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Stabilization and L_2- Gain Performance of Periodic Piecewise Impulsive Linear Systems

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ABSTRACT This paper deals with the stability, stabilization and L_2- gain problem of periodic piecewise impulsive linear system. Firstly, by developing a new Lyapunov function called multiple piecewise time-varying Lyapunov function, a sufficient condition that guarantee the system λ^*- exponentially stable is established. Moreover, a state-feedback controller is designed to stabilize the system. Then L_2- gain analysis is investigated to obtain a result that guarantee the system λ^*- exponentially stable and with L_2- gain performance. Finally, numerical simulation examples are provided to verify the effectiveness of the results proposed in this paper.

INDEX TERMS Periodic piecewise impulsive linear system, stability, stabilization, L_2-gain performance, time-varying Lyapunov function.

I. INTRODUCTION

Switched systems consist of a family of continuous-time or discrete-time subsystems and a switching rule that governs the transitions among these subsystems. Switched systems have been extensively investigated in the last two decades since that the switching behavior occurs extensively in many practical processes, such as power systems [1], network control [2], aircraft control [3] and etc. [4]. A variety of theoretic contributions have been reported, see [5]–[7] for details.

Stability and stabilization are the research focuses of switched systems. Stability analysis is related closely with the form of switching rule. Generally that the switching rules can be classified into arbitrary switching and constrained switching [8]. The latter could be further divided into time-dependent switching, state-dependent switching, and switching under stochastic constraints. The basic approach to investigate the stability under arbitrary switching is to find a common Lyapunov function [5] for the switched systems or to adopt joint spectral radius method [9]. However, in many practical cases, switching rules are often under some constraints, which makes switched system stable under arbitrary switching is too conservative. For the cases that state of a switched system is difficult to measure, a time-dependent switching rule is a viable choice for stabilization [10], [11]. Time-dependent switching usually restrain the dwell time (DT) of each subsystem or the average dwell time (ADT) of a switching sequence to be within a certain bound to make the switched system stable [12], [13]. In a sense, the method of ADT is more flexible compared with the DT method in that it does not have constraints on the dwell time of each subsystem. Therefore, there are many contributions which have applied the method of ADT to the control of varieties of systems, e.g. [14]–[16]. Recently, some new extended results based on ADT have been proposed, namely mode-dependent average dwell time (MDADT) [17], [18] and persistent dwell time (PDT) [19]. In short, the idea of MDADT is to allow each subsystem to have its own ADT [20], [21]. [22] used MDADT approach to analyze the stability of discrete switched systems. PDT is a more general concept, which covers the DT and ADT from a certain point of view [19]. In [23], the fault detection filter design for a class of nonlinear switched systems is investigated by PDT. For the case that the systems have abrupt random changes

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in structures or parameters [24], it adopts the switching under stochastic constraints to model this occasion, which generally refers to stochastic Markovain switching. In [25], the finite-time control problem is investigated for a class of discrete-time markovian jump systems with deterministic switching and time-delay. Forthermore, sufficient conditions of the exponential almost sure stability for a switching markov jump linear system are presented [24]. The stabilization analysis is closely related with the realization of control inputs such as the state-feedback control[26], adaptive output feedback control [27], observer-based nonlinear control [28], sliding mode control [29] and fuzzy control [30].

As a special but important class of switched system [31], periodic piecewise system has received great attention in the past decade. Periodic piecewise systems are often applied to describe systems in many fields, which have periodically switching sequence and fixed dwell time of subsystems, such as DC-DC converters, conveyors with periodically changing loads and vibration systems [32]–[34]. Some new results on the stability and control problems of periodic piecewise system have been presented in recent years. The problem of guaranteed cost control is investigated for continuous-time periodic piecewise linear systems with delay [35]. In [31], the stability and L_2- gain analysis of periodic piecewise linear systems is presented by discontinuous Lyapunov function which does not need to monotonically decrease in one interval.

On the other hand, the impulsive effect, i.e. state jump, exists extensively in various practical systems, such as frequency-modulated systems, optimal control models in economics, neural networks [36]–[38]. Accordingly, switched system is also subject to impulsive effect. The co-existence of impulses and switching may cause severer oscillation and instability, and result in poor performance. Therefore, in the past decade, the impulsive switched systems have attracted much attention and obtained many theoretical results [39]. In [40], it investigates the finite-time stability problem for a class of switched linear systems with impulse effects. Furthermore, the finite-time stability for impulsive switched linear time-varying systems is presented [41]. On the other hand, the exponential stability and stabilization problems of a nonlinear impulsive switched systems with time-varying disturbances are investigated [42]. To the best of our knowledge, although periodic piecewise systems is an important class of switched system, there are no results considering impulse effect in it. In fact, periodic piecewise systems with impulsive effects are widely used in practical applications, such as DC-DC converters with non-ideal switching diodes, conveyors with periodically jumping loads and electrical circuits with switches. This motivate our research.

This paper deals with the stability, stabilization and L_2- gain analysis of periodic piecewise impulsive linear system. The main contributions lie in: this paper propose a Lyapunov function called multiple piecewise time-varying Lyapunov function (MPTLF), which is then applied to derive a sufficient condition of λ^*- exponential stability for periodic piecewise linear system with impulse effects. The sufficient condition can be degenerated to the case without impulsive effect, and the degenerated result is more flexible compared with [31]. Then based on the proposed exponentially stability result, state feedback controller is designed. Furthermore, L_2- gain analysis of the system is also investigated.

Notations: $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the space of n-dimensional real vectors and of $n \times n$ matrices with real entries. $\|\cdot\|$ denotes the Euclidean vector norm. Z, Z^+ and $\mathbb{N}$ denote the sets of integers, of positive integers and of natural numbers, respectively. $\mathbb{N}_M$ denotes the set of natural numbers between 1 and M. $P > 0(\geqslant 0)$ denotes the real symmetric and positive definite (semi-positive definite) matrix. $\lambda_{\max}(P)$, $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of the matrix P. The superscript T denotes matrix transposition.

II. PROBLEM FORMULATION

Consider a continuous-time periodic piecewise impulsive linear system:

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + C_{\sigma(t)}\omega(t), \quad t \neq t_k \\ \Delta x(t) &= D_k x(t), \quad t = t_k,\ k = 0,1,2,\cdots \\ y(t) &= E_{\sigma(t)}x(t) + F_{\sigma(t)}u(t) + G_{\sigma(t)}\omega(t) \end{aligned}$$

where $x(t)$, $u(t)$ and $\omega(t)$ are system state, control input and disturbance input, respectively. $\Delta x(t_k) = x(t_k^+) - x(t_k)$, $x(t_k^+) = \lim_{h\to 0^+} x(t_k + h)$, $x(t_k) = x(t_k^-) = \lim_{h\to 0^+} x(t_k - h)$. In this paper, we assume that the switching of subsystem and the jump of state happen simultaneously at t_k, periodic switching signal $\sigma(t) \in \mathbb{N}_M$ is right continuous and satisfies $\sigma(t+T) = \sigma(t)$. In the following, the impulse/switching instant t_k is rewritten by $t_k = lT + t_i$, $l = 0, 1, \cdots$, $i = 0, 1, \cdots, M$, as shown in Figure 1. Accordingly, the above system model can be rewritten as

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + C_{\sigma(t)}\omega(t), \\ &\quad t \neq lT + t_i, l = 0, 1, 2, \cdots, \quad i = 0, 1, \cdots, M, \\ \Delta x(t) &= D_i x(t), t = lT + t_i, \quad l = 0, 1, 2, \cdots, \\ &\quad i = 0, 1, \cdots, M-1, \\ y(t) &= E_{\sigma(t)}x(t) + F_{\sigma(t)}u(t) + G_{\sigma(t)}\omega(t) \end{aligned} \tag{1}$$

Since the switching signal is periodic, the dwell time of each subsystem is fixed, which is denoted by $T_i = t_i - t_{i-1}$, $i \in \mathbb{N}_M$. Clearly, $T = \sum_{i=1}^{M} T_i$. Without loss of generality, the subsystems $S_1 : (A_1, B_1, C_1)$, $S_2\,(A_2, B_2, C_2)$, $\cdots$, $S_M : (A_M, B_M, C_M)$ are assumed to be activated successively in any period T.

Definition 1 [31]: The periodic piecewise impulsive linear system (1) with $u(t) = 0$ and $\omega(t) = 0$ is said to be λ^*-exponentially stable if $\|x(t)\| \leqslant k e^{-\lambda^* t}\|x(0)\|$, $\forall t > 0$, for constants $k \geqslant 1$, $\lambda^* > 0$.

Definition 2 [43]: The periodic piecewise impulsive linear system (1) with $u(t) = 0$ is said to have weighted L_2- gain,

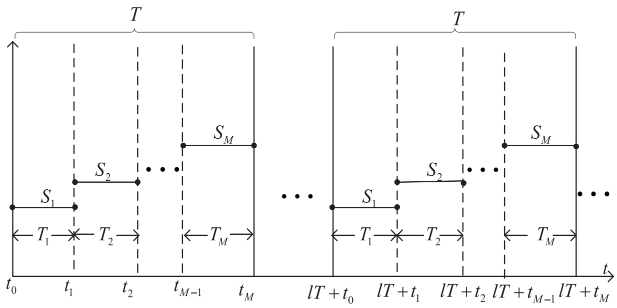

FIGURE 1. Switching law of System (1).

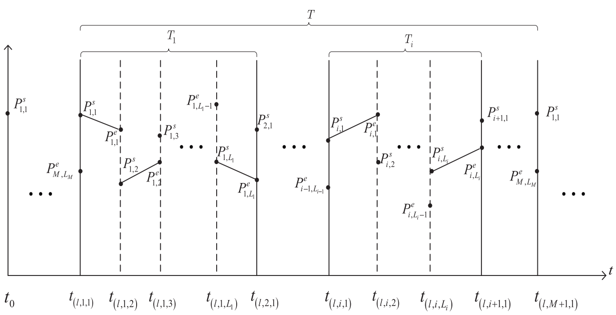

FIGURE 2. Construction of $P(t)$ in MPTLF.

if under zero initial condition(i.e., $x(t_0) = 0$), it holds that

$$\int_0^\infty e^{-\alpha s} y^T(s) y(s) ds \leqslant \gamma^2 \int_0^\infty \omega^T(s)\omega(s) ds$$

where $\alpha > 0$ and $\gamma > 0$.

III. MAIN RESULTS

In this section, we use a new Lyapunov function, called multiple piecewise time-varying Lyapunov function (MPTLF), to derive a sufficient condition for the λ^*- exponential stability of system (1). A method of designing state feedback controller and L_2- gain performance analysis are also dealt with in this section.

Divide the dwell time $[lT + t_{i-1}, lT + t_i)$ into L_i segments averagely. Denote such segments by $D_{i,m}$, where $D_{i,m} = \left[lT + t_{i-1} + J_i^{m-1}, lT + t_{i-1} + J_i^m\right)$, $J_i^m = mh_i$, $h_i = T_i/L_i$, $m \in \mathbb{N}_{L_i}$. Clearly, $t_{i-1} + J_i^{L_i} = t_i$. For simplicity of expression, we use the notation $t_{(l,i,m)}$ to denote the instant $lT + t_{i-1} + J_i^{m-1}$.

Construct the following Lyapunov function $V(t) = x(t)^T P(t) x(t)$ for $\forall t \in D_{i,m}$, where

$$P(t) = (1-\gamma) P_{i,m}^s + \gamma P_{i,m}^e$$

and $\gamma = \left(t - t_{(l,i,m)}\right)/h_i$, $P_{i,m}^s > 0$, $P_{i,m}^e > 0$.

Clearly, when $\gamma = 0$ ($/\gamma = 1$), then $P(t) = P_{i,m}^s$ ($/ P(t) = P_{i,m}^e$) respectively, as shown in Figure 2.

Remark 1: This MPTLF technique are also applicable for systems with some aperiodic switching signals, e.g. the switching signal with constrains of minimum dwell time or average dwell time.

Theorem 1: Consider periodic piecewise impulsive linear system (1) with $u(t) = 0$ and $\omega(t) = 0$, given $\lambda^* > 0$, if there exist a set of scalars L_i, $\lambda_{i,m}$, $0 < \eta_i \leqslant 1$, $\varphi_{i-1} > 0$, $u_i > 0$ and matrices $P_{i,m}^s > 0$, $P_{i,m}^e > 0$, $i \in \mathbb{N}_M$, $m \in \mathbb{N}_{L_i}$ such that

$$A_i^T P_{i,m}^s + P_{i,m}^s A_i + \left(P_{i,m}^e - P_{i,m}^s\right) L_i/T_i - \lambda_{i,m} P_{i,m}^s \leqslant 0, \quad i \in \mathbb{N}_M, \quad m \in \mathbb{N}_{L_i} \tag{2}$$

$$A_i^T P_{i,m}^e + P_{i,m}^e A_i + \left(P_{i,m}^e - P_{i,m}^s\right) L_i/T_i - \lambda_{i,m} P_{i,m}^e \leqslant 0, \quad i \in \mathbb{N}_M, \quad m \in \mathbb{N}_{L_i} \tag{3}$$

$$P_{i,m}^s - \eta_i P_{i,m-1}^e \leqslant 0, \quad i \in \mathbb{N}_M, m \in \mathbb{N}_{L_i}, m \neq 1 \tag{4}$$

$$(I + D_{i-1})^T P_{i,1}^s (I + D_{i-1}) \leqslant \varphi_{i-1} P_{i,1}^s, \quad i \in \mathbb{N}_M \tag{5}$$

$$P_{i+1,1}^s - u_{i+1} P_{i,L_i}^e \leqslant 0, \quad i \in \mathbb{N}_M, i \neq M \tag{6}$$

$$P_{1,1}^s - u_1 P_{M,L_M}^e \leqslant 0 \tag{7}$$

$$\sum_{q=1}^{M} \xi_q + \sum_{q=0}^{M-1} \nu_q + 2\lambda^* T \leqslant 0 \tag{8}$$

where $\xi_q = \frac{T_q}{L_q} \sum_{d=1}^{L_q} \lambda_{q,d}$, $\nu_q = \ln \varphi_q u_{q+1} \eta_{q+1}^{L_{q+1}-1}$, then the system (1) is λ^*- exponentially stable.

Proof: Choose Lyapunov function

$$V(t) = x(t)^T P(t) x(t), \quad t \in D_{i,m}$$

where $P(t) = (1-\gamma) P_{i,m}^s + \gamma P_{i,m}^e$, $\gamma = \left(t - t_{(l,i,m)}\right)/h_i$, $P_{i,m}^s > 0$, $P_{i,m}^e > 0$.

For $t \in D_{i,m}$, it follows that

$$\begin{aligned}
\dot{V}(t) &= x^T(t) \Big[(1-\gamma) A_i^T P_{i,m}^s + \gamma A_i^T P_{i,m}^e + \gamma P_{i,m}^e A_i \\
&\quad + (1-\gamma) P_{i,m}^s A_i + \left(P_{i,m}^e - P_{i,m}^s\right) L_i/T_i\Big] x(t) \\
&= (1-\gamma) x^T(t) \Big[A_i^T P_{i,m}^s + P_{i,m}^s A_i \\
&\quad + \left(P_{i,m}^e - P_{i,m}^s\right) L_i/T_i\Big] x(t) \\
&\quad + \gamma x^T(t) \Big[A_i^T P_{i,m}^e + P_{i,m}^e A_i \\
&\quad + \left(P_{i,m}^e - P_{i,m}^s\right) L_i/T_i\Big] x(t)
\end{aligned}$$

Then by (2) and (3), we have

$$\dot{V}(t) \leqslant \lambda_{i,m} V(t) \tag{9}$$

From (4)–(6), correspondingly, it follows that

$$V\left(t_{(l,i,m)}\right) \leqslant \eta_i V\left(t_{(l,i,m)}^-\right), \quad i \in \mathbb{N}_M, m \in \mathbb{N}_{L_i}, m \neq 1 \tag{10}$$

$$V\left(t_{(l,i,1)}^+\right) \leqslant \varphi_{i-1} V\left(t_{(l,i,1)}\right), \quad i \in \mathbb{N}_M \tag{11}$$

$$V\left(t_{(l,i+1,1)}\right) \leqslant u_{i+1} V\left(t_{(l,i+1,1)}^-\right) \tag{12}$$

Combining (9)–(12) leads to

$$\begin{aligned}
V\left(t_{(l,i+1,1)}^-\right) &\leqslant \eta_i^{L_i-1} e^{\frac{T_i}{L_i}\sum_{m=1}^{L_i} \lambda_{i,m}} V\left(t_{(l,i,1)}^+\right) \\
&\leqslant \varphi_{i-1} \eta_i^{L_i-1} e^{\frac{T_i}{L_i}\sum_{m=1}^{L_i} \lambda_{i,m}} V\left(t_{(l,i,1)}\right) \\
&\leqslant \varphi_{i-1} u_i \eta_i^{L_i-1} e^{\frac{T_i}{L_i}\sum_{m=1}^{L_i} \lambda_{i,m}} V\left(t_{(l,i,1)}^-\right)
\end{aligned} \tag{13}$$

Further from (7), we can get

$$V\left(t_{(l,1,1)}\right) \leqslant u_1 V\left(t^-_{(l,1,1)}\right) \tag{14}$$

By (13) and (14), we have

$$\begin{aligned}
&V(t_{(l+1,1,1)}) \\
&\leqslant u_1\varphi_{M-1}u_M\eta_M^{L_M-1}e^{\frac{T_M}{L_M}\sum\limits_{m=1}^{L_M}\lambda_{M,m}}V\left(t^-_{(l,\mathrm{M},1)}\right)\\
&\ \cdots\\
&\leqslant (\prod_{q=1}^{M}u_q\varphi_{q-1}\eta_q^{L_q-1})e^{\sum\limits_{q=1}^{M}\frac{T_q}{L_q}\sum\limits_{d=1}^{L_q}\lambda_{q,d}}V(t_{(l,1,1)})\\
&\ \cdots\\
&\leqslant (\prod_{q=1}^{M}u_q\varphi_{q-1}\eta_q^{L_q-1})^{l+1}e^{(l+1)\sum\limits_{q=1}^{M}\frac{T_q}{L_q}\sum\limits_{d=1}^{L_q}\lambda_{q,d}}V(t_0)
\end{aligned} \tag{15}$$

Substituting (8) into (15) leads to

$$\begin{aligned}
V((l+1)T) &\leqslant e^{(l+1)\left(\sum\limits_{q=1}^{M}\xi_q+\sum\limits_{q=0}^{M-1}\nu_q\right)}V(t_0)\\
&\leqslant e^{-2(l+1)\lambda^*T}V(t_0)
\end{aligned} \tag{16}$$

Noticing that $V\left(t_{(l+1,1,1)}\right) \geqslant \underline{\rho}\left\|x\left(t_{(l+1,1,1)}\right)\right\|^2$ and $V\left(t_0\right) \leqslant \overline{\rho}\|x\left(0\right)\|^2$, we can obtain from (16) that

$$\left\|x\left(t_{(l+1,1,1)}\right)\right\| \leqslant \sqrt{\frac{\overline{\rho}}{\underline{\rho}}}e^{-(l+1)\lambda^*T}\left\|x\left(0\right)\right\| \tag{17}$$

where $\underline{\rho} = \lambda_{\min}\left(P^s_{1,1}\right)$, $\overline{\rho} = \lambda_{\max}\left(P^s_{1,1}\right)$.

On the other hand, since $x\left(t\right) = e^{A_i\left(t-t_{(l,i,1)}\right)}x\left(t^+_{(l,i,1)}\right)$, $t \in \left(t_{(l,i,1)}, t_{(l,i+1,1)}\right)$ and $x\left(t^+_{(l,i,1)}\right) = (I+D_{i-1})x\left(t_{(l,i,1)}\right)$, we have

$$\left\|x\left(t\right)\right\| \leqslant \beta_i\left\|x\left(t^+_{(l,i,1)}\right)\right\| \tag{18}$$

$$\left\|x\left(t^+_{(l,i,1)}\right)\right\|^2 \leqslant \alpha_{i-1}\left\|x\left(t_{(l,i,1)}\right)\right\|^2 \tag{19}$$

where $\beta_i = \max\left[1, \frac{T_i}{2}\lambda_{\max}\left(A_i+A_i^T\right)\right]$, $\alpha_{i-1} = \max[1, \lambda_{\max}((I+D_{i-1})^T(I+D_{i-1}))]$.

Combining (18) with (19) and noticing that $\alpha_{i-1} \geqslant 1$, $\beta_i \geqslant 1$, one can obtain

$$\begin{aligned}
\left\|x\left(t\right)\right\| \leqslant \sqrt{\alpha_{i-1}}\beta_i\left\|x\left(t_{(l,i,1)}\right)\right\| &\leqslant \xi\left\|x\left(lT\right)\right\|,\\
&t \in [lT, (l+1)T)
\end{aligned} \tag{20}$$

where $\xi = \left(\Pi_{i=1}^M\sqrt{\alpha_{i-1}}\beta_i\right)$.

Substituting (17) into (20) leads to

$$\begin{aligned}
\left\|x\left(t\right)\right\| &\leqslant \xi\sqrt{\frac{\overline{\rho}}{\underline{\rho}}}e^{-l\lambda^*T}\left\|x\left(0\right)\right\|\\
&= \xi\sqrt{\frac{\overline{\rho}}{\underline{\rho}}}e^{\lambda^*T}e^{-(l+1)\lambda^*T}\left\|x\left(0\right)\right\|
\end{aligned}$$

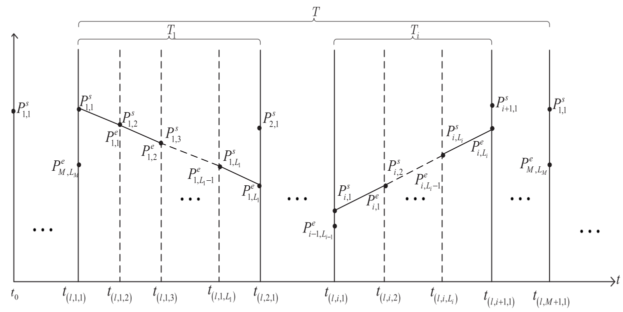

FIGURE 3. A special case of $P\left(t\right)$ in MPTLF.

Noticing that $t < (l+1)T$, $e^{-\lambda^*t} > e^{-(l+1)\lambda^*T}$, it yields that

$$\left\|x\left(t\right)\right\| \leqslant ke^{-\lambda^*t}\left\|x\left(0\right)\right\|$$

Thus the system (1) is exponentially stable.

Remark 2 [31]: investigated the stability for the periodic piecewise linear system by constructing the discontinuous Lyapunov function. Clearly, Theorem 1 can be also applied to such case without impulse effect and can obtain more flexible condition. Specifically, if we select $P^e_{i,m} = \left(1-m/L_i\right)P^s_{i,1} + mP^e_{i,L_i}\big/L_i$, $P^s_{i,m} = \left(1-(m-1)/L_i\right)P^s_{i,1} + (m-1)P^e_{i,L_i}\big/L_i$, $i \in \mathbb{N}_M$, $m \in \mathbb{N}_{L_i}$, i.e. the Lyapunov matrix $P(t)$ of each segment have linear relation as Figure 3, then the MPTLF is degenerated into the Lyapunov function in [31].

In the next, the stabilization of system (1) is dealt with by state feedback. Substituting $u\left(t\right) = K_{i,m}x\left(t\right)$ into (1) with $\omega(t) = 0$ and denote $R^s_{i,m} \triangleq \left(P^s_{i,m}\right)^{-1}$, $R^e_{i,m} \triangleq \left(P^e_{i,m}\right)^{-1}$, then the sufficient conditions (2) and (3) in Theorem 1 are converted into the following bilinear matrix inequalities(BMI),

$$\begin{aligned}
&R^s_{i,m}A_i^T + R^s_{i,m}K^T_{i,m}B_i^T + A_iR^s_{i,m} + B_iK_{i,m}R^s_{i,m}\\
&\quad +\left[R^s_{i,m}(R^e_{i,m})^{-1}R^s_{i,m} - R^s_{i,m}\right]L_i\big/T_i - \lambda_{i,m}R^s_{i,m} \leqslant 0\\
&R^e_{i,m}A_i^T + R^e_{i,m}K^T_{i,m}B_i^T + A_iR^e_{i,m} + B_iK_{i,m}R^e_{i,m}\\
&\quad +\left[R^e_{i,m} - R^e_{i,m}(R^s_{i,m})^{-1}R^e_{i,m}\right]L_i\big/T_i - \lambda_{i,m}R^e_{i,m} \leqslant 0
\end{aligned}$$

where $R^s_{i,m} > 0$, $R^e_{i,m} > 0$.

In general, such BMIs are difficult to settle since there is no polynomial-time solving algorithm [44]. In Theorem 2, the BMIs are degenerated to LMIs by choosing the piecewise Lyapunov matrices $P^s_{i,m} = P^e_{i,m} = P_{i,m}$, as shown in Figure 4.

Theorem 2: Consider periodic piecewise impulsive linear system (1) with $\omega(t) = 0$, given $\lambda^* > 0$, if there exist a set of scalars L_i, $\lambda_{i,m}$, $0 < \eta_i \leqslant 1$, $\varphi_{i-1} > 0$, $u_i > 0$ and matrices $R_{i,m} > 0$ and $Q_{i,m}$, $i \in \mathbb{N}_M$, $m \in \mathbb{N}_{L_i}$ such that

$$\begin{aligned}
&A_iR_{i,m} + B_iQ_{i,m} + R_{i,m}A_i^T + Q^T_{i,m}B_i^T - \lambda_{i,m}R_{i,m} \leqslant 0,\\
&\quad i \in \mathbb{N}_M, m \in \mathbb{N}_{L_i}
\end{aligned} \tag{21}$$

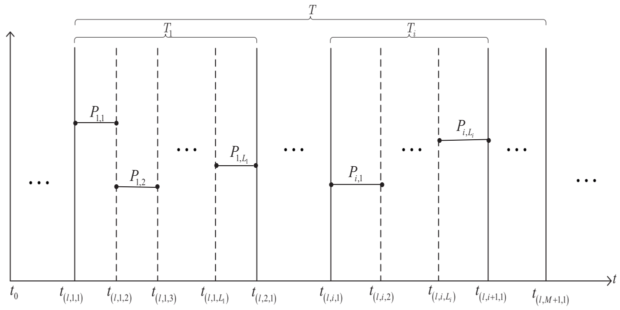

FIGURE 4. Selection of $P(t)$ for Theorem 2.

$$R_{i,m-1} - \eta_i R_{i,m} \leqslant 0, \quad i \in \mathbb{N}_M, m \in \mathbb{N}_{L_i}, m \neq 1 \tag{22}$$
$$(I + D_{i-1})R_{i,1}(I + D_{i-1})^T \leqslant \varphi_{i-1}R_{i,1}, i \in \mathbb{N}_M \tag{23}$$
$$R_{i,L_i} - u_{i+1}R_{i+1,1} \leqslant 0, \quad i \in \mathbb{N}_M, i \neq M \tag{24}$$
$$R_{M,L_M} - u_1 R_{1,1} \leqslant 0 \tag{25}$$
$$\sum_{q=1}^{M} \xi_q + \sum_{q=0}^{M-1} \nu_q + 2\lambda^* T \leqslant 0 \tag{26}$$

where ξ_q, ν_q are given as those in Theorem 1, then the system (1) is $\lambda^* -$ exponentially stable under the state feedback controllers $K_{i,m} = Q_{i,m}(R_{i,m})^{-1}$, $i \in \mathbb{N}_M$, $m \in \mathbb{N}_{L_i}$.

Proof: State feedback $u(t) = K_{i,m}x(t)$, $t \in D_{i,m}$, $i \in \mathbb{N}_M$, $m \in \mathbb{N}_{L_i}$. Let $A_{i,m} \triangleq A_i + B_i K_{i,m}$. Choose the Lyapunov function

$$V(t) = x^T(t) P_{i,m} x(t), \qquad t \in D_{i,m}$$

By the similar proof line of Theorem 1, one can conclude that if

$$A_{i,m}^T P_{i,m} + P_{i,m}A_{i,m} - \lambda_{i,m}P_{i,m} \leqslant 0, \quad i \in \mathbb{N}_M, m \in \mathbb{N}_{L_i} \tag{27}$$
$$P_{i,m} - \eta_i P_{i,m-1} \leqslant 0, \quad i \in \mathbb{N}_M,\ m \in \mathbb{N}_{L_i}, m \neq 1 \tag{28}$$
$$(I + D_{i-1})^T P_{i,1}(I + D_{i-1}) \leqslant \varphi_{i-1}P_{i,1}, \quad i \in \mathbb{N}_M \tag{29}$$
$$P_{i+1,1} - u_{i+1}P_{i,L_i} \leqslant 0, \quad i \in \mathbb{N}_M, i \neq M \tag{30}$$
$$P_{1,1} - u_1 P_{M,L_M} \leqslant 0 \tag{31}$$

hold together with (26), then the system (1) is $\lambda^* -$ exponentially stable.

Let $R_{i,m} \triangleq (P_{i,m})^{-1}$, $Q_{i,m} \triangleq K_{i,m}R_{i,m}$. It is clear that (21) implies (27). By Schur complement, (22) is equivalent to $-(R_{i,m-1})^{-1} + \eta_i{}^{-1}(R_{i,m})^{-1} \leqslant 0$ which leads to (28). By the similar proof line, (29)−(31) can be obtained from (23)−(26). Hence, system (1) can be $\lambda^* -$ exponentially stabilized by the designed controller.

Remark 3: From (26), the convergence speed λ^* of the switched system can be evaluated and controlled by the convergence speed $\lambda_{q,d}$, $q \in \mathbb{N}_M$, $d \in \mathbb{N}_{L_q}$ of subsystems. And, solving a desired λ^* is an optimization problem, which can be solved by some mature optimization algorithms.

Remark 4: The state feedback controllers are designed for subsystems during segments $D_{i,m}$, $i \in \mathbb{N}_M$, $m \in \mathbb{N}_{L_i}$ of dwell time, which can ensure the performance of subsystems during segments and hence ensure the whole performance of subsystem during the dwell time. And note that the controller design conditions are given in the form of LMIs and can be easily implemented on MATLAB.

Next, the L_2- gain problem of periodic piecewise impulsive linear system (1) is investigated by adopting MPTLF. A sufficient condition is proposed to be λ^*- exponentially stable and with L_2- gain performance.

Theorem 3: Consider periodic piecewise impulsive linear system (1) with $u(t) = 0$, given $\lambda^* > 0$, if there exist a set of scalars L_i, $\gamma > 0$, $\lambda_{i,m}$, $0 < \eta_i \leqslant 1$, $\varphi_{i-1} > 0$, $u_i > 1$ satisfying $0 < \varphi_{i-1}u_i\eta_i^{L_i-1} \leqslant 1$ and matrices $P_{i,m}^s > 0$, $P_{i,m}^e > 0$, $i \in \mathbb{N}_M$, $m \in \mathbb{N}_{L_i}$ such that

$$\begin{bmatrix} \Psi\left(P_{i,m}^s\right) + \left(P_{i,m}^e - P_{i,m}^s\right)L_i\big/T_i & P_{i,m}^s C_i & E_i^T \\ * & -\gamma^2 I & G_i^T \\ * & * & -I \end{bmatrix} < 0,$$
$$i \in \mathbb{N}_M, m \in \mathbb{N}_{L_i} \tag{32}$$
$$\begin{bmatrix} \Psi\left(P_{i,m}^e\right)A_i^T + \left(P_{i,m}^e - P_{i,m}^s\right)L_i\big/T_i & P_{i,m}^e C_i & E_i^T \\ * & -\gamma^2 I & G_i^T \\ * & * & -I \end{bmatrix} < 0,$$
$$i \in \mathbb{N}_M, m \in \mathbb{N}_{L_i} \tag{33}$$
$$P_{i,m}^s - \eta_i P_{i,m-1}^e \leqslant 0, \quad i \in \mathbb{N}_M, m \in \mathbb{N}_{L_i}, m \neq 1 \tag{34}$$
$$(I + D_{i-1})^T P_{i,1}^s (I + D_{i-1}) \leqslant \varphi_{i-1}P_{i,1}^s, \quad i \in \mathbb{N}_M \tag{35}$$
$$P_{i+1,1}^s - u_{i+1}P_{i,L_i}^e \leqslant 0, \quad i \in \mathbb{N}_M, i \neq M \tag{36}$$
$$P_{1,1}^s - u_1 P_{M,L_M}^e \leqslant 0 \tag{37}$$
$$\sum_{q=1}^{M} \xi_q + \sum_{q=0}^{M-1} \nu_q + 2\lambda^* T \leqslant 0 \tag{38}$$
$$\sum_{q=0}^{M-1} \nu_q \geqslant -\lambda^* T \tag{39}$$

where $\Psi(X) = A_i^T X + XA_i - \lambda_{i,m}X$, then the system (1) is λ^* - exponentially stable and satisfies

$$\int_0^\infty e^{\left(\lambda_{Min}^{\downarrow} - \lambda^*\right)s} y^T(s)\, y(s)\, ds \leqslant \delta\gamma^2 \int_0^\infty \omega^T(s)\, \omega(s)\, ds \tag{40}$$

where ξ_q, ν_q are given as in Theorem 1, $\varepsilon = \min_{j \in N_M}\left(\ln \eta_j^{L_j-1}\right)$, $\lambda_{Min}^{\downarrow} = \min_{q \in \mathbb{N}_M^{\downarrow}}\left(\min_{d \in \mathbb{N}_{L_q}}\left(\lambda_{q,d}\right)\right)$, $\lambda_{Max}^{\uparrow} = \max_{q \in \mathbb{N}_M^{\uparrow}}\left(\max_{d \in \mathbb{N}_{L_q}}\left(\lambda_{q,d}\right)\right)$, $\delta = \frac{\lambda^* - \lambda_{Min}^{\downarrow}}{\lambda^*} \cdot e^{-\varepsilon + \left(2\lambda_{Max}^{\uparrow} + 3\lambda^*\right)T}$, $\mathbb{N}_M^{\downarrow}$ and $\mathbb{N}_M^{\uparrow}$ indicate the set of stable subsystems and the set of unstable subsystems respectively.

Proof: It is clear that conditions (32), (33) imply (2) and (3) respectively. Based on Theorem 1, one can see from (34)-(38) that the system (1) is $\lambda^* -$ exponentially stable. In the following, we show that the system is with expected L_2- gain performance.

For $t \in D_{i,m} = \left[t_{(l,i,m)},\ t_{(l,i,m+1)}\right)$, by (32) and (33), we have

$$\dot{V}(t) \leqslant \lambda_{i,m}V(t) - \Gamma(t) \tag{41}$$

where $\Gamma(t) = y^T(t)\,y(t) - \gamma^2\omega^T(t)\,\omega(t)$. Noticing (41) and (33)–(37), it yields

$$
\begin{aligned}
&V(t)\\
&\le \varphi_{i-1}u_i\eta_i^{m-1}\prod_{q=0}^{i-2}\left(\varphi_q u_{q+1}\eta_{q+1}^{L_{q+1}-1}\right)\\
&\quad\times e^{\lambda_{i,m}\left(t-t_{(l,i,m)}\right)+\frac{T_i}{L_i}\sum\limits_{q=1}^{m-1}\lambda_{i,q}+\sum\limits_{q=1}^{i-1}\xi_q} V\left((lT)^-\right) - \varphi_{i-1}u_i\\
&\quad\times\eta_i^{m-1}e^{\lambda_{i,m}\left(t-t_{(l,i,m)}\right)+\frac{T_i}{L_i}\sum\limits_{q=1}^{m-1}\lambda_{i,q}}\sum_{z=1}^{i-1}\left[\prod_{q=z}^{i-2}\left(\varphi_q u_{q+1}\eta_{q+1}^{L_{q+1}-1}\right)\right.\\
&e^{\sum\limits_{q=z+1}^{i-1}\xi_q}\left(\sum_{k=0}^{L_z-1}\eta_z^k e^{\frac{T_z}{L_z}\sum\limits_{q=0}^{k-1}\lambda_{z,L_z-q}}\right.\\
&\quad\left.\left.\times\int_{t_{(l,z,L_z-k)}}^{t_{(l,z,L_z-k+1)}} e^{\lambda_{z,L_z-k}\left(t_{(l,z,L_z-k+1)}-s\right)}\Gamma(s)ds\right)\right]\\
&\quad-\sum_{k=1}^{m-1}\eta_i^k e^{\lambda_{i,m}\left(t-t_{(l,i,m)}\right)+\frac{T_i}{L_i}\sum\limits_{q=1}^{k-1}\lambda_{i,m-q}}\\
&\quad\times\int_{t_{(l,i,m-k)}}^{t_{(l,i,m-k+1)}} e^{\lambda_{i,m-k}\left(t_{(l,i,m-k+1)}-s\right)}\Gamma(s)ds\\
&\quad-\int_{t_{(l,i,m)}}^{t} e^{\lambda_{i,m}(t-s)}\Gamma(s)ds
\end{aligned}
$$

Integrating to initial instant and reconstructing inequality, it leads to

$$
\begin{aligned}
V(t) &\leqslant e^{\theta_0}V(0)\\
&\quad-\sum_{h=0}^{l-1}\sum_{z=1}^{M}\sum_{k=0}^{L_z-1}\int_{(l-(h+1),z,L_z-k)}^{t_{(l-(h+1),z,L_z-k+1)}} e^{\theta_1(h,z,k)}\Gamma(s)ds\\
&\quad-\sum_{z=1}^{i-1}\sum_{k=0}^{L_z-1}\int_{t_{(l,z,L_z-k)}}^{t_{(l,z,L_z-k+1)}} e^{\theta_2(z,k)}\Gamma(s)ds\\
&\quad-\sum_{k=1}^{m-1}\int_{t_{(l,i,m-k)}}^{t_{(l,i,m-k+1)}} e^{\theta_3(k)}\Gamma(s)ds\\
&\quad-\int_{t_{(l,i,m)}}^{t} e^{\theta_4}\Gamma(s)ds
\end{aligned}
\tag{42}
$$

where

$$
\begin{aligned}
\theta_0 &= \psi + \lambda_{i,m}\left(t-t_{(l,i,m)}\right) + l\left(\sum_{q=1}^{M}\xi_q\right.\\
&\quad\left.+\sum_{q=0}^{M-1}\nu_q\right) - \ln u_1\\
\theta_1(h,z,k) &= \psi + \vartheta(h,z,k) + \lambda_{i,m}\left(t-t_{(l,i,m)}\right)\\
&\quad+h\left(\sum_{q=1}^{M}\xi_q + \sum_{q=0}^{M-1}\nu_q\right) + k\ln\eta_z + \sum_{q=z}^{M-1}\nu_q\\
&\quad+\sum_{q=z+1}^{M}\xi_q + \frac{T_z}{L_z}\sum_{q=0}^{k-1}\lambda_{z,L_z-q}\\
\theta_2(z,k) &= \lambda_{i,m}\left(t-t_{(l,i,m)}\right) + \vartheta(h,z,k)|_{h=-1}\\
&\quad+(m-L_i)\ln\eta_i + k\ln\eta_z + \frac{T_i}{L_i}\sum_{q=1}^{m-1}\lambda_{i,q}\\
&\quad+\sum_{q=z}^{i-1}\nu_q + \sum_{q=z+1}^{i-1}\xi_q + \frac{T_z}{L_z}\sum_{q=0}^{k-1}\lambda_{z,L_z-q}\\
\theta_3(k) &= \lambda_{i,m}\left(t-t_{(l,i,m)}\right) + \lambda_{i,m-k}\left(t_{(l,i,m-k+1)}-s\right)\\
&\quad+k\ln\eta_i + \frac{T_i}{L_i}\sum_{q=1}^{k-1}\lambda_{i,m-q}\\
\theta_4 &= \lambda_{i,m}(t-s)
\end{aligned}
$$

with

$$
\begin{aligned}
\psi &= (m-L_i)\ln\eta_i + \frac{T_i}{L_i}\sum_{q=1}^{m-1}\lambda_{i,q} + \sum_{q=0}^{i-1}\nu_q + \sum_{q=1}^{i-1}\xi_q,\\
\vartheta(h,z,k) &= \lambda_{z,L_z-k}\left(t_{(l-h-1,z,L_z-k+1)}-s\right).
\end{aligned}
$$

Multiply both sides of (42) by e^{β}, where $\beta = (L_i - m)\ln\eta_i - \sum\limits_{q=0}^{i-1}\nu_q$. Under zero initial condition and noticing $V(t) \geqslant 0$, it follows

$$
\begin{aligned}
&\sum_{h=0}^{l-1}\sum_{z=1}^{M}\sum_{k=0}^{L_z-1}\int_{t_{(l-(h+1),z,L_z-k)}}^{t_{(l-(h+1),z,L_z-k+1)}} e^{\Omega_1(h,z,k)}y^T(s)y(s)\,ds\\
&\quad+\sum_{z=1}^{i-1}\sum_{k=0}^{L_z-1}\int_{t_{(l,z,L_z-k)}}^{t_{(l,z,L_z-k+1)}} e^{\Omega_2(z,k)}y^T(s)\,y(s)ds\\
&\quad+\sum_{k=1}^{m-1}\int_{t_{(l,i,m-k)}}^{t_{(l,i,m-k+1)}} e^{\Omega_3(k)}y^T(s)\,y(s)ds\\
&\quad+\int_{t_{(l,i,m)}}^{t} e^{\Omega_4}y^T(s)\,y(s)ds\\
&\quad\le\gamma^2\times\\
&\left\{\sum_{h=0}^{l-1}\sum_{z=1}^{M}\sum_{k=0}^{L_z-1}\int_{t_{(l-(h+1),z,L_z-k)}}^{t_{(l-(h+1),z,L_z-k+1)}} e^{\Phi_1(h,z,k)}\omega^T(s)\omega(s)\,ds\right.\\
&\quad+\sum_{z=1}^{i-1}\sum_{k=0}^{L_z-1}\int_{t_{(l,z,L_z-k)}}^{t_{(l,z,L_z-k+1)}} e^{\Phi_2(z,k)}\omega^T(s)\,\omega(s)ds\\
&\quad+\sum_{k=1}^{m-1}\int_{t_{(l,i,m-k)}}^{t_{(l,i,m-k+1)}} e^{\Phi_3(k)}\omega^T(s)\,\omega(s)ds\\
&\quad\left.+\int_{t_{(l,i,m)}}^{t} e^{\Phi_4}\omega^T(s)\,\omega(s)ds\right\}
\end{aligned}
\tag{43}
$$

where

$$
\begin{aligned}
&\Omega_1(h,z,k)\\
&\quad= \lambda_{i,m}\left(t-t_{(l,i,m)}\right) + \varpi(z,k) + \vartheta(h,z,k)\\
&\quad+h\left(\sum_{q=1}^{M}\xi_q + \sum_{q=0}^{M-1}\nu_q\right)\sum_{q=z}^{M-1}\nu_q + \sum_{q=1}^{i-1}\xi_q + \sum_{q=z+1}^{M}\xi_q
\end{aligned}
$$

$$\begin{aligned}
&\Omega_2(z,k)\\
&\quad=\lambda_{i,m}\left(t-t_{(l,i,m)}\right)+\varpi(z,k)+\vartheta(h,z,k)|_{h=-1}\\
&\qquad-\sum_{q=0}^{z-1}\nu_q+\sum_{q=z+1}^{i-1}\xi_q\\
&\Omega_3(k)\\
&\quad=\lambda_{i,m}\left(t-t_{(l,i,m)}\right)+\lambda_{i,m-k}\left(t_{(l,i,m-k+1)}-s\right)\\
&\qquad+(k-m+L_i)\ln\eta_i-\sum_{q=0}^{i-1}\nu_q+\frac{T_i}{L_i}\sum_{q=1}^{k-1}\lambda_{i,m-q}\\
&\Omega_4\\
&\quad=\lambda_{i,m}(t-s)+(L_i-m)\ln\eta_i-\sum_{q=0}^{i-1}\nu_q\\
&\Phi_1(h,z,k)\\
&\quad=\Omega_1(h,z,k)-k\ln\eta_z-\sum_{q=z}^{M-1}\nu_q\\
&\Phi_2(z,k)\\
&\quad=\Omega_2(z,k)-\sum_{q=z}^{M-1}\nu_q-k\ln\eta_z\\
&\Phi_3(k)\\
&\quad=\Omega_3(k)-\sum_{q=i}^{M-1}\nu_q-(k-m+L_i)\ln\eta_i\\
&\Phi_4\\
&\quad=\Omega_4-\sum_{q=i}^{M-1}\nu_q-(L_i-m)\ln\eta_i
\end{aligned}$$

with $\varpi(z,k)=\frac{T_i}{L_i}\sum_{q=1}^{m-1}\lambda_{i,q}+\frac{T_z}{L_z}\sum_{q=0}^{k-1}\lambda_{z,L_z-q}+k\ln\eta_z$.

Clearly, from the value range of the related parameters, one can see that $\Phi_i\geqslant\Omega_i$, $i=1,2,3,4$.

For convenience of description, $T^{\uparrow}\left(t_\iota-t_\varsigma\right)$ and $T^{\downarrow}\left(t_\iota-t_\varsigma\right)$ are used to denote the time length that $\lambda_{q,d}$ is positive and $\lambda_{q,d}$ is negative within the time interval $\left[t_\iota,t_\varsigma\right)$, respectively.

By (39), it follows that

$$\begin{aligned}
&\Omega_1(h,z,k)\\
&\geqslant-(h+1)\lambda^*T+k\ln\eta_z+\lambda_{Min}^{\downarrow}(t-s)\\
&=k\ln\eta_z+\left(\lambda_{Min}^{\downarrow}-\lambda^*\right)(t-s)+\lambda^*[t-s-(h+1)T]\\
&\geqslant k\ln\eta_z+\left(\lambda_{Min}^{\downarrow}-\lambda^*\right)(t-s)+\lambda^*[t-(l+1)T]\\
&\geqslant k\ln\eta_z+\left(\lambda_{Min}^{\downarrow}-\lambda^*\right)(t-s)-\lambda^*T\\
&\geqslant\varepsilon+\left(\lambda_{Min}^{\downarrow}-\lambda^*\right)t-\lambda^*T
\end{aligned}\tag{44}$$

$$\begin{aligned}
&\Omega_2(z,k)\\
&\geqslant k\ln\eta_z+\lambda_{Min}^{\downarrow}(t-s)\\
&\geqslant k\ln\eta_z+\left(\lambda_{Min}^{\downarrow}-\lambda^*\right)(t-s)-\lambda^*T\\
&\geqslant\varepsilon+\left(\lambda_{Min}^{\downarrow}-\lambda^*\right)t-\lambda^*T
\end{aligned}\tag{45}$$

$$\begin{aligned}
&\Omega_3(k)\\
&\geqslant\ln\eta_i^{L_i-1}+\lambda_{Min}^{\downarrow}(t-s)\\
&\geqslant\ln\eta_i^{L_i-1}+\left(\lambda_{Min}^{\downarrow}-\lambda^*\right)(t-s)-\lambda^*T\\
&\geqslant\varepsilon+\left(\lambda_{Min}^{\downarrow}-\lambda^*\right)t-\lambda^*T
\end{aligned}\tag{46}$$

$$\begin{aligned}
&\Omega_4\\
&\geqslant\ln\eta_i^{L_i-1}+\lambda_{Min}^{\downarrow}(t-s)\\
&\geqslant\varepsilon+\left(\lambda_{Min}^{\downarrow}-\lambda^*\right)t-\lambda^*T
\end{aligned}\tag{47}$$

where $\varepsilon=\min_{j\in N_M}\left(\ln\eta_j^{L_j-1}\right)$. By (41), we have

$$\begin{aligned}
&\Phi_1(h,z,k)\\
&\quad\leqslant\lambda_{Max}^{\uparrow}T^{\uparrow}(t-lT)+\lambda_{Max}^{\downarrow}T^{\downarrow}(t-lT)\\
&\qquad+\lambda_{Max}^{\uparrow}T^{\uparrow}[(l-h)T-s]+\lambda_{Max}^{\downarrow}T^{\downarrow}[(l-h)T-s]\\
&\qquad-h\sum_{q=1}^{M}\frac{T_q}{L_q}\sum_{d=1}^{L_q}\lambda_{q,d}-2\lambda^*hT+h\sum_{q=1}^{M}\frac{T_q}{L_q}\sum_{d=1}^{L_q}\lambda_{q,d}\\
&\quad\leqslant2\lambda_{Max}^{\uparrow}T-\lambda^*(t-s)+\lambda^*(t-s-2hT)\\
&\quad\leqslant2\lambda_{Max}^{\uparrow}T-\lambda^*(t-s)+\lambda^*[t-(l-1)T]\\
&\quad\leqslant-\lambda^*(t-s)+\left(\lambda_{Max}^{\uparrow}+\lambda^*\right)2T
\end{aligned}\tag{48}$$

$$\begin{aligned}
&\Phi_2(z,k)\\
&\quad\leqslant\lambda^*T+\lambda_{Max}^{\downarrow}T^{\downarrow}(t-s)+\lambda_{Max}^{\uparrow}T^{\uparrow}(t-s)\\
&\quad\leqslant-\lambda^*(t-s)+\lambda^*T+\left(\lambda_{Max}^{\uparrow}+\lambda^*\right)T\\
&\quad\leqslant-\lambda^*(t-s)+\left(\lambda_{Max}^{\uparrow}+\lambda^*\right)2T
\end{aligned}\tag{49}$$

$$\begin{aligned}
&\Phi_3(k)\\
&\quad\leqslant\lambda^*T+\lambda_{Max}^{\downarrow}T^{\downarrow}(t-s)+\lambda_{Max}^{\uparrow}T^{\uparrow}(t-s)\\
&\quad\leqslant-\lambda^*(t-s)+\left(\lambda_{Max}^{\uparrow}+\lambda^*\right)2T
\end{aligned}\tag{50}$$

$$\begin{aligned}
&\Phi_4\\
&\quad\leqslant\lambda^*T+\lambda_{Max}^{\uparrow}(t-s)\\
&\quad\leqslant-\lambda^*(t-s)+\left(\lambda_{Max}^{\uparrow}+\lambda^*\right)2T
\end{aligned}\tag{51}$$

where $\lambda_{Max}^{\downarrow}=\max_{q\in\mathbb{N}_M^{\downarrow}}\left(\max_{d\in\mathbb{N}_{L_q}}\left(\lambda_{q,d}\right)\right)$.

Substituting (44)–(51) into (43) leads to

$$\begin{aligned}
&\int_0^t e^{\varepsilon+\left(\lambda_{Min}^{\downarrow}-\lambda^*\right)t-\lambda^*T}y^T(s)\,y(s)\,ds\\
&\qquad\leqslant\gamma^2\int_0^t e^{-\lambda^*(t-s)+\left(\lambda_{Max}^{\uparrow}+\lambda^*\right)2T}\omega^T(s)\,\omega(s)\,ds
\end{aligned}$$

Integrating from 0 to ∞ on both sides of this inequality, one gets (40). This completes the proof.

Remark 5: From (40), one can see the L_2-gain performance is related to parameters, the convergence speed of the switched system, the convergence speed of subsystems during segments of dwell time and the number of segments of dwell time.These parameters can be designed to make the system meet certain L_2- performance indexes.

Remark 6: Similar to Theorem 2, the introduction of state feedback controller $u(t)=K_{i,m}x(t)$ render the sufficient condition to be BMIs in Theorem 3, which can be also converted to LMIs by adopting the piecewise Lyapunov matrices $P^s_{i,m} = P^e_{i,m} = P_{i,m}$.

Remark 7: It should be noted that in the derivation of L_2-gain problem in Theorem 3, to ensure the condition (38) and (39) are satisfied simultaneously, at least one subsystem of the switched system should be stable. However, such requirement does not need in Theorem 1 and 2 which only concern with the issue of stability.

IV. NUMERICAL EXAMPLE

Example 1: Consider a periodic piecewise impulsive linear system (1) with the two subsystems, and the dwell time of each subsystems are $T_1 = 1.5s$, $T_2 = 0.1s$ respectively.

$$A_1 = \begin{bmatrix} -0.2 & 1 \\ -0.6 & -0.2 \end{bmatrix}, \quad B_1 = 0, C_1 = \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} 0.1 & 0.5 \end{bmatrix}, \quad F_1 = 0, \ G_1 = 0,$$
$$A_2 = \begin{bmatrix} 2 & -2.8 \\ 2.5 & -1.8 \end{bmatrix}, \quad B_2 = 0, \ C_2 = \begin{bmatrix} -0.1 \\ 0.2 \end{bmatrix},$$
$$E_2 = \begin{bmatrix} 0.1 & 0.5 \end{bmatrix}, \quad F_2 = 0, \ G_2 = 0,$$
$$D_0 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0.015 & 0 \\ 0 & 0.025 \end{bmatrix}$$

Given a set of scalars as below and solving (32)-(39) in Theorem 3, $L_1 = 3$, $L_2 = 2$, $\lambda_{1,1} = \lambda_{1,2} = \lambda_{1,3} = -0.29$, $\lambda_{2,1} = \lambda_{2,2} = 3$, $\eta_1 = 0.82$, $\eta_2 = 0.68$, $\varphi_0 = 1.0405$, $\varphi_1 = 1.051$, $u_1 = 1.395$, $u_2 = 1.45$, $\gamma = \sqrt{2}$, one can obtain the MPTLF matrices as below, therefore the system (1) is λ^*- exponentially stable with $\lambda^* \leqslant 0.0387$, and with L_2- gain performance. Under $\omega(t) = 0$, Figure 5 shows the trajectory $\|x(t)\|$ of system (1) with $x(0) = [0.1, 0.3]^T$ and the corresponding state trajectory.

$$P^s_{11} = \begin{bmatrix} 8.1186 & 0.6122 \\ 0.6122 & 19.9152 \end{bmatrix} P^e_{11} = \begin{bmatrix} 8.6670 & 1.9728 \\ 1.9728 & 18.3066 \end{bmatrix}$$
$$P^s_{12} = \begin{bmatrix} 7.0931 & 1.6176 \\ 1.6176 & 14.9267 \end{bmatrix} P^e_{12} = \begin{bmatrix} 7.9832 & 1.9018 \\ 1.9018 & 12.8026 \end{bmatrix}$$
$$P^s_{13} = \begin{bmatrix} 6.5338 & 1.5572 \\ 1.5572 & 10.4356 \end{bmatrix} P^e_{13} = \begin{bmatrix} 7.1719 & 0.9250 \\ 0.9250 & 8.8000 \end{bmatrix}$$
$$P^s_{21} = \begin{bmatrix} 10.3809 & 1.3292 \\ 1.3292 & 12.6437 \end{bmatrix} P^e_{21} = \begin{bmatrix} 9.4392 & 1.2602 \\ 1.2602 & 16.4729 \end{bmatrix}$$
$$P^s_{22} = \begin{bmatrix} 6.4076 & 0.8587 \\ 0.8587 & 11.1036 \end{bmatrix} P^e_{21} = \begin{bmatrix} 5.8321 & 0.4245 \\ 0.4245 & 14.3943 \end{bmatrix}$$

Set $D_0 = D_1 = 0$, then the periodic piecewise impulsive system turns to be periodic piecewise system. Set $L_1 = L_2 = 1$, $\lambda_{1,1} = -0.29, \lambda_{2,1} = 3, u_1 = 1.395, u_2 = 1.45$. From Theorem 2 in [31], one can obtain $\lambda^* \leqslant -0.1778$, which indicates the system is not exponentially stable. Hence, compared with the Theorem 2 in [31], Theorem 1 of this paper is more efficient to obtain feasible solution.

Then the L_2- gain performance of the system (1) is analyzed. Choose the disturbance $\omega(t) = \sin(2\pi t)\, e^{-0.5t}$. Under zero-initial conditions, the system output is shown in Figure 6.

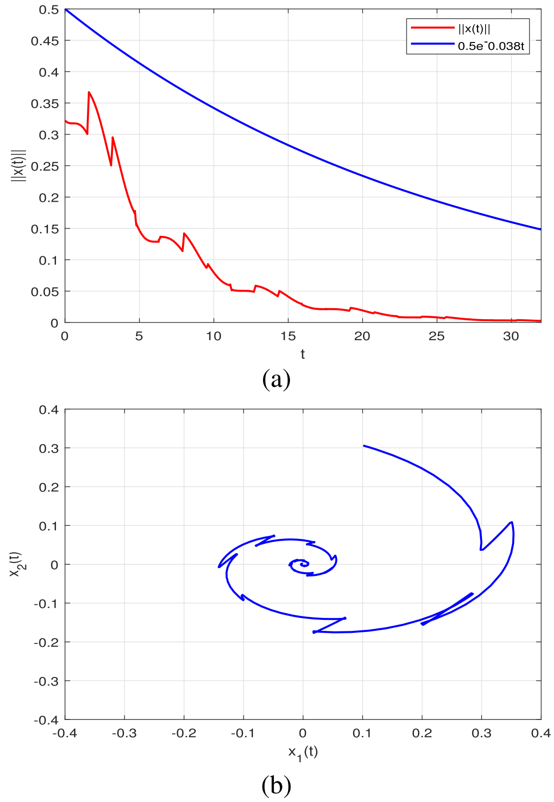

FIGURE 5. (a) The trajectory $\|x(t)\|$ of system (1) with $x(0) = [0.1, 0.3]^T$. (b) The corresponding state trajectory.

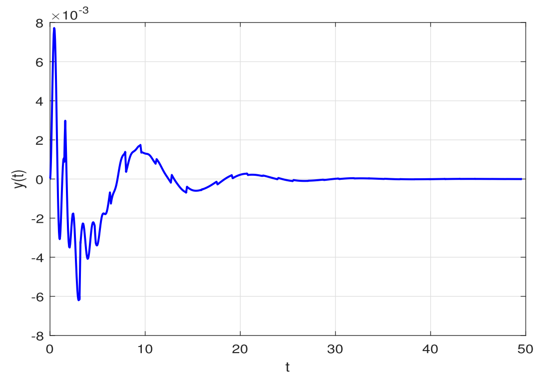

FIGURE 6. The output trajectory of $y(t)$.

Example 2: Consider a periodic piecewise impulsive linear system (1) with two subsystems, the dwell time of each subsystems, D_0 and D_1 are same as those in Example 1.

$$A_1 = \begin{bmatrix} 0.2 & 1 \\ -0.6 & -0.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 2 & -2.8 \\ 2.5 & -1.8 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

Clearly, all the subsystems are unstable. Next, the state feedback controllers are designed to stabilize the system. Given a set of scalars as below and solving (21)-(26) in Theorem 2, $L_1 = 3$, $L_2 = 2$, $\lambda_{1,1} = -0.4$, $\lambda_{1,2} = -0.5$, $\lambda_{1,3} = -0.6$,

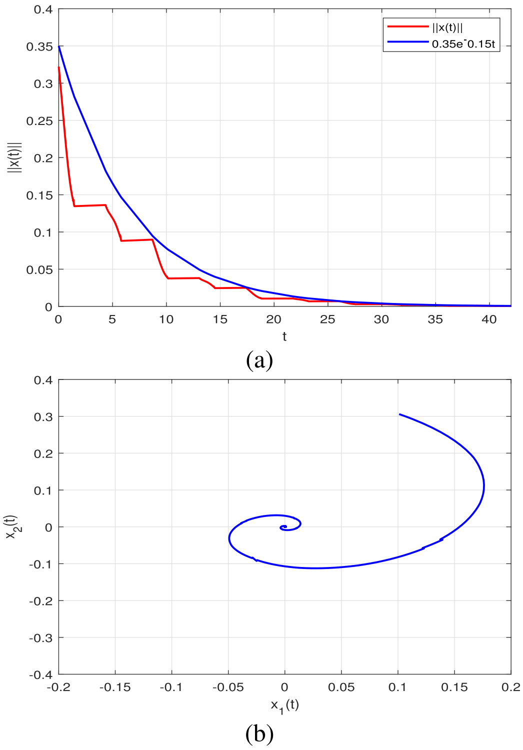

FIGURE 7. (a) The trajectory $\|x(t)\|$ of system (1) with $x(0) = [0.1, 0.3]^T$. (b) The corresponding state trajectory.

$\lambda_{2,1} = -1.4$, $\lambda_{2,2} = -1.5$, $\eta_1 = 0.82$, $\eta_2 = 0.68$, $\varphi_0 = 1.0405$, $\varphi_1 = 1.051$, $u_1 = 1.6$, $u_2 = 1.4$. One can obtain the matrices $R_{i,m}$, $Q_{i,m}$, $i \in \mathbb{N}_M$, $m \in \mathbb{N}_{L_i}$ as below,

$$R_{1,1} = \begin{bmatrix} 9.5364 & 0.6708 \\ 0.6708 & 18.1771 \end{bmatrix},$$
$$R_{1,2} = \begin{bmatrix} 11.6886 & 0.8217 \\ 0.8217 & 22.2700 \end{bmatrix}$$
$$R_{1,3} = \begin{bmatrix} 14.3200 & 1.0069 \\ 1.0069 & 27.2900 \end{bmatrix},$$
$$R_{2,1} = \begin{bmatrix} 10.2794 & 0.7299 \\ 0.7299 & 19.5891 \end{bmatrix}$$
$$R_{2,2} = \begin{bmatrix} 15.1833 & 1.0689 \\ 1.0689 & 28.9545 \end{bmatrix},$$
$$Q_{1,1} = \begin{bmatrix} -7.2238 & -2.3360 \end{bmatrix}$$
$$Q_{1,2} = \begin{bmatrix} -8.9013 & -3.4402 \end{bmatrix},$$
$$Q_{1,3} = \begin{bmatrix} -10.9982 & -4.9552 \end{bmatrix}$$
$$Q_{2,1} = \begin{bmatrix} -13.4520 & -6.1922 \end{bmatrix},$$
$$Q_{2,2} = \begin{bmatrix} -19.8209 & -8.8987 \end{bmatrix}$$

Then the gains of state feedback controllers are obtained by $K_{i,m} = Q_{i,m}\left(R_{i,m}\right)^{-1}$, $i \in \mathbb{N}_M$, $m \in \mathbb{N}_{L_i}$,

$$K_{1,1} = \begin{bmatrix} -0.7504 & -0.1008 \end{bmatrix},$$
$$K_{1,2} = \begin{bmatrix} -0.7526 & -0.1267 \end{bmatrix}$$
$$K_{1,3} = \begin{bmatrix} -0.7571 & -0.1536 \end{bmatrix},$$
$$K_{2,1} = \begin{bmatrix} -1.2897 & -0.2685 \end{bmatrix}$$
$$K_{2,2} = \begin{bmatrix} -1.2872 & -0.2598 \end{bmatrix}$$

The closed-loop system matrices are obtained by $A_{i,m} = A_i + B_i K_{i,m}$, $i \in \mathbb{N}_M$, $m \in \mathbb{N}_{L_i}$,

$$A_{1,1} = \begin{bmatrix} -0.5504 & 0.8992 \\ -1.3504 & -0.3008 \end{bmatrix},$$
$$A_{1,2} = \begin{bmatrix} -0.5526 & 0.8733 \\ -1.3526 & -0.3267 \end{bmatrix}$$
$$A_{1,3} = \begin{bmatrix} -0.5571 & 0.8464 \\ -1.3571 & -0.3536 \end{bmatrix},$$
$$A_{2,1} = \begin{bmatrix} -1.0897 & 0.7315 \\ -1.8897 & -0.4685 \end{bmatrix},$$
$$A_{2,2} = \begin{bmatrix} -1.0872 & 0.7402 \\ -1.8872 & -0.4598 \end{bmatrix}$$

Therefore the system (1) is λ^*-exponentially stable under the state feedback controllers with $\lambda^* = 0.15$, as shown in Figure 7.

V. CONCLUSION

In this paper, the λ^*- exponential stability, stabilization and L_2- gain analysis of periodic piecewise impulsive linear system are investigated. Firstly, we propose a sufficient condition of λ^*- exponentially stability for periodic piecewise impulsive linear system by constructing the so-called MPTLF. Next, the obtained result is used to design state feedback controller by solving LMIs. Then for the case that the time-varying disturbance exists, sufficient condition to guarantee L_2- gain performance is presented. Finally, numerical examples are given to verify the effectiveness of the proposed results. Note that the proposed Lyapunov function is time-varying quadratic, further research is needed to consider some more general nonlinear Lyapunov functions. On the other hand, we will consider adaptive control when the state information is unknown or SMC when system parameters are uncertain. The influence of pulse characteristics, such as impulse occur sequences and impulses gain, on system stability is also a future research direction.

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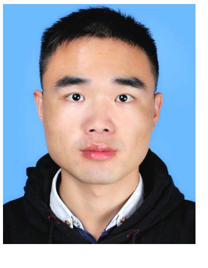

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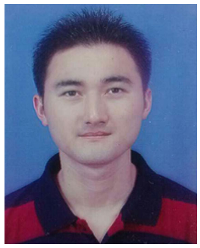

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