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Leader-Following Consensus for Multi-Agent Systems With Asynchronous Control Modes Under Nonhomogeneous Markovian Jump Network Topology

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ABSTRACT Under a nonhomogeneous Markovian jump network topology, this paper addresses the problem of leader-following consensus for multi-agent systems with an asynchronous control mode. Specifically, based on an asynchronous mode-dependent Lyapunov function and a suitable relaxation process, sufficient conditions for leader-following consensus are derived in such a way that 1) non-convex terms multiplied by time-varying signed parameters can be addressed, 2) the emergence of an asynchronous control mode can be reflected in the control synthesis of the follower agents, and 3) the constraints of two combined parameters can be explicitly imposed on the process of transforming parameterized linear matrix inequalities (PLMIs) into LMIs. Finally, two illustrative examples are given to show the validity of the proposed method.

INDEX TERMS Asynchronous control, leader-following consensus, multi-agent systems, nonhomogeneous Markov process.

I. INTRODUCTION

In the past few years, the cooperative control problem of multi-agent systems (MASs) has been widely studied in various applications such as unmanned vehicles [1], [2], formation control [3], rendezvous [4], distributed sensor networks [5], [6], and flocking and swarming [7], [8]. Especially, in the cooperative control problem, one of the most interesting issues is consensus control, which refers to designing a distributed control protocol that allows a group of agents to reach a certain agreement based on their interactive information. Furthermore, the consensus control problem can be generally divided into two types: one is the leader-less consensus control problem and the other is the leader-following consensus control problem. Of the two, the leader-following consensus algorithm under our consideration is designed according to the following scenario: the leader operates independently of the other agents and its trajectory is tracked by all other agents in a way that saves energy, reduces control costs, and improves communication orientation [9]–[12]. Following this trend, various methods have been proposed to deal with the leader-following consensus problem on the premise of a fixed network topology. However, in a practical network environment, the network topology formed among multi-agents can be randomly changed over time due to obstacles resulting from the large network size, functional connectivity disturbance, limitations in communication range, and random packet dropout.

In order to account for these network phenomena, considerable efforts have recently been made to incorporate a time-varying network topology mode into the consensus problem (see [13]–[17]). Especially, one representative method is to model the random abrupt variation of the network topology with a Markov process, and its successful use was made by [18]–[24]. However, it should be noted that these research results were obtained under the assumption that all the transition probabilities (TPs) are time-invariant. Thus, there are limitations to expanding the range of related applications because it is quite difficult to obtain the exact values of the TPs (see [25]–[29]). Thus, to make up for this weakness, one needs to utilize the concept of nonhomogeneous Markov process when representing the network switching topology in the leader-following consensus problem of MASs (see [30], [31]. Not only that, but to achieve better

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performance in the leader-following consensus problem, the control gain of each follower agent must be automatically adjusted in synchronization with the network topology mode. However, since packet loss or network delay can actually occur when analyzing the current mode of the network topology and transmitting it to the follower agent, it is almost difficult for each controller to operate in synchronization with the network topology mode. Hence, from a comprehensive point of view, it is necessary to investigate the impact of nonhomogeneous Markovian jump network topologies as well as the conditional probability resulting from the asynchronous control mode when dealing with the leader-following consensus problem [32]. However, despite this, little effort has been made to tackle such a complex problem, which is the driving force behind this study.

Based on the above discussion, this paper makes a leading attempt to address the problem of leader-following consensus for MASs with an asynchronous control mode under a nonhomogeneous Markovian jump network topology. Specifically, depending on the given procedure, the control design conditions for stochastic $\mathcal{H}_\infty$ leader-following consensus of MASs are first formulated in terms of parameterized linear matrix inequality (PLMI)-based conditions, and then transformed into LMI-based conditions. Especially, this paper focuses on addressing the following issues: **(i)** the existence of non-convex terms multiplied by time-varying signed parameters, **(ii)** the need for an asynchronous mode-dependent Lyapunov function, and **(iii)** the emergence of incompletely known transition rates combined with conditional probabilities. In other words, the main contribution of this paper can be summarized as follow.

- As mentioned in **(i)**, the underlying PLMI-based conditions are given in a form of containing non-convex terms multiplied by time-varying signed parameters such as transition rates. Thus, to deal with such negative or positive definite non-convex terms, this paper proposes a method that can simultaneously take their upper and lower bounds into account. Ultimately, with the aid of this method, the non-convex PLMI-based conditions are transformed into a convex form that depends only on positive transition rates.
- In the MASs, the controller of each follower agent needs to accurately detect the network topology mode and utilize it effectively to achieve the leader-following consensus, but it is practically difficult to estimate and use a mode perfectly synchronized with the network topology mode from the control side. Thus, it is necessary to impose an asynchronous control mode on the control synthesis of followers. For this reason, as mentioned in **(ii)**, this paper employs an asynchronous mode-dependent Lyapunov function to handle the leader-following consensus problem under the network switching topology.
- By using the asynchronous mode-dependent Lyapunov function, the PLMI-based conditions eventually become dependent on both the transition rates and conditional probabilities. Accordingly, as mentioned in **(iii)**, the incompletely known transition rates are combined with the conditional probabilities in the PLMIs. Thus, this paper proposes a suitable relaxation method that can incorporate the constraints of the combined time-varying parameters when transforming PLMIs into a finite number of solvable LMIs. Especially, to obtain less conservative conditions for control synthesis, the proposed relaxation method is developed by utilizing not only the boundary conditions of the time-varying parameters but also their equality conditions.

The rest of this paper is organized as follows. Section II presents a multi-agent system operating on a nonhomogeneous Markovian jump network topology. Section III provides asynchronous control synthesis conditions of the stochastic $\mathcal{H}_\infty$ leader-following consensus for multi-agent systems. In Section IV, two simulation examples are given to illustrate the effectiveness of our method. Finally, concluding remarks are given in Section V.

Notations: For any real symmetric matrix X, the notations $X \geq 0$ and $X > 0$ ($\Leftrightarrow 0 \leq X$ and $0 < X$) mean that X is positive semi-definite and positive definite, respectively, and vice versa. In symmetric block matrices, $(*)$ is used as an ellipsis for terms induced by symmetry. The triplet notation $(\Omega, \mathcal{F}, \mathcal{P})$ denotes a probability space, where Ω, $\mathcal{F}$, and $\mathcal{P}$ represent the sample space, the algebra of events, and the probability measure defined on $\mathcal{F}$, respectively. $\mathbf{E}\{\cdot\}$ denotes the mathematical expectation; $\mathbf{diag}(\cdot)$ indicates a diagonal matrix with diagonal entries; $\mathbf{col}(v_1, v_2, \cdots, v_n) = [v_1^T \ \ v_2^T \ \ \cdots \ \ v_n^T]^T$ for scalar or vector v_i; $\otimes$ denotes the Kronecker product; $\mathbf{N}_1 \setminus \mathbf{N}_2$ indicates the set of elements in the set $\mathbf{N}_1$, but not in the set $\mathbf{N}_2$; $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of the argument; the superscripts "-1" and "T" signify the inverse and transpose, respectively. $\mathbf{He}\{\mathcal{Q}\}$ is used to represent $\mathcal{Q} + \mathcal{Q}^T$ for any square matrix $\mathcal{Q}$; and $\mathfrak{L}_2[0, \infty)$ denotes the space of square integrable vector functions of a given dimension over $[0, \infty)$. For $\mathbb{S} = \{1, 2, \cdots, s\}$, $\big[\mathcal{Q}_i\big]_{i\in\mathbb{S}}^T = \big[\,\mathcal{Q}_1^T \ \mathcal{Q}_2^T \ \cdots \ \mathcal{Q}_s^T\,\big]$, $\big[\mathcal{Q}_i\big]_{i\in\mathbb{S}}^{\mathbf{d}} = \mathbf{diag}(\mathcal{Q}_1, \mathcal{Q}_2, \cdots, \mathcal{Q}_s)$,

$$\big[\mathcal{Q}_{ij}\big]_{i,j\in\mathbb{S}} = \begin{bmatrix} \mathcal{Q}_{11} & \mathcal{Q}_{12} & \dots & \mathcal{Q}_{1s} \\ \mathcal{Q}_{21} & \mathcal{Q}_{22} & \dots & \mathcal{Q}_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{Q}_{s1} & \mathcal{Q}_{s2} & \dots & \mathcal{Q}_{ss} \end{bmatrix},$$

where $\mathcal{Q}_i$ and $\mathcal{Q}_{ij}$ denote real submatrices with proper dimensions.

II. PRELIMINARIES AND SYSTEM MODEL

A. GRAPH THEORY AND MARKOV PROCESS

The network topology of multi-agent systems is established as a time-varying directed graph (digraph) $\mathcal{G}_{\phi(t)} = \big(\mathcal{V}, \mathcal{E}_{\phi(t)}, \mathcal{A}_{\phi(t)}\big)$, where $\phi(t) \in \mathbf{N}_\phi = \{1, 2, \cdots, n_\phi\}$ denotes the network topology mode; $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$ denotes the node set; $\mathcal{E}_{\phi(t)} \subseteq \{(v_j, v_i) \mid v_i, v_j \in \mathcal{V}, j \neq i\}$ denotes the edge set with the ordered pair (v_j, v_i) that has information flow leaving from agent v_j to agent v_i at time t; and $\mathcal{A}_{\phi(t)} = \big[a_{ij,\phi(t)}\big]_{i,j\in\{1,2,\cdots,N\}}$ denotes the adjacency matrix with $a_{ij,\phi(t)} > 0$ if and only if $(v_j, v_i) \in \mathcal{E}_{\phi(t)}$, and $a_{ij,\phi(t)} = 0$ otherwise. In this digraph, the mode switching phenomenon $\{\phi(t), t \geq 0\}$ is characterized by a continuous-time non-homogeneous Markov process with the following transition probabilities (TPs):

$$\begin{aligned}\mathbf{Pr}\big(\phi(t+\delta) = h\big|\phi(t) = g\big)\\ = \begin{cases}\pi_{gh}(t)\delta + o(\delta) & \text{if } h \neq g,\\ 1 + \pi_{gg}(t)\delta + o(\delta) & \text{if } h = g,\end{cases}\end{aligned}$$

where $\delta > 0$; $\lim_{\delta\to 0}(o(\delta)/\delta) = 0$; and $\pi_{gh}(t)$ denotes the transition rates (TRs) from mode g to mode h at time $t+\delta$, and satisfies

$$\begin{aligned}&\pi_{gg}(t) = -\sum_{h\in\mathbf{N}_\phi\setminus\{g\}} \pi_{gh}(t),\\ &\pi_{gh}(t) \geq 0, \text{ for } h \in \mathbf{N}_\phi \setminus \{g\}\,. \end{aligned}\tag{1}$$

Continuing, the neighbor set and degree of $v_i \in \mathcal{V}$ are defined, respective, as $\mathcal{N}_{i,g} = \big\{v_j \in \mathcal{V} \mid (v_j, v_i) \in \mathcal{E}_g\big\}$ and $d_{i,g} = \sum_{j=1}^{N} a_{ij,g}$, and the Laplacian matrix of $\mathcal{G}_g$ is given by $L_g = \mathcal{D}_g - \mathcal{A}_g \in \mathbb{R}^{N\times N}$, where $\mathcal{D}_g = \mathbf{diag}\big(d_{1,g}, \cdots, d_{N,g}\big)$. Moreover, a directed path leaving from node v_j to node v_i is a sequence of ordered edges, that is, (v_j, v_{i_1}), (v_{i_1}, v_{i_2}), $\cdots$, $(v_{i_{p-1}}, v_{i_p})$, (v_{i_p}, v_i), for $v_{i_k} \in \mathcal{V}$, $k = 1, 2, \cdots, p$. Meanwhile, the multi-agent systems under our consideration consist of N follower nodes and one leader node v_0, which depend on an extended graph $\mathcal{G}_{0,g} = (\mathcal{V}_0, \mathcal{E}_{0,g})$ with $\mathcal{V}_0 = \mathcal{V}\bigcup\{v_0\}$ and $\mathcal{E}_{0,g} \subseteq \{(v_0, v_i) \mid i \in \mathcal{V}\}$. Furthermore, for $\mathcal{G}_{0,g}$, the following leader adjacency matrix can be established:

$$M_g = \mathbf{diag}\big(m_{1,g}, \cdots, m_{N,g}\big) \in \mathbb{R}^{N\times N},$$

where $m_{i,g} > 0$ if and only if the leader v_0 transmits information to the follower v_i, and $m_{i,g} = 0$ otherwise. In addition, the union of the graphs is given by $\mathcal{G}_0 := \bigcup_{g=1}^{s} \mathcal{G}_{0,g}$, which has the same node set $\mathcal{V}_0$ as every graph $\mathcal{G}_{0,g}$.

Assumption 1: For all $v_i \in \mathcal{V}$, there exits a directed path from leader v_0 to follower v_i on $\mathcal{G}_{0,g}$, but all the followers transmit no information to the leader.

Next, based on the characteristics of transition rates, we consider the following three sets including mode h:

$$\begin{aligned}\mathbf{H}_g &= \big\{h \mid \pi_{gh}(t) = \pi_{gh} \text{ is time-invariant and}\\ &\qquad\qquad \text{completely known}\big\},\\ \mathbf{H}_g^{\times} &= \big\{h \mid \pi_{gh}(t) \text{ is completely unknown}\big\},\\ \widetilde{\mathbf{H}}_g &= \big\{h \mid \pi_{gh}(t) \text{ is bounded as } \underline{\pi}_{gh} \leq \pi_{gh}(t) \leq \overline{\pi}_{gh}\big\},\end{aligned}\tag{2}$$

such that $h \in \mathbf{N}_\phi = \mathbf{H}_g \bigcup \mathbf{H}_g^{\times} \bigcup \widetilde{\mathbf{H}}_g$. Hence, by (1) and (2), it follows that

- $$0 \equiv \pi_{gg}(t) + \textstyle\sum_{h\in\mathbf{N}_\phi\setminus\{g\}} \pi_{gh}(t), \tag{3}$$
- $$\pi_{gh}(t) = \lambda_{gh} + \epsilon_{gh}(t),\ \epsilon_{gh}(t) \in \big[-\overline{\epsilon}_{gh}, \overline{\epsilon}_{gh}\big], \forall h \in \widetilde{\mathbf{H}}_g, \tag{4}$$

where $\lambda_{gh} = (\underline{\pi}_{gh} + \overline{\pi}_{gh})/2$ and $\overline{\epsilon}_{gh} = (\overline{\pi}_{gh} - \underline{\pi}_{gh})/2$.

Remark 1: Similar to [33], this paper employs the zero-sum constraint of (3) to reduce the conservatism arising from incomplete knowledge of $\pi_{gh}(t)$ for $h \in \widetilde{\mathbf{H}}_g$.

Remark 2: For reference, see [36] and [37] for a detailed explanation on the $\mathcal{H}_\infty$ problem of stability analysis and control synthesis in the LMI approach.

B. MULTI-AGENT SYSTEMS AND DESIGN SPECIFICATION

The continuous-time dynamics of the ith follower are given as follows:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) + Gf(x_i(t)) + Ew_i(t), \tag{5}$$

where $x_i(t) \in \mathbb{R}^n$, $u_i(t) = \mathbb{R}^{n_u}$, $w_i(t) \in \mathbb{R}^{n_w}$, and $f(x_i(t)) \in \mathbb{R}^{n_f}$ denote the state, the control input, the external disturbance satisfying that $w_i(t) \in \mathcal{L}_2[0,\infty)$, and the nonlinear vector function, respectively. In addition, the continuous-time dynamics of the leader are given as follows:

$$\dot{x}_0(t) = Ax_0(t) + Gf(x_0(t)), \tag{6}$$

where $x_0(t) \in \mathbb{R}^n$ and $f(x_0(t)) \in \mathbb{R}^{n_f}$ denote the state and the nonlinear vector function of the leader, respectively. Based on (5) and (6), the performance output $z_i(t) \in \mathbb{R}^{n_z}$ is established as follows:

$$z_i = C\big(x_i(t) - x_0(t)\big),\ \forall i \in \mathcal{V}. \tag{7}$$

Assumption 2 ([34], [35]): The nonlinear vector function $f(x_i(t))$, for $i \in \mathcal{V}\bigcup\{0\}$, satisfies the Lipschitz condition with a constant $\rho > 0$, that is,

$$\big|\big|f(x_i(t)) - f(x_j(t))\big|\big| \leq \rho\big|\big|x_i(t) - x_j(t)\big|\big|. \tag{8}$$

Let us consider the following asynchronous mode-dependent control protocol:

$$u_i(t) = F(\varphi(t))\big(\tilde{x}_i(t) + m_{i,g}\big(x_i(t) - x_0(t)\big)\big),\ \forall i \in \mathcal{V}, \tag{9}$$

where $\varphi(t) \in \mathbf{N}_\varphi = \{1, 2, \cdots, n_\varphi\}$ denotes the control mode that is asynchronous to the network topology mode, $F(\varphi(t) = s) = F_s$ denotes the control gain to be designed later, and $\tilde{x}_i(t)$ denotes the synthesized signal given as follows:

$$\tilde{x}_i(t) = \sum_{j=1}^{N} \ell_{ij,g} x_j(t) = \sum_{j=1}^{N} \ell_{ij,g}\big(x_j(t) - x_0(t)\big). \tag{10}$$

Now, let us define the error state and the error nonlinear vector of the ith follower, respectively, as follows:

$$\begin{aligned}e_i(t) &= x_i(t) - x_0(t),\\ v_i(t) &= f(x_i(t)) - f(x_0(t)),\ \forall i \in \mathcal{V}.\end{aligned}$$

Then, based on (5), (6), and (9), the error system dynamics of the ith follower is represented as follows:

$$\begin{aligned}\dot{e}_i(t) &= Ae_i(t) + BF_s\left(\sum_{j=1}^{N} \ell_{ij,g} e_j(t) + m_{i,g} e_i(t)\right)\\ &\quad e + G\, v_i(t) + Ew_i(t).\end{aligned}\tag{11}$$

Hence, letting $e(t) = \big[e_i(t)\big]_{i\in\mathcal{V}}$, $v(t) = \big[v_i(t)\big]_{i\in\mathcal{V}}$, $w(t) = \big[w_i(t)\big]_{i\in\mathcal{V}}$, and $z(t) = \big[z_i(t)\big]_{i\in\mathcal{V}}$, the resultant augmented closed-loop system is given by

$$\begin{aligned}\dot{e}(t) &= \Big((I_N\otimes A)+(L_g+M_g)\otimes BF_s\Big)e(t)\\ &\quad +(I_N\otimes G)v(t)+(I_N\otimes E)w(t), \qquad (12)\\ z(t) &= (I_N\otimes C)e(t). \qquad (13)\end{aligned}$$

In parallel, from Assumption 2, it follows that $||v_i(t)|| = ||f(x_i(t))-f(x_0(t))|| \le \rho||x_i(t)-x_0(t)|| = \rho||e_i(t)||$, which leads to

$$v^T(t)v(t) \le e^T(t)(I_N\otimes \rho^2 I_n)e(t). \qquad (14)$$

Definition 1: For any initial condition $e(0)=e_0$, $\phi(0)=\phi_0$, and $\varphi(0)=\varphi_0$, the leader-following consensus of (5) and (6) is said to be stochastically achieved with γ-disturbance attenuation if it holds that
(i) for $w(t)\equiv 0$, the response $e(t)$ satisfies

$$\lim_{T\to\infty}\mathbf{E}\left\{\int_0^T ||e(t)||^2dt \,\Big|\, e_0,\phi_0,\varphi_0\right\} < \infty, \qquad (15)$$

(ii) for $0\neq w(t)\in\mathcal{L}_2[0,\infty)$, the response $z(t)$ satisfies

$$\lim_{T\to\infty}\mathbf{E}\left\{\int_0^T ||z(t)||_2^2-\gamma^2||w(t)||_2^2dt \,\Big|\, e_0\equiv 0,\phi_0,\varphi_0\right\} < 0. \qquad (16)$$

In light of Definition 1, this paper will design the asynchronous mode-dependent control gain F_s that ensures (15) and (16) for continuous-time multi-agent systems with (5) and (6) under a nonhomogeneous Markovian jump network topology.

Before ending this section, let us recall the following useful lemma.

Lemma 1: For any matrices $X\in\mathbb{R}^{n\times m}$, $Y\in\mathbb{R}^{n\times m}$, and $0<Q=Q^T\in\mathbb{R}^{n\times n}$, the following inequality holds:

$$\mathbf{He}\big\{X^TY\big\} \le X^TQX+Y^TQ^{-1}Y. \qquad (17)$$

III. CONTROL DESIGN

Let us choose the following control-mode-dependent Lyapunov function:

$$V(t,\varphi(t)=s) = e^T(t)\big(I_N\otimes P_s\big)e(t), \qquad (18)$$

where the symmetric matrix $P_s = P(\varphi(t)=s)\in\mathbb{R}^{n\times n}$ is positive definite. i.e.,

$$I_N\otimes P_s = \mathbf{diag}(\underbrace{P_s,P_s,\cdots,P_s}_{N}) > 0.$$

Then, since the weak infinitesimal operator acting on $V(t,\varphi(t))$ provides

$$\begin{aligned}&\nabla V(t)\\ &= \lim_{\delta\to 0}\frac{1}{\delta}\mathbf{E}\Big\{V(t+\delta,\varphi(t+\delta)=r|\phi(t)=g)\\ &\quad -V(t,\varphi(t)=s|\phi(t)=g)\Big\}\\ &= \lim_{\delta\to 0}\frac{1}{\delta}\Bigg(\sum_{r\in\mathbf{N}_\varphi}\sum_{h\in\mathbf{N}_\phi\setminus\{g\}}\big(\pi_{gh}(t)\delta+o(\delta)\big)\varpi_{hr}V(t+\delta,r)\\ &\quad +\sum_{r\in\mathbf{N}_\varphi}\big(1+\pi_{gg}(t)\delta+o(\delta)\big)\varpi_{gr}V(t+\delta,r)\\ &\quad -\sum_{s\in\mathbf{N}_\varphi}\varpi_{gs}V(t,s)\Bigg)\\ &= \lim_{\delta\to 0}\Bigg(\sum_{r\in\mathbf{N}_\varphi}\Big(\sum_{h\in\mathbf{N}_\phi\setminus\{g\}}\pi_{gh}(t)\varpi_{hr}V(t+\delta,r)\\ &\quad +\pi_{gg}(t)\varpi_{gr}V(t+\delta,r)\Big)\\ &\quad +\sum_{s\in\mathbf{N}_\varphi}\varpi_{gs}\frac{1}{\delta}\big(V(t+\delta,s)-V(t,s)\big)\Bigg)\\ &= \sum_{h\in\mathbf{N}_\phi}\sum_{r\in\mathbf{N}_\varphi}\pi_{gh}(t)\varpi_{hr}V(t,r)+\sum_{s\in\mathbf{N}_\varphi}\varpi_{gs}\dot{V}(t,s), \qquad (19)\end{aligned}$$

it follows from (18) that

$$\begin{aligned}\nabla V(t) &= \sum_{s\in\mathbf{N}_\varphi}\varpi_{gs}\mathbf{He}\Big\{e^T(t)(I_N\otimes P_s)\dot{e}(t)\Big\}\\ &\quad +\sum_{h\in\mathbf{N}_\phi}\sum_{r\in\mathbf{N}_\varphi}\pi_{gh}(t)\varpi_{hr}e^T(t)(I_N\otimes P_r)e(t)\\ &= \mathbf{He}\Big\{e^T(t)(I_N\otimes\mathbf{P}_g)\dot{e}(t)\Big\}+e^T(t)(I_N\otimes\widetilde{\mathbf{P}}_g)e(t), \qquad (20)\end{aligned}$$

where

$$\mathbf{P}_g = \sum_{s\in\mathbf{N}_\varphi}\varpi_{gs}P_s,\quad \widetilde{\mathbf{P}}_g = \sum_{h\in\mathbf{N}_\phi}\sum_{r\in\mathbf{N}_\varphi}\pi_{gh}(t)\varpi_{hr}P_r.$$

Further, substituting (12) into (20) leads to

$$\begin{aligned}\nabla V(t) &= e^T(t)\mathbf{He}\Big\{(I_N\otimes\mathbf{P}_g)\big((I_N\otimes A)\\ &\quad +(L_g+M_g)\otimes BF_s\big)\Big\}e(t)\\ &\quad +\underbrace{\mathbf{He}\Big\{e^T(t)(I_N\otimes\mathbf{P}_g)(I_N\otimes G)v(t)\Big\}}_{\mathbf{(a)}}\\ &\quad +\mathbf{He}\Big\{e^T(t)(I_N\otimes\mathbf{P}_g)(I_N\otimes E)w(t)\Big\}\\ &\quad +e^T(t)\big(I_N\otimes\widetilde{\mathbf{P}}_g\big)e(t), \qquad (21)\end{aligned}$$

and Lemma 1 and (14) allow

$$\begin{aligned}\mathbf{(a)} &= \sum_{s\in\mathbf{N}_\varphi}\varpi_{gs}\mathbf{He}\Big\{e^T(t)(I_N\otimes P_sG)v(t)\Big\}\\ &\le \sum_{s\in\mathbf{N}_\varphi}\varpi_{gs}\Big(e^T(t)(I_N\otimes P_sGG^TP_s)e(t)+v^T(t)v(t)\Big)\\ &\le \sum_{s\in\mathbf{N}_\varphi}\varpi_{gs}e^T(t)\Big(I_N\otimes(P_sGG^TP_s+\rho^2I_n)\Big)e(t).\end{aligned}$$

As a result, letting

$$\xi^T(t) = \big[\,e^T(t)\big|w^T(t)\,\big],$$

(21) satisfies

$$\nabla V(t)\leq \xi^T(t)\left[\begin{array}{c|c}\Psi_{11}+(I_N\otimes\widetilde{\mathbf{P}}_g)+(I_N\otimes\rho^2I_n) & (*)\\ (I_N\otimes E^T)(I_N\otimes\mathbf{P}_g) & 0\end{array}\right]\xi(t), \quad (22)$$

where

$$\Psi_{11}=\mathbf{He}\Big\{(I_N\otimes\mathbf{P}_g)\big((I_N\otimes A)+(L_g+M_g)\otimes BF_s\big)\Big\} + \left(I_N\otimes\sum_{s\in\mathbf{N}_\varphi}\varpi_{gs}P_sGG^TP_s\right).$$

The following lemma provides the stochastic $\mathcal{H}_\infty$ leader-following consensus analysis condition for multi-agent systems with (12).

Lemma 2: For a prescribed scalar $\gamma > 0$, suppose that there exists $0<\bar{P}_s=\bar{P}_s^T\in\mathbb{R}^{n\times n}$ such that the following conditions hold: for $g\in\mathbf{N}_\phi$,

$$0>\left[\begin{array}{c|c}\begin{array}{c}\Psi_{11}+(I_N\otimes\widetilde{\mathbf{P}}_g)\\+(I_N\otimes\rho^2I_n)+(I_N\otimes C^TC)\end{array} & (*)\\ \hline I_N\otimes E^T\mathbf{P}_g & -\gamma^2I\end{array}\right], \quad (23)$$

where

$$\Psi_{11}=I_N\otimes\left(\mathbf{He}\{\mathbf{P}_gA\}+\sum_{s\in\mathbf{N}_\varphi}\varpi_{gs}P_sGG^TP_s\right) +\mathbf{He}\Big\{(L_g+M_g)\otimes\mathbf{P}_gBF_s\Big\}.$$

Then the $\mathcal{H}_\infty$ leader-following consensus of multi-agent systems with (12) is stochastically reached.

Proof: First, let us consider the case where $w(t)\equiv 0$. Then, from (22), it follows that $\nabla V(t)\leq e^T(t)\big(\Psi_{11}+(I_N\otimes\widetilde{\mathbf{P}}_g)+(I_N\otimes\rho^2I_n)\big)e(t)$. That is, since (23) implies $\Psi_{11}+(I_N\otimes\widetilde{\mathbf{P}}_g)+(I_N\otimes\rho^2I_n)<0$, there exists a sufficiently small $\epsilon>0$ such that $\nabla V(t)\leq-\epsilon\|e(t)\|_2^2$, and the generalized Dynkin's formula offers

$$\mathbf{E}\{V(T)\}-V(0)=\mathbf{E}\left\{\int_0^T\nabla V(t)dt\;\Big|\;e_0,\phi_0,\varphi_0\right\} \quad (24)$$
$$\leq-\epsilon\mathbf{E}\left\{\int_0^T\|e(t)\|_2^2\,dt\;\Big|\;e_0,\phi_0,\varphi_0\right\},$$

which results in

$$\lim_{T\to\infty}\mathbf{E}\left\{\int_0^T\|e(t)\|_2^2\,dt\;\Big|\;e_0,\phi_0,\varphi_0\right\}\leq\frac{1}{\varepsilon}V(0)$$

corresponding to (15) in Definition 1. Next, note that from (13) and (22), it follows that

$$||z(t)||_2^2-\gamma^2||w(t)||_2^2+\nabla V(t) \leq\xi^T(t)\left[\begin{array}{c|c}\begin{array}{c}\Psi_{11}+(I_N\otimes\widetilde{\mathbf{P}}_g)\\+(I_N\otimes\rho^2I_n)\\+(I_N\otimes C)^T(I_N\otimes C)\end{array} & (*)\\ \hline I_N\otimes E^T\mathbf{P}_g & -\gamma^2I\end{array}\right]\xi(t).$$

Hence, condition (23) guarantees

$$0>\mathbf{E}\left\{\int_0^T||z(t)||_2^2-\gamma^2||w(t)||_2^2+\nabla V(t)dt\;\Big|\;e_0,\phi_0,\varphi_0\right\}.$$

Furthermore, for $e_0\equiv0$ (i.e., $V(0)\equiv0$), (24) allows

$$0>\mathbf{E}\{V(T)\} +\mathbf{E}\left\{\int_0^T\|z(t)\|_2^2-\gamma^2\|w(t)\|_2^2dt\;\Big|\;e_0\equiv0,\phi_0,\varphi_0\right\},$$

which ensures

$$\mathbf{E}\left\{\int_0^T\|z(t)\|_2^2-\gamma^2\|w(t)\|_2^2dt\;\Big|\;e_0\equiv0,\phi_0,\varphi_0\right\}<0. \quad (25)$$

Therefore, according to Definition 1, it can be seen that the leader-following consensus of multi-agent systems with (12) is stochastically reached with γ-disturbance attenuation. ∎

The following lemma provides the asynchronous control synthesis conditions of the stochastic $\mathcal{H}_\infty$ leader-following consensus for multi-agent systems with (12), formulated in terms of PLMIs.

Lemma 3: For prescribed scalars $\gamma>0$ and μ, suppose that there exist $0<\bar{P}_s=\bar{P}_s^T\in\mathbb{R}^{n\times n}$, $W_{sr}=W_{sr}^T\in\mathbb{R}^{n\times n}$, and $\bar{F}_s\in\mathbb{R}^{n_u\times n}$ such that the following conditions hold: for $g\in\mathbf{N}_\phi$, $s\in\mathbf{N}_\varphi$,

$$0>\left[\begin{array}{c|ccc}\overline{\Psi}_{11,s} & (*) & (*) & (*)\\ \hline (I_N\otimes E^T) & -\gamma^2I_{Nn_w} & 0 & 0\\ (I_N\otimes\rho\bar{P}_s) & 0 & -I_{Nn} & 0\\ (I_N\otimes C\bar{P}_s) & 0 & 0 & -I_{Nn_z}\end{array}\right] +\Upsilon^T\left(\sum_{h\in\mathbf{N}_\phi\backslash\{g\}}\pi_{gh}(t)\big(I_N\otimes\mathcal{W}_h(\mu)\big)\right)\Upsilon, \quad (26)$$

$$0\leq\begin{bmatrix}W_{sr} & (*)\\ \bar{P}_s & \bar{P}_r\end{bmatrix},\ \forall r\in\mathbf{N}_\varphi, \quad (27)$$

where

$$\overline{\Psi}_{11,s}=I_N\otimes\Big(\mathbf{He}\{A\bar{P}_s\}+GG^T\Big) +\mathbf{He}\{(L_g+M_g)\otimes B\bar{F}_s\},$$
$$\mathcal{W}_h(\mu)=\sum_{r\in\mathbf{N}_\varphi}\Big(\varpi_{hr}W_{sr}+\mu^2\varpi_{gr}\bar{P}_r\Big)-2\mu\bar{P}_s,$$
$$\Upsilon=\big[I_{Nn}|0\ 0\ 0\big]\in\mathbb{R}^{Nn\times N(2n+n_w+n_z)}.$$

Then, the leader-following consensus of multi-agent systems with (12) is stochastically reached with γ-disturbance attenuation, where the control gains are designed as follows:

$$F_s=\bar{F}_s\bar{P}_s^{-1},\ \forall s\in\mathbf{N}_\varphi. \quad (28)$$

Proof: Since (23) can be represented as $0>\sum_{s\in\mathbf{N}_\varphi}\varpi_{gs}\Psi_s$, where

$$\Psi_s=\left[\begin{array}{c|c}\begin{array}{c}\Psi_{11,s}+(I_N\otimes\widetilde{\mathbf{P}}_g)\\+(I_N\otimes\rho^2I_n)+(I_N\otimes C^TC)\end{array} & (*)\\ \hline I_N\otimes E^TP_s & -\gamma^2I_{Nn_w}\end{array}\right],$$

$$\Psi_{11,s}=I_N\otimes\Big(\mathbf{He}\{P_sA\}+P_sGG^TP_s\Big) +\mathbf{He}\{(L_g+M_g)\otimes P_sBF_s\},$$

condition $\Psi_s < 0$ implies (23). Further, by the Schur complement, condition $\Psi_s < 0$ is converted into

$$0 > \left[\begin{array}{c|ccc} \Psi_{11,s} + \left(I_N \otimes \widetilde{\mathbf{P}}_g\right) & (*) & (*) & (*) \\ \hline \left(I_N \otimes E^T P_s\right) & -\gamma^2 I_{Nn_w} & 0 & 0 \\ \left(I_N \otimes \rho I_n\right) & 0 & -I_{Nn} & 0 \\ \left(I_N \otimes C\right) & 0 & 0 & -I_{Nn_z} \end{array}\right], \quad (29)$$

and performing a congruence transformation on (29) by $\mathbf{diag}(I_N \otimes \bar{P}_s, I_{Nn_w}, I_{Nn}, I_{Nn_z})$ becomes

$$0 > \left[\begin{array}{c|ccc} \Psi_{11,s} + \left(I_N \otimes \bar{P}_s \widetilde{\mathbf{P}}_g \bar{P}_s\right) & (*) & (*) & (*) \\ \hline \left(I_N \otimes E^T\right) & -\gamma^2 I_{Nn_w} & 0 & 0 \\ \left(I_N \otimes \rho \bar{P}_s\right) & 0 & -I_{Nn} & 0 \\ \left(I_N \otimes C\bar{P}_s\right) & 0 & 0 & -I_{Nn_z} \end{array}\right], \quad (30)$$

where $\bar{P}_s = P_s^{-1}$ and $\bar{F}_s = F_s \bar{P}_s$. In addition, using $\pi_{gg}(t) = -\sum_{h \in \mathbf{N}_\phi \setminus \{g\}} \pi_{gh}(t)$, the nonconvex term $\bar{P}_s \widetilde{\mathbf{P}}_g \bar{P}_s$ is formulated as follows:

$$\begin{aligned} &\bar{P}_s \widetilde{\mathbf{P}}_g \bar{P}_s \\ &= \sum_{h \in \mathbf{N}_\phi \setminus \{g\}} \pi_{gh}(t) \sum_{r \in \mathbf{N}_\varphi} \varpi_{hr} \bar{P}_s P_r \bar{P}_s + \pi_{gg}(t) \sum_{r \in \mathbf{N}_\varphi} \varpi_{gr} \bar{P}_s P_r \bar{P}_s \\ &= \sum_{h \in \mathbf{N}_\phi \setminus \{g\}} \pi_{gh}(t) \left(\sum_{r \in \mathbf{N}_\varphi} \varpi_{hr} \bar{P}_s P_r \bar{P}_s - \sum_{r \in \mathbf{N}_\varphi} \varpi_{gr} \bar{P}_s P_r \bar{P}_s \right). \end{aligned}$$

Hence, since condition (27) and Lemma 1 ensure that $-\mu^2 \bar{P}_r + 2\mu \bar{P}_s \leq \bar{P}_s P_r \bar{P}_s \leq W_{sr}$, it is obvious that

$$\begin{aligned} &\bar{P}_s \widetilde{\mathbf{P}}_g \bar{P}_s \\ &\leq \sum_{h \in \mathbf{N}_\phi \setminus \{g\}} \pi_{gh}(t) \left(\sum_{r \in \mathbf{N}_\varphi} \varpi_{hr} W_{sr} - \sum_{r \in \mathbf{N}_\varphi} \varpi_{gr} \left(2\mu \bar{P}_s - \mu^2 \bar{P}_r\right) \right) \\ &= \sum_{h \in \mathbf{N}_\phi \setminus \{g\}} \pi_{gh}(t) \mathcal{W}_h(\mu), \end{aligned}$$

and it holds that

$$I_N \otimes \bar{P}_s \widetilde{\mathbf{P}}_g \bar{P}_s \leq \sum_{h \in \mathbf{N}_\phi \setminus \{g\}} \pi_{gh}(t) \left(I_N \otimes \mathcal{W}_h(\mu) \right). \quad (31)$$

Therefore, from (30) and (31), it can be seen that (26) and (27) imply (23). ∎

The following theorem provides the relaxed asynchronous control synthesis conditions of the stochastic $\mathcal{H}_\infty$ leader-following consensus for multi-agent systems with (12), formulated in terms of linear matrix inequalities (LMIs).

Theorem 1: For prescribed scalars $\gamma > 0$, μ, and ρ, suppose that there exist $0 < \bar{P}_s = \bar{P}_s^T \in \mathbb{R}^{n \times n}$, $W_{sr} = W_{sr}^T \in \mathbb{R}^{n \times n}$, $\bar{F}_s \in \mathbb{R}^{n_u \times n}$, $X_{gh} = X_{gh}^T \in \mathbb{R}^{n \times n}$, and $Z_g \in \mathbb{R}^{n \times n}$ such that the following conditions hold: for $g \in \mathbf{N}_\phi$, $s \in \mathbf{N}_\varphi$,

$$0 > \left[\begin{array}{c|c} \mathfrak{L}_g + \mathbb{X}_g^{(1)} + \mathbb{Z}_g^{(1)} & (*) \\ \hline \left[I_N \otimes \frac{1}{2} \mathcal{W}_h(\mu) \middle| 0\,0\,0 \right]_{h \in \widetilde{\mathbf{H}}_g \setminus \{g\}} + \mathbb{Z}_g^{(2)} & \mathbb{X}_g^{(3)} \end{array}\right], \quad (32)$$

$$0 \geq \mathcal{W}_h(\mu), \ \forall h \in \mathbf{H}_g^\times \setminus \{g\}, \ \text{if } g \in \mathbf{H}_g^\times, \quad (33)$$

$$0 \leq \begin{bmatrix} W_{sr} & (*) \\ \bar{P}_s & \bar{P}_r \end{bmatrix}, \ \forall r \in \mathbf{N}_\varphi, \quad (34)$$

where

$$\mathfrak{L}_g = \left[\begin{array}{c|c|cc} \begin{array}{l} I_N \otimes \left(\mathbf{He}\{A\bar{P}_s\} + GG^T \right) \\ +(L_g + M_g) \otimes \mathbf{He}\{B\bar{F}_s\} \\ +I_N \otimes \sum_{h \in \mathbf{H}_g \setminus \{g\}} \pi_{gh} \mathcal{W}_h(\mu) \\ +I_N \otimes \sum_{h \in \widetilde{\mathbf{H}}_g \setminus \{g\}} \lambda_{gh} \mathcal{W}_h(\mu) \end{array} & (*) & (*) & (*) \\ \hline I_N \otimes E^T & -\gamma^2 I_{Nn_w} & 0 & 0 \\ \hline I_N \otimes \rho \bar{P}_s & 0 & -I_{Nn} & 0 \\ I_N \otimes C\bar{P}_s & 0 & 0 & -I_{Nn_z} \end{array}\right],$$

$$\begin{aligned} \mathcal{W}_h(\mu) &= \sum_{r \in \mathbf{N}_\varphi} \left(\varpi_{hr} W_{sr} + \mu^2 \varpi_{gr} \bar{P}_r \right) - 2\mu \bar{P}_s, \\ \Upsilon &= \left[I_{Nn} | 0 | 0\ 0 \right] \in \mathbb{R}^{Nn \times N(2n + n_w + n_z)}, \\ \Pi_g &= \sum_{h \in \mathbf{H}_g} \pi_{gh} + \sum_{h \in \widetilde{\mathbf{H}}_g \setminus \{g\}} \lambda_{gh}, \\ \mathbb{X}_g^{(1)} &= \Upsilon^T \left(I_N \otimes \sum_{h \in \widetilde{\mathbf{H}}_g \setminus \{g\}} -\epsilon_{gh}^2 X_{gh} \right) \Upsilon, \\ \mathbb{X}_g^{(3)} &= \left[\left(I_N \otimes X_{gh} \right) \right]^{\mathbf{d}}_{h \in \widetilde{\mathbf{H}}_g \setminus \{g\}}, \\ \mathbb{Z}_g^{(1)} &= \begin{cases} \Upsilon^T \left(I_N \otimes \mathbf{He}\{\Pi_g Z_g\} \right) \Upsilon, & \text{if } g \in \mathbf{H}_g, \\ 0, & \text{otherwise}, \end{cases} \\ \mathbb{Z}_g^{(2)} &= \begin{cases} \left[I_N \otimes Z_g \middle| 0\ 0\ 0 \right]_{h \in \widetilde{\mathbf{H}}_g \setminus \{g\}}, & \text{if } g \in \mathbf{H}_g, \\ 0, & \text{otherwise}. \end{cases} \end{aligned}$$

Then, the leader-following consensus of multi-agent systems is stochastically reached with γ-disturbance attenuation, where the control gains are designed as follows:

$$F_s = \bar{F}_s \bar{P}_s^{-1}, \forall s \in \mathbf{N}_\varphi. \quad (35)$$

Proof: Based on $\mathbf{N}_\phi = \mathbf{H}_g + \widetilde{\mathbf{H}}_g + \mathbf{H}_g^\times$, condition (26) can be rearranged as follows:

$$0 > \mathbb{T} + \Upsilon^T \left(\sum_{h \in \widetilde{\mathbf{H}}_g \bigcup \mathbf{H}_g^\times \setminus \{g\}} \pi_{gh}(t) \left(I_N \otimes \mathcal{W}_h(\mu) \right) \right) \Upsilon, \quad (36)$$

where

$$\mathbb{T} = \left[\begin{array}{c|ccc} \mathbb{T}_{11} & (*) & (*) & (*) \\ \hline I_N \otimes E^T & -\gamma^2 I_{Nn_w} & 0 & 0 \\ I_N \otimes \rho \bar{P}_s & 0 & -I_{Nn} & 0 \\ I_N \otimes C\bar{P}_s & 0 & 0 & -I_{Nn_z} \end{array}\right],$$

$$\mathbb{T}_{11} = \overline{\Psi}_{11,s} + \sum_{h \in \mathbf{H}_g \setminus \{g\}} \pi_{gh} \left(I_N \otimes \mathcal{W}_h(\mu) \right).$$

Thus, to relax (36) on the basis of (3) and (4), we can consider the following two specific cases: **(i)** $\mathbf{H}_g^{\times} = \emptyset$ (that is, $g \in \mathbf{H}_g \bigcup \widetilde{\mathbf{H}}_g$), and **(ii)** $\mathbf{H}_g^{\times} \neq \emptyset$ (that is, $g \in \mathbf{H}_g^{\times}$).

(i) Let us consider the first case where $\mathbf{H}_g^{\times} = \emptyset$. Then condition (36) reduces to

$$0 > \mathbb{T} + \Upsilon^T \left(\sum_{h \in \widetilde{\mathbf{H}}_g \setminus \{g\}} \pi_{gh}(t) \big(I_N \otimes \mathcal{W}_h(\mu) \big) \right) \Upsilon. \tag{37}$$

Further, using

$$\pi_{gh}(t) = \lambda_{gh} + \epsilon_{gh}(t), \ \forall h \in \widetilde{\mathbf{H}}_g, \tag{38}$$

condition (37) is represented as follows:

$$\begin{aligned} 0 &> \mathfrak{L}_g + \left(\sum_{h \in \widetilde{\mathbf{H}}_g \setminus \{g\}} \epsilon_{gh}(t) \Upsilon^T \big(I_N \otimes \mathcal{W}_h(\mu) \big) \Upsilon \right) \\ &= \mathfrak{L}_g + \mathbf{He} \left\{ \big(\Xi_g(t) \otimes \Upsilon \big)^T \left[\left(I_N \otimes \frac{1}{2} \mathcal{W}_h(\mu) \right) \Upsilon \right]_{h \in \widetilde{\mathbf{H}}_g \setminus \{g\}} \right\}, \end{aligned} \tag{39}$$

where $\Xi_g(t) = \big[\epsilon_{gh}(t) \big]_{h \in \widetilde{\mathbf{H}}_g \setminus \{g\}} \in \mathbb{R}^{n_h}$, and n_h denotes the number of elements belonging to the set $\widetilde{\mathbf{H}}_g \setminus \{g\}$. Thus, letting

$$\chi^T(t) = \Big[I \Big| \big(\Xi_g(t) \otimes \Upsilon \big)^T \Big],$$

condition (39) is rewritten as follows:

$$0 > \chi^T(t) \left[\begin{array}{c|c} \mathfrak{L}_g & (*) \\ \hline \left[\left(I_N \otimes \frac{1}{2} \mathcal{W}_h(\mu) \right) \Upsilon \right]_{h \in \widetilde{\mathbf{H}}_g \setminus \{g\}} & 0 \end{array} \right] \chi(t). \tag{40}$$

In addition, since (32) implies $0 > \mathbb{X}_g^{(3)}$, it is ensured that $X_{gh} < 0$, for all $h \in \widetilde{\mathbf{H}}_g \setminus \{g\}$, and from (4), it follows that

$$\begin{aligned} 0 &\le \sum_{h \in \widetilde{\mathbf{H}}_g \setminus \{g\}} (\epsilon_{gh}(t) + \bar{\epsilon}_{gh})(\epsilon_{gh}(t) - \bar{\epsilon}_{gh}) \Upsilon^T \big(I_N \otimes X_{gh} \big) \Upsilon \\ &= \mathbb{X}_g^{(1)} + \sum_{h \in \widetilde{\mathbf{H}}_g \setminus \{g\}} \epsilon_{gh}^2(t) \Upsilon^T \big(I_N \otimes X_{gh} \big) \Upsilon \\ &= \mathbb{X}_g^{(1)} + \big(\Xi_g(t) \otimes \Upsilon \big)^T \mathbb{X}_g^{(3)} \big(\Xi_g(t) \otimes \Upsilon \big). \end{aligned} \tag{41}$$

Especially, for $g \in \mathbf{H}_g$, (3) and (38) lead to

$$\begin{aligned} 0 &= \left(\pi_{gg} + \sum_{h \in \mathbf{H}_g \setminus \{g\}} \pi_{gh} + \sum_{h \in \widetilde{\mathbf{H}}_g \setminus \{g\}} \big(\lambda_{gh} + \epsilon_{gh}(t) \big) \right) \\ &\quad \times \Upsilon^T \mathbf{He} \Big\{ \big(I_N \otimes Z_g \big) \Big\} \Upsilon \\ &= \left(\Pi_g + \sum_{h \in \widetilde{\mathbf{H}}_g \setminus \{g\}} \epsilon_{gh}(t) \right) \Upsilon^T \mathbf{He} \Big\{ \big(I_N \otimes Z_g \big) \Big\} \Upsilon \\ &= \mathbb{Z}_g^{(1)} + \mathbf{He} \left\{ \sum_{h \in \widetilde{\mathbf{H}}_g \setminus \{g\}} \epsilon_{gh}(t) \Upsilon^T \big(I_N \otimes Z_g \big) \Upsilon \right\} \\ &= \mathbb{Z}_g^{(1)} + \mathbf{He} \left\{ \big(\Xi_g(t) \otimes \Upsilon \big)^T \mathbb{Z}_g^{(2)} \right\}. \end{aligned} \tag{42}$$

Hence, by the S-procedure, combining (40) with (41) and (42) results in (32).

(ii) Let us consider the second case where $\mathbf{H}_g^{\times} \neq \emptyset$. Then, by (33), condition (36) reduces to (37) and thus can be formulated in terms of (40). Hence, as in case **(i)**, condition (41) can be used for the S-procedure of (40), which leads to (32) with $\mathbb{Z}_g^{(1)} = 0$ and $\mathbb{Z}_g^{(2)} = 0$. ∎

IV. ILLUSTRATIVE EXAMPLES

In this section, two examples are presented to illustrate the effectiveness of the proposed method.

Example 1: Let us consider the following multi-agent system consisting of three followers and one leader, adopted in [38]:

$$A = \begin{bmatrix} -3 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ G = \begin{bmatrix} 0.2 & 0 \\ 0.1 & 0.1 \\ 0.1 & 0 \end{bmatrix},$$
$$E = \begin{bmatrix} 1.0 \\ 0.1 \\ 1.0 \end{bmatrix}, \quad C = \begin{bmatrix} 0.2 & 0.8 & 1.0 \end{bmatrix}. \tag{43}$$

Fig. 1 shows the applied network topology that satisfies the following network-mode-dependent Laplacian and leader adjacency matrices:

$$L_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix},$$
$$L_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}, \quad L_4 = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$M_1 = \mathbf{diag}(1, 1, 1), \quad M_2 = \mathbf{diag}(1, 1, 0),$$
$$M_3 = \mathbf{diag}(1, 0, 1), \quad M_4 = \mathbf{diag}(1, 0, 1),$$

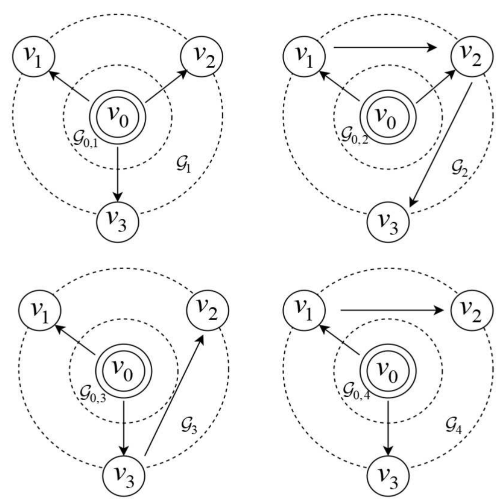

FIGURE 1. (Example 1) Network-mode-dependent digraphs with $\mathcal{G}_{0,g}$ and $\mathcal{G}_g$, for $g \in \mathbf{N}_\phi = \{1, 2, 3, 4\}$.

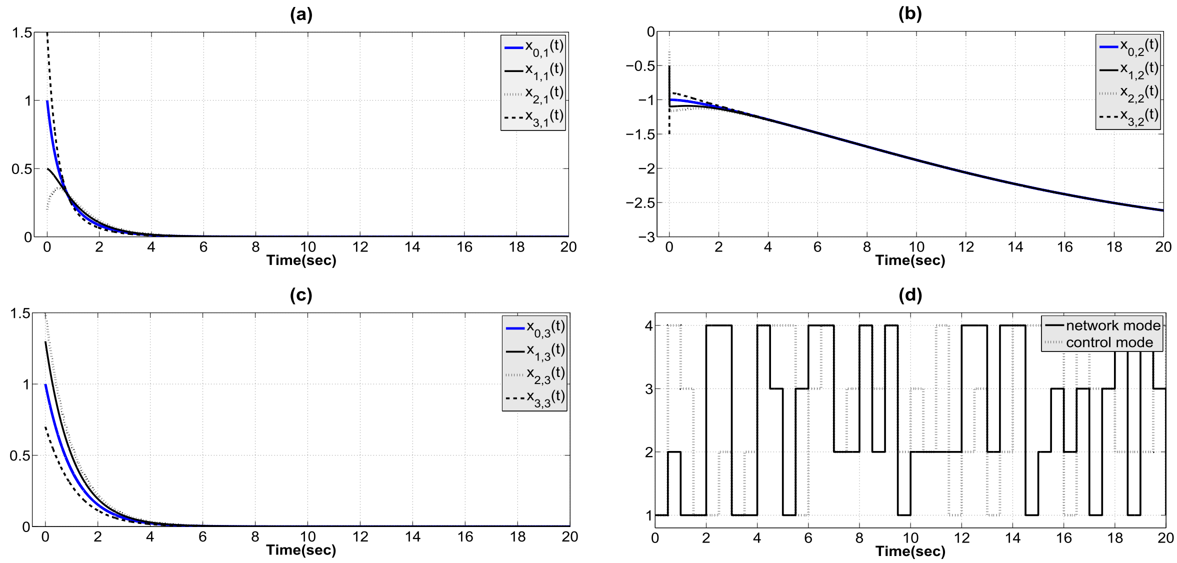

FIGURE 2. (Example 1) State responses of (43) with $w(t) \equiv 0$, and evolution of asynchronous network and control modes.

where $g \in \mathbf{N}_\phi = \{1, 2, 3, 4\}$. Further, the lower and upper bounds of $\pi_{gh}(t) \in [\underline{\pi}_{gh}, \overline{\pi}_{gh}]$ are given as follows:

$$\left[\underline{\pi}_{gh}\right]_{g,h\in\mathbf{N}_\phi} = \begin{bmatrix} -2.0 & 0.1 & \times & \times \\ \times & \times & 0.1 & 0.1 \\ 0.2 & \times & -2.0 & \times \\ 0.1 & \times & \times & \times \end{bmatrix},$$

$$\left[\overline{\pi}_{gh}\right]_{g,h\in\mathbf{N}_\phi} = \begin{bmatrix} -1.0 & 0.5 & \times & \times \\ \times & \times & 0.6 & 0.8 \\ 0.9 & \times & -1.0 & \times \\ 0.8 & \times & \times & \times \end{bmatrix},$$

where "$\times$" denotes the incompletely known transition rate. That is, it is known that $\mathbf{H}_1 = \emptyset$, $\widetilde{\mathbf{H}}_1 = \{1, 2\}$, $\mathbf{H}_1^\times = \{3, 4\}$, $\mathbf{H}_2 = \emptyset$, $\widetilde{\mathbf{H}}_2 = \{3, 4\}$, $\mathbf{H}_2^\times = \{1, 2\}$, $\mathbf{H}_3 = \emptyset$, $\widetilde{\mathbf{H}}_3 = \{1, 3\}$, $\mathbf{H}_3^\times = \{2, 4\}$, $\mathbf{H}_4 = \emptyset$, $\widetilde{\mathbf{H}}_4 = \{1\}$, $\mathbf{H}_4^\times = \{2, 3, 4\}$,

$$\left[\lambda_{gh}\right]_{g,h\in\mathbf{N}_\phi} = \begin{bmatrix} -1.5 & 0.3 & \times & \times \\ \times & \times & 0.35 & 0.45 \\ 0.55 & \times & -1.5 & \times \\ 0.45 & \times & \times & \times \end{bmatrix},$$

$$\left[\bar{\epsilon}_{gh}\right]_{g,h\in\mathbf{N}_\phi} = \begin{bmatrix} 0.5 & 0.2 & \times & \times \\ \times & \times & 0.25 & 0.35 \\ 0.35 & \times & 0.5 & \times \\ 0.35 & \times & \times & \times \end{bmatrix}.$$

In addition, to represent the degree of asynchronism between the network and control modes, the following conditional probability matrix is taken into account:

$$\left[\varpi_{gs}\right]_{g\in\mathbf{N}_\phi, s\in\mathbf{N}_\rho} = \begin{bmatrix} 0.5 & 0.2 & 0.1 & 0.2 \\ 0.2 & 0.5 & 0.1 & 0.2 \\ 0.1 & 0.2 & 0.5 & 0.2 \\ 0.2 & 0.2 & 0.1 & 0.5 \end{bmatrix}.$$

As numerical results, Table 1 shows the optimal $\mathcal{H}_\infty$ performance levels for various ρ, obtained from Theorem 1 with $\mu = 1.0$. As a result, from Table 1, it can be found that Theorem 1 achieves better performances as the value of ρ decreases. Moreover, despite the use of an asynchronous mode (which represents a worse situation compared to the synchronous mode), the proposed method provides a better or similar performance level for $\rho \leq 1$ than that of [38] dealing with the synchronous mode, which means that the proposed method can be effectively used to compensate for the effects of such a worse situation. Especially, for $f(x_i(t)) = \left[\sin(x_{i,1}(t)) \quad \sin(x_{i,2}(t))\right]^T$, Theorem 1 offers the following asynchronous control gains:

TABLE 1. Comparison of $\mathcal{H}_\infty$ performance levels according to ρ.

ρ	0.1	0.5	1.0	1.5
γ	0.1850	0.8534	1.5139	2.1201

$$\begin{aligned} F_1 &= \begin{bmatrix} -51.8684 & -532.9827 & -260.1962 \end{bmatrix}, \\ F_2 &= \begin{bmatrix} -54.0508 & -555.2251 & -271.0367 \end{bmatrix}, \\ F_3 &= \begin{bmatrix} -52.5292 & -539.7340 & -263.4876 \end{bmatrix}, \\ F_4 &= \begin{bmatrix} -56.1646 & -576.7878 & -281.5487 \end{bmatrix}. \end{aligned}$$

Based on these control gains, Figs. 2-(a), (b), and (c) show the state responses of (43) with $w(t) \equiv 0$, $x_0(0) = [1.0\ -1.0\ 1.0]^T$, $x_1(0) = [0.5\ -0.5\ 1.3]^T$, $x_2(0) = [0.2\ -0.3\ 1.5]^T$, and $x_3(0) = [1.5\ -1.5\ 0.7]^T$; and Fig. 2-(d) shows the evolution of the asynchronous network and control modes. Hence, from Fig. 2, it can be seen that the consensus is asymptotically reached despite the occurrence of asynchronous control modes.

Example 2: Let us consider the following multi-agent systems consisting of three single-link manipulators (with

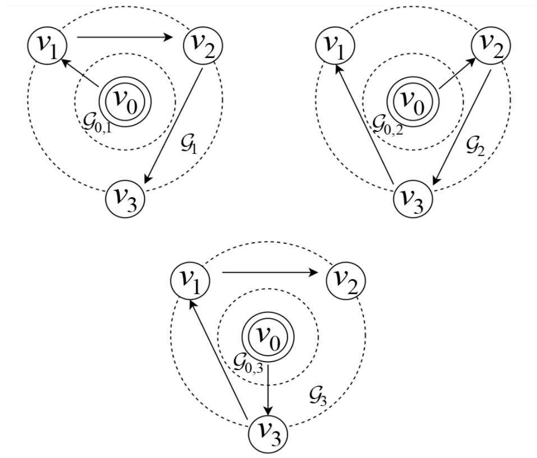

FIGURE 3. **(Example 2) Network-mode-dependent digraphs with $\mathcal{G}_{0,g}$ and $\mathcal{G}_g$, for $g \in \mathbb{N}_\phi = \{1, 2, 3\}$.**

flexible joints actuated by DC motors) and one leader:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5 & 0 & -19.5 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix},$$
$$G = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -3.33 \end{bmatrix}, \quad E = \begin{bmatrix} 0.1 \\ 0.05 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \tag{44}$$

Fig.3 shows the applied network topology that satisfies the following network-mode-dependent Laplacian and leader adjacency matrices:

$$L_1 = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}, \; L_2 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix},$$
$$L_3 = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; M_1 = \mathbf{diag}(1, 0, 0),$$
$$M_2 = \mathbf{diag}(0, 1, 0), \; M_3 = \mathbf{diag}(0, 0, 1),$$

where $\phi(t) \in \mathbf{N}_\phi = \{1, 2, 3\}$. Further, the lower and upper bounds of $\pi_{gh}(t) \in [\underline{\pi}_{gh}, \overline{\pi}_{gh}]$ are given as follows:

$$\left[\underline{\pi}_{gh}\right]_{g,h\in\mathbf{N}_\phi} = \begin{bmatrix} -2.0 & \times & 1.0 \\ \times & \times & 0.8 \\ 0.2 & \times & -3.5 \end{bmatrix},$$
$$\left[\overline{\pi}_{gh}\right]_{g,h\in\mathbf{N}_\phi} = \begin{bmatrix} -1.0 & \times & 1.5 \\ \times & \times & 0.8 \\ 1.0 & \times & -1.5 \end{bmatrix},$$

where "$\times$" denotes the incompletely known transition rate. That is, it is known that $\mathbf{H}_1 = \emptyset$, $\widetilde{\mathbf{H}}_1 = \{1, 3\}$, $\mathbf{H}_1^\times = \{2\}$, $\mathbf{H}_2 = \{3\}$, $\widetilde{\mathbf{H}}_2 = \emptyset$, $\mathbf{H}_2^\times = \{1, 2\}$, $\mathbf{H}_3 = \emptyset$, $\widetilde{\mathbf{H}}_3 = \{1, 3\}$, $\mathbf{H}_3^\times = \{2\}$,

$$\left[\lambda_{gh}\right]_{g,h\in\mathbf{N}_\phi} = \begin{bmatrix} -1.5 & \times & 1.25 \\ \times & \times & 0.8 \\ 0.6 & \times & -2.5 \end{bmatrix},$$
$$\left[\bar{\epsilon}_{gh}\right]_{g,h\in\mathbf{N}_\phi} = \begin{bmatrix} 0.5 & \times & 0.25 \\ \times & \times & 0.0 \\ 0.4 & \times & 1.0 \end{bmatrix}.$$

In addition, to represent the degree of asynchronism between the network and control modes, the following conditional

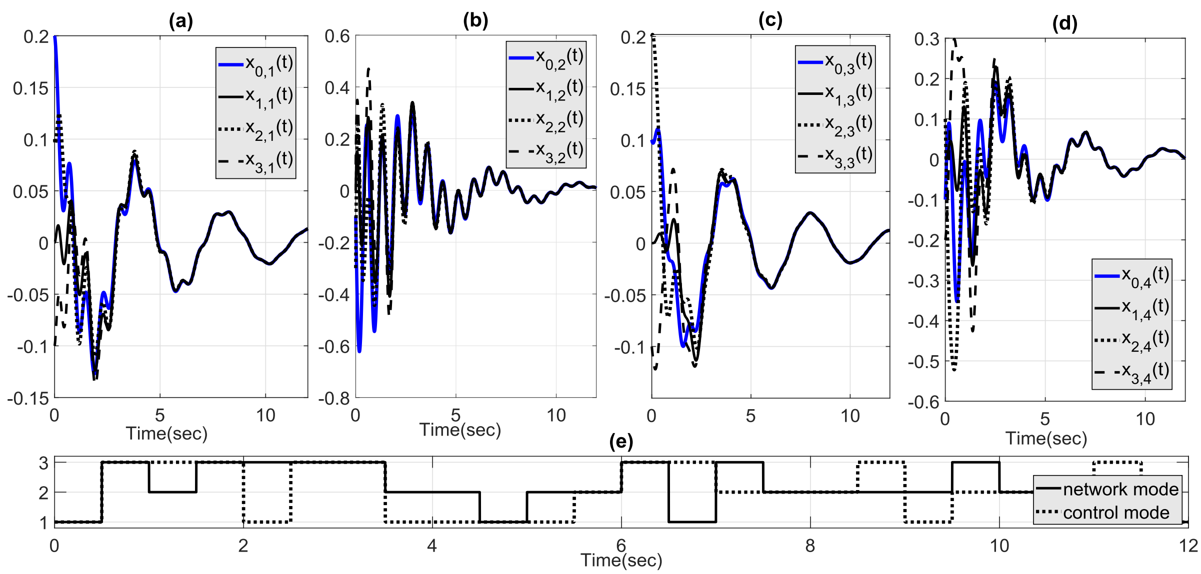

FIGURE 4. **(Example 2) State responses of (44) with $w(t) \equiv 0$, and evolution of asynchronous network and control modes.**

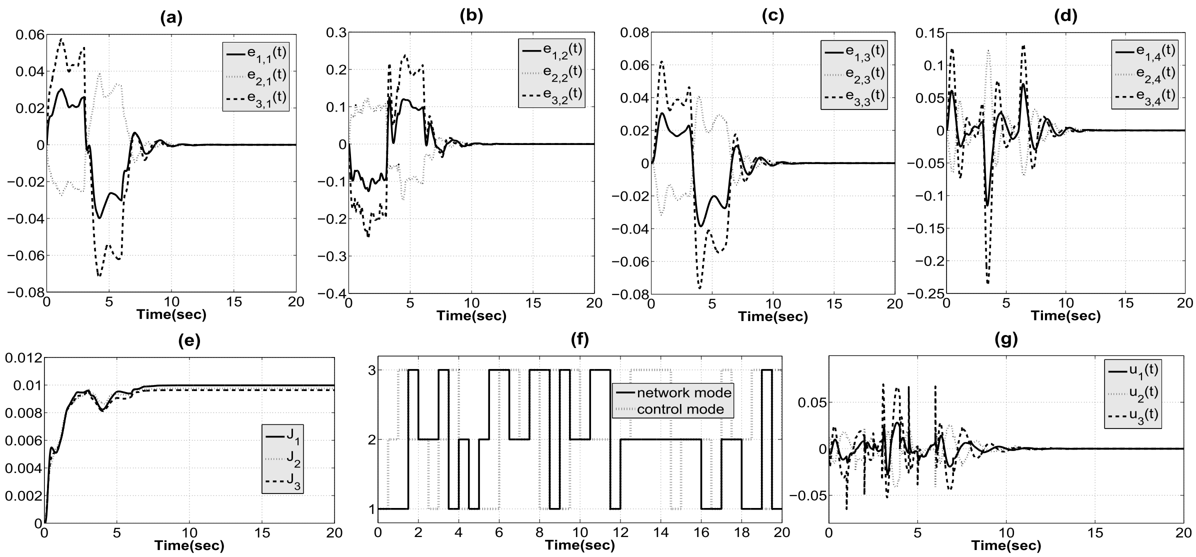

FIGURE 5. (Example 2) Error responses $e_i(t) = x_i(t) - x_0(t)$ of (44), values of $J_i(t) = \int_0^t ||z_i(\tau)||^2 \, d\tau / \int_0^t ||w_i(\tau)||^2 \, d\tau$, mode evolution and control input.

probability matrix is taken into account:

$$\left[\varpi_{gs}\right]_{g\in\mathbf{N}_\phi, s\in\mathbf{N}_\rho} = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.2 & 0.7 \end{bmatrix}.$$

Then, from Theorem 1, the minimized $\mathcal{H}_\infty$ performance level is given as $\gamma = 0.1554$, and the asynchronous control gains are obtained as follows:

$$F_1 = \begin{bmatrix} -1.4218 & -0.3151 & 0.4682 & -0.0510 \end{bmatrix},$$
$$F_2 = \begin{bmatrix} -2.7983 & -0.4633 & 0.7009 & -0.1050 \end{bmatrix},$$
$$F_3 = \begin{bmatrix} -1.9498 & -0.3855 & 0.5765 & -0.0713 \end{bmatrix},$$

where $\mu = 1.0$ and $f(x_i(t)) = \sin(x_{i,3}(t))$. Based on these control gains, Figs. 4-(a), (b), (c), and (d) show the state responses of (44) with $w(t) \equiv 0$, $x_0(0) = [0.2 \ -0.1 \ 0.1 \ -0.1]^T$, $x_1(0) = [0.0 \ 0.0 \ 0.0 \ 0.0]^T$, $x_2(0) = [0.1 \ -0.3 \ 0.2 \ 0.1]^T$, and $x_3(0) = [-0.1 \ 0.1 \ -0.1 \ -0.2]^T$; and Fig. 4-(e) shows the evolution of the asynchronous network and control modes. Accordingly, from Fig. 4, it can be seen that consensus is asymptotically reached despite the occurrence of asynchronous control modes. Moreover, for $x_0(0) = x_1(0) = x_2(0) = x_3(0) = 0$, Figs. 5-(a), (b), (c), and (d) show the error responses $e_i(t) = x_i(t) - x_0(t)$ of (44) with

$$w_1(t) = \begin{cases} 1, & \text{for } t \in [0,3) \\ -1, & \text{for } t \in [3,6), \\ 0, & \text{otherwise} \end{cases} \quad w_2(t) = \begin{cases} -1, & \text{for } t \in [0,3) \\ 1, & \text{for } t \in [3,6), \\ 0, & \text{otherwise} \end{cases}$$

$$w_3(t) = \begin{cases} 2, & \text{for } t \in [0,3) \\ -2, & \text{for } t \in [3,6), \\ 0, & \text{otherwise} \end{cases}$$

and Fig 5-(f) shows the generated mode evolution for simulation, and Fig. 5-(g) shows the control input applied to each agent for leader-following consensus. In addition, Fig 5-(e) shows the value of $J_i(t) = \int_0^t ||z_i(\tau)||^2 \, d\tau / \int_0^t ||w_i(\tau)||^2 \, d\tau$, which reveals that those values are less than the obtained $\mathcal{H}_\infty$ performance level.

V. CONCLUDING REMARKS

In this paper, the asynchronous control problem of leader-following consensus of multi-agent systems has been addressed with consideration of nonhomogeneous Markovian jump network topologies. Based on an asynchronous mode-dependent Lyapunov function and a proper relaxation method, we obtained a set of LMI-based conditions for leader-following consensus by overcoming the difficulties caused by non-convex terms, asynchronous control, and two correlated parameters. Finally, our future work will deal with various asynchronous control problems for leader-following consensus of nonlinear multi-agent systems with consideration of i) mismatched terms (see [39]), ii) network-induced delays, and iii) quantization errors, which will be integral in considering the real situations.

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