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Online Neuro-Fuzzy Controller: Design for Robust Stability

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ABSTRACT The Online Neuro-Fuzzy Controller (ONFC) is a fuzzy-based adaptive control that uses a very simple structure and can control nonlinear, time-varying and uncertain systems. Its efficiency and low computational cost allowed applications in several industrial plants successfully. However, none of the previous works on the ONFC provided a design procedure endowed with formal guarantees of robust closed-loop stability. In this paper, some conditions for ONFC robust stability, considering system polytopic uncertainties, are presented using the Lyapunov method. A new adaptation rule is proposed that dynamically varies the adaptation gain and incorporates the dead-zone technique to ensure robustness to the noise measurement. A reference model is also introduced, in order to allow a direct specification of the closed-loop dynamics. Simulation results show that the new design conditions present good performance in the control of several types of systems.

INDEX TERMS Adaptive control, fuzzy control, ONFC, robust stability.

I. INTRODUCTION

The fuzzy control is typically seen as adaptive. It usually deals with nonlinear dynamics using weighted interpolation, either considering local models or simply combining controllers for different operational points. However, some structures based on fuzzy systems bring the capability of recursive adaptation as a tool to manage nonlinearity, uncertainty, and parametric variation [1]–[3]. Neuro-fuzzy networks combine fuzzy logic with neural networks and are widely used in the context of adaptive control [4], [5]. Due to their feature of universal approximators [6] they are powerful tools in the areas of identification, control, and especially adaptive systems [7]–[9]. Nevertheless, in order to control nonlinear systems, a neuro-fuzzy controller usually has a large number of fuzzy sets in the antecedents or consequents. This reduces interpretability and renders the stability analysis rather complex.

Stability is a central problem in adaptive control, as an implicit feedback loop performs the automatic tuning of the controller parameters. This dual feedback makes the closed-loop system nonlinear and time-varying. In the case of Neuro-Fuzzy controllers the stability analysis is usually complex, in spite of its known simplicity of design. When stability conditions are found, they are sometimes so restrictive or complex that adaptive control is discouraged when the problem is solvable by simpler techniques [8].

Several works deal with different approaches for adaptive control based on fuzzy systems. For instance, [10] presents a state-feedback fuzzy controller for a flexible robotic arm. In that work, an explicit fuzzy model of the plant is built, and the robust stability is established via Linear Matrix Inequality (LMI) conditions. More recently, [11] presents an adaptive fuzzy output-feedback controller for switched nonlinear systems. That work deals with unknown dynamics using fuzzy systems for performing approximations, and an observer for performing state estimation. The Lyapunov method is employed for establishing the closed-loop stability. The reference [12] studies the issue of model uncertainties introduced by communication failure in networked systems. The control design is an output feedback dissipative tracking

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controller of T-S fuzzy systems with a reference model, and the corresponding design conditions are given in the form of LMIs.

This paper is concerned with a new version of the ONFC controller (Online Neuro-Fuzzy Controller), a fuzzy controller which has a much simpler structure than most of the recently proposed fuzzy controllers. The first ONFC controller has been proposed in [13] as an adjustable parameter controller that uses the output error information to tune its consequents. This kind of technique is known in the adaptive control literature as a direct adaptation method (does not require model identification). This is a significant advantage in the context of fuzzy systems as it reduces the universe of discourse to the maximum error value and not to the operating range. Indeed, ONFC is capable of being applied to systems with wide operating ranges using only two rules and it needs no expert knowledge to create the initial structure.

Despite the simplicity of ONFC, its efficiency in solving complex problems is noticeable. In general, control engineering problems involving non-linear plants or with uncertain models are significantly common in practice. The ONFC has its strongest potential in those problems. Using the gradient method to determine the parameter adaptation law, Gouvea *et al.* applied it to the control of induction motors [14]. The ONFC low computational cost expands its application potential, and several authors have used it in various practical control problems. ONFC has been successfully applied to plants such as a two-wheeled robot, a coupled tank level control and industrial applications, including the temperature control of a cooking plant in a Brazilian oil industry [15]–[19].

Some authors have proposed significant modifications to the original ONFC that have improved it in some respects. Pires [20] has shown the equivalence of ONFC with an adaptive PI (Proportional-Integral) controller and applied sliding modes for increasing closed-loop robustness. Reference [17] proposed a modification for the treatment of parametric drift that causes the undesirable increasing of consequent values when operating in the presence of measurement noise. Moreover, the authors added a derivative feature to ONFC. References [16] and [15] proposed a version of ONFC employing three inferential rules, obtaining improvements in the control of nonlinear systems. Santos *et al.* [21] performed a comparative study between the ONFC, the PID (Proportional-Integral-Derivative) and PID-NN (a PID-Neural Network), in the control of a two-wheeled robot. The three controllers were implemented in an embedded microcontroller with limited memory capacity and low processing speed. That study showed that all those controllers could be implemented easily in that situation of hardware limitations, and the ONFC presented superior performance. It should be noticed that several recently proposed observer-based fuzzy controllers cannot be implemented on that hardware. Finally, Gomes *et al.* [19] applied a variable adaptation rate to increase the ONFC performance in a coupled tank control problem.

Notwithstanding the many successful applications mentioned above, the ONFC controller lacks a formal analysis of the conditions for robust stability. This makes the task of evaluating which plants can be controlled by the ONFC difficult for the designer. In addition, in those former works there are no design rules for choosing the adaptation parameters. In general, this choice is performed in a heuristic way, largely based on the designer's prior knowledge.

This paper aims to develop formal robust stability conditions for a generalized version of the ONFC controller which employs a reference model, a new adaptation rule and a dead zone technique. The discrete-time linear case with model uncertainty, time-varying parameters and measurement noise is considered here. The Lyapunov method is applied to establish robust stability and to determine design conditions that are stated as Linear Matrix Inequalities (LMI's). This generalized version of ONFC is a case of a Fuzzy Model Reference Adaptive Control (FMRAC). Actually, [2] shows that an FMRAC is superior to MRAC for some kinds of problems. Simulation results corroborate the conditions found analytically.

Some specific contributions of this paper are:

- An enhanced version of ONFC controller is proposed. This controller is now endowed with a mechanism for the explicit specification of the closed-loop dynamics by means of a reference model.
- There is a formal proof of robust stability of the closed-loop for the proposed controller, which relies on the boundedness of the difference between the actual system dynamics and the model reference dynamics. The stability condition is stated in terms of Linear Matrix Inequalities (LMIs), allowing the choice of the design parameters which ensure robust stability for polytope-bounded system model uncertainties.
- Even in the case of few available information about the plant model, the proposed ONFC controller can be put in operation easily, because the design parameters have simple interpretation and do not require a fine-tuning for deployment on a real plant.

This paper is organized as follows. In Section II the generalized ONFC controller is presented, with a new the adaptation law, and Linear Matrix Inequalities (LMI) conditions for robust stability are proposed. The proof of stability based on Lyapunov analysis is presented in Section III. Section IV sumarizes the proposed ONFC design procedure. Simulation results are presented in Section V, highlighting the theoretical results. Finally, some conclusions are drawn in Section VI.

II. ONFC CONTROLLER

The ONFC controller can be seen as a Takagi-Sugeno Fuzzy Inference System (TS-FIS) with zero order consequent (singleton) consisting of only two rules. The weights of the consequents are adjusted to minimize the output error of the closed-loop, as described below.

Consider the following discrete-time system:

$$x(k+1) = Ax(k) + Bu(k) + \xi(k)$$
$$y(k) = Cx(k) + \upsilon(k) \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times 1}$, $C \in \mathbb{R}^{1\times n}$ are matrices with constant coefficients, $u(k)$ is the control input, $\xi(k)$ and $\upsilon(k)$ are external disturbances with zero mean whose sum $C\xi(k)+\upsilon(k+1)$ is limited by the open interval $(-\xi_0, \xi_0)$, and $y(k)$ is the measured output. The following assumptions are required here:

(A1) the matrix C is precisely known and the pair (A, B) is uncertain, and belongs to a convex polytope with n_p known vertices (A_i, B_i):

$$(A, B) = \sum_{i=1}^{n_p} \beta_i (A_i, B_i)$$
$$\beta_i \geq 0\,, \quad \sum_{i=1}^{n_p} \beta_i = 1 \tag{2}$$

(A2) the triple (A, B, C) is output stabilizable for all instances of the unknown system within the polytope;

(A3) $C \cdot B_i > 0 \;\; \forall\, i \in \{1, \ldots, n_p\}$. For simplicity, only the case $C \cdot B_i > 0$ is considered in this paper. By analogy, the construction for the case $C \cdot B_i < 0$ is trivial.

(A4) The signals $|\xi(k)|$ and $|\upsilon(k)|$ are bounded, and the respective upper bounds are known.

The first step of the controller design involves the choice, by the designer, of a stable and time-invariant reference model given by:

$$x_m(k+1) = A_m x_m(k) + B_m r(k),$$
$$y_m(k) = Cx_m(k) \tag{3}$$

where $r(k)$ is any limited reference signal, $A_m \in \mathbb{R}^{n\times n}$ is Schur, and B_m is defined so that the reference model system has unitary gain. The ONFC controller will be defined in order to enforce the plant closed-loop dynamics, given by:

$$x(k+1) = (A - BK_1)x(k) + BK_2 r(k) \tag{4}$$

where K_1 and K_2 are appropriate scalars, to approximate the reference system dynamics (3).

Definition 1: If there are values of K_1 and K_2 that satisfy:

(1) $A_m = A - BK_1$

(2) $B_m = BK_2$

then the matching conditions are met. □

The control problem is defined as the problem of synthesis of a bounded signal $u(k)$ such that the error e_y given by

$$e_y(k) = y(k) - y_m(k) \tag{5}$$

becomes bounded by

$$|e_y(k)| < \epsilon, \tag{6}$$

for a given $\epsilon > 0$ and for all $k > k_0$, with k_0 a finite integer.

Let the output error $e_r(k)$ be defined by:

$$e_r(k) = r(k) - y(k) \tag{7}$$

The ONFC controller illustrated in the diagram of Fig. 1 is defined by the following rules:

1. IF e_r is Z_1 THEN u is $u_1 = w_1$.
2. IF e_r is Z_2 THEN u is $u_2 = w_2$.

The sets Z_1 and Z_2 are the antecedents of the inference system represented in Fig. 2 and the scalars w_1 and w_2 are the consequents. The degrees of membership of the error e_r to fuzzy sets Z_1 and Z_2 are given by μ_1 and μ_2 functions, respectively, which are complementary functions to each other, defined by:

$$\mu_1(k) = \begin{cases} 1, & \text{if } e_r(k) \leq -E_M \\ \dfrac{E_M - e_r(k)}{2E_M}, & \text{if } -E_M < e_r(k) < E_M \\ 0, & \text{if } e_r(k) \geq E_M \end{cases} \tag{8}$$

$$\mu_2(k) = 1 - \mu_1(k). \tag{9}$$

where E_M is a fixed parameter. Since $\mu_2(k) + \mu_1(k) = 1$, the $u(k)$ control action, which is given by a weighted average (according to the Takagi-Sugeno method), becomes:

$$u(k) = \mu_1(k)w_1(k) + \mu_2(k)w_2(k). \tag{10}$$

The $u(k)$ control action at time k as a function of error is illustrated by Fig. 2. The adaptation law of the weight w_1 and w_2 is designed to decrease the $e_y(k)$ error in order to

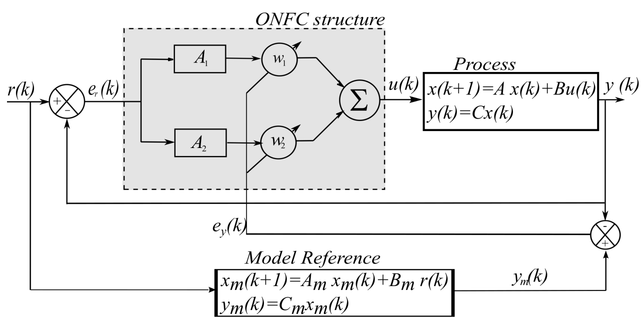

FIGURE 1. ONFC controller scheme.

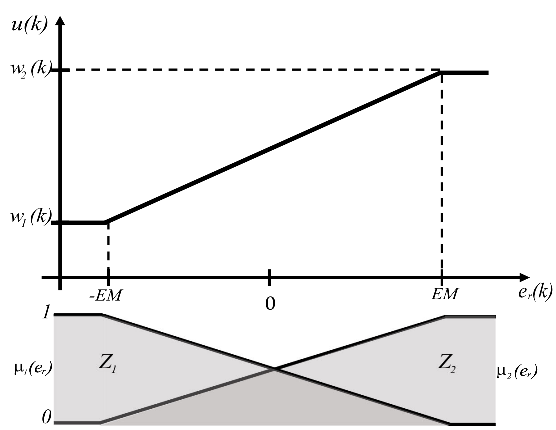

FIGURE 2. Membership functions and output controller at instant *k*.

enforce the output of the closed-loop model to approximate the reference model output. This adaptation law employed here is given by:

$$\begin{aligned} w_1(k) &= w_1(k-1) - \gamma e_y(k)\mu_1(k-1) \\ w_2(k) &= w_2(k-1) - \gamma e_y(k)\mu_2(k-1) \end{aligned} \tag{11}$$

where γ is the update rate and has the same signal of the plant loop gain.

Define Λ as the maximum value of scalar CB in the uncertainty polytope:

$$\Lambda = \max_{i=1,\ldots,n_p} C \cdot B_i \tag{12}$$

The following adaptation rule for γ is defined:

$$\gamma = \begin{cases} \dfrac{\alpha}{\Lambda[\mu_1^2(k) + \mu_2^2(k)]}, & if \ \ |e_y(k)| > \xi_0 \\ 0, & if \ \ |e_y(k)| \leq \xi_0 \end{cases} \tag{13}$$

where $0 < \alpha < 2$. The design constant ξ_0 is defined such that:

$$\xi_0 > max(|C\xi(k) + \upsilon(k+1)|) \tag{14}$$

The value of ξ_0 should be as small as possible, so an accurate estimate of $max(|\xi(k)|)$ and $max(|\upsilon(k)|)$ is desirable. In addition it is desirable for practical purposes that $\xi_0 \ll \epsilon$.

Now, consider the matrices M_i defined by:

$$M_i = \begin{bmatrix} Q_i & S_i^T \\ S_i & R_i \end{bmatrix} \tag{15}$$

with the submatrices:

$$\begin{aligned} Q_i &= A_m^T[P + \alpha C^T C/\Lambda]A_m - \rho_1 P \\ S_i &= A_m^T P B_i - A_m^T C^T(\rho_2 - \alpha) \\ R_i &= B_i^T P B_i - C B_i(2\rho_2 - \alpha) + 1 - \rho_2 \end{aligned} \tag{16}$$

for $i = 1, \ldots, n_p$ and $\rho_1, \rho_2 \in (0, 1)$.

Within the interval $e_r(k) \in \{-E_M, E_M\}$, the ONFC controller is stable according to the following Theorem:

Theorem 1: Consider the system (1) for which assumptions (A1) to (A4) hold, with output feedback defined by the control law (7), (8), (9), (10), and the parameter adaptation law given by (3), (5), (11), (12), (13), (14). Suppose that there exist a symmetric matrix P and three scalars $\rho_1, \rho_2 \in (0, 1)$ and $\alpha \in (0, 2)$ such that the following Linear Matrix Inequalities hold:

$$\begin{aligned} &P > 0 \\ &M_i < 0 \quad \forall\, i \in \{1, \ldots, n_p\} \end{aligned} \tag{17}$$

with the matrices M_i defined by (15) and (16). Then the closed-loop system is stable for that value of α and for any instance of the plant within the uncertainty polytope (2), and $|e_y(k)|$ converges to the interval $(0, \xi_0)$ in finite time. □

The proof of this theorem is presented in Section III.

III. PROOF OF STABILITY

To perform the stability analysis by the Lyapunov method, it is necessary to obtain the model of the augmented error of the controlled system. The output error $e_y(k)$ defined by (5) may be rewritten as:

$$e_y(k) = Ce_x(k), \tag{18}$$

where:

$$e_x(k) = x(k) - x_m(k) \tag{19}$$

is the error between the system and the reference model state variables.

Consider (19) on time $k+1$, the term $A_m x(k)$ is added and subtracted, leading to:

$$e_x(k+1) = x(k+1) + A_m x(k) - A_m x(k) - x_m(k+1) \tag{20}$$

Substituting (1) and (3) in (20) and considering (11), it follows that:

$$\begin{cases} e_x(k+1) = A_m e_x(k) + (A - A_m)x(k) \\ \qquad\qquad -B_m r(k) + Bu(k) \\ w_1(k+1) = w_1(k) - \gamma\, C\, e_x(k+1)\mu_1(k) \\ w_2(k+1) = w_2(k) - \gamma\, C\, e_x(k+1)(1 - \mu_1(k)) \end{cases} \tag{21}$$

This model describes the dynamics of closed-loop system error relative to the reference model (3), considering the variables $r(k)$, $u(k)$, $x(k)$ and $\mu_1(k)$ as exogenous inputs. Any point of equilibrium of the system (21), defined by:

$$\begin{bmatrix} e_x(k+1) \\ w_1(k+1) \\ w_2(k+1) \end{bmatrix} = \begin{bmatrix} e_x(k) \\ w_1(k) \\ w_2(k) \end{bmatrix} = \begin{bmatrix} e_x^* \\ w_1^* \\ w_2^* \end{bmatrix} \tag{22}$$

must satisfy:

$$|Ce_x^*| \leq \xi_0 \tag{23}$$

Condition (23) is necessary for the equilibrium because, according to (21), $w_1(k+1) = w_1(k)$ and $w_2(k+1) = w_2(k)$ either when $\gamma = 0$, which occurs when $|Ce_x^*| \leq \xi_0$, see (13), or when $Ce_x(k+1) = 0$, which is also included in (23). Notice that the possible values of w_1^* and w_2^* are not unique.

It should be noticed that (1) (with $\xi(k) = \upsilon(k) = 0$) implies that for any fixed $u(k) = u^*$, there will be a fixed point $x(k) = x^*$ and (3) implies that for any fixed value $r(k) = r^*$ there will be a fixed point $x_m(k) = x_m^*$. This means that, given u^* and r^*, there will be an $e_x(k) = e_x^*$ that represents a fixed point of the first equation of (21). The issue to be shown in this proof is the stability of the fixed point indicated in (22).

Define the matrices $\Gamma_A(k) \in \mathbb{R}^{n\times n}$ and $\Gamma_B(k) \in \mathbb{R}^{n\times 1}$ such that the closed-loop system is represented by:

$$\begin{aligned} x(k+1) &= (A_m + \Gamma_A(k))x(k) + (B_m - \Gamma_B(k))r(k) + \xi(k), \\ y(k) &= Cx(k) + \upsilon(k) \end{aligned} \tag{24}$$

Under this definition, the part of the system dynamics which represents the difference from the system to the reference model can be represented by a new variable $\nu(k)$ which includes the external disturbance:

$$\nu(k) = \Gamma_A(k)x(k) + \Gamma_B(k)r(k) + \xi(k) \tag{25}$$

In an equilibrium point, the following relations hold:

$$\begin{cases} A_m + \Gamma_A^* = A + B\dfrac{(w_2^* - w_1^*)}{2E_M}C \\ B_m - \Gamma_B^* = B\left[\dfrac{(w_2^* + w_1^*)}{2r^*} - \dfrac{(w_2^* - w_1^*)}{2E_M}\right] \end{cases} \tag{26}$$

in which Γ_A^* and Γ_B^* correspond to the equilibrium of $\Gamma_A(k)$ and $\Gamma_B(k)$.

In order to verify the stability of the fixed point, the point (22) is subtracted from both sides of system (21) and the relationship (26) is assumed. Defining $\tilde{w}_j = w_j - w_j^*$, and $\tilde{u} = u - u^*$, the following model is obtained:

$$\begin{cases} e_x(k+1) = A_m e_x(k) + B\tilde{u}(k) + \Gamma_A(k)x(k) \\ \qquad\qquad +\Gamma_B(k)r(k) \\ \tilde{w}_1(k+1) = \tilde{w}_1(k) - \gamma e_y(k+1)\mu_1(k) \\ \tilde{w}_2(k+1) = \tilde{w}_2(k) - \gamma e_y(k+1)\mu_2(k) \end{cases} \tag{27}$$

in which:

$$\tilde{u}(k) = \begin{cases} \tilde{w}_1 - \dfrac{(w_2^* - w_1^*)}{2E_M}(e_r + E_M), & \text{if } e_r \le -E_M \\ \mu_1(k)\tilde{w}_1 + \mu_2(k)\tilde{w}_2, & \text{if } -E_M < e_r < E_M \\ \tilde{w}_2 - \dfrac{(w_2^* - w_1^*)}{2E_M}(e_r - E_M), & \text{if } e_r \ge E_M \end{cases} \tag{28}$$

Equation (27) represents the dynamics of the state error e_x relative to the equilibrium point. Since w_j^* is a constant, then $\tilde{w}_j(k+1) - \tilde{w}_j(k) = w_j(k+1) - w_j(k)$.

To evaluate the stability of closed-loop system (1), consider the complete dynamic model of the error e_x:

$$e_x(k+1) = A_m e_x(k) + B\tilde{u} + \nu(k), \tag{29}$$

where $\nu(k)$ is given by (25), and the following candidate Lyapunov function:

$$V(k) = \rho_1 e_x^T(k)Pe_x(k) + \frac{\rho_2\tilde{w}_1^2(k)}{\gamma} + \frac{\rho_2\tilde{w}_2^2(k)}{\gamma} \tag{30}$$

where $P = P^T > 0$, $\gamma > 0$, and $\rho_1, \rho_2 \in (0, 1)$.

The system will be stable if, for all $k > 0$:

$$\Delta V = V(k+1) - V(k) < 0 \tag{31}$$

Define:

$$\Delta V = \Delta V_e(k) + \Delta V_w(k) \tag{32}$$

where:

$$\begin{aligned} \frac{\Delta V_e(k)}{\rho_1} &= e_x^T(k+1)Pe_x(k+1) - e_x^T(k)Pe_x(k) \\ &= \left[A_m e_x(k) + B\tilde{u}(k) + \nu(k)\right]^T P[A_m e_x(k) \\ &\quad + B\tilde{u}(k) + \nu(k)] - e_x^T(k)Pe_x(k) \end{aligned} \tag{33}$$

and, consider the interval $e_r(k) \in \{-E_M, E_M\}$ where $\tilde{u}(k) = \mu_1(k)\tilde{w}_1(k) + \mu_2(k)\tilde{w}_2(k)$:

$$\begin{aligned} \frac{\Delta V_w(k)}{\rho_2} &= \frac{\tilde{w}_1^2(k+1)}{\gamma} + \frac{\tilde{w}_2^2(k+1)}{\gamma} - \frac{\tilde{w}_1^2(k)}{\gamma} - \frac{\tilde{w}_2^2(k)}{\gamma} \\ &= -2e_y(k+1)[\mu_1(k)\tilde{w}_1(k) + \mu_2(k)\tilde{w}_2(k)] \\ &\quad +\gamma e_y^2(k+1)[\mu_1^2(k) + \mu_2^2(k)] \\ &\le -2[CA_m e_x(k) + B\tilde{u}(k) + \nu(k)) + \upsilon(k+1)]\tilde{u}(k) \\ &\quad + \frac{\alpha}{CB}\{C[A_m e_x(k) + B\tilde{u}(k) + \nu(k)] + \upsilon(k+1)\}^2 \end{aligned} \tag{34}$$

where α is a constant that sets the velocity of adaptation according to (13).

Applying in (33) the relationship:

$$X^TY + Y^TX \le aX^TX + \frac{Y^TY}{a}, \tag{35}$$

$\forall a > 0$, it comes that:

$$\begin{aligned} &\left[A_m e_x(k) + B\tilde{u}(k)\right]^T P[\nu(k)] + [\nu(k)]^T P[A_m e_x(k) + B\tilde{u}(k)] \\ &\quad + [\nu(k)]^T P[\nu(k)] \le \frac{1}{1-\rho_1}[\nu(k)]^T P[\nu(k)] \\ &\quad + \frac{1-\rho_1}{\rho_1}\left[A_m e_x(k) + B\tilde{u}(k)\right]^T P\left[A_m e_x(k) + B\tilde{u}(k)\right] \end{aligned} \tag{36}$$

with $0 < \rho_1 < 1$, then

$$\begin{aligned} \Delta V_e(k) \le &\begin{bmatrix} e_x(k) \\ \tilde{u}(k) \end{bmatrix}^T \begin{bmatrix} A_m^TPA_m - \rho_1 P & A_m^TPB \\ B^TPA_m & B^TPB \end{bmatrix} \begin{bmatrix} e_x(k) \\ \tilde{u}(k) \end{bmatrix} \\ &+ \frac{\rho_1}{1-\rho_1}[\nu(k)]^T P[\nu(k)] \end{aligned} \tag{37}$$

Applying the relationship (35) in (34), it follows that

$$\begin{aligned} \frac{\Delta V_w(k)}{\rho_2} &\le -2C[A_m e_x(k) + B\tilde{u}(k)]\tilde{u}(k) \\ &\quad + \frac{\alpha}{\rho_2 CB}\{C[A_m e_x(k) + B\tilde{u}(k)]\}^2 + \frac{1-\rho_2}{\rho_2}\tilde{u}(k)^2 \\ &\quad + \left(\frac{\alpha}{CB} + \frac{2\rho_2}{1-\rho_2}\right) \\ &\quad \times [C\nu(k) + \upsilon(k+1)]^T[C\nu(k) + \upsilon(k+1)] \end{aligned} \tag{38}$$

with $0 < \rho_2 < 1$, then

$$\begin{aligned} \Delta V_w \le &\begin{bmatrix} e_x(k) \\ \tilde{u}(k) \end{bmatrix}^T \begin{bmatrix} \frac{\alpha}{CB}A_m^TC^TCA_m & (\alpha-\rho_2)A_m^TC^T \\ (\alpha-\rho_2)CA_m & 1-\rho_2+[\alpha-2\rho_2]CB \end{bmatrix} \begin{bmatrix} e_x(k) \\ \tilde{u}(k) \end{bmatrix} \\ &+ \left(\frac{\alpha\rho_2}{CB} + \frac{2\rho_2^2}{1-\rho_2}\right)[C\nu(k) \\ &+ \upsilon(k+1)]^T[C\nu(k) + \upsilon(k+1)] \end{aligned} \tag{39}$$

Thus, consider (32), (37), and (39):

$$\Delta V \leq \begin{bmatrix} e_x(k) \\ \tilde{u}(k) \end{bmatrix}^T M \begin{bmatrix} e_x(k) \\ \tilde{u}(k) \end{bmatrix} + \frac{\rho_1}{1-\rho_1}[\nu(k)]^T P[\nu(k)] + [C\nu(k) + \upsilon(k+1)]^T W[C\nu(k) + \upsilon(k+1)] \tag{40}$$

where

$$M = \sum_{i=1}^{n_p} \beta_i M_i, \tag{41}$$

and

$$W = \sum_{i=1}^{n_p} \beta_i W_i, \tag{42}$$

with β_i given by (2), the matrix M_i defined by (15) and the scalar W_i defined by:

$$W_i = \frac{\alpha\rho_2}{CB_i} + \frac{2\rho_2^2}{1-\rho_2} \tag{43}$$

for $i = 1, \ldots, n_p$.

The analysis of (43) shows that $W_i > 0$ for $i = 1, \ldots, n_p$ and therefore, the last terms of (40) are positive definite. However, additional conditions can guarantee stability even in the presence of disturbance or uncertainty.

Consider a strictly positive function $f(e_x(k), \tilde{u}(k))$ such that:

$$f(e_x(k), \tilde{u}(k)) < e_x(k)^T e_x(k) + \tilde{u}(k)^T \tilde{u}(k) \tag{44}$$

and an unknown constant ν_0 such that:

$$\nu_0 > \frac{\rho_1}{1-\rho_1} \frac{\lambda_{max}(P)}{\|\lambda_{max}(M)\|} max(\nu^T(k)\nu(k)) + \frac{max(W)}{\|\lambda_{max}(M)\|} max([C\nu(k) + \upsilon(k+1)]^T [C\nu(k) + \upsilon(k+1)]) \tag{45}$$

with $\lambda_{max}(P)$ and $\lambda_{max}(M)$ the largest eigenvalues of all P and M, respectively, and $max(W)$ the largest W_i for $i = 1, \ldots, n_p$.

Assuming that M is negative definite, it comes that:

$$\begin{bmatrix} e_x(k) \\ \tilde{u}(k) \end{bmatrix}^T M \begin{bmatrix} e_x(k) \\ \tilde{u}(k) \end{bmatrix} < -\|\lambda_{max}(M)\|[e_x(k)^T e_x(k) + \tilde{u}(k)^T \tilde{u}(k)] < -\|\lambda_{max}(M)\| f[e_x(k), \tilde{u}(k)] \tag{46}$$

where $\lambda_{max}(M)$ is the largest eigenvalue of M. Then, from (45), it follows that:

$$\nu_0\|\lambda_{max}(M)\| > \frac{\rho_1}{1-\rho_1}\nu^T(k)P\nu(k) + W[C\nu(k) + \upsilon(k+1)]^T \times [C\nu(k) + \upsilon(k+1)] \tag{47}$$

Therefore, from (40), (46), and (47):

$$\Delta V < \|\lambda_{max}(M)\|\{-f[e_x(k), \tilde{u}(k)] + \nu_0(k)\} \tag{48}$$

Thus, a sufficient condition to ensure that $\Delta V(k) < 0$ is:

$$f[e_x(k), \tilde{u}(k)] > \nu_0(k) \tag{49}$$

This shows that the signals $e_x^T e_x$ and $\tilde{u}^T \tilde{u}$ are bounded and converge to a closed set.

We can even observe the following. Consider an f for which the condition (44) assumes the form:

$$f[e_x(k), \tilde{u}(k)] = \tilde{u}^2(k) \leq e_x(k)^T e_x(k) + \tilde{u}(k)^T \tilde{u}(k) \tag{50}$$

Due to the implicit integration involved in the definition of $w_1(k)$ and $w_2(k)$, in the adaptation law (11), $\tilde{u}(k)$ is dependent on the sum of e_y and $f(e_x, \tilde{u})$ just stops increasing when $e_y(k) < \xi_0$. Therefore, the rule (13) ensures that, when $f > \nu_0$, the norm of the model output error converges to ξ_0 in order to make $\tilde{u}(k)$ bounded.

In spite of this, in the presence of measurement noise, the weights w_1 and w_2 could increase indefinitely if the adaptation law remains active for $e_y(k) < \xi_0$, which could lead those parameters to drift [17]. In this way, the parameters w_1 and w_2 do not change when $|e_y(k)| < \xi_0$, and start to adapt when $|e_y(k)|$ exceeds ξ_0. The adaptation rule (13), known as the Dead Zone Modification technique, guarantees the boundedness of the weights under measurement noise. □

The values of ρ_1 and ρ_2 can be adjusted to minimize the eigenvalues of M_i, and thus the system requirements. However, this may increase the eigenvalues of W_i, the norm of ν_0 and the maximum absolute value of e_x. In the design stage, ρ_1 and ρ_2 are chosen arbitrarily close to 1.0 when the matching condition is satisfied or when only the output error needs to be minimized, without the need of guaranteeing the convergence of the closed-loop dynamics to the reference model.

IV. CONTROLLER DESIGN PROCEDURE

The design procedure for the ONFC controller may be stated as follows:

1) Choose the reference model matrices A_m and B_m;
2) Choose $E_M \simeq max\|y(k) - r(k)\|$, $\rho_1, \rho_2 \in (0, 1)$, and ξ_0 according to (14);
3) Consider matrix M given by (42). Choose α such that the following LMIs are satisfied:

$$\begin{cases} lP > 0 \\ M_i < 0 \ , \ \ \forall\, i = 1, \ldots, n_p \end{cases}$$

The matrix definitions are repeated here for convenience:

$$M_i = \begin{bmatrix} Q_i & S_i^T \\ S_i & R_i \end{bmatrix}$$
$$Q_i = A_m^T[P + \alpha C^T C/\Lambda]A_m - \rho_1 P$$
$$S_i = A_m^T P B_i - A_m^T C^T(\rho_2 - \alpha)$$
$$R_i = B_i^T P B_i - CB_i(2\rho_2 - \alpha) + 1 - \rho_2$$

4) The controller implementation follows (8), (9), (10) and (11), with γ given by (13). Those formulas are repeated here for convenience:

The reference model:

$$x_m(k+1) = A_m x_m(k) + B_m r(k)$$
$$y_m(k) = C x_m(k)$$

The error signals:

$$e_r(k) = r(k) - y(k)$$
$$e_y(k) = y(k) - y_m(k)$$

The adaptation rule:

$$\gamma = \begin{cases} \dfrac{\alpha}{\Lambda[\mu_1^2(k-1)+\mu_2^2(k-1)]}, & \text{if } |e_y(k)| \geq \xi_0 \\ 0, & \text{if } |e_y(k)| \leq \xi_0 \end{cases}$$

$$w_1(k) = w_1(k-1) - \gamma e_y(k)\mu_1(k-1)$$

$$w_2(k) = w_2(k-1) - \gamma e_y(k)\mu_2(k-1)$$

The control signal computation:

$$\mu_1(k) = \begin{cases} 1, & \text{if } e_r(k) \leq -E_M \\ \dfrac{E_M - e_r(k)}{2E_M}, & \text{if } -E_M < e_r(k) < E_M \\ 0, & \text{if } e_r(k) \geq E_M \end{cases}$$
$$\mu_2(k) = 1 - \mu_1(k)$$
$$u(k) = \mu_1(k)w_1 + \mu_2(k)w_2$$

V. NUMERICAL RESULTS

In this section, some simulation results are presented in order to evaluate the performance of ONFC controller.

A. UNCERTAIN PLANT

The first simulations consider the following uncertain discrete-time system subject to measurement noise:

$$x(k+1) = \begin{bmatrix} 0.88 & -0.25 & -0.10 \\ 0.06 & 0.39 & -0.09 \\ -0.49 & -0.12 & 0.69 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ b_1 \\ b_2 \end{bmatrix} u(k)$$
$$y(k) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x(k) + \upsilon(k)$$

where $\upsilon(k)$ is a Gaussian measurement noise with variance $\sigma^2 = 0.1$ and the uncertain matrix B is such that $b_1 \in [0.8, 1]$ and $b_2 \in [0.6, 0.8]$. The chosen reference model has the form:

$$x_m(k+1) = \begin{bmatrix} \lambda_{Am} & 0 & 0 \\ 0 & \lambda_{Am} & 0 \\ 0 & 0 & \lambda_{Am} \end{bmatrix} x_m(k) + \begin{bmatrix} 0 \\ 0 \\ b_m \end{bmatrix} r(k)$$
$$y_m(k) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x_m(k)$$

where $|\lambda_{Am}| \in (-1, 1)$ and b_m is defined in such a way that the gain of the reference model is unitary.

In order to illustrate the feasibility of the LMI problem (17), 10,000 combinations of $\lambda_{Am} \in [-1.0, 1.0]$ and $\alpha \in [0, 4.0]$ are evaluated, with the antecedent parameter $E_M = 15$. To compare, the LMI problem (17) is solved employing the same previous parameters and considering two situations: a) $\rho_1 = \rho_2 = 0.98$ and b) $\rho_1 = \rho_2 = 0.75$. The Fig. 3 shows the region of the $\lambda_{Am} \times \alpha$ plane that exhibits feasibility of the inequalities $M < 0$ and $P = P^T > 0$ over the whole B polytope.

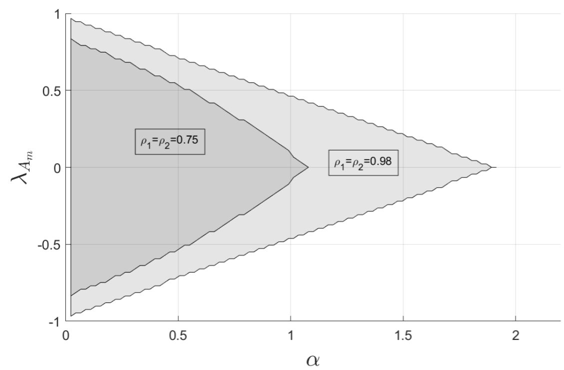

FIGURE 3. Region of feasibility of the LMI problem of Theorem 1 for the example system $\rho_1 = \rho_2 = 0.98$ (larger area) and $\rho_1 = \rho_2 = 0.75$ (smaller area). If the combination of α and λ_{Am} is within the highlighted areas, the closed-loop stability is guaranteed.

As expected, the LMI feasibility requires $0 < \alpha < 2$.

The control law (10) is implemented using the dead zone technique (13) with the parameters $\alpha = 0.4$, $E_M = 15$, and $\xi_0 = 0.8$. The initial conditions are defined as $w_1(0) = w_2(0) = 1.0$, $x(0) = [0, 0, 0]^T$ and the parameters of the reference model are chosen as $\lambda_{A_m} = 0.5$ and $b_m = 0.5$.

Note that in this case it is not possible to meet the matching condition. The signals $y(k)$ and $r(k)$ are shown in Fig. 4.*a*. It can be observed that the controller is able to conduct the plant response to the reference model, despite the presence of noise. Fig. 4.*b* shows the evolution of the consequents ω_1 and ω_2 over time. It is noticed the robustness of the adaptation algorithm concerning the presence of measurement noise and parametric uncertainty, preventing the consequent weights to grow indefinitely. Adaptation Law (13) turns off the variation of w_1 and w_2 when the system is in a steady state, to avoid the drif of the consequents due to noise.

The initial conditions $w_1(0)$ and $w_2(0)$ affect the performance and the final values of the consequents ($w_1(\infty)$ and $w_2(\infty)$). To show this phenomenon, some simulations are performed for different initial conditions of w_1 and w_2 close to the curve defined by (51):

$$w_2^* + w_1^* = \frac{2\,r^*}{C(I-A)^{-1}B} \tag{51}$$

This curve defines a constraint for the possible values of the fixed points w_1^* and w_2^*.

The design parameters, in all cases, are $\lambda_{Am} = 0.5$, $E_M = 15$, and $\alpha = 0.6$. The evolution of the consequents for those different initial conditions is shown in Fig. 5.

The signals w_1 and w_2 converge to an equilibrium curve. Each pair of initial condition values evolves differently and is attracted to a different point on the curve. For unstable systems, it is even possible that initial conditions excessively far

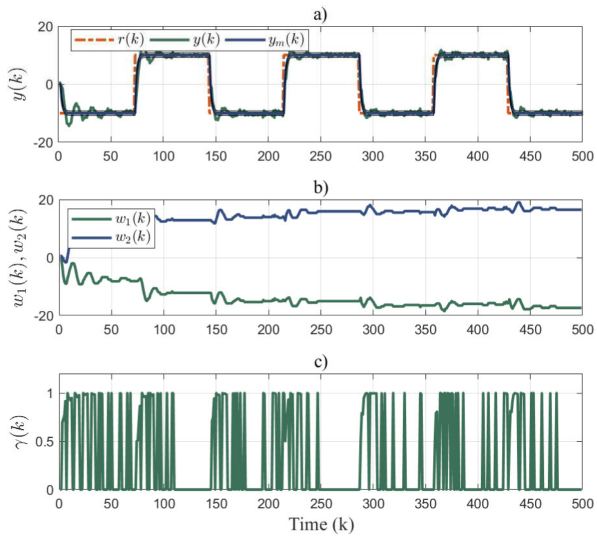

FIGURE 4. **Simulation of a system with noise disturbance. (a) Output from the controlled system. (b) Variation of consequents. (c) Dynamic adaptation rate of parameters w_1 and w_2.**

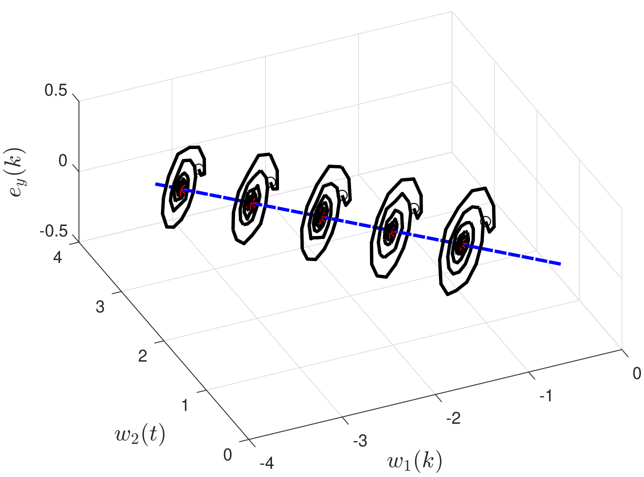

FIGURE 5. **Evolution of variables of a stable system. (o) Initial Condition, (*) Equilibrium, (-) Trajectory, (- -) Curve** (51)**.**

from the equilibrium curve lead the error to diverge before the system properly adapts the values of w_1 and w_2. Furthermore, when $e_r \gg E_M$ the controller loses the guarantees of stability and the error may diverge.

B. TIME-VARYING PLANT

Another test to evaluate the ability to adapt to a time-varying plant is implemented as follows. Consider the linear system with a time-varying parameter:

$$x(k+1) = \begin{bmatrix} 0.5 & 0.1 \\ 0.4 & a(k) \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$
$$y(k) = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} x(k) + \upsilon(k)$$

where $a(k)$ is an entry of A matrix whose time variation is shown by Fig. 6.*c* and $\upsilon(k)$ is a disturbance with variance

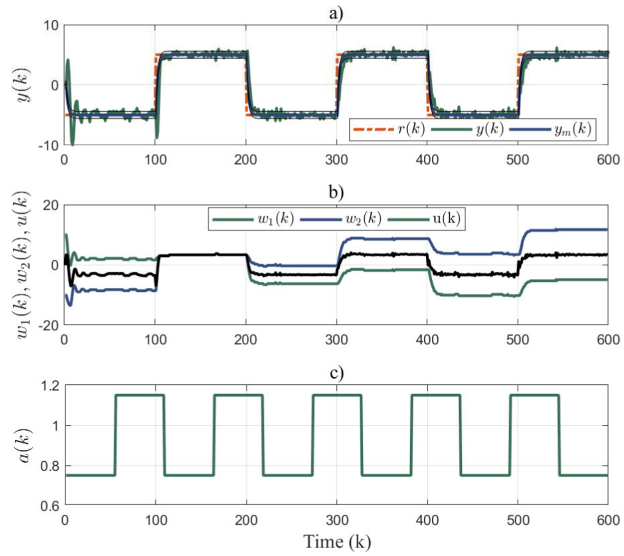

FIGURE 6. **Control of a system with parametric variation. *a*) Output signals of the reference and system models. *b*) Evolution of consequents. c)Variation of system parameter.**

$\sigma^2 = 0.1$. The design parameters are chosen as $\alpha = 0.7$ and $E_M = 5$ and the initial conditions are defined as $w_1(0) = w_2(0) = 0$ and $x(0) = [0, 0, 0]^T$. Over time, $a(k)$ makes the open-loop system to switch between a stable and an unstable format. In this example, the following reference model is used:

$$x_m(k+1) = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix} x_m(k) + \begin{bmatrix} 0 \\ 0.6 \end{bmatrix} r(k)$$
$$y_m(k) = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} x_m(k)$$

The signals of the controlled system are shown in Fig. 6.*a* and 6.*b*. The system output $y(k)$ remains close to the reference signal $y_m(k)$ even under abrupt changes in parameter $a(k)$.

VI. CONCLUSION

Adaptive controllers are especially useful for solving control problems involving nonlinearities, uncertainties, and parametric variation. One of the main challenges in their design is the issue of robust stability, as the interdependence between parameters and states results in a time-varying nonlinear system. This task becomes even more complicated when using the so-called intelligent adaptive controllers, due to their complex structures.

This paper proposes an extended version of the ONFC controller for SISO discrete-time linear systems, which can be seen as a direct adaptive reference model control, implemented as a zero-order Takagi-Sugeno system. This controller includes a reference model which can be used for the specification of the closed-loop dynamics. LMI conditions for the stability of the closed-loop system are derived, allowing the assessment of robust stability for plant models with polytope-bounded uncertainties.

The most important advantages of the ONFC controller are the simplicity of design and low computational cost. Those advantages still hold for the proposed extended ONFC controller, since no fine-tuning is needed for the design of control parameters. The control design procedure is rather intuitive, being performed on a trade-off between the adaptation rate and the size of the stable bounded convergence region.

Future works by the authors include: the examination of the sigma modification (a smooth rule) for replacing the dead-zone in ONFC, the usage of predictors within ONFC for dealing with dead time processes, the extension of ONFC for MIMO linear systems, and the evaluation of the computational resource requirements (memory and processing time) of ONFC in comparison with other recent adaptive control methods.

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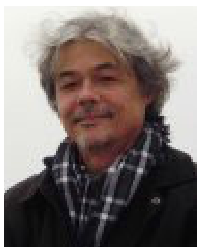

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