evaluation is presented in IV section. In addition, in order to verify proposed codebook applicability, two different multi-user detection algorithms are used to compare convergence behavior under different codebook schemes. Finally, V section concludes this paper.

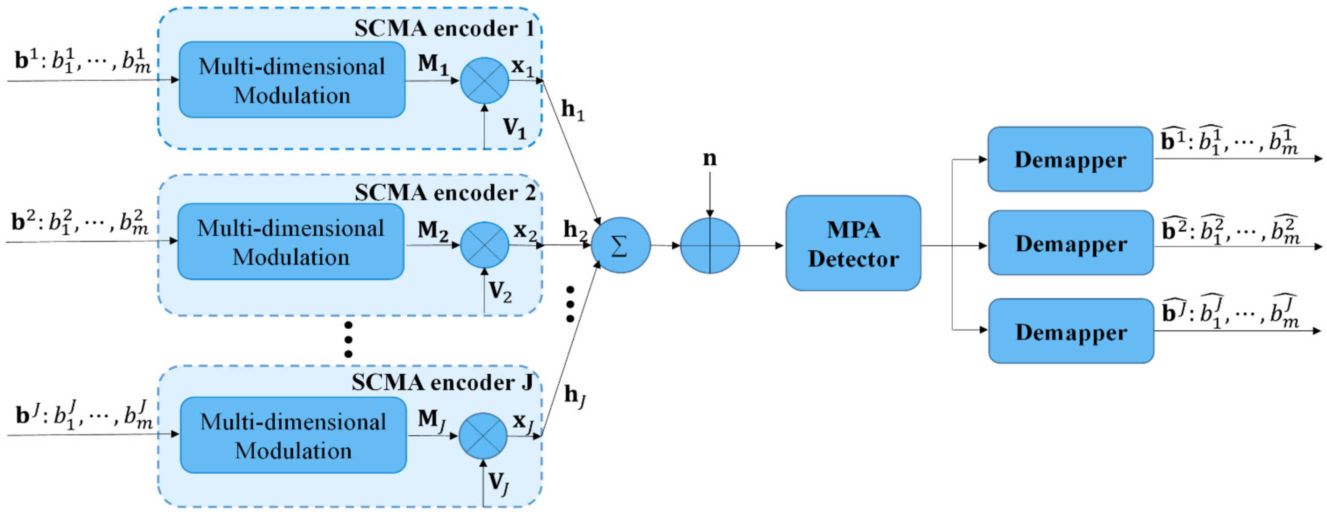

FIGURE 1. Uplink SCMA system model.

II. SCMA UPLINK SYSTEM MODEL

In this paper, we consider an uplink SCMA system with J users sharing K orthogonal REs, where the overload ratios is $\lambda = J/K(J > K)$. The uplink SCMA system mainly consists of encoder and detector as shown in Fig. 1. The encoded procedure for user j can be written as $f_j : \mathbb{B}^{\log_2(M)} \rightarrow \chi_j$, where $\chi_j \subset \mathbb{C}^K$ is the codebook of user j with cardinality M. A vector $\mathbf{b}_j$ of $\log_2(M)$ coded bits for user j is mapped to a K-dimensional codeword $\mathbf{x}_j$ selected from codebook set χ_j. Each complex codeword $\mathbf{x}_j$ is a sparse vector with $N<K$ non-zero elements. The sparsity of the codewords limits the number of users superimposed on the same resource which turns out to reduce complexity of multi-user detection. At the receiver, the received signal after synchronous user multiplexing can be mathematically expressed as:

$$\mathbf{y} = \sum_{j=1}^{J} \mathbf{h}_j \mathbf{x}_j + \mathbf{n} \tag{1}$$

where $\mathbf{x}_j = \left[x_j^1, x_j^2, \cdots x_j^K\right]^{\mathrm{T}}$ denotes a K-dimensional codeword of user j, $\mathbf{h_j} = [h_j^1, h_j^2, \cdots, h_j^K]^{\mathrm{T}}$ is corresponding channel coefficient of user j, $\mathbf{n} \sim \mathcal{CN}(0, \sigma^2)$ is the additive white Gaussian noise (AWGN) with zero mean and $N_0\mathbf{I}_K$ variance.

III. SCMA CODEBOOK DESIGN

In this section, we proposed detailed criterions and scheme of the codebook design. The SCMA codebook design process of the user j is shown in encoder j of Fig. 1, which includes following steps:

Step1: We define a vector $\mathbf{b}_j$ of $\log_2(M)$ coded bits for user j, and $\mathbf{b}_j$ is mapped to user-specific constellation $\mathbf{M}_j \in \mathbb{C}^{N\times M}$.

$$g_j : \mathbb{B}^{\log_2(M)} \rightarrow \mathbf{M}_j \tag{2}$$

The multi-dimensional mother constellation $\mathbf{M} \in \mathbb{C}^{N\times M}$ should be designed for J shared users. The mother constellation $\mathbf{M}$ can be rotated by constellation operator $\Delta_j \in \mathbb{C}^{N\times N}$ to obtain the constellation $\mathbf{M}_j$ of user j.

Step2: The binary mapping matrix $\mathbf{V}_j \in \mathbb{B}^{K\times N}$ represents the spreading operation of the constellation $\mathfrak{M}_j$ of user j. And the $\mathbf{V}_j$ can be obtained from the factor graph matrix $\mathbf{F} = [\mathbf{f}_1, \mathbf{f}_2, \cdots, \mathbf{f}_J] \in \mathbb{B}^{K\times J}$, where $\mathbf{f}_j = \mathrm{diag}(\mathbf{V}_j \cdot \mathbf{V}_j^{\mathrm{T}})$. The k^{th} row of the matrix $\mathbf{F}$ represents the k^{th} RE with d_f non-zero elements, indicating the d_f users assigned to the k^{th} RE, and the j^{th} column of matrix $\mathbf{F}$ represents the user j with d_v non-zero elements, indicating d_v REs are assigned to the user j.

The structure of SCMA can be represented as a factor graph, as shown in Fig. 2, where RNs and VNs are resource nodes and variable nodes, respectively. The overload ratio of the Fig. 2 is $\lambda = 150\%$, and its factor graph matrix is equation (4).

Based on the above steps, the user j codebook can be expressed as:

$$\chi_j^{K\times N} = \mathbf{v}_j^{K\times N}\mathbf{M}_j^{N\times M} = \mathbf{v}_j^{K\times N}\Delta_j^{N\times N}\mathbf{M}^{N\times M} \tag{3}$$

$$\mathbf{F}_{4\times 6} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \tag{4}$$

A. MOTHER CONSTELLATION DESIGN

1) PROBLEM STATEMENT

We construct the optimization problem of the mother constellation based on the lattice theory. The mother constellation $\mathbf{M}$ can be defined as:

$$\mathbf{M} = [\mathbf{c}_1, \mathbf{c}_2, \cdots, \mathbf{c}_N]^{\mathrm{T}} \tag{5}$$

where $\mathbf{c}_n = [c_n^1, c_n^2, \cdots, c_n^M]$ denotes the complex constellation points of n^{th} dimensional mother constellation.

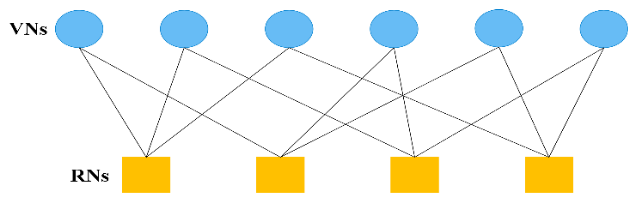

FIGURE 2. SCMA factor graph with $J = 6$, $K = 4$.

The mother constellation with best performance can be obtained by maximizing the MED and minimizing the average energy $E_{avg}^{\mathbf{M}}$ [23]. Therefore, the optimization problem of constructing the N-dimensional mother constellation can be constructed as follows:

$$\begin{aligned} &\min\ E_{avg}^{\mathbf{M}} \\ &s.t.\ d_{\min} \geq 1 \end{aligned} \tag{6}$$

where $E_{\text{avg}}^{\mathbf{M}}$ can be written as:

$$E_{avg}^{\mathbf{M}} = \frac{1}{MN} \sum_{i=1}^{M} \sum_{n=1}^{N} \left\| c_n^i \right\|^2 \tag{7}$$

We defined the normalized MED of mother constellation as:

$$d_{E,\min}^{\mathbf{M}} = \frac{d_{\min}}{\sqrt{E_{avg}^{\mathbf{M}}}} \tag{8}$$

The optimization problem of (6) can be further expressed as:

$$\begin{aligned} &\min\ \sum_{i=1}^{M} \sum_{n=1}^{N} \left\| c_n^i \right\|^2, \quad c_n^i \in \mathbf{c}_n \\ &s.t.\ d_{\min} \geq 1 \end{aligned} \tag{9}$$

Specifically, assume that $\{\mathbf{c}_n, n = 1, 2, \cdots, N-1\}$ has been designed, so the optimization problem is to find the best constellation $\mathbf{c}_N$ to satisfy restriction in (9), the optimization problem can be rewritten as:

$$\begin{aligned} &\min\ \sum_{i=1}^{M} \left\| c_N^i \right\|^2 \\ &s.t.\ d_{\min}^N \geq 1 \end{aligned} \tag{10}$$

where $d_{\min}^N = \left\| c_N^i - c_N^j \right\|$ means MED of the constellation $\mathbf{c}_N$, which is a one-dimensional complex constellation. So we can transfer the one-dimensional complex constellation $\mathbf{c}_N$ to a two-dimensional real constellation $\mathbf{Z}_N = [\mathbf{z}^1, \mathbf{z}^2, \cdots, \mathbf{z}^M] \in \mathbb{R}^{2\times M}$, where $\mathbf{z}^m = [real(\mathbf{c}_N^m), imag(\mathbf{c}_N^m)]^T$, $1 < m < M$.We use the lattice theory to solve the optimization problem of (10).

The constellation $\mathbf{Z}_v$ in v-dimensional lattice Λ can be expressed by generator matrix $\mathbf{G}(\Lambda)$, where $\mathbf{G}(\Lambda) = [\mathbf{g}_1, \mathbf{g}_2, \cdots, \mathbf{g}_v]^{\mathrm{T}}$.

$$\mathbf{Z}_v = \sum_{i=1}^{v} \alpha_i \mathbf{g}_i, \quad \alpha_i \in \mathbb{Z} \tag{11}$$

The performance of lattice constellation is measured by coding gain and shaping gain, then coding gain of the lattice constellation can be given as:

$$\gamma_c = \frac{(d_{\min}^G)^2}{[\mathbf{V}(\Lambda)]^{\frac{2}{v}}} \tag{12}$$

where $\mathbf{V}(\Lambda)$ represents the reciprocal of lattice points number per unit volume, and relationship between $\mathbf{G}(\Lambda)$ and $\mathbf{V}(\Lambda)$ can be written as:

$$\mathbf{V}(\Lambda) = |\det(\mathbf{G}(\Lambda))| \tag{13}$$

If a constellation based on lattice has the smallest average energy, and it will have large coding gain. Coding gain γ_c can be increased by fixing $d_{\min}^G$ and decreasing $\mathbf{V}(\Lambda)$. So the optimization problem of (10) can be transformed into:

$$\begin{aligned} &\min\ |\det(\mathbf{G}(\Lambda))| \\ &s.t.\ d_{\min}^G = 1 \end{aligned} \tag{14}$$

We consider uplink Rayleigh fading channel in SCMA system. As shown in Fig. 2, SCMA system can be taken as a MISO channel with d_f transmit antennas and a single receive antenna, and received signal at the k^{th} RE can be written as:

$$y_k = \sum_{\substack{i=1,\\ l_i \in \xi_k}}^{d_f} h_{l_i}^k x_{l_i}^k + n_k \tag{15}$$

where $l_i \in \xi_k$ represents user index connected to the k^{th} RE, for example, the user indexes connected to the 1^{th} RE are 1, 2, 3 as shown in the factor graph matrix. The $h_{l_i}^k$ denotes the channel coefficient between the l_i^{th} user connected to the k^{th} RE. We use $\mathbf{s}_k = [x_{l_1}^k, x_{l_2}^k, \cdots, x_{l_{d_f}}^k]^{\mathrm{T}}$ to denote the transmit signal vector at the k^{th} RE. Since the user indexes vary with that of the REs, the uplink SCMA system can be regarded as a fast Rayleigh fading MISO channel model. We define $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_K]$, and the upper bound of the pair-wise error probability (PEP) between two possible transmit signal matrices $\mathbf{S}^{(a)}$ and $\mathbf{S}^{(b)}$ can be can be expressed as [24]:

$$\begin{aligned} P(\mathbf{S}^{(a)} \to \mathbf{S}^{(b)}) &\leq \prod_{k \in \rho(\mathbf{S}^{(a)}, \mathbf{S}^{(b)})} \left| \mathbf{s}_k^{(a)} - \mathbf{s}_k^{(b)} \right|^{-2} \left(\frac{E}{4 d_f N_0}\right)^{-\delta} \\ &= \mathrm{d}_P^{-2} \left(\frac{E_s}{4 d_f N_0}\right)^{-\delta} \end{aligned} \tag{16}$$

where $\rho(\mathbf{S}^{(a)}, \mathbf{S}^{(b)})$ means the set of REs index in which $\mathbf{s}_k^{(a)} \neq \mathbf{s}_k^{(b)}$, and $\delta \equiv \left| \rho(\mathbf{S}^{(a)}, \mathbf{S}^{(b)}) \right|$. The d_P^2 represents the minimum squared product distance (MSPD) between the two signal vector, it can be expressed as:

$$d_P^2 = \prod_{k \in \rho(\mathbf{S}^{(a)}, \mathbf{S}^{(b)})} \left| \mathbf{s}_k^{(a)} - \mathbf{s}_k^{(b)} \right|^2 \tag{17}$$

Since each user occupies d_v REs to transmit information in the SCMA system, then the minimum value of δ is fixed as d_v. So we can increase the MPD to minimize PEP, as so to improve the bit error rate performance (BER) performance on the Rayleigh channel. It is equivalent to increasing the MPD of the mother constellation $\mathbf{M}$.

Then the MPD of the mother constellation is expressed as [25]:

$$d_{P,\min}^{\mathbf{M}} = \min\left\{\prod_{n=1}^{N}\left|c_n^i - c_n^j\right|\right\}, \quad 1 \le i \le j \le M \quad (18)$$

2) PROBLEM SOLVING

The optimization problem of (14) is transformed into a series of linear inequality constraint problems [26]. And orthogonal transformation method is used to transform $\mathbf{G}(\Lambda)$ into lower triangle matrix $\mathbf{G}'(\Lambda)$. Thus the equation (14) can be rewritten as:

$$\begin{aligned} &\min \ \left|g'_{22}, g'_{33}, \cdots, g'_{vv}\right| \\ &s.t.\ d_{\min}^{G} = 1 \end{aligned} \quad (19)$$

The problem (19) can be solved by the interior point method, the generator matrix $\mathbf{G}'(\Lambda)$ of two-dimensional real-valued lattice constellation can be obtained.

Therefore, a constellation with infinite lattice points can be obtained. Any lattice point $\mathbf{Z}_v$ in the constellation can be constructed by the generator matrix $\mathbf{G}'(\Lambda)$. It can be represented as:

$$\mathbf{Z}_v = \sum_{v=1}^{2}\alpha_{nv}\mathbf{g}'_v, \alpha_{nv} \in \mathbb{Z}, \quad n = 1, 2, \cdots, \infty \quad (20)$$

In [20], author presents the first-dimension of the mother constellation using an irregular M points pulse amplitude modulation (PAM) constellation, and the other dimensions are obtained by rotating and interleaving the first dimension constellation points. The method can improve BER performance of system under downlink Rayleigh fading channel. However, the BER performance of the codebook becomes worse as the codebook size gradually increases. This can be explained by the fact that the amplitude of mother constellation in [20] increases linearly, so the mother constellation cannot guarantee to obtain maximum coding gain. Motivated by this, in this paper, the first-dimension of mother constellation can be constructed by the subset $\mathbf{A}$ with M points selected from optimal two-dimensional lattice constellation with large code gain. In order to improve BER performance under Rayleigh fading channel, the multi-dimensional mother constellation can be constructed as:

a) Determine the number of mother constellation points M, and select a subset $\mathbf{A}$ with M points from the two-dimensional lattice constellation;
b) We count the number of constellation points m_l on the same ring l so that $M = \sum_{l=1}^{L} m_l, l = 1, 2, \cdots, L$ to generate subset $\mathbf{A}$. It should be noticed that energy of the constellation points in the same ring is equal;
c) Symmetry of the subset $\mathbf{A}$ is preferred so that it has a zero mean;
d) The subset $\mathbf{A}$ is used to construct an N-dimensional mother constellation by reordering constellation point. The odd-dimension of mother constellation is the subset $\mathbf{A}$. The even-dimension of mother constellation can be obtained by reordering the constellation points of the subset $\mathbf{A}$. The reordering operation is performed by the SSA algorithm;

The complex multi-dimensional mother constellation can be expressed as $\mathbf{M}^{N\times M} = [\mathfrak{M}_1(\mathbf{A}), \cdots, \mathfrak{M}_N(\mathbf{A})]^{\mathrm{T}}$ by reordering operation, where $\mathfrak{M}_1, \mathfrak{M}_2 \cdots, \mathfrak{M}_N$ is reorder function of the mother constellation. Thus the reordering criteria of the mother constellation can be expressed as:

$$[\mathfrak{M}_1, \cdots, \mathfrak{M}_N] = \mathop{\arg\max}_{\mathfrak{M}_1^*, \mathfrak{M}_2^*, \cdots, \mathfrak{M}_N^*} (d_{\mathrm{p},\min}^{\mathbf{M}}) \quad (21)$$

In this paper, we focus on two-dimensional mother constellations ($N = 2$). The generation process of the mother constellation is shown in Algorithm 1.

Algorithm 1 Symbol Switching Algorithm

1: Fix the symbol index of the subset $\mathbf{A}$ with M points arranged in a natural sequence as $\mathfrak{M}_1 = [1, 2, \cdots, M]^{\mathrm{T}}$, thus the odd-dimension of mother constellation can be constructed.
2: Randomly assign index orders for subset $\mathbf{A}$ with M points, therefore even-dimension of mother constellation can be obtained. Then the multi-dimensional mother constellation can be expressed as $\mathbf{M}^{2\times M} = [\mathfrak{M}_1(\mathbf{A}), \mathfrak{M}_2(\mathbf{A})]^{\mathrm{T}}$. Calculate $d_{P,\min}^{\mathbf{M}}$ of the mother constellation using (18).
3: **for** $s = 1 : M - 1$ **do**
4: **for** $t = t + 1 : M$ **do**
5: Switch the index of the s^{th} points with that of the t^{th} points in the even-dimension of mother constellation.
6: Calculate $d_{P,\min}^{*}$ using (18).
7: **if** $d_{P,\min}^{*} > d_{p,\min}^{\mathbf{M}}$ **then**
8: $d_{P,\min}^{\mathbf{M}} = d_{P,\min}^{*}$
9: **Set** $t = s+1$. **Return to** 5.
10: **else**
11: Switch back the two symbols.
12: **End if**
13: **End for** s, t
14: **Obtain** $\mathfrak{M}_1^*, \mathfrak{M}_2^*$ **and** $\mathbf{M}^{2\times M} = \left[\mathfrak{M}_1^*(\mathbf{A}), \mathfrak{M}_2^*(\mathbf{A})\right]^{\mathrm{T}}$

B. MAPPING MATRIX DESIGN

The factor graph matrix of Latin structure can improve BER performance, and is an important factor for the SCMA system to achieve overload [27]. In general, the factor graph matrix in SCMA system is not very large, so it can be obtained by adopting a manual design method. However, it is hard to manually get factor graph matrix as the number of users and resource elements of SCMA system increases.

In [21], an automatic method is used to generate the factor graph matrix. The binary matrix $\mathbf{f}_j = \left[f_1^j, f_2^j, \cdots, f_K^j\right]$ can be convert to decimal, it is defined as:

$$D(\mathbf{f}_j) = \sum_{k=1}^{K} f_k^j \cdot 2^{K-k} \tag{22}$$

The factor graph matrix $\mathbf{F}$ can be constructed by reordering column vectors under the criterion that

$$D(\mathbf{f}_1) > D(\mathbf{f}_2) > \cdots > D(\mathbf{f}_J) \tag{23}$$

Then find the location of i^{th} non-zero value in the k^{th} row of $\mathbf{F}$, and notes its index as (k, i), where $1 \leq k \leq K, 1 \leq i \leq d_f$. The d_f user codewords colliding in the same RE can be distinguished effectively by phase rotation under AWGN channel. The rotation angle θ_k^i can be defined as:

$$\theta_k^i = \alpha_i \cdot (\theta^*) = \beta_{\alpha_i}, \alpha_i = (k + i - N) \bmod (d_f) \tag{24}$$

The Latin factor matrix $\mathbf{F}_{4\times 6}^{\mathrm{L}}$ can be obtained by the (4) and (24):

$$\mathbf{F}_{4\times 6}^{\mathrm{L}} = \begin{bmatrix} \beta_0 & \beta_1 & \beta_2 & 0 & 0 & 0 \\ \beta_1 & 0 & 0 & \beta_2 & \beta_0 & 0 \\ 0 & \beta_2 & 0 & \beta_0 & 0 & \beta_1 \\ 0 & 0 & \beta_0 & 0 & \beta_1 & \beta_2 \end{bmatrix} \tag{25}$$

Then the mapping matrix $\mathbf{V}_1$ and the constellation operator Δ_1 of the user 1 are expressed as:

$$\mathbf{V}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} e^{j*\beta_0} & 0 \\ 0 & e^{j*\beta_1} \end{bmatrix} \tag{26}$$

Finally, the codebook for the user 1 can be constructed by equation (3).

$$\boldsymbol{\chi}_1 = \mathbf{v}_1 \Delta_1 \mathbf{M} \tag{27}$$

The resource block constellation $\mathbf{Q}_k$ can be constructed by d_f constellations multiplexed on the k^{th} RE [28], which can be expressed as:

$$\mathbf{Q}_k^{(w)} = \sum_{1 \leq m_1, m_2, \cdots, m_{d_f} \leq M} (\chi_{\xi_k^1}^{m_1} + \chi_{\xi_k^2}^{m_2} + \cdots + \chi_{\xi_k^{d_f}}^{m_{d_f}})$$
$$w = 1, 2, \cdots, M^{d_f} \tag{28}$$

where ξ_k means user index set connected to the k^{th} RE, the i^{th} elements of ξ_k is denoted by ξ_k^i. And the m_1^{th} codeword of codebook $\boldsymbol{\chi}_{\xi_k^1}$ is denoted by $\chi_{\xi_k^1}^{m_1}$. The MED between constellation points of resource block constellation $\mathbf{Q}_k$ is considered as a key performance determining parameter for AWGN channel. The optimization problem of rotation angle θ can be constructed as:

$$\begin{aligned} &\arg\max_{\theta*} \ (d_{\min}^{\theta}) \\ &s.t. \ d_{k,\min}^{\theta} = \min\left\{ d_k^{\theta} \,\middle|\, p \in [1, |\mathbf{Q}_k|], q \in [1, |\mathbf{Q}_k|], p \neq q \right\} \\ &\quad d_k^{\theta} = \frac{\| Q_k^p - Q_k^q \|}{\sqrt{E_{Q_k}}}, \quad \theta \in [0, \pi] \end{aligned} \tag{29}$$

where d_k^{θ} is normalize Euclidean distance of resource block constellation $\mathbf{Q}_k$, and $d_{k,\min}^{\theta}$ is normalize MED of the resource block constellation $\mathbf{Q}_k$. Because each RE contributes in the decoding process and therefore determines the system performance. Where $d_{\min}^{\theta}$ represents mean of the normalized MED of the $Kd_{k,\min}^{\theta}$ after rotating by θ degrees. The optimal rotation angle θ^* can be obtained by equation (29). The rotation angle of the first user can be fixed to 0 degree on the k^{th} resource block, and the optimal rotation angle of the remaining $d_f - 1$ users can be generated by equation (24).

IV. NUMERICAL SIMULATIONS AND RESULT ANALYSES

In this section, we compare the BER performance of the proposed codebook and the codebook [8], the codebook [21], the codebook [22] the codebook [23] under the AWGN channel and uplink Rayleigh channel. The transmission power of the different codebooks schemes is same. In order to investigate the overload capacity of SCMA system, the factor graph matrices showed in (4), (30) and (31) are used to generate factor matrices for the overloading ratio $\lambda = 150\%$, $\lambda = 200\%$ and $\lambda = 250\%$, respectively. Two strategies multi-user detection algorithms, i.e., parallel (P-MPA) [29] and serial (S-MPA) [30] are used to compare the convergence performance of different codebook schemes. In the subsequent simulation, the each dimensional average energy of mother constellation is normalized to 1, the $d_{E,\min}^{\mathbf{M}}$ means the normalized MED of mother constellation as shown in the Table 1 and $d_{P,\min}^{\mathbf{M}}$ is the MPD of mother constellation as shown in Table 2. The simulation parameter is shown in Table 3.

$$\mathbf{F}_{6\times 12} = \begin{bmatrix} 1 & & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{30}$$

$$\mathbf{F}_{6\times 15} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{31}$$

A. PERFORMANCE SIMULATION OF THE PROPOSED CODEBOOK UNDER AWGN CHANNEL

Under the AWGN channel, we provide the parameters of several codebooks under different codebook sizes as shown in Table 1, and make corresponding the BER performance comparison from Fig. 3 to Fig. 5.

When the overloading ratio $\lambda = 150\%$, codebook size $M = 4$, the BER performance of the proposed codebook under AWGN channel is shown in Fig. 3. Because the proposed codebook has the largest MED of mother constellation among the five codebooks as shown in Table 1. The BER performance of the proposed codebook is the best under

TABLE 1. When $\lambda = 150\%$, $F_{4\times6}$, the parameter comparison of different codebooks under AWGN channel.

category	$d_{E,\min}^{\mathbf{M}}$ [8]	$d_{E,\min}^{\mathbf{M}}$ [21]	$d_{E,\min}^{\mathbf{M}}$ [22]	$d_{E,\min}^{\mathbf{M}}$ [23]	$d_{E,\min}^{\mathbf{M}}$ prop
M = 4	1.4142	1.4142	1.2886	1.0000	1.5811
M = 8	0.8164	0.8164	0.5240	1.0000	1.1547
M = 16	0.6324	0.6324	0.7207	1.0000	0.8340

TABLE 2. When $\lambda = 150\%$, $F_{4\times6}$, the parameter comparison of different codebooks under uplink Rayleigh fading channel.

category	$d_{P,\min}^{\mathbf{M}}$ [8]	$d_{P,\min}^{\mathbf{M}}$ [21]	$d_{P,\min}^{\mathbf{M}}$ [22]	$d_{P,\min}^{\mathbf{M}}$ [23]	$d_{P,\min}^{\mathbf{M}}$ prop
M = 4	2.0000	2.0000	1.6451	2.0000	2.0000
M = 8	0.6666	0.6666	0.3020	0.2896	1.1547
M = 16	0.4000	0.4000	0.4123	0.1256	0.6024

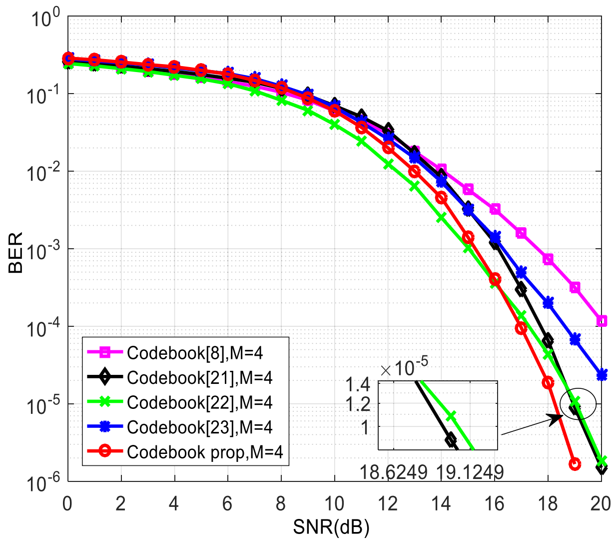

FIGURE 3. BER performance comparison for the AWGN channel where $\lambda = 150\%$, $M = 4$.

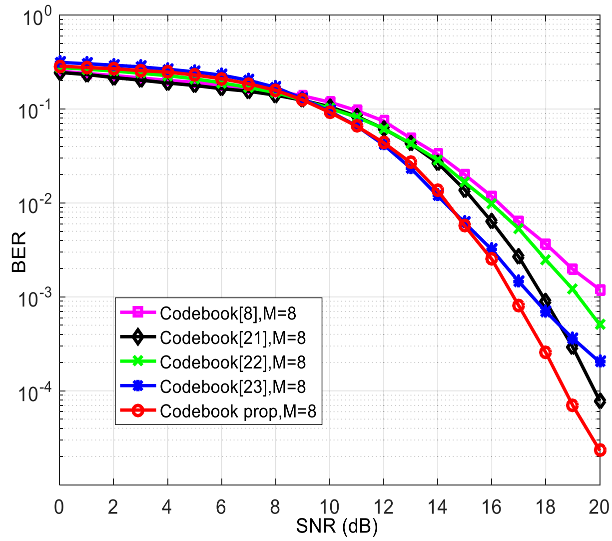

FIGURE 4. BER performance comparison for the AWGN channel where $\lambda = 150\%$, $M = 8$.

AWGN channel. The codebook [21] and the LDS have same MED of mother constellation, but the LDS has smaller MED of the resource block constellation than the codebook [21]. Thus the BER performance of LDS is poor than codebook [21]. When the BER is 10^{-4}, our proposed codebook can obtain 0.3dB, 0.7dB, 1.6dB and 3dB performance gain compared to the GAM codebook [22], the CR codebook [21], the 4D codebook [23] and the LDS respectively. And the BER value of our scheme decreases faster than other codebooks as signal-to-noise ratio (SNR) increases.

Fig. 4 shows comparison of several schemes with the case that has an overloading ratio $\lambda = 150\%$, and a codebook size of $M = 8$ under AWGN channel. As shown in Table 1, the MED of the proposed mother constellation is larger than other codebooks. So our scheme exhibits best performance among the five codebooks. When BER is 10^{-3}, the proposed codebook can obtain about 0.6dB, 1dB, 2.3dB and 3.2dB SNR gains compared with the codebook [23], the codebook [21], the codebook [22] and the codebook [8], respectively.

Fig. 5 reveals the BER performance of the case when codebook size $M = 16$, overloading ratio $\lambda = 150\%$ under the AWGN channel. When the BER is 10^{-3}, our proposed codebook can obtain 0.3dB, 0.9dB, 1.4dB and 1.6dB compared to the codebook [22], the codebook [23], the codebook [8] and the codebook [21], respectively. As shown in Table 1, the codebook [23] has the largest the MED of mother constellation, but resource block constellation points are overlapped. So the BER performance of the codebook [23] has degenerated when SNR range is 22∼24dB.

B. PERFORMANCE SIMULATION OF THE PROPOSED CODEBOOK UNDER UPLINK RAYLEIGH FADING CHANNEL

We provide the parameters of several codebooks under different codebook sizes as shown in Table 2, and make

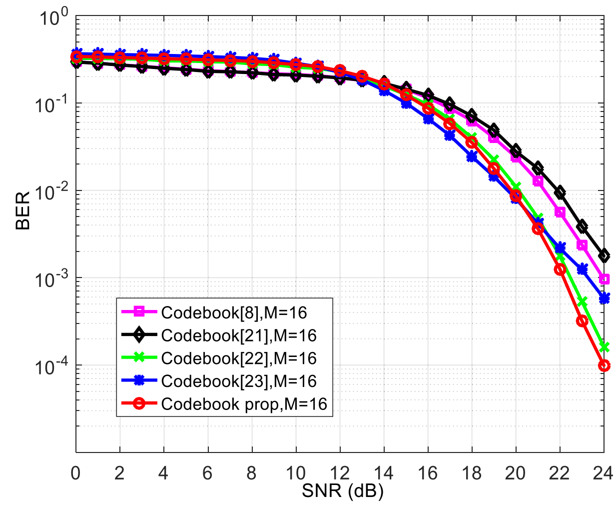

FIGURE 5. BER performance comparison for the AWGN channel where $\lambda = 150\%$, $M = 16$.

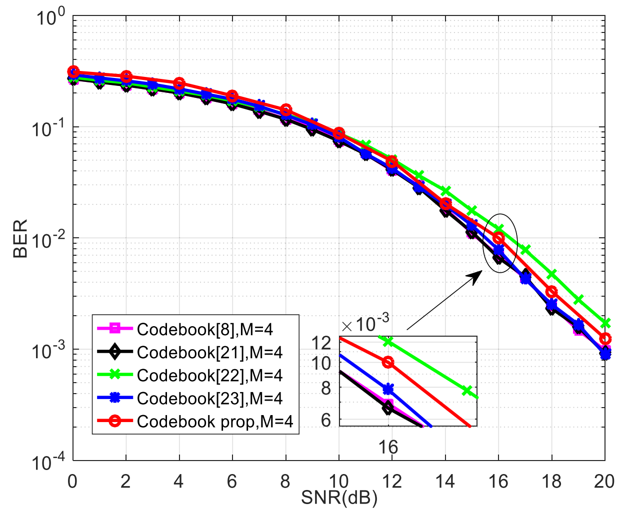

FIGURE 6. BER performance comparison for the uplink Rayleigh fading channel where $\lambda = 150\%$, $M = 4$.

TABLE 3. Simulation parameter.

Parameters	Values
Number of frame	1e4
Number of Iteration	6
Number of users	6, 12, 15
Number of REs	4, 6
Codebook size	4, 8, 16
Overloading ratios	150%, 200%, 250%
LDS modulation type	QPSK, 8QAM, 16QAM
Channels	AWGN, Rayleigh fading

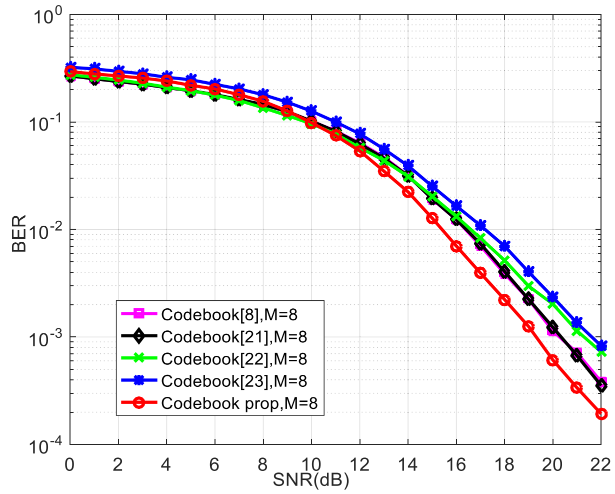

FIGURE 7. BER performance comparison for the uplink Rayleigh fading channel where $\lambda = 150\%$, $M = 8$.

corresponding the BER performance simulation under uplink Rayleigh fading channel from Fig. 6 to Fig. 8.

Fig.6 describes the case with overloading ratio $\lambda = 150\%$, codebook size $M = 4$ under uplink Rayleigh fading channel. As shown in Table 2, our scheme has larger the MPD of the mother constellation which is the critical factor for the BER performance under uplink the Rayleigh fading channel than the codebook [22], while MED of the mother constellation influences weakly BER performance under uplink Rayleigh channel. When the BER is 10^{-2}, our scheme achieves 0.4 dB SNR gain as compared with the codebook [22].

In Fig. 7, the BER performance of several codebooks are compared when codebook size $M = 8$, overloading ratio $\lambda = 150\%$ under uplink Rayleigh channel. Since the MPD value of the proposed codebook is the largest among the five codebooks, which can be gotten from Table 2. Our proposed codebook has smaller BER value than other codebooks under uplink Rayleigh channel. When the BER is 10^{-2}, our scheme can obtain 0.8dB, 0.9dB, 1dB and 1.6dB compared to the codebook [21], the codebook [8], the codebook [22] and the codebook [23], respectively.

As shown in Fig. 8, the BER performance of the proposed codebook outperforms other codebooks under uplink Rayleigh fading channel, because the MPD of the mother constellation of the proposed codebook is the largest among the five codebooks in Table 2. When BER is 10^{-3}, the proposed codebook can obtain about 0.2dB, 0.6dB, 0.7dB and 2.3dB SNR gains compared with the codebook [22], the codebook [21], the codebook [8] and the codebook [23], respectively.

C. OVERLOADING CAPACITY COMPARISON OF THE PROPOSED CODEBOOK

When the codebook size is $M = 4$, under the AWGN channel, we compare the overloading capacity performance of the codebook [23] and the proposed codebook.

When overload ratio $\lambda = 150\%, 200\%, 250\%$, the BER performance of the proposed codebook and codebook [23] are shown in Fig. 9. It is known that the larger overload ratio is, the worse BER performance of the proposed codebook and the codebook [23]. This is one of the common phenomena in NOMA technology, not just for SCMA. The main reason is

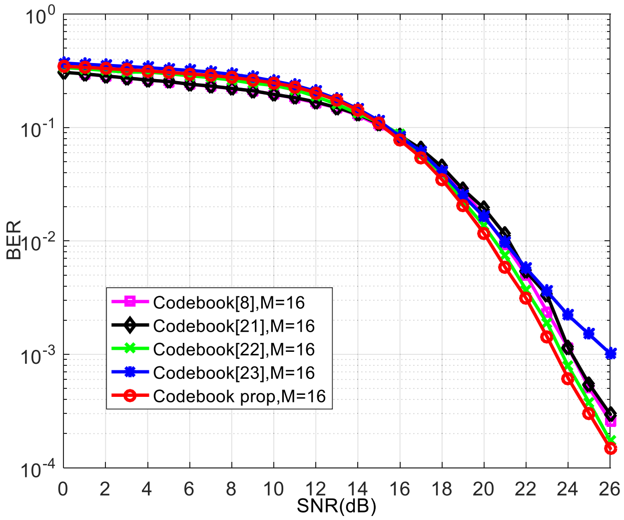

FIGURE 8. BER performance comparison for the uplink Rayleigh fading channel where $\lambda = 150\%$, $M = 16$.

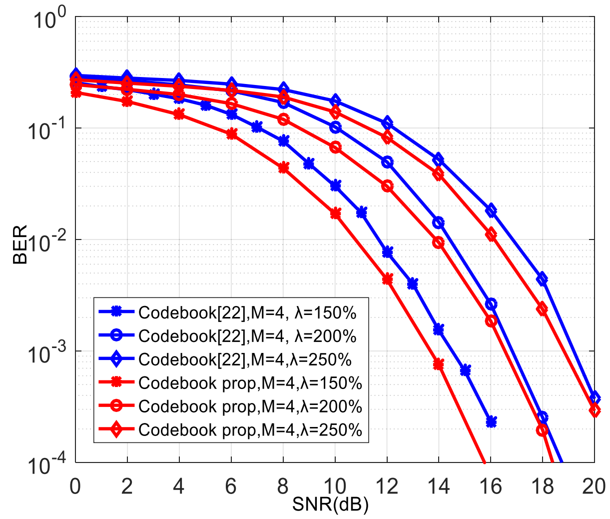

FIGURE 9. BER performance comparison of proposed codebook and codebook [23], where $M = 4$, $\lambda = 150\%$, 200%, 250%.

that the more users is superimposed on a RE for transmission, the more difficult for multi-user detection. However, the proposed codebook still outperforms the codebook [23] under different overloading condition. When BER is 10^{-3}, our proposed codebook can obtain 0.8dB, 0.3dB and 0.4dB SNR gains as compared with the codebook [23], respectively.

D. CONVERGENCE COMPARISON OF THE PROPOSED CODEBOOK UNDER THE TWO STRATEGIES MPA ALGORITHM

As mentioned above, the two main ways which are superior codebook and low complexity detection algorithm improve the BER performance of SCMA systems. When signal-to-noise ratio is 18dB, codebook size $M = 4, 8, 16$, the convergence behavior of the proposed codebook and the codebook [23] are compared. We use two different multi-user detection algorithms which are P-MPA and S-MPA for the proposed codebook and the codebook [23], the results are shown in Fig.10 and Fig.11, respectively.

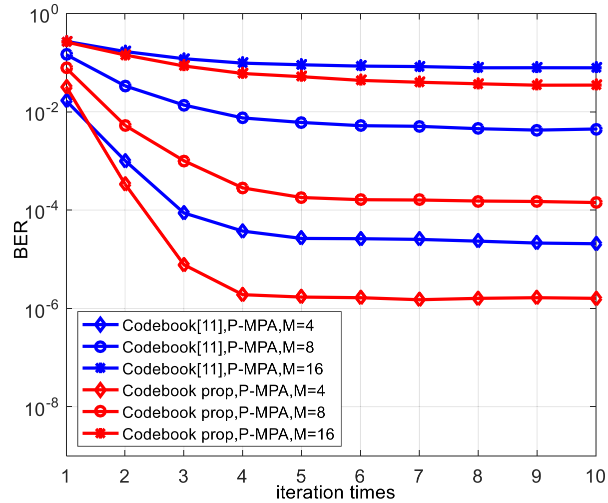

FIGURE 10. Convergence behavior comparison of proposed codebook and codebook [23] with P-MPA algorithm.

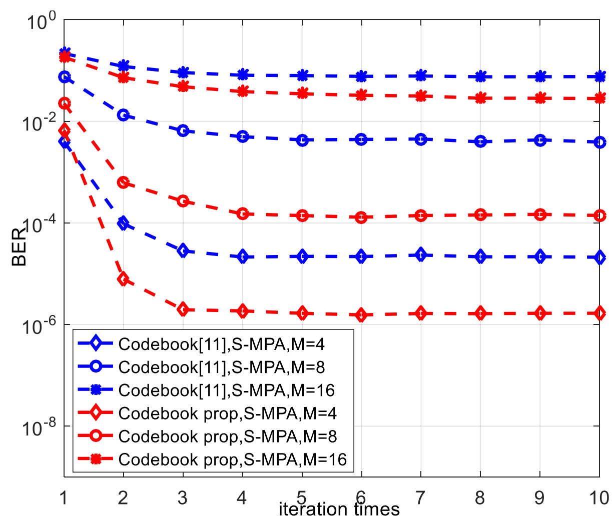

FIGURE 11. Convergence behavior comparison of proposed codebook and codebook [23] with S-MPA algorithm.

When codebook size is gradually increasing, the convergence speed of both the proposed codebook and the codebook [23] will be slow down, which can be gotten from Fig.10 and Fig. 11.

This is the common phenomena of SCMA technology. The main reason is that the more users is superimposed on same RE, which causes the complexity of detection increased when M is large. Fig.10 illustrates the convergence speed of the proposed codebook and the codebook [23] by using P-MPA detection algorithm. The proposed codebook converges faster than the codebook [23] under different codebook sizes. It can be observed from Fig.11, the convergence speed of the S-MPA is much faster than the P-MAP algorithm. When codebook size $M = 4$, and S-MPA algorithm converges at the third iteration point, while the P-MAP algorithm converges at the fifth iteration point. The convergence speed of the proposed codebook under two different detection algorithms is improved greatly.

V. CONCLUSION

In this paper, an efficient SCMA codebook design with better BER performance, faster convergence speed is introduced for uplink SCMA system. The main procedures of the codebook design is considered from two perspectives: i) Construct mother constellation by maximizing the MED and MPD criterions ii) Generate mapping matrix by maximizing MED of resource block constellation. The main innovation of this algorithm is the multi-dimensional mother constellation optimization which is decomposed into N two-dimensional real constellation optimization. Based on the lattice theory, a two-dimensional real lattice constellation with the largest coding gain can be obtained. The first dimension of mother constellation can be constructed by a subset of the two-dimensional real lattice constellation, and the others can be obtained by using SSA algorithm for the subset. And the user-specific mapping matrix is obtained by optimizing the rotation degree of the user constellation. Then the codebooks are generated by combing the multi-dimensional mother constellation and the mapping matrix. Thus, the BER performances of our proposed codebooks is greatly outperform to the reference codebooks under different channels, different codebook sizes and different overloading ratios Moreover, the proposed codebook exhibits faster convergence speed than existing codebook by using two different multi-user detection algorithms under different codebook sizes.

ABBREVIATIONS

Symbol	Description
SCMA	Sparse Code Multiple Access
NOMA	Non-orthogonal Multiple Access
5G	Fifth Generation
SSA	Symbol Switching Algorithm
REs	Resource Elements
BER	Bit Error Rate
MPA	Message Passing Algorithm
OMA	Orthogonal Multiple Access
PD-NOMA	Power-domain Non-orthogonal Multiple Access
CD-NOMA	Code-domain Non-orthogonal Multiple Access
PSMA	Power-domain Sparse Code Multiple Access
LDS	Low-density Signature
LDPC	Low-density Parity-check Coding
star-QAM	Star-Quadrature Amplitude Modulation
GAM	Golden Angle Modulation
MED	Minimum Euclidean Distance
MPD	Minimum Product Distance
AWGN	Additive White Gaussian Noise
RNs	Resource Nodes
VNs	Variable Nodes
PEP	Pair-wise Error Probability
PAM	Pulse Amplitude Modulation
P-MPA	Parallel Message Passing Algorithm
S-MPA	Serial Message Passing Algorithm

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