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Adaptive Dynamic Control for Manipulator Actuated Integrated Translation and Rotation Stabilization of Spacecraft

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ABSTRACT This paper presents the adaptive control problem of manipulator actuated integrated position and attitude stabilization of spacecraft in proximity operations. Towards this end, explicit kinematics and dynamics are formulated for a spacecraft with multiple manipulators, where two vector factorizations are proposed to ensure the skew-symmetric property of the system matrices. In what follows, a reference trajectory adaptive tracking control scheme is designed following backstepping procedure to drive the manipulator motion causing reactions on the spacecraft for the integrated position and attitude stabilization in the presence of unknown kinematic and dynamic parameters. The prescribed reference trajectory is designed by polynomial methods enabling a well-behaviored stabilization performance, and a second-order filter is introduced to estimate the joint acceleration ensuring the control scheme available. Meanwhile, to apply adaptive technique, the explicit-form regressor matrices of manipulator dynamics are derived and the adaptation law is hence designed to update the estimate of unknown system parameters. The closed-loop stability is guaranteed within the Lyapunov framework. At last, numerical simulations are given to demonstrate the effect of the designed adaptive control scheme.

INDEX TERMS Manipulator actuation, adaptive control, integrated translation and rotation control.

NOMENCLATURE

$\boldsymbol{a}_k^i$	position vector from the mass center of J_k^i to C_k^i
B_0, B_k^i	spacecraft and kth link of the ith manipulator, respectively
$\boldsymbol{b}_k^i$	position vector from C_k^i to the mass center of J_{k+1}^i
$\boldsymbol{b}_0^i$	position vector of joint 1 of the ith manipulator with respect to the mass center of spacecraft
C_k^i	mass center of B_k^i
$\boldsymbol{E}_k$	k × k identity matrix
$\boldsymbol{h}_k^i$	unit vector of the rotation direction of joint J_k^i
$\boldsymbol{I}_0, \boldsymbol{I}_k^i$	inertial tensor of B_0 and B_k^i, respectively
J_k^i	kth joint of the ith manipulator
m_0, m_k^i	mass of spacecraft and B_k^i, respectively
$\boldsymbol{p}_k^i$	position vector of J_k^i
$\boldsymbol{q}$	attitude of spacecraft represented by unit quaternion
$\boldsymbol{r}_k^i$	position vector of B_k^i
$\boldsymbol{r}_0$	position vector of spacecraft
$\boldsymbol{v}_0$	velocity vector of spacecraft
$\boldsymbol{\omega}_0$	angular velocity vector of spacecraft
θ_k^i	rotational angle of joint J_k^i
$\tau_{c,k}^i$	control torque acting on J_k^i

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I. INTRODUCTION

Integrated translation and rotation control is considered as a promising technology for proximity operation missions and thus has been studied for the past decade. To develop this technology, earlier works mainly focused on the feasibility and reasonability [1]–[3]. Afterwards, more practical issues were then taken into account, including fuel-optimal integrated control problem [4], [5], finite-time technique based responsive integrated maneuver [6], [7], collision-free six degree-of-freedom (DOF) control for spacecraft rendezvous [8], [9], and velocity-free integrated control [10].

Besides the above, recent years have witnessed the increased concentrations on the effect of control actuator, because its characteristics greatly affect the control system performance, and what's more, actuator layout and resulting

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control allocation straightforwardly determine the capability of integrated translation and rotation control of spacecraft [11]–[16]. In view of configuration flexibility and design convenience, thruster was regarded as a routine choice to provide both enough control force and torque enabling the designed integrated control algorithm in previous works and applications. Moreover, various thruster-relevant problems were handled, involving thruster layout design problem [11], [12], thruster saturation [13], [14], thruster misalignment [15], and thruster failure problem [16]. However, the utilization of thruster has to cost limited fuel of spacecraft and heavily restricts the control accuracy due to its hard nonlinearity caused by the on-off working mode of thrusters.

In such situation, advanced actuation strategies are necessary for future proximity operation missions requiring higher control accuracy and better system performance. Different from the working principle of thrusters, momentum exchange devices have increasingly been studied and gained applications. Flywheels and control moment gyroscopes are two such good examples to enable accurate attitude control of spacecraft based on rotating masses inside the body [17]–[19]. Similarly, strain-actuated solar arrays were proposed and utilized to ensure an ultra-quiet spacecraft attitude control system, which had also been experimentally demonstrated in a 1DOF testbed [20].

Momentum exchange philosophy is thus a good alternative to the traditional thruster actuation methods. However, if it is used to enforce integrated spacecraft position and attitude control, both linear angular and linear momentum exchange should be conducted. By virtue of this, space manipulators, especially ones mounted on spacecraft body to complete various proximity operations such as capture and movement, were deemed as possible control actuators in [21]–[23] for the integrated translation and rotation stabilization of a spacecraft. In fact, a spacecraft with multiple manipulators captures the dynamic characteristics of a multi-body system, enabling both angular and linear momentum exchange. The feasibility and effectiveness of the manipulator actuated control strategy had been initially verified in kinematic level by using a single manipulator [21], [22] and dual manipulator actuation [23].

Specifically, the previous studies [21]–[23] formulated and mainly focused on the kinematic couplings between the motions of spacecraft and manipulators. The command joint motion enabling manipulator actuation was developed from this kinematic coupling equation, and thus the strategy was in fact proposed only from the kinematic view point. Naturally, the impact of manipulator dynamics was neglected and the joint rates of manipulators were directly treated as control inputs, in the light of the high servo-capability of joint control-loop. This treatment was only suitable for initially verifying the feasibility of the manipulator actuation method. However, the actual control input should be joint control torque rather than the joint rate, though the joint control-loop performs a good performance. The impact of the manipulator dynamics should be considered in the control-loop of the whole system comprising both kinematics and dynamics.

Dynamic modeling of space manipulator has been always a research focus for many years [24]–[30]. To facilitate the controller design and closed-loop stability analysis, the recursive Newton-Euler method and the Euler-Lagrange method are two prevalent dynamic modeling approaches. The former one ensures an explicit chain-form dynamic formulation of manipulator dynamics [24]–[27], while the later one leads to a compact-form, albeit implicit, dynamic formulation holding the skew-symmetric property [28]–[30]. In view of missions with multiple manipulators with higher DOF, it is necessary and better to incorporate two above methods and develop an explicit dynamic formulation possessing skew-symmetric property for spacecraft whose motions actuated by multiple manipulators.

Another thing should be addressed is unknown system parameter. Recalling thruster-actuated spacecraft system with unknown mass property, adaptive control had been utilized in the integrated translation and rotation control system design [13], [15], [31]–[33]. Compared with a single spacecraft, the spacecraft containing manipulators often has much more unknown parameters, because manipulators consist of several links and the relevant parameter, including mass property and position of center mass are hard to be exactly determined or measured. In spite of this, adaptive control [34], [35] was still a natural and preferred approach and the resulting closed-loop stability analysis was almost mature and rigorous based on an premise that the uncertain kinematics and dynamics were linear in a set of kinematic and dynamic parameters [36]–[40], although there were other endeavors mainly focusing on ground manipulators such as approximate Jacobian control method [41], [42], and observer-based controller design [43], [44]. However, for adaptive control, the regressor matrices in the previous studies [36]–[40] only possessed implicit forms and thus may be hardly applied in a general scenario, especially the one containing multiple manipulators with higher DOF. Thus, for manipulator actuated control missions subject to unknown kinematic and dynamic parameters, it is indispensible to derive explicit-form regressor matrices based on the explicit dynamic modeling. To the best of author's knowledge, few studies have been made on the adaptive dynamic control problem of the manipulator actuated integrated position and attitude stabilization of spacecraft subject to the unknown parameters

In this paper, a multiple-manipulator actuated adaptive nonlinear control scheme is proposed to deal with the integrated translation and rotation stabilization problems for spacecraft in proximity operations subject to unknown system parameters, including not only the mass properties of manipulator links and spacecraft body but also the position of the mass center of each manipulator link. To do so, recursive Newton-Euler modeling philosophy is first utilized to formulate an explicit kinematics and dynamics within the Lagrangian framework for a multiple manipulator actuated spacecraft position and attitude control system, where, as a key technique, two vector factorizations are proposed to incorporate the above two classic dynamic modeling methods. Then, an adaptive control scheme is synthesized by backstepping philosophy such that the position and attitude error of the spacecraft can be stabilized subject to the above unknown parameters, by solving an equivalent reference trajectory tracking problem. The reference trajectory is pre-designed by polynomial methods for a well-behaviored stabilization performance. A second-order filter is introduced to estimate the joint and spacecraft acceleration ensuring the control scheme computable. To apply adaptive technique, this paper for the first time presents the explicit-form regressor matrices of manipulator dynamics, and derives the adaptation law to update the estimates of unknown system parameters. The closed-loop stability is analyzed within the Lyapunov framework. Finally, numerical simulations are given to demonstrate the effect of the designed adaptive control scheme.

The remainder of this paper is organized as follows: In Sec. II, the kinematics and dynamics are formulated and the control problem is stated. Then, a reference trajectory tracking based adaptive control scheme is designed for joint control torques enabling manipulator actuation in Sec. III, where the closed-loop stability analysis is given as well. Numerical simulation results applying the proposed adaptive control law to a scenario are presented in Sec. IV. Finally, Sec. V draws the conclusion.

II. SYSTEM MODELING AND PROBLEM FORMULATION

A. BASIC FRAMES

In the present paper, a spacecraft mounted with N space manipulators is considered and the ith manipulator with n_i DOF is illustrated as an example in Fig. 1.

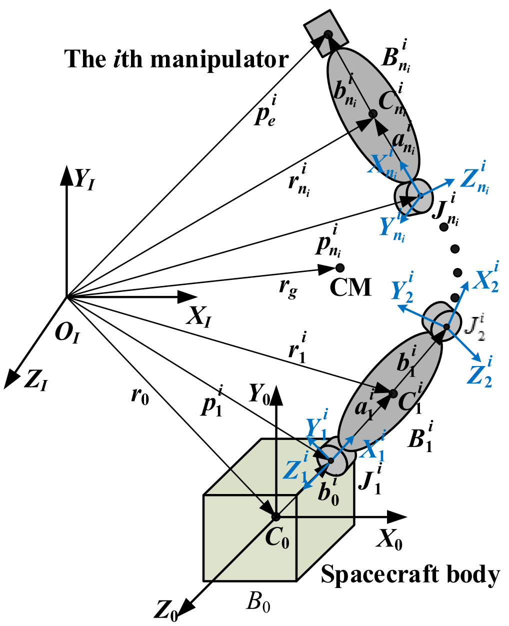

FIGURE 1. Multiple-manipulator actuated spacecraft system.

Let $O_I X_I Y_I Z_I$ and $O_0 X_0 Y_0 Z_0$ be an inertial frame and the body frame of the spacecraft, respectively. In view of space manipulators, let $O_k^i X_k^i Y_k^i Z_k^i$ be the joint frame with origin at the mass center of joint J_k^i, $O_k^i Z_k^i$ along the rotational axis of joint J_k^i, $O_k^i X_k^i$ along the vector from the mass center of joint J_k^i to the mass center of link B_k^i, and $O_k^i Y_k^i$ completing the right-handed frame. All vectors and vector derivatives appeared in this paper are described in the inertial frame, if not pointed out specially.

B. DYNAMICS FORMULATION

The translational and rotational kinematics of a spacecraft is described by [2], [17]

$$\begin{cases} \dot{\boldsymbol{q}} = \dfrac{1}{2}\boldsymbol{N}_q(\boldsymbol{q})\omega_0 \\ \dot{\boldsymbol{r}}_0 = \boldsymbol{v}_0, \end{cases} \quad \text{with } \boldsymbol{N}_q(\boldsymbol{q}) = \begin{bmatrix} -\boldsymbol{q}_v^{\mathrm{T}} \\ q_0\boldsymbol{E}_3 + \boldsymbol{q}_v^{\times} \end{bmatrix} \tag{1}$$

where $\boldsymbol{q} = [q_0\ \boldsymbol{q}_v^{\mathrm{T}}]^{\mathrm{T}}$ is an unit quaternion representing the spacecraft attitude, composed of a scalar component q_0 and a vector component $\boldsymbol{q}_v$; $\zeta^{\times}$ represents the cross product matrix for any vector $\boldsymbol{\zeta} \in \mathbb{R}^3$. By defining the generalized velocity and position of the spacecraft as $\boldsymbol{V}_0 = [\boldsymbol{\omega}_0^{\mathrm{T}}\ \boldsymbol{v}_0^{\mathrm{T}}]^{\mathrm{T}}$ and $\boldsymbol{X} = [\boldsymbol{q}^{\mathrm{T}}\ \boldsymbol{r}_0^{\mathrm{T}}]^{\mathrm{T}}$, respectively, (1) can be simplified as

$$\dot{\boldsymbol{X}} = \boldsymbol{N}(\boldsymbol{q})\boldsymbol{V}_0 \tag{2}$$

where $\boldsymbol{N}(\boldsymbol{q}) = \mathrm{diag}\{0.5\boldsymbol{N}_q(\boldsymbol{q}), \boldsymbol{E}_3\}$.

In what follows, by using the spatial operator in [42], [43] and following the recursive modeling philosophy, the explicit kinematics and dynamics are formulated for a N-manipulator mounted spacecraft and the main formulations are given as follows. Notice that, the mathematical derivation of dynamic modeling in fact follows [28], [29] and thus the details are omitted herein, due to the paper length limit. Meanwhile, the explicit expressions of the system matrices appeared in the dynamic formulations are summarized in Appendix A.

Supposing any external force or torque is not exerted on the system, the linear and angular momentum of the total system are conserved, which results in that

$$\boldsymbol{H}_{bb}\boldsymbol{V}_0 + \boldsymbol{H}_{bm}\dot{\boldsymbol{\Theta}} = \boldsymbol{G}(-\boldsymbol{r}_0)\boldsymbol{H}_0 \tag{3}$$

where $\boldsymbol{\Theta} = [(\boldsymbol{\Theta}^1)^{\mathrm{T}}\ (\boldsymbol{\Theta}^2)^{\mathrm{T}} \cdots\ (\boldsymbol{\Theta}^N)^{\mathrm{T}}]^{\mathrm{T}}$ is the joint angle vector and the component $\boldsymbol{\Theta}^i$ denotes the joint angle vector of the ith manipulator, yielding $\boldsymbol{\Theta}^i = [\theta_1^i\ \theta_2^i\ \cdots \theta_{n_i}^i]^{\mathrm{T}}$; $\boldsymbol{G}(\cdot)$ is an operator defined by

$$\boldsymbol{G}(\boldsymbol{\zeta}) = \begin{bmatrix} \boldsymbol{E}_3 & \boldsymbol{\zeta}^{\times} \\ O & \boldsymbol{E}_3 \end{bmatrix}, \quad \forall \boldsymbol{\zeta} \in \mathbb{R}^3 \tag{4}$$

and $\boldsymbol{H}_0 = [\boldsymbol{L}^{\mathrm{T}}(0)\ \boldsymbol{P}^{\mathrm{T}}(0)]^{\mathrm{T}}$, where $\boldsymbol{L}(0)$ and $\boldsymbol{P}(0)$ are constant vectors representing initial angular and linear momentum of the total system, respectively. Equation (3) in fact reveals the couplings between manipulator motions and position/attitude motions of spacecraft in the kinematic level.

As for the system dynamics, on the one hand, the dynamics of the ith manipulator can be formulated as

$$\begin{aligned} &(\boldsymbol{H}^i)^{\mathrm{T}}\boldsymbol{G}^i\boldsymbol{M}^i(\boldsymbol{G}_0^{\mathrm{T}})\dot{\boldsymbol{V}}_0 + (\boldsymbol{H}^i)^{\mathrm{T}}\boldsymbol{G}^i\boldsymbol{M}^i(\boldsymbol{G}^i)^{\mathrm{T}}\boldsymbol{H}^i\ddot{\boldsymbol{\Theta}}^i \\ &\qquad + (\boldsymbol{H}^i)^{\mathrm{T}}\boldsymbol{G}^i\boldsymbol{M}^i(\boldsymbol{G}^i)^{\mathrm{T}}\boldsymbol{\beta}^i + (\boldsymbol{H}^i)^{\mathrm{T}}\boldsymbol{G}^i\boldsymbol{b}^i = \boldsymbol{\tau}_c^i \end{aligned} \tag{5}$$

where $\boldsymbol{\tau}_c^i = [\boldsymbol{\tau}_{c,1}^i \cdots \boldsymbol{\tau}_{c,k}^i]^{\mathrm{T}}$ denotes the control torque vector for the ith manipulator, $\boldsymbol{\beta}^i = [\boldsymbol{\beta}^{\mathrm{T}}(J_1^i)\ \boldsymbol{\beta}^{\mathrm{T}}(J_2^i) \cdots \boldsymbol{\beta}^{\mathrm{T}}(J_{n_i}^i)]^{\mathrm{T}}$ and $\boldsymbol{b}^i = [\boldsymbol{b}^{\mathrm{T}}(J_1^i)\ \boldsymbol{b}^{\mathrm{T}}(J_2^i)\ \cdots \boldsymbol{b}^{\mathrm{T}}(J_{n_i}^i)]^{\mathrm{T}}$, yielding

$$\boldsymbol{\beta}(J_k^i) = \begin{bmatrix} \boldsymbol{\omega}(J_{k-1}^i)^{\times}\boldsymbol{h}_k^i\dot{\theta}_k^i \\ \boldsymbol{\omega}(J_{k-1}^i)^{\times}\boldsymbol{\omega}(J_{k-1}^i)^{\times}\boldsymbol{l}_{k-1}^i \end{bmatrix}$$
$$\boldsymbol{b}(J_k^i) = \begin{bmatrix} \boldsymbol{\omega}(J_k^i)^{\times}(\boldsymbol{I}_k^i - m_k^i\boldsymbol{a}_k^{i\times}\boldsymbol{a}_k^{i\times})\boldsymbol{\omega}(J_k^i) \\ m_k^i\boldsymbol{\omega}(J_k^i)^{\times}\boldsymbol{\omega}(J_k^i)^{\times}\boldsymbol{a}_k^i \end{bmatrix} \tag{6}$$

in which $\boldsymbol{\omega}(J_k^i)$ denotes the angular velocity of joint J_k^i and $\boldsymbol{l}_k^i = \boldsymbol{a}_k^i + \boldsymbol{b}_k^i$. On the other hand, the spacecraft body dynamics can be given by

$$\left(\boldsymbol{M}_0 + \sum_{i=1}^{N}\boldsymbol{G}_0^i\boldsymbol{M}^i(\boldsymbol{G}_0^i)^{\mathrm{T}}\right)\dot{\boldsymbol{V}}_0 + \sum_{i=1}^{N}\left(\boldsymbol{G}_0^i\boldsymbol{M}^i(\boldsymbol{G}_0^i)^{\mathrm{T}}\boldsymbol{H}^i\right)\ddot{\boldsymbol{\Theta}}^i$$
$$+\boldsymbol{b}_0 + \sum_{i=1}^{N}\left(\boldsymbol{G}_0^i\boldsymbol{M}^i(\boldsymbol{G}^i)^{\mathrm{T}}\boldsymbol{\beta}^i + \boldsymbol{G}_0^i\boldsymbol{b}^i\right) = 0 \tag{7}$$

It can be seen that (5) and (7) constitute a chain-form explicit dynamic formulation of the total system, yet it is necessary to make a further derivation in an explicit way in order to 1) derive the system matrices satisfying the skew-symmetric property held in dynamic equations derived by the Euler-Lagrange method [24]–[27], and 2) clearly reveal the dynamic couplings between the motions of spacecraft body and the mounted manipulators. To this end, two vector factorizations are proposed for $\boldsymbol{\beta}(J_k^i)$ and $\boldsymbol{b}(J_k^i)$ as

$$\boldsymbol{\beta}(J_k^i) = \dot{\boldsymbol{H}}_k^i\dot{\theta}_k^i + \dot{\boldsymbol{G}}^{\mathrm{T}}(\boldsymbol{l}_{k-1}^i)\boldsymbol{V}_0 + \dot{\boldsymbol{G}}^{\mathrm{T}}(\boldsymbol{l}_{k-1}^i)\bar{\boldsymbol{H}}_{k-1}^i\dot{\boldsymbol{\Theta}}^i$$
$$\boldsymbol{b}(J_k^i) = \boldsymbol{B}(J_k^i)\boldsymbol{H}_k^i\dot{\theta}_k^i + \boldsymbol{B}(J_k^i)\boldsymbol{V}_0 + \boldsymbol{B}(J_k^i)\bar{\boldsymbol{H}}_{k-1}^i\dot{\boldsymbol{\Theta}}^i \tag{8}$$

and hence vectors $\boldsymbol{\beta}^i$ and $\boldsymbol{b}^i$ can be summarized as

$$\boldsymbol{\beta}^i = \dot{\boldsymbol{H}}\dot{\boldsymbol{\Theta}}^i + (\boldsymbol{G}_l^i)^{\mathrm{T}}\boldsymbol{V}_0 + (\boldsymbol{G}_L^i)^{\mathrm{T}}\bar{\boldsymbol{H}}_{k-1}^i\dot{\boldsymbol{\Theta}}^i$$
$$\boldsymbol{b}^i = B_V\boldsymbol{V}_0 + \boldsymbol{B}^i(\bar{\boldsymbol{H}}^i + \boldsymbol{H}^i)\dot{\boldsymbol{\Theta}}^i \tag{9}$$

which simplifies the dynamics composed of (5) and (7) as

$$\boldsymbol{H}_{bb}\dot{\boldsymbol{V}}_0 + \boldsymbol{H}_{bm}\ddot{\boldsymbol{\Theta}} + \boldsymbol{C}_{bb}\boldsymbol{V}_0 + \boldsymbol{C}_{bm}\dot{\boldsymbol{\Theta}} = 0$$
$$\boldsymbol{H}_{bm}^{\mathrm{T}}\dot{\boldsymbol{V}}_0 + \boldsymbol{H}_{mm}\ddot{\boldsymbol{\Theta}} + \boldsymbol{C}_{mb}\boldsymbol{V}_0 + \boldsymbol{C}_{mm}\dot{\boldsymbol{\Theta}} = \boldsymbol{\tau}_c \tag{10}$$

Moreover, given explicit terms in Appendix A, it can be proved that the system matrices in (10) hold the skew-symmetric property as follows.

$$\boldsymbol{x}^{\mathrm{T}}(\dot{\boldsymbol{H}}_{bb} - 2\boldsymbol{C}_{bb})\boldsymbol{x} = 0, \quad \forall x \in \mathbb{R}^6$$
$$\boldsymbol{x}^{\mathrm{T}}(\dot{\boldsymbol{H}}_{mm} - 2\boldsymbol{C}_{mm})\boldsymbol{x} = 0, \quad \forall x \in \mathbb{R}^{n_1+\cdots+n_N}$$
$$\dot{\boldsymbol{H}}_{bm} - \boldsymbol{C}_{mb}^{\mathrm{T}} - \boldsymbol{C}_{bm} = O \tag{11}$$

Remark 1: It can be concluded that two proposed vector factorizations in (8) successfully incorporates the recursive Newton-Euler and the Euler-Lagrange methods, and hence the resulting dynamics (10) possesses an explicit form and holds the skew-symmetric property, capable of covering more complicated scenarios with multiple manipulators.

C. CONTROL PROBLEM

As mentioned in Section I, the control objective of the present paper is to enable integrated translation and rotation stabilization of spacecraft by appropriate motions of mounted manipulators. Meanwhile, unknown system parameters are considered herein. Specifically, to cover more practical situations, besides the mass property parameters of spacecraft body and mounted manipulators, the position parameters of mass center of all manipulator links are also regarded as the unknowns. Meanwhile, this makes the initial system momentum $\boldsymbol{H}_0$ become unknown in view of (3). Therefore, the control problem can be formulated as follows.

Problem 1: For the system dynamics composed of (2), (3) and (10), design appropriate control torque τ_c to actuate the joint motions of mounted manipulators such that the position and attitude errors are driven to zero as closely as possible, in the presence of unknown parameters including $m_0, m_k^i, \boldsymbol{I}_0, \boldsymbol{I}_k^i$ and $\boldsymbol{a}_k^i$, and unknown initial system momentum $\boldsymbol{H}_0$.

III. CONTROL SCHEME

It can be found that the dynamics formulation composed of (2), (3) and (10) essentially holds a quasi-cascaded structure, which makes the backstepping philosophy a natural and preferable method to cope with Problem 1. Before this, a reference integrated translational and rotational stabilization trajectory is designed ensuring a satisfactory performance by polynomial method. Then, making full use of the structure of dynamics in Eq. (10), a second-order filter is proposed such that the unavailable accelerations are replaced by computable terms. At last, an appropriate joint control torque together with adaptation laws are derived within the Laypunov framework, where the regressor matrices are derived in an explicit way.

A. REFERENCE TRAJECTORY

Two steps are given to determine a reference stabilization trajectory of the spacecraft.

First, define the reference attitude as $\boldsymbol{q}_d = [q_{d0}\ \boldsymbol{q}_{dv}^{\mathrm{T}}]^{\mathrm{T}}$ and the vector component $\boldsymbol{q}_{dv}$ can be designed as

$$\boldsymbol{q}_{dv}(t) = \begin{cases} \sum_{k=0}^{n_q}\boldsymbol{a}_{qk}t^k, & t < T_r \\ 0 & t \geq T_r \end{cases} \tag{12}$$

where the pre-determined time T_r and the coefficient vectors a_{qk} are chosen to at least satisfy

$$\boldsymbol{q}_{dv}(0) = \boldsymbol{a}_{q0} = \boldsymbol{q}_v(0)$$
$$\dot{\boldsymbol{q}}_{dv}(0) = \boldsymbol{a}_{q1} = \dot{\boldsymbol{q}}_v(0)$$
$$\boldsymbol{q}_{dv}(T_r) = \sum_{k=0}^{n_q}\boldsymbol{a}_{qk}T_r^k = 0$$
$$\dot{\boldsymbol{q}}_{dv}(T_r) = \sum_{k=1}^{n_q}k\boldsymbol{a}_{qk}T_r^{k-1} = 0$$
$$\ddot{\boldsymbol{q}}_{dv}(T_r) = \sum_{k=2}^{n_q}k(k-1)\boldsymbol{a}_{qk}T_r^{k-2} = 0 \tag{13}$$

and

$$\max_{0\leq t\leq T_r}\boldsymbol{q}_{dv}^{\mathrm{T}}(t)\boldsymbol{q}_{dv}(t) \leq 1 \tag{14}$$

which ensure a feasible second-order continuous rotational stabilization trajectory due to 1) $\boldsymbol{q}_d(0) = \boldsymbol{q}(0)$ and $\dot{\boldsymbol{q}}_d(0) = \dot{\boldsymbol{q}}(0)$; 2) the vector function $\boldsymbol{q}_{dv}(t) \in \mathbb{C}^2$; 3)$\boldsymbol{q}_{dv}(T_r) = 0$ and $\dot{\boldsymbol{q}}_d(T_r) = 0$; and 4) $||\boldsymbol{q}_{dv}|| \leq 1$, and meanwhile this requires the polynomial order should $n_q \geq 4$. More constraints could be satisfied by increasing n_q and adding extra coefficient vectors. And the reference scalar component q_{d0} is be governed by

$$q_{d0} = \mathrm{sgn}(q_0(t))\sqrt{1 - \boldsymbol{q}_{dv}^{\mathrm{T}}(t)\boldsymbol{q}_{dv}(t)} \tag{15}$$

where $\mathrm{sgn}(\cdot)$ is the sign function, yielding

$$\mathrm{sgn}(a) = \begin{cases} 1, & a > 0 \\ 0, & a = 0 \\ -1, & a < 0 \end{cases} \quad \forall a \in \mathbb{R}$$

moreover, the reference angular velocity can be obtained by recalling the attitude kinematics in (1) as

$$\boldsymbol{\omega}_d(t) = 2\boldsymbol{N}_q^T(\boldsymbol{q}_d(t))\dot{\boldsymbol{q}}_d(t) \tag{16}$$

Then, the reference position trajectory of the spacecraft can be designed by

$$\boldsymbol{r}_d(t) = \begin{cases} \sum_{k=0}^{n_r} \boldsymbol{a}_{rk} t^k, & t < T_r \\ 0 & t \geq T_r \end{cases} \tag{17}$$

Similarly, the coefficient vectors $\boldsymbol{a}_{rk}$ are chosen to satisfy

$$\begin{aligned} \boldsymbol{r}_d(0) &= \boldsymbol{a}_{r0} = \boldsymbol{q}_v(0) \\ \dot{\boldsymbol{r}}_d(0) &= \boldsymbol{a}_{r1} = \dot{\boldsymbol{q}}_v(0) \\ \boldsymbol{r}_d(T_r) &= \sum_{k=0}^{n_r} \boldsymbol{a}_{rk} T_r^k = 0 \\ \dot{\boldsymbol{r}}_d(T_r) &= \sum_{k=1}^{n_r} k\boldsymbol{a}_{rk} T_r^{k-1} = 0 \\ \ddot{\boldsymbol{r}}_d(T_r) &= \sum_{k=2}^{n_r} k(k-1)\boldsymbol{a}_{rk} T_r^{k-2} = 0 \end{aligned} \tag{18}$$

and the reference velocity yields

$$\boldsymbol{v}_d(t) = \dot{\boldsymbol{r}}_d(t) \tag{19}$$

B. ADAPTIVE CONTROL LAW

Notice that the designed reference stabilization trajectory promises a well-behaviored performance with a finite-time and continuous convergence in the pre-determined time T_r, so *Problem 1* will be solved if an appropriate control law is proposed such that the actual position and attitude motion of the spacecraft is driven to track the reference trajectory. It can be seen that the stabilization problem described by *Problem 1* can be in fact transformed into a reference trajectory tracking problem. To deal with this problem, an adaptive trajectory tracking control scheme is developed by backstepping philosophy.

First, let position and attitude tracking errors be $\tilde{\boldsymbol{r}} = \boldsymbol{r}_0 - \boldsymbol{r}_d$ and $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d^{-1} \circ \boldsymbol{q} = [\tilde{q}_0 \ \ \tilde{\boldsymbol{q}}_v^{\mathrm{T}}]^{\mathrm{T}}$, respectively, where the operator $\circ$ denotes the quaternion multiplication. Here, $\tilde{\boldsymbol{q}}$ is also a unit quaternion representing the attitude tracking error and yields $\dot{\tilde{\boldsymbol{q}}} = (1/2)\boldsymbol{N}_q(\tilde{\boldsymbol{q}})(\omega_0 - \boldsymbol{R}(\tilde{\boldsymbol{q}})\omega_d)$, where $\boldsymbol{R}(\tilde{\boldsymbol{q}})$ is the rotation matrix from the reference attitude to the real one and satisfies $\boldsymbol{R}(\tilde{\boldsymbol{q}}) = (\tilde{q}_0^2 - \tilde{\boldsymbol{q}}_v^{\mathrm{T}}\tilde{\boldsymbol{q}}_v)\boldsymbol{E}_3 + 2\tilde{\boldsymbol{q}}_v\tilde{\boldsymbol{q}}_v^{\mathrm{T}} - 2\tilde{q}_0\tilde{\boldsymbol{q}}_v^{\times}$. It should be noticed that, the error quaternion $\tilde{\boldsymbol{q}}$ has two equilibrium points, i.e., $[\pm 1 \ \ 0 \ \ 0 \ \ 0]^{\mathrm{T}}$, representing the same attitude. For the sake of simplicity and minimizing the path length, the equilibrium point of $\tilde{\boldsymbol{q}}$ can be determined by the given initial condition [2], [44] and thus it is reasonable to assume the scalar parameter of $\tilde{\boldsymbol{q}}$ does not change sign. Moreover, without loss of generality, the point $[1 \ 0 \ 0 \ 0]^{\mathrm{T}}$ is chosen as the equilibrium of $\tilde{\boldsymbol{q}}$, ensuring

$$(\tilde{q}_0 - 1)^2 + \tilde{\boldsymbol{q}}_v^{\mathrm{T}}\tilde{\boldsymbol{q}}_v = 2(1 - \tilde{q}_0) \leq 2(1 - \tilde{q}_0^2) = 2\tilde{\boldsymbol{q}}_v^{\mathrm{T}}\tilde{\boldsymbol{q}}_v \tag{20}$$

Then, define $\tilde{\boldsymbol{X}} = [\tilde{q}_0 - 1 \ \ \tilde{\boldsymbol{q}}_v^{\mathrm{T}} \ \ \tilde{\boldsymbol{r}}^{\mathrm{T}}]^{\mathrm{T}}$ and $\boldsymbol{V}_d = [(\boldsymbol{R}(\tilde{\boldsymbol{q}})\omega_d)^{\mathrm{T}} \ \boldsymbol{v}_d^{\mathrm{T}}]^{\mathrm{T}}$, and thus the error kinematics can be derived from (2) that

$$\dot{\tilde{\boldsymbol{X}}} = \boldsymbol{N}(\tilde{\boldsymbol{q}})(\boldsymbol{V}_0 - \boldsymbol{V}_d), \quad \text{with } \boldsymbol{N}^{\mathrm{T}}(\tilde{\boldsymbol{q}})\tilde{\boldsymbol{X}} = \Lambda\tilde{\boldsymbol{X}}_r \tag{21}$$

where $\tilde{\boldsymbol{X}}_r = [\tilde{\boldsymbol{q}}_v^{\mathrm{T}} \ \tilde{\boldsymbol{r}}^{\mathrm{T}}]^{\mathrm{T}}$ and $\Lambda = \mathrm{diag}\{0.5\boldsymbol{E}_3, \boldsymbol{E}_3\}$. This leads to that the command velocity $\boldsymbol{V}_c$ can be designed as

$$\boldsymbol{V}_c = \boldsymbol{V}_d - \boldsymbol{K}_x\boldsymbol{N}^{\mathrm{T}}(\tilde{\boldsymbol{q}})\tilde{\boldsymbol{X}} \tag{22}$$

where $\boldsymbol{K}_x$ is a positive definite matrix to be given. Recalling that (3) reveals the kinematic couplings between the motions of spacecraft body and manipulators, it is preferable to design appropriate joint motion to actuate the spacecraft motion. By doing so, define the command joint rate as $\dot{\boldsymbol{\Theta}}_c$ and let the joint tracking error be $\tilde{\boldsymbol{\Theta}} = \boldsymbol{\Theta} - \boldsymbol{\Theta}_c$, so (3) can be re-written as

$$\begin{aligned} \hat{\boldsymbol{H}}_{bb}\boldsymbol{V}_0 = \boldsymbol{G}(-\boldsymbol{r}_0)\hat{\boldsymbol{H}}_0 - \hat{\boldsymbol{H}}_{bm}\dot{\boldsymbol{\Theta}}_c - \hat{\boldsymbol{H}}_{bm}\dot{\tilde{\boldsymbol{\Theta}}} \\ +\tilde{\boldsymbol{H}}_{bb}\boldsymbol{V}_0 + \tilde{\boldsymbol{H}}_{bm}\dot{\boldsymbol{\Theta}} - \boldsymbol{G}(-\boldsymbol{r}_0)\tilde{\boldsymbol{H}}_0 \end{aligned} \tag{23}$$

where $\hat{\boldsymbol{H}}_{bb}, \hat{\boldsymbol{H}}_{bm}$ and $\hat{\boldsymbol{H}}_0$ are utilized to represent the estimates of $\boldsymbol{H}_{bb}, \boldsymbol{H}_{bm}$ and $\boldsymbol{H}_0$, respectively, attributing to containing unknown parameters; $\tilde{\boldsymbol{H}}_{bb}, \tilde{\boldsymbol{H}}_{bm}$ and $\tilde{\boldsymbol{H}}_0$ are the corresponding estimate errors, defined by $\tilde{\boldsymbol{\zeta}} = \hat{\boldsymbol{\zeta}} - \boldsymbol{\zeta}$, $\boldsymbol{\zeta} = \{\boldsymbol{H}_{bb}, \boldsymbol{H}_{bm}, \boldsymbol{H}_0\}$.This expression will be utilized for other matrices or vectors including unknown parameters throughout the paper.

The command joint rate $\dot{\boldsymbol{\Theta}}_c$ can be thus designed as

$$\dot{\boldsymbol{\Theta}}_c = -\hat{\boldsymbol{H}}_{bm}^*\hat{\boldsymbol{H}}_{bb}\boldsymbol{V}_c + \hat{\boldsymbol{H}}_{bm}^*\boldsymbol{G}(-\boldsymbol{r}_0)\hat{\boldsymbol{H}}_0 + (\boldsymbol{E} - \hat{\boldsymbol{H}}_{bm}^*\hat{\boldsymbol{H}}_{bm})\boldsymbol{\eta} \tag{24}$$

where $\hat{\boldsymbol{H}}_{bm}^*$ is the Moore-Penrose inverse of the matrix $\hat{\boldsymbol{H}}_{bm}$, η is an arbitrary vector existing only if $\sum_{i=1}^{N} n_i > 6$. Then, construct a Lyapunov function $W = (1/2)\tilde{\boldsymbol{X}}^{\mathrm{T}}\tilde{\boldsymbol{X}}$ and its time derivative can be obtained by using (3), (23) and (24) as

$$\begin{aligned} \dot{\boldsymbol{W}} = -\tilde{\boldsymbol{X}}^{\mathrm{T}}\boldsymbol{N}(\tilde{\boldsymbol{q}})\boldsymbol{K}_x\boldsymbol{N}^{\mathrm{T}}(\tilde{\boldsymbol{q}})\tilde{\boldsymbol{X}} - \tilde{\boldsymbol{X}}^{\mathrm{T}}\boldsymbol{N}(\tilde{\boldsymbol{q}})\hat{\boldsymbol{H}}_{bb}^{-1}\hat{\boldsymbol{H}}_{bm}\dot{\tilde{\boldsymbol{\Theta}}} \\ +\tilde{\boldsymbol{X}}^{\mathrm{T}}\boldsymbol{N}(\tilde{\boldsymbol{q}})\hat{\boldsymbol{H}}_{bb}^{-1}(\tilde{\boldsymbol{H}}_{bb}\boldsymbol{V}_0 + \tilde{\boldsymbol{H}}_{bm}\dot{\boldsymbol{\Theta}} - \boldsymbol{G}(-\boldsymbol{r}_0)\tilde{\boldsymbol{H}}_0) \end{aligned} \tag{25}$$

Next, to track the designed command joint rate $\dot{\boldsymbol{\Theta}}_c$, control torque will be designed based on the dynamics (10). To avoid appearance of $\ddot{\boldsymbol{\Theta}}$ and $\dot{\boldsymbol{V}}_0$ in the control scheme, a second-order filter is proposed as

$$\hat{\boldsymbol{H}}_{bb}\dot{\boldsymbol{V}}_c + \hat{\boldsymbol{H}}_{bm}\ddot{\boldsymbol{\Theta}}_c + \hat{\boldsymbol{C}}_{bb}\boldsymbol{V}_c + \hat{\boldsymbol{C}}_{bm}\dot{\boldsymbol{\Theta}}_c = \boldsymbol{K}_b\tilde{\boldsymbol{V}} \tag{26}$$

where $\boldsymbol{K}_v$ is a positive definite matrix and $\tilde{\boldsymbol{V}} = \boldsymbol{V}_0 - \boldsymbol{V}_c$. Then, combining (10) and (26) results in

$$\begin{aligned} &\boldsymbol{H}_{bb}\dot{\tilde{\boldsymbol{V}}} + \boldsymbol{H}_{bm}\ddot{\tilde{\boldsymbol{\Theta}}} + \boldsymbol{C}_{bb}\tilde{\boldsymbol{V}} + \boldsymbol{C}_{bm}\dot{\tilde{\boldsymbol{\Theta}}} \\ &\quad = \tilde{\boldsymbol{H}}_{bb}\dot{\boldsymbol{V}}_c + \tilde{\boldsymbol{H}}_{bm}\ddot{\boldsymbol{\Theta}}_c + \tilde{\boldsymbol{C}}_{bb}\boldsymbol{V}_c + \tilde{\boldsymbol{C}}_{bm}\dot{\boldsymbol{\Theta}}_c - \boldsymbol{K}_b\tilde{\boldsymbol{V}} \quad (27) \\ &\boldsymbol{H}_{bm}^{\mathrm{T}}\dot{\tilde{\boldsymbol{V}}} + \boldsymbol{H}_{mm}\ddot{\tilde{\boldsymbol{\Theta}}} + \boldsymbol{C}_{mb}\tilde{\boldsymbol{V}} + \boldsymbol{C}_{mm}\dot{\tilde{\boldsymbol{\Theta}}} \\ &\quad = \boldsymbol{\tau}_c - (\boldsymbol{H}_{bm}^{\mathrm{T}}\dot{\boldsymbol{V}}_c + \boldsymbol{H}_{mm}\ddot{\boldsymbol{\Theta}}_c + \boldsymbol{C}_{mb}\boldsymbol{V}_c + \boldsymbol{C}_{mm}\dot{\boldsymbol{\Theta}}_c) \quad (28) \end{aligned}$$

The control torque τ_c can be designed as

$$\begin{aligned} \boldsymbol{\tau}_c = &-\boldsymbol{K}_m\dot{\tilde{\boldsymbol{\Theta}}} + \hat{\boldsymbol{H}}_{bm}^{\mathrm{T}}\hat{\boldsymbol{H}}_{bb}^{-\mathrm{T}}\boldsymbol{N}^{\mathrm{T}}(\tilde{\boldsymbol{q}})\tilde{\boldsymbol{X}} \\ &+ \hat{\boldsymbol{H}}_{bm}^{\mathrm{T}}\dot{\boldsymbol{V}}_c + \hat{\boldsymbol{H}}_{mm}\ddot{\boldsymbol{\Theta}}_c + \hat{\boldsymbol{C}}_{mb}\boldsymbol{V}_c + \hat{\boldsymbol{C}}_{mm}\dot{\boldsymbol{\Theta}}_c \end{aligned} \quad (29)$$

At last, construct a Lyapunov function U as

$$W = W_1 + \frac{1}{2}\begin{bmatrix} \tilde{\boldsymbol{V}} \\ \dot{\tilde{\boldsymbol{\Theta}}} \end{bmatrix}^{\mathrm{T}} \underbrace{\begin{bmatrix} \boldsymbol{H}_{bb} & \boldsymbol{H}_{bm} \\ \boldsymbol{H}_{bm}^{\mathrm{T}} & \boldsymbol{H}_{mm} \end{bmatrix}}_{H} \begin{bmatrix} \tilde{\boldsymbol{V}} \\ \dot{\tilde{\boldsymbol{\Theta}}} \end{bmatrix} \quad (30)$$

and then by using (25), (27)-(29) and the skew-symmetric property in (11), the time derivative of W can be obtained as

$$\begin{aligned} \dot{W} = &-\tilde{\boldsymbol{X}}^{\mathrm{T}}\boldsymbol{N}(\tilde{\boldsymbol{q}})\boldsymbol{K}_x\boldsymbol{N}^{\mathrm{T}}(\tilde{\boldsymbol{q}})\tilde{\boldsymbol{X}} - \tilde{\boldsymbol{V}}^{\mathrm{T}}\boldsymbol{K}_b\tilde{\boldsymbol{V}} - \dot{\tilde{\boldsymbol{\Theta}}}^{\mathrm{T}}\boldsymbol{K}_m\dot{\tilde{\boldsymbol{\Theta}}} \\ &+ \tilde{\boldsymbol{X}}^{\mathrm{T}}\boldsymbol{N}(\tilde{\boldsymbol{q}})\hat{\boldsymbol{H}}_{bb}^{-1}(\tilde{\boldsymbol{H}}_{bb}\boldsymbol{V}_0 + \tilde{\boldsymbol{H}}_{bm}\dot{\boldsymbol{\Theta}} - \boldsymbol{G}(-\boldsymbol{r}_0)\tilde{\boldsymbol{H}}_0) \\ &+ \tilde{\boldsymbol{V}}^{\mathrm{T}}\left(\tilde{\boldsymbol{H}}_{bb}\dot{\boldsymbol{V}}_c + \tilde{\boldsymbol{H}}_{bm}\ddot{\boldsymbol{\Theta}}_c + \tilde{\boldsymbol{C}}_{bb}\boldsymbol{V}_c + \tilde{\boldsymbol{C}}_{bm}\dot{\boldsymbol{\Theta}}_c\right) \\ &+ \dot{\tilde{\boldsymbol{\Theta}}}^{\mathrm{T}}\left(\tilde{\boldsymbol{H}}_{bm}^{\mathrm{T}}\boldsymbol{V}_c + \tilde{\boldsymbol{H}}_{mm}\ddot{\boldsymbol{\Theta}}_c + \tilde{\boldsymbol{C}}_{mb}\boldsymbol{V}_c + \tilde{\boldsymbol{C}}_{mm}\dot{\boldsymbol{\Theta}}_c\right) \end{aligned} \quad (31)$$

Remark 2: For cases yielding $\sum_{i=1}^{N} n_i > 6$, the vector $\boldsymbol{\eta}$ represents the redundant design freedom for solving command joint rate and can be made full use of to satisfy various constraints. What's more, by left multiplying $\hat{\boldsymbol{H}}_{bm}$ on both sides of (24), it can be further found that, the vector $\boldsymbol{\eta}$ is eliminated in the expression of $\hat{\boldsymbol{H}}_{bm}\dot{\boldsymbol{\Theta}}_c$ and thus has no impact on the variation of the position and attitude of the spacecraft, by recalling (23). This implies that the regulation of the vector $\boldsymbol{\eta}$ is more effective to handle constraints relevant to manipulators such as collision avoidance and joint motion limitation, rather than those relevant to the motion of the spacecraft body.

C. REGRESSOR MATRIX AND ADAPTATION LAW

To complete the control scheme, adaptive technique is utilized to deal with estimate errors in (31). By doing so, it is necessary to first determine unknown parameter vector and regressor matrix based on explicit expressions in Section II. On the one hand, since $m_0, m_k^i, \boldsymbol{I}_0, \boldsymbol{I}_k^i$ and $\boldsymbol{a}_k^i$ are unknown, define $\boldsymbol{\xi}_0 = \boldsymbol{H}_0$, $\boldsymbol{\xi}_{m0} = m_0$, $\boldsymbol{\xi}_{m,k}^i = m_k^i$, $\boldsymbol{\xi}_{ma,k}^i = m_k^i\boldsymbol{a}_k^i$, and

$$\boldsymbol{\xi}_{I0} = \begin{bmatrix} I_{0xx} \\ I_{0yy} \\ I_{0zz} \\ I_{0yz} \\ I_{0xz} \\ I_{0xy} \end{bmatrix}, \ \boldsymbol{\xi}_{I,k}^i = \begin{bmatrix} I_{kxx}^i \\ I_{kyy}^i \\ I_{kzz}^i \\ I_{kyz}^i \\ I_{kxz}^i \\ I_{kxy}^i \end{bmatrix}, \ \boldsymbol{\xi}_{maa,k}^i = m_k^i \begin{bmatrix} a_{kx}^i a_{kx}^i \\ a_{ky}^i a_{ky}^i \\ a_{kz}^i a_{kz}^i \\ a_{ky}^i a_{kz}^i \\ a_{kx}^i a_{kz}^i \\ a_{kx}^i a_{ky}^i \end{bmatrix} \quad (32)$$

where a_{kx}^i, a_{ky}^i and a_{kz}^i are the components of the vector $\boldsymbol{a}_k^i$ in the joint frame of J_k^i, the components of $\boldsymbol{\xi}_{I0}$ are the elements of $\boldsymbol{I}_0$, and the components of $\boldsymbol{\xi}_{I,k}^i$ are the elements of $\boldsymbol{I}_k^i$ in the joint frame of J_k^i. Then, the unknown system parameter vector is chosen as $\boldsymbol{\xi} = [\boldsymbol{\xi}_{m0}^{\mathrm{T}}\ \boldsymbol{\xi}_{I0}^{\mathrm{T}}\ \boldsymbol{\xi}_{I}^{\mathrm{T}}\ \boldsymbol{\xi}_{ma}^{\mathrm{T}}\ \boldsymbol{\xi}_{maa}^{\mathrm{T}}\ \boldsymbol{\xi}_{m}^{\mathrm{T}}]^{\mathrm{T}}$, where $\boldsymbol{\xi}_x = [(\boldsymbol{\xi}_x^1)^{\mathrm{T}} \cdots (\boldsymbol{\xi}_x^N)^{\mathrm{T}}]^{\mathrm{T}}$ with $\boldsymbol{\xi}_x^i = [(\boldsymbol{\xi}_{x,1}^1)^{\mathrm{T}} \cdots (\boldsymbol{\xi}_{x,n_i}^N)^{\mathrm{T}}]^{\mathrm{T}}$, for $x = I, ma, maa, m$. On the other hand, after a complicated mathematical derivations, it can be developed from (31) that

$$\begin{aligned} \hat{\boldsymbol{H}}_{bb}^{-1}(\tilde{\boldsymbol{H}}_{bb}\boldsymbol{V}_0 + \tilde{\boldsymbol{H}}_{bm}\dot{\boldsymbol{\Theta}}) &= \hat{\boldsymbol{H}}_{bb}^{-1}\boldsymbol{Y}_a\tilde{\boldsymbol{\xi}} \\ \tilde{\boldsymbol{H}}_{bb}\dot{\boldsymbol{V}}_c + \tilde{\boldsymbol{H}}_{bm}\ddot{\boldsymbol{\Theta}}_c + \tilde{\boldsymbol{C}}_{bb}\boldsymbol{V}_c + \tilde{\boldsymbol{C}}_{bm}\dot{\boldsymbol{\Theta}}_c &= \boldsymbol{Y}_b\tilde{\boldsymbol{\xi}} \\ \tilde{\boldsymbol{H}}_{bm}^{\mathrm{T}}\dot{\boldsymbol{V}}_c + \tilde{\boldsymbol{H}}_{mm}\ddot{\boldsymbol{\Theta}}_c + \tilde{\boldsymbol{C}}_{mb}\boldsymbol{V}_c + \tilde{\boldsymbol{C}}_{mm}\dot{\boldsymbol{\Theta}}_c &= \boldsymbol{Y}_m\tilde{\boldsymbol{\xi}} \\ \hat{\boldsymbol{H}}_{bb}^{-1}\boldsymbol{G}(-r_0)\tilde{\boldsymbol{H}}_0 &= \boldsymbol{Y}_h\tilde{\boldsymbol{H}}_0 \end{aligned} \quad (33)$$

where $\boldsymbol{Y}_a, \boldsymbol{Y}_b, \boldsymbol{Y}_m$ and $\boldsymbol{Y}_h$ are regressor matrices and the explicit expressions are given in Appendix B.

By utilizing (33), the adaption law is thus designed as

$$\begin{cases} \dot{\hat{\boldsymbol{\xi}}} = -\boldsymbol{\Gamma}_1(\boldsymbol{Y}_a^{\mathrm{T}}\hat{\boldsymbol{H}}_{bb}^{-\mathrm{T}}\boldsymbol{N}^{\mathrm{T}}(\tilde{\boldsymbol{q}})\tilde{\boldsymbol{X}} + \boldsymbol{Y}_b^{\mathrm{T}}\tilde{\boldsymbol{V}} + \boldsymbol{Y}_m^{\mathrm{T}}\dot{\tilde{\boldsymbol{\Theta}}}) \\ \dot{\hat{\boldsymbol{H}}}_0 = \boldsymbol{\Gamma}_2\boldsymbol{Y}_h^{\mathrm{T}}\boldsymbol{N}^{\mathrm{T}}(\tilde{\boldsymbol{q}})\tilde{\boldsymbol{X}} \end{cases} \quad (34)$$

where $\boldsymbol{\Gamma}_1$ and $\boldsymbol{\Gamma}_2$ are two positive definite matrices to be given.

D. STABILITY ANALYSIS

Fig.2 illustrates the closed-loop system structure based on the aforementioned design and analysis. To handle the stability

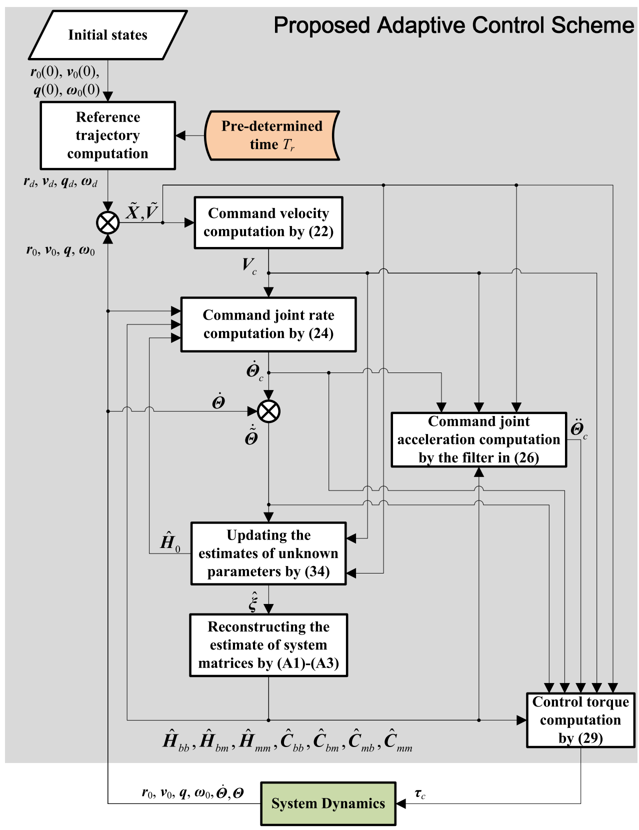

FIGURE 2. The structure of the closed-loop system.

analysis, a composite Lyapunov function of the whole system is given as $U = W_2 + (1/2)\tilde{\boldsymbol{\xi}}^{\mathrm{T}}\boldsymbol{\Gamma}_1^{-1}\tilde{\boldsymbol{\xi}} + (1/2)\tilde{\boldsymbol{H}}_0^{\mathrm{T}}\boldsymbol{\Gamma}_2^{-1}\tilde{\boldsymbol{H}}_0$, which is proved to satisfy $\underline{k}_U\tilde{\boldsymbol{x}}^{\mathrm{T}}\tilde{\boldsymbol{x}} \leq U \leq \bar{k}_U\tilde{\boldsymbol{x}}^{\mathrm{T}}\tilde{\boldsymbol{x}}$, where $\tilde{x}$ is the summarized error defined by $\tilde{\boldsymbol{x}} = [\tilde{\boldsymbol{X}}^{\mathrm{T}}\ \tilde{\boldsymbol{V}}^{\mathrm{T}}\ \dot{\tilde{\boldsymbol{\Theta}}}^{\mathrm{T}}\ \tilde{\boldsymbol{\xi}}^{\mathrm{T}}\ \tilde{\boldsymbol{H}}_0^{\mathrm{T}}]^{\mathrm{T}}$, $\underline{k}_U$, $\bar{k}_U$ yield $\underline{k}_U = \min\{1, \lambda_{\min}(\boldsymbol{H}), \lambda_{\min}(\boldsymbol{\Gamma}_1^{-1}), \lambda_{\min}(\boldsymbol{\Gamma}_2^{-1})\}/2$ and $\underline{k}_U = \max\{1, \lambda_{\max}(\boldsymbol{H}), \lambda_{\max}(\boldsymbol{\Gamma}_1^{-1}), \lambda_{\max}(\boldsymbol{\Gamma}_2^{-1})\}/2$, respectively. Then, for a compact set $B_c = \{\tilde{\boldsymbol{x}} : U(\tilde{\boldsymbol{x}}) \leq \bar{k}_U\tilde{\boldsymbol{x}}^{\mathrm{T}}\tilde{\boldsymbol{x}} \leq c\}$, an assumption is made as follows to make the control scheme computable.

Assumption 1: The matrix $\hat{\boldsymbol{H}}_{bm}$ has full row rank and the matrix $\hat{\boldsymbol{H}}_{bb}$ is non-singular for any $\tilde{\boldsymbol{x}} \in B_c$.

Thus, the following theorem is given to solve *Problem 1*.

Theorem 1: Given the system dynamics described by (2) and (10), and the reference trajectory governed by (12)-(19), if Assumption 1 hold, then for any initial states in B_c, the control torque τ_c in (29) with the filter in (26) and the adaption law in (34) enables the tracking error states $\tilde{\boldsymbol{X}}$, $\tilde{\boldsymbol{V}}$ and $\dot{\tilde{\boldsymbol{\Theta}}}$ converge to zero as $t \to +\infty$, in the presence of unknown system parameters $m_0, m_k^i, \boldsymbol{I}_0, \boldsymbol{I}_k^i, \boldsymbol{a}_k^i$, and unknown initial system momentum $\boldsymbol{H}_0$.

Proof: Differentiating U with respect to time and using (31) and (34) lead to that

$$\begin{aligned}\dot{U} &= \dot{W} + \tilde{\boldsymbol{\xi}}^{\mathrm{T}}\boldsymbol{\Gamma}_1^{-1}\dot{\hat{\boldsymbol{\xi}}} + \tilde{\boldsymbol{H}}_0^{\mathrm{T}}\boldsymbol{\Gamma}_2^{-1}\dot{\hat{\boldsymbol{H}}}_0 \\ &= -\tilde{\boldsymbol{X}}^{\mathrm{T}}\boldsymbol{N}\boldsymbol{K}_x\boldsymbol{N}^{\mathrm{T}}\tilde{\boldsymbol{X}} - \tilde{\boldsymbol{V}}^{\mathrm{T}}\boldsymbol{K}_v\tilde{\boldsymbol{V}} - \dot{\tilde{\boldsymbol{\Theta}}}^{\mathrm{T}}\boldsymbol{K}_\theta\dot{\tilde{\boldsymbol{\Theta}}} \leq 0 \quad (35)\end{aligned}$$

It is obvious that the set $B_0 = \{\tilde{x} : \tilde{\boldsymbol{X}} = 0,\ \tilde{\boldsymbol{V}} = 0,\ \dot{\tilde{\boldsymbol{\Theta}}} = 0\}$ is the largest invariant set in the set of all points in B_c where $\dot{U} = 0$, and therefore, according to LaSalle's theorem [46], the states $\tilde{\boldsymbol{X}}$, $\tilde{\boldsymbol{V}}$ and $\dot{\tilde{\boldsymbol{\Theta}}}$ approach B_0 as $t \to +\infty$.

The proof is completed. ■

IV. NUMERICAL SIMULATION

This section is to verify the effect of the proposed adaptive control scheme via numerical simulations, where a cubic spacecraft with two 4-DOF manipulators is considered, as shown in Fig. 3. The cubic spacecraft is supposed to be with side length $L = 1.2$m, and its mass properties are given as $m_0 = 50$kg and $\boldsymbol{I}_0 = \mathrm{diag}\{12, 12, 12\}\mathrm{kg}\cdot\mathrm{m}^2$. Each manipulator link is assumed to be a cylinder type with length of 1.5m, radius of 0.05m, and mass of 8kg, which implies that, in the joint frame of J_k^i, $m_k^i = 8$kg, $\boldsymbol{I}_k^i = \mathrm{diag}\{0.01, 1.5, 1.5\}\mathrm{kg}\cdot\mathrm{m}^2$, and $\boldsymbol{a}_k^i = [0.75\,0\,0]^{\mathrm{T}}$ m, for $i = A, B$, $k = 1,2,3,4$. These parameters determine the true value of the parameter vector ξ. The mounting position vectors of joint 1 for both manipulators are given in the body frame as $\boldsymbol{b}_0^A = (1/2)[L\ \ -L\ L]^{\mathrm{T}}$ and $b_0^B = (1/2)[-L\ \ L\ \ -L]^{\mathrm{T}}$. Besides, based on the given joint layout and manipulator configuration in Fig. 3, the initial joint angles of two manipulators are set to $\boldsymbol{\Theta}^A(0) = [30\ 120\ 60\ -60]^{\mathrm{T}}$ deg and $\boldsymbol{\Theta}^B(0) = [-60\ 60\ 120\ -30]^{\mathrm{T}}$ deg, respectively. The initial joint velocity of two manipulators are set to zero, i.e., $\dot{\boldsymbol{\Theta}}^A(0) = \dot{\boldsymbol{\Theta}}^B(0) = 0$.

The initial position and attitude error of the spacecraft are set as $\boldsymbol{r}_0(0) = [0.2\ -0.1\ 0.2]^{\mathrm{T}}$ m and $\boldsymbol{\Theta}_0(0) = [-10\ 10\ 10]^{\mathrm{T}}$ deg. Notice that the Euler angle form is adopted in illustrations to describe the spacecraft attitude for a clear physical meaning and the equivalent quaternion is utilized during the numerical calculation and simulation. The initial spacecraft velocity and angular velocity are set as $\boldsymbol{v}_0(0) = 10^{-3}\times[1.5\ 1.5\ -1.5]^{\mathrm{T}}$ m/s and $\omega_0(0) = [0.1\ -0.1\ 0.1]^{\mathrm{T}}$ deg/s, respectively. Furthermore, the initial angular and linear momentums can be computed out by using (3) as $\boldsymbol{L}(0) = [0.625\ -0.565\ 0.723]^{\mathrm{T}}$ kg $\cdot$ m^2/s and $\boldsymbol{P}(0) = [0.175\ 0.280\ -0.066]^{\mathrm{T}}$ kg $\cdot$ m/s, respectively.

As for the adaptive control scheme, choose $\boldsymbol{K}_x = 20\,\boldsymbol{E}_6$, $\boldsymbol{K}_b = \mathrm{diag}\{5E_3, \boldsymbol{E}_3\}$, $\boldsymbol{K}_m = 10\,\boldsymbol{E}_8$, $\boldsymbol{\Gamma}_1 = 100\,\boldsymbol{E}_{135}$, and $\boldsymbol{\Gamma}_2 = 10\,\boldsymbol{E}_6$. The initial value of the estimate is set to be $\hat{\xi}(0) = 0.95\xi$. The polynomial order is selected as $n_r = n_q = 4$ for the reference trajectory design. Moreover the pre-determined time T_r is a key parameter to the system convergence performance and it will be studied by choosing $T_r = 30$, 40, and 50s, respectively.

Fig. 4 shows the time histories of the spacecraft position and velocity with different T_r. Meanwhile, the time histories of the spacecraft attitude and angular velocity are illustrated in Fig. 5. It can be seen from these simulation results that the spacecraft position and attitude error can be stabilized with a satisfactory dynamic performance regardless of unknown system parameters and initial system momentum. Moreover,

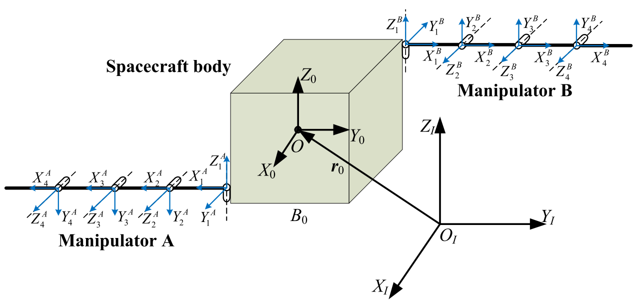

FIGURE 3. Simulation scenario with a spacecraft with dual 4-DOF manipulator.

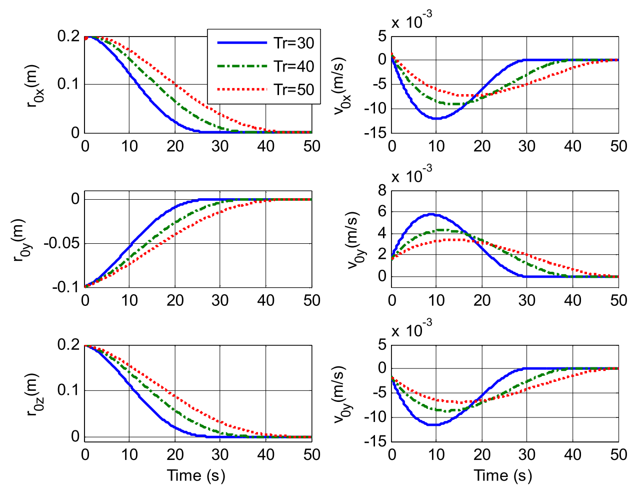

FIGURE 4. The time history of spacecraft position and velocity.

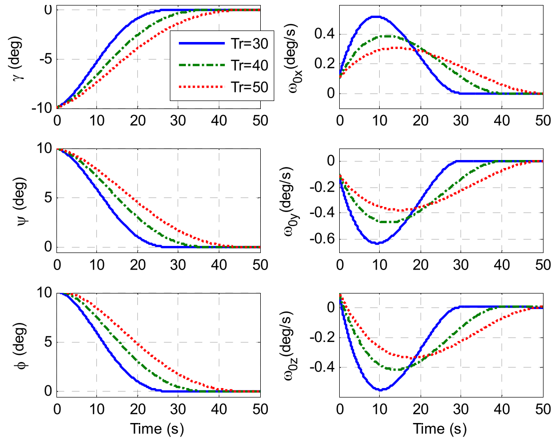

FIGURE 5. The time history of spacecraft attitude and angular velocity.

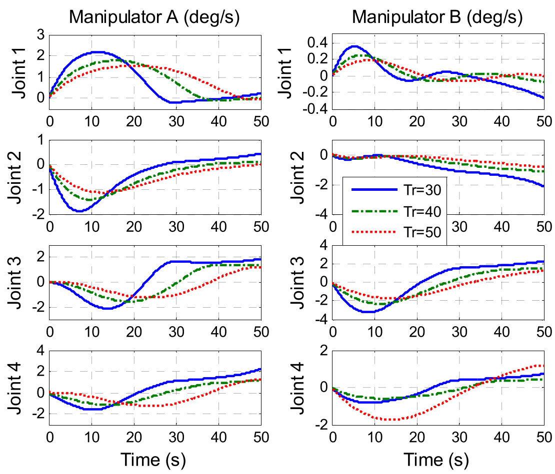

FIGURE 6. The time history of joint rate of two manipulators.

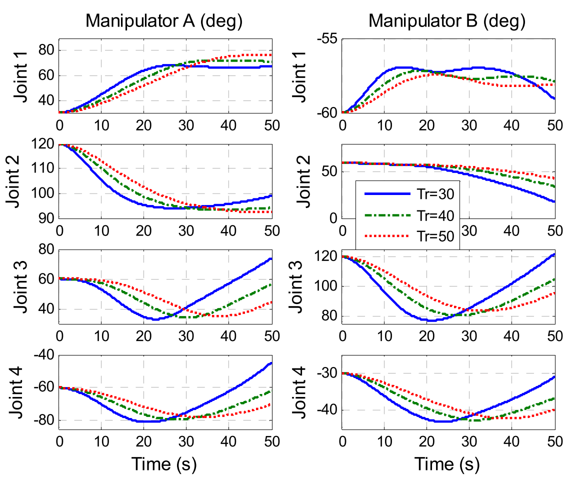

FIGURE 7. The time history of joint angle of two manipulators.

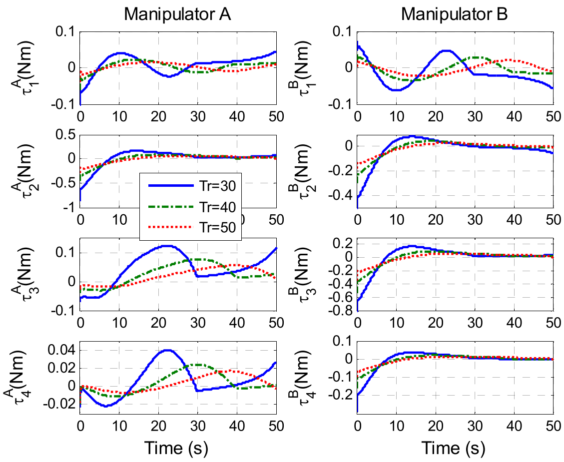

FIGURE 8. The time history of control torque of two manipulators.

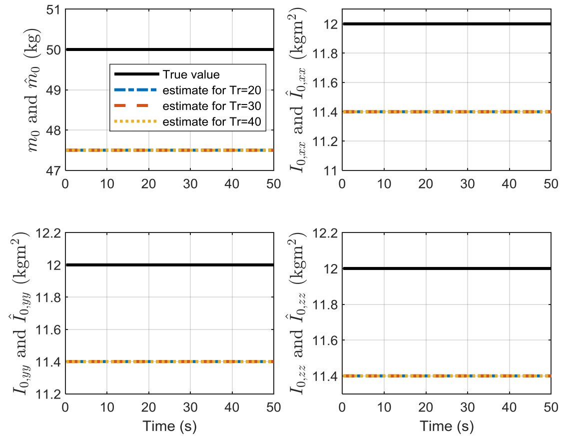

FIGURE 9. The estimate and true value of the spacecraft mass property.

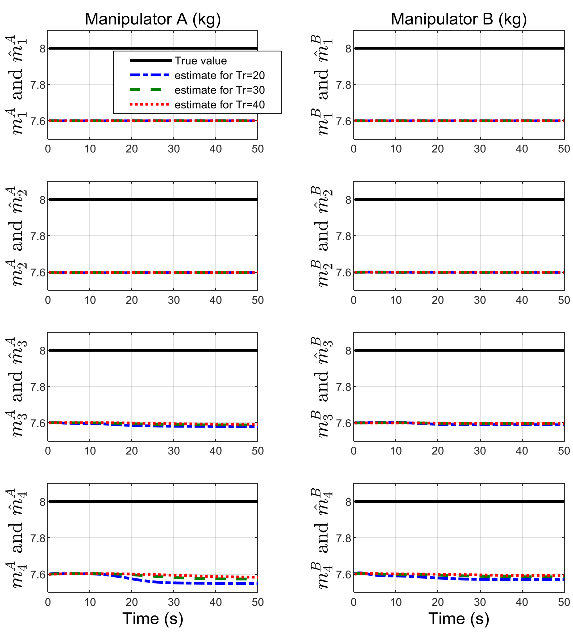

FIGURE 10. The estimate and true value of the manipulator link mass.

all components of the translational and rotational motion of the spacecraft possess a similar performance for different T_r, because they obey the same polynomial function once the polynomial order is determined, by recalling the design procedure of the reference trajectory in Section III.A. This also results in that a larger value of T_r causes a smaller regulation of spacecraft velocity and angular velocity.

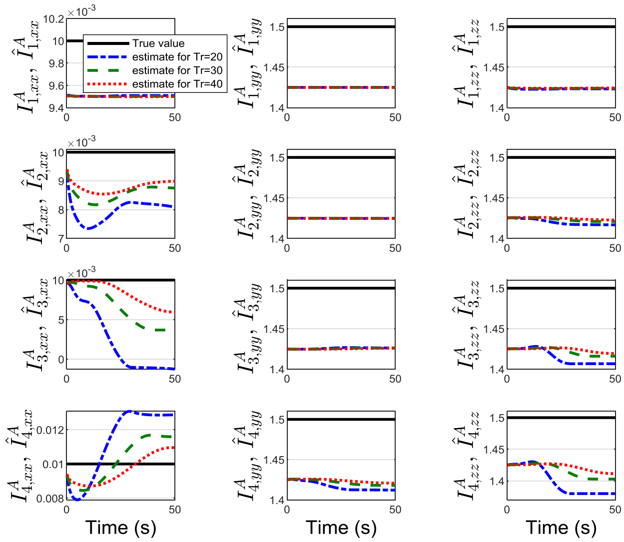

FIGURE 11. The estimate and true value of the main elements of the inertia matrix of Manipulator A (Unit: kg · m^2).

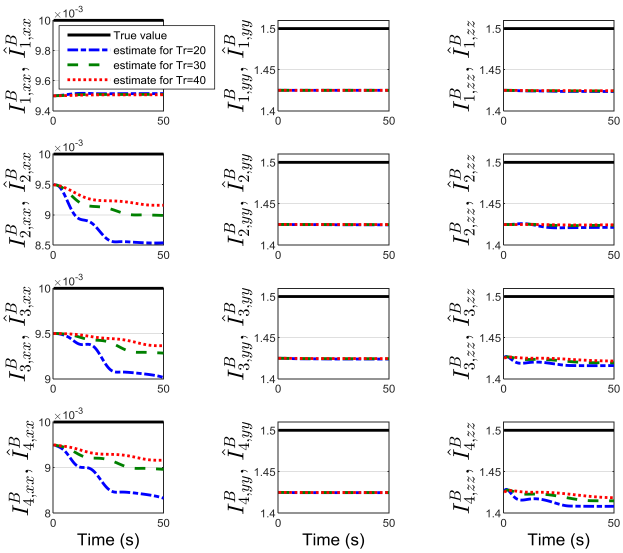

FIGURE 12. The estimate and true value of the main elements of the inertia matrix of Manipulator B (Unit: kg · m^2).

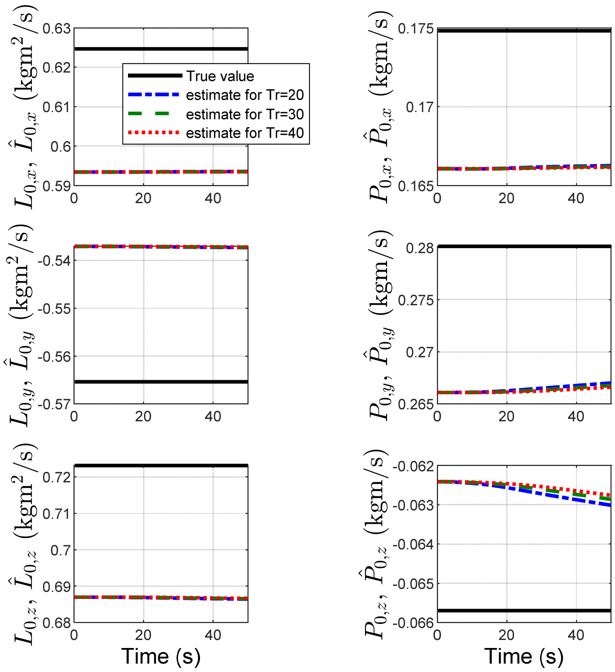

FIGURE 13. The estimate and true value of initial system momentum.

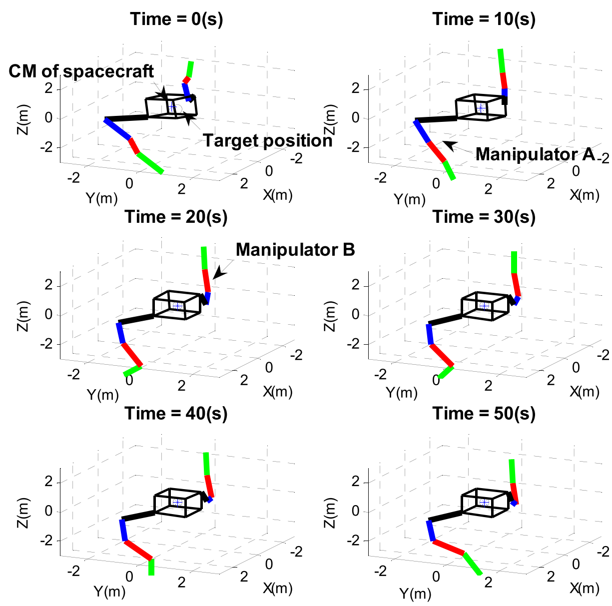

FIGURE 14. System variation at representative times ($T_r = 30$).

Figs 6-7 illustrate the time histories of the joint rates and angles of two manipulators, respectively. It can be found that the joint angles change in a good performance. Besides, by comparison, a smaller value of T_r will lead to a larger joint angle variation, which implies that T_r cannot be set arbitrarily small due to the physical limit of the joint motion.

Another thing should be noted from Fig. 6 that not all joint rates of manipulators converge to zero. This is because that the joint motion of two manipulators should be driven to continuously compensate for the initial residual system momentum to keep the position and attitude stabilization of the spacecraft. It can be also concluded that the joint motion would be stopped if there is no initial system momentum.

Fig. 8 describes the time histories of joint control torques for both manipulators. It can be seen from the curves that the proposed reference trajectory based control scheme not only guarantees a satisfactory system performance but also a well-behaviored control variation with less overshoot. Moreover, as shown in the figure, a smaller value of T_r ensures a fast stabilization, but causes a larger control effort.

Figs. 9-13 illustrate the simulation results of the estimates of the spacecraft mass property, the link mass of both

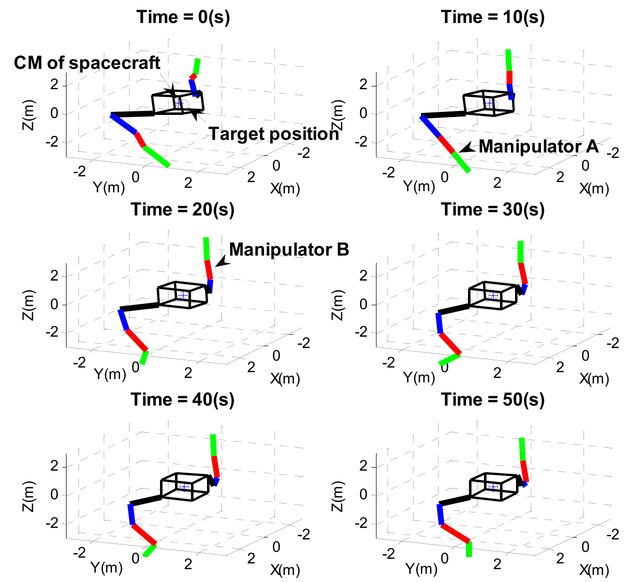

FIGURE 15. System variation at representative times ($T_r = 40$).

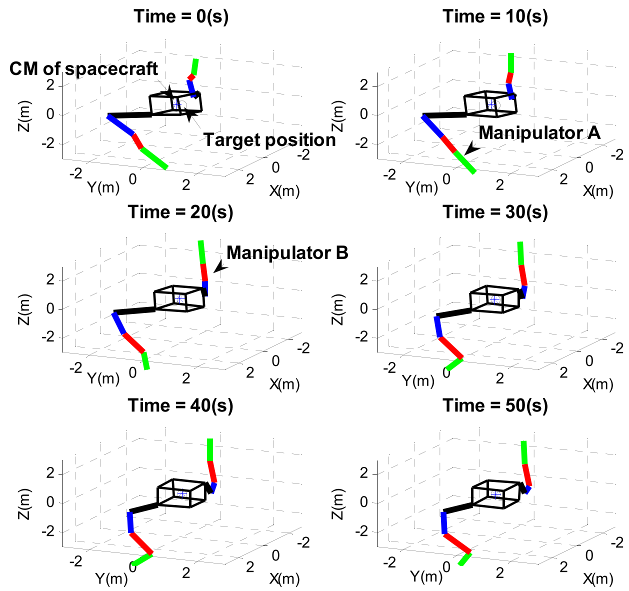

FIGURE 16. System variation at representative times ($T_r = 50$).

manipulators, the inertia matrix of Manipulator A, the inertia matrix of Manipulator B, and the initial system momentum, respectively. Due to the length limitation, the estimates of ξ_{ma} and ξ_{maa} are omitted here. It can be found from the simulation results that the estimates are bounded but they do not converge to the true values, which is mainly due to sufficient frequency components in the tracked states is not guaranteed, i.e., the persistent excitation (PE) condition is not satisfied [34]. Besides, it can be found that the value of T_r affects the estimation to a little extent.

Figs. 14-16 describe the system component variations at several representative times in a 3-dimensional manner for cases with T_r = 30, 40, and 50, respectively. It can be observed obviously from these figures that 1) the mass center of the spacecraft reaches the target position; 2) the spacecraft is regulated to the desired attitude via the proposed manipulator actuation; and 3) both of two manipulators have conducted a reasonable motion since the joints motion within their admissible range and there is no possible collision.

V. CONCLUSION

This study deals with the adaptive control problem for manipulator actuated integrated translational and rotational stabilization of spacecraft in proximity operations. By doing so, the kinematics and dynamics are successively formulated in an explicit form by incorporating two classic methods. The kinematic coupling equation in (3) formulated based on momentum conservation connects the motions of joint and spacecraft and makes the whole dynamics possess a cascaded structure. Then, an adaptive control scheme is proposed following the backstepping procedure to stabilize the position and attitude error of the spacecraft subject to unknown mass properties of system components and unknown mass center positions of each manipulator link. The prescribed reference trajectory is designed by polynomial methods for a good stabilization performance. The second-order filter in (26) is constructed based on the structure of spacecraft dynamics to ensure the control scheme computable. The general explicit-form regressor matrices of space manipulator dynamics are derived for the first time, and the resulting adaptation law is to update the estimate of unknown parameters. The closed-loop stability analysis guarantees the asymptotic convergence of the position and attitude error of the spacecraft. Finally, numerical simulations are given to demonstrate the effect of the designed adaptive control scheme.

APPENDIX

A. EXPLICIT EXPRESSIONS OF MATRICES IN SEC. II.B

The system matrices in kinematic and dynamic formulations described by (3) and (10) in Sec. II.B have the form as

$$\begin{aligned}
\boldsymbol{H}_{bb} &= \boldsymbol{M}_0 + \sum\nolimits_{i=1}^{N} \boldsymbol{G}_0^i \boldsymbol{M}^i (\boldsymbol{G}_0^i)^{\mathrm{T}}, \boldsymbol{H}_{bm} = [\boldsymbol{H}_{bm}^1 \ \cdots \ \boldsymbol{H}_{bm}^N],\\
\boldsymbol{H}_{bm}^i &= \boldsymbol{G}_0^i \boldsymbol{M}^i (\boldsymbol{G}_0^i)^{\mathrm{T}} \boldsymbol{H}^i, \ \boldsymbol{H}_{mm} = \mathrm{diag}\{\boldsymbol{H}_{mm}^1, \ldots, \boldsymbol{H}_{mm}^N\},\\
\boldsymbol{H}_{mm}^i &= (\boldsymbol{H}^i)^{\mathrm{T}} \boldsymbol{G}^i \boldsymbol{M}^i (\boldsymbol{G}^i)^{\mathrm{T}} \boldsymbol{H}^i
\end{aligned} \tag{A1}$$

$$\begin{aligned}
\boldsymbol{C}_{bb} &= \boldsymbol{B}_0 + \sum\nolimits_{i=1}^{N} \boldsymbol{C}_{bb}^i, \ \boldsymbol{C}_{bm} = [\boldsymbol{C}_{bm}^1 \ \cdots \ \boldsymbol{C}_{bm}^N]\\
\boldsymbol{C}_{mb} &= [(\boldsymbol{C}_{mb}^1)^{\mathrm{T}} \cdots (\boldsymbol{C}_{mb}^1)^{\mathrm{T}}]^{\mathrm{T}}, \boldsymbol{C}_{mm} = \mathrm{diag}\{\boldsymbol{C}_{mm}^1, \ldots, \boldsymbol{C}_{mm}^N\}\\
\boldsymbol{C}_{bb}^i &= \boldsymbol{G}_0^i \boldsymbol{M}^i (\boldsymbol{G}^i)^{\mathrm{T}} (\boldsymbol{G}_l^i)^{\mathrm{T}} + \boldsymbol{G}_0^i \boldsymbol{B}_V^i\\
\boldsymbol{C}_{mb}^i &= (\boldsymbol{H}^i)^{\mathrm{T}} (\boldsymbol{G}^i)^{\mathrm{T}} (\boldsymbol{M}^i (\boldsymbol{G}^i)^{\mathrm{T}} (\boldsymbol{G}_l^i)^{\mathrm{T}} + \boldsymbol{B}_V^i\\
\boldsymbol{C}_{bm}^i &= \boldsymbol{G}_0^i \boldsymbol{M}^i (\boldsymbol{G}^i)^{\mathrm{T}} (\dot{\boldsymbol{H}}^i + (\boldsymbol{G}_L^i)^{\mathrm{T}} \bar{\boldsymbol{H}}^i) + \boldsymbol{G}_0^i \boldsymbol{B}^i (\bar{\boldsymbol{H}}^i + \boldsymbol{H}^i)\\
\boldsymbol{C}_{mm}^i &= (\boldsymbol{G}^i \boldsymbol{H}^i)^{\mathrm{T}} (\boldsymbol{M}^i (\boldsymbol{G}^i)^{\mathrm{T}} (\dot{\boldsymbol{H}}^i + (\boldsymbol{G}_L^i)^{\mathrm{T}} \bar{\boldsymbol{H}}^i) + \boldsymbol{B}^i (\bar{\boldsymbol{H}}^i + \boldsymbol{H}^i))
\end{aligned} \tag{A2}$$

where

$$\boldsymbol{M}^i = \mathrm{diag}\{\boldsymbol{M}(J_1^i), \ldots, \boldsymbol{M}(J_{n_i}^i)\}, \ \boldsymbol{H}^i = \mathrm{diag}\{\boldsymbol{H}_1^i, \ldots, \boldsymbol{H}_{n_i}^i\}$$

$$\boldsymbol{M}_0 = \mathrm{diag}\{\boldsymbol{I}_0, m_0 \boldsymbol{E}_3\}, \ \boldsymbol{G}_0^i = [\boldsymbol{G}(l_0^i) \ \boldsymbol{G}(l_1^i) \cdots \boldsymbol{G}(l_{0(n_i-1)}^i)]$$

$$G^i = \begin{bmatrix} E_6 & G(l_1^i) & \cdots & G(l_{1(n-1)}^i) \\ & E_6 & \ddots & \vdots \\ & & \ddots & G(l_{(n-1)}^i) \\ & & & E_6 \end{bmatrix}, \ \bar{H}^i = \begin{bmatrix} \bar{H}_0^i \\ \bar{H}_1^i \\ \vdots \\ \bar{H}_{n_i-1}^i \end{bmatrix}$$

$$G_l^i = [\dot{G}(l_0^i) \cdots \dot{G}(l_{n_i-1}^i)],\ G_L^i = \mathrm{diag}\{\dot{G}(l_0^i), \ldots, \dot{G}(l_{n_i-1}^i)\}$$

$$B_V^i = [B^{\mathrm{T}}(J_1^i) \cdots B^{\mathrm{T}}(J_{n_i}^i)]^{\mathrm{T}},\ B^i = \mathrm{diag}\{B(J_1^i), \ldots, B(J_{n_i}^i)\}$$

$$B(J_k^i) = \begin{bmatrix} \omega(J_k^i)\left(I_k^i - m_k^i a_k^{i\times} a_k^{i\times}\right) & O \\ -m_k^i \dot{a}_k^{i\times} & O \end{bmatrix}, \quad B_0 = \begin{bmatrix} \omega_0^\times I_0 & O \\ O & O \end{bmatrix}$$

$$M(J_k^i) = \begin{bmatrix} I_k^i - m_k^i a_k^{i\times} a_k^{i\times} & m_k^i a_k^{i\times} \\ -m_k^i a_k^{i\times} & m_k^i E_3 \end{bmatrix}, \quad H_k^i = \begin{bmatrix} h_k^i \\ O \end{bmatrix}$$

$$\bar{H}_{k-1}^i = [H_1^i \cdots H_{k-1}^i \ O_{6\times1} \cdots O_{6\times1}] \tag{A3}$$

B. EXPLICIT EXPRESSIONS OF MATRICES IN SEC. III.C

Explicit expressions of regressor matrices in Sec. III.C are given as follows.

(1) The regressor matrix Y_a has the explicit form as

$$Y_a = \begin{bmatrix} O & Y_{a,I0}^\omega & Y_{a,I}^\omega & Y_{a,ma}^\omega & -Y_{a,maa}^\omega & Y_{a,m}^\omega \\ Y_{a,m0}^v & O & O & -Y_{a,ma}^v & O & Y_{a,m}^v \end{bmatrix} \tag{B1}$$

with

$$\begin{aligned} Y_{a,I0}^\omega &= L_I(\omega_0), \quad Y_{a,I}^\omega = L_{I\omega}(\omega_0) + L_{Iu2}(\dot{\Theta}, h) \\ Y_{a,ma}^\omega &= L_{au}(v_0) - L_{lau}(\omega_0) + L_{lau2}(\dot{\Theta}, h) - L_{alu2}(\dot{\Theta}, h) \\ Y_{a,maa}^\omega &= L_{aau}(\omega_0) + L_{aau2}(\dot{\Theta}, h), \\ Y_{a,m}^\omega &= L_{lu}(v_0) - L_{llu}(\omega_0) - L_{llu2}(\dot{\Theta}, h) \\ Y_{a,m0}^v &= v_0, \quad Y_{a,ma}^v = L_{au}(\omega_0) - L_{au2}(\dot{\Theta}, h) \\ Y_{a,m}^v &= L_m(v_0) - L_{lu}(\omega_0) - L_{lu2}(\dot{\Theta}, h) \end{aligned} \tag{B2}$$

where, $h = [(h^1)^{\mathrm{T}} \cdots (h^N)^{\mathrm{T}}]^{\mathrm{T}}$ with $h^i = [(h_1^i)^{\mathrm{T}} \cdots (h_{n_i}^i)^{\mathrm{T}}]^{\mathrm{T}}$; for any $u \in \mathbb{R}^3$, the matrices $L_I(u)$ and $L_m(u)$ yield

$$L_I(u) = \begin{bmatrix} u_x & 0 & 0 & 0 & u_z & u_y \\ 0 & u_y & 0 & u_z & 0 & u_x \\ 0 & 0 & u_z & u_y & u_x & 0 \end{bmatrix}, \quad L_m(u) = \underbrace{[u \cdots u]}_{n_1+\cdots+n_N}$$

and moreover, on the one hand, the matrices $L_x(u)$, $x = I\omega$, au, lau, aau, lu, llu, au, are governed by

$$L_x(u) = \left[L_{I\omega}^1(u) \cdots L_{I\omega}^N(u)\right], L_x^i(u) = \left[L_{x,1}^i(u) \cdots L_{x,n_i}^i(u)\right]$$

with

$$L_{I\omega,k}^i(u) = R_{Jk}^{i,0} L_I((R_{Jk}^{i,0})^{\mathrm{T}} u), \quad L_{au,k}^i(u) = -u^\times R_{Jk}^{i,0}$$

$$L_{lau,k}^i(u) = -(l_{0(k-1)}^{i\times} u^\times + (l_{0(k-1)}^{i\times} u)^\times) R_{Jk}^{i,0}$$

$$L_{aau,k}^i(u) = \begin{bmatrix} \left((u^\times R_{Jk}^{i0} e_x)^\times R_{Jk}^{i0} e_x\right)^{\mathrm{T}} \\ \left((u^\times R_{Jk}^{i0} e_y)^\times R_{Jk}^{i0} e_y\right)^{\mathrm{T}} \\ \left((u^\times R_{Jk}^{i0} e_z)^\times R_{Jk}^{i0} e_z\right)^{\mathrm{T}} \\ \left(((u^\times R_{Jk}^{i0} e_z)^\times - (R_{Jk}^{i0} e_z)^\times u^\times) R_{Jk}^{i0} e_y\right)^{\mathrm{T}} \\ \left(((u^\times R_{Jk}^{i0} e_z)^\times - (R_{Jk}^{i0} e_z)^\times u^\times) R_{Jk}^{i0} e_x\right)^{\mathrm{T}} \\ \left(((u^\times R_{Jk}^{i0} e_y)^\times - (R_{Jk}^{i0} e_y)^\times u^\times) R_{Jk}^{i0} e_x\right)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$$

$$L_{lu,k}^i(u) = l_{0(k-1)}^{i\times} u, \quad L_{llu,k}^i(u) = l_{0(k-1)}^{i\times} l_{0(k-1)}^{i\times} u,$$
$$L_{au,k}^i(u) = -u^\times R_{Jk}^{i0}$$

in which $e_x = [1\ 0\ 0]^{\mathrm{T}}$, $e_y = [0\ 1\ 0]^{\mathrm{T}}$, $e_z = [0\ 0\ 1]^{\mathrm{T}}$, and R_{Jk}^{i0} represents the transformation matrix from joint frame to the inertial frame; on the other hand, for any two vectors $\varphi \in \mathbb{R}^{n_1+\cdots+n_N}$ and $\rho \in \mathbb{R}^{3(n_1+\cdots+n_N)}$, comprising the scalar $\varphi_k^i \in \mathbb{R}$ and the vector $\rho_k^i \in \mathbb{R}^3$ with the structure as

$$\begin{aligned} \varphi &= [(\varphi^1)^{\mathrm{T}}\ (\varphi^2)^{\mathrm{T}} \cdots (\varphi^N)^{\mathrm{T}}]^{\mathrm{T}}, \ \varphi^i = [\varphi_1^i\ \varphi_2^i \cdots \varphi_{n_i}^i]^{\mathrm{T}} \\ \rho &= [(\rho^1)^{\mathrm{T}}\ (\rho^2)^{\mathrm{T}} \cdots (\rho^N)^{\mathrm{T}}]^{\mathrm{T}}, \\ \rho^i &= [(\rho_1^i)^{\mathrm{T}}\ (\rho_2^i)^{\mathrm{T}} \cdots (\rho_{n_i}^i)^{\mathrm{T}}]^{\mathrm{T}} \end{aligned} \tag{B3}$$

the matrices $L_y(\varphi, \rho)$, $y = Iu$, $lau2$, $alu2$, $aau2$, $llu2$, $au2$, $lu2$, are governed by

$$L_y(\varphi, \rho) = \left[L_y^1(\varphi^1, \rho^1) \cdots L_y^N(\varphi^N, \rho^N)\right]$$
$$L_y^i(\varphi^i, \rho^i) = \sum\nolimits_{k=1}^{n_i} L_{y,k}^i(\varphi_k^i, \rho_k^i)$$

with

$$L_{Iu2,k}^i(\varphi_k^i, \rho_k^i) = \varphi_k^i \cdot \left[O\ R_{Jk}^{i,0} L_I((R_{Jk}^{i,0})^{\mathrm{T}} \rho_k^i) \cdots R_{Jn_i}^{i,0} L_I((R_{Jn_i}^{i,0})^{\mathrm{T}} \rho_k^i)\right]$$

$$L_{lau2,k}^i(\varphi_k^i, \rho_k^i) = \varphi_k^i \left[O\ l_{0(k-1)}^{i\times} \rho_k^{i\times} R_{Jk}^{i0} \cdots l_{0(n_i-1)}^{i\times} \rho_k^{i\times} R_{Jn_i}^{i0}\right]$$

$$L_{alu2,k}^i(\varphi_k^i, \rho_k^i) = \varphi_k^i \cdot \left[O\ (\rho_k^{i\times} l_{k(k-1)}^i)^\times R_{Jk}^{i0} \cdots (\rho_k^{i\times} l_{k(n_i-1)}^i)^\times R_{Jn_i}^{i0}\right]$$

$$L_{aau2,k}^i(\varphi_k^i, \rho_k^i) = \left[O\ L_{aau,k}^i(\varphi_k^i \rho_k^i) \cdots L_{aau,n_i}^i(\varphi_k^i \rho_k^i)\right]$$

$$L_{llu2,k}^i(\varphi_k^i, \rho_k^i) = \varphi_k^i \left[O\ l_{0(k-1)}^{i\times} l_{k(k-1)}^{i\times} \rho_k^i \cdots l_{0(n_i-1)}^{i\times} l_{k(n_i-1)}^{i\times} \rho_k^i\right]$$

$$L_{au2,k}^i(\varphi_k^i, \rho_k^i) = \varphi_k^i \left[O\ \rho_k^{i\times} R_{Jk}^{i0} \cdots \rho_k^{i\times} R_{Jn_i}^{i0}\right]$$

$$L_{lu2,k}^i(\varphi_k^i, \rho_k^i) = \varphi_k^i \left[O\ l_{k(k-1)}^{i\times} \rho_k^i \cdots l_{k(n_i-1)}^{i\times} \rho_k^i\right]$$

(2) The regressor matrix Y_b has the explicit form as

$$Y_b = \begin{bmatrix} O & Y_{b,I0}^\omega & Y_{b,I}^\omega & Y_{b,ma}^\omega & -Y_{b,maa}^\omega & Y_{b,m}^\omega \\ Y_{b,m0}^v & O & O & -Y_{b,ma}^v & O & Y_{b,m}^v \end{bmatrix} \tag{B4}$$

with

$$\begin{aligned} Y_{b,I0}^\omega &= L_I(\dot{\omega}_c) + \omega_0^\times L_I(\omega_c),\ Y_{b,m0}^v = \dot{v}_c \\ Y_{b,I}^\omega &= L_{I\omega}(\dot{\omega}_c) + L_{Iu2}(\ddot{\Theta}_c, h) + L_{I\omega5}(\omega_c) \\ &\quad + L_{Iu2}(\dot{\Theta}_c, \dot{h}) + L_{Iu6}(\dot{\Theta}_c, h) \\ Y_{b,ma}^\omega &= L_{au}(\dot{v}_c) - L_{lau}(\dot{\omega}_c) + L_{lau2}(\ddot{\Theta}_c, h) + L_{lau2}(\dot{\Theta}_c, \dot{h}) \\ &\quad + L_{lau5}(\omega_c) + L_{lau6}(\dot{\Theta}_c, \dot{h}_k^i) - L_{alu2}(\ddot{\Theta}_c, h) \\ &\quad - L_{alu2}(\dot{\Theta}_c, \dot{h}) + L_{alu5}(\omega_c) - L_{alu6}(\dot{\Theta}_c, \dot{h}) \\ Y_{b,maa}^\omega &= L_{aau}(\dot{\omega}_c) + L_{aau2}(\ddot{\Theta}_c, h) + L_{maa5}(\omega_c) \end{aligned}$$

$$\begin{aligned}
&\quad + \boldsymbol{L}_{aau2}(\dot{\boldsymbol{\Theta}}_c, \dot{\boldsymbol{h}}) + \boldsymbol{L}_{aau6}(\dot{\boldsymbol{\Theta}}_c, \boldsymbol{h}) \\
\boldsymbol{Y}_{b,m}^{\omega} &= \boldsymbol{L}_{lu}(\dot{\boldsymbol{v}}_c) - \boldsymbol{L}_{llu}(\dot{\boldsymbol{\omega}}_c) - \boldsymbol{L}_{llu2}(\ddot{\boldsymbol{\Theta}}_c, \boldsymbol{h}) - \boldsymbol{L}_{llu2}(\dot{\boldsymbol{\Theta}}_c, \dot{\boldsymbol{h}}) \\
&\quad + \boldsymbol{L}_{llu5}(\boldsymbol{\omega}_c) - \boldsymbol{L}_{llu6}(\dot{\boldsymbol{\Theta}}_c, \boldsymbol{h}) \\
\boldsymbol{Y}_{b,ma}^{v} &= \boldsymbol{L}_{au}(\dot{\boldsymbol{\omega}}_c) - \boldsymbol{L}_{au2}(\ddot{\boldsymbol{\Theta}}_c, \boldsymbol{h}) - \boldsymbol{L}_{au5}(\boldsymbol{\omega}_c) \\
&\quad - \boldsymbol{L}_{au2}(\dot{\boldsymbol{\Theta}}_c, \dot{\boldsymbol{h}}) - \boldsymbol{L}_{au6}(\dot{\boldsymbol{\Theta}}_c, \boldsymbol{h}) \\
\boldsymbol{Y}_{b,m}^{v} &= \boldsymbol{L}_{m}(\dot{\boldsymbol{v}}_c) - \boldsymbol{L}_{lu}(\dot{\boldsymbol{\omega}}_c) - \boldsymbol{L}_{lu2}(\ddot{\boldsymbol{\Theta}}_c, \boldsymbol{h}) + \boldsymbol{L}_{lu5}(\boldsymbol{\omega}_c) \\
&\quad - \boldsymbol{L}_{lu2}(\dot{\boldsymbol{\Theta}}_c, \dot{\boldsymbol{h}}) - \boldsymbol{L}_{lu6}(\dot{\boldsymbol{\Theta}}_c, \boldsymbol{h})
\end{aligned} \tag{B5}$$

where, for any $u \in \mathbb{R}^3$, the matrices $\boldsymbol{L}_x(\boldsymbol{u})$, $x = I\omega5$, $maa5$, $lau5$, $alu5$, $lu5$, $llu5$, $au5$, are governed by

$$\begin{aligned}
\boldsymbol{L}_x(\boldsymbol{u}) &= \left[\boldsymbol{L}_{I\omega}^1(\boldsymbol{u}) \cdots \boldsymbol{L}_{I\omega}^N(\boldsymbol{u}) \right], \\
\boldsymbol{L}_x^i(\boldsymbol{u}) &= \left[\boldsymbol{L}_{x,1}^i(\boldsymbol{u}) \cdots \boldsymbol{L}_{x,n_i}^i(\boldsymbol{u}) \right]
\end{aligned}$$

with

$$\begin{aligned}
\boldsymbol{L}_{I\omega5,k}^i(\boldsymbol{u}) &= \boldsymbol{\omega}^{\times}(J_k^i)\boldsymbol{R}_{Jk}^{i0}\boldsymbol{L}_I(\boldsymbol{R}_{Jk}^{i0,\mathrm{T}}\boldsymbol{u}), \\
\boldsymbol{L}_{llu5,k}^i(\boldsymbol{u}) &= (\dot{\boldsymbol{l}}_{0(k-1)}^{i\times}\boldsymbol{u})^{\times}\boldsymbol{l}_{0(k-1)}^i \\
\boldsymbol{L}_{maa5,k}^i(\boldsymbol{u}) &= \boldsymbol{\omega}^{\times}(J_k^i)\boldsymbol{L}_{aau,k}^i(\boldsymbol{u}),\ \boldsymbol{L}_{alu5,k}^i(\boldsymbol{u}) = (\dot{\boldsymbol{l}}_{0(k-1)}^{i\times}\boldsymbol{u})^{\times}\boldsymbol{R}_{Jk}^{i0} \\
\boldsymbol{L}_{lau5,k}^i(\boldsymbol{u}) &= \boldsymbol{l}_{0(k-1)}^{i\times}\boldsymbol{u}^{\times}\boldsymbol{\omega}^{\times}(J_k^i)\boldsymbol{R}_{Jk}^{i0},\ \boldsymbol{L}_{lu5,k}^i(\boldsymbol{u}) = \boldsymbol{u}^{\times}\dot{\boldsymbol{l}}_{0(k-1)}^i \\
\boldsymbol{L}_{au5,k}^i(\boldsymbol{u}) &= \boldsymbol{u}^{\times}\boldsymbol{\omega}^{\times}(J_k^i)\boldsymbol{R}_{Jk}^i
\end{aligned}$$

and, for any two vectors $\boldsymbol{\varphi} \in \mathbb{R}^{n_1+\cdots+n_N}$ and $\boldsymbol{\rho} \in \mathbb{R}^{3(n_1+\cdots+n_N)}$, comprising the scalar $\boldsymbol{\varphi}_k^i \in \mathbb{R}$ and the vector $\boldsymbol{\rho}_k^i \in \mathbb{R}^3$ with the structure in (B3), the matrices $\boldsymbol{L}_y(\varphi, \rho)$, $y = Iu6$, $lau6$, $alu6$, $aau6$, $llu6$, $au6$, $lu6$, are governed by

$$\begin{aligned}
\boldsymbol{L}_y(\boldsymbol{\varphi}, \boldsymbol{\rho}) &= \left[\boldsymbol{L}_y^1(\boldsymbol{\varphi}^1, \boldsymbol{\rho}^1) \cdots \boldsymbol{L}_y^N(\boldsymbol{\varphi}^N, \boldsymbol{\rho}^N) \right] \\
\boldsymbol{L}_y^i(\boldsymbol{\varphi}^i, \boldsymbol{\rho}^i) &= \sum\nolimits_{k=1}^{n_i} \boldsymbol{L}_{y,k}^i(\varphi_k^i, \boldsymbol{\rho}_k^i)
\end{aligned}$$

with

$$\begin{aligned}
&\boldsymbol{L}_{Iu6,k}^i(\varphi_k^i, \boldsymbol{\rho}_k^i) \\
&= \varphi_k^i \cdot \left[O\ \boldsymbol{\omega}^{\times}(J_k^i)\boldsymbol{R}_{Jk}^{i0}\boldsymbol{L}_I(\boldsymbol{R}_{Jk}^{i0,\mathrm{T}}\boldsymbol{\rho}_k^i) \cdots \boldsymbol{\omega}^{\times}(J_{n_i}^i)\boldsymbol{R}_{Jn_i}^{i0}\boldsymbol{L}_I(\boldsymbol{R}_{Jn_i}^{i0,\mathrm{T}}\boldsymbol{\rho}_k^i) \right] \\
&\boldsymbol{L}_{lau6,k}^i(\varphi_k^i, \boldsymbol{\rho}_k^i) \\
&= \varphi_k^i \cdot \left[O\ \boldsymbol{l}_{0(k-1)}^{i\times}\boldsymbol{\rho}_k^{i\times}\boldsymbol{\omega}^{\times}(J_k^i)\boldsymbol{R}_{Jk}^{i0} \cdots \boldsymbol{l}_{0(n_i-1)}^{i\times}\boldsymbol{\rho}_k^{i\times}\boldsymbol{\omega}^{\times}(J_{n_i}^i)\boldsymbol{R}_{Jn_i}^{i0} \right] \\
&\boldsymbol{L}_{alu6,k}^i(\varphi_k^i, \boldsymbol{\rho}_k^i) \\
&= \varphi_k^i \cdot \left[O\ (\boldsymbol{\rho}_k^{i\times}\dot{\boldsymbol{l}}_{k(k-1)}^i)^{\times}\boldsymbol{R}_{Jk}^{i0} \cdots (\boldsymbol{\rho}_k^{i\times}\dot{\boldsymbol{l}}_{k(n_i-1)}^i)^{\times}\boldsymbol{R}_{Jn_i}^{i0} \right] \\
&\boldsymbol{L}_{aau6,k}^i(\varphi_k^i, \boldsymbol{\rho}_k^i) \\
&= \varphi_k^i \cdot \left[O\ \boldsymbol{\omega}^{\times}(J_k^i)\boldsymbol{L}_{aau,k}^i(\boldsymbol{\rho}_k^i) \cdots \boldsymbol{\omega}^{\times}(J_{n_i}^i)\boldsymbol{L}_{aau,n_i}^i(\boldsymbol{\rho}_k^i) \right] \\
&\boldsymbol{L}_{llu6,k}^i(\varphi_k^i, \boldsymbol{\rho}_k^i) \\
&= \varphi_k^i \left[O\ \boldsymbol{l}_{0(k-1)}^{i\times}\dot{\boldsymbol{l}}_{k(k-1)}^{i\times}\boldsymbol{\rho}_k^i \cdots \boldsymbol{l}_{0(n_i-1)}^{i\times}\dot{\boldsymbol{l}}_{k(n_i-1)}^{i\times}\boldsymbol{\rho}_k^i \right] \\
&\boldsymbol{L}_{au6,k}^i(\varphi_k^i, \boldsymbol{\rho}_k^i) \\
&= \varphi_k^i \cdot \left[O\ \boldsymbol{\rho}_k^{i\times}\boldsymbol{\omega}^{\times}(J_k^i)\boldsymbol{R}_{Jk}^{i0} \cdots \boldsymbol{\rho}_k^{i\times}\boldsymbol{\omega}^{\times}(J_{n_i}^i))\boldsymbol{R}_{Jn_i}^{i0} \right] \\
&\boldsymbol{L}_{lu6,k}^i(\varphi_k^i, \boldsymbol{\rho}_k^i) \\
&= \varphi_k^i \left[O\ \dot{\boldsymbol{l}}_{k(k-1)}^{i\times}\boldsymbol{\rho}_k^i \cdots \dot{\boldsymbol{l}}_{k(n_i-1)}^{i\times}\boldsymbol{\rho}_k^i \right]
\end{aligned}$$

(3) The regressor matrix $\boldsymbol{Y}_m$ has the explicit form as

$$\boldsymbol{Y}_m = \left[O\ O\ \boldsymbol{Y}_{m,I}\ \boldsymbol{Y}_{m,ma}\ -\boldsymbol{Y}_{m,maa}\ -\boldsymbol{Y}_{m,m} \right] \tag{B6}$$

with

$$\begin{aligned}
\boldsymbol{Y}_{m,I} &= \boldsymbol{L}_{I3}(\dot{\boldsymbol{\omega}}_c) + \boldsymbol{L}_{I4}(\ddot{\Theta}_c) - \boldsymbol{L}_{I8}(\boldsymbol{\omega}_c) + \boldsymbol{L}_{I9}(\dot{\Theta}_c) \\
\boldsymbol{Y}_{m,ma} &= \boldsymbol{L}_{ma3}(\dot{\boldsymbol{\omega}}_c, \dot{v}_c) + \boldsymbol{L}_{ma4}(\ddot{\Theta}_c) + \boldsymbol{L}_{ma8}(\boldsymbol{\omega}_c) + \boldsymbol{L}_{ma9}(\dot{\Theta}_c) \\
\boldsymbol{Y}_{m,maa} &= \boldsymbol{L}_{maa3}(\dot{\boldsymbol{\omega}}_c) + \boldsymbol{L}_{maa4}(\ddot{\Theta}_c) - \boldsymbol{L}_{maa8}(\boldsymbol{\omega}_c) + \boldsymbol{L}_{maa9}(\dot{\Theta}_c) \\
\boldsymbol{Y}_{m,m} &= \boldsymbol{L}_{m3}(\dot{\boldsymbol{\omega}}_c, \dot{v}_c) + \boldsymbol{L}_{m4}(\ddot{\Theta}_c) + \boldsymbol{L}_{m8}(\boldsymbol{\omega}_c) + \boldsymbol{L}_{m9}(\dot{\Theta}_c)
\end{aligned} \tag{B7}$$

for any $u, w \in \mathbb{R}^3$, the matrices $\boldsymbol{L}_x(\boldsymbol{u})$, $x = I3$, $maa3$, $I8$, $ma8$, $maa8$, $m8$, and $\boldsymbol{L}_y(\boldsymbol{u}, \boldsymbol{w})$, $y = ma3$, $m3$, are governed by

$$\begin{aligned}
\boldsymbol{L}_x(\boldsymbol{u}) &= \mathrm{diag}\{\boldsymbol{L}_x^1(\boldsymbol{u}), \ldots, \boldsymbol{L}_x^N(\boldsymbol{u})\} \\
\boldsymbol{L}_y(\boldsymbol{u}, \boldsymbol{w}) &= \mathrm{diag}\{\boldsymbol{L}_x^1(\boldsymbol{u}, \boldsymbol{w}), \ldots, \boldsymbol{L}_x^N(\boldsymbol{u}, \boldsymbol{w})\} \\
\boldsymbol{L}_x^i(\boldsymbol{u}) &= [(\boldsymbol{L}_{x,1}^i(\boldsymbol{u}))^{\mathrm{T}} \cdots (\boldsymbol{L}_{x,n_i}^i(\boldsymbol{u}))^{\mathrm{T}}]^{\mathrm{T}} \\
\boldsymbol{L}_y^i(\boldsymbol{u}, \boldsymbol{w}) &= [(\boldsymbol{L}_{x,1}^i(\boldsymbol{u}, \boldsymbol{w}))^{\mathrm{T}} \cdots (\boldsymbol{L}_{x,n_i}^i(\boldsymbol{u}, \boldsymbol{w}))^{\mathrm{T}}]^{\mathrm{T}}
\end{aligned}$$

with

$$\begin{aligned}
\boldsymbol{L}_{I3,k}^i(\boldsymbol{u}) &= \boldsymbol{u}^{\mathrm{T}}\boldsymbol{R}_{Jj}^{i0}\boldsymbol{L}_{Iu2,k}^i(1, (\boldsymbol{R}_{Jk}^{i0})^{\mathrm{T}}\boldsymbol{h}_k^i) \\
\boldsymbol{L}_{ma3,k}^i(\boldsymbol{u}, \boldsymbol{w}) &= \boldsymbol{u}^{\mathrm{T}}(\boldsymbol{L}_{lau2,k}^i(1, \boldsymbol{h}_k^i) - \boldsymbol{L}_{alu2,k}^i(1, \boldsymbol{h}_k^i)) \\
&\quad + \boldsymbol{w}^{\mathrm{T}}\boldsymbol{L}_{au2,k}^i(1, \boldsymbol{h}_k^i) \\
\boldsymbol{L}_{I3,k}^i(\boldsymbol{u}) &= \boldsymbol{u}^{\mathrm{T}}\boldsymbol{R}_{Jj}^{i0}\boldsymbol{L}_{Iu2,k}^i(1, (\boldsymbol{R}_{Jk}^{i0})^{\mathrm{T}}\boldsymbol{h}_k^i) \\
\boldsymbol{L}_{ma3,k}^i(\boldsymbol{u}, \boldsymbol{w}) &= \boldsymbol{u}^{\mathrm{T}}\boldsymbol{L}_{lau2,k}^i(1, \boldsymbol{h}_k^i) \\
&\quad - \boldsymbol{u}^{\mathrm{T}}\boldsymbol{L}_{alu2,k}^i(1, \boldsymbol{h}_k^i) + \boldsymbol{w}^{\mathrm{T}}\boldsymbol{L}_{au2,k}^i(1, \boldsymbol{h}_k^i) \\
\boldsymbol{L}_{maa3,k}^i(\boldsymbol{u}) &= \boldsymbol{u}^{\mathrm{T}}\boldsymbol{L}_{aau2,k}^i(1, \boldsymbol{h}_k^i) \\
\boldsymbol{L}_{m3,k}^i(\boldsymbol{u}, \boldsymbol{w}) &= \boldsymbol{u}^{\mathrm{T}}\boldsymbol{L}_{llu2,k}^i(1, \boldsymbol{h}_k^i) + \boldsymbol{w}^{\mathrm{T}}\boldsymbol{L}_{lu2,k}^i(1, \boldsymbol{h}_k^i) \\
\boldsymbol{L}_{ma8,k}^i(\boldsymbol{u}) &= \boldsymbol{u}^{\mathrm{T}} \left[O\ \dot{l}_{0(k-1)}^{i\times}\boldsymbol{h}_k^{i\times}\boldsymbol{R}_{Jk}^{i0} + (l_{k(k-1)}^{i\times}\boldsymbol{h}_k^i)^{\times}\omega^{\times}(J_k^i)\boldsymbol{R}_{Jk}^{i0} \right. \\
&\qquad \left. \cdots \dot{l}_{0(n_i-1)}^{i\times}\boldsymbol{h}_k^{i\times}\boldsymbol{R}_{Jn_i}^{i0} + (l_{k(n_i-1)}^{i\times}\boldsymbol{h}_k^i)^{\times}\omega^{\times}(J_{n_i}^i)\boldsymbol{R}_{Jn_i}^{i0} \right] \\
\boldsymbol{L}_{m8,k}^i(\boldsymbol{u}) &= \boldsymbol{u}^{\mathrm{T}} \left[O\ \dot{l}_{0(k-1)}^{i\times}l_{k(k-1)}^{i\times}\boldsymbol{h}_k^i \cdots \dot{l}_{0(n_i-1)}^{i\times}l_{k(n_i-1)}^{i\times}\boldsymbol{h}_k^i \right] \\
\boldsymbol{L}_{I8,k}^i(\boldsymbol{u}) &= \boldsymbol{u}^{\mathrm{T}} \left[O\ \boldsymbol{R}_{Jk}^{i0}\boldsymbol{L}_I(\boldsymbol{R}_{Jk}^{i0,\mathrm{T}}\omega^{\times}(J_k^i)\boldsymbol{h}_k^i) \right. \\
&\qquad \left. \cdots \boldsymbol{R}_{Jn_i}^{i0}\boldsymbol{L}_I(\boldsymbol{R}_{Jn_i}^{i0,\mathrm{T}}\omega^{\times}(J_{n_i}^i)\boldsymbol{h}_k^i) \right] \\
\boldsymbol{L}_{maa8,k}^i(\boldsymbol{u}) &= \boldsymbol{u}^{\mathrm{T}} \left[O\ \boldsymbol{L}_{aau,k}^i(\omega^{\times}(J_k^i)\boldsymbol{h}_k^i) \right. \\
&\qquad \left. \cdots \boldsymbol{L}_{aau,n_i}^i(\omega^{\times}(J_{n_i}^i)\boldsymbol{h}_k^i) \right]
\end{aligned}$$

and, for any two vectors $\varphi \in \mathbb{R}^{n_1+\cdots+n_N}$ comprising the scalar $\varphi_k^i \in \mathbb{R}$ with the structure in (B3), the matrices $\boldsymbol{L}_z(\varphi)$, $z = I4$, $maa4$, $ma4$, $m4$, $I9$, $maa9$, $ma9$, $m9$ are governed by

$$\begin{aligned}
\boldsymbol{L}_z(\boldsymbol{\varphi}) &= \mathrm{diag}\{\boldsymbol{L}_z^i(\boldsymbol{\varphi}^1), \ldots, \boldsymbol{L}_z^N(\boldsymbol{\varphi}^N)\}, \\
\boldsymbol{L}_z^i(\boldsymbol{\varphi}^i) &= \sum\nolimits_{k=1}^{n_i} \boldsymbol{L}_{z,k}^i(\varphi_k^i)
\end{aligned}$$

$$\boldsymbol{L}_{z,k}^i(\varphi_k^i) = \varphi_k^i \left[\begin{array}{c|ccc}
 & \boldsymbol{L}_{z,k}^i[1,k] & \cdots & \boldsymbol{L}_{z,k}^i[1,n_i] \\
O & \vdots & \ddots & \vdots \\
 & \boldsymbol{L}_{z,k}^i[k-1,k] & \cdots & \boldsymbol{L}_{z,k}^i[k-1,n_i] \\
\hline
 & \boldsymbol{L}_{z,k}^i[k,k] & \cdots & \boldsymbol{L}_{z,k}^i[k,n_i] \\
O & & \ddots & \\
 & O & & \boldsymbol{L}_{z,k}^i[n_i,n_i]
\end{array} \right]$$

where $L_{z,k}^{i}[j, r]$ is the element at the jth row and the rth column of the matrix $L_{z,k}^{i}(\varphi_k^i)$, $j = 1,\ldots, n_i$, $r = k,\ldots,n_i$, and satisfies

$$
\begin{aligned}
\boldsymbol{L}_{I4,k}^{i}[j,r] &= \boldsymbol{h}_j^{i,\mathrm{T}} \boldsymbol{R}_{Jr}^{i0} \boldsymbol{L}_I (\boldsymbol{R}_{Jr}^{i0,\mathrm{T}} \boldsymbol{h}_k^i) \\
\boldsymbol{L}_{maa4,k}^{i}[j,r] &= \boldsymbol{h}_j^{i,\mathrm{T}} \boldsymbol{L}_{aau,j}^{i} (\boldsymbol{h}_k^i) \\
\boldsymbol{L}_{ma4,k}^{i}[j,r] &= \boldsymbol{h}_j^{i,\mathrm{T}} \left(\boldsymbol{l}_{j(k-1)}^{i\times} \boldsymbol{h}_k^{i\times} + (\boldsymbol{l}_{k(r-1)}^{i\times} \boldsymbol{h}_k^i)^{\times} \right) \boldsymbol{R}_{Jr}^{i0} \\
\boldsymbol{L}_{m4,k}^{i}[j,r] &= \boldsymbol{h}_j^{i,\mathrm{T}} \boldsymbol{l}_{j(r-1)}^{i\times} \boldsymbol{l}_{k(r-1)}^{i\times} \boldsymbol{h}_k^i \\
\boldsymbol{L}_{I9,k}^{i}[j,r] &= \boldsymbol{h}_j^{i,\mathrm{T}} \left(\boldsymbol{R}_{Jr}^{i0} \boldsymbol{L}_I (\boldsymbol{R}_{Jr}^{i0,\mathrm{T}} \dot{\boldsymbol{h}}_k^i) + \omega^{\times}(J_r^i) \boldsymbol{R}_{Jr}^{i0} \boldsymbol{L}_I (\boldsymbol{R}_{Jr}^{i0,\mathrm{T}} \boldsymbol{h}_k^i) \right) \\
\boldsymbol{L}_{maa9,k}^{i}[j,r] &= \boldsymbol{h}_j^{i,\mathrm{T}} \left(\boldsymbol{L}_{auu,r}^{i} (\dot{\boldsymbol{h}}_k^i) + \omega^{\times}(J_r^i) \boldsymbol{L}_{auu,r}^{i} (\boldsymbol{h}_k^i) \right) \\
\boldsymbol{L}_{ma9,k}^{i}[j,r] &= \boldsymbol{h}_j^{i,\mathrm{T}} (\boldsymbol{l}_{k(r-1)}^{i\times} \dot{\boldsymbol{h}}_k^i + \dot{\boldsymbol{l}}_{k(r-1)}^{i\times} \boldsymbol{h}_k^i)^{\times} \boldsymbol{R}_{Jr}^{i0} \\
&\quad + \boldsymbol{h}_j^{i,\mathrm{T}} \boldsymbol{l}_{j(r-1)}^{i\times} (\dot{\boldsymbol{h}}_k^{i\times} + \boldsymbol{h}_k^{i\times} \omega^{\times}(J_r^i)) \boldsymbol{R}_{Jr}^{i0} \\
\boldsymbol{L}_{m9,k}^{i}[j,r] &= \boldsymbol{h}_j^{i,\mathrm{T}} \boldsymbol{l}_{j(r-1)}^{i\times} (\boldsymbol{l}_{k(r-1)}^{i\times} \dot{\boldsymbol{h}}_k^i + \dot{\boldsymbol{l}}_{k(r-1)}^{i\times} \boldsymbol{h}_k^i)
\end{aligned}
$$

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REFERENCES

[1] F. Terui, "Position and attitude control of a spacecraft by sliding mode control," in *Proc. Amer. Control Conf. (ACC)*, Philadelphia, PA, USA, Jun. 1998, pp. 217–221.

[2] R. Kristiansen, P. J. Nicklasson, and J. T. Gravdahl, "Spacecraft coordination control in 6DOF: Integrator backstepping vs passivity-based control," *Automatica*, vol. 44, no. 11, pp. 2896–2901, Nov. 2008, doi: 10.1016/j.automatica.2008.04.019.

[3] J. Shan, "Six-degree-of-freedom synchronised adaptive learning control for spacecraft formation flying," *IET Control Theory Appl.*, vol. 2, no. 10, pp. 930–949, Oct. 2008, doi: 10.1049/iet-cta:20080063.

[4] C. Pukdeboon, "Inverse optimal sliding mode control of spacecraft with coupled translation and attitude dynamics," *Int. J. Syst. Sci.*, vol. 46, no. 13, pp. 2421–2438, 2015, doi: 110.1080/00207721.2015.1011251.

[5] D. Zhou, Y. Zhang, and S. Li, "Receding horizon guidance and control using sequential convex programming for spacecraft 6-DOF close proximity," *Aerosp. Sci. Technol.*, vol. 87, pp. 459–477, Apr. 2019, doi: 10.1016/j.ast.2019.02.041.

[6] Y. Huang and Y. Jia, "Robust adaptive fixed-time tracking control of 6-DOF spacecraft fly-around mission for noncooperative target," *Int. J. Robust Nonlinear Control*, vol. 28, no. 6, pp. 2598–2618, Apr. 2018, doi: 10.1002/rnc.4038.

[7] Q. Hu, X. Shao, and W.-H. Chen, "Robust fault-tolerant tracking control for spacecraft proximity operations using time-varying sliding mode," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 54, no. 1, pp. 2–17, Feb. 2018, doi: 10.1109/TAES.2017.2729978.

[8] H. Dong, Q. Hu, and M. R. Akella, "Dual-Quaternion-Based spacecraft autonomous rendezvous and docking under Six-Degree-of-Freedom motion constraints," *J. Guid., Control, Dyn.*, vol. 41, no. 5, pp. 1150–1162, May 2018, doi: 10.2514/1.G003094.

[9] Z. Xu, Y. Chen, and Z. Xu, "Optimal guidance and collision avoidance for docking with the rotating target spacecraft," *Adv. Space Res.*, vol. 63, no. 10, pp. 3223–3234, May 2019, doi: 10.1016/j.asr.2019.01.026.

[10] H. Gui and G. Vukovich, "Finite-time output-feedback position and attitude tracking of a rigid body," *Automatica*, vol. 74, pp. 270–278, Dec. 2016, doi: 10.1016/j.automatica.2016.08.003.

[11] F. Curti, M. Romano, and R. Bevilacqua, "Lyapunov-based Thrusters' selection for spacecraft control: Analysis and experimentation," *J. Guid., Control, Dyn.*, vol. 33, no. 4, pp. 1143–1160, Jul. 2010, doi: 10.2514/1.47296.

[12] F. Zhang and G. R. Duan, "Integrated translational and rotational finite-time maneuver of a rigid spacecraft with actuator misalignment," *IET Control Theory Appl.*, vol. 6, no. 9, pp. 1192–1204, 2012, doi: 10.1049/iet-cta.2011.0413.

[13] F. Zhang and G. Duan, "Robust adaptive integrated translation and rotation control of a rigid spacecraft with control saturation and actuator misalignment," *Acta Astronautica*, vol. 86, pp. 167–187, May 2013, doi: 10.1016/j.actaastro.2013.01.010.

[14] G. Wu and G. Song, "Antisaturation attitude and orbit-coupled control for spacecraft final safe approach based on fast nonsingular terminal sliding mode," *J. Aerosp. Eng.*, vol. 32, no. 2, 2019, Art. no. 04019002, doi: 10.1061/(ASCE)AS.1943-5525.0000993.

[15] F. Zhang and G.-R. Duan, "Robust adaptive integrated translation and rotation finite-time control of a rigid spacecraft with actuator misalignment and unknown mass property," *Int. J. Syst. Sci.*, vol. 45, no. 5, pp. 1007–1034, May 2014, doi: 10.1080/00207721.2012.743618.

[16] H. Dong, Q. Hu, M. I. Friswell, and G. Ma, "Dual-Quaternion-Based fault-tolerant control for spacecraft tracking with finite-time convergence," *IEEE Trans. Control Syst. Technol.*, vol. 25, no. 4, pp. 1231–1242, Jul. 2017, doi: 10.1109/TCST.2016.2603070.

[17] M. J. Sidi, *Spacecraft Dynamics and Control: A Practical Engineering Approach*, 1st ed. New York, NY, USA: Cambridge Univ. Press, 1997.

[18] S. P. Bhat and P. K. Tiwari, "Controllability of spacecraft attitude using control moment gyroscopes," *IEEE Trans. Autom. Control*, vol. 54, no. 3, pp. 585–590, Mar. 2009, doi: 10.1109/TAC.2008.2008324.

[19] T. Sasaki, T. Shimomura, and H. Schaub, "Robust attitude control using a double-gimbal variable-speed control moment gyroscope," *J. Spacecraft Rockets*, vol. 55, no. 5, pp. 1235–1247, 2018, doi: 10.2514/1.A34120.

[20] Y. K. Nakka, S.-J. Chung, J. T. Allison, J. B. Aldrich, and O. S. Alvarez-Salazar, "Nonlinear attitude control of a spacecraft with distributed actuation of solar arrays," *J. Guid., Control, Dyn.*, vol. 42, no. 3, pp. 458–475, Mar. 2019, doi: 10.2514/1.G003478.

[21] Z. Feng and Y. Ning, "Manipulator-actuated adaptive integrated translational and rotational stabilization for proximity operations of spacecraft," in *Proc. 43rd Annu. Conf. IEEE Ind. Electron. Soc. (IECON)*, Beijing, China, Nov. 2017, pp. 6223–6228, doi: 10.1109/IECON.2017.8217081.

[22] F. Zhang and G.-R. Duan, "Manipulator-actuated adaptive integrated translational and rotational stabilization for spacecraft in proximity operations with control constraint," *Int. J. Control, Autom. Syst.*, vol. 16, no. 5, pp. 2103–2113, Oct. 2018, doi: 10.1007/s12555-017-0689-7.

[23] F. Zhang, Y. Li, and N. Yan, "Dual manipulator-actuated integrated translational and rotational stabilization of spacecraft in proximity operations," *Trans. Jpn. Soc. Aerosp. Sci.*, vol. 63, no. 5, pp. 195–205, 2020, doi: 10.2322/tjsass.63.195.

[24] Y. Umetani and K. Yoshida, "Resolved motion rate control of space manipulators with generalized Jacobian matrix," *IEEE Trans. Robot. Automat.*, vol. 5, no. 3, pp. 303–314, Jun. 1989, doi: 10.1109/70.34766.

[25] K. Yoshida, "Engineering test satellite VII flight experiments for space robot dynamics and control: Theories on laboratory test beds ten years ago, now in orbit," *Int. J. Robot. Res.*, vol. 22, no. 5, pp. 321–335, May 2003, doi: 10.1177/0278364903022005003.

[26] H. Wang, D. Guo, H. Xu, W. Chen, T. Liu, and K. K. Leang, "Eye-in-Hand tracking control of a free-floating space manipulator," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 53, no. 4, pp. 1855–1865, Aug. 2017, doi: 10.1109/TAES.2017.2674218.

[27] S. A. Ali Moosavian and E. Papadopoulos, "Explicit dynamics of space free-flyers with multiple manipulators via SPACEMAPLE," *Adv. Robot.*, vol. 18, no. 2, pp. 223–244, 2004, doi: 10.1163/156855304322758033.

[28] A. Jain, "Unified formulation of dynamics for serial rigid multibody systems," *J. Guid., Control, Dyn.*, vol. 14, no. 3, pp. 531–542, May 1991, doi: 10.2514/3.20672.

[29] G. Rodriguez, A. Jain, and K. Kreutz-Delgado, "A spatial operator algebra for manipulator modeling and control," *Int. J. Robot. Res.*, vol. 10, no. 4, pp. 371–381, 1991, doi: 10.1177/027836499101000406.

[30] Q. Hu, C. Guo, Y. Zhang, and J. Zhang, "Recursive decentralized control for robotic manipulators," *Aerosp. Sci. Technol.*, vol. 76, pp. 374–385, May 2018, doi: 10.1016/j.ast.2018.02.018.

[31] L. Sun and W. Huo, "6-DOF integrated adaptive backstepping control for spacecraft proximity operations," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 51, no. 3, pp. 2433–2443, Jul. 2015, doi: 10.1109/TAES.2015.140339.

[32] L. Sun and Z. Zheng, "Adaptive relative pose control for autonomous spacecraft rendezvous and proximity operations with thrust misalignment and model uncertainties," *Adv. Space Res.*, vol. 59, no. 7, pp. 1861–1871, Apr. 2017, doi: 10.1016/j.asr.2017.01.005.

[33] L. Sun, W. Huo, and Z. Jiao, "Adaptive backstepping control of spacecraft rendezvous and proximity operations with input saturation and full-state constraint," *IEEE Trans. Ind. Electron.*, vol. 64, no. 1, pp. 480–492, Jan. 2017, doi: 10.1109/TIE.2016.2609399.

[34] K. J. Astrom and B. Wittenmark, *Adaptive Control*, 2nd ed. New York, NY, USA: Addison-Wesley, 1995.

[35] W. Sun, Y. Liu, and H. Gao, "Constrained sampled-data ARC for a class of cascaded nonlinear systems with applications to motor-servo systems," *IEEE Trans. Ind. Informat.*, vol. 15, no. 2, pp. 766–776, Feb. 2019, doi: 10.1109/TII.2018.2821677.

[36] G. Feng and M. Palaniswami, "Adaptive control of robot manipulators in task space," *IEEE Trans. Autom. Control*, vol. 38, no. 1, pp. 100–104, 1993, doi: 10.1109/9.186316.

[37] X. Liang, H. Wang, Y.-H. Liu, W. Chen, G. Hu, and J. Zhao, "Adaptive task-space cooperative tracking control of networked robotic manipulators without task-space velocity measurements," *IEEE Trans. Cybern.*, vol. 46, no. 10, pp. 2386–2398, Oct. 2016, doi: 10.1109/TCYB.2015.2477606.

[38] H. Wang and Y. Xie, "Prediction error based adaptive jacobian tracking for free-floating space manipulators," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 48, no. 4, pp. 3207–3221, Oct. 2012, doi: 10.1109/TAES.2012.6324694.

[39] H. Wang, "Adaptive control of robot manipulators with uncertain kinematics and dynamics," *IEEE Trans. Autom. Control*, vol. 62, no. 2, pp. 948–954, Feb. 2017, doi: 10.1109/TAC.2016.2575827.

[40] X.-Y. Yao, H.-F. Ding, and M.-F. Ge, "Task-space tracking control of multi-robot systems with disturbances and uncertainties rejection capability," *Nonlinear Dyn.*, vol. 92, no. 4, pp. 1649–1664, Jun. 2018, doi: 10.1007/s11071-018-4152-y.

[41] C. Chern Cheah, M. Hirano, S. Kawamura, and S. Arimoto, "Approximate jacobian control for robots with uncertain kinematics and dynamics," *IEEE Trans. Robot. Autom.*(1989–June 2004)*, vol. 19, no. 4, pp. 692–702, Aug. 2003, doi: 10.1109/TRA.2003.814517.

[42] C. C. Cheah, M. Hirano, S. Kawamura, and S. Arimoto, "Approximate jacobian control with task-space damping for robot manipulators," *IEEE Trans. Autom. Control*, vol. 49, no. 5, pp. 752–757, May 2004, doi: 10.1109/TAC.2004.825971.

[43] B. Xiao, S. Yin, and O. Kaynak, "Tracking control of robotic manipulators with uncertain kinematics and dynamics," *IEEE Trans. Ind. Electron.*, vol. 63, no. 10, pp. 6439–6449, Oct. 2016, doi: 10.1109/TIE.2016.2569068.

[44] B. Xiao and S. Yin, "Exponential tracking control of robotic manipulators with uncertain dynamics and kinematics," *IEEE Trans. Ind. Informat.*, vol. 15, no. 2, pp. 689–698, Feb. 2019, doi: 10.1109/TII.2018.2809514.

[45] J. T.-Y. Wen and K. Kreutz-Delgado, "The attitude control problem," *IEEE Trans. Autom. Control*, vol. 36, no. 10, pp. 1148–1162, Oct. 1991, doi: 10.1109/9.90228.

[46] H. K. Khalil, *Nonlinear Systems*, 3rd ed. Pearson, NJ, USA, Pearson, 2002.

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