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# **Achievable Physical-Layer Security Over Composite Fading Channels**

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**ABSTRACT** We investigate the physical layer security limits of Wyner's wiretap model over Fisher-Snedecor  $\mathcal{F}$  composite fading channels.  $\mathcal{F}$  fading conditions have been recently shown to provide an accurate characterization of multipath fading and shadowing effects in emerging wireless transmission scenarios such as body centric, cellular and vehicular communications. To this end, we utilize a redefined analytic expression for the Fisher-Snedecor  $\mathcal{F}$  distribution in order to ensure unconstrained validity and reliability when used in the analysis of various performance metrics of interest. In this context, we assume that the main channel (i.e., between the source and the legitimate destination) and the eavesdropper's channel (i.e., between the source and the illegitimate destination) undergo independent quasi-static Fisher-Snedecor F composite fading. Novel exact analytic expressions are then derived for the corresponding average secrecy capacity (ASC), secure outage probability (SOP) and probability of strictly positive secrecy capacity (SPSC) along with their insightful asymptotic representations. In addition, analytical expressions for the ASC, SOP and SPSC over mixed fading channels such as Nakagami-m/Nakagami-m, Nakagami-m/Fisher-Snedecor  $\mathcal{F}$ and Fisher-Snedecor  $\mathcal{F}/N$ akagami-m are derived. The new formulations are validated through comparisons with Monte-Carlo simulations and analyzed to gain useful insights into the impact of the fading parameters on the achievable accuracy and the overall system performance.

**INDEX TERMS** Average secrecy capacity, Fisher-Snedecor  $\mathcal{F}$  distribution, physical layer security, secure outage probability.

# I. INTRODUCTION

Due to the inherent broadcast nature of wireless communication systems, the message-bearing signal may be eavesdropped and decoded by illegitimate users. As a result, the information security in wireless communication systems has recently received considerable attention even at the physical layer of the Open System Interconnection (OSI) model. In his seminal contribution in [2], Shannon introduced the concept of information-theoretic secrecy by investigating the

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transmission of a secret message between a source, a legitimate receiver and an eavesdropper, where the source and the legitimate receiver share a secret key. To achieve perfect secrecy, it was shown that the key rate should be at least as large as the message rate. Subsequently, in [3], Wyner introduced the notion of a wiretap channel that enables a source to send a secret message to a legitimate receiver in the presence of an eavesdropper without using a shared key, such that almost no information about the secret message is leaked to the eavesdropper.

Based on the above considerations, earlier studies have analyzed the performance of physical-layer security (PLS)



for several wireless applications, such as point-to-point, dual-hop and multi-hop communications, and cognitive radio networks (see e.g., [4]–[26] and the references therein). From these studies, a number of important performance measures such as the average secrecy capacity (ASC), secure outage probability (SOP), probability of strictly positive secrecy capacity (SPSC) and probability of nonzero secrecy capacity (PNSC) have been derived. In this context, a general and compact analysis framework for the wireless information-theoretic security of arbitrarily-distributed fading channels was presented in [4], whereas in [5], the authors investigated PLS in a random wireless network where both the legitimate and eavesdropping nodes are randomly deployed. The secrecy capacity of a wiretap broadcast channel with an external eavesdropper, where the source sends two private messages over a broadcast channel while keeping them secret from the eavesdropper, was studied in [6]. In addition, the authors in [7] evaluated the ASC performance of the classic Wyner's model over  $\alpha$ - $\mu$  fading channels, while the authors in [8] derived closed-form expressions for the ASC, SOP and the SPSC assuming a classic Wyner's wiretap model over generalized-K fading channels.

In the same context, the performance of SOP and SPSC over generalized gamma fading channels was analyzed in [9]. Novel analytical expressions for the SPSC and a lower bound on the SOP were derived over independent and non-identically distributed (i.n.i.d)  $\kappa$ - $\mu$  fading channels in [10]. The derived expressions were then used to study the performance of different emerging wireless applications, such as cellular device-to-device, peer-to-peer, vehicle-tovehicle and body centric communications. In [11], the same authors derived analytical expressions for the SPSC and a lower bound on the SOP over  $\alpha - \mu/\kappa - \mu$  and  $\kappa - \mu/\alpha - \mu$  fading channels, whereas the SPSC performance for wireless communication systems under Rician fading was analyzed in [12]. Furthermore, the effect of in-phase and quadrature imbalance on the ASC was studied in [13], whereas secure communications in terms of the ASC, SOP and PNSC over independent and correlated lognormal shadowing channels as well as composite fading channels were investigated in [14]. Finally, the wireless PLS performance in the presence of multiple eavesdroppers was evaluated in [15]–[17].

Recently, numerous studies have analyzed PLS performance in the context of dual- and multi-hop relaying systems [18]–[20], [27]–[30]. More specifically, the ergodic secrecy capacity performance in a dual-hop multiple-antenna amplify-and-forward (AF) relaying system was analyzed in [18]. To further enhance the ergodic secrecy capacity for a dual-hop decode-and-forward (DF) relaying system in the presence of an eavesdropper, a random phase shifting scheme was proposed in [19]. Similarly, an upper bound, lower bound and an approximate expression for the SOP of a dual-hop AF relaying system with relay selection and without the knowledge of the eavesdropper's instantaneous channel state information (CSI) were derived in [20]. Likewise, the impact of correlated Rayleigh fading on the security of multiple

AF relaying networks was quantified in [31], whereas the performance of SOP in a dual-hop DF relaying system with diversity combining at the eavesdropper was considered in [32]. In [27], the authors jointly considered full-duplex and physical layer security by formulating a cross-layer optimization problem to maximize the secrecy rate in a multi-hop wireless network. The performance of PLS in DF full-duplex multi-hop relaying systems was investigated in [28], whereas the performance of PLS of a multi-hop DF relay network was analyzed in [29] in the presence of multiple passive eavesdroppers over Nakagami-*m* fading channels. Finally, the PNSC performance of a multi-hop relay network over Nakagami-*m* fading channels with hardware impairments was quantified in [30].

Multipath fading and shadowing phenomena are encountered in most practical wireless communication scenarios, such as in congested downtown areas with slow mobility or stationary users [33]–[35]. In addition, this type of composite fading is encountered in land-mobile satellite communication systems, subject to urban and/or vegetative shadowing [36]–[39]. However, even though the existing composite fading models can provide a fair characterization of the underlying fading phenomena, quite often their mathematical representation turns out to be inconvenient, leading to cumbersome or intractable analytic results for most critical performance metrics of interest. Unfortunately, this common analytical limitation impacts the formulation of important statistical metrics such as the probability density function, the cumulative distribution function and the moment generating function, which are required to evaluate important performance metrics, such as spectral efficiency, probability of error, channel capacity and outage probability. With this motivation, a composite fading model which is both accurate and tractable was recently proposed for characterizing the combined effects of multipath fading and shadowing. The so-called Fisher-Snedecor  $\mathcal F$  fading model was introduced in [40] under the assumption that the scattered multipath follows a Nakagami-m distribution, while the root-meansquare signal is shaped by an inverse Nakagami-*m* random variable. The validity of this model was verified through extensive comparisons with field measurements obtained for a number of emerging wireless communications scenarios. Owing to its generality, several well-known fading models such as the Nakagami-m, Rayleigh and one-sided Gaussian can be obtained as special cases. In addition, the Fisher-Snedecor  $\mathcal{F}$  distribution has been shown, both theoretically and experimentally, to outperform other frequently used composite fading distributions, such as the generalized-K distribution both in terms of modeling accuracy and computational complexity. For example, in [40], the authors demonstrated an important application of the Fisher-Snedecor  $\mathcal{F}$  distribution for characterizing the composite fading observed in deviceto-device (D2D) communications channels. It is worth mentioning here that D2D communication is expected to play an indispensable role in beyond fifth-generation (5G) wireless networks, see [41] and the references therein. In addition,

Notation/Symbol	Definition/Explanation
$y_D, y_E$	Received signal at legitimate destination and eavesdropper's, respectively
$h_D, h_E$	Main channel and Eavesdropper's channel coefficients, respectively
$n_D, n_E$	AWGN at legitimate destination and eavesdropper's, respectively
$\sigma_D^2, \sigma_E^2$	Noise variances at legitimate destination and eavesdropper's, respectively
$P_t$	Transmitted power
$\overline{\gamma}_D, \overline{\gamma}_E$	Average SNR of the main and eavesdropper's links, respectively
$m_D, m_E$	Fading parameter of the main and eavesdropper's links, respectively
$m_{s_D}, m_{s_E}$	Shadowing parameter of the main and eavesdropper's links, respectively

TABLE 1. Notations and symbols used throughout the paper.

it has been extensively shown that composite fading conditions are encountered in applications relating to both conventional and emerging wireless communication technologies, such as body area networks and vehicle-to-vehicle communications [42], [43].

Motivated by the above, in the present contribution we analyze a wireless communication system in the context of wireless PLS and Fisher-Snedecor  $\mathcal{F}$  composite fading. Specifically, we consider a wireless system in which the main channel and the eavesdropper's channel experience independent quasi-static Fisher-Snedecor  $\mathcal{F}$  fading effects. The main contributions of this work can be summarized as follows<sup>1</sup>:

- We utilize a redefined analytic expression for the probability density function (PDF) of the Fisher-Snedecor F fading distribution, which is a slight modification of the underlying inverse Nakagami-m PDF from that is used in [40]. While the PDF given in [40] corresponds to a valid model for physical channel characterization, the formulation of the underlying inverse Nakagami-m model renders the determination of the parameter range of numerous performance measures of interest problematic. On the contrary, the redefined PDF of the F model that is utilized in the present analysis is more generic and robust when employed in the analysis of digital communication systems.
- With the aid of the redefined PDF for the Fisher-Snedecor F fading model, we derive novel closed-form analytical expressions for the ASC, SOP and SPSC of the classic Wyner's model over Fisher-Snedecor F composite fading channels.
- Capitalizing on the derivation of the exact analytic expressions, we derive new asymptotic expressions for performance metrics of interest such as the ASC, SOP and SPSC when the destination node is located close

 $^{1}$ It is noted that during the submission of this manuscript, it became apparent that [44] addressed in parallel some of the topics under study in the present work. However, the analytic and numerical results in the present contribution are different and more valid compared to those in [44] because they are based exclusively on a redefined analytic expression for the probability density function of the Fisher-Snedecor  $\mathcal{F}$  fading distribution. Furthermore, this manuscript reports results that were not addressed in the independently carried out contribution.

- to the source node (i.e., in the high signal-to-noise ratio (SNR) regime).
- Useful and simple closed-form analytical expressions for the ASC, SOP and SPSC are also derived for some specific cases of interest, such as Fisher-Snedecor F/Nakagami-m, Nakagami-m/Fisher-Snedecor F and Nakagami-m/Nakagami-m distributions, which also include as a special case the standard Rayleigh fading conditions when the Nakagami fading parameter is set to unity (i.e., m = 1).
- Considering the derived results for the cases of interest introduced above, new asymptotic expressions for the ASC, SOP and SPSC are derived. These expressions provide useful insights into the effect of the involved system parameters on the overall security performance.

The rest of the paper is organized as follows: In Section II we present the considered system and channel models, whereas in Section III we derive the corresponding ASC, SOP and SPSC representations. The asymptotic analysis for the aforementioned metrics is provided in Section IV, whereas respective special cases are obtained from our general analysis in Section V. In Section VI, we investigate the impact of different system parameters on the overall security performance. Finally, useful concluding remarks are given in Section VII.

# **II. SYSTEM AND CHANNEL MODELS**

# A. SYSTEM MODEL

We consider a wiretap channel model [45], which consists of a source (S), a legitimate destination (D) and an eavesdropper (E). In this system, S sends a confidential message to D over the main channel while E attempts to decode this message from its received signal, as depicted in Fig. 1. Also, the main channel  $(S \rightarrow D)$  and the eavesdropper's channel  $(S \rightarrow E)$  experience independent quasi-static Fisher-Snedecor F fading conditions [40]. The fading coefficients for both links remain constant during the transmission time of a block but vary independently from one block to another. The CSI is assumed to be known at both the transmitter and the destination while we assume that the noise over all channels is zero-mean and unit-variance complex additive white



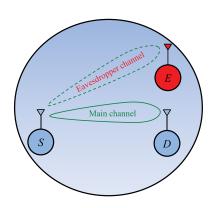
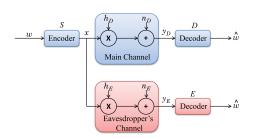


FIGURE 1. Illustration of a wireless communication system with potential eavesdropping.



**FIGURE 2.** System model of the wiretap channel in the presence of Fisher-Snedecor  $\mathcal{F}$  fading.

Gaussian noise (AWGN). Based on the system model of the wiretap channel shown in Fig. 2, the received signals at D and E can be expressed, respectively, as

$$y_D = \sqrt{P_t} h_D x + n_D, \tag{1}$$

and

$$y_E = \sqrt{P_t} h_E x + n_E, \qquad (2)$$

where x is the normalized transmitted signal (i.e.,  $\mathbb{E}\{|x|^2\} = 1$ , with  $\mathbb{E}[\cdot]$  denoting statistical expectation). Based on the above, the instantaneous output SNRs at D and E are, respectively, given by

$$\gamma_D = \overline{\gamma}_D |h_D|^2 \tag{3}$$

and

$$\gamma_E = \overline{\gamma}_E |h_E|^2, \tag{4}$$

where  $\overline{\gamma}_D = P_t/\sigma_D^2$  and  $\overline{\gamma}_E = P_t/\sigma_E^2$ .

In this work, we use the redefined version of the  $\mathcal{F}$  distribution presented in [46]. As such, the PDF of the instantaneous SNR,  $\gamma_k$  (where  $k \in \{D, E\}$ ), of the Fisher-Snedecor  $\mathcal{F}$  fading distribution can be expressed as follows

$$f_k(\gamma_k) = \frac{m_k^{m_k} (m_{s_k} - 1)^{m_{s_k}} \overline{\gamma}_k^{m_{s_k}} \gamma^{m_k - 1}}{B(m_k, m_{s_k})(m_k \gamma + (m_{s_k} - 1) \overline{\gamma}_k)^{m_k + m_{s_k}}}, \quad (5)$$

where  $m_k$  and  $m_{s_k} > 1$  denote the fading severity and shadowing parameters, respectively,  $\overline{\gamma}_k = \mathbb{E}[\gamma_k]$  is the corresponding mean power and  $B(\cdot, \cdot)$  denotes the beta function [47, Eq. (8.384.1)].

The representation in (5) is based on a slight modification of the underlying inverse Nakagami-m PDF from that used in [40]. While the PDF given in [40] corresponds to a valid model for physical channel characterization, the form of the underlying inverse Nakagami-m model renders the determination of the parameter range of many performance measures of interest problematic. On the contrary, the redefined PDF of the  $\mathcal{F}$  model in (5) is more generic and robust when employed in the analysis of digital communication systems.

Based on the above and with the aid of [48, Eq. (8.4.2.5)] and [48, Eq. (8.2.2.15)], it follows that

$$f_k(\gamma_k) = \frac{1}{\Gamma(m_k)\Gamma(m_{s_k})} \gamma_k^{-1} G_{1,1}^{1,1} \left( a_k \gamma_k \, \middle| \, \frac{1 - m_{s_k}}{m_k} \right), \quad (6)$$

where  $a_k = \frac{m_k}{(m_{s_k}-1)\overline{\gamma}_k}$  and  $G^{s,t}_{u,v}(\cdot)$  is the Meijer's G-function defined in Appendix A Eq. (37). It is also worth noting that the Fisher-Snedecor  $\mathcal{F}$  distribution includes as special cases the Nakagami-m distribution  $(m_s \to \infty)$ , Rayleigh distribution  $(m_s \to \infty)$  and m = 1) and one-sided Gaussian distribution  $(m_s \to \infty)$  and  $m = \frac{1}{2}$ ). In addition, the corresponding cumulative distribution function (CDF) of  $\gamma_k$  can be expressed as follows:

$$F_k(\gamma_k) = \frac{1}{\Gamma(m_k)\Gamma(m_{s_k})} G_{2,2}^{1,2} \left( a_k \gamma_k \middle| \begin{array}{c} 1 - m_{s_k}, 1 \\ m_k, 0 \end{array} \right). \quad (7)$$

The above redefined measures are essential for performing a valid evaluation of the PHY layer security over  $\mathcal{F}$  composite fading channels.

# III. PHY LAYER SECURITY OVER ${\mathcal F}$ FADING CHANNELS

Based on the new analytic expressions for the PDF and CDF of the Fisher-Snedecor  $\mathcal{F}$  fading distribution given in (6) and (7), we analyze the average secrecy capacity, the secrecy outage probability, and the strictly-positive secrecy capacity.

# A. AVERAGE SECRECY CAPACITY (ASC)

We consider an active eavesdropping scenario [49], where the CSI of the eavesdropper's channel is known at S. Hence, S can adapt the achievable secrecy rate R such that  $R \le R_s$ . In this context, the maximum achievable secrecy rate  $C_s = R_s$  can be written as [49]

$$C_s = [C_D - C_E]^+, \tag{8}$$

where  $C_D = \ln(1 + \gamma_D)$  and  $C_E = \ln(1 + \gamma_E)$  denote the capacities in the main and eavesdropper channels with AWGN, respectively, whereas  $[z]^+ = \max\{z, 0\}$ .

Theorem 1 (Average Secrecy Capacity): The ASC for the Fisher-Snedecor  $\mathcal{F}$  composite fading channels is given by (9), at the bottom of the next page, where  $G_{p_1,q_1:p_2,q_2:p_3,q_3}^{m_1,n_1:m_2,n_2:m_3,n_3}(\cdot)$  is the bivariate Meijer's G-function (BVMGF) defined in Appendix A Eq. (35).

*Proof:* The proof is provided in Appendix B.1.

Notably, even though the ASC in (9) is expressed in terms of the BVMGF, which is not a built-in function in standard scientific software packages, it can be efficiently computed with the aid of the methods outlined in [50]–[52].



# B. SECURE OUTAGE PROBABILITY (SOP)

In this subsection, we consider a passive eavesdropper where the source S and destination D have no CSI knowledge about the eavesdropper. In this scenario, the SOP can be used as a key performance metric to characterize the wireless fading channel [49] since it denotes the probability that the instantaneous secrecy capacity falls below a target secrecy rate  $R_s \geq 0$ .

Theorem 2 (Secure Outage Probability): The SOP for Fisher-Snedecor  $\mathcal{F}$  composite fading channels is given by

$$C_s^{SOP} = \sum_{n=0}^{\infty} \frac{(-1)^n a_D^n (\Psi - 1)^n}{n! \Gamma(m_D) \Gamma(m_{S_D}) \Gamma(m_E) \Gamma(m_{S_E})} \times G_{4,4}^{3,3} \left( \frac{a_E}{a_D \Psi} \middle| 1, 1 - m_{S_E}, 1 + n - m_D, 1 + n \right),$$
(10)

where  $\Psi = \exp(R_s) > 1$ .

Although (10) is expressed in terms of an infinite series, it converges rapidly and requires few terms (as little as three in certain cases) to achieve an acceptable target accuracy.

*Proof:* The proof is provided in Appendix B.2.

# C. STRICTLY POSITIVE SECRECY CAPACITY (SPSC)

Likewise, the SPSC is another important metric in secure communications as it accounts for the probability that the secrecy rate for the  $S \rightarrow D$  link is positive, i.e.,  $C_s > 0$ .

Corollary 1 (Strictly Positive Secrecy Capacity): The SPSC for Fisher-Snedecor  $\mathcal{F}$  composite fading channels is given by

$$C_s^{SPSC} = 1 - \frac{G_{3,3}^{3,2} \left(\frac{a_E}{a_D} \middle| \frac{1 - m_{s_E}, 1 - m_D, 1}{m_E, m_{s_D}, 0}\right)}{\Gamma(m_D)\Gamma(m_{s_D})\Gamma(m_E)\Gamma(m_{s_E})}. \tag{11}$$

function, which is a well known special built-in function in most scientific software packages such as Matlab and Mathematica. *Proof:* The proof is provided in Appendix B.3.

The validity of the new results were verified through comparisons with respective results from computer simulations. To the best of the authors' knowledge, these results have not been reported in the open technical literature.

# **IV. ASYMPTOTIC ANALYSIS**

Capitalizing on the exact analytic results derived in Sec. III, we quantify the asymptotic performance when  $\overline{\gamma}_D \to \infty$ , which can be also viewed as the scenario where the destination D is located close to the source S. As such, the asymptotic behavior can be derived based on the behavior of the PDF of  $\gamma_k$  around the origin. Thus, the PDF, given in (5) or (6), and the CDF, given in (7), at high SNR regime can be rewritten as

$$f_k(\gamma_k) = \frac{a_k^{m_k}}{B(m_k, m_{s_k})} \gamma_k^{m_k - 1} + \mathcal{O}\left((a_k \gamma_k)^{m_k}\right), \qquad (12)$$

and

$$F_k(\gamma_k) = \frac{a_k^{m_k} \Gamma(m_k + m_{s_k})}{\Gamma(m_k + 1)\Gamma(m_{s_k})} \gamma_k^{m_k} + \mathcal{O}\left((a_k \gamma_k)^{m_k}\right), \quad (13)$$

respectively, where  $\mathcal{O}(\cdot)$  stands for higher-order terms.

# A. ASYMPTOTIC ANALYSIS OF AVERAGE SECRECY **CAPACITY**

Proposition 1 (Asymptotic Average Secrecy Capacity): The asymptotic ASC for Fisher-Snedecor  $\mathcal{F}$  composite fading channels is given by (14), at the bottom of the next page.

*Proof:* The proof is provided in Appendix B.4.

# B. ASYMPTOTIC ANALYSIS OF SECURE OUTAGE **PROBABILITY**

Likewise, the asymptotic analysis for the corresponding SOP is provided in the following Proposition.

Proposition 2 (Asymptotic Secure Outage Probability): The asymptotic SOP is given by

$$C_s^{SOP,asym} = \frac{(a_D \Psi)^{m_D} \Gamma(m_D + m_{s_D})}{B(m_E, m_{s_E}) \Gamma(m_D + 1) \Gamma(m_{s_D})} \sum_{n=0}^{m_D} {m_D \choose n} \times \left(\frac{\Psi - 1}{\Psi}\right)^{m_D - n} a_E^{-n} B(m_E + n, m_{s_E} - n).$$
(15)

The above representation provides useful insights into the impact of the involved parameters on the overall performance. For example, it demonstrates that the diversity gain is proportional to the fading parameter of the main link,  $m_D$ .

*Proof:* The proof is provided in Appendix B.5.

isher-Snedecor 
$$\mathcal{F}/\text{Fisher-Snedecor }\mathcal{F}$$
:
$$\overline{C}_{s}^{ASC} = \frac{G_{2,2:1,1:2,2}^{2,1:1,1:1,2} \left(a_{D}, a_{E} \begin{vmatrix} 0,1 & 1-m_{s_{D}} & 1-m_{s_{E}}, 1\\ 0,0 & m_{D} & m_{E}, 0 \end{pmatrix}}{\Gamma(m_{D})\Gamma(m_{s_{D}})\Gamma(m_{E})\Gamma(m_{s_{E}})} + \frac{G_{2,2:1,1:2,2}^{2,1:1,1:1,2} \left(a_{E}, a_{D} \begin{vmatrix} 0,1 & 1-m_{s_{E}} & 1-m_{s_{D}}, 1\\ 0,0 & m_{E} & m_{D}, 0 \end{pmatrix}}{\Gamma(m_{D})\Gamma(m_{s_{D}})\Gamma(m_{E})\Gamma(m_{s_{E}})} - \frac{G_{3,3}^{3,2} \left(a_{E} \begin{vmatrix} 1-m_{s_{E}}, 0, 1\\ m_{E}, 0, 0 \end{pmatrix}}{\Gamma(m_{E})\Gamma(m_{s_{E}})}$$

$$(9)$$



# C. ASYMPTOTIC ANALYSIS OF STRICTLY POSITIVE SECRECY CAPACITY

Finally, a simple asymptotic expression is also derived for the corresponding SPSC.

Proposition 3 (Asymptotic Strictly Positive Secrecy Capacity): The asymptotic SPSC is given by

$$C_s^{SPSC,asym} = \frac{B(m_E + m_D, m_{s_E} - m_D)\Gamma(m_D + m_{s_D})a_D^{m_D}}{B(m_E, m_{s_E})\Gamma(m_D + 1)\Gamma(m_{s_D})a_E^{m_D}}$$

$$= \frac{\Gamma(m_E + m_D)\Gamma(m_{s_E} - m_D)\Gamma(m_D + m_{s_D})}{\Gamma(m_D + 1)\Gamma(m_D)\Gamma(m_E)\Gamma(m_{s_E})a_E^{m_D}a_D^{-m_D}}.$$
(16)

Equation (16) is also insightful since it shows that the diversity gain is proportional to the fading parameter of the main link,  $m_D$ .

*Proof:* The proof is provided in Appendix B.6.

# **V. SPECIAL CASES**

The  $\mathcal{F}$  composite fading model is extremely versatile and includes as special cases other well-known fading models. Therefore, following the derivation of the exact and asymptotic analytical framework in the previous section, we also derive analytic expressions for a number of special cases of interest which are encountered in practical wireless communication scenarios.

### A. FISHER-SNEDECOR F/Nakagami-m

First, we provide exact and asymptotic expressions for the ASC, SOP and SPSC when the legitimate link experiences Fisher-Snedecor  $\mathcal{F}$  fading, while the eavesdropper's link experiences Nakagami-*m* fading (i.e.,  $m_{s_E} \to \infty$ ).

Corollary 2 (Average Secrecy Capacity): The ASC when the legitimate link experiences Fisher-Snedecor  $\mathcal{F}$  fading, while the eavesdropper's link experiences Nakagami-m fading is given by (17), as shown at the bottom of the page.

*Proof:* The proof is provided in Appendix B.7.

Corollary 3 (Secure Outage Probability): The SOP when the legitimate link experiences Fisher-Snedecor  $\mathcal{F}$  fading, while the eavesdropper's link experiences Nakagami-m fading is given by

$$C_s^{SOP} = \frac{1}{\Gamma(m_D)\Gamma(m_{s_D})\Gamma(m_E)} \sum_{n=0}^{\infty} \frac{(-1)^n a_D^n (\Psi - 1)^n}{n!} \times G_{3,4}^{3,2} \left( \frac{m_E}{a_D \Psi \overline{\gamma}_E} \middle| 1, 1 + n - m_D, 1 + n \atop m_E, n + m_{s_D}, n, 1 + n \right).$$
(18)

Proof: The proof follows with the aid of equa-

tions (10), [48, Eq. (8.2.2.14)], [48, Eq. (8.2.2.12)] and [47, Eq. (8.328.2)].

Corollary 4 (Strictly Positive Secrecy Capacity): The SPSC when the legitimate link experiences Fisher-Snedecor  $\mathcal{F}$ fading, while the eavesdropper's link experiences Nakagamim fading is expressed as

$$C_s^{SPSC} = 1 - \frac{G_{2,3}^{3,1} \left(\frac{m_E}{a_D \overline{\gamma}_E} \middle| 1 - m_D, 1 \atop m_E, m_{s_D}, 0\right)}{\Gamma(m_D) \Gamma(m_{s_D}) \Gamma(m_E)}.$$
 (19)

*Proof:* The proof follows with the aid of equations (11), [48, Eq. (8.2.2.14)] and [48, Eq. (8.2.2.12)].

As in the previous section, we capitalize on the exact analytic results derived for the aforementioned special case to derive asymptotic representations that provide useful insights into the impact of the involved parameters on the overall system performance.

Proposition 4 (Asymptotic Average Secrecy Capacity): The asymptotic ASC when the legitimate link experiences Fisher-Snedecor  $\mathcal{F}$  fading, while the eavesdropper's link experiences Nakagami-m fading is given by (20), shown at the bottom of the next page.

*Proof*: The first and third terms in (14), when  $m_{s_E} \rightarrow$  $\infty$ , reduce to the Nakagami-*m* case using [48, Eq. (8.2.2.12)] and [48, Eq. (8.2.2.14)]. While, the second term reduces to

$$\overline{C}_{s}^{ASC,asym} = \underbrace{\frac{a_{D}^{m_{D}}G_{4,4}^{3,3}\left(a_{E} \left| \begin{array}{c} 1 - m_{SE}, 1, -m_{D}, 1 - m_{D} \\ m_{E}, -m_{D}, -m_{D}, 0 \end{array}\right)}_{B(m_{D}, m_{S_{D}})\Gamma(m_{E})\Gamma(m_{S_{E}})} + \underbrace{\frac{G_{2,2:1,1:2,2}^{2,1:1,1:1,2}\left(a_{E}, a_{D} \left| \begin{array}{c} 0, 1 \\ 0, 0 \end{array} \right| \begin{array}{c} 1 - m_{SE} \left| 1 - m_{S_{D}}, 1 \\ m_{D}, 0 \end{array}\right)}_{2nd \text{ term}} - \underbrace{\frac{G_{3,2}^{3,2}\left(a_{E} \left| \begin{array}{c} 1 - m_{SE}, 0, 1 \\ m_{E}, 0, 0 \end{array}\right)}_{\Gamma(m_{E})\Gamma(m_{S_{E}})}}_{3rd \text{ term}}. \tag{14}$$

$$\overline{C}_{s}^{ASC} = \frac{G_{2,2:1,1:1,2}^{2,1:1,1:1,1} \left( a_{D}, \frac{m_{E}}{\overline{\gamma}_{E}} \middle| 0, 1 \middle| 1 - m_{s_{D}} \middle| 1 \atop m_{D}, 0 \middle| m_{E}, 0 \right)}{\Gamma(m_{D})\Gamma(m_{s_{D}})\Gamma(m_{E})} + \frac{G_{2,2:0,1:2,2}^{2,1:1,0:1,2} \left( \frac{m_{E}}{\overline{\gamma}_{E}}, a_{D} \middle| 0, 1 \middle| - \middle| 1 - m_{s_{D}}, 1 \atop m_{D}, 0 \middle| m_{D}$$



the Nakagami-*m* case when the BVMGF is represented by its integral form using (35) in Appendix A, followed by applying [47, Eq. (8.328.2)], and then using (35) in Appendix A, (20) is obtained, which completes the proof.

Proposition 5 (Asymptotic Secure Outage Probability): The asymptotic SOP when the legitimate link experiences Fisher-Snedecor  $\mathcal{F}$  fading, while the eavesdropper's link experiences Nakagami-m fading is given by

$$C_s^{SOP,asym} = \frac{(a_D \Psi)^{m_D} \Gamma(m_D + m_{s_D})}{\Gamma(m_D + 1)\Gamma(m_{s_D})\Gamma(m_E)} \sum_{n=0}^{m_D} \binom{m_D}{n} \times \left(\frac{\Psi - 1}{\Psi}\right)^{m_D - n} \frac{\overline{\gamma}_E^n \Gamma(m_E + n)}{m_E^n}. \quad (21)$$

$$Proof: \quad \text{The proof follows using (15) and} [47, Eq. (8.328.2)].$$

Proposition 6 (Asymptotic Strictly Positive Secrecy Capacity): The asymptotic SPSC when the legitimate link experiences Fisher-Snedecor  $\mathcal{F}$  fading, while the eavesdropper's link experiences Nakagami-m fading is given by

$$C_s^{SPSC,asym} = 1 - \frac{a_D^{m_D} \overline{\gamma}_E^{m_D} \Gamma(m_D + m_{s_D}) \Gamma(m_D + m_E)}{m_E^{m_D} \Gamma(m_D + 1) \Gamma(m_{s_D}) \Gamma(m_E)}.$$
(22)
$$Proof: \text{ The proof follows using equations (16) and }$$
[47, Eq. (8.328.2)].

# B. NAKAGAMI-m/FISHER-SNEDECOR $\mathcal{F}$

Conversely, in this subsection we derive exact and asymptotic expressions for the case of Nakagami-m/Fisher-Snedecor  $\mathcal{F}$  fading conditions.

Corollary 5 (Average Secrecy Capacity): The ASC when the legitimate link experiences Nakagami-m fading, while the eavesdropper's link suffers Fisher-Snedecor  $\mathcal{F}$  fading is given by (23), as shown at the bottom of the page.

*Proof:* The proof is provided in Appendix B.8.

Corollary 6 (Secure Outage Probability): The SOP when the legitimate link experiences Nakagami-m fading, while the eavesdropper's link experiences Fisher-Snedecor F fading is given by

$$C_s^{SOP} = \sum_{n=0}^{\infty} \frac{m_D^n (1 - \Psi)^n}{n! \overline{\gamma}_D^n \Gamma(m_D) \Gamma(m_E) \Gamma(m_{s_E})} \times G_{4,3}^{2,3} \left( \frac{a_E \overline{\gamma}_D}{m_D \Psi} \middle| 1, 1 - m_{s_E}, 1 + n - m_D, 1 + n \right).$$
(24)

*Proof:* The proof follows with the aid of equations (10), [48, Eq. (8.2.2.14)] and [48, Eq. (8.2.2.12)].

Corollary 7 (Strictly Positive Secrecy Capacity): The SOP when the legitimate link experiences Nakagami-m fading, while the eavesdropper's link experiences Fisher-Snedecor  $\mathcal{F}$  fading is given by

$$C_s^{SPSC} = 1 - \frac{G_{3,2}^{2,2} \left(\frac{a_E \overline{\gamma}_D}{m_D} \middle| 1 - m_{s_E}, 1 - m_D, 1\right)}{\Gamma(m_D) \Gamma(m_E) \Gamma(m_{s_E})}. \tag{25}$$
Proof: The proof follows using

*Proof:* The proof follows using (11) and (48) Eq. (8.2.2.12)].

Likewise, the corresponding asymptotic representations for this scenario are derived in the following propositions.

Proposition 7 (Asymptotic Average Secrecy Capacity): The asymptotic ASC when the legitimate link experiences Nakagami-m fading, while the eavesdropper's link suffers Fisher-Snedecor  $\mathcal{F}$  fading is given by (26), as shown at the bottom of the next page.

Proof: The proof is provided in Appendix B.9.

Proposition 8 (Asymptotic Secure Outage Probability):

The asymptotic SOP when the legitimate link experiences
Nakagami-m fading, while the eavesdropper's link suffers
Fisher-Snedecor F fading is given by

$$C_s^{SOP,asym} = \frac{1}{B(m_E, m_{s_E})\Gamma(m_D + 1)} \left(\frac{m_D \Psi}{\overline{\gamma}_D}\right)^{m_D} \times \sum_{n=0}^{m_D} {m_D \choose n} \left(\frac{\Psi - 1}{\Psi}\right)^{m_D - n} \times a_F^{-n} B(m_E + n, m_{s_E} - n).$$
(27)

*Proof:* The proof follows using (15) and [47, Eq. (8.328.2)].

Fisher-Snedecor  $\mathcal{F}/N$ akagami-m:

$$\overline{C}_{s}^{ASC,asym} = \frac{a_{D}^{m_{D}}G_{3,4}^{3,2} \left(\frac{m_{E}}{\overline{\gamma}_{E}} \middle| 1, -m_{D}, 1 - m_{D}}{m_{E}, -m_{D}, -m_{D}, 0}\right)}{B(m_{D}, m_{s_{D}})\Gamma(m_{E})} + \frac{G_{2,2:0,1:2,2}^{2,1:1,0:1,2} \left(\frac{m_{E}}{\overline{\gamma}_{E}}, a_{D} \middle| 0, 1 \middle| - \middle| 1 - m_{s_{D}}, 1 \right)}{\Gamma(m_{D})\Gamma(m_{s_{D}})\Gamma(m_{E})} - \frac{G_{2,3}^{3,1} \left(\frac{m_{E}}{\overline{\gamma}_{E}} \middle| 0, 1 \middle| m_{E}, 0, 0\right)}{\Gamma(m_{E})}$$

$$(20)$$

Nakagami-m/Fisher-Snedecor  $\mathcal{F}$ :

$$\overline{C}_{s}^{ASC} = \frac{G_{2,2:0,1:2,2}^{2,1:1,0:1,2} \left(\frac{m_{D}}{\overline{\gamma}_{D}}, a_{E} \begin{vmatrix} 0,1 & - & 1-m_{s_{E}}, 1\\ 0,0 & m_{D} & m_{E}, 0 \end{vmatrix}\right)}{\Gamma(m_{D})\Gamma(m_{E})\Gamma(m_{s_{E}})} + \frac{G_{2,2:1,1:1,2}^{2,1:1,1:1,1} \left(a_{E}, \frac{m_{D}}{\overline{\gamma}_{D}} \begin{vmatrix} 0,1 & 1-m_{s_{E}} & 1\\ 0,0 & m_{E} & m_{D}, 0 \end{vmatrix}\right)}{\Gamma(m_{D})\Gamma(m_{E})\Gamma(m_{s_{E}})} - \frac{G_{3,3}^{3,2} \left(a_{E} \begin{vmatrix} 1-m_{s_{E}}, 0, 1\\ m_{E}, 0, 0 \end{vmatrix}\right)}{\Gamma(m_{E})\Gamma(m_{s_{E}})} \tag{23}$$



Proposition 9 (Asymptotic Strictly Positive Capacity): The asymptotic SSPC when the legitimate link experiences Nakagami-m fading, while the eavesdropper's link suffers Fisher-Snedecor  $\mathcal{F}$  fading is given by

$$C_s^{SPSC,asym} = 1 - \frac{m_D^{m_D} \Gamma(m_D + m_E) \Gamma(m_{s_E} - m_D)}{a_E^{m_D} \overline{\gamma}_D^{m_D} \Gamma(m_D + 1) \Gamma(m_E) \Gamma(m_{s_E})}.$$
(28)
$$Proof: \quad \text{The proof follows using (16)} \quad \text{and}$$
[47, Eq. (8.328.2)].

# C. NAKAGAMI-m/NAKAGAMI-m

Finally, capitalizing on the above results, we derive exact and asymptotic expressions for the classical case of Nakagami*m*/Nakagami-*m* fading conditions.

In this case, the ASC, SOP and SPSC can be obtained either from the Fisher-Snedecor  $\mathcal{F}/N$ akagami-m case or the Nakagami-m/Fisher-Snedecor  $\mathcal{F}$  case.

Corollary 8 (Average Secrecy Capacity): The ASC when both the legitimate and the eavesdropper's links experience Nakagami-m fading is given by (29), as shown at the bottom of the page.

*Proof:* The proof is provided in Appendix B.10.

Corollary 9 (Secure Outage Probability): The SOP when both the legitimate and the eavesdropper's links experience Nakagami-m fading is given by

$$C_s^{SOP} = \frac{1}{\Gamma(m_D)\Gamma(m_E)} \sum_{n=0}^{\infty} \frac{\left(-\frac{m_D}{\overline{\gamma}_D} (\Psi - 1)\right)^n}{n!} \times G_{3,3}^{2,2} \left(\frac{m_E \overline{\gamma}_D}{m_D \overline{\gamma}_E \Psi} \middle| 1, 1 + n - m_D, 1 + n \right). \tag{30}$$

*Proof:* With the help of [48, Eq. (8.2.2.12)] and [48, Eq. (8.2.2.14)], (24) reduces to (30), which completes the proof.

Corollary 10 (Strictly Positive Secrecy Capacity): The SPSC when both the legitimate and the eavesdropper's links suffer Nakagami-m fading is given by

$$C_s^{SPSC} = 1 - \frac{G_{2,2}^{2,1} \left(\frac{m_E \overline{\gamma}_D}{m_D \overline{\gamma}_E} \middle| 1 - m_D, 1 \atop m_E, 0\right)}{\Gamma(m_D) \Gamma(m_E)}.$$
 (31)

Proof: With the help of [48, Eq. (8.2.2.12)] and

[48, Eq. (8.2.2.14)], (25) reduces to (31), which completes the proof.

Proposition 10 (Asymptotic Average Secrecy Capacity): The asymptotic ASC when both the legitimate and the eavesdropper's links undergo Nakagami-m fading is given by (32), as shown at the bottom of the page.

*Proof:* The proof is provided in Appendix B.11. Proposition 11 (Asymptotic Secure Outage Probability): The asymptotic SOP when both the legitimate and the eavesdropper's links suffer Nakagami-m fading is given by

$$C_s^{SOP,asym} = \frac{1}{\Gamma(m_D + 1)\Gamma(m_E)} \left(\frac{m_D \Psi}{\overline{\gamma}_D}\right)^{m_D} \sum_{n=0}^{m_D} {m_D \choose n} \times \left(\frac{\Psi - 1}{\Psi}\right)^{m_D - n} \frac{\overline{\gamma}_E^n \Gamma(m_E + n)}{m_E^n}.$$
(33)

Proof: With the aid of [47, Eq. (8.328.2)], (21) reduces

to (33), which completes the proof.

Proposition 12 (Asymptotic Strictly Positive Secrecy Capacity): The asymptotic SPSC when both the legitimate and eavesdropper's links experience Nakagami-m fading is

Nakagami-m/Fisher-Snedecor  $\mathcal{F}$ 

$$\overline{C}_{s}^{ASC,asym} = \frac{\left(\frac{m_{D}}{\overline{Y}_{D}}\right)^{m_{D}} G_{4,4}^{3,3} \left(a_{E} \left| \begin{array}{c} 1 - m_{s_{E}}, 1, -m_{D}, 1 - m_{D} \\ m_{E}, -m_{D}, -m_{D}, 0 \end{array} \right)}{\Gamma(m_{D})\Gamma(m_{E})\Gamma(m_{s_{E}})} + \frac{G_{2,2:1,1:1,2}^{2,1:1,1:1,1} \left(a_{E}, \frac{m_{D}}{\overline{Y}_{D}} \left| \begin{array}{c} 0, 1 \\ 0, 0 \end{array} \right| \frac{1 - m_{s_{E}}}{m_{D}} \left| \begin{array}{c} 1 \\ m_{D}, 0 \end{array} \right)}{\Gamma(m_{D})\Gamma(m_{E})\Gamma(m_{s_{E}})} - \frac{G_{3,3}^{3,2} \left(a_{E} \left| \begin{array}{c} 1 - m_{s_{E}}, 0, 1 \\ m_{E}, 0, 0 \end{array} \right)}{\Gamma(m_{E})\Gamma(m_{s_{E}})} \right) (26)$$

Nakagami-m/Nakagami-m:

$$\overline{C}_{s}^{ASC} = \frac{G_{2,2:0,1:1,2}^{2,1:1,0:1,1} \left(\frac{m_{D}}{\overline{\gamma}_{D}}, \frac{m_{E}}{\overline{\gamma}_{E}} \middle| 0, 1 \middle| - \middle| 1 \atop 0, 0 \middle| m_{D} \middle| m_{E}, 0\right)}{\Gamma(m_{D})\Gamma(m_{E})} + \frac{G_{2,2:0,1:1,2}^{2,1:1,0:1,1} \left(\frac{m_{E}}{\overline{\gamma}_{E}}, \frac{m_{D}}{\overline{\gamma}_{D}} \middle| 0, 1 \middle| - \middle| 1 \atop 0, 0 \middle| m_{E} \middle| m_{D}, 0\right)}{\Gamma(m_{D})\Gamma(m_{E})} - \frac{G_{2,3}^{3,1} \left(\frac{m_{E}}{\overline{\gamma}_{E}} \middle| 0, 1 \atop \overline{\gamma}_{E} \middle| m_{E}, 0, 0\right)}{\Gamma(m_{E})}.$$
(29)

Nakagami-m/Nakagami-m

$$\overline{C}_{s}^{ASC,asym} = \frac{G_{3,4}^{3,2} \left(\frac{m_{E}}{\overline{\gamma}_{E}} \middle| \frac{1, -m_{D}, 1 - m_{D}}{m_{E}, -m_{D}, -m_{D}, 0}\right)}{\Gamma(m_{D})\Gamma(m_{E})} \left(\frac{m_{D}}{\overline{\gamma}_{D}}\right)^{m_{D}} + \frac{G_{2,2:0,1:1,2}^{2,1:1,0:1,1} \left(\frac{m_{E}}{\overline{\gamma}_{E}}, \frac{m_{D}}{\overline{\gamma}_{D}} \middle| 0, 1 \middle| - \middle| 1 \atop 0, 0 \middle| m_{E} \middle| m_{D}, 0\right)}{\Gamma(m_{D})\Gamma(m_{E})} - \frac{G_{2,3}^{3,1} \left(\frac{m_{E}}{\overline{\gamma}_{E}} \middle| 0, 1 \atop m_{E}, 0, 0\right)}{\Gamma(m_{E})} \tag{32}$$



given by

$$C_s^{SPSC,asym} = 1 - \frac{\Gamma(m_D + m_E) m_D^{m_D} \overline{\gamma}_E^{m_D}}{\Gamma(m_D + 1) \Gamma(m_E) m_E^{m_D} \overline{\gamma}_D^{m_D}}.$$
 (34)

*Proof:* Using [47, Eq. (8.328.2)], (22) reduces to (34), which completes the proof.

The above-mentioned scenarios include the classical Rayleigh case which can be obtained by setting the corresponding fading parameters to unity.

# **VI. NUMERICAL RESULTS AND DISCUSSIONS**

In this section, we present numerical results corresponding to the derived analytical representations in order to quantify the achievable performance and limitations of the considered setup under different multipath and shadowing conditions. Monte-Carlo simulations are also provided to demonstrate the validity of the derived analytical results. It can be observed from the figures that there is an excellent agreement between the analytical and simulation results, thereby confirming the

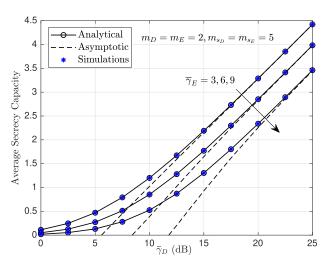
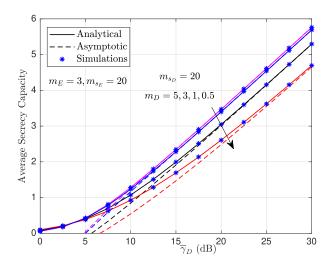


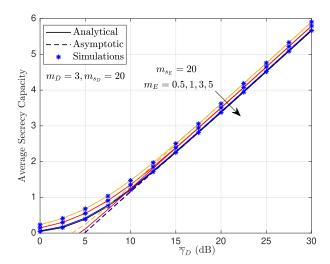
FIGURE 3. Impact of the average SNR of the eavesdropper's link,  $\overline{\gamma}_E$ , on the ASC as a function of  $\overline{\gamma}_D$ .



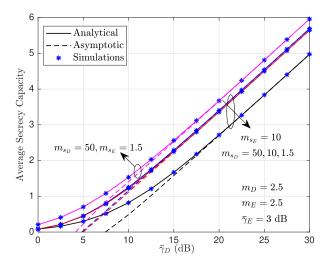
**FIGURE 4.** Impact of fading of the main link,  $m_D$ , on the ASC versus  $\overline{\gamma}_D$ .

validity of the derived expressions. Furthermore, it can be observed that the curves that correspond to the asymptotic representations are in close agreement with those that correspond to the exact representations in the high average SNR regime.

Fig. 3 illustrates the impact of the average SNR of the eavesdropper's link,  $\overline{\gamma}_E$ , on the ASC as a function of  $\overline{\gamma}_D$  for  $m_D = m_E = 2$  and  $m_{s_D} = m_{s_E} = 5$ . The results demonstrate that the ASC increases as  $\overline{\gamma}_E$  decreases, which means that the transmitter S can transmit at a higher rate to the legitimate destination D. In the same context, the impact of various fading conditions on the ASC performance is illustrated in Fig. 4 for  $m_E = 3$ ,  $m_{s_E} = 20$ ,  $m_D = \{5, 3, 1, 0.5\}$  and  $m_{s_D} = 20$ . It is evident that as the severity of the fading conditions decreases, i.e., the fading parameter of the main link  $m_D$  increases, the ASC improves. This is due to the fact that as  $m_D$  increases, the number of multipath clusters arriving

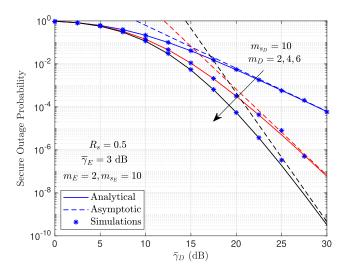


**FIGURE 5.** Impact of fading of the eavesdropper's link,  $m_E$ , on the ASC versus  $\overline{\gamma}_D$ .

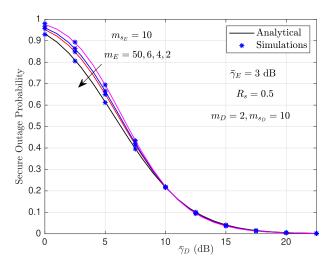


**FIGURE 6.** Impact of shadowing of the main link,  $m_{S_D}$ , and eavesdropper's link,  $m_{S_F}$ , on the ASC versus  $\overline{\gamma}_D$ .

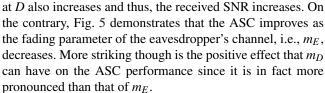




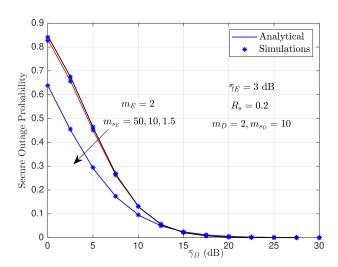
**FIGURE 7.** Impact of fading of the main link,  $m_D$ , on the SOP versus  $\overline{\gamma}_D$ .



**FIGURE 8.** Impact of fading of the eavesdropper's link,  $m_E$ , on the SOP versus  $\overline{\gamma}_D$ .



The influence that the shadowing effects encountered on the main link,  $m_{s_D}$ , and eavesdropper's link,  $m_{s_E}$ , can have on the ASC is demonstrated in Fig. 6 for  $m_D = m_E = 2.5$  and  $\overline{\gamma}_E = 3 \, dB$ . When  $m_{s_E} = 10$  and  $m_{s_D}$  increases from  $m_{s_D} = 1.5$  (heavy shadowing) to  $m_{s_D} = 50$  (light shadowing), the ASC performance improves noticeably. The results also show that when  $m_{s_D}$  changes from characterizing moderate shadowing (i.e.,  $m_{s_D} = 10$ ) towards light shadowing (i.e.,  $m_{s_D} = 50$ ), the ASC performance becomes independent of  $m_{s_D}$ . However, in contrast to  $m_{s_D}$ , when  $m_{s_E}$  decreases from  $m_{s_E} = 10$  (moderate shadowing) to  $m_{s_E} = 1.5$  (heavy



**FIGURE 9.** Impact of shadowing of the eavesdropper's link on the SOP.  $m_D = 2$ ,  $m_{SD} = 10$ ,  $m_E = 2$ ,  $\overline{\gamma}_E = 3$  dB,  $R_S = 0.2$ ..

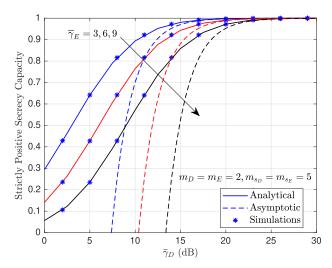


FIGURE 10. Impact of the average SNR of the eavesdropper's link,  $\overline{\gamma}_E$ , on the SPSC as a function of  $\overline{\gamma}_D$ .

shadowing), the ASC improves, i.e., S can transmit at a higher rate to D.

The results depicted in Fig. 7 illustrate the effect the encountered fading conditions in the main link,  $m_D$ , have on the SOP when  $R_s = 0.5$  and  $\overline{\gamma}_D = 3$  dB. It can be seen that as the value of  $m_D$  increases, the SOP performance improves. This is due to increasing  $m_D$  which leads to the diversity order in (15). On the other hand, Fig. 8 shows the effect of the fading conditions in the eavesdropper's link,  $m_E$ , on the SOP when  $R_s = 0.5$  and  $\overline{\gamma}_D = 3$  dB. Here too, it can be seen that as the value of  $m_E$  increases, the eavesdropper's link undergoes less severe fading conditions, which in turn reduces the overall SOP performance. It is also noted that the influence of increasing  $m_D$  is more pronounced than that of  $m_E$ .

Fig. 9 illustrates the impact of the shadowing conditions of the eavesdropper's link on the corresponding SOP



performance. It is evident that the SOP improves considerably as the severity of shadowing in the eavesdropper's link changes from light to heavy, i.e., when  $m_{s_E}$  decreases. This can be related to the fact that as the severity of shadowing increases, the eavesdropper's link experiences increasingly poorer reception. In the same context, Fig. 10 shows the impact of varying the average SNR of the eavesdropper's link,  $\overline{\gamma}_E$ , upon the SPSC for moderate multipath fading and shadowing conditions, namely  $m_D = m_E = 2$  and  $m_{s_D} = m_{s_E} = 5$ . It is observed that as  $\overline{\gamma}_E$  increases, the corresponding SPSC performance exhibits a noticeable decrease. Hence, the greater  $\overline{\gamma}_E$  is, the better the quality of the eavesdropper's channel will be and thus the SPSC performance degrades, as expected.

# VII. CONCLUSION

In this article, we have presented a systematic analysis of the Physical Layer Security of digital communications over Fisher-Snedecor  $\mathcal{F}$  composite fading channels in the presence of an eavesdropper. Exact and asymptotic expressions for a number of important related performance metrics, namely the ASC, SOP and SPSC have been derived using a redefined form for the Fisher Snedecor  $\mathcal{F}$  fading model, which leads to more unconstrained and reliable results. Using these formulations and considering both general and special cases, it has been demonstrated that the system performance improves considerably when the value of the fading parameter of the main link,  $m_D$ , increases. A similar observation has been made when the shadowing parameter of the main link,  $m_{SD}$ , increases, i.e., its severity changes from heavy shadowing to light shadowing. It was also shown that performance improvements occur when the value of the fading parameter of the eavesdropper's link,  $m_E$  and the average SNR of the eavesdropper's link,  $\overline{\gamma}_E$ , decrease. Additionally, the performance deteriorates when the shadowing parameter of the eavesdropper's link,  $m_{SE}$  increases, i.e., it changes from heavy shadowing to light shadowing. Lastly, the results have shown that the positive effect of  $m_D$  on the system performance is more pronounced than that of  $m_E$ .

As a final note, with the prevalence of multiple-input multiple-output in many emerging wireless applications such as those used in 5G cellular, a direct extension of this work will be to consider PLS for multi-antenna scenarios. Additionally, to further improve the practical applicability of the proposed framework, situations were the PLS is impacted by Fisher-Snedecor  $\mathcal F$  composite fading and artificial noise should be considered.

# APPENDIX PRELIMINARIES

In this appendix, we provide some preliminary results that assist in the analysis of the system under consideration. To this end, it is recalled that the BVMGF is defined in terms of a two-fold contour integral in the complex plane [53,

Eq. (13.1)], namely

$$G_{p_{1},q_{1}:p_{2},q_{2}:p_{3},q_{3}}^{m_{1},n_{1}:m_{2},n_{2}:m_{3},n_{3}}\left(x,y\begin{vmatrix}\mathbf{a_{1}p_{1}}\\\mathbf{b_{1}q_{1}}\end{vmatrix}\begin{vmatrix}\mathbf{a_{2}p_{2}}\\\mathbf{b_{2}q_{2}}\end{vmatrix}\begin{vmatrix}\mathbf{a_{3}p_{3}}\\\mathbf{b_{3}q_{3}}\right)$$

$$=\frac{1}{(2\pi i)^{2}}\int_{L_{1}}\int_{L_{2}}\Psi_{1}(s+t)\Psi_{2}(s)\Psi_{3}(t)x^{-s}y^{-t}dsdt,$$
(35)

where

$$\Psi_{k}(\tau) = \frac{\prod_{j=1}^{m_{k}} \Gamma(b_{kj} + \tau) \prod_{j=1}^{n_{k}} \Gamma(1 - a_{kj} - \tau)}{\prod_{j=n_{k}+1}^{p_{k}} \Gamma(a_{kj} + \tau) \prod_{j=m_{k}+1}^{q_{k}} \Gamma(1 - b_{kj} - \tau)}$$
(36)

with  $k \in \mathbb{N}$ . The contour  $L_1$ , in the complex s-plane, separates the poles of  $\Gamma(1-a_{1j}-s)$   $(j=1,\ldots,n_1)$  and  $\Gamma(1-a_{2j}-s)$   $(j=1,\ldots,n_2)$  from the poles of  $\Gamma(b_{1j}+s)$   $(j=1,\ldots,m_1)$  and  $\Gamma(b_{2j}+s)$   $(j=1,\ldots,m_2)$ , when  $t \in L_2$ . Likewise, the contour  $L_2$ , in the complex s-plane, separates the poles of  $\Gamma(1-a_{1j}-s)$   $(j=1,\ldots,n_1)$  and  $\Gamma(1-a_{3j}-s)$   $(j=1,\ldots,n_3)$  from the poles of  $\Gamma(b_{1j}+s)$   $(j=1,\ldots,m_1)$  and  $\Gamma(b_{3j}+s)$   $(j=1,\ldots,m_3)$ , when  $s \in L_1$ . Note that the BVMGF converges if the conditions listed in [53, Sec. 13.2] are satisfied. However, It is straightforward to show that the parameters of the BVMGF in (9), (14), (17), (20), (23), (26), (29), and (32) satisfy these sufficient conditions, and therefore the BVMGF converges.

The Meijer's G-function is defined by means of a one-fold contour integral in the complex plane [48, Eq. (8.3.1.1)], i.e.,

$$G_{p_0,q_0}^{m_0,n_0}\left(x\begin{vmatrix} a_1,\cdots,a_{p_0}\\b_1,\cdots,b_{q_0} \end{pmatrix} = \frac{1}{2\pi i} \int_L \Psi(s) x^{-s} ds,$$
 (37)

where

$$\Psi_k(s) = \frac{\prod_{j=1}^{n_0} \Gamma(b_j + s) \prod_{k=1}^{n_0} \Gamma(1 - a_k - s)}{\prod_{k=n_0+1}^{n_0} \Gamma(a_k + s) \prod_{j=m_0+1}^{q_0} \Gamma(1 - b_j - s)}.$$

The contour L is a suitable closed contour in the complex s-plane which can be chosen among three types of integration paths. Also, it is noted that the poles  $\Gamma(b_j + s)$  must not coincide with the poles of  $\Gamma(1 - a_k - s)$  (with  $j = 1, \ldots, m_0$  and  $k = 1, \ldots, n_0$ ).

# **APPENDIX**

### A. PROOF OF THEOREM 1

Since both the main and eavesdropper channels experience independent fading, the corresponding ASC is given by

$$\overline{C}_{s}^{ASC} = \mathbb{E}\left[C_{s}(\gamma_{D}, \gamma_{E})\right] 
= \int_{0}^{\infty} \int_{0}^{\infty} C_{s}(\gamma_{D}, \gamma_{E}) f(\gamma_{D}, \gamma_{E}) d\gamma_{D} d\gamma_{E} 
= \mathcal{J}_{1} + \mathcal{J}_{2} - \mathcal{J}_{3},$$
(38)

where  $f(\gamma_D, \gamma_E)$  is the joint PDF of  $\gamma_D$  and  $\gamma_E$ , and

$$\mathcal{J}_{1} = \int_{0}^{\infty} \ln\left(1 + \gamma_{D}\right) f_{D}\left(\gamma_{D}\right) F_{E}\left(\gamma_{D}\right) d\gamma_{D}, \qquad (39)$$



$$\mathcal{J}_2 = \int_0^\infty \ln\left(1 + \gamma_E\right) f_E\left(\gamma_E\right) F_D\left(\gamma_E\right) d\gamma_E, \qquad (40)$$

and

$$\mathcal{J}_3 = \int_0^\infty \ln\left(1 + \gamma_E\right) f_E\left(\gamma_E\right) d\gamma_E. \tag{41}$$

Performing a change of variables in (6) and (7),  $\mathcal{J}_1$  becomes

$$\mathcal{J}_{1} = \frac{1}{\Gamma(m_{D})\Gamma(m_{s_{D}})\Gamma(m_{E})\Gamma(m_{s_{E}})} \int_{0}^{\infty} \gamma_{D}^{-1} \ln(1 + \gamma_{D})$$

$$\times G_{1,1}^{1,1} \left( a_{D} \gamma_{D} \middle| 1 - m_{s_{D}} \right) G_{2,2}^{1,2} \left( a_{E} \gamma_{D} \middle| 1 - m_{s_{E}}, 1 \right) d\gamma_{D}.$$
(42)

Representing the logarithmic function in terms of Meijer's G-function [48, Eq. (3.4.6.5)], (42) becomes

$$\mathcal{J}_{1} = \frac{1}{\Gamma(m_{D})\Gamma(m_{s_{D}})\Gamma(m_{E})\Gamma(m_{s_{E}})} \int_{0}^{\infty} \gamma_{D}^{-1} G_{2,2}^{1,2} \left(\gamma_{D} \begin{vmatrix} 1, 1 \\ 1, 0 \end{pmatrix} \times G_{1,1}^{1,1} \left(a_{D}\gamma_{D} \begin{vmatrix} 1 - m_{s_{D}} \\ m_{D} \end{pmatrix} G_{2,2}^{1,2} \left(a_{E}\gamma_{D} \begin{vmatrix} 1 - m_{s_{E}}, 1 \\ m_{E}, 0 \end{pmatrix} d\gamma_{D}.$$
(43)

With the aid of [54],  $\mathcal{J}_1$  can be obtained in closed-form as

$$\mathcal{J}_{1} = \frac{G_{2,2:1,1:2,2}^{2,1:1,1:1,2} \left( a_{D}, a_{E} \begin{vmatrix} 0, 1 \\ 0, 0 \end{vmatrix} \frac{1 - m_{s_{D}}}{m_{D}} \begin{vmatrix} 1 - m_{s_{E}}, 1 \\ m_{E}, 0 \end{vmatrix} \right)}{\Gamma(m_{D})\Gamma(m_{s_{D}})\Gamma(m_{E})\Gamma(m_{s_{E}})}$$
(44)

Likewise, the integral  $\mathcal{J}_2$  can be obtained as

$$\mathcal{J}_{2} = \frac{G_{2,2:1,1:2,2}^{2,1:1,1:1,2} \left( a_{E}, a_{D} \middle| 0, 1 \middle| 1 - m_{s_{E}} \middle| 1 - m_{s_{D}}, 1 \right)}{\Gamma(m_{D})\Gamma(m_{s_{D}})\Gamma(m_{E})\Gamma(m_{s_{E}})}$$
(45)

Similarly, using (6) and (41), the integral  $\mathcal{J}_3$  can be rewritten as

$$\mathcal{J}_{3} = \int_{0}^{\infty} \frac{\gamma_{E}^{-1} G_{2,2}^{1,2} \left(\gamma_{E} \begin{vmatrix} 1,1\\1,0 \end{pmatrix} G_{1,1}^{1,1} \left(a_{E} \gamma_{E} \begin{vmatrix} 1-m_{s_{E}}\\m_{E} \end{pmatrix} d \gamma_{E}}{\Gamma(m_{E}) \Gamma(m_{s_{E}})},$$
(46)

which can be solved with the help of [48, Eq. (2.24.1.1)] as

$$\mathcal{J}_{3} = \frac{G_{3,3}^{3,2} \left( a_{E} \middle| 1 - m_{s_{E}}, 0, 1 \right)}{\Gamma(m_{E})\Gamma(m_{s_{E}})}.$$
 (47)

Finally, substituting (44), (45) and (47) into (38), yields (9).

# **B. PROOF OF THEOREM 2**

The SOP in the considered set up can be obtained via

$$C_s^{SOP} = \Pr\{C_s(\gamma_D, \gamma_E) < R_s\} = \Pr\{\gamma_D < \Psi \gamma_E + \Psi - 1\}$$

$$= \int_0^\infty f_E(\gamma_E) \left( \int_0^{\Psi \gamma_E + \Psi - 1} f_D(\gamma_D) d\gamma_D \right) d\gamma_E$$

$$= \int_0^\infty F_D(\Psi \gamma_E + \Psi - 1) f_E(\gamma_E) d\gamma_E, \tag{48}$$

where it is recalled that  $\Psi = \exp(R_s) \ge 1$ . To this effect and using (6) along with (7), it follows that

$$C_s^{SOP} = \frac{\int_{0}^{\infty} \gamma_E^{-1} G_{1,1}^{1,1} \left( a_E \gamma_E \begin{vmatrix} 1 - m_{s_E} \\ m_E \end{vmatrix} \right)}{\Gamma(m_D) \Gamma(m_{s_D}) \Gamma(m_E) \Gamma(m_{s_E})} \times G_{2,2}^{1,2} \left( a_D \left( \Psi \gamma_E + \Psi - 1 \right) \begin{vmatrix} 1 - m_{s_D}, 1 \\ m_D, 0 \end{vmatrix} \right) d\gamma_E.$$
(49)

After making the change of variable  $x = a_D \Psi \gamma_E$  in (49) and using [48, Eq. (2.24.1.3)], then the SOP in (10) is obtained, which completes the proof.

# C. PROOF OF COROLLARY 1

The corresponding SPSC can be obtained using

$$C_s^{SPSC} = \Pr\{C_s(\gamma_D, \gamma_E) > 0\} = 1 - \Pr\{C_s(\gamma_D, \gamma_E) < 0\}$$
  
= 1 - C\_s^{SOP}|\_{R\_s=0}. (50)

Substituting  $R_s = 0$  (i.e.,  $\Psi = 1$ ) and n = 0 into (10), and using [48, Eq. (8.2.2.8)], the SPSC in (11) is obtained, which completes the proof.

# D. PROOF OF PROPOSITION 1

The asymptotic ASC can be evaluated via

$$C_s^{ASC,asym} = \mathcal{J}_1^{asym} + \mathcal{J}_2 - \mathcal{J}_3, \tag{51}$$

where  $\mathcal{J}_2$  and  $\mathcal{J}_3$  are defined in (45) and (47), respectively, whereas  $\mathcal{J}_1^{asym}$  can be obtained using (39) and (12) as

$$\mathcal{J}_{1}^{asym} = \frac{a_{D}^{m_{D}}}{B(m_{D}, m_{s_{D}})\Gamma(m_{E})\Gamma(m_{s_{E}})} \int_{0}^{\infty} \gamma_{D}^{m_{D}-1} \ln(1 + \gamma_{D}) \times G_{2,2}^{1,2} \left( a_{E} \gamma_{D} \middle| 1 - m_{s_{E}}, 1 \atop m_{E}, 0 \right) d\gamma_{D}. \quad (52)$$

The above integral can be solved in closed-form as follows:

$$\mathcal{J}_{1}^{asym} = \frac{a_{D}^{m_{D}}G_{4,4}^{3,3}\left(a_{E} \left| \begin{array}{c} 1 - m_{SE}, 1, -m_{D}, 1 - m_{D} \\ m_{E}, -m_{D}, -m_{D}, 0 \end{array} \right)}{B(m_{D}, m_{SD})\Gamma(m_{E})\Gamma(m_{SE})}. (53)$$

Based on this, substituting (53), (45) and (47) into (51) yields (14), which completes the proof.



### E. PROOF OF PROPOSITION 2

The asymptotic SOP can be obtained using (5), (48), and (13), yielding

$$C_{s}^{SOP,asym} = \frac{a_{D}^{m_{D}} a_{E}^{m_{E}} \Psi^{m_{D}} \Gamma(m_{D} + m_{s_{D}})}{B(m_{E}, m_{s_{E}}) \Gamma(m_{D} + 1) \Gamma(m_{s_{D}})} \int_{0}^{\infty} \gamma_{E}^{m_{E} - 1} \times (1 + a_{E} \gamma_{E})^{-(m_{E} + m_{s_{E}})} \left(\frac{\Psi - 1}{\Psi} + \gamma_{E}\right)^{m_{D}} d\gamma_{E}.$$
(54)

Using the power series expansion in [47, Eq. (1.111)], one obtains

$$C_s^{SOP,asym} = \frac{a_D^{m_D} a_E^{m_E} \Psi^{m_D} \Gamma(m_D + m_{s_D})}{B(m_E, m_{s_E}) \Gamma(m_D + 1) \Gamma(m_{s_D})} \sum_{n=0}^{m_D} {m_D \choose n} \times \left(\frac{\Psi - 1}{\Psi}\right)^{m_D - n} \int_0^\infty \gamma_E^{m_E + n - 1} \times (1 + a_E \gamma_E)^{-(m_E + m_{s_E})} d\gamma_E.$$
 (55)

Notably, the above integral can be expressed in closed-form using [47, Eq. (3.194.3)] as in (15). This completes the proof.

### F. PROOF OF PROPOSITION 3

The asymptotic SPSC can be obtained using

$$C_s^{SPSC,asym} = 1 - C_s^{SOP,asym} \left|_{R_s = 0, n \to m_D} \right|. \tag{56}$$

Thus, with the aid of (15) and after some algebraic manipulations, the asymptotic SPSC is obtained as in (16), which completes the proof.

### G. PROOF OF COROLLARY 2

For the specific case of Nakagami-m fading conditions, i.e.,  $m_{s_E} \to \infty$ ,  $\mathcal{J}_1$  and  $\mathcal{J}_2$  in (44) and (45), reduce to (57) and (58), namely

$$\mathcal{J}_{1} = \frac{G_{2,2:1,1:1,2}^{2,1:1,1:1,1} \left( a_{D}, \frac{m_{E}}{\overline{\gamma}_{E}} \begin{vmatrix} 0, 1 & 1 - m_{s_{D}} & 1 \\ 0, 0 & m_{D} & m_{E}, 0 \end{vmatrix} \right)}{\Gamma(m_{D})\Gamma(m_{s_{D}})\Gamma(m_{E})}$$
(57)

and

$$\mathcal{J}_{2} = \frac{G_{2,2:0,1:2,2}^{2,1:1,0:1,2} \left(\frac{m_{E}}{\overline{\gamma}_{E}}, a_{D} \begin{vmatrix} 0, 1 \\ 0, 0 \end{vmatrix} \frac{- 1 - m_{s_{D}}, 1}{m_{D}, 0}\right)}{\Gamma(m_{D})\Gamma(m_{s_{D}})\Gamma(m_{E})}. (58)$$

It is noted here that (57) is obtained by first representing the BVMGF in (44) by its integral representation using (35) in Appendix A and then applying [47, Eq. (8.328.2)]. Finally, by using (35) in Appendix A again, one can obtain (57). Likewise, the evaluation of the integral  $\mathcal{J}_2$ , i.e., (58) can be obtained in the same manner.

However, a closed-form expression for  $\mathcal{J}_3$  is obtained differently since [48, Eq. (8.2.2.14)] is first used and then [48, Eq. (8.2.2.12)] is applied. To this effect and by using [48, Eq. (8.2.2.14)] once more, it follows that

$$\mathcal{J}_{3} = \frac{G_{2,3}^{3,1} \left(\frac{m_{E}}{\gamma_{E}} \middle| 0, 1 \atop m_{E}, 0, 0\right)}{\Gamma(m_{E})}.$$
 (59)

Finally, substituting (57), (58) and (59) into (38), (17) is obtained, which completes the proof.

# H. PROOF OF COROLLARY 5

For the special case of Nakagami-m fading, i.e.,  $m_{s_D} \to \infty$ ,  $\mathcal{J}_1$  and  $\mathcal{J}_2$  in (44) and (45) reduce to (60) and (61), namely

$$\mathcal{J}_{1} = \frac{G_{2,2:0,1:2,2}^{2,1:1,0:1,2} \left(\frac{m_{D}}{\overline{\gamma}_{D}}, a_{E} \begin{vmatrix} 0, 1 & - & 1 - m_{s_{E}}, 1 \\ 0, 0 & m_{D} & m_{E}, 0 \end{vmatrix}}{\Gamma(m_{D})\Gamma(m_{E})\Gamma(m_{s_{E}})}$$
(60)

and

$$\mathcal{J}_{2} = \frac{G_{2,2:1,1:1,2}^{2,1:1,1:1,1} \left( a_{E}, \frac{m_{D}}{\overline{\gamma}_{D}} \middle| 0, 1 \middle| 1 - m_{s_{E}} \middle| 1 \atop m_{D}, 0 \middle| m_{E} \middle| m_{D}, 0 \right)}{\Gamma(m_{D}) \Gamma(m_{E}) \Gamma(m_{s_{E}})}. (61)$$

It is noted that the above expressions are obtained by first representing the BVMGF in (44) by its integral representation using (35) in Appendix A, then applying [47, Eq. (8.328.2)], and finally using (35) in Appendix A again. To this effect, substituting (47), (60), and (61) into (38), then (23) is obtained, which completes the proof.

# I. PROOF OF PROPOSITION 7

The first term in (14) reduces to the Nakagami-*m* case after using the relation in [47, Eq. (8.328.2)], i.e.,

$$\mathcal{J}_{1}^{asym} = \frac{\left(\frac{m_{D}}{\overline{\gamma}_{D}}\right)^{m_{D}} G_{4,4}^{3,3} \left(a_{E} \left| \begin{array}{c} 1 - m_{s_{E}}, 1, -m_{D}, 1 - m_{D} \\ m_{E}, -m_{D}, -m_{D}, 0 \end{array}\right)}{\Gamma(m_{D})\Gamma(m_{E})\Gamma(m_{s_{E}})}.$$
(62)

The second term in (14) can be reduced using the definition of the BVMGF in (35) in Appendix A and the relation in [47, Eq. (8.328.2)], which yields

$$\mathcal{J}_{2} = \frac{G_{2,2:1,1:1,2}^{2,1:1,1:1,1} \left( a_{E}, \frac{m_{D}}{\overline{\gamma}_{D}} \middle| 0, 1 \middle| 1 - m_{s_{E}} \middle| 1 \atop m_{D}, 0 \middle| m_{E} \middle| m_{D}, 0 \right)}{\Gamma(m_{D})\Gamma(m_{E})\Gamma(m_{s_{E}})}.$$
(63)

It is noted here that there is no change in the third term in (14). Thus, the asymptotic ASC is obtained as in (26), which completes the proof.

# J. PROOF OF COROLLARY 8

The first and second terms in (17) can be reduced to the Nakagami-*m* case with the aid of (35) in Appendix A and [47, Eq. (8.328.2)], yielding (64) and (65) below

$$\mathcal{J}_{1} = \frac{G_{2,2:0,1:1,2}^{2,1:1,0:1,1} \left(\frac{m_{D}}{\overline{\gamma}_{D}}, \frac{m_{E}}{\overline{\gamma}_{E}} \begin{vmatrix} 0, 1 \\ 0, 0 \end{vmatrix} \frac{-}{m_{D}} \begin{vmatrix} 1 \\ m_{D} \end{vmatrix} \right)}{\Gamma(m_{D})\Gamma(m_{E})}, \quad (64)$$

and

$$\mathcal{J}_{2} = \frac{G_{2,2:0,1:1,2}^{2,1:1,0:1,1} \left(\frac{m_{E}}{\overline{\gamma}_{E}}, \frac{m_{D}}{\overline{\gamma}_{D}} \middle| \begin{array}{c} 0, 1 \middle| - \middle| 1 \\ 0, 0 \middle| m_{E} \middle| m_{D}, 0 \end{array}\right)}{\Gamma(m_{D}) \Gamma(m_{E})}. \quad (65)$$

Based on the above and after some algebraic manipulations, (29) is deduced, which completes the proof.



### K. PROOF OF PROPOSITION 10

The first term in (20) can be reduced to the Nakagami-m case, with the aid of [47, Eq. (8.328.2)], namely

$$\mathcal{J}_{1}^{asym} = \frac{\left(\frac{m_{D}}{\overline{\gamma}_{D}}\right)^{m_{D}}}{\Gamma(m_{D})\Gamma(m_{E})} G_{3,4}^{3,2} \left(\frac{m_{E}}{\overline{\gamma}_{E}} \middle| 1, -m_{D}, 1 - m_{D}, 0 \right).$$
(66)

Likewise, the second term in (20) can be reduced to the Nakagami-*m* case using (35) in Appendix A and [47, Eq. (8.328.2)], which yields

$$\mathcal{J}_{2}^{asym} = \frac{G_{2,2:0,1:1,2}^{2,1:1,0:1,1} \left( \frac{m_{E}}{\overline{\gamma}_{E}}, \frac{m_{D}}{\overline{\gamma}_{D}} \middle| 0, 1 \middle| - \middle| 1 \middle| m_{D}, 0 \right)}{\Gamma(m_{D})\Gamma(m_{E})}. (67)$$

Therefore, using (66) and (67), the asymptotic ASC for the case of Nakagami-*m*/Nakagami-*m* fading conditions is readily deduced in (32), which completes the proof.

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