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New Acceptance Sampling Plans Based on Truncated Life Tests for Akash Distribution With an Application to Electric Carts Data

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ABSTRACT In this paper, we develop new acceptance sampling plans based on truncated life tests for the Akash distribution. With various values of the Akash distribution parameter, the minimum sample sizes required to assert the specified mean life are obtained, also the operating characteristic function values and producer's risk of the proposed sampling plan are presented. The results are illustrated by different examples for different values of the sampling plans. A real data of 20 small electric carts is used to illustrate the power of the new sampling plans. The results revealed that the suggested acceptance sampling plan is useful for researchers and engineering in producing lots.

INDEX TERMS Acceptance sampling plan, truncated life test, operating characteristic function, producer's risk, Akash distribution, consumer's risk.

I. INTRODUCTION

In statistical process control, even used most effective statistical techniques, defective or not satisfying some standard requirements products are inevitable. For this reason, it is required to check the lot after production and producers have to prevent them from reaching consumer. Acceptance sampling plans have been widely used to see if the lot is acceptable or not by inspecting sample. The acceptance of a lot is decided when the number of failures exceed acceptance number and one can terminate the test. Now, the process started by obtaining the minimum sample size that is necessary to emphasize a certain average life when the life test is terminated at a predetermined time. Such tests are called truncated lifetime tests.

An acceptance sampling plan based on truncated life tests consists of the following quantities:

1) The number of units (n) on test.

2) An acceptance number (c), where if c or less failures happened within the test time (t), the lot is accepted.

3) The maximum test duration time, t.

4) The ratio $d = t/\mu_0$, where μ_0 is the specified average life.

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The acceptance sampling plan based on truncated life tests is studied by many authors for a variety of distribution. We can list some key studies in acceptance sampling literature considered special distributions as follows: exponential distribution by Sobel and Tischendrof [31], log-logistic distribution Kantam *et al.* [21] generalized Rayleigh distribution by Tsai and Wu [32], generalized exponential distribution by Aslam *et al.* [14], Maxwell distribution by Lu [23], inverse Rayleigh distribution by Rao *et al.* [26]. Govindaraju and Kissling [17] proposed sampling plans for beta distributed compositional fractions.

Braimah *et al.* [16] investigated single truncated acceptance sampling plans for Weibull product life distributions. Malathi and Muthulakshmi [2] proposed an economic design of acceptance sampling plans for truncated life test using Fréchet distribution. Gogah and Al-Nasser [18] developed a ranked acceptance sampling plan by attribute for exponential distribution. Mahdy *et al.* [24] considered skew-generalized inverse Weibull distribution in acceptance sampling. Al-Omari *et al.* [10] and [11] proposed acceptance sampling plans based on truncated life tests for Rama distribution and three-parameter Lindley distribution. Al-Omari and Al-Nasser [9] introduced new acceptance sampling plans for two parameter quasi Lindley distribution.

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In the recent years about acceptance sampling plans see for exponentiated Fréchet distribution Al-Nasser and Al-Omari [1], for transmuted inverse Rayleigh distribution Al-Omari [2], for exponentiated inverse Rayleigh distribution Sriramachandran and Palanivel [30], for generalized inverted exponential distribution Al-Omari [3], for generalized inverse Weibull distribution Al-Omari [4], Al-Omari and Alhadrami [8] for extended exponential distribution, and for weighted exponential distribution Gui and Aslam [19] proposed acceptance sampling plans based on truncated lifetime tests.

Also, in newly published studies, Al-Omari [5] has studied the Garima distribution, transmuted generalized inverse Weibull distribution, and Sushila distribution respectively, Al-Omari *et al.* [6] used Marshall-Olkin Esscher transformed Laplace distribution and Al-Omari *et al.* [7] considered new Weibull-Pareto distribution in acceptance sampling plans based on truncated life tests.

The rest of this paper is organized as follows: Section 2 provides the Akash distribution as well as some statistical properties. In Section 3, we illustrated the new sampling plans based on the Akash distribution and its properties as the minimum sample size, the operating characteristic function and the producer's risk. The important tables and illustrated examples are given in Section 4. An application of real data is given in Section 5. Finally, the paper is concluded in Section 6.

II. AKASH DISTRIBUTION

Shanker (2015) suggested Akash distribution probability density function (PDF) defined as

$$f_{AD}(x,\delta) = \frac{\delta^3}{\delta^2+2}\left(1+x^2\right)e^{-\delta x}, \quad \delta > 0,\ x > 0, \quad (1)$$

and cumulative distribution function (CDF) given by

$$F_{AD}(x,\delta) = 1-\left[1+\frac{\delta x}{\delta^2+2}(\delta x+2)\right]e^{-\delta x}, \quad (2)$$

The following three figures are the PDF, CDF, and reliability and Hazard functions of the Akash distribution, respectively.

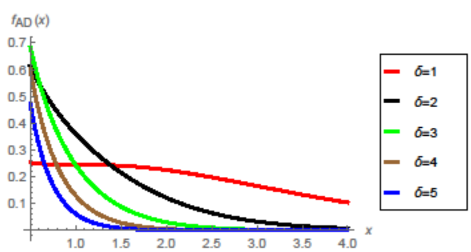

FIGURE 1. The PDF of the Akash distribution for $\delta = 1, 2, 3, 4, 5$.

The hazard and reliability functions of the Akash distribution, respectively are given in Figure 3 and Figure 4, respectively.

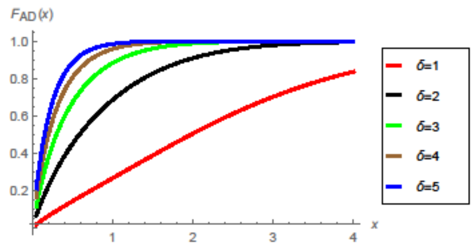

FIGURE 2. The CDF of the Akash distribution for $\delta = 1, 2, 3, 4, 5$.

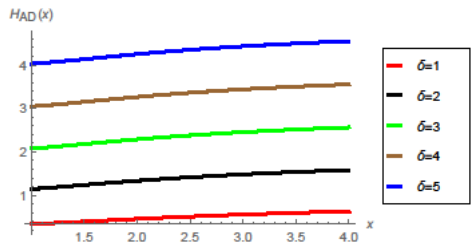

FIGURE 3. The hazard function of the Akash distribution for $\delta = 1, 2, 3, 4, 5$.

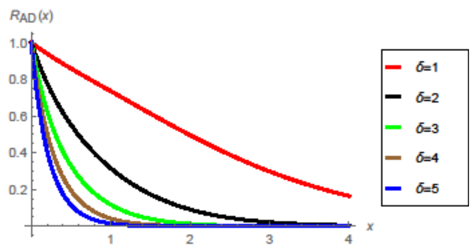

FIGURE 4. The reliability function of the Akash distribution for $\delta = 1, 2, 3, 4, 5$.

The r^{th} moment of the AD distribution is given by

$$E\left(X^k\right) = \frac{k!\left[\delta^2+(k+1)(k+2)\right]}{\delta^k\left(\delta^2+2\right)}; \quad k = 1, 2, \ldots, \quad (3)$$

and the mean of the AD distribution is $E(X) = \frac{\delta^2+6}{\delta(\delta^2+2)}$. The coefficient of variation (CV) and coefficient of skewness (Sk), respectively, are

$$CV = \frac{\sigma}{\mu} = \frac{\sqrt{\delta^4+16\,\delta^2+12}}{\delta^2+6}$$

and

$$Sk = \frac{2\left(\delta^6+30\delta^4+36\delta^2+24\right)}{\sqrt{\left(\delta^4+16\delta^2+24\right)^3}}.$$

The hazard rate function and mean residual life function are

$$h(x,\delta) = \frac{f(x,\delta)}{1-F(x,\delta)} = \frac{\delta^3\left(1+x^2\right)}{\delta x(\delta x+2)+\left(\delta^2+2\right)}, \quad (4)$$

and

$$m(x,\delta) = E\{X-x|X>x\} = \frac{1}{1-F(x)}\int_x^{\infty}[1-F(z)]dz$$

TABLE 1. Minimum sample sizes to be tested for a time t to assert with probability P^* and acceptance number c that $\mu \geq \mu_0$ for $\delta = 2$ in the Akash distribution.

P^*	c	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	3	2	2	1	1	1	1	1
0.75	1	5	4	3	3	2	2	2	2
0.75	2	8	6	5	4	4	3	3	3
0.75	3	10	8	7	6	5	4	4	4
0.75	4	13	10	8	7	6	5	5	5
0.75	5	15	12	10	8	7	6	6	6
0.75	6	18	13	11	10	8	7	7	7
0.75	7	20	15	13	11	9	9	8	8
0.75	8	22	17	14	13	10	10	9	9
0.75	9	25	19	16	14	12	11	10	10
0.75	10	27	21	17	15	13	12	11	11
0.90	0	4	3	2	2	1	1	1	1
0.90	1	7	5	4	4	3	2	2	2
0.90	2	10	7	6	5	4	4	3	3
0.90	3	13	10	8	7	5	5	4	4
0.90	4	16	12	9	8	7	6	5	5
0.90	5	18	14	11	10	8	7	6	6
0.90	6	21	15	13	11	9	8	7	7
0.90	7	23	17	14	13	10	9	9	8
0.90	8	26	19	16	14	11	10	10	9
0.90	9	28	21	18	15	13	11	11	10
0.90	10	31	23	19	17	14	12	12	11
0.95	0	5	4	3	2	2	1	1	1
0.95	1	9	6	5	4	3	3	2	2
0.95	2	12	9	7	6	4	4	3	3
0.95	3	15	11	9	7	6	5	5	4
0.95	4	17	13	10	9	7	6	6	5
0.95	5	20	15	12	10	8	7	7	6
0.95	6	23	17	14	12	9	8	8	7
0.95	7	26	19	15	13	11	9	9	8
0.95	8	28	21	17	15	12	11	10	9
0.95	9	31	23	19	16	13	12	11	11
0.95	10	33	25	20	18	14	13	12	12
0.99	0	8	6	4	4	2	2	2	1
0.99	1	12	8	7	5	4	3	3	3
0.99	2	15	11	9	7	5	4	4	4
0.99	3	18	13	11	9	7	6	5	5
0.99	4	21	15	12	11	8	7	6	6
0.99	5	24	18	14	12	9	8	7	7
0.99	6	27	20	16	14	11	9	8	8
0.99	7	30	22	18	15	12	10	9	9
0.99	8	33	24	20	17	13	11	10	10
0.99	9	36	26	21	18	14	12	12	11
0.99	10	38	28	23	20	16	14	13	12

$$= \frac{\delta^2 x^2 + 4\delta x + \left(\delta^2 + 6\right)}{\delta\left[\delta x(\delta x + 2) + \left(\delta^2 + 2\right)\right]}, \tag{5}$$

and the reliability function of the AD is

$$R(x, \delta) = 1 - F(x, \delta) = \frac{\left(\delta^2 + 2\right) e^{x\epsilon}}{\epsilon(x(x\epsilon + 2) + \epsilon) + 2} - 1. \tag{6}$$

The maximum likelihood estimator of δ is the solution of the equation

$$\bar{x}\delta^3 - \delta^2 + 2\bar{x}\delta - 6 = 0.$$

Skanker and Shukla (2-17) have modified Akash distribution to two parameters Akash distribution. Shukla *et al.* [29] suggested a new generalization of the Akash distribution which includes Akash and exponential distributions as particular cases.

III. DESIGN OF THE ACCEPTANCE SAMPLING PLAN

Assume that life time of the product follow the Akash distribution defined in Equation (1). Let the life test terminates at a predetermined time t_0 and the number of failures within this time interval [0,t] are obtained. The lot occurs is accepted if the number of failures at the end of the time t_0 does not exceed the acceptance number c.

During the experiment the researchers assume that the lot size is infinitely large so that the theory of binomial

TABLE 2. Operating characteristic function values for the sampling plan $(n, c = 2, t/\mu_0)$ with a given probability P^* for $\delta = 2$ in the Akash distribution.

P*	n		μ/μ_0					
		t/μ_0	2	4	6	8	10	12
0.75	8	0.628	0.611124	0.887763	0.954771	0.977617	0.987363	0.992188
0.75	6	0.942	0.591434	0.876869	0.949172	0.974478	0.985452	0.990946
0.75	5	1.257	0.570894	0.866130	0.943554	0.971275	0.983478	0.989650
0.75	4	1.571	0.634089	0.892316	0.955283	0.977380	0.987027	0.991887
0.75	4	2.356	0.388736	0.769042	0.892362	0.941689	0.965002	0.977392
0.75	3	3.141	0.525801	0.847411	0.932549	0.964270	0.978798	0.986393
0.75	3	3.927	0.379301	0.771045	0.892968	0.941260	0.964255	0.976622
0.75	3	4.712	0.261509	0.689475	0.847380	0.913624	0.946266	0.964262
0.90	10	0.628	0.447198	0.809375	0.917517	0.957602	0.975494	0.984607
0.90	7	0.942	0.470311	0.818419	0.921058	0.959244	0.976363	0.985114
0.90	6	1.257	0.417330	0.785869	0.903481	0.949079	0.970042	0.980941
0.90	5	1.571	0.435562	0.795852	0.907939	0.951311	0.971286	0.981695
0.90	4	2.356	0.388736	0.769042	0.892362	0.941689	0.965002	0.977392
0.90	4	3.141	0.212555	0.634260	0.812567	0.892386	0.932796	0.955316
0.90	3	3.927	0.379301	0.771045	0.892968	0.941260	0.964255	0.976622
0.90	3	4.712	0.261509	0.689475	0.847380	0.913624	0.946266	0.964262
0.95	12	0.628	0.312651	0.721554	0.871017	0.931207	0.959311	0.974036
0.95	9	0.942	0.277020	0.687876	0.850213	0.918383	0.951035	0.968438
0.95	7	1.257	0.293414	0.698811	0.855277	0.920919	0.952433	0.969273
0.95	6	1.571	0.281530	0.688044	0.847876	0.916004	0.949097	0.966935
0.95	4	2.356	0.388736	0.769042	0.892362	0.941689	0.965002	0.977392
0.95	4	3.141	0.212555	0.634260	0.812567	0.892386	0.932796	0.955316
0.95	3	3.927	0.379301	0.771045	0.892968	0.941260	0.964255	0.976622
0.95	3	4.712	0.261509	0.689475	0.84738	0.913624	0.946266	0.964262
0.99	15	0.628	0.171489	0.586239	0.788944	0.881260	0.927360	0.952562
0.99	11	0.942	0.152154	0.555614	0.766195	0.865853	0.916835	0.945165
0.99	9	1.257	0.133147	0.525701	0.743292	0.849975	0.905803	0.937316
0.99	7	1.571	0.173971	0.579607	0.779295	0.872967	0.920947	0.947696
0.99	5	2.356	0.194562	0.607701	0.795930	0.882489	0.926672	0.951335
0.99	4	3.141	0.212555	0.634260	0.812567	0.892386	0.932796	0.955316
0.99	4	3.927	0.105614	0.504378	0.724326	0.833541	0.892344	0.926541
0.99	4	4.712	0.048577	0.388736	0.634203	0.769042	0.845894	0.892362

distribution can be applied. Assume that the consumer's risk (the probability of acceptance a bad lot) is determined to be at most $1 - P^*$, i.e., the probability that the real mean life μ is less than μ_0, not exceeds $1 - P^*$. Our problem is to get the smallest sample size n necessary to satisfy the inequality

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \leq 1 - P^*, \tag{7}$$

where c is the acceptance number for given values of $P^* \in (0, 1)$, where $p = F(t; \mu_0)$ is the probability of a failure observed within the time t which depends only on the ratio t/μ_o, where

$$p = 1 - \left[\begin{array}{c} 1 + \dfrac{\delta^2+6}{(\delta^2+2)^2} \dfrac{d}{\mu/\mu_0} \\ \times \left(\dfrac{\delta^2+6}{\delta^2+2} \dfrac{d}{\mu/\mu_0} + 2 \right) \end{array} \right] e^{-\left(\frac{\delta^2+6}{\delta^2+2} \frac{d}{\mu/\mu_0} \right)}, \tag{8}$$

and $\mu_0 = \frac{\delta_0^2+6}{\delta_0(\delta_0^2+2)}$.

If the number of observed failures within the time t is at most c, then from (7) we can confirm with probability P that $F(t; \mu) \leq F(t; \mu_0)$, which implies $\mu_0 \leq \mu$.

TABLE 3. Minimum ratio of μ/μ_0 for the acceptability of a lot with producer's risk of 0.05 for $\delta = 2$ in the Akash distribution.

P*	c	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	40.596	40.509	54.055	33.565	50.337	67.109	83.902	100.674
0.75	1	8.596	9.927	9.262	11.575	9.741	12.986	16.236	19.482
0.75	2	5.751	6.043	6.328	5.713	8.568	6.889	8.612	10.334
0.75	3	4.124	4.638	5.150	5.128	5.697	4.829	6.037	7.244
0.75	4	3.686	3.920	3.786	3.819	4.332	3.828	4.786	5.743
0.75	5	3.131	3.483	3.560	3.073	3.550	3.244	4.055	4.866
0.75	6	2.982	2.865	2.956	3.149	3.046	2.861	3.577	4.291
0.75	7	2.691	2.709	2.901	2.725	2.697	3.595	3.239	3.886
0.75	8	2.475	2.588	2.546	2.801	2.441	3.254	2.987	3.584
0.75	9	2.439	2.491	2.541	2.517	2.768	2.993	2.791	3.349
0.75	10	2.292	2.412	2.300	2.295	2.554	2.787	2.635	3.161
0.90	0	54.186	60.894	54.055	67.558	50.337	67.109	83.902	100.674
0.90	1	12.536	12.893	13.246	16.555	17.359	12.986	16.236	19.482
0.90	2	7.465	7.336	8.064	7.909	8.568	11.423	8.612	10.334
0.90	3	5.664	6.186	6.189	6.436	5.697	7.595	6.037	7.244
0.90	4	4.754	4.993	4.510	4.732	5.726	5.776	4.786	5.743
0.90	5	3.936	4.292	4.105	4.449	4.608	4.732	4.055	4.866
0.90	6	3.623	3.510	3.823	3.695	3.893	4.061	3.577	4.291
0.90	7	3.219	3.241	3.258	3.625	3.400	3.595	4.495	3.886
0.90	8	3.073	3.038	3.151	3.182	3.041	3.254	4.068	3.584
0.90	9	2.826	2.881	3.063	2.847	3.275	2.993	3.742	3.349
0.90	10	2.746	2.754	2.760	2.874	3.001	2.787	3.484	3.161
0.95	0	67.776	81.278	81.256	67.558	101.32	67.109	83.902	100.674
0.95	1	16.469	15.851	17.205	16.555	17.359	23.143	16.236	19.482
0.95	2	9.176	9.912	9.790	10.078	8.568	11.423	8.612	10.334
0.95	3	6.689	6.957	7.223	6.436	7.690	7.595	9.495	7.244
0.95	4	5.109	5.528	5.230	5.637	5.726	5.776	7.221	5.743
0.95	5	4.472	4.696	4.647	4.449	4.608	4.732	5.916	4.866
0.95	6	4.049	4.153	4.254	4.237	3.893	4.061	5.077	4.291
0.95	7	3.747	3.771	3.614	3.625	4.086	3.595	4.495	3.886
0.95	8	3.371	3.488	3.453	3.561	3.624	4.054	4.068	3.584
0.95	9	3.214	3.270	3.324	3.175	3.275	3.690	3.742	4.489
0.95	10	2.973	3.096	2.989	3.162	3.001	3.404	3.484	4.180
0.99	0	108.55	122.05	108.46	135.55	101.32	135.073	168.873	100.674
0.99	1	22.361	21.754	25.092	21.502	24.827	23.143	28.934	34.718
0.99	2	11.739	12.48	13.227	12.235	11.86	11.423	14.281	17.136
0.99	3	8.223	8.496	9.283	9.027	9.652	10.252	9.495	11.393
0.99	4	6.529	6.597	6.663	7.433	7.096	7.634	7.221	8.664
0.99	5	5.543	5.904	5.727	5.808	5.645	6.143	5.916	7.099
0.99	6	4.901	5.114	5.113	5.316	5.541	5.190	5.077	6.092
0.99	7	4.450	4.564	4.678	4.517	4.764	4.533	4.495	5.393
0.99	8	4.116	4.161	4.354	4.315	4.200	4.054	4.068	4.881
0.99	9	3.858	3.852	3.844	3.828	3.774	3.690	4.613	4.489
0.99	10	3.540	3.608	3.675	3.736	3.877	4.001	4.256	4.180

The minimum sample size values satisfying inequality (7) have been calculated for $P^* = 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712$ and $d = t_0/\mu_0 = 0.75, 0.90, 0.95, 0.99$. The values of $d = t_0/\mu_0$ and P^* are consist with the corresponding values of Al-Nasser and Al-Omari [1], Baklizi and El Masri [15], Kantam *et al.* [21], and Gupta and Groll [20].

The operating characteristic function of the sampling plan $(n, c, t/\mu_0)$ is the probability of accepting the lot. Indeed, it can be can be considered as a source for choosing the minimum sample size, n, or the acceptance number, c. The operating characteristic function of the suggested acceptance sampling plan is defined as

$$OC(p) = P\,(\text{Accepting a lot}\,|\mu < \mu_0) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i}, \tag{9}$$

where $p = F(t_0; \mu)$.

The producer's risk (PR) is the probability of rejection of the lot when it is good, i.e., $\mu > \mu_0$. It is defined as

$$PR = P\,(\text{Rejecting a lot}) = \sum_{i=c+1}^{n} \binom{n}{i} p^i (1-p)^{n-i}. \tag{10}$$

For the suggested sampling plan and a given value for the producer's risk, $\Re$, the experimenter is interesting in knowing the value of $\mu > \mu_0$ that will assert the PR to be at most $\Re$. Since $p = F\left(\frac{t}{\mu_0}\frac{\mu_0}{\mu}\right)$ is a function of μ/μ_0, then μ/μ_0 is the smallest positive number for which p satisfies the inequality given by

$$\sum_{i=c+1}^{n} \binom{n}{i} p^i (1-p)^{n-i} \leq \Re, \tag{11}$$

TABLE 4. Minimum sample sizes to be tested for a time t to assert with probability P^* and acceptance number c that $\mu \geq \mu_0$ for $\delta = 5$ in the Akash distribution.

P^*	c	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	3	2	2	1	1	1	1	1
0.75	1	5	4	3	3	2	2	2	2
0.75	2	8	6	5	4	4	3	3	3
0.75	3	10	8	6	6	5	4	4	4
0.75	4	12	9	8	7	6	5	5	5
0.75	5	15	11	9	8	7	7	6	6
0.75	6	17	13	11	10	8	8	7	7
0.75	7	19	15	12	11	9	9	8	8
0.75	8	22	16	14	12	11	10	9	9
0.75	9	24	18	15	14	12	11	10	10
0.75	10	26	20	17	15	13	12	11	11
0.90	0	4	3	2	2	1	1	1	1
0.90	1	7	5	4	4	3	2	2	2
0.90	2	10	7	6	5	4	4	3	3
0.90	3	12	9	8	7	5	5	4	4
0.90	4	15	11	9	8	7	6	6	5
0.90	5	18	13	11	9	8	7	7	6
0.90	6	20	15	12	11	9	8	8	7
0.90	7	22	17	14	12	10	9	9	8
0.90	8	25	19	15	14	11	10	10	9
0.90	9	27	20	17	15	13	11	11	11
0.90	10	30	22	19	16	14	13	12	12
0.95	0	5	4	3	2	2	1	1	1
0.95	1	8	6	5	4	3	3	2	2
0.95	2	11	8	7	6	5	4	4	3
0.95	3	14	10	8	7	6	5	5	4
0.95	4	17	12	10	9	7	6	6	6
0.95	5	19	14	12	10	8	7	7	7
0.95	6	22	16	13	12	10	9	8	8
0.95	7	25	18	15	13	11	10	9	9
0.95	8	27	20	17	15	12	11	10	10
0.95	9	30	22	18	16	13	12	11	11
0.95	10	32	24	20	17	14	13	12	12
0.99	0	8	5	4	3	2	2	2	2
0.99	1	11	8	6	5	4	3	3	3
0.99	2	15	10	8	7	5	5	4	4
0.99	3	18	13	10	9	7	6	5	5
0.99	4	21	15	12	10	8	7	6	6
0.99	5	23	17	14	12	9	8	8	7
0.99	6	26	19	15	13	11	9	9	8
0.99	7	29	21	17	15	12	11	10	9
0.99	8	32	23	19	16	13	12	11	10
0.99	9	34	25	21	18	14	13	12	11
0.99	10	37	27	22	19	16	14	13	12

where

$$p = 1 - \left[1 + \frac{\delta^2+6}{(\delta^2+2)^2} \frac{d}{\mu/\mu_0} \times \left(\frac{\delta^2+6}{\delta^2+2} \frac{d}{\mu/\mu_0} + 2 \right) \right] e^{-\left(\frac{\delta^2+6}{\delta^2+2} \frac{d}{\mu/\mu_0}\right)}. \quad (12)$$

For a given value of the producer's risk, say λ, under this sampling plan, one may be interested in knowing what is the smallest value of the ratio μ/μ_0 that will assert the producer's risk is at most λ. This value is the minimum positive number for which $p = F\left(\frac{t}{\mu_0}\frac{\mu_0}{\mu}\right)$ satisfies the inequality

TABLE 5. Operating characteristic function values for the sampling plan $(n, c = 2, t/\mu_0)$ with a given probability P^* for $\delta = 5$ in the Akash distribution.

P*	n	t/μ_0	μ/μ_0					
			2	4	6	8	10	12
0.75	8	0.628	0.601638	0.890324	0.957027	0.979091	0.988327	0.992841
0.75	6	0.942	0.569814	0.875935	0.950448	0.975640	0.986313	0.991569
0.75	5	1.257	0.540749	0.861603	0.943679	0.972028	0.984183	0.990214
0.75	4	1.571	0.601868	0.885998	0.954427	0.977568	0.987383	0.992222
0.75	4	2.356	0.352804	0.749119	0.886050	0.939890	0.964694	0.97758
0.75	3	3.141	0.501591	0.829736	0.926111	0.961905	0.977930	0.986112
0.75	3	3.927	0.369850	0.746072	0.881085	0.935999	0.961888	0.975556
0.75	3	4.712	0.268014	0.660737	0.829701	0.904575	0.941668	0.961896
0.90	10	0.628	0.436619	0.813307	0.921381	0.960272	0.977299	0.985857
0.90	7	0.942	0.446947	0.817144	0.922948	0.961037	0.977724	0.986116
0.90	6	1.257	0.385386	0.779338	0.903682	0.950354	0.971275	0.981949
0.90	5	1.571	0.398867	0.785317	0.906312	0.951693	0.972040	0.982424
0.90	4	2.356	0.352804	0.749119	0.886050	0.939890	0.964694	0.977580
0.90	4	3.141	0.191866	0.602051	0.797461	0.886075	0.930291	0.954462
0.90	3	3.927	0.369850	0.746072	0.881085	0.935999	0.961888	0.975556
0.90	3	4.712	0.268014	0.660737	0.829701	0.904575	0.941668	0.961896
0.95	11	0.628	0.365289	0.770773	0.900037	0.948539	0.970255	0.981324
0.95	8	0.942	0.341623	0.752889	0.890324	0.942984	0.966841	0.979091
0.95	7	1.257	0.263791	0.690568	0.855560	0.922808	0.95432	0.970851
0.95	6	1.571	0.248094	0.673992	0.845401	0.916625	0.950373	0.968203
0.95	5	2.356	0.166472	0.580284	0.785402	0.879179	0.926071	0.951717
0.95	4	3.141	0.191866	0.602051	0.797461	0.886075	0.930291	0.954462
0.95	4	3.927	0.100140	0.466682	0.69977	0.820799	0.886029	0.923505
0.95	3	4.712	0.268014	0.660737	0.829701	0.904575	0.941668	0.961896
0.99	15	0.628	0.163650	0.592724	0.797320	0.887902	0.932242	0.956136
0.99	10	0.942	0.187943	0.618881	0.813307	0.897601	0.938419	0.960272
0.99	8	1.257	0.174971	0.601164	0.801521	0.890134	0.933535	0.956943
0.99	7	1.571	0.147308	0.563205	0.776002	0.873851	0.922836	0.949627
0.99	5	2.356	0.166472	0.580284	0.785402	0.879179	0.926071	0.951717
0.99	5	3.141	0.063552	0.399071	0.648129	0.785445	0.861739	0.906378
0.99	4	3.927	0.100139	0.466682	0.699770	0.820799	0.886029	0.923505
0.99	4	4.712	0.051113	0.352804	0.601990	0.749119	0.834541	0.886050

$$\sum_{i=c+1}^{n} \binom{n}{i} p^i (1-p)^{n-i} \le \lambda. \tag{13}$$

For a given acceptance sampling plan $(n,\ c,\ t/\mu_0)$ based on the AD at a specified confidence level P^*, the smallest values of μ/μ_0 satisfying Inequality (13) are given in Table (3) for $\delta = 2$.

IV. ILLUSTRATION OF TABLES AND EXAMPLES

In this section, we studied the performance of the proposed sampling plans in terms of the minimum sample sizes, operating characteristic function and minimum ratio. Various values of the Akash distribution parameter values $\delta = 2, 5$. The results for $\delta = 2$ are presented in Tables (1-3) and for $\delta = 5$ are given in Tables (4-6).

A. TABLES FOR $\delta = 2$

For an acceptance number c, the smallest sample sizes necessary to assert that the mean life exceeds μ_0 with probability greater than or equal P^* for $\delta = 2$ in Akash distribution are presented in Table (1).

TABLE 6. Minimum ratio of μ/μ_0 for the acceptability of a lot with producer's risk of 0.05 for $\delta = 5$ in the Akash distribution.

P*	c	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	38.981	38.967	51.998	32.46	48.679	64.898	81.138	97.358
0.75	1	8.361	9.700	9.126	11.406	9.786	13.047	16.312	19.572
0.75	2	5.636	5.978	6.31	5.771	8.655	7.128	8.912	10.693
0.75	3	4.077	4.628	4.164	5.205	5.867	5.080	6.351	7.621
0.75	4	3.313	3.416	3.857	3.931	4.522	4.058	5.073	6.087
0.75	5	3.123	3.122	3.103	3.198	3.737	4.982	4.308	5.169
0.75	6	2.774	2.917	3.049	3.273	3.224	4.298	3.796	4.555
0.75	7	2.529	2.766	2.641	2.852	2.862	3.816	3.429	4.114
0.75	8	2.491	2.428	2.645	2.543	3.219	3.456	3.152	3.782
0.75	9	2.332	2.363	2.379	2.643	2.936	3.178	2.934	3.521
0.75	10	2.205	2.309	2.401	2.419	2.712	2.956	2.758	3.310
0.90	0	51.983	58.471	51.998	64.986	48.679	64.898	81.138	97.358
0.90	1	12.132	12.541	12.944	16.177	17.104	13.047	16.312	19.572
0.90	2	7.278	7.218	7.977	7.886	8.655	11.538	8.912	10.693
0.90	3	5.062	5.373	6.176	6.469	5.867	7.821	6.351	7.621
0.90	4	4.340	4.454	4.558	4.821	5.895	6.028	7.537	6.087
0.90	5	3.897	3.906	4.166	3.878	4.795	4.982	6.229	5.169
0.90	6	3.391	3.542	3.473	3.810	4.083	4.298	5.374	4.555
0.90	7	3.039	3.281	3.344	3.300	3.586	3.816	4.770	4.114
0.90	8	2.923	3.086	2.943	3.305	3.219	3.456	4.321	3.782
0.90	9	2.706	2.744	2.897	2.973	3.458	3.178	3.974	4.768
0.90	10	2.643	2.644	2.856	2.712	3.178	3.615	3.696	4.435
0.95	0	64.986	77.974	78.023	64.986	97.459	64.898	81.138	97.358
0.95	1	14.014	15.372	16.734	16.177	17.104	22.803	16.312	19.572
0.95	2	8.097	8.454	9.632	9.969	11.826	11.538	14.425	10.693
0.95	3	6.044	6.115	6.176	6.469	7.805	7.821	9.778	7.621
0.95	4	5.021	4.970	5.253	5.697	5.895	6.028	7.537	9.043
0.95	5	4.154	4.296	4.691	4.546	4.795	4.982	6.229	7.474
0.95	6	3.800	3.852	3.893	4.340	4.908	5.444	5.374	6.448
0.95	7	3.546	3.538	3.690	3.742	4.276	4.780	4.770	5.724
0.95	8	3.210	3.303	3.533	3.679	3.813	4.291	4.321	5.185
0.95	9	3.078	3.121	3.153	3.299	3.458	3.914	3.974	4.768
0.95	10	2.862	2.976	3.081	3.001	3.178	3.615	3.696	4.435
0.99	0	103.99	97.478	104.049	97.513	97.459	129.93	162.445	194.917
0.99	1	19.655	21.021	20.512	20.914	24.26	22.803	28.509	34.208
0.99	2	11.369	10.916	11.28	12.038	11.826	15.766	14.425	17.309
0.99	3	8.004	8.329	8.159	8.960	9.701	10.406	9.778	11.733
0.99	4	6.382	6.509	6.632	6.565	7.229	7.860	7.537	9.043
0.99	5	5.181	5.458	5.732	5.862	5.816	6.393	7.992	7.474
0.99	6	4.618	4.778	4.726	4.865	5.714	5.444	6.806	6.448
0.99	7	4.221	4.303	4.378	4.612	4.949	5.701	5.976	5.724
0.99	8	3.926	3.953	4.117	4.049	4.390	5.083	5.364	5.185
0.99	9	3.574	3.684	3.913	3.941	3.963	4.610	4.894	4.768
0.99	10	3.408	3.472	3.527	3.569	4.067	4.236	4.520	4.435

Assume that the life time of the products follows the Akash distribution with parameter $\delta = 2$, and that the researcher like to establish that the mean life is greater than or equal to at least $\mu_0 = 1000$ hours with probability $P^* = 0.99$. Also, assume that the life test will be terminated at $t_0 = 1257$ hours. Since Table (1) provides the smallest sample size, then when $P^* = 0.99$, $d = t_0/\mu_0 = 1.257$, and $c = 2$, the corresponding Table (1) entry is $n = 9$ units.

Now, these 9 units should be tested and if out of the 9 items if no more than two items are fail within 1257 hours, the researcher can confirm that the true mean life μ of the items is at least 1000 hours with confidence level of 0.99.

The operating characteristic function values for the suggested sampling plan based on the Akash distribution adopted in Table (1) for various values of P^* and $d = t_0/\mu_0 = 1.257$ with acceptance number $c = 2$ are presented in Table (2). For illustration, when $P^* = 0.99$, $c = 2$, $t_0/\mu_0 = 1.257$, $\mu/\mu_0 = 4$, the corresponding table entry is 0.525701, it implies, based on the above acceptance sampling plan, that is the lot is accepted if out of 9 items, less than or equal 2 items fail before time point $t_0 = 1257$ hours, then if $\mu \geq 4 \times t_0/1.257 = 3.1822 t_0 = 4000$ hours, then the product will be accepted with probability of at least 0.525701.

For the above example, from Table (2) the operating characteristic values for the sampling plan $(n, c, t/\mu_0) = (9, 2, 1.257)$ when $P^* = 0.99$ are:

μ/μ_0	2	4	6	8	10	12
OC	0.13	0.53	0.74	0.85	0.91	0.94

These values show if the true average life is twice the specified average life ($\mu/\mu_0 = 2$), the producer's risk is about 0.866853, and the producer's risk are 0.474299, 0.256708, 0.150025, 0.094197 and 0.062684 for $\mu/\mu_0 = 4, 6, 8, 10, 12$, respectively. Hence, the producer's risk goes to zero.

The minimum ratio of the true mean lifetime to the specified one for the proposed acceptance plan of a lot with producer's risk $\Re = 0.05$ are given in Table (3). For illustration, when $P^* = 0.99$ (consumer's risk is 0.01), $c = 2, d = t/\mu_0 = 1.257$, the corresponding table entry $\mu/\mu_0 = 13.277$, which implies that if $\mu \geq 13.277 \times t_0/1.257 = 10.5625\, t_0 = 13277$ hours, then with $c = 2$ and sample size $n = 9$, the lot will be rejected with probability less than or equal to 0.05. That is, the product is accepted with probability of at least 0.95.

B. RESULTS FOR $\delta = 5$

Minimum sample sizes to be tested for a time t to assert with probability P^* and an acceptance number c that $\mu \geq \mu_0$ for $\delta = 5$ in the Akash distribution are summarized in Table 4. The operating characteristic function values for the new sampling plan and the minimum ratio of μ/μ_0 for the acceptability of a lot with producer's risk of 0.05 for $\delta = 5$ in the Akash distribution are provided in Tables 4 and 5, respectively.

When are compared the minimum sample sizes presented in Tables (1) and (4) to investigate the effect of the life time distribution parameter, it is found that the minimum sample sizes calculated when $\delta = 5$ are less than their counterparts of $\delta = 2$ for fixed P^* and t/μ_0.

Also, when we compare the results obtained based on the suggested acceptance sampling plans for the Akash distribution with their competitive in Al-Nasser and Al-Omari [1], Baklizi and El Masri [15], and Kantam *et al.* [21], it turns out that the samples size obtained in this paper are smaller than their counterparts.

V. AN APPLICATION OF ELECTRIC CARTS DATA

A real data set is considered in this section to investigate the performance of the suggested acceptance sampling plans. These data was already considered by Zimmer *et al.* (1998), Gui and Aslam (2017), and Lio *et al.* (2010). The data consists of the lifetime (in months) to first failure of 20 small electric carts used for internal transportation and delivery in a large manufacturing facility. The data are 0.9, 1.5, 2.3, 3.2, 3.9, 5, 6.2, 7.5, 8.3, 10.4, 11.1, 12.6, 15, 16.3, 19.3, 22.6, 24.8, 31.5, 38.1, 53. The analysis of the data is given below in Table 7.

TABLE 7. The analysis of the electronic carts data.

Min	0.9	**Skewnwess**	1.25
Max	53	**Kurtosis**	0.86
Mean	14.68	**Range**	52.1
Median	10.75	**Standard error of the mean**	3.06
First Quartile	4.73	**Standard deviation**	13.66
Third Quartile	20.13	**Variance**	186.7

Now, we check whether the Akash distribution can be used or not. Hence we considered the criteria: Anderson-Darling criterion (AD), Cramér–von Mises criterion (CM), Akaike Information criterion (AIC), Bayesian Information criterion (BIC), the maximized log-likelihood (MLL), Consistent Akaike Information criterion (CAIC), and Hannan-Quinn Information criterion (HQIC) are obtained and summarized in Table (7) where

$$AIC = -2MLL + 2w, \quad CAIC = -2MLL + \frac{2wn}{n-w-1},$$
$$BIC = -2MLL + wLog(n),$$
$$HQIC = 2Log\left[Log(n)(w - 2MLL)\right],$$

where w is the number of parameters and n is the sample size. The results are presented in Table 8.

TABLE 8. The AIC, CAIC, BIC, HQIC, CM, A, KS, and -2MLL for the electric carts data.

AIC	**BIC**	**CAIC**	**HQIC**	**CM**
160.4	161.4	160.6	160.6	0.014
AD	**-2MLL**	**KS**	**P-Value**	
0.124	79.2	0.2	0.3	

The maximum likelihood estimates (MLE) of δ is $\hat{\delta} = 0.2017$ with standard deviation 0.0259. Hence, $\hat{\mu} = \frac{\hat{\delta}^2+6}{\hat{\delta}(\hat{\delta}^2+2)} = 14.6535$. The Kolmogorov-Smirnov distance between the fitted and observed distributions is 0.207 with P-Value of 0.313. Thus, the Akash distribution shows a very good fit.

Let the specified mean lifetime and the testing time are $\mu_0 = 14.6535$ and $t = 9.202$ months, respectively. Therefore, for $P^* = 0.75$ and $d = t/\mu_0 = 0.628$, the acceptance number and the corresponding minimum sample sizes are given in Table 9, which is found to be $c = 4$. Hence, if the number of failures before $t = 9.202$ months, is less than or equal to 4, we can accept the lot with the assured mean lifetime 14.6535 months with probability 0.70. Since the number of failures before $t = 9.202$ months is 9, then the lot is rejected.

TABLE 9. The minimum sample sizes for $t/\mu_0 = 0.628$, $P^* = 0.75$, $\hat{\delta} = 0.2017$ for 20 small electric carts data.

c	0	1	2	3	4	5
n	4	9	13	13	20	24
c	6	7	8	9	10	
n	28	32	35	39	43	

The values of OC for the ASP ($n = 20, c = 4, t/\mu_0 = 0.628$) and the corresponding producer's risk are presented in Table 10, while the minimum ratios for this example are given in Table 11.

TABLE 10. Values for the function of the operating characteristic and the corresponding producer risk for the ASP ($n = 20, c = 4, t/\mu_0 = 0.628$) with $P^* = 0.75$, $\hat{\delta} = 0.2017$ for 20 small electric carts data.

μ/μ_0	2	4	6
OC	0.9828	0.9999	0.9999
Producer's risk	0.0172	0.0001	0.0001
μ/μ_0	8	10	12
OC	0.9999	1	1
Producer's risk	0.0001	0	0

TABLE 11. Minimum ratio of μ/μ_0 for the acceptability of a lot with producer's risk of 0.05 with $P^* = 0.75$, $c = 4$, $\hat{\delta} = 0.2017$ for the 20 small electric carts data.

t/μ_0			
0.628	0.942	1.257	1.571
1.746	1.877	2.032	2.008
t/μ_0			
2.356	3.141	3.927	4.712
2.452	3.269	4.087	4.903

VI. CONCLUSION

In this paper, new acceptance sampling plans based on truncated life tests for the Akash distribution are proposed. The necessary tables are presented for the minimum sample size needed to guarantee a certain mean life of the test units. The operating characteristic function values as well as the associated producer's risks are also provided. The suggested sampling plans are applied for real data set. The outcomes of this paper can be used to develop other kinds of acceptance sampling plans such as group and double acceptance sampling plans for Akash and other distributions.

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