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# On Topological Indices for Swapped Networks Modeled by Optical Transpose Interconnection System

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**ABSTRACT** The optical transpose interconnection system (OTIS) network has many application in architecture for parallel as well as in distributed network. The optical translate interconnection system utilizes a straightforward pair of lenslet clusters to execute a coordinated interconnection that is valuable for mix based multistage interconnection networks, work of-trees network processors, and hypercubes. In Cancan (2019) and Hayat *et al.* (2014), different interconnection network has studied related to topological indices. In this article we have computed the Ve-degree and Ev-degree base topological indices of swapped network by taking path and complete graph as original graphs. We have included some dedicated formulas for different types of topological indices for the OTIS swapped network by taking the path and complete graph on $n$ vertices as basis of graph.

**INDEX TERMS** Optical transpose interconnection system (OTIS), swapped network, ev-degree, ve-degree, path, complete graph, topological indices.

## I. INTRODUCTION

The role of graph theory is rapidly increasing day by day especially in chemistry. There is a branch related to chemistry, mathematics and computer science and their components includes quantitative structure-property relationship (*QSPR*) and quantitative structure-activity relationship (*QSAR*) and the component can contribute to the research on physicochemical properties of chemical compound. Assigning numbers to a molecular graph has very nice properties in chemistry. A drawing, a sequence of numbers, a polynomial, a numeric number, or a matrix are all ways to recognized a graph [1]–[4], [9].

A topological index is a numeric quantity associated with a graph that characterizes the topology of the graph and is invariant under the graph automorphism. To study of quantitative structure-property relationship (*QSPR*) and quantitative structure-activity relationship (*QSAR*), many topological indices are widely used and are of great importance in modern chemistry and biochemistry [7], [8]. To obtain a significance correlation, it is essential that appropriate descriptors are employed, whether they are theoretical, empirical or derived from readily available experimental characteristics of structure. Numerous application of graph theory can be found in networking. Its first and popular application in chemistry was the boiling point of the paraffin by Wiener [29].

The primary goal of the OTIS is to develop competency Contact the new optoelectronic computer engineering. This is the property of this network gives benefit to both optical and electronic technologies [15]. In OTIS networks, processors they are organized by groups. Electronic connections are used between processors within the same group, while optical links used to communicate between groups. There are many algorithms for routing, selection/sorting [12], [14], [21], [26], for numerical computation [20], Fourier transformation [13], matrix multiplication [11], and image processing [10].

A network can be represented by an interconnected structure mathematically by graphing. As the graph has vertices that can be represented by processor nodes and edges represent links between these nodes / processors [15]. Determine the topology of the graph the way the heads are attached to the edges. We can have it some network properties using graph topology. The maximum distance between any two network heads is grid diameter. The number of links connected to the node determine the degree of that node [27], [28], [30].

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## II. PRELIMINARIES

In this section, we provide some basic concepts. Let $G = (V, E)$ be a graph with vertex set $V$ and edge set $E$. A **graph** $G = (V, E)$ with two nonempty sets $V$ and $E$. The elements of $V$ are called vertices and the elements of $E$ are called edges. The **degree** of a vertex $v$ denoted by $deg(v)$, is the number of different edges that are incident to a vertex $v$. The set of all vertices which are adjacent to a vertex $v$ is called the **open neighborhood** of $v$ and is denoted by $N(v)$. If we include the vertex $v$ to the set $N(v)$, then we get the **closed neighborhood** of $v$, denoted by $N[v]$.

In [5] defined the **ev-degree** of the edge $e = uv \in E$, denoted by $\widetilde{\Upsilon}_{ev}(e)$, the number of vertices of the union of the closed neighborhoods of $u$ and $v$. The **ve-degree** in [5] of the vertex $v \in V$, denoted by $\widetilde{\Upsilon}_{ve}(v)$, is the number of different edges that are incident to any vertex from the closed neighborhood of $v$. Let $G$ be a connected graph and $e = uv \in E(G)$. For definitions related to ev-degree and ve-degree topological indices, we refere [4], [5], [7], [17].

*Definition 1:* Let $G$ be a connected graph and $v \in V(G)$. The *ev*-degree Zagreb index is defined as:

$$\mathcal{M}^{ev}(G) = \sum_{e \in E(G)} \widetilde{\Upsilon}_{ev}(e)^2. \quad (1)$$

*Definition 2:* The first ve-degree Zagreb alpha index is define as:

$$\mathcal{M}_1^{\alpha ve}(G) = \sum_{v \in V(G)} \widetilde{\Upsilon}_{ve}(v)^2. \quad (2)$$

*Definition 3:* The first ve-degree Zagreb beta index is define as:

$$\mathcal{M}_1^{\beta ve}(G) = \sum_{uv \in E(G)} (\widetilde{\Upsilon}_{ve}(u) + \widetilde{\Upsilon}_{ve}(v)). \quad (3)$$

*Definition 4:* The second *ve*-degree Zagreb index is define as:

$$\mathcal{M}_2^{ve}(G) = \sum_{uv \in E(G)} (\widetilde{\Upsilon}_{ve}(u) \times \widetilde{\Upsilon}_{ve}(v)). \quad (4)$$

*Definition 5:* The *ve*-degree Randic index is define as:

$$\mathcal{R}^{ve}(G) = \sum_{uv \in E(G)} (\widetilde{\Upsilon}_{ve}(u) \times \widetilde{\Upsilon}_{ve}(v))^{-\frac{1}{2}}. \quad (5)$$

*Definition 6:* The *ev*-degree Randic index is define as:

$$\mathcal{R}^{ev}(G) = \sum_{e \in E(G)} \widetilde{\Upsilon}_{ve}(e)^{-\frac{1}{2}}. \quad (6)$$

*Definition 7:* The ve-degree atom-bond connectivity index is define as:

$$\mathcal{ABC}^{ve}(G) = \sum_{uv \in E(G)} \sqrt{\frac{\widetilde{\Upsilon}_{ve}(u) + \widetilde{\Upsilon}_{ve}(v) - 2}{\widetilde{\Upsilon}_{ve}(u) \times \widetilde{\Upsilon}_{ve}(v)}}. \quad (7)$$

*Definition 8:*The ve-degree geometric-arithmetic (ve-GA) index is define as:

$$\mathcal{GA}^{ve}(G) = \sum_{uv \in E(G)} \frac{2\sqrt{\widetilde{\Upsilon}_{ve}(u) \times \widetilde{\Upsilon}_{ve}(v)}}{\widetilde{\Upsilon}_{ve}(u) + \widetilde{\Upsilon}_{ve}(v)}. \quad (8)$$

*Definition 9:* The ve-degree harmonic (ve-H) index is defined as:

$$\mathcal{H}^{ve}(G) = \sum_{uv \in E(G)} \frac{2}{\widetilde{\Upsilon}_{ve}(u) + \widetilde{\Upsilon}_{ve}(v)}. \quad (9)$$

*Definition 10:* The ve degree sum-connectivity (ve-$\chi$) index is defined as:

$$\chi^{ve}(G) = \sum_{uv \in E(G)} (\widetilde{\Upsilon}_{ve}(u) + \widetilde{\Upsilon}_{ve}(v))^{-\frac{1}{2}}. \quad (10)$$

More results about these indices can be found in [22]–[25].

## III. AN OPTICAL TRANSPOSE INTERCONNECTION SYSTEM (OTIS) SWAPPED NETWORK

The OTIS swapped network $O_G$ is obtained from the graph $G$ with vertex set and edge set as follows:

$V(O_G) = \{(a, b)|a, b \in V(G)\}$

$E(O_G) = \{(a, b_1), (a, b_2)|a \in V(G), (b_1, b_2) \in E(G)\} \cup \{(a, b), (b, a)|a, b \in E(G),\ a \neq b\}$.

For the mutual OTIS network $O_G$, the graph $G$ is called the basis (factor) of the graph or grid. If the primary network $G$ has $n$ So, $O_G$ consists of a separate subset from the $n$ node groups are called, and they are similar to $G$. The node name $(a, b)$ in $O_G$ select the $b$ node handle in the a group [ [16]–[19]].

Now we calculate some scores *ve* and *ev* -degree Topological indicators of swapped networks.

For a given path $P_n$ on $n$ vertices and $O_{P_n}$ as its OTIS swapped network with basis network $P_n$ is shown in Figure 1.

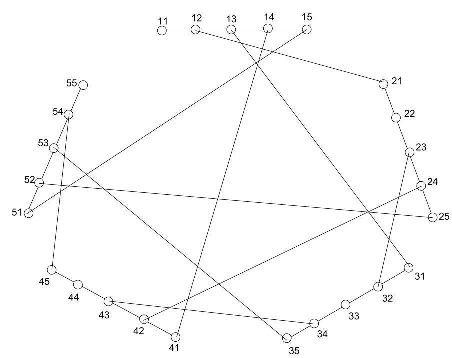


**FIGURE 1. OTIS swapped network $O_{P_5}$.**

## IV. RESULTS FOR OTIS SWAPPED NETWORK $O_{P_n}$

In the following table, $V_1$ represents the number of vertices of degree 1, $V_2$ represents the number of vertices of degree 2 and $V_3$ represents the number of vertices of degree 3 see table 1.

Thus finally we calculate the number of vertices of degree 1 are 2, the number of vertices of degree 2 are $3n - 4$ and the number of vertices of degree 3 are $n^2 - 3n + 2$. Similarly, we will partition the edges using same methodolgy. Table 2 shows the edge partition of $O_{P_n}$ with $n \geq 5$.

**TABLE 1.** Vertex partition of $O_{P_n}$.

| $n$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| $V_1$ | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $V_2$ | 2 | 5 | 8 | 11 | 14 | 17 | 20 |
| $V_3$ | 0 | 2 | 6 | 12 | 20 | 30 | 42 |

**TABLE 2.** Edge partition of $O_{P_n}$, $n \geq 5$.

| $(deg(u), deg(v))$ | Number of edges |
|---|---|
| $(1,3)$ | 2 |
| $(2,2)$ | 3 |
| $(2,3)$ | $6n-14$ |
| $(3,3)$ | $\frac{3}{2}(n^2-5n+6)$ |

**TABLE 3.** Vertex and edges of $O_{P_n}$, $n \geq 5$.

| Total Vertices | Total Edges |
|---|---|
| $n^2$ | $\frac{3n(n-1)}{2}$ |

**TABLE 4.** Number of vertices with corresponding degrees.

| $deg(u)$ | Number of vertices |
|---|---|
| 1 | 2 |
| 2 | $3n-4$ |
| 3 | $n^2-3n+2$ |

**TABLE 5.** Edge partition of $O_{Pn}$.

| Number of edges | Degree of its end vertices | $ev$-degrees |
|---|---|---|
| 2 | $(1,3)$ | 4 |
| 3 | $(2,2)$ | 4 |
| $6n-14$ | $(2,3)$ | 5 |
| $\frac{3}{2}(n^2-5n+6)$ | $(3,3)$ | 6 |

**TABLE 6.** Vertex partition of $O_{Pn}$.

| Number of vertices | Degrees | $ve$-degrees |
|---|---|---|
| 2 | 1 | 3 |
| 6 | 2 | 5 |
| $3n-10$ | 2 | 6 |
| 2 | 3 | 6 |
| 4 | 3 | 7 |
| $6n-24$ | 3 | 8 |
| $n^2-9n+20$ | 3 | 9 |

The order and size of $O_{P_n}$ network are presented in Table 3.

The types of vertices and their partition for $O_{P_n}$ network are presented in Table 4.

In Table 4, We partition the edges, based on *ev*-degree of the $O_{Pn}$ for $n \geq 5$. In Table 6 and 7, we partition the vertices, based on *ve*-degree of $O_{Pn}$ for $n \geq 5$.

The ve-degree based partition are presented in Table 6.

The ve-degree based partition based on degree of end vertices are presented in Table 7.

Now we calculated ev-degree and ve-degree based indices such as the *ev*-degree Zagreb index, first ve-degree Zagreb alpha index, first ve-degree Zagreb beta index, the second ve-degree Zagreb index, ve-degree Randic index, *ev*-degree Randic index, ve-degree atom-bond connectivity

**TABLE 7.** The *ve*-degree of the end vertices of edges of $O_{Pn}$.

| No. of edges | Degree of end vertices | $ve$-deg of end vertices |
|---|---|---|
| 2 | $(1,2)$ | $(3,6)$ |
| 3 | $(2,2)$ | $(5,5)$ |
| 2 | $(2,2)$ | $(5,6)$ |
| 2 | $(2,3)$ | $(5,7)$ |
| 2 | $(2,3)$ | $(5,8)$ |
| 6 | $(2,3)$ | $(6,7)$ |
| $6n-24 \begin{cases} 6n-26 \\ 2 \end{cases}$ | $\begin{cases} (2,3) \\ (3,3) \end{cases}$ | $(6,8)$ |
| 4 | $(3,3)$ | $(7,8)$ |
| $3n-12$ | $(3,3)$ | $(8,8)$ |
| $6n-30$ | $(3,3)$ | $(8,9)$ |
| $\frac{3}{2}(n^2-11n+30)$ | $(3,3)$ | $(9,9)$ |

(ve-$\mathcal{ABC}$) index, ve-degree geometric-arithmetic (ve-$\mathcal{GA}$) index, ve-degree harmonic (ve-$\mathcal{H}$) index and ve degree sum-connectivity (ve-$\chi$) for $O_{Pn}$ formulas.

### A. EV-DEGREE ZAGREB INDEX

By using ev-degree from edges partition of $O_{Pn}$ given in table (5), we compute the ev-degree based Zagreb index:

$$\begin{aligned} \mathcal{M}^{ev}(O_{Pn}) &= \sum_{e \in E(O_{Pn})} \widetilde{\Upsilon}_{ev}(e)^2 \\ &= 2 \times 4^2 + 3 \times 4^2 + (6n-14) \times 5^2 \\ &\quad + \frac{3}{2}(n^2-5n+6) \times 6^2 \\ &= 32 + 48 + 150n - 350 + 54n^2 - 270n + 324 \\ &= 54n^2 - 120n + 54. \end{aligned}$$

### B. THE FIRST VE-DEGREE ZAGREB ALPHA INDEX

By using ve-degree from vertices partition of $O_{Pn}$, given in table (6), we compute the first ve-degree Zagreb alpha index:

$$\begin{aligned} \mathcal{M}_1^{\alpha ve}(O_{Pn}) &= \sum_{v \in V(O_{Pn})} \widetilde{\Upsilon}_{ve}(v)^2 \\ &= 2 \times 3^2 + 6 \times 5^2 + (3n-8) \times 6^2 + 4 \times 7^2 \\ &\quad + (6n-24) \times 8^2 \\ &\quad + (n^2-9n+20) \times 9^2 \\ &= 18 + 150 + 108n - 288 + 196 + 384n \\ &\quad - 1536 + 81n^2 - 729n + 1620 \\ &= 81n^2 - 237n + 160. \end{aligned}$$

### C. THE FIRST VE-DEGREE ZAGREB BETA INDEX

By using ve-degree of end vertices of edges partition of $O_{Pn}$, given in table (7), we compute the first ve-degree Zagreb beta index:

$$\begin{aligned} &\mathcal{M}_1^{\beta ve}(O_{Pn}) \\ &= \sum_{uv \in E(O_{Pn})} (\widetilde{\Upsilon}_{ve}(u) + \widetilde{\Upsilon}_{ve}(v)) \\ &= 2 \times 9 + 3 \times 10 + 2 \times 11 + 2 \times 12 + 2 \times 13 + 6 \times 13 \\ &\quad + (6n-24) \times 14 \\ &\quad + 4 \times 15 + (3n-12) \times 16 + (6n-30) \times 17 \end{aligned}$$

$$
\begin{aligned}
&+\frac{3}{2}(n^2-11n+30)\times 18\\
&=18+30+22+24+26+78+84n-336+60\\
&\quad+48n-192+102n\\
&\quad-510+27n^2-297n+810\\
&=27n^2-63n+30.
\end{aligned}
$$

### D. THE SECOND VE-DEGREE ZAGREB INDEX

By using ve-degree of end vertices of edges partition of $O_{Pn}$, given in table (7), we compute the second ve-degree based Zagreb index:

$$
\begin{aligned}
&\mathcal{M}_2^{ve}(O_{Pn})\\
&=\sum_{uv\in E(O_{Pn})}(\widetilde{\Upsilon}_{ve}(u)\times\widetilde{\Upsilon}_{ve}(v))\\
&=2\times 18+3\times 25+2\times 30+2\times 35+2\times 40+6\times 42\\
&\quad+(6n-24)\times 48+4\times 56+(3n-12)\times 64\\
&\quad+(6n-30)\times 72\\
&\quad+\frac{3}{2}(n^2-11n+30)\times 81\\
&=36+75+60+70+80+252+288n-1152+224\\
&\quad+192n\\
&\quad-768+432n-2160+\frac{243}{2}n^2-\frac{2673}{2}n+3645\\
&=\frac{243}{2}n^2-\frac{849}{2}n+362.
\end{aligned}
$$

### E. THE VE-DEGREE RANDIC INDEX

By using ve-degree of end vertices of edges partition of $O_{Pn}$, given in table (7), we compute the ve-degree Randic index:

$$
\begin{aligned}
&\mathcal{R}^{ve}(O_{Pn})\\
&=\sum_{uv\in E(O_{Pn})}(\widetilde{\Upsilon}_{ve}(u)\times\widetilde{\Upsilon}_{ve}(v))^{-\frac{1}{2}}\\
&=2\times 18^{-\frac{1}{2}}+3\times 25^{-\frac{1}{2}}\\
&\quad+2\times 30^{-\frac{1}{2}}+2\times 35^{-\frac{1}{2}}+2\times 40^{-\frac{1}{2}}\\
&\quad+6\times 42^{-\frac{1}{2}}+(6n-24)\times 48^{-\frac{1}{2}}+4\times 56^{-\frac{1}{2}}\\
&\quad+(3n-12)\times 64^{-\frac{1}{2}}\\
&\quad+(6n-30)\times 72^{-\frac{1}{2}}+\frac{3}{2}(n^2-11n+30)\times 81^{-\frac{1}{2}}\\
&=\frac{2}{3\sqrt{2}}+\frac{3}{5}+\frac{2}{\sqrt{30}}+\frac{2}{\sqrt{35}}+\frac{1}{\sqrt{10}}+\frac{6}{\sqrt{42}}+\frac{3}{2\sqrt{3}}n\\
&\quad-\frac{6}{\sqrt{3}}\\
&\quad+\frac{2}{\sqrt{14}}+\frac{3}{8}n-\frac{3}{2}+\frac{1}{\sqrt{2}}n-\frac{5}{\sqrt{2}}+\frac{1}{6}n^2-\frac{11}{6}n+5\\
&=\frac{1}{6}n^2+\left(\frac{3}{2\sqrt{3}}+\frac{3}{8}+\frac{1}{\sqrt{2}}-\frac{11}{6}\right)n+\frac{2}{3\sqrt{2}}+\frac{3}{5}+\frac{2}{\sqrt{30}}\\
&\quad+\frac{2}{\sqrt{35}}+\frac{1}{\sqrt{10}}+\frac{6}{\sqrt{42}}-\frac{6}{\sqrt{3}}+\frac{2}{\sqrt{14}}-\frac{3}{2}-\frac{5}{\sqrt{2}}+5\\
&=\frac{1}{6}n^2+\left(\frac{3}{2\sqrt{3}}+\frac{1}{\sqrt{2}}-\frac{35}{24}\right)n+\frac{2}{3\sqrt{2}}+\frac{2}{\sqrt{30}}+\frac{2}{\sqrt{35}}\\
&\quad+\frac{1}{\sqrt{10}}+\frac{6}{\sqrt{42}}-\frac{6}{\sqrt{3}}+\frac{2}{\sqrt{14}}-\frac{5}{\sqrt{2}}+\frac{41}{10}.
\end{aligned}
$$

### F. THE EV-DEGREE RANDIC INDEX

By using ev-degree from edges partition of $O_{Pn}$, given in table (5), we compute the ev-degree based Randic index:

$$
\begin{aligned}
&\mathcal{R}^{ev}(O_{Pn})\\
&=\sum_{e\in E(O_{Pn})}\widetilde{\Upsilon}_{ev}(e)^{-\frac{1}{2}}\\
&=2\times 4^{-\frac{1}{2}}+3\times 4^{-\frac{1}{2}}+(6n-14)\times 5^{-\frac{1}{2}}\\
&\quad+\frac{3}{2}(n^2-5n+6)\times 6^{-\frac{1}{2}}\\
&=\frac{3}{2\sqrt{6}}n^2+\left(\frac{6}{\sqrt{5}}-\frac{15}{2\sqrt{6}}\right)n+\left(\frac{5}{2}-\frac{14}{\sqrt{5}}+\frac{9}{\sqrt{6}}\right).
\end{aligned}
$$

### G. THE ATOM-BOND CONNECTIVITY INDEX

By using ve-degree of end vertices of edges partition of $O_{Pn}$, given in table (7), we compute the Atom-bond connectivity index:

$$
\begin{aligned}
&\mathcal{ABC}^{ve}(O_{Pn})\\
&=\sum_{uv\in E(O_{Pn})}\sqrt{\frac{\widetilde{\Upsilon}_{ve}(u)+\widetilde{\Upsilon}_{ve}(v)-2}{\widetilde{\Upsilon}_{ve}(u)\times\widetilde{\Upsilon}_{ve}(v)}}\\
&=2\times\sqrt{\frac{9-2}{18}}+3\times\sqrt{\frac{10-2}{25}}+2\times\sqrt{\frac{11-2}{30}}\\
&\quad+2\times\sqrt{\frac{12-2}{35}}+2\times\sqrt{\frac{13-2}{40}}+6\times\sqrt{\frac{13-2}{42}}\\
&\quad+(6n-24)\times\sqrt{\frac{14-2}{48}}+4\\
&\quad\times\sqrt{\frac{15-2}{56}}+(3n-12)\times\sqrt{\frac{16-2}{64}}\\
&\quad+(6n-30)\times\sqrt{\frac{17-2}{72}}+\frac{3}{2}(n^2-11n+30)\\
&\quad\times\sqrt{\frac{18-2}{81}}\\
&=\frac{2\sqrt{7}}{3\sqrt{2}}+\frac{3\sqrt{8}}{5}+\frac{6}{\sqrt{30}}+\frac{2\sqrt{2}}{\sqrt{7}}+\frac{\sqrt{11}}{\sqrt{10}}+\frac{6\sqrt{11}}{\sqrt{42}}\\
&\quad+3n-12\\
&\quad+\frac{2\sqrt{13}}{\sqrt{14}}+\frac{3\sqrt{14}}{8}n-\frac{3\sqrt{14}}{2}+\frac{\sqrt{15}}{\sqrt{2}}n-\frac{5\sqrt{15}}{\sqrt{2}}+\frac{2}{3}n^2\\
&\quad-\frac{22}{3}n+20\\
&=\frac{2}{3}n^2+\left(3+\frac{3\sqrt{14}}{8}-\frac{5\sqrt{15}}{\sqrt{2}}-\frac{22}{3}\right)n+\frac{2\sqrt{7}}{3\sqrt{2}}+\frac{3\sqrt{8}}{5}\\
&\quad+\frac{6}{\sqrt{30}}+\frac{2\sqrt{2}}{\sqrt{7}}+\frac{\sqrt{11}}{\sqrt{10}}+\frac{6\sqrt{11}}{\sqrt{42}}-12+\frac{2\sqrt{13}}{\sqrt{14}}\\
&\quad-\frac{3\sqrt{14}}{2}-\frac{5\sqrt{15}}{\sqrt{2}}+20\\
&=\frac{2}{3}n^2+\left(\frac{3\sqrt{14}}{8}-\frac{5\sqrt{15}}{\sqrt{2}}-\frac{13}{3}\right)n+\frac{2\sqrt{7}}{3\sqrt{2}}+\frac{3\sqrt{8}}{5}
\end{aligned}
$$

$$+\frac{6}{\sqrt{30}}+\frac{2\sqrt{2}}{\sqrt{7}}+\frac{\sqrt{11}}{\sqrt{10}}+\frac{6\sqrt{11}}{\sqrt{42}}+\frac{2\sqrt{13}}{\sqrt{14}}-\frac{3\sqrt{14}}{2}-\frac{5\sqrt{15}}{\sqrt{2}}+8.$$

### H. THE GEOMETRIC-ARITHMETIC INDEX

By using ve-degree of end vertices of edges partition of $O_{Pn}$, given in table (7), we compute the Geometric-arithmetic index:

$$\begin{aligned}
&\mathcal{GA}^{ve}(O_{Pn})\\
&=\sum_{uv\in E(O_{Pn})}\frac{2\sqrt{\widetilde{\Upsilon}_{ve}(u)\times\widetilde{\Upsilon}_{ve}(v)}}{\widetilde{\Upsilon}_{ve}(u)+\widetilde{\Upsilon}_{ve}(v)}\\
&=2\times\frac{2\sqrt{18}}{9}+3\times\frac{2\sqrt{25}}{10}+2\times\frac{2\sqrt{30}}{11}+2\times\frac{2\sqrt{35}}{12}\\
&\quad+2\times\frac{2\sqrt{40}}{13}\\
&\quad+6\times\frac{2\sqrt{42}}{13}+(6n-24)\times\frac{2\sqrt{48}}{14}+4\times\frac{2\sqrt{56}}{15}\\
&\quad+(3n-12)\times\frac{2\sqrt{64}}{16}\\
&\quad+(6n-30)\times\frac{2\sqrt{72}}{17}+\frac{3}{2}(n^2-11n+30)\times\frac{2\sqrt{81}}{18}\\
&=\frac{4\sqrt{2}}{3}+3+\frac{4\sqrt{30}}{11}+\frac{4\sqrt{35}}{12}+\frac{8\sqrt{10}}{13}+\frac{12\sqrt{42}}{13}\\
&\quad+\frac{24\sqrt{3}}{7}n\\
&\quad-\frac{96\sqrt{3}}{7}+\frac{16\sqrt{14}}{15}+3n-12+\frac{72\sqrt{2}}{17}n-\frac{360\sqrt{2}}{17}\\
&\quad+\frac{3}{2}n^2-\frac{33}{2}n+45\\
&=\frac{3}{2}n^2+\left(\frac{24\sqrt{3}}{7}+3+\frac{72\sqrt{2}}{17}-\frac{33}{2}\right)n+\frac{4\sqrt{2}}{3}+3\\
&\quad+\frac{4\sqrt{30}}{11}\\
&\quad+\frac{\sqrt{35}}{3}+\frac{8\sqrt{10}}{13}+\frac{12\sqrt{42}}{13}-\frac{96\sqrt{3}}{7}+\frac{16\sqrt{14}}{15}-12\\
&\quad-\frac{360\sqrt{2}}{17}+45\\
&=\frac{3}{2}n^2+\left(\frac{24\sqrt{3}}{7}+\frac{72\sqrt{2}}{17}-\frac{27}{2}\right)n+\frac{4\sqrt{2}}{3}+\frac{4\sqrt{30}}{11}\\
&\quad+\frac{\sqrt{35}}{3}\\
&\quad+\frac{8\sqrt{10}}{13}+\frac{12\sqrt{42}}{13}-\frac{96\sqrt{3}}{7}+\frac{16\sqrt{14}}{15}-\frac{360\sqrt{2}}{17}+36.
\end{aligned}$$

### I. THE HARMONIC INDEX

By using ve-degree of end vertices of edges partition of $O_{Pn}$, given in table (7), we compute the Harmonic index:

$$\begin{aligned}
&\mathcal{H}^{ve}(O_{Pn})\\
&=\sum_{uv\in E(O_{Pn})}\frac{2}{\widetilde{\Upsilon}_{ve}(u)+\widetilde{\Upsilon}_{ve}(v)}\\
&=2\times\frac{2}{9}+3\times\frac{2}{10}+2\times\frac{2}{11}+2\times\frac{2}{12}+2\times\frac{2}{13}+6\times\frac{2}{13}\\
&\quad+(6n-24)\times\frac{2}{14}+4\times\frac{2}{15}+(3n-12)\times\frac{2}{16}\\
&\quad+(6n-30)\times\frac{2}{17}\\
&\quad+\frac{3}{2}(n^2-11n+30)\times\frac{2}{18}\\
&=\frac{4}{9}+\frac{3}{5}+\frac{4}{11}+\frac{1}{3}+\frac{4}{13}+\frac{6}{7}n-\frac{24}{7}+\frac{8}{15}+\frac{12}{13}+\frac{3}{8}n\\
&\quad-\frac{3}{2}\\
&\quad+\frac{12}{17}n-\frac{60}{17}+\frac{1}{6}n^2-\frac{11}{6}n+5\\
&=\frac{1}{6}n^2+\frac{299}{2856}n-\frac{72799}{1531530}.
\end{aligned}$$

### J. THE SUM-CONNECTIVITY INDEX

By using ve-degree of end vertices of edges partition of $O_{Pn}$, given in table (7), we compute the Sum-connectivity index:

$$\begin{aligned}
&\chi^{ve}(O_{Pn})\\
&=\sum_{uv\in E(O_{Pn})}(\widetilde{\Upsilon}_{ve}(u)+\widetilde{\Upsilon}_{ve}(v))^{-\frac{1}{2}}\\
&=2\times 9^{-\frac{1}{2}}+3\times 10^{-\frac{1}{2}}+2\times 11^{-\frac{1}{2}}+2\times 12^{-\frac{1}{2}}+2\times 13^{-\frac{1}{2}}\\
&\quad+6\times 13^{-\frac{1}{2}}+(6n-24)\times 14^{-\frac{1}{2}}+4\times 15^{-\frac{1}{2}}\\
&\quad+(3n-12)\times 16^{-\frac{1}{2}}\\
&\quad+(6n-30)\times 17^{-\frac{1}{2}}+\frac{3}{2}(n^2-11n+30)\times 18^{-\frac{1}{2}}\\
&=\frac{2}{3}+\frac{3}{\sqrt{10}}+\frac{2}{\sqrt{11}}+\frac{1}{\sqrt{3}}+\frac{8}{\sqrt{13}}+\frac{6}{\sqrt{14}}n-\frac{24}{\sqrt{14}}\\
&\quad+\frac{4}{\sqrt{15}}\\
&\quad+\frac{3}{4}n-3+\frac{6}{\sqrt{17}}n-\frac{30}{\sqrt{17}}+\frac{1}{2\sqrt{2}}n^2-\frac{11}{2\sqrt{2}}n+\frac{15}{\sqrt{2}}\\
&=\frac{1}{2\sqrt{2}}n^2+\left(\frac{6}{\sqrt{14}}+\frac{3}{4}+\frac{6}{\sqrt{17}}-\frac{11}{2\sqrt{2}}\right)n+\frac{2}{3}+\frac{3}{\sqrt{10}}\\
&\quad+\frac{2}{\sqrt{11}}+\frac{1}{\sqrt{3}}+\frac{8}{\sqrt{13}}-\frac{24}{\sqrt{14}}+\frac{4}{\sqrt{15}}-3-\frac{30}{\sqrt{17}}+\frac{15}{\sqrt{2}}\\
&=\frac{1}{2\sqrt{2}}n^2+\left(\frac{6}{\sqrt{14}}+\frac{3}{4}+\frac{6}{\sqrt{17}}-\frac{11}{2\sqrt{2}}\right)n+\frac{3}{\sqrt{10}}+\frac{2}{\sqrt{11}}\\
&\quad+\frac{1}{\sqrt{3}}+\frac{8}{\sqrt{13}}-\frac{24}{\sqrt{14}}+\frac{4}{\sqrt{15}}-\frac{30}{\sqrt{17}}+\frac{15}{\sqrt{2}}-\frac{7}{3}.
\end{aligned}$$

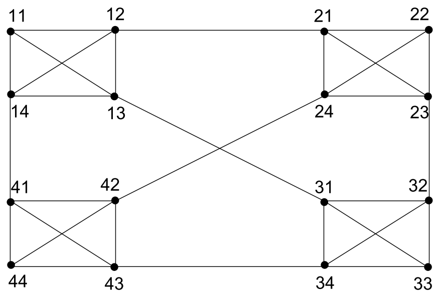


**FIGURE 2.** OTIS swapped network $O_{K_4}$.

**TABLE 8.** Vertex partition of $O_{K_n}$.

| $n$ | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|
| $V_1$ | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| $V_2$ | 6 | 12 | 20 | 30 | 42 | 56 | 72 |

**TABLE 9.** Edge partition of $O_{K_n}$, $n \geq 5$.

| Number of edges | $(deg(u), deg(v))$ |
|---|---|
| $n(n-1)$ | $(n, n-1)$ |
| $\frac{n(n-1)^2}{2}$ | $(n, n)$ |

**TABLE 10.** Vertex and edges of $O_{K_n}$, $n \geq 5$.

| Total Vertices | Total Edges |
|---|---|
| $n^2$ | $\frac{n^3-n}{2}$ |

**TABLE 11.** Number of vertices with corresponding degrees.

| $deg(u)$ | Number of vertices |
|---|---|
| $n-1$ | $n$ |
| $n$ | $n(n-1)$ |

## V. RESULTS FOR OTIS SWAPPED NETWORK $O_{K_n}$

The complete graph denoted by $K_n$ with $n$ vertices and $O_{K_n}$ be the OTIS swapped network for $O_{K_4}$ as example shown in figure 2.

In table 8, $V_1$ represents the number of vertices of degree $n-1$, and $V_2$ represents the number of vertices of degree $n$.

Thus finally we calculate the number of vertices of degree $n-1$ are $n$, and the number of vertices of degree $n$ are $n(n-1)$. Similarly, we will partition the edges using same methodology. Table 9 shows the edge partition of $O_{k_n}$ with $n \geq 4$.

The order and size of $O_{K_n}$ network are presented in Table 10.

The degree based partition of vertices of $O_{K_n}$ network are presented in Table 11.

In Table 12 we partition the edges, based on *ev*-degree of the $O_{K_n}$. In Table 13 and Table 13 we partition the vertices, based on *ve*-degree of $O_{K_n}$.

Now we will calculate ev-degree and ve-degree based indices such as the *ev*-degree Zagreb index, first ve-degree Zagreb alpha index, first ve-degree Zagreb beta index, the second ve-degree Zagreb index, ve-degree Randic index, *ev*-degree Randic index, ve-degree atom-bond connectivity

**TABLE 12.** Edge partition of $O_{Kn}$.

| Number of edges | Degree of its end vertices | $ev$-degrees |
|---|---|---|
| $n(n-1)$ | $(n, n-1)$ | $n+1$ |
| $\frac{n(n-1)(n-2)}{2}$ | $(n, n)$ | $n+2$ |
| $\frac{n(n-1)}{2}$ | $(n, n)$ | $2n$ |

**TABLE 13.** Vertex partition of $O_{Kn}$.

| Number of vertices | Degrees | $ve$-degrees |
|---|---|---|
| $n$ | $n-1$ | $(n-1)(\frac{n+2}{2})$ |
| $n(n-1)$ | $n$ | $(n-1)(\frac{n+4}{2})$ |

**TABLE 14.** The *ve*-degree of the end vertices of edges of $O_{Kn}$.

| No. of edges | Degree | $ve$-degrees |
|---|---|---|
| $n(n-1)$ | $(n-1, n)$ | $((n-1)(\frac{n+2}{2}), (n-1)(\frac{n+4}{2}))$ |
| $\frac{n(n-1)^2}{2}$ | $(n, n)$ | $((n-1)(\frac{n+4}{2}), (n-1)(\frac{n+4}{2}))$ |

(ve-$\mathcal{ABC}$) index, ve-degree geometric-arithmetic (ve-$\mathcal{GA}$) index, ve-degree harmonic (ve-$\mathcal{H}$) index and ve degree sum-connectivity (ve-$\chi$) for $O_{K_n}$ formulas.

### A. EV-DEGREE ZAGREB INDEX

By using ev-degree from edges partition of $O_{K_n}$ given in table (12), we compute the ev-degree based Zagreb index:

$$\begin{aligned}
&\mathcal{M}^{ev}(O_{K_n}) \\
&= \sum_{e \in E(O_{K_n})} \widetilde{\Upsilon}_{ev}(e)^2 \\
&= n(n-1) \times (n+1)^2 + \frac{n(n-1)(n-2)}{2} \times (n+2)^2 \\
&\quad + \frac{n(n-1)}{2} \times (2n)^2 \\
&= \frac{1}{2}[2n(n^2-1)(n+1) + n(n-1)(n^2-4)(n+2) \\
&\quad + 4n^3(n-1)] \\
&= \frac{1}{2}n^5 + \frac{7}{2}n^4 - 4n^3 - 3n^2 + 3n.
\end{aligned}$$

### B. THE FIRST VE-DEGREE ZAGREB ALPHA INDEX

By using ve-degree from vertices partition of $O_{K_n}$, given in table (13), we compute the first ve-degree Zagreb alpha index:

$$\begin{aligned}
&\mathcal{M}_1^{\alpha ve}(O_{K_n}) \\
&= \sum_{v \in V(O_{K_n})} \widetilde{\Upsilon}_{ve}(v)^2 \\
&= n \times ((n-1)(\frac{n+2}{2}))^2 + n(n-1) \times ((n-1)(\frac{n+4}{2}))^2 \\
&= \frac{1}{4}(n^2+1-2n)(n^4+8n^3+12n^2-12n) \\
&= \frac{1}{4}n^6 + \frac{3}{2}n^5 - \frac{3}{4}n^4 - 7n^3 + 9n^2 - 3n.
\end{aligned}$$

### C. THE FIRST VE-DEGREE ZAGREB BETA INDEX

By using ve-degree of end vertices of edges partition of $O_{K_n}$, given in table (14), we compute the first ve-degree Zagreb beta index:

$$\mathcal{M}_1^{\beta ve}(O_{K_n}) = \sum_{uv\in E(O_{K_n})} (\widetilde{\Upsilon}_{ve}(u) + \widetilde{\Upsilon}_{ve}(v))$$
$$= n(n-1) \times (n-1)(n+3) + \frac{n(n-1)^2}{2} \times (n-1)(n+4)$$
$$= (n-1)^2 \left[n^2 + 3n + \frac{1}{2}(n^2-n)(n+4)\right]$$
$$= \frac{1}{2}n^5 + \frac{3}{2}n^4 - \frac{7}{2}n^3 + \frac{1}{2}n^2 + n.$$

### D. THE SECOND VE-DEGREE ZAGREB INDEX

By using ve-degree of end vertices of edges partition of $O_{K_n}$, given in table (14), we compute the second ve-degree based Zagreb index:

$$\mathcal{M}_2^{ve}(O_{K_n}) = \sum_{uv\in E(O_{K_n})} (\widetilde{\Upsilon}_{ve}(u) \times \widetilde{\Upsilon}_{ve}(v))$$
$$= n(n-1) \times \frac{1}{4}(n-1)^2(n+2)(n+4) + \frac{n(n-1)^2}{2} \times (n-1)^2\frac{(n+4)^2}{4}$$
$$= \frac{1}{4}(n-1)^3[(n^2+2n)(n+4) + \frac{(n^2-n)}{2}(n^2+8n+16)]$$
$$= \frac{1}{8}n^7 + \frac{3}{4}n^6 - \frac{1}{2}n^5 - \frac{17}{4}n^4 + \frac{51}{8}n^3 - \frac{5}{2}n^2.$$

### E. THE VE-DEGREE RANDIC INDEX

By using ve-degree of end vertices of edges partition of $O_{K_n}$, given in table (14), we compute the ve-degree Randic index:

$$\mathcal{R}^{ve}(O_{K_n}) = \sum_{uv\in E(O_{K_n})} (\widetilde{\Upsilon}_{ve}(u) \times \widetilde{\Upsilon}_{ve}(v))^{-\frac{1}{2}}$$
$$= n(n-1) \times (\frac{1}{4}(n-1)^2(n+2)(n+4))^{-\frac{1}{2}} + \frac{n(n-1)^2}{2} \times ((n-1)^2\frac{(n+4)^2}{4})^{-\frac{1}{2}}$$
$$= \frac{2n}{\sqrt{(n+2)(n+4)}} + \frac{n(n-1)}{(n+4)}.$$

### F. THE EV-DEGREE RANDIC INDEX

By using ev-degree from edges partition of $O_{K_n}$, given in table (12), we compute the ev-degree based Randic index:

$$\mathcal{R}^{ev}(O_{K_n}) = \sum_{e\in E(O_{K_n})} \widetilde{\Upsilon}_{ev}(e)^{-\frac{1}{2}}$$
$$= n(n-1) \times (n+1)^{-\frac{1}{2}} + \frac{n(n-1)(n-2)}{2} \times (n+2)^{-\frac{1}{2}} + \frac{n(n-1)}{2} \times (2n)^{-\frac{1}{2}}.$$

### G. THE ATOM-BOND CONNECTIVITY INDEX

By using ve-degree of end vertices of edges partition of $O_{K_n}$, given in table (14), we compute the Atom-bond connectivity index:

$$\mathcal{ABC}^{ve}(O_{K_n}) = \sum_{uv\in E(O_{K_n})} \sqrt{\frac{\widetilde{\Upsilon}_{ve}(u) + \widetilde{\Upsilon}_{ve}(v) - 2}{\widetilde{\Upsilon}_{ve}(u) \times \widetilde{\Upsilon}_{ve}(v)}}$$
$$= n(n-1) \times \sqrt{\frac{(n-1)(n+3)-2}{\frac{1}{4}(n-1)^2(n+2)(n+4)}} + \frac{n(n-1)^2}{2} \times \sqrt{\frac{(n-1)(n+4)-2}{((n-1)^2\frac{(n+4)^2}{4})}}$$
$$= 2n\sqrt{\frac{(n-1)(n+3)-2}{(n+2)(n+4)}} + \frac{n(n-1)\sqrt{(n-1)(n+4)-2}}{(n+4)}.$$

### H. THE GEOMETRIC-ARITHMETIC INDEX

By using ve-degree of end vertices of edges partition of $O_{K_n}$, given in table (14), we compute the Geometric-arithmetic index:

$$\mathcal{GA}^{ve}(O_{K_n}) = \sum_{uv\in E(O_{K_n})} \frac{2\sqrt{\widetilde{\Upsilon}_{ve}(u) \times \widetilde{\Upsilon}_{ve}(v)}}{\widetilde{\Upsilon}_{ve}(u) + \widetilde{\Upsilon}_{ve}(v)}$$
$$= n(n-1) \times \frac{2\sqrt{\frac{1}{4}(n-1)^2(n+2)(n+4)}}{(n-1)(n+3)} + \frac{n(n-1)^2}{2} \times \frac{2\sqrt{(n-1)^2\frac{(n+4)^2}{4}}}{(n-1)(n+4)}$$
$$= \frac{n(n-1)\sqrt{(n+2)(n+4)}}{(n+3)} + \frac{n(n-1)^2}{2}.$$

### I. THE HARMONIC INDEX

By using ve-degree of end vertices of edges partition of $O_{K_n}$, given in table (14), we compute the Harmonic index:

$$\mathcal{H}^{ve}(O_{K_n}) = \sum_{uv\in E(O_{K_n})} \frac{2}{\widetilde{\Upsilon}_{ve}(u) + \widetilde{\Upsilon}_{ve}(v)}$$
$$= n(n-1)\times\frac{2}{(n-1)(n+3)} + \frac{n(n-1)^2}{2} \times \frac{2}{(n-1)(n+4)}$$
$$= \frac{2n}{n+3} + \frac{n(n-1)}{(n+4)}.$$

### J. THE SUM-CONNECTIVITY INDEX

By using ve-degree of end vertices of edges partition of $O_{K_n}$, given in table (14), we compute the Sum-connectivity index:

$$\chi^{ve}(O_{K_n}) = \sum_{uv\in E(O_{K_n})} (\widetilde{\Upsilon}_{ve}(u) + \widetilde{\Upsilon}_{ve}(v))^{-\frac{1}{2}}$$
$$= n(n-1) \times ((n-1)(n+3))^{-\frac{1}{2}} + \frac{n(n-1)^2}{2} \times ((n-1)(n+4))^{-\frac{1}{2}}.$$

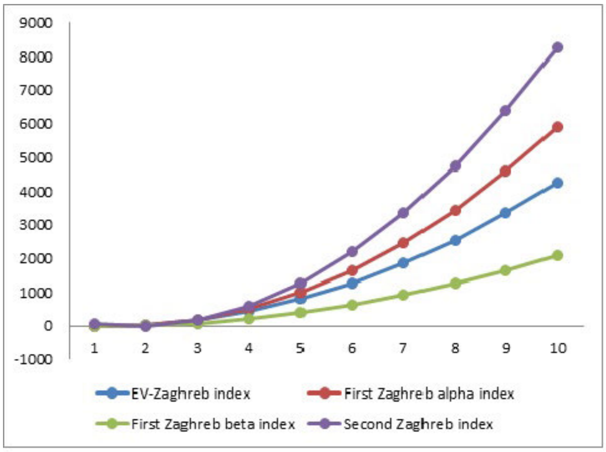


**FIGURE 3.** Graphical Comparison of $M^{ev}$, $M_1^{\alpha ve}$, $M_1^{\beta ve}$ and $M_2^{ve}$.

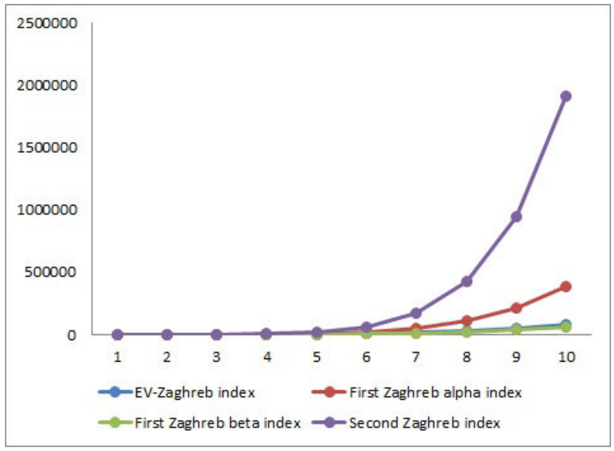


**FIGURE 5.** Graphical Comparison of $M^{ev}$, $M_1^{\alpha ve}$, $M_1^{\beta ve}$ and $M_2^{ve}$.

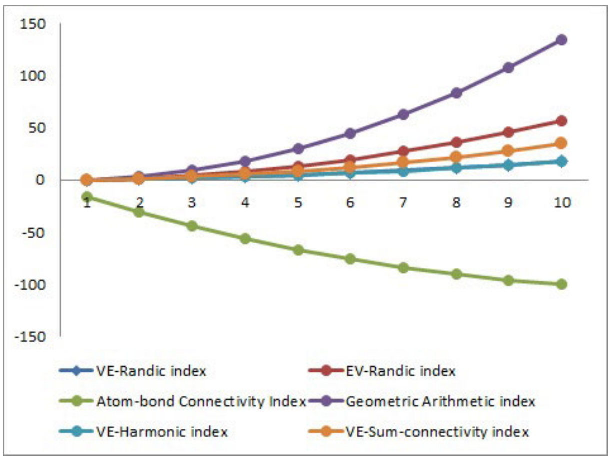


**FIGURE 4.** Graphical Comparison of $R^{ev}$, $GA^{ve}$, $ABC^{ve}$ $H^{ve}$, $R^{ve}$ and $\chi^{ve}$.

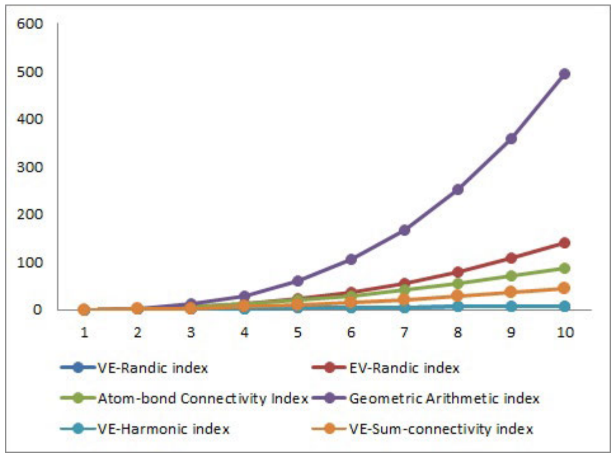


**FIGURE 6.** Graphical Comparison of $R^{ev}$, $GA^{ve}$, $ABC^{ve}$ $H^{ve}$, $R^{ve}$ and $\chi^{ve}$.

## VI. GRAPHICAL REPRESENTATION AND DISCUSSION

In this section we discuss graphically representation related to the ev-degree and ve-degree based topological descriptors for the OTIS swapped network with the basis graph as path and complete graph.

We determined the explicit formulas for the ev-degree and ve-degree based topological indices such as the *ev*-degree Zagreb index, first ve-degree Zagreb alpha index, first ve-degree Zagreb beta index, the second ve-degree Zagreb index, ve-degree Randic index, *ev*-degree Randic index, ve-degree atom-bond connectivity (ve-$\mathcal{ABC}$) index, ve-degree geometric-arithmetic (ve-$\mathcal{GA}$) index, ve-degree harmonic (ve-$\mathcal{H}$) index and ve degree sum-connectivity (ve-$\chi$) for the OTIS swapped network.

- The graphical representation of OTIS swapped network of path are shown in Figures 3, 4. It can be observe that the values of all indices increase with increasing value of $n$ except Atom Bond Connectivity index. The values of $ABC^{ve}$ decrease with the increase of $n$.
- Similarly, the graphical representation of OTIS swapped network of complete graph are shown in Figures 5, 6. It can be observe that the values of all indices increase with increasing value of $n$.

## VII. CONCLUSION

The study of graphs and networks through topological descriptors is important to understand their underlying topologies. Such investigations have a wide range of applications in computer science where various graph invariants based assessments are used to deal with several challenging schemes. In the analysis of the quantitative structure-property relationships (QSPRs) and the quantitative structure-activity relationships (QSARs), graph invariants are important tools to approximate and predicate the properties of the biological structures.

In this article, we have provided results related to the ev-degree and ve-degree based indices such as the *ev*-degree Zagreb index, first ve-degree Zagreb alpha index, first ve-degree Zagreb beta index, the second ve-degree Zagreb index, ve-degree Randic index, *ev*-degree Randic index, ve-degree atom-bond connectivity (ve-$\mathcal{ABC}$) index, ve-degree geometric-arithmetic (ve-$\mathcal{GA}$) index, ve-degree harmonic (ve-$\mathcal{H}$) index and ve degree sum-connectivity (ve-$\chi$) for the OTIS swapped network of path and complete graph as basis graph.

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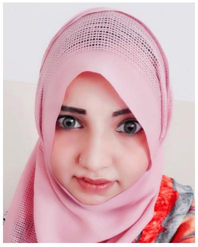

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