A New Nonparametric Tukey MA-EWMA Control Charts for Detecting Mean Shifts

RATTIKARN TABORAN, SAOWANIT SUKPARUNGSEE[ID], AND YUPAPORN AREEPONG[ID]
Department of Applied Statistics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok, Bangkok 10800, Thailand
Corresponding author: Saowanit Sukparungsee (saowanit.s@sci.kmutnb.ac.th)

ABSTRACT Control charts are a type of statistical tool used to control a production process in order to obtain the quality products that can fulfill the demands of both the manufacturer and the consumers. In this paper, we propose the Tukey Moving Average-Exponentially Weighted Moving Average control chart (MME-TCC) to detect the change of average of the process with symmetric and asymmetric distribution and to compare the efficiency in detecting the change of the MME-TCC to the MA, MME, MEM, MA-TCC and MEM-TCC at the various change levels of the parameter. The criteria to measure the efficiency were average run length (ARL), standard deviation of run length (SDRL), and median run length (MRL) which evaluated by using Monte Carlo simulation (MC), The research results showed that the proposed control chart has the highest efficiency in detecting the change when the change level was at $-0.75 \leq \delta \leq 0.75$. However, if the change of parameter increased ($\delta \geq 1.00$), the MME had more efficiency. In the case where the observation was logistic distributions, the MA-TCC had more efficiency to detect the change. Moreover, from applying the proposed control chart to two sets of real data, the mine explosion period in the UK during 1875-1951 and data of diameter of the workpiece from an industrial factory, it was found that the MME-TCC was able to more quickly detect the change than the other control charts.

INDEX TERMS Mixed control chart, Tukey moving average-exponentially weighted moving average control chart (MME-TCC), average run length (ARL), Monte Carlo simulation (MC).

I. INTRODUCTION

Each type of business fulfills the demand of consumers differently for their highest satisfaction, highest utilization and most effective response to the customer demand. Therefore, producing the quality products according to the standard is a key factor of the business success. Quality control is significant for mass production. Sometimes, the various sources of raw materials make the quality different, which affects the product quality directly, or the difference in the workmen or their skills can influence the quality of products as well. In any case, variation is the opponent of quality, which is classified into two types: a common causes variation, which is the mild variation that is normally occurs, and an assignable causes variation that has severe impacts on the production process. The cause of the second type of variation, which might be machine, material, man, or method, must be identified [1].

The associate editor coordinating the review of this manuscript and approving it for publication was Jenny Mahoney.

Statistical Process Control (SPC) plays an important role in product inspection and quality control. The popular tool that is used to control the SPC is the control chart, which is applied in order to examine or control the production process so that the variation is within the limit level. If the variation of the production process is over the limit, the cause should be identified. In 1931, Shewhart proposed the use of the control chart [2], which was an efficient tool to examine the change of a production process. It was classified into two types: control charts for variables and attributes. The assumption of Shewhart's control chart was that the process should have the normally and independently distributed, otherwise the control chart would not detect the small changes. Thus, the Exponentially Weighted Moving Average control chart (EWMA) was proposed by Roberts [3] in 1959 and the Moving Average control chart (MA) by Khoo [4] in 2004 as alternative control charts in order to detect the small changes. In addition, many researchers attach importance to the designing of MA and EWMA charts with situations presented, for example, Sukparungsee and Areepong [5], Chananet et al. [6],

Sunthornwat *et al.* [7], Khan *et al.* [8], Aslam *et al.* [9], Alghamdi *et al.* [10], etc.

Currently, the parametric control chart, which is a control chart that uses a parameter, is popular to detect the change of mean or standard deviation of the process. The disadvantage of the parametric control chart is that the data depends on this assumption about normality, homogeneity of variances, independence, and as such the properties of these control charts. In practical terms, the distribution of the collected data from the production process is unknown or only the distribution that has been identified but not the parameters of a distribution. As a result, the non-parametric control chart was proposed to resolve such problems.

In 2000, Ryan [11] proposed the Arcsine, which is a control chart that can detect the mean in a process very well. Later on, Alemi [12] proposed the Tukey's Control Chart (TCC) in 2004, which is a control chart designed for observing the data with one observation or subgroup size. Moreover, it can efficiently detect the change of mean. Afterwards, Yang *et al.* [13] proposed the Exponentially Weighted Moving Average Sign control chart (EWMA Sign) in 2011, which is a control chart to detect a small change without using the parameter, and the Arcsine Exponentially Weighted Moving Average Sign control chart (Arcsine EWMA Sign), which was adapted from the EWMA Sign control chart using the Arcsine method to transformation the data to the standard normal distribution in order to detect the error of mean in a process. In 2012, Sukparungsee [14] studied the strength of TCC for the data with normal and non-normal distributions and found that TCC had better efficiency to detect the larger changes than Shewhart's chart and EWMA. Then, in 2014, Khaliq and Riaz [15] proposed the Tukey-Cumulative Control Chart (TCC-CUSUM) to detect the change of average when the process had the symmetric and asymmetric distribution. The proposed control chart had higher efficiency in detecting the change than TCC and CUSUM. In 2016, Khaliq *et al.* [16] presented the Exponentially Weighted Moving Average-Tukey's Control Chart (TUKEY-EWMA), which was developed from the TCC; it is a combination of EWMA and TCC using the average run length as the criteria. The results showed that TUKEY-EWMA has better efficiency that both TCC and EWMA. Later, in 2017, Muhammad Riaz *et al.* [17] proposed the Tukey EWMA-CUSUM by comparison with the Shewhart, EWMA, CUSUM, and TCC charts, as well as several other variants such as the mixed EWMA-CUSUM, Tukey EWMA and Tukey CUSUM charts. The results indicated that the proposed control chart had higher efficiency in detection of the change. Then, Mongkoltawat *et al.* [18] proposed the Exponentially Weighted Moving Average-Tukey's control chart for Moving Range and Range to detect the change of variation of processes with the symmetric and asymmetric distribution using ARL as the criteria. It was found that the efficiency of the proposed control chart was better than that of the EWMA and TCC at all change levels. Besides, many authors developed and designed nonparametric control chart in several situations including, for instance, Abbas *et al.* [19],

Riaz *et al.* [20], Shafqat *et al.* [21], Chakraborti and Graham [22], and Mabude *et al.* [23].

In the previous research studies, none of them combined the advantages of MA, EWMA and TCC in order to detect the change of mean in the process. Hence, the researcher has proposed the MME-TCC control chart to detect the change of mean of the process have the symmetric and asymmetric distribution and to compare it with the MA, MME, MEM, MA-TCC and MEM-TCC control charts regarding the efficiency in detecting the change of ARL_1, SDRL and MDRL. The control chart having the lowest ARL_1, SDRL and MDRL are considered as the control chart with the best efficiency. Furthermore, it could be applied to the real data, which were the mine explosion period in the UK during 1875-1951 and data of diameter of the workpiece from an industrial factory.

II. DESIGN STRUCTURES OF CONTROL CHARTS
The control charts used in this research included the parametric control charts, which were EWMA, MA, MME and MEM charts and the non-parametric control charts, which were MA-TCC, MEM-TCC and MME-TCC. This research studied the efficiency in detecting change of the MA, MME, MEM, MA-TCC, MEM-TCC and MME-TCC as follows.

A. EXPONENTIALLY WEIGHTED MOVING AVERAGE (EWMA) CONTROL CHART
The EWMA control chart was proposed by Robert. It is a chart to quickly detect the change of parameter in the process. The statistic of EWMA is as follows.

$$Z_i = \lambda X_i + (1 - \lambda) Z_{i-1}, \quad i = 1, 2, \ldots \quad (1)$$

where Z_i is the statistic of the EWMA control chart at i, λ is the weighted parameter of the previous data ($0 \le \lambda \le 1$) and X_i is the observation at time i.. The calculation for the upper control limit (UCL) and lower control limit (LCL) of EWMA can be calculated as follows.

$$UCL/LCL = \mu_0 \pm K\sigma \sqrt{\left(\frac{\lambda}{2 - \lambda}\right)\left[1 - (1 - \lambda)^{2i}\right]} \quad (2)$$

From equation (2), $i \to \infty$, then $(1 - \lambda)^{2i} \to 0$. The control limits of the EWMA control chart are:

$$UCL/LCL = \mu_0 \pm K\sigma \sqrt{\left(\frac{\lambda}{2 - \lambda}\right)} \quad (3)$$

where K is the coefficient of the control limits of EWMA control chart, μ_0 is the mean of the process and σ is the standard deviation of the process when it is under control.

B. MOVING AVERAGE (MA) CONTROL CHART
The MA control chart as the most appropriate tool for detecting a small change. In the moving average control chart, the width (w) and the statistics of the MA control chart at i are calculated from the moving average at each w. There are

two cases as follows:

$$MA_i = \begin{cases} \dfrac{X_i + X_{i-1} + X_{i-2} + \dots}{i}, & i < w \\ \dfrac{X_i + X_{i-1} + \dots + X_{i-w+1}}{w}, & i \geq w. \end{cases} \quad (4)$$

The calculation of the upper control limit (UCL) and the lower control limit (LCL) of MA control chart are following

$$UCL/LCL = \begin{cases} \mu_0 \pm \dfrac{K_1 \sigma}{\sqrt{i}}, & i < w \\ \mu_0 \pm \dfrac{K_1 \sigma}{\sqrt{w}}, & i \geq w. \end{cases} \quad (5)$$

where K_1 is a coefficient of the control limits of the MA control chart, μ_0 is the mean of the process, and σ is the standard deviation of the process when it is under control.

C. MIXED MOVING AVERAGE - EXPONENTIALLY WEIGHTED MOVING AVERAGE CONTROL CHART (MME CHART)

This chart is a combination of the MA and EWMA control charts [24], [25]. In the mathematical model developed for the MME chart design, the plot statistic of the MA chart is used as an input to the EWMA chart (equation 1). Therefore, the statistic of the MME chart is as follows:

$$Z_i = \lambda MA_i + (1 - \lambda) Z_{i-1}, \quad i = 1, 2, \dots \quad (6)$$

where λ is the weighting parameter of the data in the past having the values from 0 to 1, Z_0 is the starting value and is set to be equal to the target mean μ_0, and the UCL and LCL of the MME chart are as follows:

$$UCL/LCL = \mu_{MA} \pm K_2 \sqrt{\left(\dfrac{\sigma_{MA}^2}{w}\right)\left(\dfrac{\lambda}{2-\lambda}\right)} \quad (7)$$

where K_2 is the coefficient of the control limits for the MME chart, μ_{MA} is the mean of the process and the variance is σ_{MA}^2.

D. MIXED EXPONENTIALLY WEIGHTED MOVING AVERAGE -MOVING AVERAGE CONTROL CHART (MEM CHART)

Similarly, the MEM chart was generated from combining the EWMA and MA charts. Therefore, the statistic of the MEM chart is as follows:

$$MA_i = \begin{cases} \dfrac{Z_i + Z_{i-1} + Z_{i-2} + \dots}{i}, & i < w \\ \dfrac{Z_i + Z_{i-1} + \dots + Z_{i-w+1}}{w}, & i \geq w. \end{cases} \quad (8)$$

Thus, the control limits for MEM chart can be represented as follows.

$$UCL/LCL = \begin{cases} \mu_z \pm K_3 \sqrt{\left(\dfrac{\sigma_z^2}{i}\right)\left(\dfrac{\lambda}{2-\lambda}\right)}, & i < w \\ \mu_z \pm K_3 \sqrt{\left(\dfrac{\sigma_z^2}{w}\right)\left(\dfrac{\lambda}{2-\lambda}\right)}, & i \geq w. \end{cases} \quad (9)$$

where K_3 is a coefficient of control limits of MEM control chart, μ_z is the mean of the process, and variance is σ_z^2.

E. TUKEY'S CONTROL CHART (TCC)

This is a non-parametric control chart where the distribution of the data is unknown or the subsample (n) is 1. The control limits are shown in equation (10).

$$UCL = Q_3 + K(\text{IQR})$$
$$LCL = Q_1 - K(\text{IQR}) \quad (10)$$

where Q_1 and Q_3 are the first and third quartiles, IQR is the quartile range $(Q_3 - Q_1)$ and K is a coefficient of the control limits of TCC.

F. MIXED MOVING AVERAGE-TUKEY'S CONTROL CHART (MA-TCC)

The MA-TCC control chart was proposed by Taboran et al. [26]. It is a non-parametric control chart that combines the MA and TCC control charts which use the statistic of MA, as shown in equation (4), and the control limit is of the TCC. Therefore, the control limits of the MA-TCC control chart for $i \geq w$ are as given by equation (11).

$$UCL = Q_3 + K_4(\text{IQR})\dfrac{1}{\sqrt{w}}$$
$$LCL = Q_1 - K_4(\text{IQR})\dfrac{1}{\sqrt{w}} \quad (11)$$

where $i < w$, then the control limits are as in equation (12).

$$UCL = Q_3 + K_4(\text{IQR})\dfrac{1}{\sqrt{i}}$$
$$LCL = Q_1 - K_4(\text{IQR})\dfrac{1}{\sqrt{i}} \quad (12)$$

where K_4 is the coefficient of the control limits of MA-TCC, Q_1 and Q_3 are the first and third quartiles, IQR is the quartile range $(Q_3 - Q_1)$.

G. MIXED TUKEY EXPONENTIALLY WEIGHTED MOVING AVERAGE - MOVING AVERAGE CONTROL CHART (MEM-TCC)

The MEM-TCC control chart was design from combining the MEM and TCC control charts which use the statistic of MEM, as in equation (8), the UCL and LCL are of MEM-TCC, which results in the expectation that the data is the same as that of TCC, and the variance will be applied between MEM and TCC. Therefore, the control limits of the MEM-TCC control chart for $i \geq w$ are as given by equation (13)

$$UCL = Q_3 + K_5(IQR)\sqrt{\left(\dfrac{1}{w}\right)\left(\dfrac{\lambda}{2-\lambda}\right)}$$
$$LCL = Q_1 - K_5(IQR)\sqrt{\left(\dfrac{1}{w}\right)\left(\dfrac{\lambda}{2-\lambda}\right)} \quad (13)$$

where $i < w$, then the control limits are as in equation (14).

$$UCL = Q_3 + K_5(IQR)\sqrt{\left(\dfrac{1}{i}\right)\left(\dfrac{\lambda}{2-\lambda}\right)}$$

TABLE 1. Comparative ARL, SDRL and MRL performance of MA, MME, MEM, MA-TCC, MEM-TCC and MME-TCC control charts for normal distribution.

δ	MA $K_1 = 2.884$			MME $K_2 = 7.632$			MEM $K_3 = 6.480$			MA-TCC $K_4 = 3.667$			MEM-TCC $K_5 = 8.692$			MME-TCC $K_6 = 8.765$		
	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL
-4.00	1.00	0.00	1.00	**0.00**	0.00	**0.00**	1.02	0.00	1.00	0.06	0.00	0.00	1.16	0.00	1.00	0.20	0.00	0.00
-3.00	1.10	0.00	1.00	**0.05**	0.00	**0.00**	1.27	0.00	1.00	0.35	0.00	0.00	1.75	0.00	2.00	0.62	0.00	1.00
-2.00	1.99	0.00	2.00	**0.47**	0.00	**0.00**	2.47	0.00	2.00	1.40	0.00	2.00	3.08	0.00	3.00	1.53	0.00	1.00
-1.50	3.76	0.01	3.00	**1.51**	0.01	**0.00**	4.19	0.01	4.00	3.12	0.01	3.00	4.52	0.00	4.00	2.69	0.00	2.00
-1.00	10.17	0.02	7.00	**5.83**	0.02	**1.00**	9.26	0.02	7.00	9.03	0.02	7.00	8.27	0.01	7.00	6.08	0.01	5.00
-0.75	20.61	0.04	15.00	13.52	0.03	**6.00**	17.00	0.04	15.00	18.62	0.04	13.00	13.68	0.02	11.00	**11.15**	0.02	8.00
-0.50	51.62	0.11	36.00	35.51	0.08	18.00	40.10	0.11	36.00	46.54	0.10	29.00	29.23	0.06	22.00	**26.09**	0.06	**18.00**
-0.25	162.27	0.36	113.00	124.59	0.29	61.00	134.98	0.35	113.00	143.90	0.32	95.00	92.11	0.20	65.00	**88.12**	0.20	**60.00**
-0.10	311.20	0.69	216.00	243.70	0.63	137.00	294.27	0.65	216.00	268.48	0.61	204.00	206.05	0.47	144.00	**201.94**	**0.45**	**118.00**
-0.05	353.55	0.79	246.00	277.22	0.77	166.00	347.11	0.77	246.00	300.74	0.68	242.00	248.10	0.57	173.00	**244.25**	**0.55**	**133.00**
0.00	370.36	0.83	258.00	370.05	0.82	254.00	370.32	0.82	257.00	370.83	0.85	257.00	370.11	0.83	255.00	370.71	0.87	250.00
0.05	354.09	0.79	246.00	276.83	0.77	169.00	346.89	0.77	246.00	293.37	0.66	240.00	251.87	0.58	176.00	**248.43**	**0.55**	**134.00**
0.10	311.23	0.69	216.00	242.74	0.63	141.00	293.07	0.68	216.00	256.49	0.58	204.00	211.42	0.48	147.00	**207.44**	**0.46**	**119.00**
0.25	162.75	0.36	113.00	124.39	0.29	57.00	134.23	0.35	112.00	135.20	0.31	94.00	95.27	0.21	68.00	**91.19**	**0.20**	**56.00**
0.50	51.64	0.11	36.00	37.20	0.08	17.00	39.99	0.11	36.00	44.03	0.10	29.00	30.04	0.06	22.00	**26.71**	0.06	**16.00**
0.75	20.58	0.04	15.00	13.52	0.03	**6.00**	16.92	0.04	13.00	17.81	0.04	12.00	13.96	0.02	11.00	**11.32**	0.02	8.00
1.00	10.13	0.02	7.00	**5.87**	0.01	**1.00**	9.21	0.02	7.00	8.74	0.02	7.00	8.39	0.01	7.00	6.16	0.01	5.00
1.50	3.76	0.01	3.00	**1.53**	0.01	**0.00**	4.18	0.01	4.00	3.06	0.01	3.00	4.58	0.00	4.00	2.73	0.00	2.00
2.00	1.99	0.00	2.00	**0.47**	0.00	**0.00**	2.48	0.00	2.00	1.38	0.00	2.00	3.11	0.00	3.00	1.55	0.00	1.00
3.00	1.10	0.00	1.00	**0.05**	0.00	**0.00**	1.27	0.00	1.00	0.34	0.00	0.00	1.77	0.00	2.00	0.63	0.00	1.00
4.00	1.00	0.00	1.00	**0.00**	0.00	**0.00**	1.02	0.00	1.00	0.06	0.00	0.00	1.16	0.00	1.00	0.20	0.00	0.00

Note: The bold is minimal of ARL, SDRL and MRL

$$LCL = Q_1 - K_5 (IQR) \sqrt{\left(\frac{1}{i}\right)\left(\frac{\lambda}{2-\lambda}\right)} \qquad (14)$$

where K_5 is the coefficient of the control limits for the MEM-TCC chart, λ is the weighting parameter of the data in the past having the values from 0 to 1, w is the width control chart, Q_1 and Q_3 are the lower and upper quartiles respectively, and IQR is the inter-quartiles range.

H. MIXED TUKEY MOVING AVERAGE-EXPONENTIALLY WEIGHTED MOVING AVERAGE CONTROL CHART (MME-TCC)

Likewise, the MME-TCC control chart is the combines MME with TCC control charts which uses the statistics of MME, as showed in equation (6). Therefore, the control limits of the MME-TCC control chart are as in equation (15)

$$UCL = Q_3 + K_6 (IQR) \sqrt{\left(\frac{1}{w}\right)\left(\frac{\lambda}{2-\lambda}\right)}$$

$$LCL = Q_1 - K_6 (IQR) \sqrt{\left(\frac{1}{w}\right)\left(\frac{\lambda}{2-\lambda}\right)} \qquad (15)$$

where K_6 is the coefficient of the control limits of MME-TCC chart which is consistent with $ARL_0 = 370$ obtained from MC simulation.

III. EFFICIENCY COMPARISON CRITERIA

The popular criteria used to evaluate the efficiency of a control chart is the Average Run Length (ARL) [27]. The ARL value measures the efficiency of the control chart with regard to detecting the amount of waste in the production process, which is determined by the quickness of the detection of the observed value outside the control when the average of the process changes. The control chart that is able to detect the change quickly is the efficient one because it allows for the quick detection, so the causes and solutions can be identified immediately. The ARL is the average number of points that must be plotted before a point indicates an out-of-control condition when ARL_0 is at the in-control state and ARL_1 is at the out-of-control state. ARL_0 and ARL_1 can be calculated as follows. At the in-control state, $ARL_0 = 1/\rho$, where ρ is a probability of the Type I error, which means the probability that any point exceeds the control limits and $ARL_1 = 1/(1-\gamma)$, where γ is the probability of the Type II error, which means the probability that the process in under the control limits when the process changes. In addition, the criteria for measuring the efficiency of the control chart are the standard deviation of the run length (SDRL) and median run length (MRL). For performance comparisons in cases the observation was asymmetric distribution, the MRL should be used as a measure because it is less affected by

TABLE 2. Comparative ARL, SDRL and MRL performance of MA, MME, MEM, MA-TCC, MEM-TCC and MME-TCC control charts for logistic distribution.

δ	MA $K_1 = 7.661$			MME $K_2 = 20.292$			MEM $K_3 = 18.340$			MA-TCC $K_4 = 3.965$			MEM-TCC $K_5 = 10.052$			MME-TCC $K_6 = 12.120$		
	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL
-4.00	9.14	0.02	7.00	1.16	0.01	1.00	2.44	0.02	1.00	2.65	0.02	1.00	1.54	0.01	1.00	**0.32**	**0.00**	**0.00**
-3.00	19.77	0.04	14.00	3.12	0.03	1.00	6.24	0.04	7.00	4.38	0.04	6.00	2.73	0.01	2.00	**1.04**	0.01	**1.00**
-2.00	52.79	0.12	37.00	9.62	0.09	2.00	24.10	0.11	22.00	8.09	0.10	19.00	9.16	0.06	3.00	**5.02**	**0.05**	**2.00**
-1.50	93.56	0.21	65.00	43.03	0.19	10.00	52.77	0.20	42.00	17.53	0.18	37.00	21.90	0.14	60.00	**13.38**	**0.10**	**9.00**
-1.00	169.55	0.38	118.00	117.08	0.38	46.00	119.58	0.39	80.00	46.64	0.33	70.00	59.60	0.37	45.00	**40.62**	**0.26**	**40.00**
-0.75	226.65	0.51	157.00	173.75	0.50	80.00	176.50	0.55	110.00	81.94	0.44	94.00	100.43	0.63	60.00	**73.11**	**0.40**	**59.00**
-0.50	291.18	0.65	201.00	238.00	0.64	127.00	250.70	0.75	142.00	147.98	**0.56**	122.00	167.39	0.67	92.00	**129.85**	0.63	**90.00**
-0.25	346.92	0.78	240.00	285.01	0.73	171.00	326.45	0.93	180.00	**196.26**	**0.67**	**147.00**	254.48	0.75	150.00	211.39	0.71	149.00
-0.10	366.16	0.82	253.00	301.07	0.76	181.00	358.56	1.00	204.00	**251.03**	0.73	**156.00**	301.77	0.79	157.00	256.46	0.75	156.00
-0.05	369.05	0.83	255.00	303.70	0.78	183.00	365.60	1.02	211.00	299.18	**0.74**	176.00	312.82	0.80	178.00	**267.44**	0.80	**175.00**
0.00	370.19	0.83	256.00	370.31	0.83	256.00	370.39	1.02	216.00	370.02	0.85	253.00	370.15	0.93	255.00	370.41	0.92	254.00
0.05	369.43	0.82	256.00	303.39	0.80	183.00	371.75	1.02	219.00	303.61	**0.74**	177.00	323.38	0.85	181.00	**280.30**	0.79	**176.00**
0.10	366.20	0.82	254.00	301.27	0.79	181.00	371.20	1.01	221.00	**255.29**	**0.71**	**156.00**	326.46	0.79	179.00	282.17	0.76	172.00
0.25	346.99	0.78	241.00	285.52	0.77	171.00	356.74	0.95	218.00	**199.96**	**0.68**	**149.00**	310.30	0.72	168.00	266.93	0.69	167.00
0.50	290.34	0.65	202.00	238.49	0.65	142.00	298.33	0.77	188.00	**151.35**	**0.57**	**124.00**	247.17	0.69	129.00	205.73	0.61	127.00
0.75	226.53	0.51	157.00	184.02	0.50	109.00	230.08	0.57	150.00	**83.50**	**0.45**	**96.00**	178.62	0.61	102.00	142.76	0.49	82.00
1.00	170.06	0.38	118.00	136.77	0.37	81.00	170.97	0.41	113.00	**47.45**	0.34	**71.00**	126.06	0.57	79.00	97.54	**0.33**	75.00
1.50	93.46	0.21	65.00	72.69	0.20	42.00	93.72	0.21	65.00	**17.80**	0.18	**37.00**	66.15	0.25	46.00	47.52	**0.15**	42.00
2.00	52.75	0.11	37.00	39.56	0.11	22.00	54.31	0.11	40.00	**8.16**	0.10	**19.00**	38.77	0.12	29.00	26.23	**0.08**	21.00
3.00	19.75	0.04	14.00	13.13	0.04	7.00	23.13	0.04	18.00	**4.40**	0.04	**6.00**	18.35	0.04	15.00	11.34	0.03	8.00
4.00	9.12	0.02	7.00	5.13	0.02	1.00	13.01	0.02	11.00	**2.67**	0.02	**1.00**	11.64	0.02	10.00	6.94	0.01	3.00

Note: The bold is minimal of ARL, SDRL and MRL.

the skewness of the run length distribution [28]. Besides, there are other criteria using for comparing the efficiency of the control chart, such as chattering, false detection rate, missed detection rate, average detection delay, and etc. Some researchers utilized such criteria in their studies e.g. Xu *et al.* [29] and [30], Aslansefat *et al.* [31], and Naghoosi *et al.* [32].

This research applied Monte Carlo Simulation (MC) to evaluate of ARL, SDRL and MRL, MC is the basic approach that is understandable. The MC can be calculated as follows.

$$ARL = \frac{\sum_{i=1}^{M} RL_i}{M} \qquad (16)$$

$$SDRL = \sqrt{E(RL)^2 - ARL^2} \qquad (17)$$

$$MRL = Median(RL) \qquad (18)$$

where RL_i represents the examined sample before the process is out of the control limits for the first time. In the simulation at round i, M represents the repetition number of the experiment where $M = 200,000$.

The ARL at the in-control process is represented with ARL_0, which means the number of average sample sets to be used in the examination until any of the statistics is out of the control limits in the situation where the average of process does not change or is at the default value (α_0) The ARL of

the out-of-control process is represented with ARL_1, which means the number of the average sample sets to be used in the examination until any of the statistics are out of the control limits under the situation that the average of process changes at different levels (α_1) .

IV. RESEARCH RESULTS

The objectives of this research were to compare the quickness in detecting the change of mean of the process between the proposed control chart and MA, MME, MEM, MA-TCC and MEM-TCC control charts under the four distributions processes, which were symmetrical distributions: Normal(0,1) and Logistic(6,2) distributions, and asymmetric distributions: Exponential(1) and Gamma(4,1) distributions. The parameter (α) of the process was set at $\alpha = \alpha_0$ when it was an in-control process and at $\alpha = \alpha_1$ when it was an out-of-control process and $\alpha_1 = \alpha_0 + \delta\sigma_0$, where δ referred to the amount of shift, α_1 was the shifted mean, α_0 was the in-control mean and, σ_0 was the controlled value of the process standard deviation. The evaluation criteria for the efficiency of the control chart was considered from the ARL, SDRL and MRL. The control chart with the lowest ARL, SDRL and MRL was the most efficient control chart.

When the observation was Normal(0,1) distribution, $ARL_0 = 370$, $\lambda = 0.25$ it was found that the MME-TCC

TABLE 3. Comparative ARL, SDRL and MRL performance of MA, MME, MEM, MA-TCC, MEM-TCC and MME-TCC control charts for exponential distribution.

δ	MA $K_1 = 3.339$			MME $K_2 = 4.424$			MEM $K_3 = 8.251$			MA-TCC $K_4 = 4.781$			MEM-TCC $K_5 = 10.756$			MME-TCC $K_6 = 11.090$		
	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL
0.00	370.82	0.83	257.00	370.28	0.68	223.00	370.30	0.82	258.00	370.13	0.84	254.00	370.02	0.81	259.00	370.88	0.91	191.00
0.05	253.40	0.57	175.00	228.20	0.55	149.00	253.18	0.56	178.00	276.51	0.68	181.00	232.42	0.55	164.00	**207.27**	**0.50**	**128.00**
0.10	180.02	0.40	124.00	160.43	0.39	104.00	180.74	0.40	127.00	192.23	0.48	125.00	163.96	0.38	116.00	**144.92**	**0.35**	**90.00**
0.25	78.99	0.18	54.00	67.96	0.17	42.00	81.21	0.17	58.00	79.05	0.20	49.00	72.90	0.16	52.00	**62.79**	**0.15**	**40.00**
0.50	31.40	0.07	22.00	25.56	0.07	**15.00**	33.76	0.07	25.00	28.59	0.08	17.00	31.18	0.06	23.00	**25.41**	0.06	18.00
0.75	17.01	0.04	12.00	13.84	0.04	**7.00**	19.41	0.04	15.00	14.35	0.04	8.00	18.65	0.03	15.00	**13.04**	0.03	10.00
1.00	10.95	0.02	8.00	**8.03**	0.02	**4.00**	13.18	0.02	10.00	8.75	0.03	5.00	13.20	0.02	11.00	9.95	0.02	7.00
1.50	6.09	0.01	4.00	**4.11**	0.01	**1.00**	7.86	0.01	6.00	4.40	0.01	2.00	8.46	0.01	7.00	6.00	0.01	5.00
2.00	4.15	0.01	3.00	**2.59**	0.01	1.00	5.60	0.01	5.00	2.75	0.01	1.00	6.35	0.01	6.00	4.31	0.01	3.00
3.00	2.59	0.01	2.00	**1.40**	0.01	0.00	3.60	0.01	3.00	1.47	0.01	0.00	4.35	0.01	4.00	2.78	0.01	2.00
4.00	1.97	0.00	1.00	**0.93**	0.00	0.00	2.72	0.00	2.00	0.97	0.00	0.00	3.36	0.00	3.00	2.06	0.00	2.00

Note: The bold is minimal of ARL, SDRL and MRL.

TABLE 4. Comparative ARL, SDRL and MRL performance of MA, MME, MEM, MA-TCC, MEM-TCC and MME-TCC control charts for gamma distribution.

δ	MA $K_1 = 3.023$			MME $K_2 = 2.0005$			MEM $K_3 = 7.600$			MA-TCC $K_4 = 4.015$			MEM-TCC $K_5 = 11.466$			MME-TCC $K_6 = 13.88$		
	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL	ARL	SDRL	MRL
0.00	370.52	0.83	258.00	370.29	0.84	161.00	370.60	1.00	194.00	370.25	0.84	254.00	370.15	0.90	255.00	370.12	0.89	254.00
0.05	283.92	0.63	197.00	239.33	0.63	138.00	286.90	0.87	147.00	284.43	0.74	179.00	382.64	0.86	184.00	**234.09**	**0.62**	**137.00**
0.10	219.56	0.49	152.00	184.63	0.49	114.00	224.43	0.66	117.00	217.85	0.56	137.00	285.77	0.76	90.00	**167.09**	**0.48**	**76.00**
0.25	108.91	0.24	76.00	89.16	0.24	52.00	114.93	0.31	70.00	103.37	0.27	64.00	130.33	0.38	68.00	**69.37**	**0.20**	**51.00**
0.50	41.48	0.09	29.00	31.97	0.09	**1.00**	47.17	0.10	33.00	36.14	0.10	21.00	48.50	0.11	34.00	**22.34**	0.12	32.00
0.75	19.52	0.04	14.00	13.92	0.04	**1.00**	24.76	0.05	19.00	15.44	0.04	8.00	24.86	0.04	20.00	**10.37**	0.05	18.00
1.00	10.75	0.02	8.00	6.99	0.02	**1.00**	15.65	0.02	13.00	7.63	0.02	3.00	15.94	0.02	13.00	**6.37**	0.03	9.00
1.50	4.53	0.01	3.00	**2.31**	0.01	**1.00**	8.49	0.01	8.00	2.48	0.01	1.00	9.32	0.01	9.00	3.89	0.01	5.00
2.00	2.53	0.00	2.00	**0.93**	0.00	**0.00**	5.74	0.01	5.00	0.99	0.00	0.00	6.94	0.00	7.00	3.18	0.01	3.00
3.00	1.32	0.00	1.00	**0.20**	0.00	**0.00**	3.41	0.00	3.00	0.21	0.00	0.00	5.01	0.00	5.00	2.61	0.00	3.00
4.00	1.06	0.00	1.00	**0.04**	0.00	**0.00**	2.37	0.00	2.00	0.05	0.00	0.00	3.88	0.00	4.00	2.10	0.00	2.00

Note: The bold is minimal of ARL, SDRL and MRL.

control chart where $K_6 = 8.765$ was the most efficient control chart to detect the change at the ±0.05, ±0.10, ±0.25, ±0.50 and ±0.75 levels. while the MME control chart where $K_2 = 7.632$ was the most efficient control chart to detect the change at ±1.00, ±1.50, ±2.00, ±3.00 and ±4.00, when considering the result of the SDRL and MDRL, it was consistent with ARL_1, as shown in Figure 1 and Table 1.

In Figure 2 and Table 2, where the observation was Logistic (6,2) distribution, $ARL_0 = 370$ and $\lambda = 0.25$, it is shown that the MME-TCC control chart where $K_6 =12.120$was the most efficient control chart to detect the change at ±0.05, -0.50, -0.75, -1.00, -1.50, -2.00, -3.00, and -4.00, whereas the MA-TCC control chart where $K_4 = 3.965$ was the most efficient one to detect the change at ±0.10, ±0.25, 0.50, 0.75, 1.00, 1.50, 2.00, 3.00 and 4.00, the result of the SDRL and MDRL, it was correspond to ARL_1 values.
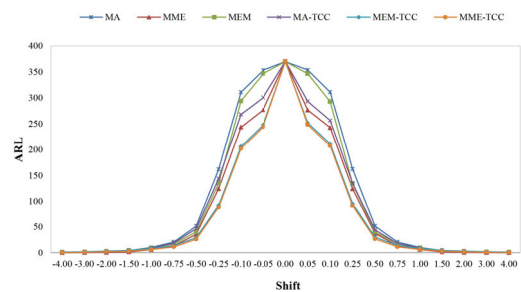

FIGURE 1. ARL curves of MA, MME, MEM, MA-TCC, MEM-TCC and MME-TCC control charts at $ARL_0 = 370$, $w = 5$ and $\lambda = 0.25$ for normal distribution.

As seen in Table 3 and Figure 3, where the observation was Exponential (1) distribution, $ARL_0 = 370$ and $\lambda = 0.25$, it was found that the MME-TCC control chart where
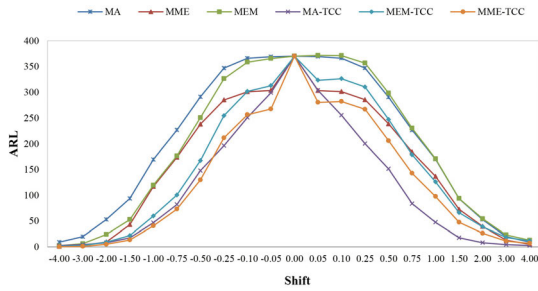
TABLE 5. ARL performance of MA, MME, MEM, MA-TCC, MEM-TCC and MME-TCC charts for normal distribution by varying w and set $ARL_0 = 370$.

δ	MA K_1			MME K_2			MEM K_3			MA-TCC K_4			MEM-TCC K_5			MME-TCC K_6		
	2.981	2.884	2.746	7.894	7.632	7.267	4.098	6.480	9.165	2.420	3.667	4.876	5.940	8.692	11.316	5.945	8.765	11.625
	W=2	W=5	W=10	W=2	W=5	W=10	W=2	W=5	W=10	W=2	W=5	W=10	W=2	W=5	W=10	W=2	W=5	W=10
-4.00	1.00	1.00	1.00	**0.03**	**0.00**	**0.00**	1.02	1.02	1.02	0.11	0.06	0.04	1.04	1.16	1.28	0.42	0.20	0.07
-3.00	1.15	1.10	1.00	**0.28**	**0.05**	**0.02**	1.27	1.27	1.27	0.53	0.35	0.27	1.40	1.75	1.97	0.93	0.62	0.32
-2.00	2.80	1.99	1.08	**1.88**	**0.47**	**0.18**	2.47	2.47	2.47	2.41	1.40	1.11	2.50	3.08	3.42	1.98	1.53	1.02
-1.50	6.61	3.76	1.46	5.33	**1.51**	**0.56**	4.18	4.19	4.19	6.12	3.12	2.40	3.95	4.52	5.04	**3.37**	2.69	1.90
-1.00	20.98	10.17	3.62	18.89	**5.83**	**2.24**	9.26	9.26	9.26	20.00	9.03	6.46	8.09	8.27	9.00	**7.46**	6.08	4.47
-0.75	43.07	20.61	7.71	40.20	13.52	**5.16**	17.00	17.00	17.01	41.17	18.62	12.72	14.10	13.68	14.34	**13.39**	**11.15**	8.31
-0.50	97.67	51.62	21.27	93.60	35.51	**14.70**	40.09	40.10	40.11	93.12	46.54	31.62	31.17	29.23	30.08	**30.37**	**26.09**	19.49
-0.25	230.57	162.27	83.14	222.58	124.59	**58.53**	134.90	134.98	135.05	216.66	143.90	114.62	96.76	92.11	100.71	**95.95**	**88.12**	68.99
-0.10	339.27	311.20	205.26	328.55	243.70	**144.44**	294.12	294.27	294.54	314.09	268.48	270.20	202.30	206.05	250.45	**201.88**	**201.94**	175.44
-0.05	362.40	353.55	251.11	350.98	277.22	**176.69**	346.89	347.11	347.43	333.51	300.74	324.77	238.22	248.10	308.53	**238.06**	**244.25**	221.93
0.00	370.11	370.36	370.46	370.07	370.05	**370.50**	370.10	370.32	370.67	370.46	370.83	370.44	370.16	370.11	370.76	**370.01**	370.71	370.55
0.05	361.80	354.09	250.54	351.20	276.83	**176.88**	346.67	346.89	330.66	330.28	293.37	311.27	242.74	251.87	290.50	**242.37**	**248.43**	231.19
0.10	338.91	311.23	204.39	328.10	242.74	**144.20**	292.87	293.07	280.90	308.15	256.49	252.54	207.57	211.42	227.04	**207.10**	**207.44**	187.08
0.25	231.23	162.75	83.15	222.46	124.39	**58.50**	134.16	134.23	137.97	209.00	135.20	105.46	100.14	95.27	89.58	**99.28**	**91.19**	74.20
0.50	97.65	51.64	21.13	93.23	37.20	**14.64**	39.98	39.99	40.48	89.50	44.03	29.71	31.95	30.04	27.92	**31.15**	**26.71**	20.53
0.75	43.10	20.58	7.69	40.13	13.52	**5.12**	16.92	16.92	16.67	39.61	17.81	12.15	14.38	13.96	13.64	**13.64**	**11.32**	8.60
1.00	21.11	10.13	3.62	18.94	**5.87**	**2.21**	9.21	9.21	9.11	19.32	8.74	6.25	8.22	8.39	8.67	**7.54**	6.16	4.61
1.50	6.59	3.76	1.46	5.31	**1.53**	**0.56**	4.18	4.18	4.25	5.99	3.06	2.35	3.99	4.58	4.89	**3.41**	2.73	1.95
2.00	2.80	1.99	1.08	**1.86**	**0.47**	**0.18**	2.48	2.48	2.51	2.37	1.38	1.10	2.52	3.11	3.33	2.00	1.55	1.06
3.00	1.15	1.10	1.00	**0.28**	**0.05**	**0.02**	1.27	1.27	1.29	0.53	0.34	0.26	1.41	1.77	1.93	0.95	0.63	0.34
4.00	1.00	1.00	1.00	**0.03**	**0.00**	**0.00**	1.02	1.02	1.02	0.11	0.06	0.04	1.05	1.16	1.25	0.44	0.20	0.07

Note: The bold is minimal of ARL, SDRL and MRL.

TABLE 6. ARL performance of MA, MME, MEM, MA-TCC, MEM-TCC and MME-TCC charts for exponential distribution by varying w and set $ARL_0 = 370$.

δ	MA K_1			MME K_2			MEM K_3			MA-TCC K_4			MEM-TCC K_5			MME-TCC K_6		
	4.133	3.339	2.860	5.470	4.424	3.794	5.220	8.251	11.667	4.210	4.781	5.435	8.410	10.756	12.370	8.420	11.090	15.350
	W=2	W=5	W=10	W=2	W=5	W=10	W=2	W=5	W=10	W=2	W=5	W=10	W=2	W=5	W=10	W=2	W=5	W=10
0.00	370.36	370.82	370.38	370.93	370.28	370.15	370.84	370.30	370.09	370.69	370.13	370.42	370.18	370.02	370.71	370.47	370.88	370.69
0.05	268.79	253.40	200.94	264.13	228.20	**166.98**	253.56	253.18	253.02	**178.45**	276.51	229.35	233.72	232.42	222.84	233.65	**207.27**	178.45
0.10	201.81	180.02	134.18	198.03	160.43	**110.95**	180.97	180.74	180.62	**116.89**	192.23	149.12	167.27	163.96	155.47	166.93	**144.92**	116.89
0.25	98.62	78.99	51.87	95.30	67.96	**42.54**	81.31	81.21	81.18	**44.58**	79.05	54.55	75.86	72.90	67.37	75.19	**62.79**	44.58
0.50	41.96	31.40	18.62	39.69	25.56	**14.85**	33.78	33.76	33.75	**16.50**	28.59	18.13	32.19	31.18	29.56	31.45	**25.41**	16.50
0.75	23.02	17.01	9.64	21.41	13.84	**7.51**	19.43	19.41	19.41	**9.20**	14.35	8.93	18.83	18.65	18.09	18.13	**13.04**	9.20
1.00	14.79	10.95	6.10	13.45	**8.03**	**4.63**	13.18	13.18	13.17	**6.23**	8.75	5.37	12.98	13.20	13.08	12.31	9.95	6.23
1.50	8.00	6.09	3.37	6.95	**4.11**	**2.36**	7.86	7.86	7.86	**3.78**	4.40	2.68	7.93	8.46	8.40	7.33	6.00	3.78
2.00	5.30	4.15	2.38	4.42	**2.59**	**1.50**	5.60	5.60	5.60	**2.76**	2.75	1.69	5.74	6.35	6.13	5.19	4.31	2.76
3.00	3.16	2.59	1.64	2.40	**1.40**	**0.83**	3.60	3.60	3.60	**1.85**	1.47	0.92	3.79	4.35	3.93	3.28	2.78	1.85
4.00	2.32	1.97	1.37	1.60	**0.93**	**0.57**	2.72	2.72	2.72	**1.41**	0.97	0.63	2.91	3.36	2.87	2.42	2.06	1.41

Note: The bold is minimal of ARL, SDRL and MRL.

$K_7 = 11.090$ was the most efficient control chart to detect the change at 0.05, 0.10, 0.25, 0.50 and 0.75, while the MME control chart where $K_2 = 4.424$ was the most efficient one to detect the change at 1.00, 1.50, 2.00, 3.00, and 4.00, the result of the SDRL and MDRL, it was consistent to ARL_1 values.

FIGURE 2. ARL curves of MA, MME, MEM, MA-TCC, MEM-TCC and MME-TCC control charts at ARL$_0$ = 370, w = 5 and λ = 0.25 for logistic distribution.
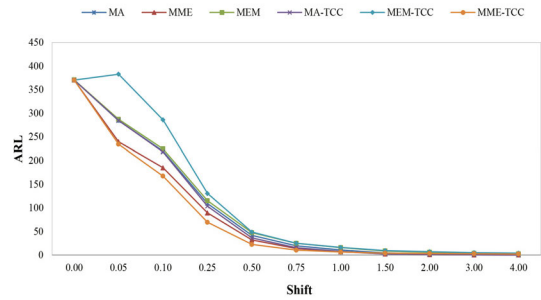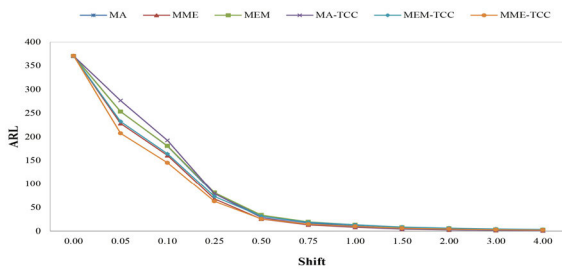

FIGURE 3. ARL curves of MA, MME, MEM, MA-TCC, MEM-TCC and MME-TCC control charts at ARL$_0$ = 370, w = 5 and λ = 0.25 for exponential distribution.

FIGURE 4. ARL curves of MA, MME, MEM, MA-TCC, MEM-TCC and MME-TCC control charts at ARL$_0$ = 370, w = 5 and λ = 0.25 for gamma distribution.

As shown in Table 4 and Figure 4, where the observation was Gamma(4,1) distribution, ARL$_0$ = 370 and λ = 0.25, the MME-TCC control chart where =13.880 was the most efficient control chart to detect the change at 0.05, 0.10, 0.25, 0.50, 0.75 and 1.00, whereas the MME control chart where K_2 = 2.0005 was the most efficient one to detect the change at 1.50, 2.00, 3.00, and 4.00, the result of the SDRL and MDRL, it was consistent to ARL$_1$ values.

Furthermore, the researcher considered the efficiency of average change value detection of each control chart by varying, w = 2, 5 and 10 since all control charts used the moving average based designs. The results in Table 5 and Table 6 showed that if the observed value had normal distribution and exponential distribution, the results were consistent; when the value of w increased, ARL of all control charts decreased. When comparing the efficiency of the proposed control chart to other control charts, it was found that the increasing value of w provided the lower efficiency of average change value detection. In contrast, MME chart detected the average change value quicker, especially when w = 10. It was obvious that MME chart was more efficient in change detection than other control charts at all change levels.

V. APPLICATION
In this section, the proposed control chart will be performed, considering two set data as follows.

A. THE DIAMETER OF THE WORKPIECE FROM AN INDUSTRIAL FACTORY
The researcher applied the proposed control chart to data of 40 sets of a diameter of the workpiece from an industrial
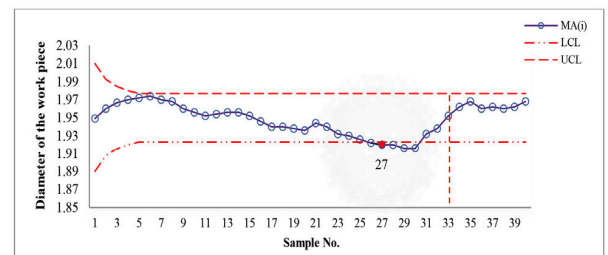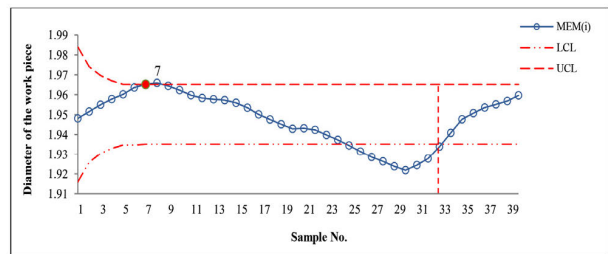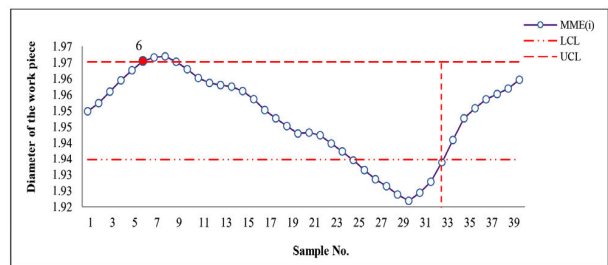
FIGURE 5. Graphical displays of the illustrative of the diameter of the workpiece from an industrial factory of parametric control charts: (a) MA chart, (b) MEM chart and (c) MME chart.

factory that had the normal distribution and target value was 2.0 mm. The mean diameter of the workpiece has been estimated parameter was 1.95 mm., the standard deviation was 0.03 mm., and at the 33rd change of the process, it showed the mean change of diameter was 1.67 mm [33].

The data sets: 1.94, 1.98, 1.98, 1.98, 1.98, 1.95, 1.96, 1.97, 1.94, 1.96, 1.93, 1.97, 1.98, 1.94, 1.94, 1.90, 1.94, 1.98, 1.93,
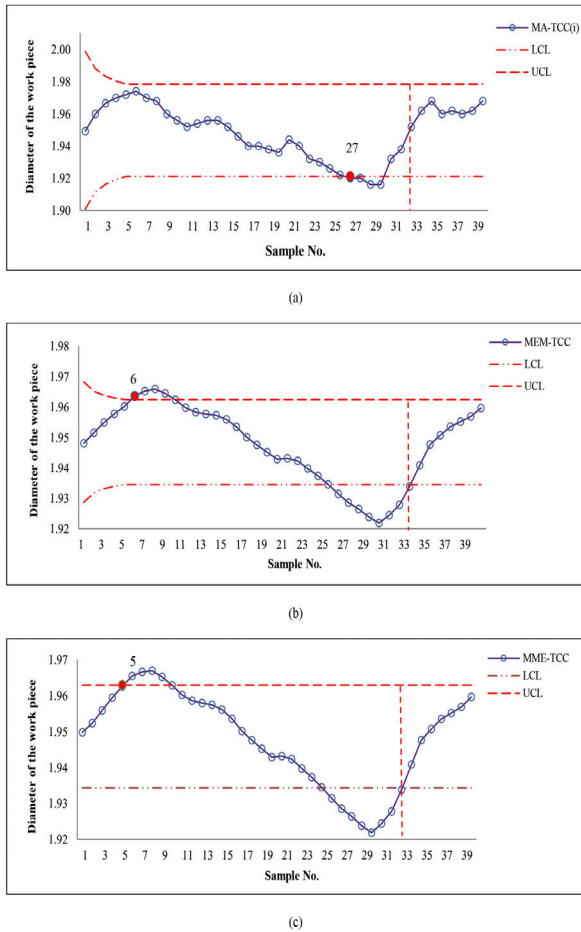
FIGURE 6. Graphical displays of the illustrative of the diameter of the workpiece from an industrial factory of non-parametric control charts: (a) MA-TCC chart, (b) MEM-TCC chart and (c) MME-TCC chart.

1.93, 1.94, 1.92, 1.94, 1.92, 1.91, 1.92, 1.91, 1.94, 1.90, 1.91, 2.00, 1.94, 2.01, 1.95, 1.94, 1.96, 1.95, 2.00, 1.96, 1.97.

The data generated the MA, MME, MEM, MA-TCC, MEM-TCC, and MME-TCC control charts from equations (5), (7), (9), (12), (13), (14) and (15) as shown in graphical results as Figures 5-6. For this case study, the set of data that had the normal distribution showed that the proposed chart was able to detect the change from the 5th, the MME and MEM-TCC charts were able to detect at 6th, the MEM chart can be detected at 7th, the MA and MA-TCC charts were able to detect at 27th.

B. THE MINE EXPLOSION PERIOD IN THE UK DURING 1875-1951

The second set was 100 sets of data of the mine explosion period in the UK during 1875-1951 with exponential distribution showed that there were ten or more people died from the process. When the process had not changed, the mean was 129 days/time. At the 51st change of the process, it showed 339 days/time [34].
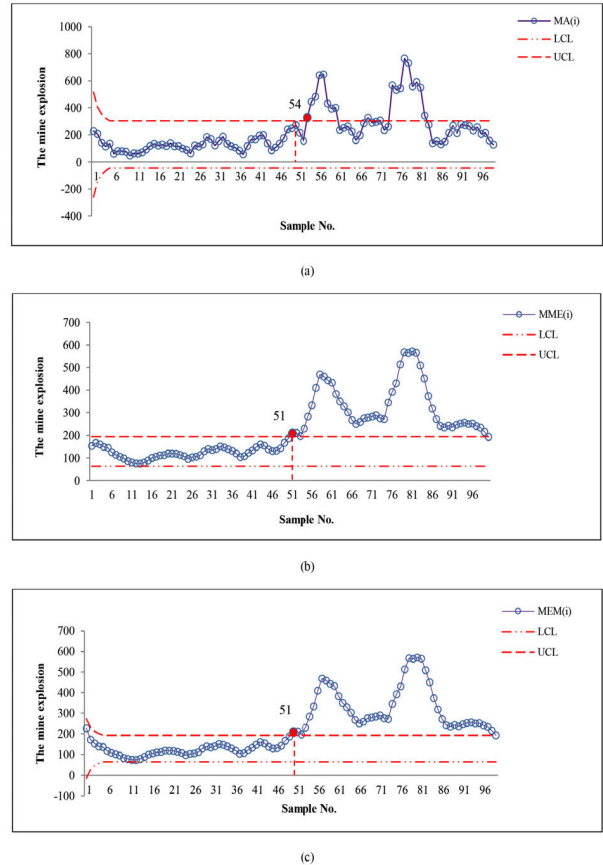
FIGURE 7. Graphical displays of the illustrative of the mine explosion period in the UK during 1875-1951 of parametric control charts: (a) MA chart, (b) MEM chart and (c) MME chart.

The data sets: 378, 36, 15, 31, 215, 11, 137, 4, 15, 72, 96, 124, 50, 120, 203, 176, 55, 93, 59, 315, 59, 61, 1, 13, 189, 345, 20, 81, 286, 114, 108, 188, 233, 28, 22, 61, 78, 99, 326, 275, 54, 217, 113, 32, 23, 151, 361, 312, 354, 58, 275, 78, 17, 1205, 644, 467, 871, 48, 123, 457, 498, 49, 131, 182, 255, 195, 224, 566, 390, 72, 228, 271, 208, 517, 1613, 54, 326, 1312, 348, 745, 217, 120, 275, 20, 66, 291, 4, 369, 338, 336, 19, 329, 330, 312, 171, 145, 75, 364, 37, 19.

The performance in detecting a mean of the mine explosion period in the UK from 1875-1951 of the MA, MME, MEM, MA-TCC, MEM-TCC, and MME-TCC control charts are demonstrated in term of graphical results as Figures 7-8. The performance of the proposed chart can detect a mean change of the mine explosion period in the 11th is superior to MEM-TCC control chart which can detect at 12th whereas the MME and MEM charts were able to detect a change of the mine explosion at 51th, the MA chart was able to detect change at 54th, the MA-TCC control chart was able to detect a change of the mine explosion at 55th. Therefore, it could be concluded that MME-TCC was the quickest control chart to detect the change of the mine explosion period in the UK between 1875-1951, where $w = 5$ and $\lambda = 0.25$.
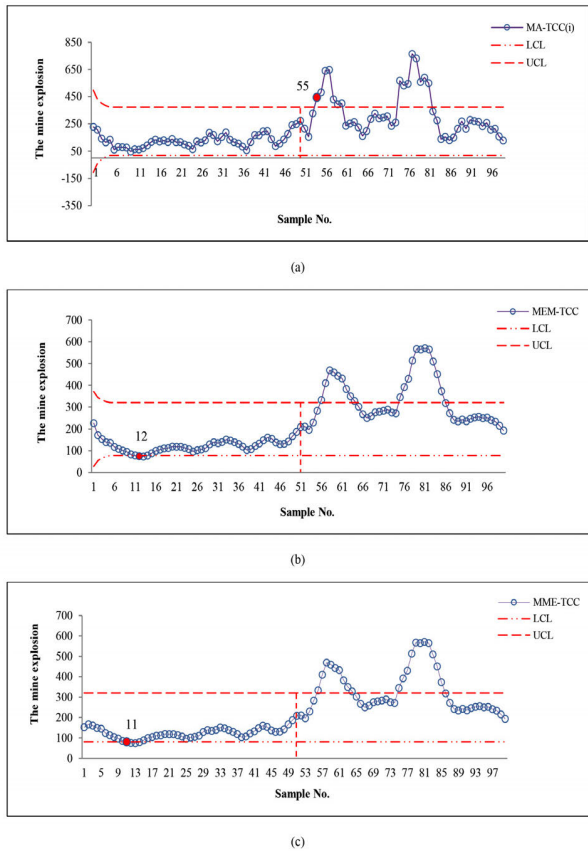
FIGURE 8. Graphical displays of the illustrative of the mine explosion period in the UK during 1875-1951 of non-parametric control charts: (a) MA-TCC chart, (b) MEM-TCC chart and (c) MME-TCC chart.

TABLE 7. Comparison between proposed chart with MA, MME, MEM, MA-TCC and MEM-TCC charts for symmetric and asymmetric distribution.

Distribution	ARL$_0$ = 370			
	$\delta \leq -1$	$-0.75 \leq \delta \leq -0.05$	$0.05 \leq \delta \leq 0.75$	$\delta \geq 1$
Normal distribution	MME	MME-TCC	MME-TCC	MME
Logistic distribution	MME-TCC	MME-TCC	MA-TCC	MA-TCC
Exponential distribution	-	-	MME-TCC	MME
Gamma distribution	-	-	MME-TCC	MME

VI. CONCLUSION AND RECOMMENDATIONS

From the comparison of the performance of the proposed control chart with the MA, MME, MEM, MA-TCC and MEM-TCC control charts based on the ARL$_1$, SDRL and MRL as the criteria to measure the efficiency, the results illustrated that if the process has the symmetric and asymmetric distribution, the average run length (ARL) was 370 when the process was in the in-control process. The research results could be summarized in Table 7.

With the normal distribution, MME-TCC was more efficient to detect the change than other control charts at $-0.75 \leq \delta \leq 0.75$. However, when $\delta \leq -1.00$ and $\delta \geq 1.00$, it was found out that MME chart had better efficiency than other control charts.

For the exponential distribution and gamma distribution, MME-TCC was more efficient detection than other control charts at $0.05 \leq \delta \leq 0.75$. However, when $\delta \geq 1.00$, MME chart was more efficient to detect the change than other control charts, which was in line with the normal distribution.

For the logistic distribution, it provided the different results. When the parameter changes negatively ($\delta \leq -0.05$), MME-TCC had better detection efficiency than other control charts. On the other hand, it the parameter change positively ($\delta \geq 0.05$), MA-TCC was the quickest chart to detect the change of average at all change levels. Additionally, it was found out that if w value increased, ARL of all control charts comparing to the proposed chart would decreased. It was in line with the previous research [25] which discovered that if w value was higher, the detection efficiency of the proposed control chart would decrease when comparing to other control charts. In contrast, the higher value of w supported the better efficiency of MME chart. In particular, when $w = 10$, MME chart was the quickest control chart to detect the change at all change level. The application results of the proposed control chart to the two sets of data indicated that the proposed control chart was applicable to various data and was efficient to detect the change of both data sets (normal and non-normal distributions). However, it depended on set the parameter of each control chart.

Besides, the researchers compared ARL performance of proposed MME-TCC chart vs. Tukey-EWMA [16], Tukey-CUSUM [15] and Tukey EWMA-CUSUM [17] charts under the Normal (0,1) distribution, ARL$_0$ = 370 and the numerical ARL$_1$ where δ is a shift sizes: $-4.00 \leq \delta \leq 4.00$. The numerical results found the performance of the proposed chart performed better than the Tukey-EWMA, Tukey-CUSUM and Tukey EWMA-CUSUM charts for all magnitudes of change, except for at $\delta = 0.25$, the Tukey EWMA-CUSUM performed better than the proposed chart. This would be an alternative to the non-parametric control charts that can be applied to other fields such as health care, epidemiology, environmental science, etc. The further research studies may extend the scope to examine the method of efficiency comparison of the control charts, which can then be applied to the data with different distributions.

REFERENCES

[1] J. Banks, *Principle of Quality Control*. New York, NY, USA: Wiley, 1989.

[2] W. A. Shewhart, *Economic Control of Quality of manufactured Product.* New York, NY, USA: D. Van Nostrand Company, 1931.

[3] S. W. Roberts, "Control chart tests based on geometric moving average," *Techmometrics*, vol. 42, no. 1, pp. 239–250, 1959.

[4] M. B. C. Khoo, "A moving average control chart for monitoring the fraction non-conforming," *Qual. Rel. Eng. Int.*, vol. 20, no. 6, pp. 617–635, Oct. 2004.

[5] S. Sukparungsee and Y. Areepong, "Combining martingale and integral equations for finding optimal designs of EWMA procedure," in *Proc. Pyrenees Int. Workshop Statist., Probab. Oper. Res.*, 2009, pp. 81–91.

[6] C. Chananet, Y. Areepong, and S. Sukparungsee, "An approximate formula for ARL in moving average chart with ZINB data," *Thailand Statistician*, vol. 13, no. 2, pp. 209–222, 2015.

[7] R. Sunthornwat, Y. Areepong, and S. Sukparungsee, "Average run length with a practical investigation of estimating parameters of the EWMA control chart on the long memory AFRIMA process," *Thailand Statistician*, vol. 16, no. 2, pp. 191–203, 2018.

[8] N. Khan, M. Aslam, and C.-H. Jun, "A EWMA control chart for exponential distributed quality based on moving average statistics," *Qual. Rel. Eng. Int.*, vol. 32, no. 3, pp. 1179–1190, Apr. 2016.

[9] M. Aslam, M. Azam, and C.-H. Jun, "Mixed sampling plan based on exponentially weighted moving average statistic," *Commun. Statist. Theory Methods*, vol. 45, no. 22, pp. 6709–6719, Nov. 2016.

[10] S. A. D. Alghamdi, M. Aslam, K. Khan, and C.-H. Jun, "A time truncated moving average chart for the weibull distribution," *IEEE Access*, vol. 5, pp. 7216–7222, 2017.

[11] T. P. Ryan, *Statistical Methods for Quality Improvement*, 2nd ed. New York, NY, USA: Wiley, 2000.

[12] F. Alemi, "Tukey's control chart," *Qual Manag Healthcare*, vol. 13, no. 4, pp. 216–221, 2004.

[13] S.-F. Yang, J.-S. Lin, and S. W. Cheng, "A new nonparametric EWMA sign control chart," *Expert Syst. Appl.*, vol. 38, no. 5, pp. 6239–6243, May 2011.

[14] S. Sukparungsee, "Robustness of Tukey's control chart in detecting a changes of parameter of skew distributions," *Int. J. Appl. Phys. Math.*, vol. 2, no. 5, pp. 379–382, 2012.

[15] Q.-U.-A. Khaliq and M. Riaz, "Robust tukey-CUSUM control chart for process monitoring," *Qual. Rel. Eng. Int.*, vol. 32, no. 3, pp. 933–948, Apr. 2016.

[16] Q.-U.-A. Khaliq, M. Riaz, and S. Ahmad, "On designing a new tukey-EWMA control chart for process monitoring," *Int. J. Adv. Manuf. Technol.*, vol. 82, nos. 1–4, pp. 1–23, Jan. 2016.

[17] M. Riaz, Q. Khaliq, and S. Gul, "Mixed Tukey EWMA-CUSUM control chart and its applications," *Qual. Technol. Quant. Manage.*, to be published, doi: 10.1080/16843703.2017.1304034.

[18] P. Mongkoltawat, S. Sukparungsee, and Y. Areepong, "Exponentially weighted moving average—Tukey's control chart for moving rage and rang," *J. King Mongkut's Univ. Technol. North Bangkok*, vol. 27, no. 4, pp. 843–854, 2017.

[19] A. Abbasi, M. Aslam, and A. Saghir, "A mixed nonparametric control chart for efficient process monitoring," *Int. J. Adv. Manuf. Technol.*, vol. 99, nos. 9–12, pp. 2549–2561, Dec. 2018, doi: 10.1007/s00170-018-2545-1.

[20] M. Riaz, M. Abid, H. Z. Nazir, and S. A. Abbasi, "An enhanced nonparametric EWMA sign control chart using sequential mechanism," *PLOS One*, to be published, doi: 10.1371/journal.pone.0225330.

[21] A. Shafqat, Z. Huang, M. Aslam, and M. S. Nawaz, "A nonparametric repetitive sampling DEWMA control chart based on linear prediction," *IEEE Access*, vol. 8, pp. 74977–74990, 2020, doi: 10.1109/ACCESS.2020.2989132.

[22] S. Chakraborti and M. A. Graham, "Nonparametric (distribution-free) control charts: An updated overview and some results," *Qual. Eng.*, vol. 31, no. 4, pp. 523–544, Oct. 2019, doi: 10.1080/08982112.2018.1549330.

[23] K. Mabude, J.-C. Malela-Majika, P. Castagliola, and S. C. Shongwe, "Distribution-free mixed GWMA-CUSUM and CUSUM-GWMA mann–whitney charts to monitor unknown shifts in the process location," *Commun. Statist. Simul. Comput.*, pp. 1–24, Aug. 2020, doi: 10.1080/03610918.2020.1811331.

[24] R. Taboran, S. Sukparungsee, and Y. Areepong, "Mixed moving average-exponentially weighted moving average control charts for monitoring of parameter change," in *Proc. Int. Multi Conf. Eng. Comput. Scientists*, Hong Kong, 2019, pp. 411–415.

[25] S. Sukparungsee, Y. Areepong, and R. Taboran, "Exponentially weighted moving average—Moving average charts for monitoring the process mean," *PLOS One*, to be published, doi: 10.1371/journal.pone.0228208.

[26] R. Taboran, S. Sukparungsee, and Y. Areepong, "Moving average-Tukey's control chart for monitoring of parameter change," in *Proc. Universal Academic Cluster Int. March Conf. Bangkok*, Bangkok, Thailand, 2018, pp. 125–132.

[27] D. C. Montgomery, *Introduction to Statistical Quality Control*, 6th ed. New York, NY, USA: Wiley, 2009.

[28] F. F. Gan, "An optimal design of cumulative sum control chart based on median run length," *Commun. Statist. - Simul. Comput.*, vol. 23, no. 2, pp. 485–503, Jan. 1994.

[29] J. Xu, J. Wang, I. Izadi, and T. Chen, "Performance assessment and design for univariate alarm systems based on FAR, MAR, and AAD," *IEEE Trans. Autom. Sci. Eng.*, vol. 9, no. 2, pp. 296–307, Apr. 2012.

[30] X. Xu, X. Weng, D. Xu, H. Xu, Y. Hu, and J. Li, "Evidence updating with static and dynamical performance analyses for industrial alarm system design," *ISA Trans.*, vol. 99, pp. 110–122, Apr. 2020.

[31] K. Aslansefat, M. Bahar Gogani, S. Kabir, M. A. Shoorehdeli, and M. Yari, "Performance evaluation and design for variable threshold alarm systems through semi-Markov process," *ISA Trans.*, vol. 97, pp. 282–295, Feb. 2020.

[32] E. Naghoosi, I. Izadi, and T. Chen, "Estimation of alarm chattering," *J. Process Control*, vol. 21, no. 9, pp. 1243–1249, Oct. 2011.

[33] D. Buntam, "A comparison of the efficiency of shewhart control charts and tabular cumulative sum control charts: Case study monitor of diameter of rod," *J. Kmutnb*, vol. 23, no. 3, pp. 560–568, 2013.

[34] D. C. Montgomery, *Introduction to Statistical Quality Control Case Study*. New York, NY, USA: Wiley, 2008.

RATTIKARN TABORAN is currently pursuing the Ph.D. degree in applied statistics program with the King Mongkut's University of Technology North Bangkok. She is also a Lecturer of Statistics with the Department of Mathematics, Muban Chombueng Rajabhat University, Thailand. Her research interests include statistical process control, statistics analysis, regression analysis, and cluster analysis.
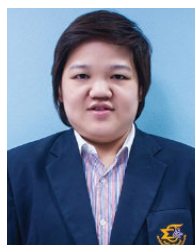
SAOWANIT SUKPARUNGSEE is currently an Associate Professor of Statistics and the Vice President of Academic Affairs, King Mongkut's University of Technology North Bangkok, Thailand. Her research interests include statistical process control, sequential change-point analysis, operations research, and simulation.

YUPAPORN AREEPONG is currently an Associate Professor of Statistics and the Vice President of Academic Affairs, King Mongkut's University of Technology North Bangkok, Thailand. Her research interests include statistical process control, sequential change-point analysis, and forecasting and time series analysis.

• • •