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Algebraic Characteristics of Anti-Intuitionistic Fuzzy Subgroups Over a Certain Averaging Operator

DILSHAD ALGHAZZAWI[1], UMER SHUAIB[2], TAZEEM FATIMA[2], ABDUL RAZAQ[3], AND MUHAMMAD AHSAN BINYAMIN[2]

[1]Department of Mathematics, King Abdul Aziz University, Jeddah 21589, Saudi Arabia
[2]Department of Mathematics, Government College University Faisalabad, Faisalabad 38000, Pakistan
[3]Division of Science and Technology, Department of Mathematics, University of Education, Lahore 54000, Pakistan

Corresponding author: Umer Shuaib (mumershuaib@gcuf.edu.pk)

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ABSTRACT In this paper, we propose the concept of ρ anti-intuitionistic fuzzy sets, ρ anti – intuitionistic fuzzy subgroups and prove some of their algebraic properties. We investigate a necessary and sufficient condition for a ρ-anti intuitionistic fuzzy set to be a ρ-anti intuitionistic fuzzy subgroup. We extend this ideology by defining the notions of ρ anti-intuitionistic fuzzy coset, ρ anti-intuitionistic fuzzy normal subgroup and derive some of their key algebraic characteristics. In addition, we study the quotient group of a group induced by ρ-anti intuitionistic fuzzy normal subgroup and establish a group isomorphism between this newly defined quotient group and the quotient group of group G relative to its particular normal subgroup G_{S_ρ}.

INDEX TERMS Intuitionistic fuzzy set (IFS), ρ-anti intuitionistic fuzzy set (ρ-AIFS), ρ-anti intuitionistic fuzzy subgroup (ρ-AIFSG), ρ-anti intuitionistic fuzzy coset, ρ-anti intuitionistic fuzzy normal subgroup (ρ-AIFNSG).
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I. INTRODUCTION

Crisp set theory deals with the situations which are inevitable and precise and the elements have a Boolean state of nature. In real life, this deterministic theory does not appropriately work due to its limitations to deal many physical phenomena such as small or tall, less or more and so on. Fuzzy logic gives explanations and measurements of vagueness in these situations. Fuzzy logic theory is based on the concept of graded membership function in which there is a gradual transition from zero to unity. An important extension of fuzzy sets is the theory of intuitionistic fuzzy sets because it has more ability to tackle with vagueness and imprecision. Another approach to deal with the problems and situations which are vague and not concise is anti-intuitionistic fuzz set. The anti-intuitionistic fuzzy sets have distinguished feature of allocating a membership and non-membership value to each element. Owing to the complicated pattern of executive surroundings and judgmental obstacles themselves, judgment composers can present evaluations or judgments to a few specific extent however, there may occur an element of mistake, because sometimes they are also not sure regarding to the decisions namely, there may exist some reluctancy degree, such kind of reluctancy is perfectly expressed in the framework of anti-intuitionistic fuzzy sets. This particular theory deals with the arrangement of instructions and processing in human brains and such crucial traits as it has the capability to deal with inconclusiveness and vagueness. It has a very indispensable role in most of professional and scientific fields. This phenomenon has useful applications in statistical investigation, medical screening, transportation, nuclear and solid state physics.

Zadeh [1] initiated the concept of fuzzy sets in 1965. The idea of fuzzy subgroups was introduced by Rosenfeld [2] by utilizing perception of fuzzy sets in 1971. Gupta and Ragade [3] presented many useful aspects of fuzzy set

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theory along with their implementations in different disciplines in 1977. Das [4] defined the level subgroups of a fuzzy subgroup in 1981. To see more development on fuzzy subgroups, we refer to [5], [6]. Atanassov [7] characterized the concept of IFSs and described its essential features in 1986. This specific theory has effectively been used in the formulation of IFS iterated function system to image analysis [8], topological spaces [9], medical sciences [10], fractal image construction [11], matrix theory [12] and graph theory [13]. Yan effectively applied this theory in subring in [14]. Fathi and Salleh [15] defined the intuitionistic fuzzy subgroups based over intuitionistic fuzzy space. For more development on the theory of intuitionistic fuzzy subgroups, we refer to [16]–[19]. The notion of anti-fuzzy subgroup was proposed by Biswas [20] in 1990. Kim and Jun [21] introduced the concept of anti-fuzzy R-subgroups of near-ring in 1999. Li *et al.* [22] studied anti intuitionistic fuzzy subgroup (AIFSG) and anti-intuitionistic fuzzy normal subgroup (AIFNSG) with their important appliances in 2009. Palaniappan *et al.* [23] proposed the concept of homomorphism and anti-homomorphism of lower level subgroups of an AIFSG in 2009. Many important properties of AIFSG were discussed in [24], [25]. Massa'deh [26] explained the structural properties of intuitionistic anti fuzzy M-subgroups in 2013. Muthuraj and Balamurugan [27] studied the intuitionistic multi anti fuzzy subgroups in 2014. The author [28] explored the concept of intuitionistic anti L-fuzzy normal M-subgroups. Wang [29] explained the intuitionistic anti fuzzy subincline of incline algebra in 2018. Kausar [30] proposed the concept of direct product of finite intuitionistic anti fuzzy normal subrings over non-associative rings in 2019. For more on intuitionistic fuzzy sets, we suggest reading of [31]–[40].

The utmost aim of this article is to obtain a class of ρ-AIFSG that corresponds to a given AIFSG. The conception of ρ-AIFSG based on ρ-AIFS along with their discrete analytical aspects has been presented. We explore the ideas of ρ-anti-intuitionistic fuzzy coset and ρ-AIFNSG together with some of their important properties. In addition, the research of this anomaly has been extended through the concept of quotient group of a group induced by ρ-AIFNSG and an important group isomorphism has been formulated in the framework of ρ-AIFSG.

The rest of paper is designed in this way. In section 2, essential interpretations related to AIFSGs and their corresponding sequels have been assembled. We propose the notions of ρ-AIFS, ρ-AIFSG and prove some of their algebraic properties in section 3. In section 4, by practicing these ideas, we define ρ-anti intuitionistic fuzzy coset, ρ-AIFNSG and investigate many fundamental algebraic characteristics of these phenomena. In addition, we extend this study to define quotient group of a group induced by ρ-AIFNSG and establish a group isomorphism between this newly defined quotient group and the quotient group of group G with respect to its particular normal subgroup G_{S_ρ}.

II. PRELIMINARIES

This section carries some basic notions and the consequences associated to the philosophy of anti-intuitionistic fuzzy subgroups which are essential to apprehend this article.

Definition 1 [7]: The IFS S of a universe X is an entity in a pattern like $S = \{< x, \mu_S(x), \nu_S(x) >: x \in X, 0 \le \mu_S(x) + \nu_S(x) \le 1\}$ where $\mu_S : X \to [0,1]$ and $\nu_S : X \to [0,1]$ prescribe the membership and non-membership grade of an element x in X respectively.

Definition 2 [24]: For any two fixed positive real numbers β and γ within the closed unit interval such that $0 \le \beta + \gamma \le 1$, the (β, γ)-cut set of an IFS S is denoted by $S_{(\beta,\gamma)}$ and is defined as:

$$S_{(\beta,\gamma)} = \{x \in X : \mu_S(x) \le \beta, \nu_S(x) \ge \gamma\}.$$

Definition 3 [24]: An IFS S of a group G is termed as AIFSG of G if it assures the subsequent axioms for all $x, y \in G$.

(i) $\mu_S(xy) \le max\{\mu_S(x), \mu_S(y)\}$ and $\nu_S(xy) \ge min\{\nu_S(x), \nu_S(y)\}$
(ii) $\mu_S(x^{-1}) \le \mu_S(x)$ and $\nu_S(x^{-1}) \ge \nu_S(x)$.

Definition 4 [24]: Consider an IFS S of a group G. Then S is an AIFSG if all of its (β,γ)-cut sets are subgroups of G.

Definition 5 [22]: For any AIFSG S and an element x of a group G, the anti-intuitionistic fuzzy left coset of G is denoted by xS and is defined as: $x(g) = \{\mu_{xS}(g), \nu_{xS}(g)\} = \{(\mu_S(x^{-1}g), \nu_S(x^{-1}g)) : g \in G\}$. Similarly, one can define the anti-intuitionistic fuzzy right coset of S.

Definition 6 [22]: An AIFSG S of a group G is called an AIFNSG of G if it satisfies the following assertion:

$\mu_S(xy) = \mu_S(yx)$ and $\nu_S(xy) = \nu_S(yx)$ for all $x, y \in G$.

Definition 7 [19]: Let S and T be any two IFS's of a universe X. An averaging operator $S@T$ is defined as follows:

$S@T = \left\{< x, \frac{\mu_S(x)+\mu_T(x)}{2}, \frac{\nu_S(x)+\nu_T(x)}{2} >: x \in X\right\}$.

III. ALGEBRAIC PROPERTIES OF ρ-ANTI INTUITIONISTIC FUZZY SUBGROUPS

This section is devoted to initiate the study of ρ-AIFSG along with various fundamental algebraic postulates of this ideology.

Definition 8: Let S be an IFS of a universe X and $\rho \in [0,1]$. Then the ρ-AIFS of X with respect to S is defined as:

$$S_\rho = \left\{ \begin{array}{c} \langle x, \mu_{S_\rho}(x), \nu_{S_\rho}(x)\rangle : x \in X, \\ 0 \leqslant \mu_{S_\rho}(x) + \nu_{S_\rho}(x) \leqslant 1 \end{array} \right\}$$

Definition 9: For any two fixed positive real numbers β and γ within the closed unit interval such that $0 \le \beta + \gamma \le 1$, the (β,γ)-cut set of a ρ-AIFS S_ρ is denoted by $S_{\rho(\beta,\gamma)}$ and is defined as:

$S_{\rho(\beta,\gamma)} = \{x \in X : \mu_{S_\rho}(x) \le \beta, \nu_{S_\rho}(x) \ge \gamma\}$.

Definition 10: A ρ-AIFS S_ρ of a group G is called ρ-AIFSG of G if it satisfies the subsequent conditions for all $x, y \in G$.

(i) $\mu_{S_\rho}(xy) \le max\{\mu_{S_\rho}(x), \mu_{S_\rho}(y)\}$ and $\nu_{S_\rho}(xy) \ge min\{\nu_{S_\rho}(x), \nu_{S_\rho}(y)\}$

(ii) $\mu_{S_\rho}\left(x^{-1}\right) \leq \mu_{S_\rho}(x)$ and $\nu_{S_\rho}\left(x^{-1}\right) \geq \nu_{S_\rho}(x)$.

Example 11: Consider quaternion group, that is, $Q_8 = \{\pm 1, \pm i, \pm j, \pm k$. The IFS of Q_8 is given as:

$$S = \left\{ \begin{array}{l} \langle 1, 0.30, 0.60\rangle, \langle -1, 0.30, 0.60\rangle, \\ \langle i, 0.50, 0.40\rangle, -\langle i, 0.50, 0.40\rangle, \\ \langle j, 0.70, 0.20\rangle, -\langle j, 0.70, 0.20\rangle, \\ \langle k, 0.70, 0.20\rangle, -\langle k, 0.70, 0.20\rangle \end{array} \right\}.$$

The ρ-AIFS of Q_8 corresponding to the value $\rho = 0.3$ is defined as:

$$S_\rho = \left\{ \begin{array}{l} \langle 1, 0.50, 0.45\rangle, -\langle 1, 0.50, 0.45\rangle, \\ \langle i, 0.60, 0.35\rangle, -\langle i, 0.60, 0.35\rangle, \\ \langle j, 0.70, 0.25\rangle, -\langle j, 0.70, 0.25\rangle \\ \langle k, 0.70, 0.25\rangle, -\langle k, 0.70, 0.25\rangle \end{array} \right\}.$$

In view of Definition (3.3), it is quite evident that S_ρ is ρ-AIFSG of Q_8.

Applications 12: One can view some of the important applications of the phenomenon of ρ-AIFSG in real world problems like:

(i) An important application of ρ-AIFS is that these sets are used in project evaluation. Our methodology provides a significant range of evaluating a project by choosing a suitable parameter ρ.

(ii) The ρ-AIFSs play an important role in developing a routing algorithm, which is used by each router to make its own routing decision on the basis of a appropriate value of the parameterρ.

(iii) In medical decision making problems, we apply this phenomenon to determine the amount of dose which is not harmful for patients by choosing a suitable value of ρ according to the medical history of patients.

Remark 13: Consider a ρ-AIFS S_ρ of a group G, it is said to be ρ-AIFSG if each (β, γ)-cut set is a subgroup of G.

The following result describes three important features of any ρ-AIFSG.

Theorem 14: Let S_ρ be a ρ-AIFSG of a group G and $x, y \in G$, then

(i) $\mu_{S_\rho}\left(x^{-1}\right) = \mu_{S_\rho}(x)$ and $\nu_{S_\rho}\left(x^{-1}\right) = \nu_{S_\rho}(x)$

(ii) $\mu_{S_\rho}(e) \leq \mu_{S_\rho}(x)$ and $\nu_{S_\rho}(e) \geq \nu_{S_\rho}(x)$

(iii) $S_\rho(xy^{-1}) = S_\rho(e)$ implies $S_\rho(x) = S_\rho(y)$.

Proof: **(i)** In view of Definition (3.3), we have

$$\mu_{S_\rho}\left(x^{-1}\right) \leq \mu_{S_\rho}(x), \quad \text{for all } x \in G. \tag{3.1}$$

which means that

$$\mu_{S_\rho}\left(x^{-1}\right)^{-1} \leq \mu_{S_\rho}\left(x^{-1}\right). \tag{3.2}$$

By the combination of (3.1) and (3.2), we obtain $\mu_{S_\rho}\left(x^{-1}\right) = \mu_{S_\rho}(x)$. Similarly, one can also prove for non-membership function.

(ii) The application of Definition (3.3) on an element x of G yields that

$$\begin{aligned} \mu_{S_\rho}(e) &= \mu_{S_\rho}\left(xx^{-1}\right) \\ &\leq \max\left\{\mu_{S_\rho}(x), \mu_{S_\rho}\left(x^{-1}\right)\right\} \\ &= max\left\{\mu_{S_\rho}(x), \mu_{S_\rho}(x)\right\}. \end{aligned}$$

This implies that $\mu_{S_\rho}(e) \leq \mu_{S_\rho}(x)$. The other required inequality can also be proved in the same way.

(iii) For any $x, y \in G$, we have

$$\begin{aligned} \mu_{S_\rho}(x) &= \mu_{S_\rho}\left(xy^{-1}y\right) \\ &\leq max\left\{\mu_{S_\rho}\left(xy^{-1}\right), \mu_{S_\rho}(y)\right\} \end{aligned}$$

The application of given condition in the above relation yields that

$$\mu_{S_\rho}(x) \leq \mu_{S_\rho}(y). \tag{3.3}$$

Moreover,

$$\begin{aligned} \mu_{S_\rho}(y) &= \mu_{S_\rho}\left(yx^{-1}x\right) \\ &\leq max\left\{\mu_{S_\rho}\left(yx^{-1}\right), \mu_{S_\rho}(x)\right\}. \end{aligned}$$

By using Theorem (3.7) (i), in the above relation, we get

$$\mu_{S_\rho}(y) \leq max\left\{\mu_{S_\rho}(xy^{-1}), \mu_{S_\rho}(x)\right\}.$$

By applying the given condition in the above expression, we get

$$\mu_{S_\rho}(y) \leq \mu_{S_\rho}(x). \tag{3.4}$$

The comparison of the Relations (3.3) and (3.4) gives the required equality. Similarly, one can prove the following relations;

$$\nu_{S_\rho}(x) \leq \nu_{S_\rho}(y) \quad \text{and } \nu_{S_\rho}(y) \leq \nu_{S_\rho}(x).$$

Consequently, $S_\rho(x) = S_\rho(y)$, for all $x, y \in G$.

The following theorem presents an important relation between an AIFSG and ρ-AIFSG of a group G.

Theorem 15: Every AIFSG(G) is a ρ-AIFSG(G).

Proof: Suppose S is an AIFSG(G). In view of Definition (3.1), for any $x, y \in G$, we have

$$\begin{aligned} \mu_{S_\rho}(xy) &= \psi_1\{\mu_S(xy), 1-\rho\} \\ &\leq \psi_1\{max\{\mu_S(x), \mu_S(y)\}, \quad 1-\rho\} \\ &= max\{\psi_1\{\mu_S(x), 1-\rho\}, \psi_1\{\mu_S(y), 1-\rho\}\}. \end{aligned}$$

Therefore,

$$\mu_{S_\rho}(xy) \leq max\left\{\mu_{S_\rho}(x), \mu_{S_\rho}(y)\right\}.$$

Moreover,

$$\begin{aligned} \mu_{S_\rho}\left(x^{-1}\right) &= \psi_1\mu_S\left(x^{-1}\right), \quad 1-\rho \\ &\leq \psi_1\mu_S(x), \quad 1-\rho \end{aligned}$$

Clearly, $\mu_{S_\rho}\left(x^{-1}\right) \leq \mu_{S_\rho}(x)$.

Moreover, In view of Definition (3.1) and using the fact that S is an AIFSG, we have

$$\nu_{S_\rho}(xy) = \psi_2\{\nu_S(xy), \rho\}$$

$$\geq \psi_2 \{min \{\nu_S (x), \nu_S (y)\}, \rho\}$$
$$= min \{\psi_2 \{\nu_S (x), \rho\}, \psi_1 \{\nu_S (y), \rho\}\}.$$

Therefore,

$$\nu_{S_\rho} (xy) \geq min \left\{\nu_{S_\rho} (x), \nu_{S_\rho} (y)\right\}.$$

Moreover,

$$\nu_{S_\rho} \left(x^{-1}\right) = \psi_2 \nu_S \left(x^{-1}\right), \quad 1-\rho$$
$$\geq \psi_2 \nu_S (x), \quad 1-\rho.$$

Clearly, $\nu_{S_\rho} \left(x^{-1}\right) \geq \nu_{S_\rho} (x)$.

Consequently, an AIFSG(G) is a ρ-AIFSG(G).

The subsequent example shows that the converse of previous theorem is not true.

Example 16: Consider a unit group under modulo 5, that is, $U_5 = \{1, 2, 3, 4\}$. The IFS of U_5 is given as:

$$S = \{\langle 1, 0.11, 0.66\rangle, \langle 2, 0.35, 0.49\rangle, \langle 3, 0.36, 0.50\rangle, \langle 4, 0.22, 0.55\rangle\}.$$

The ρ-AIFS of U_5 corresponding to the value $\rho = 0.5$ is defined as:

$$S_\rho = \{\langle 1, 0.31, 0.58\rangle, \langle 2, 0.43, 0.50\rangle, \langle 3, 0.43, 0.50\rangle, \langle 4, 0.36, 0.53\rangle\}.$$

In view of Remark (3.6), it is quite evident that S_ρ is ρ-AIFSG of U_5. Moreover, it can be observed that S is not an AIFSG of U_5. From the above discussion, we conclude that S is a ρ-AIFSG of U_5.

In the following result, we establish a necessary and sufficient condition for a ρ-AIFS to be a ρ-AIFSG of a group G.

Theorem 17: A ρ-AIFS S_ρ is ρ-AIFSG if and only if S_ρ satisfies the following assertions for all $x, y \in G$.

(i) If $\mu_{S_\rho}(x) \neq \mu_{S_\rho}(y)$ then
$\mu_{S_\rho} (xy) = \max \left\{\mu_{S_\rho} (x), \mu_{S_\rho} (y)\right\}$.

(ii) If $\nu_{S_\rho} (x) \neq \nu_{S_\rho} (y)$ then
$\nu_{S_\rho} (xy) = \min \left\{\nu_{S_\rho} (x), \nu_{S_\rho} (y)\right\}$.

Proof: Suppose S_ρ is a ρ-AIFSG of a group G and $\mu_{S_\rho}(x) \neq \mu_{S_\rho}(y)$. Take, $k = \mu_{S_\rho} (x) > \mu_{S_\rho}(y)$. Then $\mu_{S_\rho} (xy) \leq k$.

Suppose $\mu_{S_\rho} (xy) < k$. Consider

$$\mu_{S_\rho} (x) = \mu_{S_\rho}(xyy^{-1}).$$

The applications of Definition (3.3) and the assumption give that $\mu_{S_\rho} (x) < k$, which is a contradiction to supposition.

Thus, $\mu_{S_\rho} (xy) = \max \left\{\mu_{S_\rho} (x), \mu_{S_\rho} (y)\right\}$, for all $x, y \in G$.

The condition (ii) can be proved in the same way.

Conversely, suppose that any ρ-AIFS S_ρ of a group G admits (i) and (ii). Let S_ρ be not a ρ-AIFSG of G.

Then
$\mu_{S_\rho} \left(x^{-1}\right) > \mu_{S_\rho} (x)$, for all $x \in G$. Since,

$$\mu_{S_\rho} (e) = \mu_{S_\rho} \left(xx^{-1}\right).$$

By applying (i) in the above equation, we have

$$\mu_{S_\rho} (e) = \mu_{S_\rho} \left(x^{-1}\right) > \mu_{S_\rho}(x).$$

This implies that

$$\mu_{S_\rho} (e) > \mu_{S_\rho}(x). \tag{3.5}$$

Consider

$$\mu_{S_\rho} (x) = \mu_{S_\rho} (ex).$$

The application of (i) in the above relation gives us:

$$\mu_{S_\rho} (x) = \mu_{S_\rho}(e). \tag{3.6}$$

Thus, the Relations (3.5) and (3.6) lead to the contradiction.

Consequently, $\mu_{S_\rho} \left(x^{-1}\right) \leq \mu_{S_\rho} (x)$, for all $x \in G$.

Similarly, one can also establish the following inequality in the framework of condition (i) $\nu_{S_\rho} \left(x^{-1}\right) \geq \nu_{S_\rho} (x)$.

Moreover, assume that $\mu_{S_\rho} (xy) > \mu_{S_\rho} (x)$. Consider

$$\mu_{S_\rho}(y) = \mu_{S_\rho} \left(x^{-1}xy\right).$$

By using condition (i) and the assumption in above relation, we get

$$\mu_{S_\rho} (y) \geq \mu_{S_\rho} (x), \quad \text{for all } x, y \in G \tag{3.7}$$

Now assume that $\mu_{S_\rho} (xy) > \mu_{S_\rho} (y)$.

Consider

$$\mu_{S_\rho}(x) = \mu_{S_\rho} \left(xyy^{-1}\right).$$

By applying condition (i) and the assumption in above relation, we obtain

$$\mu_{S_\rho} (x) \geq \mu_{S_\rho} (y), \quad \text{for all } x, y \in G. \tag{3.8}$$

The comparison of Relations (3.7) and (3.8) establishes a contradiction against (i).

Thus,

$$\mu_{S_\rho} (xy) \leq \max \left\{\mu_{S_\rho} (x), \mu_{S_\rho} (y)\right\}.$$

Similarly, one can apply (ii) in the light of above mentioned arguments to prove the case for non-membership function of S_ρ.

Consequently, S_ρ is a ρ-AIFSG of G.

In the following theorem, we investigate a condition under which a ρ-AIFS is a ρ-AIFSG.

Theorem 18: Let S be an IFS of a group G and for all $x \in G$, $\mu_S (x) = \mu_S \left(x^{-1}\right)$ and $\nu_S (x) = \nu_S(x^{-1})$. Moreover, $\rho < \max \{1-q, r\}$ where $q = \max \mu_S (x)$, *for all* $x \in G$ and $r = \min\{\nu_S (x)$, *for all* $x \in G$. Then S is a ρ-AIFSG of G.

Proof: In view of given condition, we have $q < 1-\rho$ and $r > \rho$. It follows that $\mu_S (x) < 1-\rho$ and $\nu_S (x) > \rho$, for all $x \in G$. Therefore, $\mu_{S_\rho}(xy) \leq \max \mu_{S_\rho} (x), \mu_{S_\rho} (y)$ and $\nu_{S_\rho}(xy) \geq \min \nu_{S_\rho} (x), \nu_{S_\rho} (y)$, for all $x, y \in G$.

Moreover, we have $\mu_S \left(x^{-1}\right) = \mu_S (x)$ and $\nu_{S_\rho} \left(x^{-1}\right) = \nu_{S_\rho}(x)$. Thus,

$$\mu_{S_\rho} \left(x^{-1}\right) = \mu_{S_\rho} (x) \text{ and } \nu_{S_\rho} \left(x^{-1}\right) = \nu_{S_\rho}(x).$$

Thus, a ρ-AIFS is a ρ-AIFSG of G.

The following theorem describes that the union of two ρ-AIFSG's is a ρ-AIFSG.

Theorem 19: Union of two ρ-AIFSG's of a group G is a ρ-AIFSG of G.

Proof: Let S_ρ and T_ρ be any two ρ-AIFSG's of a group G. Then by Definition (3.1) for each $x, y \in G$, we have

$$\begin{aligned}\mu_{(S\cup T)_\rho}(xy) &= \psi_1\left\{\mu_{(S\cup T)}(xy), 1-\rho\right\}\\ &= \psi_1\left\{\max\{\mu_S(xy), \mu_T(xy)\}, 1-\rho\right\}\\ &= \begin{bmatrix} max\{\psi_1\{\mu_S(xy), 1-\rho\}\},\\ max\{\psi_1\{\mu_T(xy), 1-\rho\}\}\end{bmatrix}\\ &= max\left\{\mu_{S_\rho}(xy), \mu_{T_\rho}(xy)\right\}.\end{aligned}$$

The application of Definition (3.3) in above relation yields that

$$\begin{aligned}\mu_{(S\cup T)_\rho}(xy) &\le max\begin{bmatrix} max\left\{\mu_{S_\rho}(x), \mu_{S_\rho}(y)\right\},\\ max\left\{\mu_{T_\rho}(x), \mu_{T_\rho}(y)\right\}\end{bmatrix}\\ &= max\begin{bmatrix} max\left\{\mu_{S_\rho}(x), \mu_{T_\rho}(x)\right\},\\ max\left\{\mu_{S_\rho}(y), \mu_{T_\rho}(y)\right\}\end{bmatrix}.\end{aligned}$$

This implies that

$$\mu_{(S\cup T)_\rho}(xy) \le max\left\{\mu_{S_\rho\cup T_\rho}(x), \mu_{S_\rho\cup T_\rho}(y)\right\}.$$

Moreover,

$$\begin{aligned}\mu_{(S\cup T)_\rho}\left(x^{-1}\right) &= \psi_1\left\{\mu_{(S\cup T)}\left(x^{-1}\right), 1-\rho\right\}\\ &= \psi_1\left\{\max\{\mu_S\left(x^{-1}\right), \mu_T\left(x^{-1}\right)\}, 1-\rho\right\}\\ &= max\left\{\mu_{S_\rho}\left(x^{-1}\right), \mu_{T_\rho}\left(x^{-1}\right)\right\}\\ &\le max\left\{\mu_{S_\rho}(x), \mu_{T_\rho}(x)\right\}.\end{aligned}$$

It follows that

$$\mu_{(S\cup T)_\rho}\left(x^{-1}\right) \le \mu_{S_\rho\cup T_\rho}(x).$$

Moreover, In view of Definition (3.1) and using the fact that S_ρ and T_ρ be ρ-AIFSG's of a group G, we have

$$\begin{aligned}\nu_{(S\cup T)_\rho}(xy) &= \psi_2\left\{\nu_{(S\cup T)}(xy), \rho\right\}\\ &= \psi_2\left\{\max\{\nu_S(xy), \nu_T(xy)\}, \rho\right\}\\ &= \begin{bmatrix} max\{\psi_1\{\nu_S(xy), \rho\}\},\\ max\{\psi_1\{\nu_T(xy), \rho\}\}\end{bmatrix}\\ &= max\left\{\nu_{S_\rho}(xy), \nu_{T_\rho}(xy)\right\}\end{aligned}$$

The application of Definition (3.3) in above relation yields that

$$\begin{aligned}\nu_{(S\cup T)_\rho}(xy) &\ge min\begin{bmatrix} max\left\{\nu_{S_\rho}(x), \nu_{S_\rho}(y)\right\},\\ max\left\{\nu_{T_\rho}(x), \nu_{T_\rho}(y)\right\}\end{bmatrix}\\ &= max\begin{bmatrix} min\left\{\nu_{S_\rho}(x), \nu_{T_\rho}(x)\right\},\\ min\left\{\nu_{S_\rho}(y), \mu_{T_\rho}(y)\right\}\end{bmatrix}.\end{aligned}$$

This implies that

$$\nu_{(S\cup T)_\rho}(xy) \ge min\left\{\nu_{S_\rho\cup T_\rho}(x), \nu_{S_\rho\cup T_\rho}(y)\right\}.$$

Moreover,

$$\begin{aligned}\nu_{(S\cup T)_\rho}\left(x^{-1}\right) &= \psi_1\left\{\nu_{(S\cup T)}\left(x^{-1}\right), \rho\right\}\\ &= \psi_1\left\{\max\{\nu_S\left(x^{-1}\right), \nu_T\left(x^{-1}\right)\}, \rho\right\}\\ &= max\left\{\nu_{S_\rho}\left(x^{-1}\right), \nu_{T_\rho}\left(x^{-1}\right)\right\}\\ &\ge min\left\{\nu_{S_\rho}(x), \nu_{T_\rho}(x)\right\}.\end{aligned}$$

It follows that

$$\nu_{(S\cup T)_\rho}\left(x^{-1}\right) \ge \nu_{S_\rho\cup T_\rho}(x).$$

Consequently, the union of any two ρ-AIFSG is a ρ-AIFSG of G.

Remark 20: The intersection of two ρ-AIFSG may not be a ρ-AIFSG of a group G.

Example 21: Consider IFS S and T of group of integers Z under addition as follows:

$$\mu_S(x) = \begin{cases} 0.40 & if\ x \in 3Z\\ 0.90 & otherwise\end{cases} \quad \text{and}$$

$$\nu_S(x) = \begin{cases} 0.50 & if\ x \in 3Z\\ 0.10 & otherwise,\end{cases}$$

$$\mu_T(x) = \begin{cases} 0.43 & if\ x \in 2Z\\ 0.73 & otherwise\end{cases} \quad \text{and}$$

$$\nu_T(x) = \begin{cases} 0.45 & if\ x \in 2Z\\ 0.25 & otherwise.\end{cases}$$

The ρ-AIFSG S_ρ and T_ρ of Z corresponding to $\rho = 0.3$ are given as:

$$\mu_{S_\rho}(x) = \begin{cases} 0.55 & if\ x \in 3Z\\ 0.80 & otherwise\end{cases} \quad \text{and}$$

$$\nu_{S_\rho}(x) = \begin{cases} 0.40 & if\ x \in 3Z\\ 0.20 & otherwise,\end{cases}$$

$$\mu_{T_\rho}(x) = \begin{cases} 0.57 & if\ x \in 2Z\\ 0.72 & otherwise\end{cases} \quad \text{and}$$

$$\nu_{T_\rho}(x) = \begin{cases} 0.33 & if\ x \in 2Z\\ 0.28 & otherwise.\end{cases}$$

Consider

$$\mu_{S_\rho\cap T_\rho}(x) = \begin{cases} 0.55 & if\ x \in 3Z\\ 0.57 & if\ x \in 2Z-3Z\\ 0.72 & otherwise\end{cases} \quad \text{and}$$

$$\nu_{S_\rho\cap T_\rho}(x) = \begin{cases} 0.50 & if\ x \in 3Z\\ 0.33 & if\ x \in 2Z-3Z\\ 0.28 & otherwise.\end{cases}$$

For $x = 9$ and $y = 4$, $\mu_{S_\rho\cap T_\rho}(x) = 0.55$ and $\nu_{S_\rho\cap T_\rho}(x) = 0.50$, $\mu_{S_\rho\cap T_\rho}(y) = 0.57$ and $\nu_{S_\rho\cap T_\rho}(y) = 0.33$.

Moreover,

$\mu_{S_\rho\cap T_\rho}$(x-y) $= \mu_{S_\rho\cap T_\rho}(9-4) = \mu_{S_\rho\cap T_\rho}(5) = 0.72 > 0.57$ and

$\mu_{S_\rho\cap T_\rho}$(x-y) $= \mu_{S_\rho\cap T_\rho}(9-4) = \nu_{S_\rho\cap T_\rho}(5) < 0.33$.

It is quite evident from the above discussion that the intersection of two ρ-AIFSG is not a ρ-AIFSG.

IV. CHARACTERIZATIONS OF ρ-ANTI INTUITIONISTIC FUZZY NORMAL SUBGROUPS

In this section, we describe the ideas of ρ-anti intuitionistic fuzzy cosets and ρ-AIFNSG and extend the study of these concepts to define the notion of quotient group of a group induced by ρ-AIFNSG along with their many algebraic properties.

Definition 22: For any ρ-AIFSG S_ρ and an element x of a group G. The ρ anti-intuitionistic fuzzy left coset of G is denoted by xS_ρ and

$$xS_\rho(g) = \{\mu_{xS_\rho}(g), \nu_{xS_\rho}(g)\} = \left\{\left(\mu_{S_\rho}\left(x^{-1}g\right), \nu_{S_\rho}\left(x^{-1}g\right)\right)\right\}.$$

Similarly, one can define the anti-intuitionistic fuzzy right coset of S_ρ.

Definition 23: A ρ-AIFSG of a group G is said to be ρ-AIFNSG of G if

$$xS_\rho = S_\rho x, \quad \text{for all } x \in G.$$

In the subsequent theorem, we show that every AIFNSG is a ρ-AIFNSG of G.

Theorem 24: Every AIFNSG(G) is a ρ-AIFNSG(G).

Proof: Let S be an AIFNSG of a group G. In view of Theorem (3.8), S_ρ is a ρ-AIFSG of a group G. To complete the proof it is sufficient to show that S_ρ satisfies definition (4.2).

The application of Definition (2.5) yields that

$$\mu_S\left(x^{-1}g\right) = \mu_S\left(gx^{-1}\right).$$

This implies that $\psi_1\left\{\mu_S\left(x^{-1}g\right), 1-\rho\right\} = \psi_1\mu_S\left(gx^{-1}\right)$, $1-\rho$.

Therefore,

$$\mu_{xS_\rho} = \mu_{S_\rho x}, \quad \text{for all } g \in G.$$

Moreover, in view of Definition (2.5) and using the fact that S is AIFNSG of, we have

$$\nu_S\left(x^{-1}g\right) = \nu_S\left(gx^{-1}\right).$$

This implies that $\psi_2\left\{\nu_S\left(x^{-1}g\right), 1-\rho\right\} = \psi_2\nu_S\left(gx^{-1}\right), 1-\rho$.

Therefore,

$$\nu_{xS_\rho} = \nu_{S_\rho x}, \quad \text{for all } g \in G.$$

Thus, $xS_\rho = S_\rho x$, for all $g \in G$.

The converse of above theorem is not true which can be viewed in the following example.

Example 25: Consider the dihedral group $D_3 = \{e, \alpha, \alpha^2, \beta, \alpha\beta, \alpha^2\beta$. The AIFSG of D_3 is given as:

$$S = \left\{\begin{array}{c}\langle e, 0.59, 0.20\rangle, \langle \alpha, 0.60, 0.19\rangle, \\ \langle \alpha^2, 0.60, 0.19\rangle, \langle \beta, 0.59, 0.20\rangle, \\ \langle \alpha\beta, 0.60, 0.19\rangle, \langle \alpha^2\beta, 0.60, 0.19\rangle\end{array}\right\}.$$

The ρ-AIFSG of D_3 corresponding to the value $\rho = 0.6$ is defined as:

$$S_\rho = \left\{\begin{array}{c}\langle e, 0.50, 0.40\rangle, \langle \alpha, 0.50, 0.40\rangle, \\ \langle \alpha^2, 0.50, 0.40\rangle, \langle \beta, 0.50, 0.40\rangle, \\ \langle \alpha\beta, 0.50, 0.40\rangle, \langle \alpha^2\beta, 0.50, 0.40\rangle\end{array}\right\}.$$

Clearly, S_ρ is a ρ-AIFNSG of D_3. Moreover, it can be observed that S is not an AIFNSG in the framework of Definition (2.6).

In the following result, we show another important characteristic of ρ-AIFNSG.

Theorem 26: Every ρ-AIFNSG S_ρ admits the following property.

$\mu_{S_\rho}(xy) = \mu_{S_\rho}(yx)$ and $\nu_{S_\rho}(xy) = \nu_{S_\rho}(yx)$, for all $x, y \in G$.

Proof: By applying Definition (4.2), for any fixed element $x$of a group G, we have

$$xS_\rho = S_\rho x.$$

This implies that

$$xS_\rho\left(y^{-1}\right) = S_\rho x(y^{-1}), \quad \text{for all } y^{-1} \in G.$$

Therefore, $\psi_1\left\{\mu_S\left(x^{-1}y^{-1}\right), 1-\rho\right\} = \psi_1\{\mu_S(y^{-1}x^{-1}), 1-\rho\}$.

This further shows that,

$$\mu_{S_\rho}(x^{-1}y^{-1}) = \mu_{S_\rho}(y^{-1}x^{-1}).$$

The application of Theorem (3.7) (i) yields that $\mu_{S_\rho}(yx) = \mu_{S_\rho}(xy)$, for all $y \in G$.

One can prove that $\nu_{S_\rho}(xy) = \nu_{S_\rho}(yx)$ in the similar way.

In the following result, we prove the condition for a ρ-AIFSG of a group G to be a ρ-AIFNSG.

Theorem 27: Let S_ρ be a ρ-AIFSG of G such that $\rho < \max\{1-\text{q}, \text{r}\}$, where $q = max\mu_{S_\rho}(x) : x \in G$ and $r = min\{\nu_{S_\rho}(x) : x \in G$. Then S_ρ is a ρ-AIFNSG of G.

Proof: In view of the given condition, we have $q < 1-\rho$ and $r > \rho$. It follows that $\mu_{S_\rho}(x) < 1-\rho$ and $\nu_{S_\rho}(x) > \rho$ for all $x \in G$.

Therefore, $\mu_{xS_\rho}(g) = \mu_{S_\rho}\left(x^{-1}g\right) = \alpha$ and $\nu_{xS_\rho}(g) = \nu_{S_\rho}\left(x^{-1}g\right) = \beta$.

Similarly, $\mu_{S_{\rho x}}(g) = \mu_{S_\rho}\left(gx^{-1}\right) = \alpha$ and $\nu_{S_{\rho x}}(g) = \nu_{S_\rho}\left(gx^{-1}\right) = \beta$.

Thus, $xS_\rho = S_\rho x$ for all $x \in G$.

Theorem 28: Let S_ρ be a ρ-AIFNSG of a group G. The set $G_{S_\rho} = \left\{x \in G : S_\rho(x) = S_\rho(e)\right\}$ is a normal subgroup of G.

Proof: Obviously $G_{S_\rho} \neq \emptyset$ as $e \in G_{S_\rho}$. By applying Definition (3.3) for any two elements $x, y \in G_{S_\rho}$, we have

$$\mu_{S_\rho}\left(xy^{-1}\right) \leq max\left\{\mu_{S_\rho}(x), \mu_{S_\rho}(y)\right\} = max\left\{\mu_{S_\rho}(e), \mu_{S_\rho}(e)\right\}.$$

This implies that

$$\mu_{S_\rho}\left(xy^{-1}\right) \leq \mu_{S_\rho}(e). \tag{4.1}$$

We also know that

$$\mu_{S_\rho}\left(xy^{-1}\right) \geq \mu_{S_\rho}(e). \tag{4.2}$$

By the comparison of Relations (4.1) and (4.2), we get

$$\mu_{S_\rho}\left(xy^{-1}\right) = \mu_{S_\rho}(e).$$

Similarly, one can prove that $\nu_{S_\rho}\left(xy^{-1}\right) = \nu_{S_\rho}(e)$. It follows that $xy^{-1} \in G_{S_\rho}$.

Further, in view of Definition (4.2) for any element $x \in G_{S_\rho}$ and $g \in G$, we have

$$\mu_{S_\rho}\left(gxg^{-1}\right) = \mu_{S_\rho}(x) = \mu_{S_\rho}(e).$$

Likewise, we can prove that $\nu_{S_\rho}\left(gxg^{-1}\right) = \nu_{S_\rho}(e)$.

Thus, $gxg^{-1} \in G_{S_\rho}$.

Consequently, $G_{S_\rho}$is normal subgroup of G.

Theorem 29: Every ρ-AIFNSG of a group G admits the following properties:

(i) $xS_\rho = yS_\rho$ if and only $x^{-1}y \in G_{S_\rho}$

(ii) $S_\rho x = S_\rho y$ if and only if $xy^{-1} \in G_{S_\rho}$, for all $x, y \in G$.

Proof: (i) Suppose that $xS_\rho = yS_\rho$. In view of Definition (4.1), we have

$$\mu_{S_\rho}\left(x^{-1}y\right) = \mu_{xS_\rho}(y) = \mu_{yS_\rho}(y).$$

This Implies that $\mu_{S_\rho}\left(x^{-1}y\right) = \mu_{S_\rho}(e)$.

Similarly, one can easily prove that $\nu_{S_\rho}\left(x^{-1}y\right) = \nu_{S_\rho}(e)$. Thus, $x^{-1}y \in G_{S_\rho}$.

Conversely, suppose $x^{-1}y \in G_{S_\rho}$. By applying Definition (4.1) for a fixed element x and any element g of G, we have

$$\begin{aligned}\mu_{xS_\rho}(g) &= \mu_{S_\rho}\left(x^{-1}g\right) = \mu_{S_\rho}\left(x^{-1}yy^{-1}g\right)\\ &= \mu_{S_\rho}\left(\left(x^{-1}y\right)\left(y^{-1}g\right)\right)\\ &\leq max\left\{\mu_{S_\rho}\left(x^{-1}y\right), \mu_{S_\rho}(y^{-1}g)\right\}\\ &= max\left\{\mu_{S_\rho}(e), \mu_{S_\rho}(y^{-1}g)\right\} = \mu_{S_\rho}(y^{-1}g).\end{aligned}$$

This implies that

$$\mu_{xS_\rho}(g) \leq \mu_{yS_\rho}(g). \tag{4.3}$$

Similarly, in view of above arguments, we obtain

$$\mu_{yS_\rho}(g) \leq \mu_{xS_\rho}(g). \tag{4.4}$$

The comparison of Relations (4.3) and (4.4) give that $\mu_{yS_\rho}(g) = \mu_{xS_\rho}(g)$. Similarly, one can establish that $\nu_{xS_\rho}(g) = \nu_{yS_\rho}(g)$. Consequently,$xS_\rho = yS_\rho$.

The second property can be established in the framework of the same arguments.

Definition 30: The quotient group of G induced by ρ-AIFNSG S_ρ is denoted by G/S_ρ and is defined as: $G/S_\rho = \{S_\rho x : x \in G$ under the following binary operation. $S_\rho x * S_\rho y = S_\rho xy$, for all $x, y \in G$.

Theorem 31: Let G/S_ρ be a quotient group of G induced by ρ-AIFNSG and $x \in G$. Then there is a natural epimorphism $\phi : x \rightarrow S_\rho x$ between groups G and G/S_ρ with $\ker(\phi) = G_{S_\rho}$.

Proof: Consider

$$\begin{aligned}\phi(xy) &= S_\rho xy, \quad x, y \in G.\\ &= S_\rho x * S_\rho y = \phi(x)\phi(y).\end{aligned}$$

This shows that ϕ is a natural homomorphism.

Moreover, one can easily prove the subjective property of the mapping ϕ.

This implies that ϕ is epimorphism. Now consider

$$\begin{aligned}\ker(\phi) &= \left\{x \in G : \phi(x) = S_\rho e\right\}\\ &= \left\{x \in G : S_\rho x = S_\rho e\right\}.\end{aligned}$$

The application of Theorem (4.8) (ii) in the above relation yields that

$$\ker(\phi) = \left\{x \in G : x \in G_{S_\rho}\right\} = G_{S_\rho}.$$

Theorem 32: Let S_ρ be a ρ-AIFNSG of G. Then $G/S_\rho \cong G/G_{S_\rho}$.

Proof: Define a map $\phi : G/S_\rho \rightarrow G/G_{S_\rho}$by the rule $\phi\left(xS_\rho\right) = xG_{S_\rho}$, $x \in G$. Consider

$$xS_\rho = yS_\rho$$

The application of Theorem (4.8) (i) in the above relation gives us

$$xG_{S_\rho} = yG_{S_\rho}.$$

This Implies that

$$\phi\left(xS_\rho\right) = \phi\left(yS_\rho\right).$$

It is clear from the above discussion that ϕ is a well-defined mapping.

Next, let

$$\phi\left(xS_\rho\right) = \phi\left(yS_\rho\right).$$

This implies that $xG_{S_\rho} = yG_{S_\rho}$.

It follows that $x^{-1}y \in G_{S_\rho}$. The application of Theorem (4.8) (i) in the above relation gives us

$$xS_\rho = yS_\rho.$$

Thus, ϕ is injective.

Clearly, ϕ is surjective as for each $xG_{S_\rho} \in G/G_{S_\rho}$, there exists $xS_\rho \in G/S_\rho$ such that $\phi\left(xS_\rho\right) = xG_{S_\rho}$.

Moreover, ϕ is homomorphism as for each xS_ρ, $yS_\rho \in G/S_\rho$

$$\begin{aligned}\phi\left(xS_\rho yS_\rho\right) &= \phi\left(xyS_\rho\right) = xyG_{S_\rho}\\ &= xG_{S_\rho}yG_{S_\rho} = \phi\left(xS_\rho\right)\phi(yS_\rho).\end{aligned}$$

Consequently, $G/S_\rho \cong G/G_{S_\rho}$.

V. CONCLUSION

In this paper, we proposed the concept of ρ-AIFSG defined over ρ-AIFS and proved some of their algebraic properties. We extended the study of this ideology by defining the notions of ρ-anti intuitionistic fuzzy coset, ρ-AIFNSG and presented their various algebraic characteristics. We also defined the quotient group of a group induced by ρ-AIFNSG and established a group isomorphism between this particular quotient group and the quotient group of a group G related to its normal subgroup G_{S_ρ}.

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