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Robust H-Inf Phase Control for Flexible System With Weak Damping

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ABSTRACT The characteristic of a flexible system with weak damping is that the amplitude characteristics show a large change, and the situation on the Nyquist chart is extremely complicated. Due to the existence of weak damping, the perturbation range of the flexible modal parameters allowed by the system is greatly compressed, and the robust stability is reduced, which brings challenges to the control design. To deal with this problem, the traditional H-inf loop shaping method often leads to the controller instability and poor robustness. In order to improve the stability margin of the system, increase the robustness and stability, and solve the problem of the instability of H-infcontrollers, this paper proposes a robust comprehensive design method based on phase control. In this method, by using the closed-loop pole configuration corresponding to the flexible mode at low frequency, the phase control is realized. Through the phase control, the apex of the Nyquist curve of the weakly damped mode of the system is limited to the Right half plane. The H-inf weighting function relaxes the pole configuration requirements, reduces the conservativeness of strict positive real, and combines the phase control with the H-inf optimization solution to obtain a robust and self-stable H-inf controller. A specific design example is given. The simulation results show that this control method has extremely high robustness. This design example can provide a certain degree of robust design for other weakly damp.

INDEX TERMS Robust stability, weakly damped flexible systems, phase control, H-inf optimization.

I. INTRODUCTION

With the constant seeking of precision and efficiency, a large number of flexible systems have been put into industrial production, ranging from the read/write heads of computer hard disks to all kinds of spacecraft, such as Space telescopes with Solar array or high-gain antennas [1]–[4], large truss structure satellites [5], etc. The uncertainties of flexible systems include structural uncertainties and non-structural uncertainties. Therefore, the designed control system should be able to make these two uncertainties robust, which means performance indicators can still be achieved when the system has certain parameter perturbations or has unmodeled dynamics at high frequencies. Adaptive control can also solve the control problem of systems with uncertainties [6], [7].

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H-inf control is widely used in the control problem of this kind of flexible system with uncertainty [8]–[10]. For the widely existing flexible system with weak damping, because it is more convenient to use coprime factor perturbations to describe this system so the H-inf loop forming method based on coprime factor perturbations is often used for weakly damped flexible system. But existing studies only give simple examples of the application of the H-inf loop forming method in weakly damped flexible systems, without in-depth analysis [11], [12].

The weak damping of the system is manifested in the frequency characteristic of crossing the 0dB line multiple times, which makes the situation surrounding $(-1, j0)$ on the Nyquist chart more complicated. Reference [13] conducted a research on the H-inf loop shaping design of a weakly damped flexible system, and found that the parameter (frequency) perturbation range of the flexible mode was greatly compressed due to the existence of weak damping, which caused the system to lose its robustness. For this reason, reference [14] proposes to combine H-inf loop shaping and μ synthesis to solve the problem of robust design under parameter perturbation. However, reference [15] shows that this method does not achieve the expected effect through example analysis. The addition of μ synthesis in the H-inf loop shaping design only affects the stability margin value, and cannot improve the allowable perturbation range. Therefore, weak damping is the difficulty in the design of flexible systems using the H-inf loop forming method.

When the weakly damped flexible system is designed with the H-inf loop shaping method, in addition to the above-mentioned robustness deterioration, there is also a problem of an unstable controller. H-inf control design is mainly about structure problems and weight function selection. The structural problems include two-block, four-block problems and μ synthesis, etc. At present, H-inf control has a satisfactory control effect in solving the problems of model uncertainty, model nonlinearity, actuator saturation limitation, vibration suppression and so on [16]–[19]. However, no matter what the structure is, an unstable H-inf controller will be produced for a weakly damped flexible system [14], [20]–[22]. This "unstable controller" means that the closed-loop system is stable, but the designed H-inf controller is unstable. There are relatively few studies on unstable controllers. In fact, it is difficult to invest in unstable controllers [20]. For example, the H-inf controller with high frequency components omitted is an unstable controller in [21].

In order to solve the problem of unstable controllers using H-inf control for weakly damped flexible systems and improve the robustness of the system, the phase control of the system must be considered. Taking a weak damping system with one flexible mode as an example, the Bode diagram is shown as crossing the 0dB line repeatedly, and the Nyquist diagram is shown as a large circle. When the parameters are perturbed, the situation surrounding the $(-1, j0)$ point is more likely to change, which leads to a reduction in the allowable perturbation range of the system. Whereas phase control requires that the vertices of the Nyquist curve of the weakly damped mode are all located on the positive real axis [23]. In this way, the system has better robustness when perturbation occurs. However, in the actual controller design process, there are design difficulties to achieve local regularization. Therefore, this paper starts from the theoretical basis of H-inf control, through theoretical derivation, obtains the mathematical expression of local regular design, which has a certain degree of innovation.

In this paper, the H-inf design research for changing the phase-frequency characteristics of the system. It combines the phase control idea with the H-inf control design to solve the unstable controller and poor robustness of the weakly damped flexible system. A specific design example is given, and a robust H-inf controller based on phase control of the flexible manipulator is designed. The control design has very high robustness. The simulation results show that the designed control system makes allowable parameter (damping) perturbation range of the device reaches more than 60%.

In the first section of this paper, the progress of H-inf control theory is briefly introduced, and the two problems of H-inf loop shaping method in the design of weak damping system are analyzed. The second section gives the theoretical basis of H-inf control based on phase control and the proof of related formulas. In the third section, the effectiveness of the method is illustrated by a simulation example of flexible system control with three weak damping modes. The fourth section summarizes the design difficulties and innovation of this method.

II. H-INF ROBUST CONTROL THEORY BASED ON PHASE CONTROL

The H-inf loop shaping method based on the theory of small gain only pays attention to the amplitude-frequency characteristics of the system, not the phase-frequency characteristics, and pays little attention to the Nyquist diagram. In fact, the phase information of the system is also very important. According to the Nyquist diagram drawn by the McFarlaned method proposed in [21], the unstable controller makes the closed-loop system stable by going around $(-1, j0)$. But when the resonant frequency of the system is slightly perturbed, the enclosing circle may move up or down, and cannot go around the $(-1, j0)$ point, and the closed-loop system cannot be stabilized. The McFarlaned method is used for H-inf loop shaping to control the weakly damped flexible system. The parameter perturbation range of the system is small and the robustness is poor.

If the strict positive real design is carried out, that is, the vertices of the Nyquist curve are all located on the positive real axis [24], as shown in Fig.1, the robust performance of the system will be greatly improved. But in fact, for a system with multiple weakly damped flexible modes, this strictly positive design is too conservative. Even if the controller is obtained by the H-inf loop shaping method, the order of the controller is often higher. It cannot be physically realized.

Therefore, this paper relaxes the strict positive real design requirements and proposes a local positive real design, as shown in Fig.2, that is, the vertices of the Nyquist curve of the weakly damped mode are all located on the right half plane. The digital display parameters of the system are perturbed, and the Nyquist curve of the system will not cross the critical point and cause instability.

The open loop Nyquist diagram of the system can reach the style shown in Fig.2 through the optimization design with H-inf. Compared with the [25], the controller obtained by this method has a low order and is easy to implement while ensuring robustness.

H-inf optimal design is a control theory based on coprime factor. so, the flexible controlled object $G(s)$ and the controller $K(s)$ are respectively written in the form of coprime

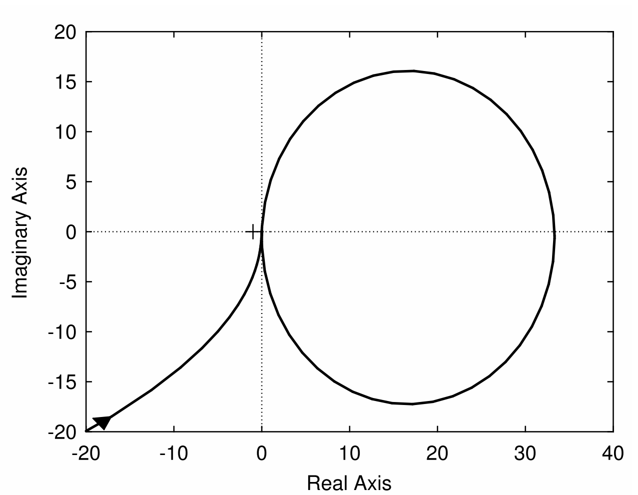

FIGURE 1. Strict positive real design.

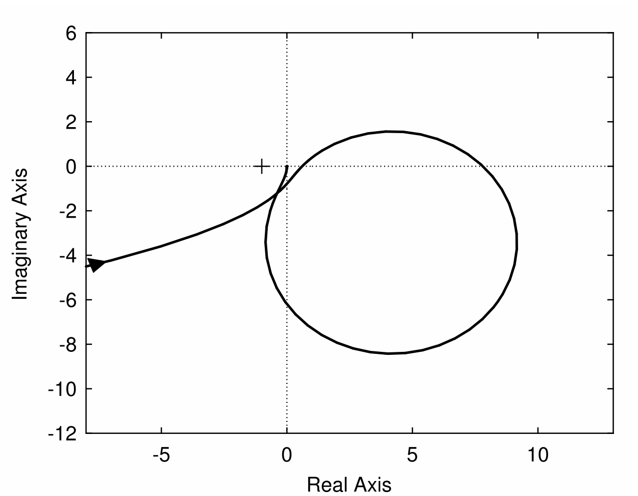

FIGURE 2. Local positive real design.

factors, refer to (1) and (2):

$$G(s) = \frac{n_g(s)/d_f(s)}{d_g(s)/d_f(s)}. \tag{1}$$

$$K(s) = \frac{N_k(s)}{D_k(s)} = \frac{n_k(s)/d_c(s)}{d_k(s)/d_c(s)}. \tag{2}$$

where, $d_f(s)$ and $d_c(s)$ are polynomials related to the characteristic equation of the system, and the coefficient of the highest order term is 1 [13]. According to (1) and (2), the closed-loop transfer function of the system can be obtained:

$$T(s) = \frac{N_g(s)D_k(s)}{N_g(s)N_k(s) + D_g(s)D_k(s)}. \tag{3}$$

Let the denominator of $T(s)$ be $P(s)$, then $P(s)$ can be expressed as shown in (4):

$$P(s) = \frac{n_g(s)n_k(s) + d_g(s)d_k(s)}{d_f(s)d_c(s)}. \tag{4}$$

where, the numerator of $P(s)$, $n_g(s)n_k(s) + d_g(s)d_k(s)$, is the closed-loop characteristic polynomial of the system, and the denominator, $n_g(s)n_k(s)+d_g(s)d_k(s)$, is the desired characteristic polynomial.

Therefore, only when $P(s) = 1$, can the closed-loop characteristic polynomial of the system be equal to the expected characteristic polynomial. If the desired characteristic polynomial is given in the design, the controller, $K(s) = \frac{n_k(s)}{d_k(s)}$, can be solved according to the condition of $P(s) = 1$. However, this condition cannot be met in all frequency ranges, so this article proposes the weighted design idea to solve the problem:

$$\min_{K(s)} \|W(s)[1 - P(s)]\|_\infty. \tag{5}$$

where, $W(s)$ is the weighting function selected to satisfy the low frequency characteristics.

According to the above analysis, by solving the optimization problems of (4) and (5), it can be satisfied that the dominant pole of the flexible mode of the system is equal to the expected pole at the low frequency band, but the high frequency is different. In this way, $P(jw) \to 1$ can be realized in the frequency band of the dominant pole. It can be seen that this method is much more flexible than the pole configuration method in the classic control theory, and it can combine the pole configuration with the optimization solution.

The optimization problem shown in (5) can be solved according to the bounded real lemma in H-inf control theory. Use $\left[A_{wp}\ B_{wp}\ C_{wp}\ 0\right]$ to represent the state space form of $W(s)[1 - P(s)]$, then according to the bounded real lemma, the necessary and sufficient condition of $W(s)[1 - P(s)] < \gamma$ is the existence of $X = X^T > 0$ makes the following matrix inequality holds. The proof is shown in Lemma 5.3 in [26]. Which is:

$$\begin{bmatrix} A_{wp}^T X + XA_{wp}^T & XB_{wp} & C_{wp}^T \\ B_{wp}^T X & -\gamma I & 0 \\ C_{wp} & 0 & \gamma \end{bmatrix} < 0. \tag{6}$$

When transformed into the state space form shown in (6), it will be found that $n_k(s)$ and $d_k(s)$ in the controller $K(s)$ only appear in C_{wp} and are linear. So (6) is a linear matrix inequality, and its optimization problem is equivalent to the solution of γ under the restriction of linear matrix inequality. Reger to (7):

$$\gamma_m = \min_{K(s)} \gamma \tag{7}$$

The above analysis shows that the main idea of the H-inf robust design method based on phase control to solve the control problem of weakly damped flexible system is:

(1) At low frequencies, make the closed-loop pole corresponding to the weakly damped mode have a negative real part while increasing the damping ratio.

(2) The high-frequency pole configuration can be appropriately relaxed through the weighting function in H infinity optimization.

(3) For other closed-loop pole configurations, it is hoped that the open-loop characteristics can be far away from the critical stable point to the right half plane.

III. ROBUST CONTROL DESIGN OF WEAKLY DAMPED FLEXIBLE SYSTEM

A. MATHEMATICAL MODEL OF WEAKLY DAMPED FLEXIBLE SYSTEM

The mathematical model of a flexible system can usually be expressed by the sum of a rigid body mode and an infinite number of flexible models, as shown in (8) [27]:

$$G(s) = \sum_{i=0}^{\infty} \frac{K_i}{s^2 + 2\zeta_i\omega_i s + \omega_i^2} \tag{8}$$

where, $i = 0$ corresponds to the rigid body mode, and all subsequent modes are all flexible modes. Generally, the higher the frequency of the flexible mode, the amplitude is often smaller. Therefore, multiple low-frequency flexible modes are often used to represent the flexibility of the entire controlled object. At the same time, the damping ζ_i in (8) is often small, and the system exhibits typical weak damping characteristics. On the open-loop Bode diagram, it appears to cross the 0dB line multiple times, and on its Nyquist diagram it appears as multiple great circles close to the $(-1, j0)$ point. Therefore, when the parameters are perturbed, it is very likely that the original surrounding $(-1, j0)$ point will be changed and the system will be unstable.

In this section, a general control example of a flexible manipulator with three weak damping modes is given to verify the effectiveness of the robust H-inf phase control method. The relevant parameters of this flexible manipulator are shown in Table.1:

TABLE 1. The relevant parameters of this flexible manipulator.

i	K_i	$\omega_i/rad \cdot s^{-1}$	ζ_i
0	1.26	0	0
1	-2.5527	5.28	0.067
2	3	10	0.02
3	2	20	0.02

The Bode diagram of the system is shown in Fig.3. It can be seen from Fig.3 that the flexible manipulator exhibits typical weak damping characteristics. Due to the existence of 3 weak damping modes, it crosses the 0dB line 6 times.

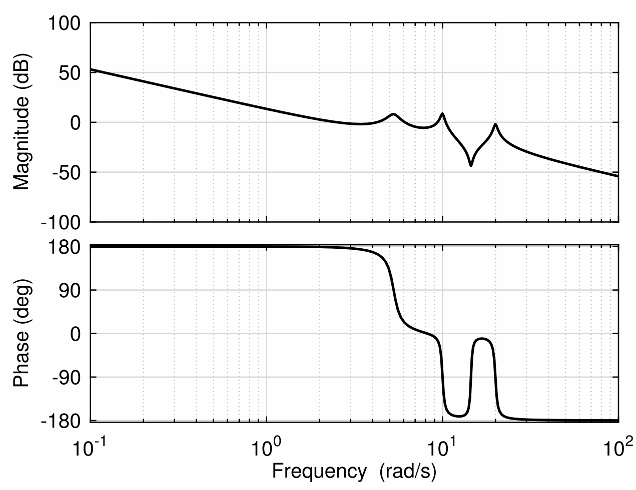

FIGURE 3. Bode diagram of the flexible controlled object.

B. ROBUST CONTROLLER DESIGN

According to the theoretical basis in the chapter 2, firstly, according to the control target requirements, determine the desired closed-loop pole, that is, the root of $d_f(s)d_c(s) = 0$.

Where, the root of $d_f(s) = 0$ represents the closed-loop pole formed by the pole of the controlled object through feedback. The first term of the controlled object (9) is double integral, which corresponds to the dominant pole of the closed-loop system. The remaining three flexible modes affect the stability of the system, so the damping ratio corresponding to the dominant pole can be set to $\zeta = 1$. According to the requirements of design indicators, the dominant pole can be selected as $(s + 1)^2$. The poles corresponding to the remaining three weakly damped modes in the transfer function are located near the imaginary axis. After the loop is closed, they should be avoided entering the right half plane. However, combined with the theory of control design, it can be known that if they are set to a farther left position, the control input will be too big again. In summary, choose $d_f(s)$ as follows:

$$d_f(s) = (s+1)^2(s^2+1.2s+25)(s^2+s+100) \times (s^2+s+400) \tag{9}$$

The root of $d_c(s) = 0$ in (4) is the closed-loop pole corresponding to the controller. Therefore, the order of $d_c(s)$ is determined by the controller. The flexible manipulator shown in (9) has a non-minimum phase zero point $s^2 - 7.932s + 36.54 = 0$. The non-minimum phase zero point make the phase lag of the system, which is beneficial for the Nyquist curve to enter the right half plane. The zero point $s^2 + 9.038s + 36.64 = 0$ in (9) is completely different. Therefore, an appropriate controller pole can be designed for pole-zero cancellation. So, $d_c(s)$ can be taken as:

$$d_c(s) = (s^2+9s+36)(s+25) \tag{10}$$

where, the pole, $s = -25$, is set to further lag the phase angle. From (10), it can be seen that the designed controller is a 3-order controller. In order to meet the low-frequency characteristics of the system, take the weighting function:

$$W(s) = \frac{0.01}{s^2+0.25s+0.01} \tag{11}$$

After determining the desired pole and weighting function, according to the optimization problem of (4) and (5), the obtained controller is:

$$K(s) = \frac{3.715(s+0.375)\left(s^2+0.63s+46.4\right)}{(s+5.65)\left(s^2+8.82s+36.2\right)} \tag{12}$$

It can be seen from (12) that the controller is stable, indicating that the design method can solve the unstable controller problem in the control design of weakly damped flexible systems. Finally, the closed-loop transfer function of the system is:

$$T(s) = \frac{17.6(s+0.375)(s^2+9.04s+36.6)(s^2-7.93s+36.5)(s^2-0.629s+46.4)(s^2-0.429s+223.1)}{(s+2.467)(s+1)(s+0.99)(s^2+8.7s+35.2)(s^2+1.11s+407.8)(s^2+0.9s+101.4)(s^2+1.2s+25.4)} \tag{13}$$

it is easy to know from (13) that the closed-loop pole is not completely consistent with the expected pole. This is due to the introduction of the weighting function $W(s)$, which also reflects the flexibility of the method.

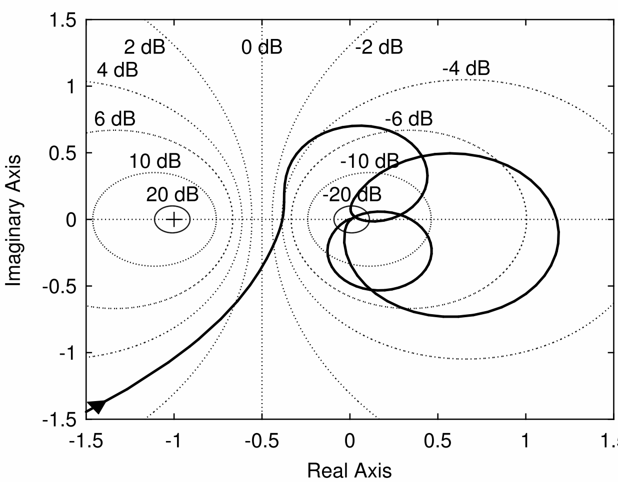

FIGURE 4. The open-loop characteristics of the system.

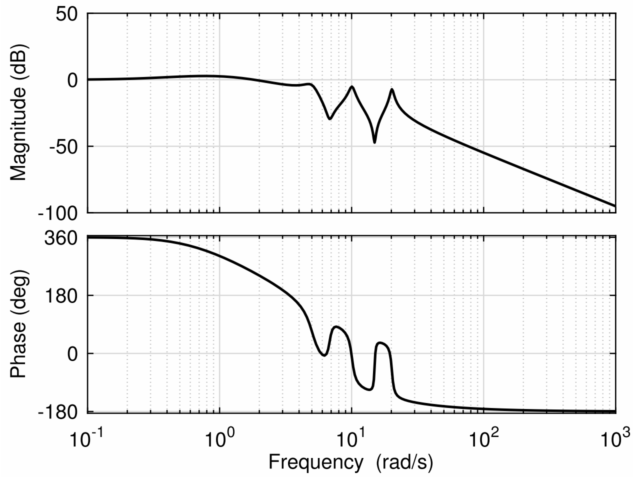

FIGURE 5. The Bode diagram of the closed-loop system.

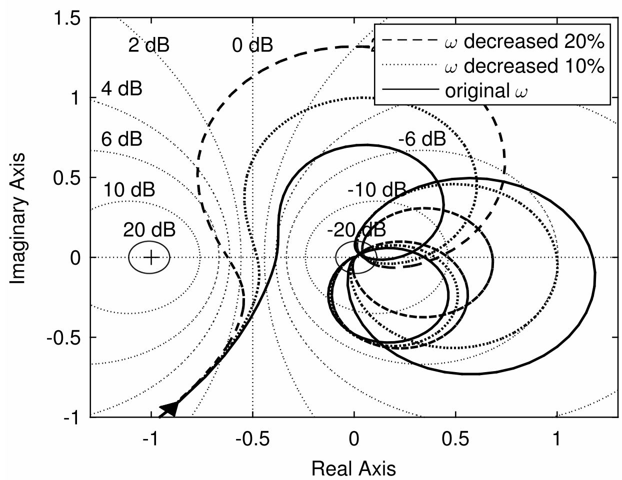

FIGURE 6. Nyquist diagram of the open loop of the system when the frequency decreases.

From the open-loop characteristics of the system shown in Fig.4, it can be seen that the three flexible modes of the controlled object correspond to the three circles whose vertices are on the right half of the Nyquist diagram, and these three circles all satisfy the preceding proposed local positive design requirements. From the Bode diagram of the closed-loop system shown in Fig.5, it can also be seen that the amplitudes corresponding to the three flexible modes do not exceed 0dB, which is also consistent with Fig.4. It shows that by configuring the closed-loop poles corresponding to the weakly damped mode in the flexible system, the open-loop characteristics of the system are far away from the critical stable point and the stability is improved.

C. SIMULATION ANALYSIS

The controller obtained in the previous section shows that the self-stable controller of (12) can stabilize the system. Next, analyze the robustness of the system.

Fig.6 is the Nyquist diagram of the open loop of the system when the frequency decreases. It can be seen from Fig.6 that as the frequency decreases, the position of the circle formed by the Nyquist curve does not change, only the diameter of the circle increases. But when the frequency is reduced by 20%, the Nyquist curve of the system is still far away from the critical stability point, and the closed-loop system can still be stable.

Fig.7 is the open-loop Nyquist diagram of the system when the frequency increases. It can be seen from the figure that as the frequency decreases, the position of the circle formed by the Nyquist curve does not change much, only the diameter of the circle decreases, as to further away from $(-1, j0)$ point, so that the closed-loop system is stable.

Fig.8 shows the adjustment process of the system in the time domain under different frequency changes. When the frequency is reduced by 20%, the system error is relatively

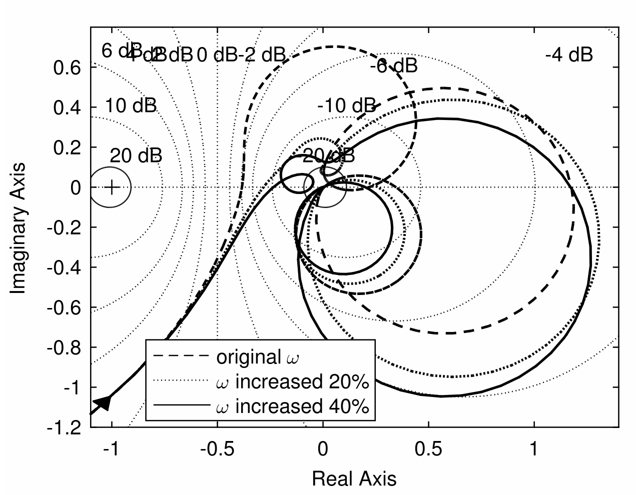

FIGURE 7. Nyquist diagram of the open loop of the system when the frequency increases.

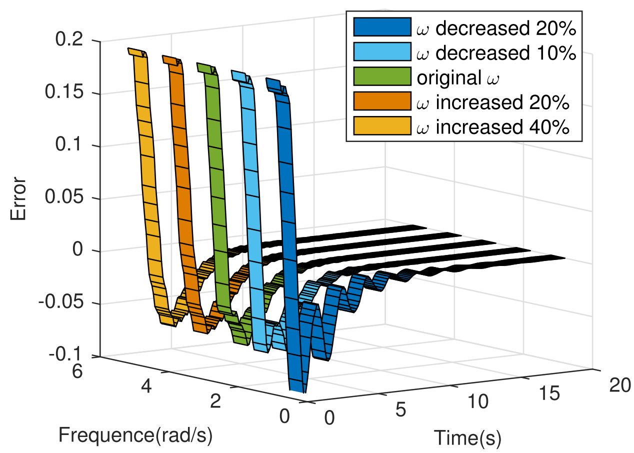

FIGURE 8. The adjustment process of the system in the time domain under different frequency changes.

large at the initial stage of system adjustment, but the final error can tend to zero. When the frequency increases, the system can eventually run stably. The only difference is that there are small vibrations of different frequencies at the initial stage of adjustment, which is related to the diameter change of the circle shown in Fig.7. In summary, for the parameter perturbation of frequency, the system has a parameter perturbation range of at least 20%, and has certain robustness.

Fig.9 is the Nyquist curve diagram with damping reduced by 30% and 60% respectively. From this figure, it can be concluded that the reduced damping makes the diameter of the circle formed by the Nyquist curve larger, and the distance from the critical stability point is reduced. But when the damping is reduced by 60%, the open-loop Nyquist curve of the system is still on the right side of the 0dB line, which has good robust stability.

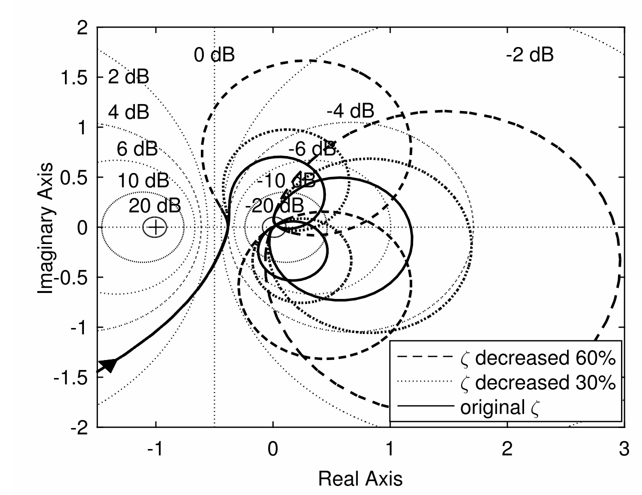

FIGURE 9. Nyquist diagram of the open loop of the system when the damping decreases.

Fig.10 is the Nyquist diagram of the system when the damping increases. It is easy to know that the increase in damping is just the opposite of the decrease in damping. The upward perturbation of damping means that its weak damping characteristic is weakened, and it becomes more stable.

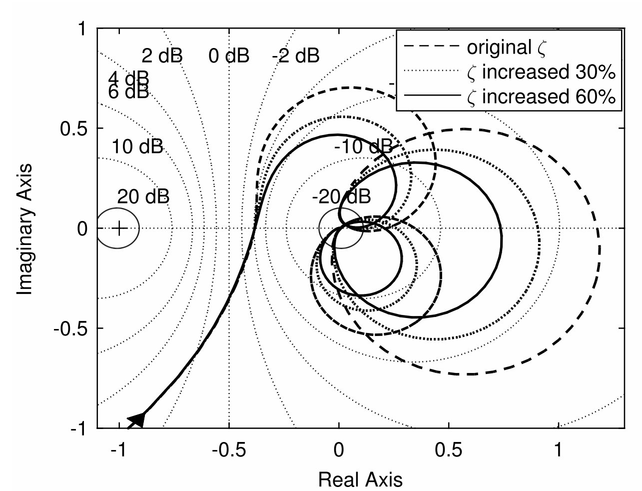

FIGURE 10. Nyquist diagram of the open loop of the system when the damping increases.

Fig.11 shows the time-domain adjustment process of the system under different damping perturbations. It can be seen that when the damping increases, the time-domain adjustment process of the system does not change much compared to the purple original mathematical model, and is even better than the original system, which is consistent to the Nyquist diagram in Fig.10. When the damping is reduced, the adjustment time of the system is increased, and there is a small amplitude vibration in the initial stage of adjustment, but when the damping is reduced to 60%, the system can still

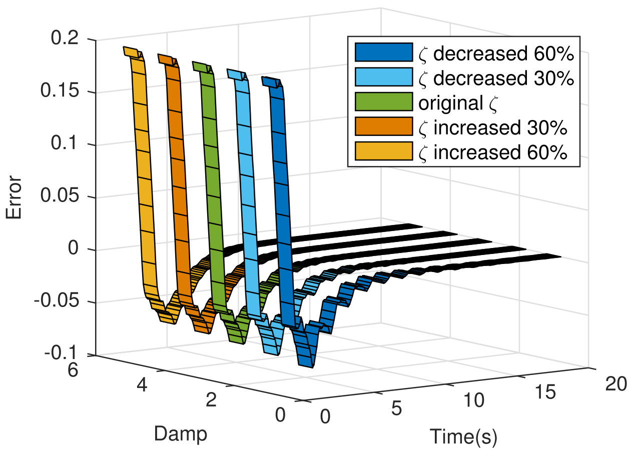

FIGURE 11. **Time-domain adjustment process of the system under different damping perturbations.**

operate stably recently. This is consistent with the Nyquist diagram shown in Fig.9.

In summary, for the parameter perturbation of damping, the system has at least 60% parameter perturbation range. The controller obtained by this method has good robustness to weakly damped flexible systems.

IV. CONCLUSION

The existing H-inf design is researched on the basis of gain control. In this paper, the phase condition is used in the H-inf design to ensure the stability and control performance of the closed-loop system. Aiming at the problem of how to design the positive real phase control with H-inf theory, an H-inf optimization method with pole placement and weighting function is given, which relaxes the strict positive real requirements, and uses the advantages of non-minimum phase zeros, making the design more flexible. A design example of a controlled plant with multiple weak damping modes shows that using this method to control the weakly damped system, the obtained controller is stable, and the robustness of the closed-loop system is significantly better than the H-inf loop forming method. Because this method is based on H-inf optimization method, and the controlled object in the design example has generality, this method can also be used in the control design of other weakly damped flexible systems.

REFERENCES

[1] Z. Wu, Y. Li, P. Wang, and M. Liu, "Dynamic head-disk interface modeling and adaptive control of a hybrid actuator for optical data storage systems," *Int. J. Optomechatronics*, vol. 9, no. 1, pp. 62–88, Jan. 2015.

[2] Y.-S. Lu, "Internal model control of lightly damped systems subject to periodic exogenous signals," *IEEE Trans. Control Syst. Technol.*, vol. 18, no. 3, pp. 699–704, May 2010.

[3] L. K. Zhu, G. F. Ma, and Q. L. Hu, "Active vibration control and attitude tracking of flexible spacecraft via robust backstepping sliding mode technique," *Control Decis.*, vol. 23, no. 5, pp. 530–534, and 540, 2008.

[4] Y. L. Wu, J. J. Li, H. B. Zeng, and G. R. Duan, "Robust H-infinity control design for spacecrafts with large flexible netted antennas," *Control Theory Appl.*, vol. 30, no. 3, pp. 365–371, 2013.

[5] S. Ding and W. X. Zheng, "Nonsmooth attitude stabilization of a flexible spacecraft," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 50, no. 2, pp. 1163–1181, Apr. 2014.

[6] T. Yang, N. Sun, H. Chen, and Y. Fang, "Observer-based nonlinear control for tower cranes suffering from uncertain friction and actuator constraints with experimental verification," *IEEE Trans. Ind. Electron.*, early access, May 12, 2020, doi: 10.1109/TIE.2020.2992972.

[7] Y. Wu, N. Sun, H. Chen, and Y. Fang, "Adaptive output feedback control for 5-DOF varying-cable-length tower cranes with cargo mass estimation," *IEEE Trans. Ind. Informat.*, early access, Jun. 30, 2020, doi: 10.1109/TII.2020.3006179.

[8] M. Sayahkarajy, Z. Mohamed, A. A. M. Faudzi, and E. Supriyanto, "Hybrid vibration and rest-to-rest control of a two-link flexible robotic arm using H^{∞} loop-shaping control design," *Forum Mathematicum*, vol. 18, no. 2, pp. 171–191, 2016.

[9] R. Sakthivel, A. Arunkumar, and K. Mathiyalagan, "Robust sampled-data H^{∞} control for mechanical systems," *Complexity*, vol. 20, no. 4, pp. 19–29, 2015.

[10] F. El Haoussi and E. H. Tissir, "An LMI-based approach for robust stabilization of time delay systems containing saturating actuators," *IMA J. Math. Control Inf.*, vol. 24, no. 3, pp. 347–356, Aug. 2006.

[11] D. Mcfarlane and K. Glover, "A loop shaping design procedure using H^{∞} synthesis," *IEEE Trans. Autom. Control*, vol. 37, no. 6, pp. 759–769, Jun. 2002.

[12] D. C. McFarlane and K. Glover, *Robust Controller Design Using Normalized Coprime Factor Plant Descriptions*, vol. 138. Cham, Switzerland: Springer, 1990.

[13] H. E. Zhen, F. W. Meng, W. Liu, and G. X. Wang, "Robustness of H^{∞} loop shaping design," *Acta Automatica Sinica*, vol. 36, no. 6, pp. 890–893, 2010.

[14] A. Lanzon and P. Tsiotras, "Tsiotras, p.: "A combined application of H_{∞} loop shaping and μ-synthesis to control high-speed flywheels," *IEEE Trans. Control Syst. Technol.*, vol. 13, no. 5, pp. 766–777, Sep. 2005.

[15] Z. He, X. M. Jiang, F. W. Meng, and G. X. Wang, "μ-synthesis in H^{∞} loop-shaping design," *Control Theory Appl.*, vol. 29, no. 3, pp. 3452–3470, 2012.

[16] L. Zhou, J. She, and S. Zhou, "Robust H_{∞} control of an observer-based repetitive-control system," *J. Franklin Inst.*, vol. 355, no. 12, pp. 4952–4969, 2018.

[17] L. B. Wu, X. Q. He, D. Q. Zhang, and H. W. Jia, "Adaptive H_{∞} fault-tolerant control for a class of uncertain switched nonlinear systems with multiple state time delays," *Int. J. Syst. Sci.*, vol. 49, no. 8, pp. 1784–1794, 2018.

[18] X. Li and W. Mao, "Finite-time H_{∞} output tracking control for a class of switched neutral systems with mode-dependent average dwell time method," *Int. J. Innov. Comput., Inf. Control*, vol. 13, no. 3, pp. 767–782, 2017.

[19] C. Li and J. Zhao, "Robust passivity-based H_{∞} control for uncertain switched nonlinear systems," *Int. J. Robust Nonlinear Control*, vol. 26, no. 14, pp. 3186–3206, Sep. 2016.

[20] H. N. K. Balini, C. W. Scherer, and J. Witte, "Performance enhancement for AMB systems using unstable H_{∞} controllers," *IEEE Trans. Control Syst. Technol.*, vol. 19, no. 6, pp. 1479–1492, Nov. 2011.

[21] F. W. Meng, Z. He, G. X. Wang, and D. Zhou, "Control design of flexible systems and h-infinity loop-shaping method," *Control Theory Appl.*, vol. 30, no. 8, pp. 1014–1020, 2013.

[22] D. C. McFarlane and K. Glover, *Robust Controller Design Using Normalized Coprime Factor Plant Descriptions*, vol. 138. Cham, Switzerland: Springer, 1989.

[23] J. Yang, M. Yu, Y. Niu, W. Zhang, and F. Meng, "H-inf phase control for flexible systems," in *Proc. 11th Int. Conf. Model., Identificat. Control (ICMIC)*. Cham, Switzerland: Springer, 2020, pp. 281–296.

[24] G. Cherubini, C. C. Chung, W. C. Messner, and S. O. R. Moheimani, "Control methods in data-storage systems," *IEEE Trans. Control Syst. Technol.*, vol. 20, no. 2, pp. 296–322, Mar. 2012.

[25] P. Coustal and J. Michelin, "Industrial application of an h-infinity design method for flexible structures H_{∞}," *IEEE Control Syst.*, vol. 14, no. 4, pp. 49–54, Aug. 1994.

[26] W. Guangxiong and H. Zhen, *Applied H-Inf Control*. Harbin, China: Harbin Institute of Technology Press, 2010.

[27] G. Franklin, *Feedback Control of Dynamic Systems*. Pearson, NJ, USA: Pearson Education, 2007.

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