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Analytical Modeling of Switching Fabrics of Elastic Optical Networks

MARIUSZ GŁĄBOWSKI[1], (Senior Member, IEEE), ERICH LEITGEB[2], (Member, IEEE), MACIEJ SOBIERAJ[1], AND MACIEJ STASIAK[1], (Member, IEEE)
[1]Institute of Communications and Computer Networks, Poznan University of Technology, 60-965 Poznan, Poland
[2]Institute of Microwave and Photonic Engineering, Graz University of Technology, 8010 Graz, Austria

Corresponding author: Maciej Sobieraj (maciej.sobieraj@put.poznan.pl)

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ABSTRACT This article proposes an analytical method for determining the traffic characteristics of multi-service switching networks in elastic optical networks. The method takes into consideration the limitations of W-S-W (wavelength-space-wavelength) switching networks in the execution of multi-slot connections in line with the concept of elastic optical networks. The results obtained on the basis of the developed analytical model are compared with the results obtained from the simulation experiments for a selected structure of a switching network. The study confirms the high accuracy of the proposed model. The model can be used to optimize and evaluate the effectiveness of blocking optical switching networks.

INDEX TERMS Elastic optical networks, energy-aware systems, multi-service traffic, optical switches, switching network, traffic engineering.

I. INTRODUCTION

In recent years, optical networks, both access and backbone networks, have become the basic solutions offering high-speed (communication) bitrates, appropriate resource allocation mechanisms, and a variety of other services. At present, along with the development of the DWDM (dense wavelength division multiplexing) technology, optical telecommunications networks of the next generation can provide transmission speed of 100 Gbps, 400 Gbps, or even 1 Tbps for a single optical path. A possibility of effectively using such high bitrates of optical paths to service users with different demands, far lower than the network capacity, is provided by the concept of elastic optical networks (EONs) [1].

To use the resources of EONs effectively, it is necessary to make use of optical switching networks that are capable of converting and transmitting optical signals in different wavelengths of light. Research and studies on optical switching networks focus mainly on the development of new non-blocking [1]–[5] and rearrangeable [6], [7] optical structures for switching networks. The construction of such systems, however, involves a huge number of switching points or complex control algorithms for setting up connections. This leaves us with a need to develop an effective solution based on a construction of blocking switching networks in which, depending on the intensity of offered traffic, blocking states can occur. To evaluate the effectiveness and efficiency of these structures, it is necessary to develop first their analytical or simulation models. However, the problem of the analytical modeling of blocking switching networks that take into consideration the properties of EONs has as yet not been sufficiently and properly addressed in the literature. This article aims to propose a new analytical model of multi-service multi-stage blocking switching networks, used in the nodes of EONs, that would make it possible to determine the influence of the traffic characteristics of these networks on the traffic effectiveness of network nodes.

The remaining part of the article is structured as follows. In Section II the current state of research on modeling optical switching networks is presented. Section III introduces EONs. It also presents the structures of offered traffic, a description of switching networks and their structures, and algorithms for the choice of connecting paths in these networks. Section IV presents the proposed analytical model of the considered switching network that services Erlang, Engset, and Pascal traffic streams. The methods for determining the point-to-group and point-to-point blocking probabilities are presented in Section V. The results of the

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analytical calculations are compared with simulation data in Section VI. Section VII sums up the article.

II. RELATED WORK

Research on optical switching networks (fabrics), including networks that take into consideration the properties of elastic networks, deal mainly with the determination of non-blocking or rearrangeable conditions in these networks. These conditions are determined depending on the capacity of inter-stage links expressed in frequency slot units (FSUs) [1]–[3], [5]–[7]. In addition, the influence of the number of switches in individual stages on non-blocking [1]–[3], [5] or rearrangeable [6], [7] conditions in these networks is investigated. Several studies have proposed new non-blocking structures of switching networks, in which non-blocking conditions are chosen in such a way as to decrease the production costs of a given network as much as possible. These studies have shown that the number of active elements should be kept as low as possible, while non-blocking conditions should be maintained [4], [8].

Numerous studies are devoted to the application of EONs in data centers [9]–[13]. A data center interconnections with EONs is a promising scenario to meet the high burstiness and high-bandwidth requirements of data center services [9]–[11]. In [10], the challenges of the time-aware data center services regarding the scheduling of elastic service time and service bandwidth according to time sensitivity are presented. A novel multi-stratum resources integration (MSRI) architecture based on network function virtualization in software-defined elastic data center optical interconnections is described in [11]. References [9]–[11] provide analyses of the effectiveness of all the connection networks between the nodes of data centers.

In EONs, the modulation technique is adapted to the distance of an optical path. A robust modulation technique with a large number of spectrum slots is used for long-distance optical paths, and a less robust modulation technique with a small number of spectrum slots is used for short-distance optical paths. In [14], the authors present a model that reduces the number of utilized spectrum slots compared with the conventional schemes. In [15], the authors have developed a flexible converter between modulation formats, which enables intelligent grooming techniques for an EON aimed at dynamical spectral efficiency allocation in optical fiber transmission.

The flexible nature of EONs effectively uses spectral resources for optical communication by allocating the minimum required bandwidth to network connections. Since the energy consumption of such networks scales with the magnitude of the bandwidth demand, addressing the issue of energy wastage is important. This fact has a profound impact on the design of efficient schemes for energy-aware optical networks, and adaptivity becomes one of the most important properties of these networks [16]–[20].

Another research area that has not yet been addressed in the literature is the determination of traffic characteristics of blocking optical switching networks to which a mixture of traffic streams with different characteristics (the size of demanded resources, the intensity of demand arrival) is offered. The first simulation studies on switching networks with a Clos structure[1] used in optical switching networks [21], [22] are discussed in [23]. A purpose-made simulator has the advantage of determining loss probabilities for individual call classes, depending on the structure of the network (the number of switches, the number of inputs/outputs, the capacity of the links) as well as the traffic structure offered to a switching network.

The standard application of three-stage Clos switching networks in optical networks prompted us to pursue studies aimed at the development of analytical models of such networks. In addition, the fact that the resources demanded by individual (particular) traffic streams can be expressed as the number of FSUs provides a good starting point for developing analytical models of switching networks in EONs, and the knowledge related to modeling electronic switching networks [24] can be used. In particular, assuming that an FSU in optical switching networks corresponds to an allocation unit (basis bandwidth unit) in electronic switching networks [24], [25], the modeling process can be based on the concept of so-called effective availability.

The first models of switching networks, based on the concept of effective availability, are proposed for single-service, two-stage switching networks in [26], [27]. In a number of studies, including [28]–[33], these models have been expanded to include switching networks with any number of stages. An appropriately modified concept of the effective availability is also used in modeling multi-service switching networks (MSNs) that service multi-service traffic streams. In [34], models of MSNs with point-to-group selection are presented. Then, [35] discusses a model of MSNs with point-to-group selection and a number of attempts to set up a connection, while [36] discusses a model of MSNs with point-to-point selection. The literature also offers examples of more complex models of MSNs that take into account, for example, Erlang, Engset, and Pascal traffic streams [37], as well as call admission control mechanisms [38], [39] or overflow links [40]–[42].

The literature offers more complex MSN models that are also based on the concept of effective availability. These models consider the possibility of supporting multi-stream traffic composed of Erlang, Engset, and Pascal traffic streams [37]. In [43] and [44], a recurrent approach to modeling MSNs by the effective availability method is proposed. Reference [38] develops models of MSNs with dynamic resource reservation for all, or selected, traffic classes. In [45], a method for modeling switching networks that have the advantage of servicing

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[1] A Clos structure (three-stage, five-stage, or seven-stage) provides the possibility of constructing a switching fabric with several dozen/hundred inputs/outputs using single switches with several inputs/outputs. A Clos structure also provides a multi-path architecture between a given input and output of a switching fabric, resulting in a situation in which switching off a certain number of elements in this structure does not lead to the total inability to set up a call but only increases the loads in the remaining modules.

multi-cast traffic is derived. Reference [39] proposes MSN models with threshold mechanisms for traffic generated by the so-called multi-service sources. Finally, [40]–[42] consider MSN models with so-called overflow links introduced to their structure.

All analytical models of multi-stage switching networks known from the literature and based on the concept of effective availability were developed at the time for electronic switching. These models did not take into consideration the limitations imposed by optical switching and, in particular, the limitations related to the conversion of wavelengths in individual switches of multi-stage switching networks and the execution of optical multi-slot connections (i.e., those that demand more than one FSU to set up a connection).

The present article discusses a new analytical model of a W-S-W (wavelength–space–wavelength) switching network that services multi-service traffic streams generated according to Erlang, Engset, and Pascal distributions. This model takes into account the availability of different types of switching in particular stages of the switching network supporting EONs. In the case of the switches of the external stage, the switching takes place both in the space domain (switching between input and output links of a switch) and in the frequency domain (change of wavelength). In the case of the switches of the middle stage, the assumption is that they execute space switching only.

III. SWITCHING NETWORKS IN ELASTIC OPTICAL NETWORKS

A. ELASTIC OPTICAL NETWORKS

In general, the DWDM technology is based mainly on C- and L-optical transmission bands because of the low optical signal attenuation in glass fibers, in which the C-band of the optical spectrum is from 1530 to 1565 nm. The requirements related to the DWDM frequency GRID are provided in the ITU-T G.694.1 standard [46]. Regarding the flexible grid, the smallest possible channel spacing is 12.5 GHz, and the maximum allowed number of separated optical channels in the C-band is 320. The flexible grid architecture combines different channel spacings in a single optical fiber. The main advantage is the implementation of transmission systems with different bitrates or modulation formats [47].

In line with the operation of the elastic frequency grid, standardized by the ITU-T [46], channel bandwidths are allocated according to the nominal central frequency f_{nom} and the channel width ω. The channel width ω is given as a multiplication factor of 12.5 GHz:

$$\omega = 12.5 \times m, \qquad (1)$$

where m is a positive integer number and 12.5 GHz is the FSU [2]. The central frequency of individual channels (slots) is defined on the basis of the following formula:

$$f_{\mathrm{nom}} = 193.1 + n \times 0.00625, \qquad (2)$$

where n is an integer number and 0.00625 is the fixed frequency shift expressed in THz. Each combination of channels (regarding their widths) is possible, provided they do not interfere with or overlap one another. The advantage of the use of the elastic frequency grid is that it provides the possibility of applying different transmission rates.

B. ARCHITECTURE OF THE SWITCHING NETWORK

Fig. 1 shows a 3-stage W-S-W switching network. The network is composed of $\upsilon \times \upsilon$ square switches. Each link of the switching network has a capacity equal to f FSUs. Additionally, one link of each of the switches of the last stage belongs to one of the υ directions.

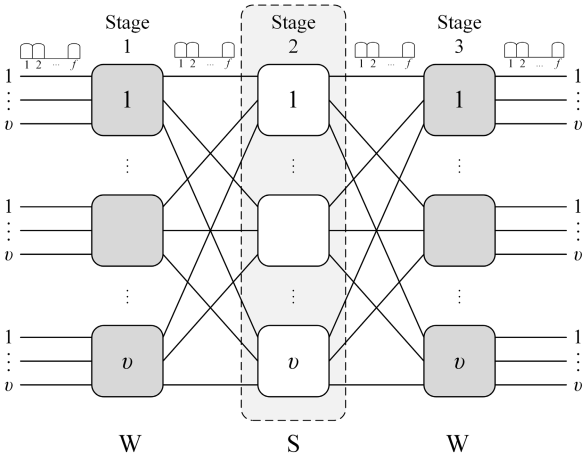


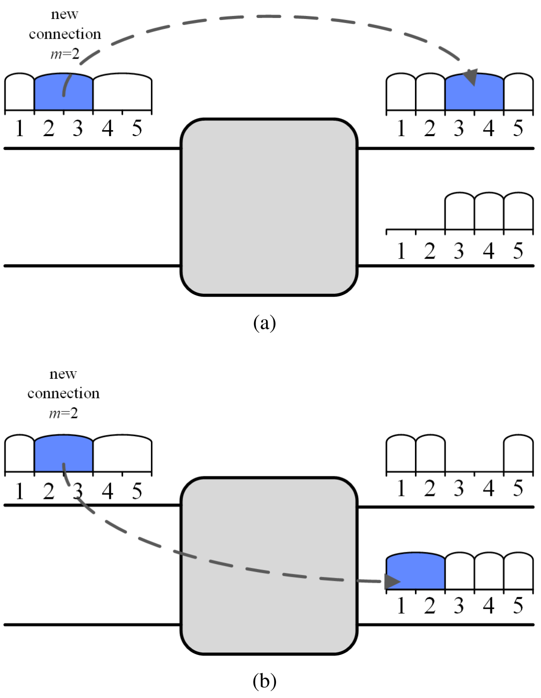
FIGURE 1. A 3-stage optical switching network.

The switches of the first and the third stage allow both the frequency slot (wavelength) and the output of a switch (optical fiber) to be changed. Fig. 2 shows an example of an operation mode of a switch of the first or the third stage. For a given connection that is being executed in a given channel, such a switch provides a change in its frequency (Fig. 2(a)), as well as a change in the output link of the switch (Fig. 2(b)). The switches of the middle stage, in turn, allow a change in the output link only (Fig. 3(a)). In the case of the switch of the middle stage, a change in the frequency of the channel is not possible (Fig. 3(b)).

The presented operation of switches requires new components, such as a bandwidth-variable waveband selective switch (BV-WSS) [2], [48], a bandwidth-variable transceiver, and a bandwidth-variable tunable waveband converter (TWBC) [2], [49]–[51]. The internal structures of the first/third-stage switches as well as the second-stage switches, used in the optical switching fabric under investigation (Fig. 1), are presented in Fig. 4(a) and Fig. 4(b), respectively.

C. STRUCTURE OF OFFERED TRAFFIC

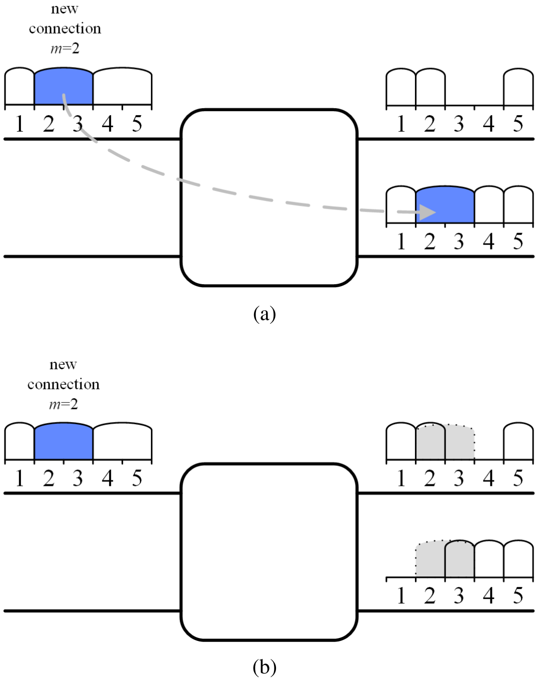
The assumption is that the switching network in Fig. 1, presented in a simplified form in Fig. 5, is offered M_I Erlang traffic streams, M_J Engset traffic streams, and M_K Pascal traffic streams. The total number of traffic streams offered to



(a)

(b)

FIGURE 2. An example of the operation of a switch of the first and the third stage.



(a)

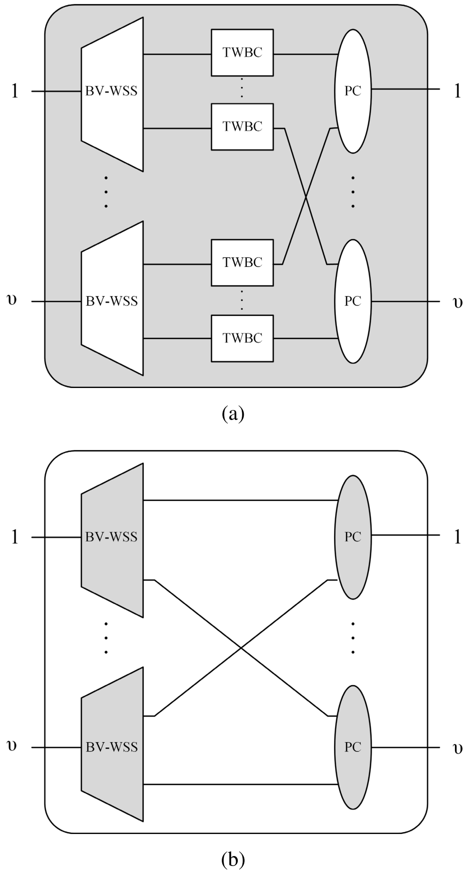
(b)

FIGURE 3. An example of the operation of a switch of the second stage.

the switching network is defined by the following formula:

$$M = M_I + M_J + M_K. \tag{3}$$

A given traffic stream (call class) c[2] is defined by the number of t_c FSUs demanded to set up a new connection and the parameter μ_c of an exponential distribution of the service time for calls of class c.



(a)

(b)

FIGURE 4. Switches: (a) a first/third-stage switch and (b) a second-stage switch. PC - passive combiner.

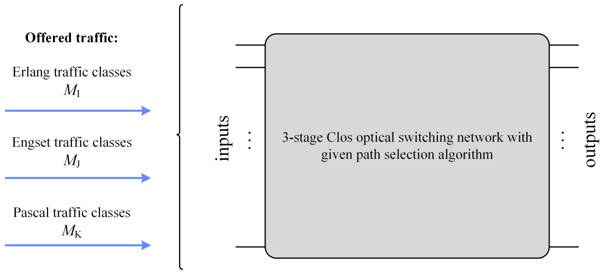


FIGURE 5. Types of traffic streams offered to a 3-stage optical switching network.

1) ERLANG TRAFFIC STREAMS

The switching network is offered M_I Erlang traffic streams. The intensities of individual call streams are the following: $\lambda_1, \lambda_2, \ldots, \lambda_i, \ldots, \lambda_{M_I}$. The number of demanded FSUs by calls of particular traffic classes is: $t_1, t_2, \ldots, t_i, \ldots, t_{M_I}$. Service times for calls of particular traffic classes are in line with an exponential distribution with the following parameters: $\mu_1, \mu_2, \ldots, \mu_i, \ldots, \mu_{M_I}$. The average intensity of traffic offered by a traffic stream of class i can then be determined

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[2] In the article, the i index defines a given Erlang traffic class, the j index a given Engset traffic class, and the k index a given Pascal traffic class. The c index denotes, in turn, any traffic class regardless of its type.

as follows:

$$A_i = \lambda_i / \mu_i. \tag{4}$$

The switching network to which Erlang traffic streams are offered provides a good example of a system in which the intensity of the arrival of new calls does not depend on the occupancy state of the system, i.e., a system with a state-independent call-arriving process [52].

2) ENGSET TRAFFIC STREAMS

The switching network is offered M_J Engset call streams. Let S_j be the number of sources that generate calls of class j that demand t_j FSUs for service. An input traffic stream of class j is the result of a superposition of S_j traffic sources that can be in either of the following two states: the active state ON (the source occupies t_j FSUs in the system) or the inactive state OFF (the source does not occupy any resources of the system). When the source is in state ON, its intensity of generating new calls is equal to zero. This means that the call-arriving process is a state-dependent process. The intensity of offered traffic α_j by a free source (in state OFF) of class j is equal to:

$$\alpha_j = \gamma_j / \mu_j, \tag{5}$$

where γ_j is the average intensity of generating new calls by a free source of class j, whereas μ_j is the parameter of the exponential distribution of service time of calls of class j. Therefore, the average intensity of traffic of class j that is offered to the system in the occupancy state of n FSUs can be determined as follows [53]:

$$A_j(n) = (S_j - n_j(n))\alpha_j, \tag{6}$$

where $n_j(n)$ is the number of serviced sources of class j in state n.

3) PASCAL TRAFFIC STREAMS

The switching network is offered M_K Pascal traffic streams. Let S_k denote the number of sources that generate calls of class k that demand t_k FSUs to set up a connection. Each free source (in state OFF) generates calls with an intensity γ_k. Service time has an exponential distribution with parameter μ_k. The intensity of offered traffic β_k by one unoccupied source of class k is equal to

$$\beta_k = \gamma_k / \mu_k. \tag{7}$$

Unlike the Engset stream, the Pascal traffic stream has a new call arrival intensity for calls of a given class that increases with an increase in the number of serviced sources (i.e., sources in state ON) of this class. The assumption in the Pascal model is that the call arrival intensity for new calls of class k in the occupancy state of n FSUs is equal to $(S_k + n_k(n))\gamma_k$, where $n_k(n)$ defines the number of serviced sources of class k (i.e., the number of sources of class k in state ON) in the occupancy state n. Therefore, the average traffic intensity of traffic offered to the system in the state of n occupied FSUs can be determined in the following way [53]:

$$A_k(n) = (S_k + n_k(n))\beta_k. \tag{8}$$

D. ALGORITHMS FOR THE CHOICE OF CONNECTION PATHS IN THE NETWORK

Output links of a switching network can be grouped into so-called directions, i.e., groups of links that lead to the same node of the next hop (next hop in the route). One of the methods for creating output directions, that provides the highest availability of routes leading to a required node of the next hop, is based on the introduction/addition of one output link from each switch of the last stage to a required direction. Depending on the intensity of traffic offered to the direction that leads to a required node of the next hop, the number of links that create this direction can vary between one link to υ links. The introduction of directions means that there are two ways to select connection paths inside the network: (1) between a given switch of the first stage and one of the many switches of the last stage whose links lead in the direction of the demanded node of the next hop (point-to-group selection), or (2) between a given switch of the first stage and a switch of the last stage that has free resources on the link that leads in the direction of the node of the next hop (point-to-point selection). Let us first consider the algorithm for setting up point-to-group connections:

Step 1: Determination of the switch of the first stage at whose input link a call of class c has arrived.

Step 2: Finding a switch of the last stage that has a free output link with t_c free neighboring FSUs in the demanded direction. If none of the switches of the last stage has t_c free neighboring FSUs in the demanded direction, the call is lost due to external blocking (Fig. 6).

Step 3: Setting up a counter for recording the number of attempts to set up a connection: $l = 1$.

Step 4: Attempting to set up a connection between a selected switch of the first and the last stage, i.e., finding the same t_c neighboring FSUs (frequency channels) in the link between the switch of the first stage and the middle stage and a switch of the middle stage and the last stage (Fig. 7).[3]

- -- If successful, the connection is set up. The run of the algorithm is terminated.
- -- If the attempt failed and the counter $l < \upsilon$, go back to **Step 3** and increase the number of attempts: $l = l + 1$.
- -- If the attempt failed and the counter $l \geq \upsilon$, then the call is lost (discarded) due to internal

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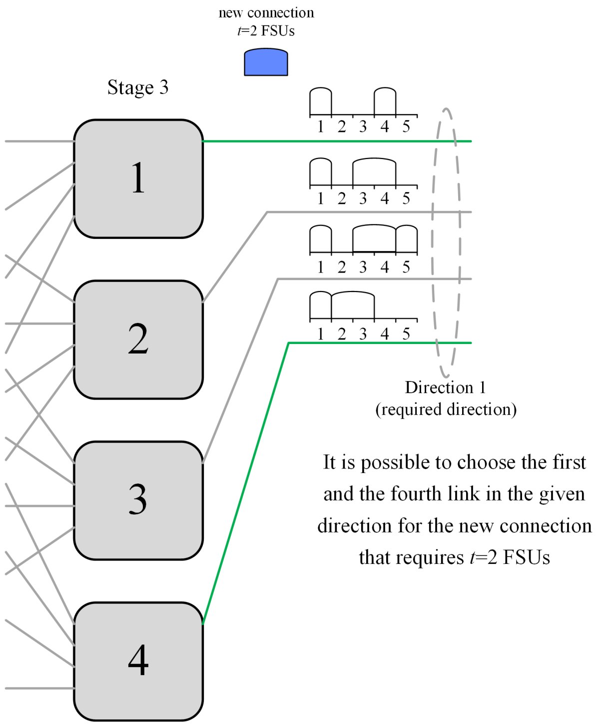
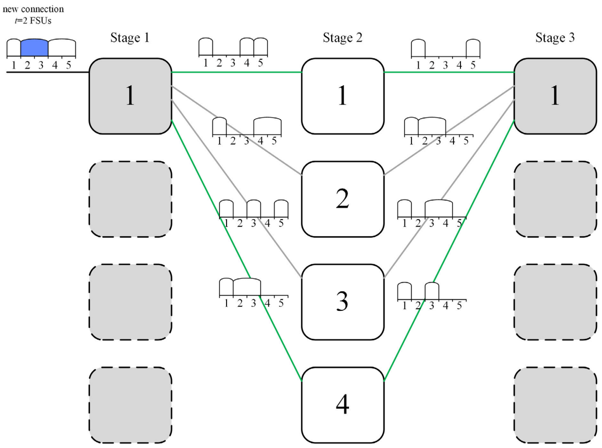
[3] The spatial switch used in the middle stage of the switching network requires as many corresponding free neighboring FSUs as required for a call of class c (t_c FSUs) to be serviced and to be found in the links between the switches of the first and the second stage and between the second and the third stage (Fig. 8).



It is possible to choose the first and the fourth link in the given direction for the new connection that requires t=2 FSUs

FIGURE 6. Choosing the switch of the last stage.



It is possible to set up a new connection that requires t=2 FSUs between first-stage switch 1 and last-stage switch 1 via middle-stage switch 1 or 4

FIGURE 7. Choosing the switch of the middle stage.

blocking. The run of the algorithm is terminated as unsuccessful.

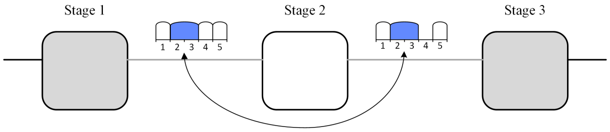
Note that the point-to-point selection is a particular case of the point-to-group selection: In the case of the point-to-point algorithm, the number of attempts in Step 4 is equal to 1.

IV. MODELING OF SWITCHING NETWORKS

The determination of traffic characteristics in multi-stage switching networks with multi-service traffic is a complex problem, both in traditional networks, in which information is transmitted by electrical signals, and in optical networks. Models of switching networks, including Clos multi-stage networks, are based primarily on the concept of so-called effective availability. These methods are run for the following aspects to be determined: (1) a model of inter-stage links, (2) a model of output links, (3) a method to determine the effective availability parameter, and (4) method to determine the distribution of available output links. Later in this section, we will present appropriate models and methods that have the advantage of developing a new method for modeling switching networks in EONs that are based on the concept of effective availability.

A. MODELING OF INTER-STAGE LINKS IN OPTICAL SWITCHING NETWORKS

According to the presented algorithm for setting up connections in the modeled switching network in EONs, setting up a connection inside the switching network for a call of class c requires at least one middle-stage switch with inter-stage links (between a selected switch of the first and third stages), in which as many successive FSUs are available as required for a call of class c (t_c FSUs) to be serviced (Fig. 8). In the case of electronic switching, all that is necessary for setting up a connection is to find t_c unoccupied allocation units in the input and output links of a given switch in the middle-stage, without the requirement that there must be the same successive "time slots." Thus, this difference in the requirements related to setting up connections by middle-stage switches implies that the modeling techniques developed for electronic switching networks cannot be directly used to model optical networks [54].



New connection requires t=2 FSUs.
The only option is to choose the second
and the third FSU.

FIGURE 8. Setting up of a connection that demands t_c neighboring FSUs inside the network.

The assumption in the article is that the requirement of finding the same free FSU slots in the input and output links of a required middle-stage switch will be solved by the concept of a virtual link (Fig. 9). The concept of a virtual link is based on a replacement of two links: The input link and the output link of a middle-stage switch are replaced by one that connects directly to a switch of the first and the third stage. In a virtual link, created for any combination of input and output links of a given middle-stage switch, those FSUs that are unoccupied in both the input link and the output link will be treated as free FSUs. The virtual link is a link with a capacity equal to the real capacity of the inter-stage links. A given number of FSUs is considered to be occupied (busy) in a virtual link if it is occupied in at least one of the real links. An example of the creation of a virtual link is shown in Fig. 9. The upper section of Fig. 9 shows – for a selected switch of the middle stage – both the input link (free: slots 2, 3, and 5; occupied: slots 1, 4, 6, and 7), and the output link (free: slots 2, 4, and 6; occupied: slots 1, 3, 5, and 7). The lower section of Fig. 9 shows the virtual link corresponding to these links. We can observe that the created virtual link has only one free FSU, i.e., FSU 2.

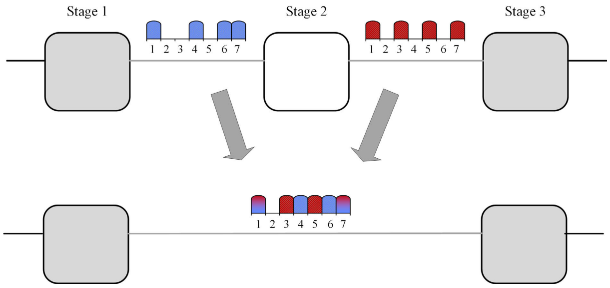


FIGURE 9. Conversion of inter-stage links into virtual links.

To determine the occupancy distribution in a virtual link, let us assume that occupancy distributions in the real links (input and output links) depend only on the total number of busy FSUs. In this way, we assume that the links are not mutually dependent and, additionally, that they do not depend on the occupancy distribution of FSUs between calls of individual classes. The occupancy distribution in individual inter-stage links that are offered Erlang, Engset, and Pascal traffic streams can be determined on the basis of the following formula [53]:

$$\begin{aligned} n\left[P_n\right]_f &= \sum_{i=0}^{M_I} A_{\text{link},i} t_i \left[P_{n-t_i}\right]_f \\ &\quad + \sum_{j=0}^{M_J} A_{\text{link},j}(n-t_j) t_j \left[P_{n-t_j}\right]_f \\ &\quad + \sum_{k=0}^{M_K} A_{\text{link},k}(n-t_k) t_k \left[P_{n-t_k}\right]_f, \end{aligned} \tag{9}$$

in which the value of $A_{\text{link},i}$ of the traffic intensity, offered to a single interstage link, for Erlang traffic of class i is

$$A_{\text{link},i} = \frac{A_i}{\upsilon^2}, \tag{10}$$

where A_i is the intensity of class i traffic offered to the whole switching fabric, while υ^2 is the number of inter-state links between successive sections of the switching fabric. In (10), it was assumed that in the considered switching fabric, the traffic flows symmetrically, i.e., the average value of the traffic offered to individual interstage links is the same.

The intensities of Engset traffic of class j, $A_{\text{link},j}(n)$, and Pascal traffic of class k, $A_{\text{link},k}(n)$, are determined by the following formulas:

$$A_{\text{link},j}(n) = \frac{A_j(n)}{\upsilon^2} = \frac{(S_j - n_j(n))\alpha_j}{\upsilon^2}, \tag{11}$$

$$A_{\text{link},k}(n) = \frac{A_k(n)}{\upsilon^2} = \frac{(S_k + n_k(n))\beta_k}{\upsilon^2}. \tag{12}$$

The parameters $n_j(n)$ and $n_k(n)$ in (11) and (12) determine, according to (6) and (8), the number of traffic sources of, respectively, class j and k that are serviced in the occupancy state of n FSUs. It is proved in [55] that these values are equal to, respectively, the average number $y_{\text{link},j}(n)$ of serviced Engset calls of class j and the average number $y_{\text{link},k}(n)$ of serviced Pascal calls of class k in a given occupancy state n:

$$n_j(n) = y_{\text{link},j}(n), \quad n_k(n) = y_{\text{link},k}(n). \tag{13}$$

The average number of serviced calls $y_{\text{link},j}(n)$ and $y_{\text{link},k}(n)$ in state n of occupied FSUs can be determined as follows [56]:

$$y_{\text{link},j}(n) = \begin{cases} A_{\text{link},j}(n-t_j)\left[P_{n-t_j}\right]_f / \left[P_n\right]_f & \\ & \text{for } t_j \le n \le f, \\ 0 & \text{for } n > f \wedge n < t_j, \end{cases} \tag{14}$$

$$y_{\text{link},k}(n) = \begin{cases} A_{\text{link},k}(n-t_k)\left[P_{n-t_k}\right]_f / \left[P_n\right]_f & \\ & \text{for } t_k \le n \le f, \\ 0 & \text{for } n > f \wedge n < t_k. \end{cases} \tag{15}$$

Note that the values of the parameters $y_{\text{link},j}(n)$ and $y_{\text{link},k}(n)$ are determined on the basis of the occupancy distribution $[P_n]_f$ ((9)), while determining the occupancy distribution requires the values of traffic intensity ((11) and (12)). This means that $y_{\text{link},j}(n)$, $y_{\text{link},k}(n)$, $A_{\text{link},j}(n)$, $A_{\text{link},k}(n)$ and $[P_n]_f$ are mutually entangled and that determining them requires applying an iterative process [55].

Once the occupancy distribution in the inter-stage link is calculated, determined on the basis of (9), it is possible to determine the blocking probability in the inter-stage link:

$$E_{\text{link},c} = \sum_{n=f-t_c+1}^{f} \left[P_n\right]_f. \tag{16}$$

Knowing the values of the blocking probabilities $E_{\text{link},i}$, $E_{\text{link},j}$, and $E_{\text{link},k}$ for calls of Erlang, Engset, and Pascal classes, respectively, we are in the position to determine the average value of traffic that is serviced by a single FSU in a real inter-stage link:

$$\begin{aligned} y &= \frac{\sum_{i=0}^{M_I} A_{\text{link},i} t_i (1 - E_{\text{link},i})}{f} \\ &\quad + \frac{\sum_{j=0}^{M_J} \frac{\sum_{n=0}^{f} A_{\text{link},j}(n) t_j}{f} (1 - E_{\text{link},j})}{f} \\ &\quad + \frac{\sum_{k=0}^{M_K} \frac{\sum_{n=0}^{f} A_{\text{link},k}(n) t_k}{f} (1 - E_{\text{link},k})}{f}, \end{aligned} \tag{17}$$

where the values of offered traffic, $A_{\text{link},i}$, $A_{\text{link},j}(n)$, and $A_{\text{link},k}(n)$, can be determined by (10), (11), and (12).

Note that the average value of traffic serviced by a single FSU in a real inter-stage link is also the occupancy probability of this FSU. This means that the occupancy probability of an FSU in a virtual link, is the result of the occupancy of an FSU in at least one of the real component links, can be interpreted as the average traffic y_{vir} serviced by an FSU of a virtual link [57]:

$$y_{\text{vir}} = 1 - (1 - y)^2. \tag{18}$$

Thus, the determined value of traffic serviced by a single FSU in a virtual link (on the basis of traffic serviced in real inter-stage links) can be used to determine the occupancy distribution $[P_n]_{\text{vir},f}$ in the virtual link. Knowing the occupancy distribution $[P_n]_{\text{vir},f}$ will allow us to determine the value of the blocking probability in the inter-stage links for calls of individual traffic classes, and consequently to estimate the values of the internal blocking probabilities.

To determine the distribution $[P_n]_{\text{vir},f}$, we assume that the value of carried traffic $y_{\text{vir},\eta}$, determined on the basis of this distribution, must be equal to the traffic value y_{vir}:

$$y_{\text{vir},\eta} = y_{\text{vir}}, \tag{19}$$

where:

$$\begin{aligned} n\,[P_n]_{\text{vir},f} &= \eta \sum_{i=0}^{M_I} A_{\text{link},i} t_i \left[P_{n-t_i}\right]_{\text{vir},f} \\ &\quad + \eta \sum_{j=0}^{M_J} A_{\text{link},j}(n) t_j \left[P_{n-t_j}\right]_{\text{vir},f} \\ &\quad + \eta \sum_{k=0}^{M_K} A_{\text{link},k}(n) t_k \left[P_{n-t_k}\right]_{\text{vir},f}. \end{aligned} \tag{20}$$

The value of parameter η in (20), determining the occupancy distribution in the virtual link, must be matched in such a way as to fulfill the criterion expressed by (19). The value of this parameter is determined in an iterative way, with the following expresions taken into consideration:

$$\begin{aligned} y_{\text{vir},\eta} &= \frac{\eta \sum_{i=0}^{M_I} A_{\text{link},i} t_i (1 - E_{\text{vir},i})}{f} \\ &\quad + \frac{\eta \sum_{j=0}^{M_J} \frac{\sum_{n=0}^{f} A_{\text{link},j}(n) t_j}{f} (1 - E_{\text{vir},j})}{f} \\ &\quad + \frac{\eta \sum_{k=0}^{M_K} \frac{\sum_{n=0}^{f} A_{\text{link},k}(n) t_k}{f} (1 - E_{\text{vir},k})}{f} \end{aligned} \tag{21}$$

and

$$E_{\text{vir},c} = \sum_{n=f-t_c+1}^{f} [P_n]_{\text{vir},f}. \tag{22}$$

The values of the blocking probabilities $E_{\text{vir},i}$, $E_{\text{vir},j}$, and $E_{\text{vir},k}$ in the virtual link of the optical switching network are considered to be final when (19) is satisfied at a required level ϵ of the relative error. The detailed process of modeling inter-stage links in an optical switching network in EONs, thereby taking into account spatial switching in the middle-stage switches, is presented as the FAG-OPT method.

Method 1 FAG-OPT Method

```
Step 1: Setting the iteration step: $l = 0$.
Step 2: Setting the initial values of the parameters $y^{(0)}_{\text{link},j}(n)$, $y^{(0)}_{\text{link},k}(n)$:
        $\forall_{0 \le j \le M_J} \forall_{0 \le n \le f}\, y^{(0)}_{\text{link},j}(n) = 0$,
        $\forall_{0 \le k \le M_K} \forall_{0 \le n \le f}\, y^{(0)}_{\text{link},k}(n) = 0$.
Step 3: Increasing the iteration step: $l = l + 1$.
Step 4: Determination of the values of traffic, $A^{(l)}_{\text{link},i}$, $A^{(l)}_{\text{link},j}(n)$, and $A^{(l)}_{\text{link},k}(n)$, on the basis of (10), (11), and (12), respectively.
Step 5: Determination of the occupancy distribution $[P_n]^{(l)}_f$ in inter-stage links on the basis of (9).
Step 6: Determination of the values $y^{(l)}_{\text{link},j}(n)$ and $y^{(l)}_{\text{link},k}(n)$ on the basis of (14) and (15).
Step 7: Repetition of Steps 3-6 until the required accuracy $\epsilon$ of the iterative process is obtained:
        $$\forall_{0 \le n \le f} \left| \frac{y^{(l-1)}_{\text{link},j} - y^{(l)}_{\text{link},j}}{y^{(l)}_{\text{link},j}} \right| \le \epsilon, \tag{23}$$
        $$\forall_{0 \le n \le f} \left| \frac{y^{(l-1)}_{\text{link},k} - y^{(l)}_{\text{link},k}}{y^{(l)}_{\text{link},k}} \right| \le \epsilon. \tag{24}$$
Step 8: Determination of the value of parameter $y_{\text{vir}}$ on the basis of (18).
Step 9: Setting the initial values: $\eta = 1$ and $l = 0$.
Step 10: Increasing the iteration step: $l = l + 1$.
Step 11: Determination of the occupancy distribution $[P_n]^{(l)}_{\text{vir},f}$ on the basis of (20).
Step 12: Determination of the values of blocking probabilities $E^{(l)}_{\text{vir},i}$, $E^{(l)}_{\text{vir},j}$, and $E^{(l)}_{\text{vir},k}$ on the basis of (22).
Step 13: Determination of the value of parameter $y^{(l)}_{\text{vir},\eta}$ on the basis of (21).
Step 14: Increasing the value of parameter $\eta$: $\eta = \eta + 0.001$.
Step 15: Repetition of Steps 10-14 until the required accuracy $\epsilon_{\text{vir}}$ of the iteration process is obtained:
         $$\left| \frac{y_{\text{vir}} - y^{(l)}_{\text{vir},\eta}}{y_{\text{vir}}} \right| \le \epsilon_{\text{vir}} \tag{25}$$
Step 16: Determination of the final value of the blocking probability:
         $$E_{\text{vir},c} = E^{(l)}_{\text{vir},c}. \tag{26}$$
```

B. MODELING OF OUTPUT DIRECTIONS IN OPTICAL SWITCHING NETWORKS

In modeling output directions in the optical switching network, it is necessary to take into consideration two dependencies that influence the possibility of admitting a call of class c, which demands t_c FSUs, for service. One dependency results from the structure of an outgoing direction that is composed of υ separated links with a capacity of f FSUs each. The other dependency results from an additional requirement in optical networks: If a given outgoing link is to be capable of servicing a call of class c (i.e., be free for calls of class c), it must have t_c free neighboring FSUs.

To determine the occupancy distribution and the blocking probability for individual traffic classes in a system under consideration (outgoing direction), we can apply a generalization of the Kaufman-Roberts formula that provides the possibility of modeling so-called product-form state-dependent systems [53]:

$$\begin{aligned} n\,[P_n]_{\upsilon f} &= \sum_{i=0}^{M_I} A_{\mathrm{dir},i} t_i \sigma_{\mathrm{str},c}(n-t_i)\sigma_{\mathrm{opt},c}(n-t_i)\left[P_{n-t_i}\right]_{\upsilon f} \\ &\quad + \sum_{j=0}^{M_J} A_{\mathrm{dir},j}(n-t_j) t_j \sigma_{\mathrm{str},c}(n-t_j)\sigma_{\mathrm{opt},c}(n-t_j)\left[P_{n-t_j}\right]_{\upsilon f} \\ &\quad + \sum_{k=0}^{M_K} A_{\mathrm{dir},k}(n-t_k) t_k \sigma_{\mathrm{str},c}(n-t_k)\sigma_{\mathrm{opt},c}(n-t_k)\left[P_{n-t_k}\right]_{\upsilon f}, \end{aligned} \tag{27}$$

where the values of traffic, $A_{\mathrm{dir},i}$, $A_{\mathrm{dir},j}(n)$, and $A_{\mathrm{dir},k}(n)$, can be determined on the basis of the following formulas:

$$A_{\mathrm{dir},i} = \frac{A_i}{\upsilon}, \tag{28}$$

$$A_{\mathrm{dir},j}(n) = \frac{A_j(n)}{\upsilon} = \frac{(S_j - n_j(n))\alpha_j}{\upsilon}, \tag{29}$$

$$A_{\mathrm{dir},k}(n) = \frac{A_k(n)}{\upsilon} = \frac{(S_k + n_k(n))\beta_k}{\upsilon}. \tag{30}$$

The parameters $A_{\mathrm{dir},i}$, $A_{\mathrm{dir},j}(n)$, and $A_{\mathrm{dir},k}(n)$ determine the value of traffic offered to a single direction of the switching fabric (assuming a symmetrical distribution of the traffic).

In (27), the above-mentioned factors that influence the possibility of admitting calls of class c are taken into consideration by the following two parameters: $\sigma_{\mathrm{str},c}(n)$ and $\sigma_{\mathrm{opt},c}(n)$. These parameters (known as conditional state-passage probabilities) define the influence of a separation of links that form the output direction and the influence of the requirements concerning the neighborhood of free FSUs, respectively. Therefore, they determine the percentage reduction in the total traffic flow between neighboring states of the service process, as a result of the impact of the above-mentioned factors.

The values $n_j(n)$ and $n_k(n)$ in (29) and (30) express, in line with (6) and (8), the number of traffic sources of, respectively, class j and k that are serviced (occupied in state ON) in the occupancy state of n FSUs. It is proved in [55] that these values are equal to the average number $y_{\mathrm{dir},j}(n)$ of serviced Engset calls of class j and the average number $y_{\mathrm{dir},k}(n)$ of serviced Pascal calls of class k, respectively, in a given occupancy state n:

$$n_j(n) = y_{\mathrm{dir},j}(n); \qquad n_k(n) = y_{\mathrm{dir},k}(n). \tag{31}$$

The average number $y_{\mathrm{dir},j}(n)$ of serviced Engset calls and the average number of serviced Pascal calls $y_{\mathrm{dir},k}(n)$ in a state of n busy FSUs can be determined as follows [53]:

$$y_{\mathrm{dir},j}(n) = \begin{cases} A_{\mathrm{dir},j}(n-t_j)\sigma_{\mathrm{str},c}(n-t_j)\sigma_{\mathrm{opt},c}(n-t_j)\dfrac{\left[P_{n-t_j}\right]_{\upsilon f}}{[P_n]_{\upsilon f}} \\ \qquad\qquad \text{for } \; t_j \le n \le \upsilon f, \\ 0 \qquad\qquad \text{for } \; n > \upsilon f \wedge n < t_j, \end{cases} \tag{32}$$

$$y_{\mathrm{dir},k}(n) = \begin{cases} A_{\mathrm{dir},k}(n-t_k)\sigma_{\mathrm{str},c}(n-t_k)\sigma_{\mathrm{opt},c}(n-t_k)\dfrac{\left[P_{n-t_k}\right]_{\upsilon f}}{[P_n]_{\upsilon f}} \\ \qquad\qquad \text{for } \; t_k \le n \le \upsilon f, \\ 0 \qquad\qquad \text{for } \; n > \upsilon f \wedge n < t_k. \end{cases} \tag{33}$$

Note that the values of the parameters $y_{\mathrm{dir},j}(n)$ and $y_{\mathrm{dir},k}(n)$ are determined on the basis of the occupancy distribution $[P_n]_{\upsilon f}$ (see (27)). This distribution can be determined if we know the traffic intensities $A_{\mathrm{dir},j}(n)$ and $A_{\mathrm{dir},k}(n)$ (see (29) and (30)). This means that the parameters $y_{\mathrm{dir},j}(n)$, $y_{\mathrm{dir},k}(n)$, $A_{\mathrm{dir},j}(n)$, $A_{\mathrm{dir},k}(n)$, and $[P_n]_{\upsilon f}$ are mutually entangled and that determining them requires applying an iterative process [55].

The value of the conditional transition coefficient $\sigma_{\mathrm{str},c}(n)$, which takes into consideration the influence of the structure of an output direction (υ links with a capacity of f FSUs each) on the possibility of call admittance for calls of individual traffic classes is taken into account in (27) by the conditional transition coefficient [34]:

$$\sigma_{\mathrm{str},c}(n) = 1 - \frac{F(\upsilon f - n, \upsilon, t_c - 1, 0)}{F(\upsilon f - n, \upsilon, f, 0)}, \tag{34}$$

where $F(x, \upsilon, f, t)$ determines the number of possible distributions (allocations) of x unoccupied FSUs in υ links, each having a capacity of f FSUs, with the additional assumption that there are at least t free FSUs in each of the links:

$$F(x, \upsilon, f, t) = \sum_{r=0}^{\left\lfloor \frac{x-\upsilon t}{f-t+1} \right\rfloor} (-1)^r \binom{\upsilon}{r} \binom{x - \upsilon(t-1) - 1 - r(f-t+1)}{\upsilon - 1}. \tag{35}$$

The parameter $\sigma_{\mathrm{str},c}(n)$ can be interpreted as the probability that a call of class c can be completely handled by any output link of a given direction. It is therefore the probability that at least one link of a given direction has t_c free FSUs.

Let us now determine the other transition coefficient that conditions the possibility of admittance of a call of class c for service in the occupancy state of n FSUs, i.e., the parameter $\sigma_{\mathrm{opt},c}(n)$. This parameter allows us to take into consideration the requirement – particularly relevant to EONs – that the admittance of a call of class c must be preceded by finding t_c free and successive (neighboring) FSUs. To determine the value of parameter $\sigma_{\mathrm{opt},c}(n)$, we propose an approximate method that is based on an appropriate transformation of the structure of an output direction. This transformation is based on a replacement of the real output direction with a capacity of $\upsilon \times f$ FSUs by a fictitious direction composed of υ_{opt} component links, each with capacity $f_{\text{opt}} = t_{\max}$:

$$\upsilon_{\text{opt}} = \left\lfloor \frac{f}{t_{\max}} \right\rfloor \upsilon, \tag{36}$$

where $t_{\max}$ determines the maximum number of demanded FSUs by calls of all classes offered to the system. The general idea of the replacement of the structure of the real direction for the purpose of determining the value of the parameter $\sigma_{\text{opt},c}(n)$ is presented in Fig. 10.

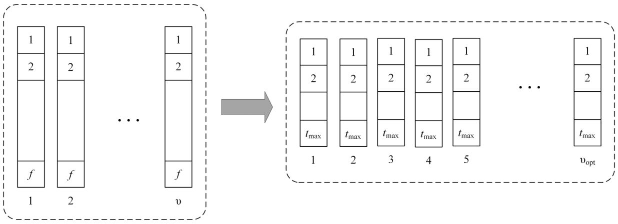


FIGURE 10. **The idea behind the model for the transformation of the structure of the output direction to include the requirements of optical switching.**

When we know the parameters f_{opt} and υ_{opt}, we can determine the value of parameter $\sigma_{\text{opt},c}(n)$ on the basis of the following formula:

$$\sigma_{\text{opt},c}(n) = 1 - \frac{F(\upsilon_{\text{opt}} f_{\text{opt}} - n, \upsilon_{\text{opt}}, t_c - 1, 0)}{F(\upsilon_{\text{opt}} f_{\text{opt}} - n, \upsilon_{\text{opt}}, f_{\text{opt}}, 0)}, \tag{37}$$

where $F(x, \upsilon_{\text{opt}}, f_{\text{opt}}, t_c)$ is determined by (35).

Thus, the conditional transition probabilities $\sigma_{\text{str},c}(n)$ and $\sigma_{\text{opt},c}(n)$ allow us to determine the occupancy distribution $[P_n]_{\upsilon f}$ in an outgoing direction of an optical switching network and, consequently, the blocking probability for calls of class c:

$$E_{\text{dir},c} = \sum_{n=0}^{\upsilon f} [P_n]_{\upsilon f} \left[1 - \sigma_{\text{str},c}(n)\sigma_{\text{opt},c}(n)\right]. \tag{38}$$

Successive steps in the determination of the occupancy distribution and the blocking probability in the limited-availability group that models output directions of an optical switching network can be written in the form of the LAG-OPT method.

Method 2 LAG-OPT Method

```
Step 1: Setting the iteration step: $l = 0$.
Step 2: Setting the initial values of parameters $y^{(0)}_{\text{dir},j}(n)$, $y^{(0)}_{\text{dir},k}(n)$:
        $\forall_{0 \le j \le M_J} \forall_{0 \le n \le f} y^{(0)}_{\text{dir},j}(n) = 0$,
        $\forall_{0 \le k \le M_K} \forall_{0 \le n \le f} y^{(0)}_{\text{dir},k}(n) = 0$.
Step 3: Determination of the values of parameters $\sigma_{\text{str},c}(n)$ and $\sigma_{\text{opt},c}(n)$ on the basis of (34) and (37).
Step 4: Increasing the iteration step: $l = l + 1$.
Step 5: Determination of the values of traffic, $A^{(l)}_{\text{dir},i}$, $A^{(l)}_{\text{dir},j}(n)$, and $A^{(l)}_{\text{dir},k}(n)$, on the basis of (28), (29), and (30), respectively.
Step 6: Determination of the occupancy distribution $[P_n]^{(l)}_{\upsilon f}$ on the basis of (27).
Step 7: Determination of the values $y^{(l)}_{\text{dir},j}(n)$ and $y^{(l)}_{\text{dir},k}(n)$ on the basis of (32) and (33).
Step 8: Repetition of Steps 4-7 until the required accuracy $\epsilon$ of the iterative process is achieved:
        $\forall_{0 \le n \le f} \left| \frac{y^{(l-1)}_{\text{dir},j} - y^{(l)}_{\text{dir},j}}{y^{(l)}_{\text{dir},j}} \right| \le \epsilon,$   (39)
        $\forall_{0 \le n \le f} \left| \frac{y^{(l-1)}_{\text{dir},k} - y^{(l)}_{\text{dir},k}}{y^{(l)}_{\text{dir},k}} \right| \le \epsilon.$   (40)
Step 9: Determination of the blocking probability in the output direction of the optical switching network on the basis of (38).
```

C. EFFECTIVE AVAILABILITY IN THE SWITCHING NETWORK

The effective availability parameter determines the average number of switches of the last stage with which a connection in a given direction can be set up.

The basis for the determination of the effective availability for a traffic stream of class c is provided by the concept of the so-called equivalent network [34]. The equivalent network is a single-service network with the same structure as the multi-service network. Each link of an equivalent network is assigned a fictitious load $e_l(c)$ that is equal to the blocking probability for traffic of class c in an inter-stage link of a real network, between stages l and $l+1$. This probability can be determined on the basis of the occupancy distribution in the virtual group that models the inter-stage links of a switching network (see (22)). To determine the effective availability $d_z(c)$ for a traffic stream of class c in a z-stage equivalent network, the following formula [34] can be used:

$$\begin{aligned} d_{z,c} = {} & [1 - \pi_z(c)]\upsilon + \pi_z(c)\eta\upsilon e_1(c) \\ & + \pi_z(c)[\upsilon - \eta\upsilon e_1(c)]e_z(c)\sigma_z(c), \end{aligned} \tag{41}$$

where

- $d_{z,c}$ – the effective availability for a traffic stream of class c in an equivalent z-stage switching network.
- $\pi_z(c)$ – the so-called probability of direct non-availability of a given switch of the last stage for a call of class c. Parameter $\pi_z(c)$ determines the probability that a connection of class c cannot be set up between a given switch of the first stage and a given switch of the last stage. The determination of this parameter is based on an analysis of the graph channel of the equivalent switching network [58].
- υ – the number of output links in a given direction.

- η – the value that determines what part of the fictitious traffic in the first-stage switch is serviced by the direction under consideration. If traffic is evenly distributed between all υ directions, we get $\eta = 1/\upsilon$.
- $\sigma_z(c)$ – the coefficient of the so-called secondary availability; it determines the number of switches in the last stage of the equivalent network [34]:

$$\sigma_z(c) = 1 - \prod_{r=2}^{z-1} \pi_r(c). \tag{42}$$

D. DISTRIBUTION OF AVAILABLE LINKS

In the effective availability method for multi-service switching networks, the so-called distribution of available links [34] is vital. This distribution determines the probability $P(c, s)$ that each link, from s randomly selected links of the output direction in a switching network, can service a call of class c. This means that any selected link has the appropriate number of demanded neighboring FSUs. This distribution can be determined in the following way:

$$P(c, s) = \sum_{n=0}^{\upsilon f} [P_n]_{\upsilon f} P(c, s|\upsilon f - n), \tag{43}$$

where $[P_n]_{\upsilon f}$ is the occupancy distribution in a limited-availability group that models the output directions; it is determined on the basis of (27) with the assumption that for each state n of the occupancy of the system, the parameter $\sigma_{\mathrm{opt},c}(n) = 1$. $P(c, s|x)$ is, in turn, the conditional distribution of available links. This distribution determines the probability of such a distribution $x = \upsilon f - n$ of free FSUs that each of the s randomly chosen links has at least t_c free, neighboring FSUs necessary to set up a connection of class c, whereas in each of the remaining $\upsilon - s$ links the number of free FSUs is lower than t_c. The conditional distribution of available links can be determined on the basis of the following formula [34]:

$$P(c, s|x) = \frac{\binom{\upsilon}{s} \sum_{w=st_c}^{\Psi} F(w, s, f, t_c) F(x - w, \upsilon - s, t_c - 1, 0)}{F(x, \upsilon, f, 0)}, \tag{44}$$

where $\Psi = sf$, if $x \geq sf$, $\Psi = x$, if $x < sf$, whereas $F(x, \upsilon, f, t)$ determines the number of possible distributions of x free FSUs in υ links, where each link has a capacity of f FSUs, with the additional assumption that in each link are at least t unoccupied FSUs. The parameter $F(x, \upsilon, f, t)$ can be determined by (35).

V. BLOCKING PROBABILITY IN SWITCHING NETWORKS OF ELASTIC OPTICAL NETWORKS

In line with the concept of the effective availability method, the methods for modeling inter-stage links and output switching networks in EONs developed in the previous section provide a basis for the process of determining the external, the internal, and the total blocking probability in these networks.

A. EXTERNAL BLOCKING PROBABILITY

The external blocking occurs when none of the output links of a demanded direction in a switching network are able to service a call of class c. This means that none of the outgoing links in the demanded direction of the switching network have at least t_c free, successive (neighboring) FSUs. Following the considerations presented in Item IV-B, we can assume that the external blocking probability is equal to the blocking probability in the output direction of the optical switching network and can be determined on the basis of the following formula:

$$E_{\mathrm{ext},c} = E_{\mathrm{dir},c} = \sum_{n=0}^{\upsilon f} [P_n]_{\upsilon f} \left[1 - \sigma_{\mathrm{str},c}(n)\sigma_{\mathrm{opt},c}(n)\right], \tag{45}$$

where $[P_n]_{\upsilon f}$ is the occupancy distribution in the limited-availability group (see (27)), whereas $\sigma_{\mathrm{str},c}(n)$ and $\sigma_{\mathrm{opt},c}(n)$ are the parameters for the conditional transition coefficient defined by (34) and (37), respectively.

B. INTERNAL BLOCKING PROBABILITY

In this article, to model internal blocking in multi-service optical switching networks the approach proposed for the PGBMT method [59] for electrical networks with point-to-group selection is used. Taking into consideration the additional limitations of the switches of the middle stage in W-S-W optical switching networks, internal point-to-group blocking occurs when all links in a demanded output direction that belong to $d_{z,c}$ available switches of the last stage do not have enough free (unoccupied) neighboring FSUs to service a call of class c. Therefore, the internal blocking probability can be determined by the following formula [60]:

$$E_{\mathrm{int},c} = \sum_{s=1}^{\upsilon - d_{z,c}} \frac{P(c, s)}{1 - P(c, 0)} \left[\binom{\upsilon - s}{d_{z,c}} \Big/ \binom{\upsilon}{d_{z,c}}\right], \tag{46}$$

where υ is the number of links in a given outgoing direction, whereas the parameters $d_{z,c}$ and $P(c, s)$ can be determined by (41) and (43), respectively. Since the internal blocking probability is determined with the assumption that at least one outgoing link is unoccupied (this assumption follows from the algorithm for setting up connections), the distribution $P(c, s)$ has been truncated by eliminating the event of the occupancy of all output links ($P(c, 0)$ in (46)).

To determine the internal blocking in optical switching networks with point-to-point selection, we can use the conclusions formulated in [61], according to which the point-to-point blocking probability in a z-stage switching networks is equal to the point-to-group blocking probability in a $(z - 1)$-stage network. According to this theorem, input links of a switch of the z stage are viewed as the links of the output direction of a switching network with point-to-point selection [59].

C. TOTAL BLOCKING PROBABILITY

The models of blocking presented above do not take into consideration the possibility of the concurrent occurrence of blocking events of different types at the analytical level. The algorithm for setting up connections in multi-stage optical networks presented in Section III operates in a sequential way, checking sequentially the possibility of external and internal blocking. Therefore, in an instance of the occurrence of, for example, an event of blocking for the switches of the last stage for a given call, the event of the internal blocking will never be registered for this particular call, even if the occupancy state of the inter-stage links corresponds to this blocking type. From the point of view of the analytical modeling of optical switching networks, the problem of the concurrent occurrence of internal and external blocking events is solved following the reasoning proposed in [62]. According to [62], to determine the total blocking probability in optical switching networks, the assumption should be that output links in a given direction can be offered only that part of the total traffic that is not lost in the inter-stage links. Similarly, in determining the internal blocking probability, the assumption is that the inter-stage links can be offered only that part of the total traffic that is not lost in the output links. Such an approach means that in determining the internal blocking probability, we take into consideration only that part of the traffic that is not lost due to external blocking. In the calculation of the external blocking probability, in turn, we use only that part of the traffic that is not lost due to internal blocking.

As a consequence, in determining the effective availability parameter (see (41)), the value of the fictitious load $e_l(c)$ (numerically equal to the blocking probability for calls of class c in an inter-stage link of a real switching network) is determined on the basis of the FAG-OPT method for the following modified values of offered traffic:

$$A_{\text{link}^*,i} = \frac{A_i(1-E_{\text{ext},i})}{\upsilon^2} = A_{\text{link},i}(1-E_{\text{ext},i}), \tag{47}$$

$$A_{\text{link}^*,j}(n) = \frac{A_j(n)(1-E_{\text{ext},j})}{\upsilon^2} = A_{\text{link},j}(n)(1-E_{\text{ext},j}), \tag{48}$$

$$A_{\text{link}^*,k}(n) = \frac{A_k(n)(1-E_{\text{ext},k})}{\upsilon^2} = A_{\text{link},k}(n)(1-E_{\text{ext},k}). \tag{49}$$

The external blocking probability can be determined on the basis of a model of a limited-availability group (the LAG-OPT method), with the assumption that the output directions are offered traffic with the following intensity values:

$$A_{\text{dir}^*,i} = \frac{A_i(1-E_{\text{int},i})}{\upsilon} = A_{\text{dir},i}(1-E_{\text{int},i}), \tag{50}$$

$$A_{\text{dir}^*,j}(n) = \frac{A_j(n)(1-E_{\text{int},j})}{\upsilon} = A_{\text{dir},j}(n)(1-E_{\text{int},j}), \tag{51}$$

$$A_{\text{dir}^*,k}(n) = \frac{A_k(n)(1-E_{\text{int},k})}{\upsilon} = A_{\text{dir},k}(n)(1-E_{\text{int},k}). \tag{52}$$

Since the exclusion of the simultaneity of the internal blocking and the external blocking in the proposed method occurs at the level of offered traffic, the total blocking probability can be written directly as the sum of the internal and the external blocking:

$$E_{\text{tot},c} = E_{\text{int},c} + E_{\text{ext},c}. \tag{53}$$

Method 3 PGB-OPT Method

Step 1: Setting the iteration step: $l = 0$.

Step 2: Setting the initial values of the internal and external probabilities:

$$\forall_{0 \le j \le M_J} E^{(0)}_{\text{int},c} = 0, \quad \forall_{0 \le k \le M_K} E^{(0)}_{\text{ext},c} = 0.$$

Step 3: Increasing the iteration step: $l = l + 1$.

Step 4: Determination of the values of traffic $A^{(l)}_{\text{link}^*,i}$, $A^{(l)}_{\text{link}^*,j}(n)$, and $A^{(l)}_{\text{link}^*,k}(n)$ offered to inter-stage links:

$$A^{(l)}_{\text{link}^*,i} = A_{\text{link},i}\left(1 - E^{(l-1)}_{\text{ext},i}\right), \tag{54}$$

$$A_{\text{link}^*,j}(n)^{(l)} = A_{\text{link},j}(n)\left(1 - E^{(l-1)}_{\text{ext},j}\right), \tag{55}$$

$$A_{\text{link}^*,k}(n)^{(l)} = A_{\text{link},k}(n)\left(1 - E^{(l-1)}_{\text{ext},k}\right). \tag{56}$$

Step 5: Determination of the occupancy distribution $[P_n]^{(l)}_{f,\text{vir}}$ (see (20)) and the blocking probability $E^{(l)}_{\text{vir},c}$ (see (22)) for calls of class c.

Step 6: Determination of the effective availability parameter $d^{(l)}_{z,c}$ (see (41)).

Step 7: Determination of the traffic intensity values $A^{(l)}_{\text{dir}^*,i}$, $A^{(l)}_{\text{dir}^*,j}(n)$, and $A^{(l)}_{\text{dir}^*,k}(n)$ offered to output directions:

$$A^{(l)}_{\text{dir}^*,i} = A_{\text{dir},i}\left(1 - E^{(l-1)}_{\text{int},i}\right), \tag{57}$$

$$A_{\text{dir}^*,j}(n)^{(l)} = A_{\text{dir},j}(n)\left(1 - E^{(l-1)}_{\text{int},j}\right), \tag{58}$$

$$A_{\text{dir}^*,k}(n)^{(l)} = A_{\text{dir},k}(n)\left(1 - E^{(l-1)}_{\text{int},k}\right). \tag{59}$$

Step 8: Determination of the occupancy distribution $[P_n]^{(l)}_{vf}$ in the limited-availability group (see (27)).

Step 9: Determination of the blocking probabilities $E^{(l)}_{\text{ext},c}$ (see (45)), $E^{(l)}_{\text{int},c}$ (see (46)) and $E^{(l)}_{\text{tot},c}$ (see (53)).

Step 10: Repetition of Steps 3-9 until the required accuracy ϵ of the iterative process is achieved:

$$\forall_{1 \le c \le M} \left| \frac{E^{(l)}_{\text{tot},c} - E^{(l-1)}_{\text{tot},c}}{E^{(l)}_{\text{tot},k}} \right| \le \epsilon. \tag{60}$$

To determine the probabilities $E_{\text{tot},c}$, $E_{\text{int},c}$, and $E_{\text{ext},c}$ in optical switching networks, it is necessary to construct an iterative process that can be written in the form of the PGB-OPT method.

VI. NUMERICAL RESULTS

The new PGB-OPT method for determining the blocking probability in optical switching networks is an approximate method. To evaluate the accuracy of the proposed method as well as the adopted assumptions, the results of the analytical calculations were compared with the data obtained in simulation experiments. The simulation studies were carried out for 3-stage optical W-S-W networks with a Clos structure [22],

[63], [64]. The switching network under consideration was composed of square switches with $\upsilon \times \upsilon$ links, each with a capacity of f FSUs.

The results of the simulation experiments are presented in the form of points plotted on a graph with marked confidence intervals determined on the basis of Student's t distribution (with a 95% confidence level) for 5 series, in the function of traffic a offered to a single FSU:

$$a = \frac{\sum_{i=1}^{M_I} t_i \frac{\lambda_i}{\mu_i} + \sum_{j=1}^{M_J} t_j \frac{\gamma_j S_j}{\mu_j} + \sum_{k=1}^{M_K} t_k \frac{\gamma_k S_k}{\mu_k}}{\upsilon \upsilon f}. \tag{61}$$

Duration time for each of the series was determined on the basis of the time needed to generate 10,000,000 calls of the class whose calls arrived with the least intensity. In each case, the confidence interval does not exceed 5% of the average value of the results of the simulation experiment. The results of the analytical calculations are presented in the graphs in the form of solid lines.

The number of FSUs required by calls of particular traffic classes depends not only on the required transmission speed, but also on the modulation format. As an example, using data provided in [65], the number of FSUs required by calls of different speeds and modulation formats is listed in Table 1.

TABLE 1. **Number of FSUs in different connections depending on the required bitrate and modulation format [65].**

Number of FSUs	Bitrate (Gb/s)	Maximum distance (km)	Modulation format
1	40	685	64-QAM
1	40	1024	32-QAM
1	40	1677.9	16-QAM
2	40	2585.2	QPSK
2	100	546	64-QAM
2	100	847.2	32-QAM
3	100	1342.5	16-QAM
5	100	2007.3	QPSK
3	160	475	64-QAM
4	160	756.5	32-QAM
4	160	1170.5	16-QAM
8	160	1710.9	QPSK
7	400	335	64-QAM
8	400	579.6	32-QAM
10	400	835.1	16-QAM
20	400	1133	QPSK
10	600	274	64-QAM
12	600	501.4	32-QAM
15	600	686.7	16-QAM
30	600	877.3	QPSK

In order to compare the accuracy of the proposed PGB-OPT method, developed for W-S-W switching networks in optical networks, with the accuracy provided by the PGBMT method, developed for electronic switching networks, Fig. 11–13 show the results of modeling (for selected traffic classes). The results take into consideration the properties of both types of switching. We can observe that the additional requirements resulting from the limitations in the execution of multi-level connections by middle-stage switches (of type S) have been properly taken into consideration in the developed analytical model. The accuracy of the calculations obtained in this study is comparable to the accuracy obtained for electronic switching networks. At the same time, we can observe that for the traffic classes that demand more FSUs/AUs, the blocking value significantly increases in optical networks as compared with electrical networks (Fig. 13, System 3). The opposite effect can be observed for calls that demand the fewest FSUs/AUs to execute a connection: In

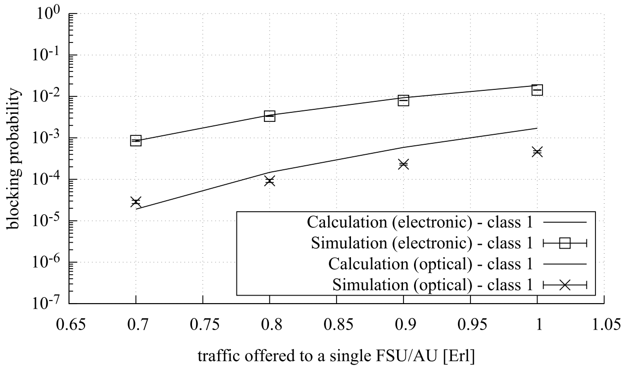


FIGURE 11. **Blocking probability for class 1 in System 1 with point-to-group selection.**

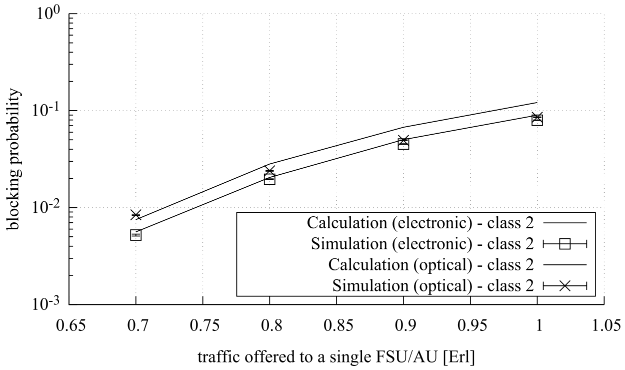


FIGURE 12. **Blocking probability for class 2 in System 2 with point-to-group selection.**

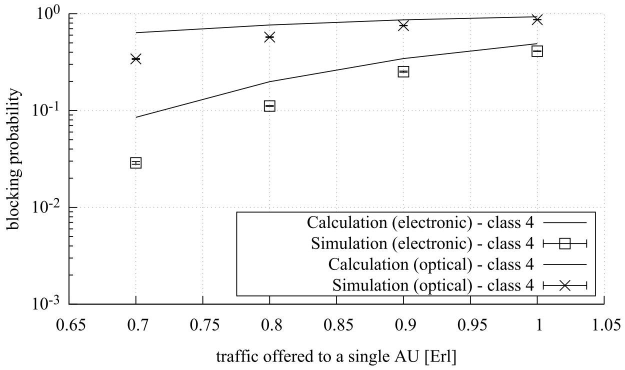


FIGURE 13. **Blocking probability for class 3 in System 3 with point-to-group selection.**

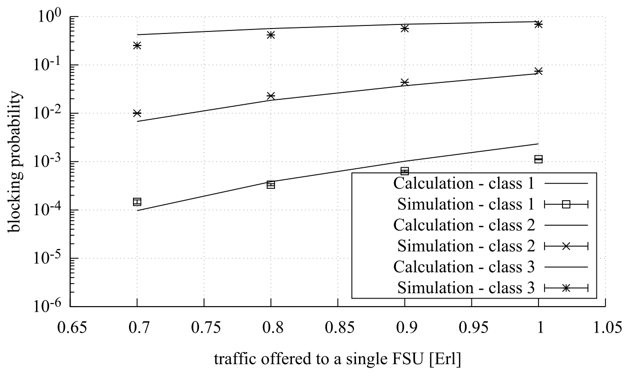


FIGURE 14. Point-to-group blocking probability in System 4 with point-to-group selection.

optical switching, the blocking value is lower than in electronic switching due to the absence of the possibility of using a single FSU by a multi-slot demand (Fig. 11, System 1).

To evaluate the accuracy of the developed analytical PBG-OPT method for all classes of offered traffic, Fig. 14 presents a comparison of the analytical results with the results of the digital simulation for System 4, in which Erlang, Engset, and Pascal traffic streams were offered. The presented results confirm the accuracy of the adopted assumptions in the PGB-OPT method.

- System 1
  - Structure of offered traffic: $M = M_I = 3$; Erlang: $t_1 = 5$ FSUs, $\mu_1^{-1} = 1$, $t_2 = 10$ FSUs, $\mu_2^{-1} = 1$, $t_3 = 20$ FSUs, $\mu_3^{-1} = 1$.
  - Structure of switching network: $\upsilon = 4, f = 120$ FSUs (38% of C-band capacity).
- System 2
  - Structure of offered traffic: $M = 3$; $M_I = 1, M_J = 1, M_K = 1$; Erlang: $t_1 = 5$ FSUs, $\mu_1^{-1} = 1$; Engset: $t_2 = 10$ FSUs, $\mu_2^{-1} = 1$, $N_2 = 2000$; Pascal: $t_3 = 20$ FSUs, $\mu_3^{-1} = 1$, $S_3 = 2000$.
  - Structure of switching network: $\upsilon = 4, f = 120$ FSUs (38% of C-band capacity).
- System 3
  - Structure of offered traffic: $M = 4$; $M_I = 2, M_J = 1, M_K = 1$; Erlang: $t_1 = 3$ FSUs, $\mu_1^{-1} = 1$, $t_2 = 12$ FSUs, $\mu_2^{-1} = 1$; Engset: $t_3 = 15$ FSUs, $\mu_3^{-1} = 1$, $N_3 = 2000$; Pascal: $t_4 = 30$ FSUs, $\mu_4^{-1} = 1$, $S_4 = 2000$.
  - Structure of switching network: $\upsilon = 4, f = 180$ FSUs (56% of C-band capacity).
- System 4
  - Structure of offered traffic: $M = 3$; $M_I = 1, M_J = 1, M_K = 1$; Erlang: $t_1 = 5$ FSUs, $\mu_1^{-1} = 1$; Engset: $t_2 = 10$ FSUs, $\mu_2^{-1} = 1, N_2 = 640$; Pascal: $t_3 = 20$ FSUs, $\mu_3^{-1} = 1$, $S_3 = 640$.
  - Structure of switching network: $\upsilon = 4, f = 32$ FSUs (10% of C-band capacity).

VII. CONCLUSION

This article proposes a new PGB-OPT method that has the advantage of modeling multi-service W-S-W switching networks operating in EONs. The method allows us to determine the blocking probability of multi-service traffic streams offered to multi-service elastic optical switching fabrics. The method also makes it possible to determine the influence of a broad range of intensities of offered traffic, the number of traffic classes, the types of traffic streams (Erlang, Engset, Pascal), and the capacity of links.

The study clearly shows that when the limitations of the middle-stage switches in W-S-W networks are taken into consideration, this significantly influences the values of the blocking probabilities in individual stream classes of traffic offered to the network.

The proposed PGB-OPT method is, to the best of our knowledge, the first analytical method for modeling multi-service blocking optical switching networks that takes into consideration the properties of EONs. The problem of the simultaneity (concurrence) of events of different types of blocking in the network is solved in our method by a separation of traffic responsible for the occurrence of blocking events of different types.

Knowing the value of the blocking probability makes it possible to apply appropriate and well-matched traffic engineering techniques that can determine the number of network elements that is necessary for traffic with the required quality parameters to be serviced. The use of such techniques can lead to a decrease in the demand for energy in data centers and switching fabrics. The blocking probability determined with the use of the proposed method may also constitute input data to the call admission control mechanisms that determine the acceptance of a new call.

REFERENCES

[1] W. Kabacinski, M. Michalski, R. Rajewski, and M. Zal, "Optical datacenter networks with elastic optical switches," in *Proc. IEEE Int. Conf. Commun. (ICC)*, May 2017, pp. 1–6.

[2] W. Kabacinski, M. Michalski, and R. Rajewski, "Strict-sense nonblocking W-S-W node architectures for elastic optical networks," *J. Lightw. Technol.*, vol. 34, no. 13, pp. 3155–3162, Jul. 1, 2016.

[3] M. Abdulsahib, M. Michalski, and W. Kabaciński, "Optimization of wide-sense nonblocking elastic optical switches," *Opt. Switching Netw.*, vol. 33, pp. 85–94, Jul. 2019. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1573427717300917

[4] W. Kabaciński, M. Michalski, and R. Rajewski, "Optimization of strict-sense nonblocking wavelength-space-wavelength elastic optical switching fabrics," *Opt. Switching Netw.*, vol. 33, pp. 76–84, Jul. 2019. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1573427717301571

[5] G. Danilewicz, "Asymmetrical space-conversion-space SCS1 strict-sense and wide-sense nonblocking switching fabrics for continuous multislot connections," *IEEE Access*, vol. 7, pp. 107058–107072, 2019.

[6] W. Kabaciński, A. Al-Tameemie, and R. Rajewski, "Necessary and sufficient conditions for the rearrangeability of WSW1 switching fabrics," *IEEE Access*, vol. 7, pp. 18622–18633, 2019.

[7] W. Kabaciński, A. Al-Tameemie, and R. Rajewski, "Rearrangeability of wavelength-space-wavelength switching fabric architecture for elastic optical switches," *IEEE Access*, vol. 7, p. 64 993–65 006, 2019.

[8] G. Danilewicz, W. Kabacinski, and R. Rajewski, "The $\log_2 N - 1$ optical switching fabrics," *IEEE Trans. Commun.*, vol. 59, no. 1, pp. 213–225, Jan. 2011.

[9] J. Zhang, H. Yang, Y. Zhao, Y. Ji, H. Li, Y. Lin, G. Li, J. Han, Y. Lee, and T. Ma, "Experimental demonstration of elastic optical networks based on enhanced software defined networking (eSDN) for data center application," *Opt. Express*, vol. 21, no. 22, pp. 26990–27002, Nov. 2013. [Online]. Available: http://www.opticsexpress.org/abstract.cfm?URI=oe-21-22-26990

[10] H. Yang, J. Zhang, Y. Zhao, Y. Ji, H. Li, Y. Lin, G. Li, J. Han, Y. Lee, and T. Ma, "Performance evaluation of time-aware enhanced software defined networking (TeSDN) for elastic data center optical interconnection," *Opt. Express*, vol. 22, no. 15, pp. 17630–17643, Jul. 2014. [Online]. Available: http://www.opticsexpress.org/abstract.cfm?URI=oe-22-15-17630

[11] H. Yang, J. Zhang, Y. Ji, R. Tian, J. Han, and Y. Lee, "Performance evaluation of multi-stratum resources integration based on network function virtualization in software defined elastic data center optical interconnect," *Opt. Express*, vol. 23, no. 24, pp. 31192–31205, Nov. 2015. [Online]. Available: http://www.opticsexpress.org/abstract.cfm?URI=oe-23-24-31192

[12] X. Luo, C. Shi, X. Chen, L. Wang, and T. Yang, "Global optimization of all-optical hybrid-casting in inter-datacenter elastic optical networks," *IEEE Access*, vol. 6, pp. 36530–36543, 2018.

[13] X. Li, T. Gao, L. Zhang, Y. Tang, Y. Zhang, and S. Huang, "Survivable K-node (edge) content connected virtual optical network (KC-VON) embedding over elastic optical data center networks," *IEEE Access*, vol. 6, pp. 38780–38793, 2018.

[14] Y. Kishi, N. Kitsuwan, H. Ito, B. C. Chatterjee, and E. Oki, "Modulation-adaptive link-disjoint path selection model for 1+ 1 protected elastic optical networks," *IEEE Access*, vol. 7, pp. 25422–25437, 2019.

[15] H. Liu, H. Wang, J. Cui, and Y. Ji, "Design of all-optical modulation format converter from one 8PSK to two QPSK signals based on phase sensitive amplification in elastic optical network," *IEEE Access*, vol. 7, pp. 51379–51385, 2019.

[16] J. Zhao, B. Bao, B. C. Chatterjee, E. Oki, J. Hu, and D. Ren, "Dispersion based highest-modulation-first last-fit spectrum allocation scheme for elastic optical networks," *IEEE Access*, vol. 6, pp. 59907–59916, 2018.

[17] G. A. Beletsioti, G. I. Papadimitriou, P. Nicopolitidis, E. Varvarigos, and S. Mavridopoulos, "A learning-automata-based congestion-aware scheme for energy-efficient elastic optical networks," *IEEE Access*, vol. 8, pp. 101978–101992, 2020.

[18] L. R. R. D. Santos and T. Abráo, "Heuristic chaotic hurricane-aided efficient power assignment for elastic optical network," *IEEE Access*, vol. 8, pp. 83359–83374, 2020.

[19] H. Liu, Q. Xiong, and Y. Chen, "Routing core and spectrum allocation algorithm for inter-core crosstalk and energy efficiency in space division multiplexing elastic optical networks," *IEEE Access*, vol. 8, pp. 70453–70464, 2020.

[20] Y. Zhou, Q. Sun, and S. Lin, "Link state aware dynamic routing and spectrum allocation strategy in elastic optical networks," *IEEE Access*, vol. 8, pp. 45071–45083, 2020.

[21] H. J. S. Dorren, N. Calabretta, and O. Raz, "A 3-stage CLOS architecture for high-throughput optical packet switching," in *Proc. Asia Commun. Photon. Conf. Exhib.*, Nov. 2009, pp. 1–6.

[22] A. Agrawal, S. Mishra, and T. Goyal, "Routing in all-optical three stage-Clos interconnection networks," *Int. J. Comput. Sci. Eng.*, vol. 1, no. 2, pp. 19–22, 2012.

[23] M. Głąbowski, H. Ivanov, E. Leitgeb, M. Sobieraj, and M. Stasiak, "Simulation studies of elastic optical networks based on 3-stage Clos switching fabric," *Opt. Switching Netw.*, vol. 36, Feb. 2020, Art. no. 100555. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1573427719301444

[24] M. Głąbowski, M. Sobieraj, and M. Stasiak, "An approach to analytical modelling of optical switching networks," in *Proc. IEICE Gen. Conf.* Tokyo, Japan: IEICE, 2018, pp. S-72–S-73.

[25] S. Kaczmarek, "Multistage optical switching networks," *J. Telecommun. Inf. Technol.*, vol. 2, pp. 21–25, 2002. [Online]. Available: https://www.il-pib.pl/pl/jtit-archive?Itemid=240&option=com_czasopisma&view=wynikiszukania&jezyk=en&nr_mies=2&rok=2002&autor=&tytul=&opis=&sl_kl=&limit=50

[26] N. Binida and W. Wend, "Die effektive erreichbarkeit für abnehmerbundel hinter zwischenleitungsanungen," *Nachrichtentechnische Zeitung*, vol. 11, no. 12, pp. 579–585, 1959.

[27] A. Charkiewicz, "An approximate method for calculating the number of junctions in a crossbar system exchange," *Elektrosvyaz*, no. 2, pp. 55–63, 1959.

[28] A. Lotze, "Bericht uber verkehrtheoretische untersuchungen CIRB," Inst. für Nachrichten-Vermittlung und Datenverarbeitung der Technischen Hochschule, Univ. Stuttgart, Stuttgart, Germany, Tech. Rep., 1963.

[29] A. Lotze, A. Roder, and G. Thierer, "PPL—A reliable method for the calculation of point-to-point loss in link systems," in *Proc. 8th Int. Teletraffic Congr.* Melbourne, VIC, Australia: Organizing Committee, 1976, pp. 1–44.

[30] A. Lotze, A. Roder, and G. Thierer, "PCM-charts for the design of economic PCM switching arrays operating in the point-to-point selection mode," Inst. Switching Data Technics, Univ. Stuttgart, Stuttgart, Germany, Paperback, 1979.

[31] E. Ershova and V. Ershov, *Digital Systems for Information Distribution* (Radio I Swiaz). Moscow, Russia: Radio Commun. (in Russian), 1983.

[32] V. Ershov, "Some further studies on effective accessibility: Fundamentals of teletraffic theory," in *Proc. 3rd Int. Seminar Teletraffic Theory*. Moscow, Russia: Organizing Committee, 1984, pp. 193–196.

[33] M. Stasiak, "Blocage interne point à point dans les réseaux de connexion," *Annales des Télécommun.*, vol. 43, nos. 9–10, pp. 561–575, 1988.

[34] M. Stasiak, "Combinatorial considerations for switching systems carrying multi-channel traffic streams," *Annales des Télécommun.*, vol. 51, nos. 11–12, pp. 611–625, 1996.

[35] M. Stasiak and M. Głąbowski, "Multi-service switching networks with point-to-group selection and several attempts of setting up a connection," in *Performance Modelling and Analysis for Heterogeneous Networks*, D. Kouvatsos, Ed. Copenhagen, Denmark: River Publishers, 2009, pp. 3–26.

[36] M. Stasiak and M. Głąbowski, "Point-to-point blocking probability in switching networks with reservation," in *Proc. 16th Int. Teletraffic Congr.*, vol. 3. Edinburgh, Scotland: Elsevier, Jun. 1999, pp. 519–528.

[37] M. Głąbowski, "Modelling of state-dependent multi-rate systems carrying BPP traffic," *Ann. Telecommun.*, vol. 63, nos. 7–8, pp. 393–407, 2008.

[38] M. Głąbowski and M. Stasiak, "Point-to-point blocking probability in switching networks with reservation," *Ann. Telecommun.*, vol. 57, nos. 7–8, pp. 798–831, 2002.

[39] M. Głąbowski and M. Sobieraj, "Analytical modelling of multiservice switching networks with multiservice sources and resource management mechanisms," *Telecommun. Syst.*, vol. 66, no. 3, pp. 559–578, Nov. 2017, doi: 10.1007/s11235-017-0305-4.

[40] M. Głąbowski and M. D. Stasiak, "Modelling of multiservice switching networks with overflow links for any traffic class," *IET Circuits, Devices Syst.*, vol. 8, no. 5, pp. 358–366, Sep. 2014.

[41] M. Głąbowski and M. D. Stasiak, "Multiservice switching networks with overflow links and resource reservation," *Math. Problems Eng.*, vol. 2016, pp. 1–17, Feb. 2016. [Online]. Available: http://www.hindawi.com/journals/mpe/2016/4090656/

[42] S. Hanczewski, M. Sobieraj, and M. D. Stasiak, "The direct method of effective availability for switching networks with multi-service traffic," *IEICE Trans. Commun.*, vol. 99, no. 6, pp. 1291–1301, 2016.

[43] M. Głąbowski, "Recurrent method for blocking probability calculation in multi-service switching networks with BPP traffic," in *Computer Performance Engineering* (Lecture Notes in Computer Science), vol. 5261, N. Thomas and C. Juiz, Eds. Berlin, Germany: Springer, 2008, pp. 152–167, doi: 10.1007/978-3-540-87412-6_12.

[44] M. Głąbowski and M. D. Stasiak, "Recurrent method for blocking probability calculation in switching networks with overflow links," *J. Telecommun. Inf. Technol.*, vols. 1–2013, pp. 56–64, Mar. 2013.

[45] M. Stasiak and P. Zwierzykowski, "Point-to-group blocking in the switching networks with unicast and multicast switching," *Perform. Eval.*, vol. 48, nos. 1–4, pp. 249–267, 2002.

[46] *Spectral Grids for WDM Applications: Dwdm Frequency Grid*, Standard G.694.1, International Telecommunication Union–Telecommunication Standardization Sector (ITU-T), 2012.

[47] Infinera. (2016). *Coherent DWDM Technologies*. [Online]. Available: https://www.infinera.com/white-paper/Coherent-DWDM-Technologies

[48] Finisar. (2019). *Dual Wavelength Selective Switch (WSS)*. [Online]. Available: https://www.finisar.com/roadms-wavelength-management/fws0120bscfal

[49] C. Politi, C. Matrakidis, and A. Stavdas, "Optical wavelength and waveband converters," in *Proc. Int. Conf. Transparent Opt. Netw.*, vol. 1, Jun. 2006, pp. 179–182.

[50] D. Vukovic, Y. Yu, M. Heuck, S. Ek, N. Kuznetsova, P. Colman, E. Palushani, J. Xu, K. Yvind, L. K. Oxenlowe, J. Mork, and C. Peucheret, "Wavelength conversion of a 9.35-Gb/s RZ OOK signal in an InP photonic crystal nanocavity," *IEEE Photon. Technol. Lett.*, vol. 26, no. 3, pp. 257–260, Feb. 1, 2014.

[51] M. Matsuura and N. Kishi, "Flexible broadband wavelength conversion in quantum-dot semiconductor optical amplifiers," *IEEE Photon. Technol. Lett.*, vol. 23, no. 15, pp. 1097–1099, Aug. 1, 2011.
[52] V. Iversen, *Teletraffic Engineering Handbook*. Lyngby, Denmark: Technical Univ. Denmark, Department of Photonics Engineering, 2010. [Online]. Available: https://orbit.dtu.dk/en/publications/teletraffic-engineeringandnetwork-planning
[53] M. Głąbowski, A. Kaliszan, and M. Stasiak, "Modeling product-form state-dependent systems with BPP traffic," *Perform. Eval.*, vol. 67, no. 3, pp. 174–197, Mar. 2010.
[54] M. Głąbowski, M. Sobieraj, and M. Stasiak, "Modified direct method for point-to-point blocking probability in multi-service switching networks with resource allocation control," in *Quality, Reliability, Security and Robustness in Heterogeneous Systems*, T. Q. Duong, N.-S. Vo, and V. C. Phan, Eds. Cham, Switzerland: Springer, 2019, pp. 109–118.
[55] M. Głąbowski, M. Stasiak, and J. Weissenberg, "Properties of recurrent equations for the full-availability group with BPP traffic," *Math. Problems Eng.*, vol. 2012, pp. 1–17, Oct. 2012.
[56] M. Stasiak and M. Głąbowski, "A simple approximation of the link model with reservation by a one-dimensional Markov chain," *Perform. Eval.*, vol. 41, nos. 2–3, pp. 195–208, Jul. 2000.
[57] S. Hanczewski, M. Stasiak, and J. Weissenberg, "Non-full-available queueing model of an EON node," *Opt. Switching Netw.*, vol. 33, pp. 131–142, Jul. 2019. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1573427717301558
[58] C. Lee, "Analysis of switching networks," *Bell Syst. Tech. J.*, vol. 34, no. 6, pp. 1287–1315, 1955.
[59] M. Stasiak, "Combinatorial considerations for switching systems carrying multi-channel traffic streams," in *Proc. 14th Int. Teletraffic Congr.*, vol. 1. Antibes, Juan-les-Pins: Elsevier, 1994, pp. 1229–1238.
[60] M. Glabowski, M. Sobieraj, and M. Stasiak, "Modeling switching networks with multi-service sources and point-to-group selection," in *Proc. 18th Asia–Pacific Conf. Commun. (APCC)*. Jeju Island, South Korea: IEEE, Oct. 2012, pp. 686–691, doi: 10.1109/APCC.2012.6388282.
[61] A. Lotze, A. Roder, and G. Thierer, "Point-to-point selection versus point-to-group selection in link systems," in *Proc. 8th Int. Teletraffic Congr.* Melbourne, VIC, Australia: Organizing Committee, 1976, p. 541.
[62] M. Głąbowski and M. D. Stasiak, "Modelling of switching networks with multiservice traffic by the IPGBMT method," in *Information Systems Architecture and Technology: Proceedings of 37th International Conference on Information Systems Architecture and Technology—ISAT 2016—Part III*, J. świątek, Z. Wilimowska, L. Borzemski, and A. Grzech, Eds. Cham, Switzerland: Springer, 2017, pp. 203–213, doi: 10.1007/978-3-319-46589-0_16.
[63] R. Proietti, Y. Yin, R. Yu, C. J. Nitta, V. Akella, C. Mineo, and S. J. B. Yoo, "Scalable optical interconnect architecture using AWGR-based TONAK LION switch with limited number of wavelengths," *J. Lightw. Technol.*, vol. 31, no. 24, pp. 4087–4097, Dec. 15, 2013.
[64] T. Ye, T. T. Lee, and W. Hu, "AWG-based non-blocking clos networks," *IEEE/ACM Trans. Netw.*, vol. 23, no. 2, pp. 491–504, Apr. 2015.
[65] C. Politi, V. Anagnostopoulos, C. Matrakidis, A. Stavdas, A. Lord, V. López, and J. P. Fernández-Palacios, "Dynamic operation of flexi-grid OFDM-based networks," in *Proc. Opt. Fiber Commun. Conf.*, 2012, pp. 1–3.

MARIUSZ GŁĄBOWSKI (Senior Member, IEEE) received the M.Sc., Ph.D., and D.Sc. (Habilitation) degrees in telecommunication from the Poznan University of Technology, Poland, in 1997, 2001, and 2010, respectively. In 2020, he was nominated as a Full Professor. Since 1997, he has been working with the Department of Electronics and Telecommunications, Poznan University of Technology. He is engaged in research and teaching in the area of performance analysis and modeling of multiservice networks and switching systems. He is the author or coauthor of four books, 12 book chapters, and over 150 papers, which have been published in communication journals and presented at national and international conferences.

ERICH LEITGEB (Member, IEEE) received the master's degree from the University of Technology Graz, in 1994, and the Ph.D. degree (Hons.) in February 1999. From 1982 to 1984, he attended the military service, including a training to become an Officer for Communications in the Austrian army, and he is still active as an Expert in Military Communications (current military rank: Lieutenant-Colonel). In 1994, he started research in optical communications at the Department of Communications and Wave Propagation, TU Graz. Since January 2000, he has been the project leader of international research projects in the field of optical communications. He has established and leads the Research Group for Optical Communications at TU Graz and joined several international projects (such as COST 270, the EU project SatNEx and SatNEx 2, COST 291, COST IC0802, and IC1101; currently, he participates in MP1401, CA15127, and CA16220) and ESA projects in different functions. Since 2011, he has been a Professor of Optical Communications and Wireless Applications at the Institute of Microwave and Photonic Engineering, TU Graz. He is the author or coauthor of seven book chapters, around 50 journal publications, 150 peer-reviewed conference papers, around 45 invited talks, and more than 70 international scientific reports.

MACIEJ SOBIERAJ received the M.Sc. and Ph.D. degrees in electronics and telecommunications from the Poznan University of Technology, Poland, in 2008 and 2014, respectively. Since 2007, he has been working at the Institute of Communications and Computer Networks, Faculty of Computing and Telecommunications, Poznan University of Technology. He is the coauthor of more than 40 scientific articles. He is engaged in research in the area of modeling multi-service cellular systems and switching networks and traffic engineering in TCP/IP networks.

MACIEJ STASIAK (Member, IEEE) received the M.Sc. and Ph.D. degrees in electrical engineering from the Institute of Communications Engineering, Moscow, Russia, in 1979 and 1984, respectively, and the D.Sc. degree in electrical engineering from the Poznan University of Technology. In 2006, he was nominated as a Full Professor. From 1983 to 1992, he worked at the Polish Industry Sector as a Designer of Electronic and Microprocessor Systems. In 1992, he joined the Poznan University of Technology, where he is currently the Director of the Institute of Communications and Computer Networks, Faculty of Computing and Telecommunications. He is the author or coauthor of more than 300 scientific articles and five books. He is engaged in research and teaching in the area of performance analysis and modeling of queuing systems, multi-service networks, and switching systems. Since 2004, he has been actively carrying out research on modeling and dimensioning of cellular networks.