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Modeling Concurrent Day-to-Day Departure Time and Route Choices With Multiple Micro-Preferences

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ABSTRACT Day-to-day traffic dynamics is to model the day-to-day evolution of travelers' travel choices, which helps to understand the aggregate traffic evolution of a non-equilibrium network and then develop scientific managements. This topic is attracting increasing interests. In the literature, day-to-day route and departure time choices are usually addressed separately; and the existing models commonly adhere to the rational behavior adjustment criterion (RBAC), but pay little attention to extra behavior preferences. In this article, we formulate the day-to-day departure time and route choices in a united model. Moreover, besides the RBAC, three microscopic behavior preferences (i.e., simplicity-seeking, proximity-prone and marginal cost preference) are suggested for modeling. The problem is formulated as a discrete-time dynamics named by the day-to-day departure time and route adjustment process (DTRAP). Basic properties of the model are verified theoretically; and numerical results indicate that the suggested micro-preferences have significant impact on traffic evolution. Thus, serious treatment and more in-depth research attention need to be laid on these micro-preferences for obtaining scientific understanding on the day-to-day network traffic evolution or for avoiding ineffective (or even wrong) traffic managements.

INDEX TERMS Day-to-day traffic dynamics, departure time choice, route choice, behavior preference.

I. INTRODUCTION

The urban transportation network is open, which can easily sustain disturbances, such as accidents, traffic controls, bad weather, and work zones. Disturbances may destroy system equilibrium, and then travelers are driven to change travel choices (including route, departure time, and mode). For saving travel cost, a fraction of travelers may swap off the current choices to cheaper ones, which then stimulate traffic evolution over time until a new system equilibrium is reached. Apparently, it is beyond the capacity of the well-established traffic equilibrium analysis theory [1]–[3] to handle such a non-equilibrium traffic evolution, and thus it is necessary and significant to establish a non-equilibrium modeling method, either for looking into the scientific law underlying day-to-day traffic evolution, or for developing effective managements or policies to improve the system efficiency more practically.

Fundamentally, given travel demand, it is the huge day-to-day individual choices (and shifting) in departure time, route and mode that drive the traffic evolution of a road network day after day. Thus, the essential problem points to travelers' day-to-day travel choices behavior modeling. This topic is initiated by Smith [4] who proposed a proportional-switch adjustment process (PAP) to examine the stability of user equilibrium, and has been attracting increasing research interests over the past decades. In the literature, the relevant models can be roughly categorized as deterministic ones and stochastic ones. For a deterministic model, the objective network state of the next day is uniquely determined, while for a stochastic (or say stochastic process) model it is probabilistically distributed. Due to limited efforts, this

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paper solely concerns with the former. For models of the latter, readers can refer to Refs. [5]–[7] and [8]. Despite falling in the deterministic category, the probabilistic models leading to the stochastic user equilibrium (such as [9]–[15] and [16], where [16] might be the unique work in the literature to discuss simultaneous day-to-day route-and-departure-time choices, but micro-preferences suggested and strengthened in this article are excluded) are beyond the scope of this article. In the literature, the deterministic models can be further divided into the path- and link-based ones. The former formulates traffic evolution by path flow, while the latter uses link flow. Considering that the path-based model can offer more fundamental capture in microscopic behaviors, this article still follows a path-based modeling method. Readers can refer to Yu *et al.* [17] and Guo *et al.* [18], for more details on link-based models, and subsequently we just review the path-based deterministic (or other closely related) models.

The literature of the path-based deterministic models is dominated by pure day-to-day route choice models. Besides the classical PAP [4], the other models still include network tatonnement process [19], projected dynamic system [20], [21], and BNN switch process [22]. Yang and Zhang [23] proved that these models are rational behavior adjustment processes [24]. For these models, PAP is the most intuitive and simplest, and has stimulated extensive extensions (e.g., [25]–[28]). Cho and Hwang [29] presented a stimulus-reaction model. Mounce and Carey [30] conducted rigorous analyses on the stability of several continuous-time rerouting dynamics with cost exponents. Zhang *et al.* [31] established a nonlinear pairwise adjusting process (NPAP) based on PAP, which can avoid traffic over-swapping. This model was later extended to the bounded rationality case [32], and algorithmic application was also explored [33]. Xiao *et al.* [34] reported a day-to-day rerouting model by treating road network as a spring system; this model was calculated and analyzed by the authors via virtual experiment data later [35]. Kumar and Peeta [36] might be the first to factor cost sensitivity into day-to-day rerouting modeling. Inspired from it, He and Peeta [37] later developed a link-based model with 1-step strategic thinking. Compared to rerouting behaviors, day-to-day departure time choice behaviors are less studied. It is empirically reported in [38] and [39] that, when sustaining congestion, travelers may change route and departure time, yet the latter choice is more stable. Iryo [40] and Iryo [41] explored the stability of day-to-day departure time choice at a single bottleneck in a PAP-based adjusting manner, and instability was observed. Xiao and Lo [42] explored the day-to-day departure time choice with social network effect. Guo *et al.* [43] reformulated the PAP model [4] and the network tatonnement process [19] to look into the day-to-day evolution of departure time choices with bounded rationality at a single bottleneck; and reported that the doubly dynamics models may lead to instability, even if their stable states are equivalent to the bounded rational user equilibrium. As a result, a pricing policy is designed in the paper to drive traffic to evolve to the system-optimal state. Zhu *et al.* [44] explored the evolution of day-to-day departure time choices at a bottleneck with stochastic capacity, where bounded rationality was included and the impact of different user information or knowledge regimes were examined. Jin [45] recently built a stable dynamics for day-to-day departure time choice at a single bottleneck, where model formulations for various road traffic flow models were presented and analyzed.

In existing path-based deterministic day-to-day dynamics models, the route and departure time choices are usually addressed separately. In addition, these models commonly adhere to the rational behavior adjustment criterion (RBAC) [24] stating that travelers can only swap onto less costly objective choices, but pay little attention to other behavior preferences, such as the simplicity-seeking, proximity-prone and marginal cost preference suggested in this article. The simplicity-seeking preference (SSP) states that people usually show explicit (rather than indifferent) preference to departure time, route and both in daily travel choices. The proximity-prone preference (PPP) states that users prefer to the more proximate one between two cost-equal departure time choices. The marginal cost preference (MCP) states that, besides anticipative cost-saving, people still concern for the expectant cost growth (i.e., marginal cost) resulted from switching decision. SSP had been empirically reported, and both SSP and PPP can be interpreted by psychology or behavioral economics. In spite of this, to the best of our knowledge, they have not been considered into day-to-day traffic dynamics modeling. MCP shows higher rationality in comparison to the former two preferences, and has long been a common behavior assumption in economics [46]. It has been included in some pure route switch literature [36], [37], but has not been factored in departure time switch models to date. To distinguish from RBAC, we refer to the three suggested behavior preferences as micro-preferences in remaining context, and detailed illustration on them will be stated in Section II.

The purpose of this study is to formulate the day-to-day traffic evolution on a road network created by concurrent departure time and route choices. For this purpose, a day-to-day traffic dynamics model, named by the departure time and route adjustment process (DTRAP), is established with more behavior consideration. Theoretical analyses are given to verify the basic properties of DTRAP, and numerical experiments are stated to explore the effect of the micro-preferences. Striking differences are observed from the traffic evolution trajectories of comparative scenarios. This implies that serious research attention deserves to be laid on the three suggested micro-preferences, either for obtaining scientific understanding on the day-to-day network traffic evolution or for avoiding ineffective (or even wrong) decision-making in traffic plans or managements.

The contribution of this article to the relevant literature is twofold. First, we formulate concurrent day-to-day departure time and route choices in a united model. In addition, besides the RBAC, three microscopic behavior preferences (i.e., SSP, PPP and MCP) are suggested and formulated in our model, and their effect is numerically analyzed.

The rest of this article is organized as follows. Section II conducts a detailed illustration for three micro-preferences. The dynamics model of DTRAP is elaborated in Section III, and necessary interpretation is also made there. Section IV conducts theoretical analyses for several key properties of DTRAP. Numerical analyses are performed in Section V to explore the effect of micro-preferences on traffic evolution. Section VI concludes the whole study, and suggests some valuable works in the future.

II. MICROSCOPIC BEHAVIOR PREFERENCES

In existing day-to-day traffic dynamics, travelers are assumed to adjust travel choices according to RBAC, i.e., travelers can only adjust to less costly options. This criterion is intuitive and reasonable, but might not be incomplete. For instance, when two or more alternatives with equal costs are available, travelers usually still show explicit preference (rather than indifference) to them; and this may attribute to the effect of other or more in-depth behavioral preferences. Subsequently, we illustrate the three suggested micro-preferences (i.e., SSP, PPP and MCP) that have been rarely factored into day-to-day traffic dynamics modeling. In order to make the illustration more rigorous, when we discuss a micro-preference, only the referred attribute differs among the candidate choices, while the remaining attributes are identical.

Definition 1 (Simplicity-Seeking Preference, SSP): Among route switch, departure time switch and concurrent switch, travelers are most likely to shift the departure time, after it the route, and finally concurrence.

This behavior preference had been empirically reported in the literature [38], [39]. It was found that, when encountering congestion, travelers are more stable to adjust departure time than route. This preference can be interpreted by human's risk aversion, familiarity bias, and the principle of least effort in psychology [47]–[50]. Among the three choices in Definition 1, departure time switch may consume the least effort and yield the least change to circumstance, which may result in the least expectable unfamiliarity, and thus generate the highest psychological safety (or the lowest risk). Hence, departure time switch is the most preferable. Along this line, the least preferable one will be the concurrent switch, since it consumes the most psychological effort and may lead to the largest unfamiliarity (and psychological risk). In the literature regarding travel behavior, familiarity is sometimes depicted as similarity [51].

Definition 2 (Proximity-Prone Preference, PPP): Given two different candidate departure time instants with identical other attributes, travelers are more likely to choose the one that is more temporally proximate to the choice.

Human's risk averse behavior, familiarity bias, and the principle of least effort found in psychology [47]–[50] can also offer explanation for this behavioral preference. More proximate candidate requires less psychological efforts, and may yield less change to future travel circumstance. Then,

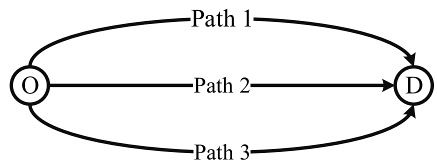

FIGURE 1. An illustrative network.

higher familiarity is expectable, and thus psychological risk will be higher. Therefore, a more proximate time instant is more preferable.

Definition 3 (Marginal Cost Preference, MCP): Given two candidate choices (departure time and routes) with identical traversed time and other attributes, travelers are more likely to swap onto the one with less marginal cost (i.e., expectant cost growth resulted from a switching behavior).

To illustrate, we perform an example on Fig. 1 which has a single OD-pair connected by three parallel paths sequentially numbered as Paths 1-3 from the top down. Suppose the traversed time for Paths 1-3 is sequentially 20, 30 and 20, and their respective marginal costs are 4, 5 and 6. Obviously, in this scenario traffic can only reroute from Path 2 to Paths 1 and 3, and Path 1 (rather than Path 3) is obviously a better option for travelers on Path 2. This natural conclusion cannot be obtained from RBAC alone, implying that MCP may offer more complete description for travelers' travel choices based on the acknowledged RBAC.

Different from the former two micro-preferences, MCP has been factored into day-to-day rerouting modeling [36], [37], where marginal cost is captured by the partial derivative of the path travel time with respect to path flow. However, it has not been factored into day-to-day departure time choice modeling. For MCP, there is a common marginal decision rule existed in economics stating that rational people think at the margin [46]. From this rule, people concern with both marginal benefit and marginal cost. To take it into day-to-day traffic dynamics modeling, it means that, given identical marginal benefits (i.e., to expectable travel cost saving) for two candidate choices, the one with less marginal travel cost is expected to cause less cost growth and thus would be more preferable by travelers.

In spite of the behavior interpretation and existing relevant studies, it is necessary to note that there still lack timely and direct evidences validating the above three micro-preferences. Thus, they are still hypothetical suggestion; and special and rigorous empirical studies are still necessary to be conducted. However, due to the limited scope of this study, we leave these works to the future study. This article just focuses on theoretically modeling them and numerically exploring their effect on day-to-day traffic evolution.

III. DYNAMICS MODEL OF DTRAP

This section elaborates on the dynamic modeling of DTRAP. Here, DTRAP is built based on NPAP [31] with an extended model formulation of revision protocol to formulate three micro-preferences. NPAP is revised from the classical PAP [4]. It inherits the state evolution equation of PAP, but adopts a nonlinear revision protocol to avoid traffic over-adjustment that existed in PAP.

Four assumptions are made for DTRAP: i) travelers hold SSP, PPP and MCP in day-to-day travel choices; ii) travelers can only adjust to the candidates with expectable cost drop, and traffic adjustment will not stop unless such candidates disappear; iii) the total travel demand for each day is fixed and known; iv) travelers adjust decisions (or choices) based on their travel experience information of the previous day, and such information is perfectly available.

Consider a fully connected road network with a set A of links indexed by a, and a set W of OD pairs indexed by w. In our research horizon, $\mathrm{T}+1$ days (begin from 0, end with T, and index a day by t) are considered, and each day comprises I time intervals (begin from 1, end with I, and index a time interval by i or j). Each OD-pair $w \in W$ is associated with a fixed demand d_w for each day, and a set K_w of effective paths indexed by r or p.

DTRAP is governed by traffic state evolution equation (1). Due to the discrete nature of day-to-day traffic evolution, only the discrete-time model is discussed here.

$$f_{wr}^{ti} - f_{wr}^{(t-1)i} = \sum_{p\in K_w}\sum_{j\le I}\left(f_{wp}^{(t-1)j}\rho_{wpr}^{tji} - f_{wr}^{(t-1)i}\rho_{wrp}^{tij}\right), \quad \forall r\in K_w,\ w\in W,\ i\in[1,I],\ t\in[1,T], \tag{1}$$

where

$$\rho_{wrp}^{tij} = \max\left(0, \frac{1-\exp\left(-\theta_{wrp}^{tij}\left(C_{wr}^{(t-1)i} - C_{wp}^{(t-1)j}\right)\right)}{\left|\mathrm{RI}_{wr}^{ti}\right| + \varepsilon}\right) \tag{2}$$

with $\mathrm{RI}_{wr}^{ti} = \left\{(p,j)|C_{wr}^{(t-1)i} - C_{wp}^{(t-1)j} > 0, p\in K_w, j\le I\right\}$.

In above equations, f_{rw}^{ti} is the state variable, denoting the traffic flow leaving home at time interval $i \in [1, I]$ of day $t \in [0, T]$ via path r between OD-pair w. ρ_{wrp}^{tij} is revision protocol which formulates the ratio of traffic flow (leaving at time instant i of previous day via route r within OD-pair w) that swaps onto route p and departure time interval j on day t. From a microscopic perspective, it can also be understood as the probability of a traveler who swaps off a previous choice to another today. C_{wr}^{ti} is the traversed cost leaving at time interval i on day t via route r between OD-pair w. RI_{wr}^{ti} is the set of candidate choices (shown as directional departure time and route pairs) whose traversed cost at previous day is strictly less than that of pair (r, i), and $\left|\mathrm{RI}_{wr}^{ti}\right|$ denotes the set cardinality. θ_{wrp}^{tij} is strictly positive, which denotes the travelers' reaction sensitivity associated with OD-pair w and day t, as well as the previous choice (r, i) and feasible candidate choice (p, j) today. In NPAP, θ_{wrp}^{tij} is defined as a constant. However, in DTRAP, it depends on travelers' micro-preferences and varies dynamically against traffic states. Its formulation is essential to this study, which will be illustrated in following context. ε is a small positive number to keep the denominator non-zero. $\exp(\cdot)$ is the exponential function of natural number.

Equation (1) gives the state evolution equation of DTRAP. The left side reflects the change of departure flow today over the previous day. On the right side, two components respectively express the total traffic flow that will adjust onto and swap off a choice today. This equation is borrowed from PAP. It is reported that this evolution equation can offer an excellent behavior approximation to the Markov evolution games, and the majority of the existing population evolution dynamics can be rewritten by it with a suitable revision protocol [52], [53]. In equation (2), function $1-\exp(\cdot)$ serves for estimating the monopolized probability that one swaps off a previous choice to another. Given an increasingly positive travel cost deviation, this probability grows from 0 to 1 in a decreasing speed, which is logical and intuitive. If candidate choices are not unique, such probability will be divided over them. In order to ensure coherent relative ratios between the estimated probabilities, a plausible method is to divide the monopolized probability by $\left|\mathrm{RI}_{wr}^{ti}\right|$. Here, it is replaced by $\left|\mathrm{RI}_{wr}^{ti}\right|+\varepsilon$ for simplifying the structure of expression, making it needless to add an extra expression for $\mathrm{RI}_{wr}^{ti} = \emptyset$.

For the state evolution equation (1), an initial path traffic flow pattern, which should meet the following feasibility condition, needs to be fed.

$$\sum_{i\le I}\sum_{r\in K_w} f_{wr}^{0i} = d_w, \quad \forall w\in W;\ f_{wr}^{0i}\ge 0 \quad \forall r\in K_w,\ w\in W,\ i\le I \tag{3}$$

Two expressions in (3) describe the constraints of demand conservation and non-negative path flow, respectively.

Compared with PAP and NPAP, an explicit revision made in (1) is adding departure time choice upon the existing route choice, gifting DTRAP for the capacity of addressing doubly day-to-day departure time and route choices. In addition to this, another essential revision is associated with the reaction sensitivity. In PAP and NPAP, it is constant. In DTRAP, three micro-preferences are included into reaction sensitivity, making it vary against dynamic traffic condition. Here θ_{wrp}^{tij} is formulated as follows.

$$\theta_{wrp}^{tij} = \frac{\mathrm{SimPrefer}_{wrp}^{ij}}{\left(s_1\cdot\mathrm{Gap}_{ij}+1\right)\left(s_2\cdot\mathrm{CostSen}_{wp}^{(t-1)j}+1\right)} \tag{4}$$

where

$$\mathrm{SimPrefer}_{wrp}^{ij} = \begin{cases}\alpha_1, & \text{if } r=p,\ i\ne j\\ \alpha_2, & \text{if } r\ne p,\ i=j\\ \alpha_3, & \text{if } r\ne p,\ i\ne j\end{cases} \tag{5}$$

$$\mathrm{Gap}_{ij} = |i-j|^{\frac{2}{3}} \tag{6}$$

$$\mathrm{CostSen}_{wp}^{tj} = \frac{\partial C_{wp}^{(t-1)j}}{\partial f_{wp}^{(t-1)j}} \tag{7}$$

In above equations, $\mathrm{SimPrefer}_{wrp}^{ij}$ is a positive constant associated with SSP, which varies against the current and the objective choice. From (5), it equals to α_1 if only shifting departure time, to α_2 if only adjusting route, and to α_3 if switching both simultaneously. Here $\alpha_1 > \alpha_2 > \alpha_3$ is set to capture SSP. Gap_{ij} is associated with PPP, which measures the distance between departure time intervals i and j. s_1 is a non-negative constant to reflect the extent of PPP. A larger s_1 implies stronger PPP, and $s_1 = 0$ means assuming away PPP. Here, Gap_{ij} is formulated by the power function in (6) to capture a natural decreasing growth rate with time distance. $CostSen_{wp}^{tj}$ is the expectant cost sensitivity of the objective choice, which is related to MCP. s_2 in (4) is a non-negative constant to reflect the extent of MCP. A larger s_2 implies stronger MCP; and $s_2 = 0$ implies ruling out the MCP. Since a larger cost sensitivity implies lower attraction, a natural formulation of it is (7), i.e., the partial derivative of the travel cost for objective choice with respect to traffic flow. Then, a larger CostSen_{wp}^{tj} implies higher marginal cost, which leads to smaller reaction sensitivity from (4), and lower swapping possibility from (2) in further. This is exactly in accord with the original intention for MCP.

Equations (5)-(7) demonstrate that the reaction sensitivity today depends on the network state of previous day, and this conclusion applies to the revision protocol from (2), and then the traffic evolution from (1). This property is in accord with the fourth assumption made in previous for the model. In addition, the above illustration means that, by reformulating the reaction sensitivity in NPAP [31] as (4), SSP, PPP and MCP are compatibly factored into present model.

IV. THEORETICAL ANALYSES

In this section, we theoretically examine several important properties for the current DTRAP model.

Proposition 1 (No Traffic Over-Adjustment): For DTRAP, the total flow that swaps off the current choice cannot exceed its initial flow; mathematically, it means that

$$0 \leq \sum_{(p,j)\in \text{RI}_{wr}^{ti}} \rho_{wrp}^{tij} \leq 1, \quad \forall r \in K_w,\ w \in W,\ i \leq I,\ t \leq T \tag{8}$$

Proof: Since $1 - \exp\left(-\theta_{wrp}^{tij}\left(C_{wr}^{(t-1)i} - C_{wp}^{(t-1)j}\right)\right) \leq 1$ holds for all $(p,j) \in \text{RI}_{wr}^{ti}$, then it follows from (2) that

$$\begin{aligned}\sum_{(p,j)\in \text{RI}_{wr}^{ti}} \rho_{wrp}^{tij} &= \sum_{(p,j)} \frac{1}{\left|\text{RI}_{wr}^{ti}\right| + \varepsilon} \max\Big(0, 1 \\ &\qquad - \exp\left(-\theta_{wrp}^{tij}\left(C_{wr}^{(t-1)i} - C_{wp}^{(t-1)j}\right)\right)\Big) \\ &\leq \sum_{(p,j)} \frac{1}{\left|\text{RI}_{wr}^{ti}\right| + \varepsilon} \times 1 = \frac{\left|\text{RI}_{wr}^{ti}\right|}{\left|\text{RI}_{wr}^{ti}\right| + \varepsilon} \leq 1\end{aligned}$$

From (2), ρ_{wrp}^{tij} must be non-negative. Then, $\sum_{(p,j)} \rho_{wrp}^{tij} \geq 0$ holds in general. Hence, Proposition 1 holds.

Proposition 1 is a necessary property for a discrete-time day-to-day travel choice dynamics model. Below we present another critical property.

Proposition 2 (Solution Set Invariance): For DTRAP, if the initial flow pattern is feasible, then the resultant flow patterns are still feasible.

Proof: Mathematically, Proposition 2 means that (3) implies (9) for DTRAP.

$$\begin{aligned}&\sum_{i\leq I}\sum_{r\in K_w} f_{wr}^{ti} = d_w, \quad \forall w \in W; \\ &\qquad f_{wr}^{ti} \geq 0, \quad \forall r \in K_w,\ w \in W, t \geq 1,\ i \leq I\end{aligned} \tag{9}$$

Along this line, proof for Proposition 2 is divided into two parts, including demand conservation and traffic flow non-negativity.

A. TRAFFIC DEMAND CONSERVATION

Since

$$\begin{aligned}&\sum\nolimits_r \sum\nolimits_i \sum\nolimits_p \sum\nolimits_j f_{wp}^{(t-1)j} \rho_{wpr}^{tji} \\ &\quad = \sum\nolimits_p \sum\nolimits_j \sum\nolimits_r \sum\nolimits_i f_{wr}^{(t-1)i} \rho_{wrp}^{tij} \\ &\quad = \sum\nolimits_r \sum\nolimits_i \sum\nolimits_p \sum\nolimits_j f_{wr}^{(t-1)i} \rho_{wrp}^{tij}\end{aligned}$$

then it follows from (1) that

$$\begin{aligned}&\sum_r\sum_i f_{wr}^{ti} \\ &= \sum_r\sum_i f_{wr}^{(t-1)i} \\ &\quad + \sum_r\sum_i\left(\sum_p\sum_j f_{wp}^{(t-1)j}\rho_{wpr}^{tji} - f_{wr}^{(t-1)i}\sum_p\sum_j \rho_{wrp}^{tij}\right) \\ &= \sum_r\sum_i f_{wr}^{(t-1)i} + 0 = \cdots = \sum_r\sum_i f_{wr}^{0i} = d_w\end{aligned}$$

B. TRAFFIC FLOW NON-NEGATIVITY

According to (1) and (2), as well as Proposition 1, we have

$$\begin{aligned}f_{wr}^{ti} &= f_{wr}^{(t-1)i} + \sum_{p\in K_w}\sum_{j\leq I}\left(f_{wp}^{(t-1)j}\rho_{wpr}^{tji} - f_{wr}^{(t-1)i}\rho_{wrp}^{tij}\right) \\ &= f_{wr}^{(t-1)i} + \sum_{p\in K_w}\sum_{j\leq I} f_{wp}^{(t-1)j}\rho_{wpr}^{tji} - f_{wr}^{(t-1)i}\sum_{(p,j)\in \text{RI}_{wr}^{ti}} \rho_{wrp}^{tij} \\ &\geq f_{wr}^{(t-1)i} + \sum_{p\in K_w}\sum_{j\leq I} f_{wp}^{(t-1)j}\rho_{wpr}^{tji} - f_{wr}^{(t-1)i} \\ &\geq \sum_{p\in K_w}\sum_{j\leq I} f_{wp}^{(t-1)j}\rho_{wpr}^{tji}\end{aligned}$$

Given $f_{wr}^{0i} \geq 0, \forall r \in K_w, w \in W, i \leq I$, it is easy to conclude that $\sum_p \sum_j f_{wp}^{(t-1)j} \rho_{wpr}^{tji} \geq 0$ by recurrence.

Solution set invariance is also a necessary property for any behavior-reasonable day-to-day traffic dynamics. In fact, it is also important to extend a dynamics model into an algorithm for solving traffic equilibrium problems, since the developed algorithm guarantees solution feasibility and thus a trial and error process (to avoid infeasibility) is unnecessary. Next, we examine the stable state of DTRAP.

Definition 1 (Stable Network Flow Pattern): Stable network flow pattern is a group of network path flow states; and if such a flow pattern is fed to DTRAP, no departure time or route adjustment will be triggered. Let $\mathbf{f}^t = \left(f_{wr}^{ti}\right)_{w,r,i}$ denote a stable network flow pattern of DTRAP, then we have $\mathbf{f}^{t+1} = \mathbf{f}^t$ hold in general.

Proposition 3: For DTRAP, its stable network flow pattern is equivalent to traffic equilibrium.

Proof: According to Definition 1 and the definition of traffic equilibrium with concurrent route and departure time choice [1], and let π_w be the minimum traversed cost on day t for OD-pair w, Proposition 3 means that $\mathbf{f}^{t+1} = \mathbf{f}^t$ implies that $C_{wr}^{ti} = \pi_w$ if $f_{wr}^{ti} > 0$, and $C_{wr}^{ti} \geq \pi_w$ if $f_{wr}^{ti} = 0$ for any $w \in W, r \in K_w, i \in [0, I]$; and vice versa. Below, we give the proofs of sufficiency and necessity separately.

C. SUFFICIENCY

We prove sufficiency by contradiction. Given a stable path flow pattern $\mathbf{f}^t$, suppose that it is not a traffic equilibrium. Then there exists at least a departure-time-and-route pair which has positive flow and non-minimum travel cost. Let pair (r, i) have positive flow and the largest non-minimum travel cost. Then, it follows from (1) and (2) that flow swapping off pair (r, i) will be positive and no traffic can swap onto it. Thus, it gives rise to $f_{wr}^{(t+1)i} < f_{wr}^{ti}$, which contradicts $\mathbf{f}^{t+1} = \mathbf{f}^t$. Therefore, the sufficiency holds.

D. NECESSITY

Given that $\mathbf{f}^t$ is a traffic equilibrium, it means that $C_{wr}^{ti} = \pi_w$ if $f_{wr}^{ti} > 0$, and $C_{wr}^{ti} \geq \pi_w$ if $f_{wr}^{ti} = 0$. Then, it follows from (3) that $\rho_{wrp}^{(t+1)ij} = 0, \forall p \in K_w, j \leq I$ if $f_{wr}^{ti} > 0$; and $\sum_p \sum_j f_{wr}^{ti} \rho_{wrp}^{(t+1)ij} = 0$ if $f_{wr}^{ti} = 0$. Then it follows from (1) that $\mathbf{f}^{t+1} = \mathbf{f}^t$, i.e., $\mathbf{f}^t$ is a stable flow pattern.

Below we examine if DTRAP follows the RBAC.

Proposition 4: DTRAP is a rational behavior adjustment process.

Proof: According to the definition of RBAC [24], we need to prove that $\sum_w \sum_r \sum_i C_{wr}^{ti} \left(f_{wr}^{(t+1)i} - f_{wr}^{ti}\right) \leq 0$ holds generally; and network reaches traffic equilibrium if it equals to zero. The whole proof is stated as follows.

$$
\begin{aligned}
&\sum_{w\in W}\sum_{r\in K_w}\sum_{i\leq I} C_{wr}^{ti}\left(f_{wr}^{(t+1)i} - f_{wr}^{ti}\right) \\
&= \sum_{w\in W}\sum_{r\in K_w}\sum_{i\leq I} C_{wr}^{ti} \sum_{p\in K_w}\sum_{j\leq I}\left(f_{wp}^{tj}\rho_{wpr}^{(t+1)ji} - f_{wr}^{ti}\rho_{wrp}^{(t+1)ij}\right) \\
&= \sum_{w}\sum_{r}\sum_{i}\sum_{p}\sum_{j} C_{wr}^{ti} f_{wp}^{tj}\rho_{wpr}^{(t+1)ji} \\
&\quad - \sum_{w}\sum_{r}\sum_{i}\sum_{p}\sum_{j} C_{wr}^{ti} f_{wr}^{ti}\rho_{wrp}^{(t+1)ij} \\
&= \sum_{w}\sum_{r}\sum_{i}\sum_{p}\sum_{j} C_{wr}^{ti} f_{wp}^{tj}\rho_{wpr}^{(t+1)ji} \\
&\quad - \sum_{w}\sum_{p}\sum_{j}\sum_{r}\sum_{i} C_{wp}^{tj} f_{wp}^{tj}\rho_{wpr}^{(t+1)ji}
\end{aligned}
$$

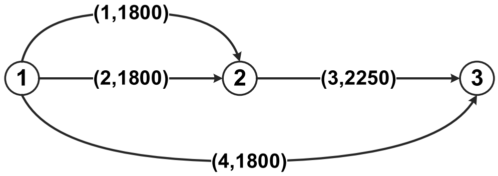

FIGURE 2. **A commuting network. Node 1 is home, and Node 3 is the company.**

$$
\begin{aligned}
&= \sum_{w}\sum_{r}\sum_{i}\sum_{p}\sum_{j} C_{wr}^{ti} f_{wp}^{tj}\rho_{wpr}^{(t+1)ji} \\
&\quad - \sum_{w}\sum_{r}\sum_{i}\sum_{p}\sum_{j} C_{wp}^{tj} f_{wp}^{tj}\rho_{wpr}^{(t+1)ji} \\
&= \sum_{w\in W}\sum_{r\in K_w}\sum_{i\leq I}\sum_{p\in K_w}\sum_{j\leq I}\left(C_{wr}^{ti} - C_{wp}^{tj}\right) f_{wp}^{tj}\rho_{wpr}^{(t+1)ji}
\end{aligned}
$$

From (2), $\rho_{wrp}^{(t+1)ij} = 0$ holds if and only if (iff) $C_{wr}^{ti} \leq C_{wp}^{tj}$; and $\rho_{wrp}^{(t+1)ij} > 0$ holds iff $C_{wr}^{ti} > C_{wp}^{tj}$. Then, it generally gives rise to $\rho_{wrp}^{(t+1)ij}\left(C_{wr}^{ti} - C_{wp}^{tj}\right) \geq 0$. Recall Proposition 2, and then $(C_{wr}^{ti} - C_{wp}^{tj}) f_{wp}^{tj}\rho_{wpr}^{(t+1)ji} \leq 0$ holds consistently. Thus, we have $\sum_w \sum_r \sum_i C_{wr}^{ti}\left(f_{wr}^{(t+1)i} - f_{wr}^{ti}\right) \leq 0$; and it equals to zero iff $(C_{wr}^{ti} - C_{wp}^{tj}) f_{wp}^{tj}\rho_{wpr}^{(t+1)ji} \equiv 0$. As a result, it is easy to prove by contradiction that $(C_{wr}^{ti} - C_{wp}^{tj}) f_{wp}^{tj}\rho_{wpr}^{(t+1)ji} \equiv 0$ holds iff $\mathbf{f}^t$ is a traffic equilibrium.

Proposition 4 demonstrates that DTRAP still follows the RBAC, which means that RBAC and three suggested micro-preferences (i.e., SSP, PPP and MCP) have been compatibly formulated in DTRAP.

The above four propositions demonstrate that DTRAP is behaviorally reasonable and more complete.

V. NUMERICAL ANALYSES

This section numerically analyzes the characteristics which are not covered in previous theoretical analyses, including the impact of three micro-preferences on day-to-day traffic evolution, and the stability.

A. EXPERIMENTAL DESIGNATION

Consider a simple day-to-day commuting road network (see Figure 2) with a single OD pair, four roads (or links) and three paths (Path 1 includes road sequences; Path 2 includes road sequences; and Path 3 includes road 4). In Figure 2, two bracketed numbers in each road are sequentially its label and capacity (unit: vehicles). For simplicity and without loss of generality, we consider one hour departure duration for each day, and divide it into 12 intervals. Therefore, we have a total of 36 travel choices (i.e., departure time interval and route pairs) for the current day-to-day commuting network. Travel demand within an hour of each day equals to 4500 vehicles, which is equally distributed among 36 travel choices (sharing 125 vehicles for each) initially.

For this commuting network, the cell transmission model (CTM) is applied to describe the within-day traf-

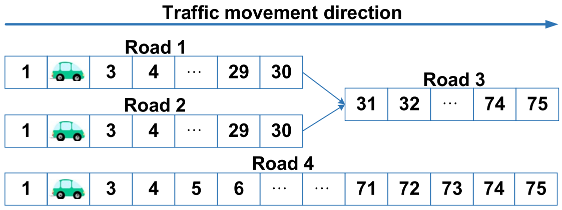

FIGURE 3. The CTM network formulation of Figure 2.

fic dynamics, and then to obtain the travel time of each departure choice. CTM is proposed by Daganzo [54], which is a discretized approximation to the kinematic wave model [55], [56]. CTM is capable of capturing the non-continuous variations of network flow pattern, but the latter cannot. What's more, CTM is computationally economic, and can offer desirable characterization with respect to within-day traffic dynamics (such as shock waves, queue formation and abreaction, and interactions between roads). For these reasons, it has been frequently applied in dynamical traffic assignment to obtain more precise travel times (e.g., [57], [58]). Since within-day traffic dynamics are not the essence of this study, the basic CTM is used here, and readers are referred to Refs. [54] and [59] more details on CTM.

To apply CTM, the primary work is to convert Figure 2 to a grid-based network (see Figure 3). Each grid in Figure 3 represents a cell, and the imbedded number indicates its label. Here, all paths are assumed to be of the same physical length, and comprise 75 cells with length of 40m for each. From Figure 2, the summation capacity of Roads 1 and 2 exceeds that of Road 3. Then, cell 31 in Road 3 becomes a bottleneck, and congestion or queue will form there if the total arrivals exceed the capacity. As to the flow assignment at the bottleneck, we allocate a probability of 0.6 for a car on Road 1 to enter Road 3, and 0.65 to enter the latter. The free flow speed and congestion wave velocity across the network are set to be 60 km/h and 19.2 km/h, respectively. The blocking density is set to be 150 vehicles/km. In addition, travelers departing at the same time interval are assumed to share the same traversed time.

Due to including both departure time and route choices, it is improper to measure the commuting travel cost by travel time solely. In the literature, a classical and widely used measure is the scheduling disutility model [60], which is mathematically formulated as follows.

$$C_r^{tn} = \kappa \cdot \tau_r^{tn} + \beta \cdot \max\left\{0, \tau_* - n \cdot \Delta - \tau_r^{tn}\right\} + \gamma \cdot \max\left\{0, n \cdot \Delta + \tau_r^{tn} - \tau_*\right\} \quad (10)$$

In (10), C_r^{tn} is the scheduling disutility (or say travel cost) resulted from leaving home at time interval n via path r on day t; τ_r^{tn} is the resultant traversed time duration; τ_* (set as 10Δ here) is the preferred arrival time interval at company; Δ is the time duration of each departure time interval (here it is 5 min); κ, β and γ are positive constants ($\kappa = 6.5$, $\beta = 0.5$, and $\gamma = 1$), which denote the value of travel time, the penalty factors related to early arrivals and late arrivals, respectively. To facilitate presentation, let $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ and $s = (s_1, s_2)$, of which their values change against the numerical experiments to be stated.

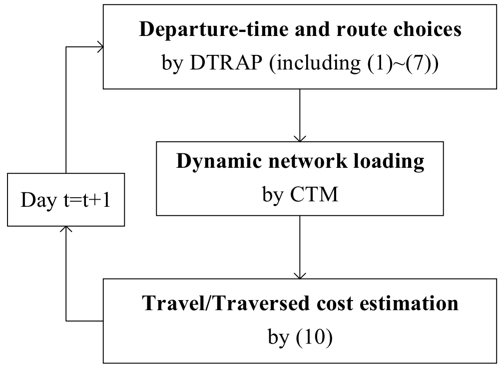

FIGURE 4. Logical structure for numerical simulation of DTRAP.

Based on CTM and the travel cost estimated by (10), the logic structure and flow for present numerical simulation of DTRAP is presented in Figure 4. As illustrated in Figure 4, in the simulation, CTM serves as the within-day dynamic network loading model to estimate the travel times of each departure time and route choice pattern created by DTRAP. Then, the resultant travel times are fed to (10) to compute the travel cost pattern which are treated as experiences by travelers for estimating reaction sensitivities, and revision protocol of the next day.

B. MICRO-PREFERENCE ANALYSIS

This subsection performs a set of numerical sensitivity analyses to explore the effect of three suggested micro-preferences (i.e., SSP, PPP and MCP) on traffic evolution. Here we set $\alpha = (0.03, 0.03, 0.03)$ to rule out SSP, and $\alpha = (0.03, 0.015, 0.005)$ is set to include SSP. Set $s_1 = 0$ to rule out PPP, and set $s_1 = 0.1$ to include PPP. Set $s_2 = 0$ to rule out MCP, and set $s_2 = 14$ to include MCP. In addition, we still set $\alpha = (0.03, 0.015, 0)$ to rule out concurrent departure time and route adjustment; and $\alpha = (0.03, 0.03, 0)$ indicates the inclusion of SSP accordingly. Due to limit space, Figure 5 selectively displays day-to-day evolution of departure flow for four choices under changing parameter combination.

Non-trivial differences are observed from the departure flow evolution trajectories (shown in different subfigures) under varying micro-preference parameters in Figure 5. By comparing the corresponding subfigures between the left and right columns, we observe that ruling out doubly switch may significantly affect the traffic evolution, e.g., impeding system convergence. This may ascribe to the sharp drop in candidate choices after ruling out the doubly switch, which can render congregate traffic switch more likely and then slower convergence or cause (or intensify) oscillations. By comparing the subfigures of each column vertically, solid differences still exist. Generally, it shows that the inclusion

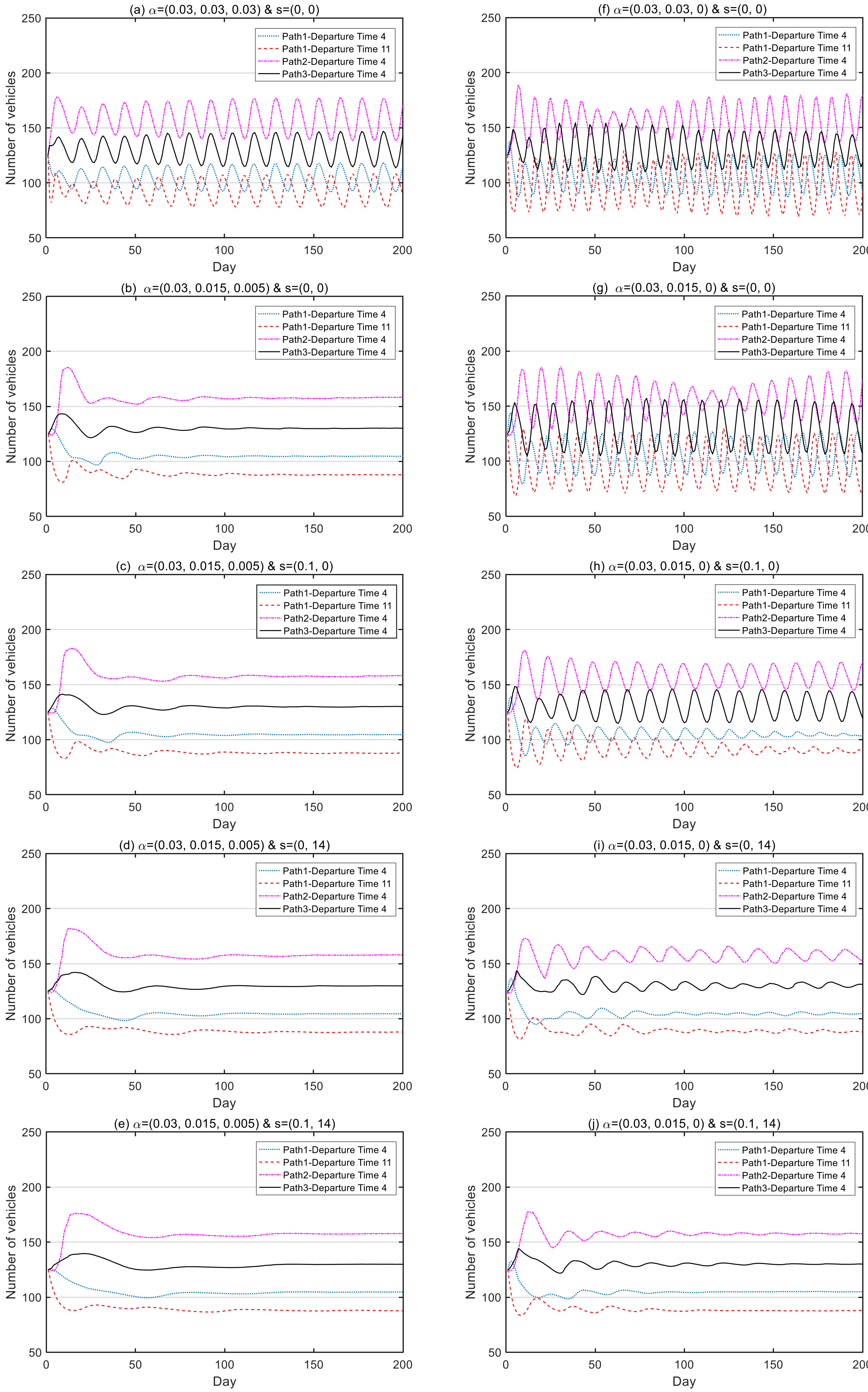

FIGURE 5. Evolution of selective departure flow under variant micro-preferences.

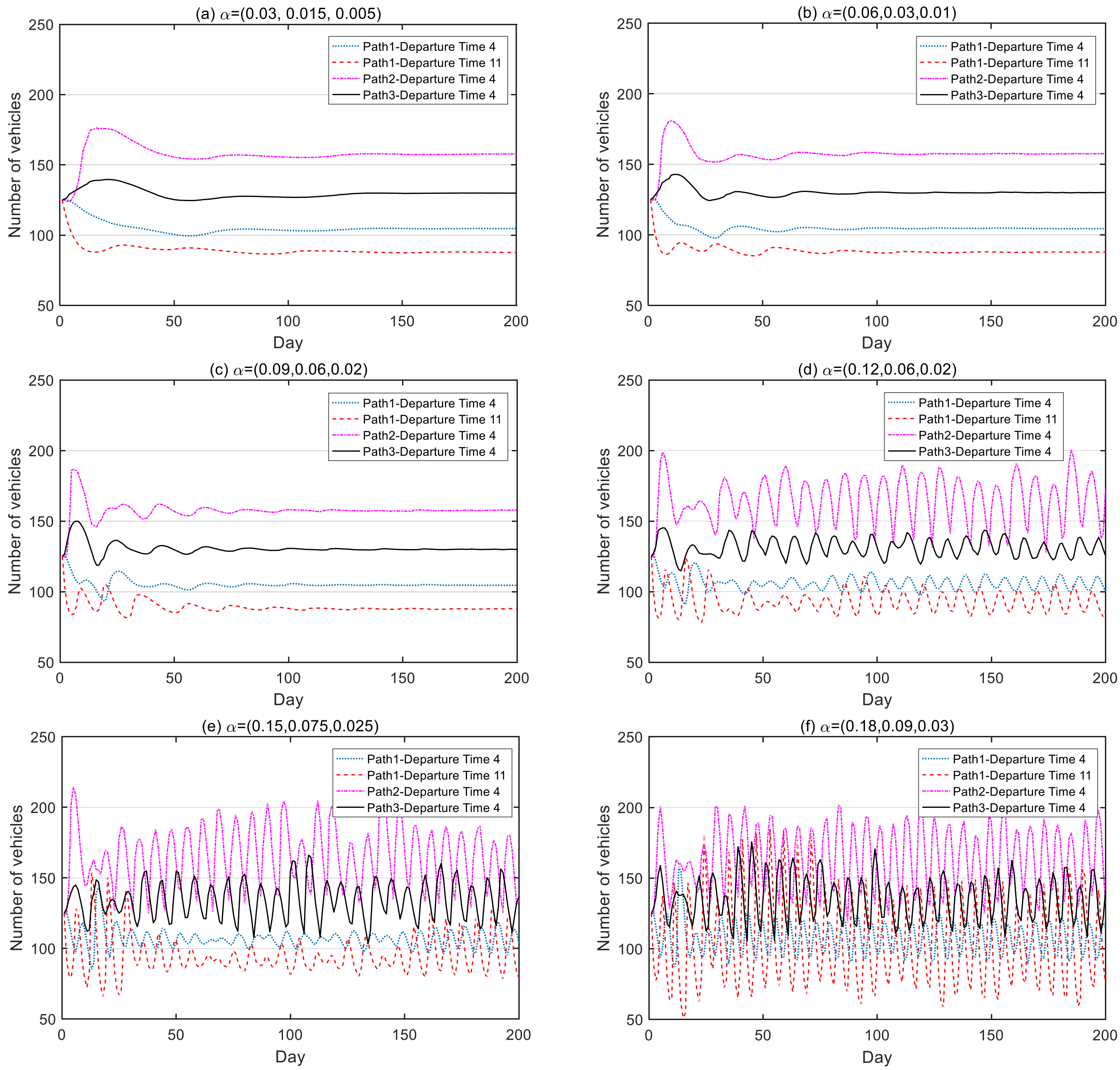

FIGURE 6. **Evolution of selective departure flow under different SSP extents.**

of PPP and MCP may help to improve the convergence of DTRAP. These differences collectively show the significant impact of three micro-preferences on traffic evolution. It implies that if they exist but are assumed away, scientific and full understanding on the day-to-day traffic evolution would be questionable, and thus the resultant managements would face the risk of inefficacy or even taking wrong effect. Consequently, serious treatment and more in-depth research attention deserve to be laid on the three suggested (or other more) micro-preferences.

Below we focus on analyzing SSP. Figure 6 displays partial traffic evolution trajectories under $s = (0.1, 14)$ with increasing α. From Figure 6, as the growth of α, traffic evolutions fall into oscillations at $\alpha = (0.2, 0.06, 0.02)$, and oscillations intensify after that. This is because a larger α means a higher reaction sensitivity, which may intensify traffic adjustment and then raise oscillation in probability. Similar reasons still interpret why including PPP and MCP helps to improve the convergence of DTRAP (see Figure 5), since they contribute to reducing the reaction sensitivity. Actually, for MCP, it may also attribute to the inclusion of marginal cost, which makes travelers less cost-sensitive and then contributes to the decrease of Ping-Pong switch, as well as oscillations.

VI. CONCLUSION AND FUTURE RESEARCH

This article models the evolution of simultaneous day-to-day departure time and route choices on a non-equilibrium road network. Three micro-preferences (i.e. SSP, PPP and MCP) are suggested here for enriching the behavior consideration in problem modeling. To develop the model (i.e., DTRAP), day-to-day route and departure time choices as well as the three micro-preferences are compatibly formulated in the framework of NPAP with incremental choice dimension and reformulated reaction sensitivity. The current DTRAP is proved to be a rational behavior adjustment process, and is able to consistently promise the feasibility of the resultant solutions. Numerical analyses suggest that the stability of DTRAP depends heavily on the reaction sensitivity; and including PPP and MCP helps to improve the stability, but a growing SSP extent may worsen it. In addition, the micro-preferences show significant impact on day-to-day traffic evolution, which implies that if they exist but are ruled out, scientific and full understanding on the day-to-day network traffic evolution would be questionable, and the resultant traffic managements would be ineffective or even wrongly effective. Accordingly, serious treatment and more in-depth research attention deserve to be laid on them.

This article presents theoretical modeling and numerical analyses with respect to simultaneous day-to-day departure time and route choice behaviors with three extra suggested micro-preferences (including SSP, PPP and MCP). Many problems are worthy of being studied in further. Primary, specific and direct empirical studies (e.g. stated preference and revealed preference) need to be conducted for either validating the suggested micro-preferences or calibrating their parameters in the model. In addition, mode choice as well as bounded rationality can be further included based on the current DTRAP model.

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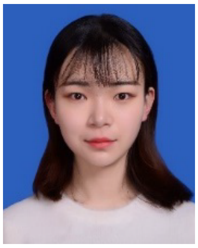

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