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A Time-Varying Bayesian Compressed Vector Autoregression for Macroeconomic Forecasting

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ABSTRACT This paper presents macroeconomic forecasting by using a time-varying Bayesian compressed vector autoregression approach. We apply a random compression by using projection matrix to randomly select predictive variables in vector autoregression (VAR), and then perform true out-of-sample forecast where the forecast values are averaged across all estimated models, containing different in both explanatory variables and number of those variables by using Bayesian model averaging (BMA). In addition to this, we allow the parameters in Bayesian compressed VAR to be time-varying by implementing dynamic model averaging (DMA) algorithm that is applicable with VAR using forgetting factor to control the degree of time-varying in the estimating parameters. We validate the performance of the proposed method via real macroeconomic data including up to 53 variables. The empirical results demonstrate that the predictive performance of time-varying Bayesian compressed VAR can beat traditional VAR types which are considered to have a potentiality to deal with large size variables.

INDEX TERMS Bayesian econometrics, macroeconomics, Bayesian model averaging, mathematical models, time-varying parameters, dynamic model averaging, compression, Kalman filter.

I. INTRODUCTION

Big data has become increasingly important in econometric field. Econometricians, from macroeconomic point of view, are typically interested in working on how to implement an efficient forecasting method for large dimensional data as much as possible. Successful traditional large dimension econometric analysis method implemented previously is typically done via principal component analysis to handle the large dimensional matrix in order to reduce data into a smaller size. From Bayesian econometric perspective, some might use the prior shrinkage to reduce computational time consumption and size of memories used in computer. Unfortunately, looking for all possible models in forecasting has not yet been considered seriously.

For a few decades, econometricians around the globe have intensively developed large number of better tools for forecasting macroeconomic data by using advanced computational approaches with high performance computer. In the existing works, most traditional macroeconomic forecasting approaches rely on regression and its variants. Recently, computational and Bayesian methods have been receiving a great attractive for tremendous number of problems including macroeconomics, forex, and stock exchange [1]–[10]. Various types of *Vector Autoregressions* (VARs) have been extremely considered as the crucial tools in macroeconomics since the seminal work of [11]. Given a large dimension of data in forecasting procedure especially in VARs, computational cumbersome is unavoidable. Specifically, Bayesian inferences which include Markov Chain Monte Carlo (MCMC) method such as Gibbs-Sampling is impossible where the number of predictors exceed over a few hundreds. In addition to that, conventional particle filtering framework has also been used for financial analysis and forecasting. Even particle filter (PF) is recognized to be one of the most efficient Bayesian algorithms as it has been

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utilized in tremendous number of science and engineering applications; but the issue of degeneracy often exists in sequential importance sampling particle filter (SIS-PF) [12]. Many attempts have been put to resolve this problem and satisfactory improvements were achieved [4], [13]–[15], but computational burden is still unavoidable. For the problem of macroeconomic forecasting, we have large number of variables to predict, the mentioned conventional PF approach may not be appropriate. A more efficient particle filtering with less computational burden may be more suitable, but it still needs extensive investigation.

In literature, number of research works based on Bayesian approach have been reported [16]–[19]. The curse of dimensionality typically arises when the number of observations is less than the number of predictors in the VARs equation. This problem often occurs and, unfortunately, it is inevitable. Hence, a rich variety of alternatives have also been proposed, such as [20] by using LASSO, [21] by using elastic net in neural network. In addition, shrinking the prior of parameters to avoid such over-parametrization was also considered as an another way to address the problem. This can be seen in a least absolute shrinkage and selection operator as Bayesian LASSO, see [22], or horseshoe [23], [24]. However, the weakness of these estimations is that the methods just focus on point forecasting rather than obtaining full predictive density.

Alternatively, research work with other methods such as random projection, or compressing the data into a smaller matrix size instead of shrinking the priors on parameters has been proposed; see [25]. Recently, [26] developed the compressing idea into Bayesian regression, where the number of predictors in the equation are randomly compressed by introducing a special matrix for computation. In addition, *Bayesian model averaging* (BMA) method was applied to weight computation for each random compressed VAR, this reduces the sensitivity of random projection matrix. Moreover, [26] have proved and obtained the posterior predictive distribution analytically without computational burden. Bayesian compressed regression is inspired by the data squashing, its utilizations can be found broadly in literature ranging from signal and image processing, machine vision, etc., see [27], [28] for examples. The objective of data squashing is to reduce a large dimensional matrix space into a smaller subspace with the attempt to yield the similar results to the analysis of from full data set. In [29], the author suggested to construct pseudo data with similar properties to the original data. More useful results can be seen in [30], this work suggested to transfer the estimated likelihood-based clustering of a largest data set into the smaller number of data points along with appropriate weights.

In [31], this work developed the idea of [26] to implement projection matrix in a Bayesian way in order to compress the size of predictors in Vector Autoregression. There are plenty of econometric models that have been used to forecast economic data especially working with Thailand's data. For example, [32] used crude oil price to investigate the dynamic movement of key macroeconomic in Thailand. Found in [33], this study applied advanced time series models such as ARIMA and ARIMAX to forecast Thailand exports to major trade partners (China, European Union and United State America), while [34] applied spatial aggregation to model tourism arrival to Thailand from East Asia. Work in [35] showed the implementation of a belief function with predictive likelihood to forecast marketing variables. Reported in [36], they found the connection between key macroeconomic variables (GNP, inflation, money supply, interest rate and exchange rate) and ASEAN stock markets. Moreover, work in [37] studied about the contagion in the stock market during the Asian financial crisis in 1997 (Thailand, Malaysia, Indonesia, Korea and Philippines) and showed that there are strong statistical evidences of having contagion between those countries. The study found in [38] applied Vector Autoregression and estimated forecast error decomposition to study the extent of contagion and interdependency across the East Asian equity markets from 1990s afterward.

The contribution of this work is that we present a way to select important variables via BMA and *dynamic model averaging* (DMA). With the selected predictive variables in compressed VAR, the huge number of models were narrowed down, and only very statistically efficient predictive models are obtained. To the best of our knowledge, there is no report in using time-varying Bayesian compressed VAR in the problem of macroeconomic forecasting. Part of this work, we applied and implemented the model to predict key macroeconomic variables of Thailand and illustrated that by using Bayesian compressed VAR, we are able to improve the predictive performance relative to the traditional VAR such as Factor Augmented VAR, Dynamic Factor Model, Bayesian VAR with Minnesota prior, and Bayesian AR(1). The prototype model used in this work was proved to be efficient one for this kind of problem, and it can be found in [31].

This paper is organized as follows. Section II presents a foundation of Bayesian compressed VAR and how it is adapted to be time-varying over the period of study using the idea of [17]. Then, section III provides competitive models to BCVAR. In section IV, we present forecasting performance from the proposed method. Future research directions are discussed in section V. Finally, section VI delivers the conclusion. All details on the data used in this work will be found in section VII.

II. BAYESIAN COMPRESSED VECTOR AUTOREGRESSION

In this section, a detailed description of macroeconomic forecasting using a time-varying Bayesian compressed vector autoregression is presented. A block diagram of the forecasting system is illustrated in Fig. 1. The forecasting process is explained as follows. We start with collected macroeconomic data (data collection is given in details in section IV) and then the data needs to be transformed (see table 16) in which to allow it suitable for the proposed framework. After data transformation, a random compression by using

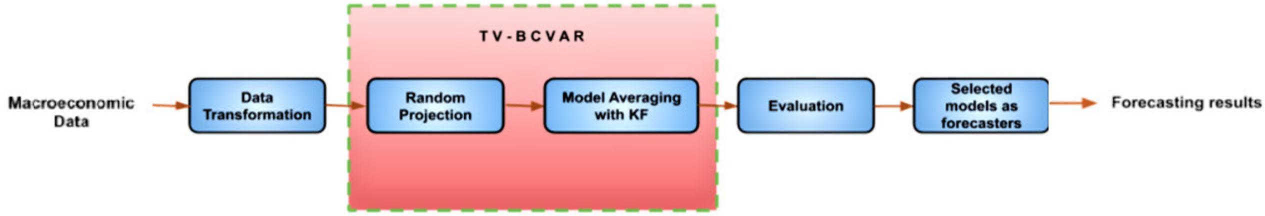

FIGURE 1. Block diagram of a Time-Varying Bayesian Compressed Vector AutoRegression (TVP-BCVAR) for macroeconomic forecasting.

projection matrix is performed, followed by selecting the best models via model averaging schemes. Further, model evaluation is employed according to the mean squared forgetting error (MSFE). Finally, macroeconomic forecasting results are obtained from the selected models.

A. CONSTANT COEFFICIENT OF BAYESIAN COMPRESSED VAR

The concept of the compression is quite similar to that of the principal component analysis (PCA). In principle, PCA scheme projects each data point onto only the first few principal components which belongs to a lower-dimensional data but data variation must be preserved as much as possible. In other words, it takes large dimensional matrix as input and produces important factors as a set of outputs where we treat these outputs as the representatives of major data variation from the original input matrix, and then use them to forecast the interested variables [39]–[47]. The first few factors contribute the most variance and the rest follows after. Likewise, compression method involves the inclusion of a projection matrix, Φ, to reduce the dimension of predictors in vector autoregressions. To be precise, suppose we have a general VAR equation given by:

$$y_t = X_t\beta + \epsilon_t, \tag{1}$$

the quantity y_t is $m \times T$ matrix consisting of m dependent variables with T observations, X_t is $k \times T$ matrix containing all predictors. Typically lags of y_t, i.e., y_{t-p} with p lags, and $\epsilon_t \sim N(0, \Omega)$ is the residual, and $K = (1 + m \times p) \times m$.

For a general VAR model, if a large dimension of VAR is being considered, for example, $m = 1000$ and $p = 1$. In this case, the uncompressed VAR will have 1,000,000 coefficients to be estimated. Therefore, the computational burden is obviously inevitable, especially when using MCMC method such as Gibbs-sampling from Bayesian fashion. This is the main reason why shrinkage algorithm is required to remedy this problem.

The main idea of compression is as follow: to shrink a large VAR to be computable using conditional posterior method such as Gibbs-Sampling, we need to reduce the size of predictors. Without loss of generality, suppose now we have VAR equation as follow:

$$Y_t = BY_{t-p} + \epsilon_t, \tag{2}$$

here Y_t is $m \times T$ variables with T observations, and Y_{t-p} is $K \times T$ where K was defined earlier in the case of constant term included, and B is $m \times k$ matrix containing VAR parameters. We shrink predictor matrix by using projection matrix Φ. Instead of using full dimension $k \times T$ of y_{t-p}, we multiply Φ with predictor matrix before the estimation procedure. The compressed VAR becomes:

$$Y_t = B^c(\Phi Y_{t-p}) + \epsilon_t \tag{3}$$

where Φ is $n \times K$, B is $m \times n$ and $n \ll m$, and Φ is subject to the be normalized, i.e., $\Phi'\Phi = I$. Thus it is precise that now B becomes more likely to be estimable via MCMC algorithm.

A random projection matrix Φ is treated to be random as suggested by [26]. In [26], it was shown by using random compressing to the data where the parameter in Φ was randomly generated. The necessary properties of a single posterior density are found by application of BMA algorithm deployed in the computation, the detail of those properties can be seen in [48]–[50] Those predictive densities will be used in forecasting procedures. The following distributions are applied for generating the quantity Φ_{ij}:

$$\mathbf{Pr}(\Phi_{ij} = \frac{1}{\varphi}) = \varphi^2 \tag{4}$$

$$\mathbf{Pr}(\Phi_{ij} = 0) = 2(1-\varphi)\varphi \tag{5}$$

$$\mathbf{Pr}(\Phi_{ij} = -\frac{1}{\varphi}) = (1-\varphi)^2 \tag{6}$$

where φ and n are unknown parameters. To address the problem of no prior information about those parameters, we apply BMA to average across the different random of projection matrices Φ_{ij}, i.e., $\Phi_{ij}^{(r)}$ where $r = 1, 2, \ldots, R$ with the total of R random draws of Φ_{ij}. Work in [26], however, suggested to generate φ from a uniform distribution:

$$\varphi \sim U[a, b], \tag{7}$$

where a and b are slightly above zero and lower than one, respectively. Finally, drawing m can be performed according to the distribution as:

$$m \sim U[2log(k), min(T, k)]. \tag{8}$$

This means that we simulate $\Phi^{(r)}$ in advance before applying MCMC method, allowing huge advantage in computational point of view and thus natural conjugate prior can be used to estimate with Gibbs-Sampling.

In [31], it was showed that there is additional issue with natural conjugate in Bayesian Compressed VAR (BCVAR) where the restriction is in the error-covariance matrix, Ω. This problem can be resolved with the re-parameterized version of the BCVAR by using triangular decomposition to Ω in which to allow it compressible. This approach has been widely used in Bayesian econometrics [51]–[53].

Now, we construct

$$A\Omega A' = \Sigma\Sigma \tag{9}$$

where Σ is diagonal element in error-covariance Ω, i.e., $\sigma_i(i = 1, 2, \ldots, m)$, and A is a lower identity matrix. Found in [31], it suggested to rewrite $A = I_m + \widetilde{A}$, where $\widetilde{A}$ is a lower triangular matrix with zeros on its diagonal. Therefore, Eq. (3) becomes:

$$Y_t = BY_{t-p} + A^{-1}\Sigma E_t \tag{10}$$

where $E_t \sim N(0, I_m)$ is normally distributed and follows homoskedasticity assumption. With further re-arranging, we obtain:

$$\begin{aligned} Y_t &= \Gamma Y_{t-p} + \widetilde{A}(-Y_{t-p}) + \Sigma E_t && (11)\\ &= \Theta Z_t + \Sigma E_t && (12) \end{aligned}$$

where $Z_t = [Y'_{t-p}, Y'_{t-p-1}, \ldots, Y'_{t-1}, -Y'_t]', \Theta = [\Gamma, \widetilde{A}]$, and $\Gamma = AB$. It should be noted that due to the structure of lower triangular character along with the diagonality of Σ, this implies that equation-by-equation estimation using Bayesian inferences can be done as suggested by [31].

Given that in the triangular specification of BCVAR where each equation has different predictors, the compression algorithm can be applied using the following setting:

$$Y_{i,t} = \Theta_i^c(\Phi_i Z_t^i) + \sigma_i E_{i,t} \tag{13}$$

here i denotes the *th*-BCVAR equation, i.e., $i = 1, 2, \ldots, m$. According to a specification mentioned above, we can apply standard Bayesian inference for a VAR equation at any instantaneous time. A standard Bayesian method for the prior distribution named "*seemingly unrelated regression model*" (SUR-Model) is applied here for drawing $\Theta_i^c|\sigma_i^2$ and σ_i^{-2} [54]: in particular,

$$\begin{aligned} \Theta_i^c|\sigma_i^2 &\sim N(\underline{\Theta}_i^c, \sigma_i^2\underline{V}_i) && (14)\\ \sigma_i^{-2} &\sim G(\underline{s}_i^{-2}, \underline{\nu}_i), && (15) \end{aligned}$$

where $G(\underline{s}_i^{-2}, \underline{\nu}_i)$ represents Gamma distribution with its mean and degrees of freedom as $\underline{s}_i^{-2}$ and $\underline{\nu}_i$, respectively. In this work, we set non-informative priors for both Θ_i^c and σ_i^{-2} where $\underline{\Theta}_i^c = 0, \underline{V}_i = 0.5 \times I$, and $\underline{\nu}_i = 0$. Finally the predictive density of one-step-ahead forecast can be done using Bayesian inference for normal linear regression model, see [55] for more detail. However, for h-step-ahead prediction, [31] mentioned about doing so by converting BCVAR from Eq. (13) to Eq. (12) and now the interested parameters can be obtained as follow:

$$\begin{aligned} \Theta = [&(\Theta_1^c\Phi_1^{(r)}, \mathbf{0}_n)', (\Theta_2^c\Phi_2^{(r)}, \mathbf{0}_{n-1})', \ldots \\ &, (\Theta_{n-1}^c\Phi_{n-1}^{(r)}, \mathbf{0}_2)', (\Theta_n^c\Phi_n^{(r)}, 0)']' \end{aligned} \tag{16}$$

where $\Phi_i^{(r)}$ stands for r number of random projection matrix of i-th equation in BCVAR. Once this transformations are performed, typical Bayesian VAR inference can now be employed to derive the required $h-$step ahead predictive densities.

We have discussed the estimating procedures and all specifications, now we need to specify random projection matrix. Like mentioning earlier that this is to reduce the computational burden, we estimate one equation at a time with the projection matrix that was simulated in advance, i.e., $\Phi_{ij}^{(r)}$ from the distribution described by Eq. (4), where r here denotes the number of iterations in drawing projection matrix. In this sense, it implies that each random projection means different explanatory variables in each VAR equation, i.e., predictive density of h-step-ahead forecast is conditional on $M_1, M_2, \ldots, M_R$. Then for each forecast horizon h, the final BMA is actually a mixture of the form given by:

$$\mathbf{P}(Y_{t+h}|D^t) = \sum_{r=1}^{R} \frac{\mathbf{exp}(-0.5\Psi_r)}{\sum_{r=1}^{R}\mathbf{exp}(-0.5\Psi_r)}\mathbf{P}(Y_{t+h}|M_r, D^t), \tag{17}$$

the quantity D^t is information available up to time t, the fraction on the right hand side of the above equation is a weight attached in BMA procedure, and $\Psi_r = BIC_r - BIC_{min}$, where BIC_r is the value of Bayesian Information Criterion (BIC) of model r, and BIC_{min} is minimum value of BIC among R models, calculated from $k_i \times ln(T) + T \times ln(\frac{SSE}{T})$. Here, k_i is the number of predictors in each i-th equation governed by Eq. (3). Lastly T represents the number of observations, and SSE is sum of square error of Eq. (3).

The two quantities: variables (elements) in the random projection matrix, φ, and the number of predictors after the compression, m, these quantities are referred to Eqs. (7) and (8), respectively. Specifically, φ and m are generated from the uniform distributions, namely

$$\varphi \sim U[0.1, 0.9], \tag{18}$$

and

$$m \sim U[1, 5ln(k_i)]. \tag{19}$$

B. TIME-VARYING BAYESIAN COMPRESSED VAR

Eq. (13) serves as a foundation of the Bayesian compress VAR, it can be extended from constant coefficient model to be time-varying model. By using Kalman filtering approach to obtain time-varying BCVAR, this algorithm is called DMA originally developed by [56], and it was adapted to be applicable for VAR via state-space model. The following recursive system forms Kalman filtering algorithm [17]:

$$Y_{i,t} = \Theta_{i,t}^c(\Phi_i Z_t^i) + \sigma_i E_{i,t} \tag{20}$$

$$\Theta_{i,t} = \Theta_{i,t-1} + \eta_t \tag{21}$$
$$\sigma^2_{i,t} = \kappa_{i,t}\sigma^2_{i,t-1} + (1-\kappa_{i,t})\widehat{E}^2_{i,t} \tag{22}$$
$$\lambda_{i,t} = \underline{\lambda} + (1-\underline{\lambda}) \times \mathbf{exp}(-0.5 \times \frac{\widehat{E}^2_{i,t-1}}{\widehat{\sigma}^2_{i,t-1}}) \tag{23}$$
$$\kappa_{i,t} = \underline{\kappa} + (1-\underline{\kappa}) \times \mathbf{exp}(-0.5 \times kurt(\widehat{E}_{i,t-12:t-1})) \tag{24}$$

where

$$\eta_t = \sqrt{\frac{(1-\lambda_{i,t})var(\Theta^c_{i,t-1|t-1})}{\lambda_{i,t}}}u_{i,t}. \tag{25}$$

The quantity $\Theta^c_{i,t}$ is allowed to evolve by following random walk using a forgetting factor approximation to its error covariance matrix. Two controlling parameters, $\lambda_{i,t}$ and $\kappa_{i,t}$ are the *forgetting factor* and *decaying factor*, respectively. Next, $u_{i,t} \sim N(0, 1)$, and $var(\theta^c_{i,t-1|t-1})$ is the variance of $\theta^c_{i,t-1}$ given the information until time $t-1$ and it is produced via Kalman filtering. In addition, $\sigma^2_{i,t}$ is also evolving via Exponentially Weighted Moving Averaging (EWMA) filter.

As mentioned previously that the key parameters which control the degree of time variation are $\lambda_{i,t}$ and $\kappa_{i,t}$, these parameters are usually set between 0.9 through 1.0. To explain this setting, if these parameters are set to 1, it implies that there is no degree of time variation and thus they will be considered to be constant coefficient BCVAR, the reader is referred to consult [56] for more detail. It is worth to note that $\widehat{\sigma}^2_{i,t-1}$ is the variance estimated at time $t-1$. Based on our monthly macroeconomic data used in this work, we performed estimation over 12 months or 1 year of observation, $kurt(\widehat{E}_{i,t-12:t-1})$ is therefore the excess kurtosis of the VAR prediction error estimated over a month ago. The two quantities $\underline{\lambda}$ and $\underline{\kappa}$ are set as the minimum values of optimal forgetting and decay factors. In this work, $\underline{\lambda}$ and $\underline{\kappa}$ are set as 0.98 and 0.94, respectively [31]. In addition to all mentioned above, the forgetting factor is allowed to evolve via Eqs. (23) and (24). For a sake of brevity, we will not provide full the Kalman filter (KF) formulae and derivation to obtain time-varying parameters in BCVAR (TVP-BCVAR), interested readers are referred directly to [17] for further detail treatment of the KF setting and derivation.

III. COMPETITIVE MODELS TO BCVAR AND TVP-BCVAR

A. BAYESIAN FACTOR-AUGMENTED VAR

A *Bayesian Factor-Augmented VAR* (BFAVAR) model by using Bayesian inference to obtain the parameters is applied [57]. The baseline BFAVAR model can be written as follow:

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = B_0 + B_1 \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + \ldots + B_p \begin{bmatrix} F_{t-p} \\ Y_{t-p} \end{bmatrix} + \epsilon^*_t \tag{26}$$

where F_t is extracted using again principal components from all selected variables depending on the size of VAR model. For instance, let the **Small-VAR** model contains 10 variables including 5 variables we want to predict. In this sense the rest 5 variables will be used to extract the *Factor*. For **Large-VAR** in this work, on the other hand, contains up to 53 variables which means that 48 of them will be used to extract the Factor. In this sense, the factor is actually a representative of variation of selected variables and it is thus contributing in predicting exercise. The maximum value of the factor is 1. It must be noted here that Y_t contains the all selected variables that we want to forecast. Finally, $\epsilon^*_t \sim N(0, \Sigma^*)$ where $E(\epsilon^*_t \epsilon^{*'}_s) = 0$ for all observations when $t \neq s$.

B. BAYESIAN DYNAMIC FACTOR MODEL

The *dynamic factor model* (DFM) is written as follow:

$$Y_t = \beta_0 + \beta_1 F_t + \epsilon_t \tag{27}$$
$$F_t = \phi_1 F_{t-1} + \ldots + \phi_p F_{t-p} + \epsilon^f_t \tag{28}$$

where F_t is $q \times 1$ vector of factor extracted from all selected n variables (with $q \ll n$). The residual follows normal distribution with zero mean and has homoskedastic characteristic, i.e., $\epsilon_t \sim N(0, \Sigma^y)$. The variance Σ^y is a diagonal matrix. In addition, β_0 and β_1 are $n \times 1$ and $n \times q$ matrices. Dynamic factor model has a time-varying factor which means that we allow them to follow a random walk process with p lags and $\epsilon^f_t \sim N(0, \Sigma^f)$. The residuals in the Eqs. (27) and (28) are mutually independent to all t and s, where $t \neq s$. Finally, we apply a non-informative prior to finish the Bayesian inferences. Please be also noted that the DFM model uses iterated forecast when performing forecasting, i.e., $h > 1$.

IV. FORECASTING RESULTS

This section delivers the empirical works in order to illustrate how the method performs on forecasting macroeconomic data. To this, we validated the performance of the proposed method via Thailand macroeconomic data. The data we used in this work contains up to 53 variables, the whole data was transformed to be approximately stationary as suggested in [17], please see Table 16 for more detail on the transformation code. We selected the core macroeconomic variable as dependent variables in VAR model including: Interest Rate, money supply (M1), consumer price index (CPI), unemployment rate and foreign direct investment (FDI). To interpret the difference between the forecasted and the data values, most common metric called root mean squared error (RMSE) is usually considered [58]–[60]. However, for the DMA framework, the forecasting values were obtained from multiple models, and the most efficient model was obtained according the algorithm described previously.

In this work, we presented the Mean Square Forecasting Error (MSFE) from three sizes of VARs. Firstly, small size VAR which contains 10 variables (Variable number 1-10 from Table 16). Medium-size VAR has 25 variables (Variable number 1-25 from Table 16) and Large-size VAR contains all 53 variables. After the transformation, the data is in monthly format ranges between 31-MAY-2012 through 31-JAN-2018. We treated half of all 69 observations as a set of training samples and the rest 50 % will be used in forecasting evaluation. MSFEs of Small-VAR, Medium-VAR and Large-VAR

TABLE 1. Mean square forecasting error Small-VAR, $h = 1, 2, 3, 6, 8, 12$.

Variable	BCVAR	TVP-BCVAR	BVAR-MINN	BFAVAR	BDFM	BVAR-OLS	B-AR	BCVAR	TVP-BCVAR	BVAR-MINN	BFAVAR	BDFM	BVAR-OLS	B-AR
	$h = 1$							$h = 2$						
Interest Rate	0.024	**0.011**	0.018	0.023	0.203	0.063	0.017	0.041	**0.010**	0.024	0.041	0.216	0.079	0.028
M1	0.106	**0.073**	0.106	0.152	0.236	0.129	0.083	0.235	**0.163**	0.196	0.223	0.326	0.181	0.174
Inflation	0.645	0.766	0.889	2.702	0.946	1.527	**0.561**	1.052	1.055	1.314	1.866	1.232	2.074	**0.923**
Unemployment	0.356	0.395	0.376	0.839	**0.303**	0.930	0.446	0.505	0.578	**0.368**	0.790	0.380	0.697	0.580
FDI	**1.023**	1.104	1.284	5.038	1.085	2.336	1.212	**1.048**	1.082	1.199	2.649	1.058	1.494	1.263
	$h = 3$							$h = 6$						
Interest Rate	0.056	**0.011**	0.029	0.096	0.240	0.106	0.037	0.087	**0.012**	0.048	0.112	0.296	0.060	0.068
M1	0.381	0.291	0.291	**0.279**	0.402	0.321	0.283	0.683	0.604	0.470	0.382	0.645	**0.381**	0.776
Inflation	1.183	1.175	1.328	1.495	1.234	2.111	**1.054**	1.086	1.076	1.307	1.281	1.271	1.012	**0.957**
Unemployment	0.507	0.586	0.314	**0.284**	0.447	0.371	0.649	0.765	0.813	0.625	0.979	**0.624**	0.748	1.008
FDI	**0.996**	1.058	1.082	1.991	1.057	1.836	1.102	0.873	0.953	**0.842**	3.173	1.024	1.150	1.013
	$h = 8$							$h = 12$						
Interest Rate	0.104	**0.014**	0.067	0.132	0.328	0.096	0.109	0.127	**0.020**	0.122	0.215	0.377	0.134	0.221
M1	0.872	0.774	0.532	**0.407**	0.786	0.548	1.102	1.430	1.256	0.838	**0.677**	1.432	0.935	1.750
Inflation	1.124	1.115	1.386	1.489	1.300	**0.938**	1.018	1.245	1.277	1.566	1.701	1.314	**1.073**	1.136
Unemployment	0.922	0.982	0.708	**0.564**	0.776	0.812	1.217	1.245	1.335	**0.672**	0.820	1.080	0.922	1.781
FDI	0.940	0.954	**0.916**	1.519	1.052	0.964	1.003	0.959	0.965	0.960	1.613	**0.957**	1.291	1.008

TABLE 2. Mean square forecasting error Medium-VAR, $h = 1, 2, 3, 6, 8, 12$.

Variable	BCVAR	TVP-BCVAR	BVAR-MINN	BFAVAR	BDFM	BVAR-OLS	B-AR	BCVAR	TVP-BCVAR	BVAR-MINN	BFAVAR	BDFM	BVAR-OLS	B-AR
	$h = 1$							$h = 2$						
Interest Rate	0.029	**0.012**	0.030	0.019	1.083	0.063	0.017	0.048	**0.013**	0.046	0.072	1.054	0.079	0.028
M1	0.082	**0.060**	0.091	0.108	0.651	0.129	0.083	0.182	**0.127**	0.140	0.207	0.729	0.181	0.174
Inflation	0.801	0.857	1.045	1.016	0.939	1.527	**0.561**	1.085	1.067	1.239	1.499	1.103	2.074	**0.923**
Unemployment	0.334	0.376	**0.321**	0.637	0.643	0.930	0.446	0.516	0.584	**0.386**	0.924	0.701	0.697	0.580
FDI	1.155	**1.117**	1.585	2.357	1.131	2.336	1.212	1.159	**1.096**	1.512	1.186	1.155	1.494	1.263
	$h = 3$							$h = 6$						
Interest Rate	0.063	**0.015**	0.049	0.078	1.173	0.106	0.037	0.111	**0.019**	0.064	0.080	1.267	0.060	0.068
M1	0.310	0.228	**0.217**	0.313	0.902	0.321	0.283	0.645	0.492	**0.365**	0.411	1.390	0.381	0.776
Inflation	1.183	1.191	1.324	1.807	**1.046**	2.111	1.054	1.095	1.018	1.292	1.252	1.037	1.012	**0.957**
Unemployment	0.537	0.609	**0.273**	0.523	0.877	0.371	0.649	0.843	0.887	0.579	**0.574**	1.175	0.748	1.008
FDI	1.129	**1.101**	1.239	2.545	1.129	1.836	1.102	0.959	0.986	**0.944**	1.239	1.030	1.150	1.013
	$h = 8$							$h = 12$						
Interest Rate	0.143	**0.021**	0.078	0.148	1.308	0.096	0.109	0.195	**0.026**	0.117	0.337	1.346	0.134	0.221
M1	0.898	0.678	**0.473**	0.506	1.721	0.548	1.102	1.547	1.171	**0.795**	0.847	2.461	0.935	1.750
Inflation	1.111	1.051	1.410	1.839	1.080	**0.938**	1.018	1.177	1.134	1.555	2.362	1.090	**1.073**	1.136
Unemployment	1.025	1.071	**0.643**	0.844	1.439	0.812	1.217	1.425	1.414	**0.693**	2.118	2.062	0.922	1.781
FDI	0.946	0.957	**0.912**	1.605	1.022	0.964	1.003	0.958	0.980	**0.946**	1.421	1.027	1.291	1.008

TABLE 3. Mean square forecasting error Large-VAR, $h = 1, 2, 3, 6, 8, 12$.

Variable	BCVAR	TVP-BCVAR	BVAR-MINN	BFAVAR	BDFM	BVAR-OLS	B-AR	BCVAR	TVP-BCVAR	BVAR-MINN	BFAVAR	BDFM	BVAR-OLS	B-AR
	$h = 1$							$h = 2$						
Interest Rate	0.026	**0.011**	0.051	0.041	0.274	0.063	0.017	0.048	**0.013**	0.078	0.035	0.329	0.079	0.028
M1	0.091	0.065	**0.064**	0.083	0.154	0.129	0.083	0.212	0.154	**0.095**	0.169	0.233	0.181	0.174
Inflation	0.802	0.864	1.014	1.013	1.237	1.527	**0.561**	1.090	1.101	1.253	1.803	1.488	2.074	**0.923**
Unemployment	0.350	0.390	**0.239**	0.566	0.349	0.930	0.446	0.489	0.566	**0.335**	0.341	0.384	0.697	0.580
FDI	1.118	1.107	1.488	1.860	**1.048**	2.336	1.212	1.144	1.110	1.540	2.598	**1.059**	1.494	1.263
	$h = 3$							$h = 6$						
Interest Rate	0.072	**0.015**	0.093	0.092	0.372	0.106	0.037	0.156	**0.022**	0.116	0.145	0.489	0.060	0.068
M1	0.359	0.274	**0.182**	0.318	0.305	0.321	0.283	0.755	0.617	**0.375**	0.445	0.586	0.381	0.776
Inflation	0.189	1.226	1.229	2.329	1.441	2.111	**1.054**	1.012	0.961	1.226	1.538	1.302	1.012	**0.957**
Unemployment	0.557	0.632	**0.278**	0.745	0.427	0.371	0.649	0.896	0.956	**0.492**	1.022	0.656	0.748	1.008
FDI	1.106	1.090	1.237	1.211	**1.078**	1.836	1.102	0.972	0.995	**0.949**	1.751	0.984	1.150	1.013
	$h = 8$							$h = 12$						
Interest Rate	0.206	**0.026**	0.129	0.226	0.547	0.096	0.109	0.276	**0.029**	0.183	0.335	0.628	0.134	0.221
M1	1.029	0.849	0.458	**0.432**	0.788	0.548	1.102	1.713	1.417	**0.661**	1.676	1.310	0.935	1.750
Inflation	1.055	1.016	1.157	1.423	1.324	**0.938**	1.018	1.159	1.159	1.354	13.922	1.280	**1.073**	1.136
Unemployment	1.102	1.178	**0.611**	1.287	0.744	0.812	1.217	1.563	1.621	**0.624**	4.389	1.188	0.922	1.781
FDI	0.978	0.984	0.999	2.888	0.984	**0.964**	1.003	**0.979**	0.989	1.015	1.476	1.006	1.291	1.008

are illustrated in Tables 1, 2 and 3, respectively. In addition, we present the Mean square forecasting error relative to the baseline model which is Bayesian-AR(4) model for $h = 1$ to $h = 12$ in Tables 4-15. The MSFE is given by

$$MSFE_{i,j,h} = \frac{\sum_{\lambda=\underline{t}}^{\bar{t}-h} e^2_{i,j,\lambda+h}}{\sum_{\lambda=\underline{t}}^{\bar{t}-h} e^2_{bcmk,j,\lambda+h}}, \tag{29}$$

where $e^2_{i,j,\lambda+h}$ and $e^2_{bcmk,j\lambda+h}$ are the squared forecasting errors of variable j at time λ and forecasting horizon h with BCVAR, TVP-BCVAR, BVAR-MINN, BFAVAR, BDFM, BVAR-OLS, B-AR models, and the Benchmark model, i.e., the Bayesian-AR(1) model, respectively. Quantities $\underline{t}$ and $\bar{t}$ represent the start and end of the out of sample forecasting periods.

Tables 1-3 demonstrate the mean square forecasting error for small-VAR, Medium VAR, and Large-VAR, respectively. According to the tables, it is observed that the best model to predict Thailand interest rate is TVP-BCVAR model for every horizontal predicting exercise. Although the MSFE results are quite close to each other, the MSFE relative to benchmark (B-AR with one to four lag predictors) are extremely different. This implies that by allowing the BCVAR model to contain time-varying parameters via the forgetting factors and exponential weighted moving average (EWMA), the predictive performance is hugely improved relative to benchmark model, please see Tables 4-15 for more detail.

For money supply prediction (M1), we found that in Small-VAR results, the first two horizons TVP-BCVAR perform very well. The longer horizon, however, BFAVAR seems to be really suitable. For the BFAVAR model, we extract one variable by using principal component from ten variables. In this sense, this implies that the one factor which is the representative of the variation from ten variables are informative in forecasting M1. These results are similar to the Medium-VAR. The large-VAR, on the other hand,

TABLE 4. Mean square forecasting error relative to Bayesian-AR(4), $h = 1$.

Variable	BCVAR	TVP-BCVAR	BVAR-MINN	BFAVAR	BDFM	BVAR-OLS
			$h = 1$			
Interest Rate	1.472	**0.640**	2.910	2.324	15.698	3.594
M1	1.086	0.779	**0.772**	0.998	1.844	1.549
Inflation	**1.430**	1.539	1.808	1.805	2.206	2.722
Unemployment	0.786	0.874	**0.536**	1.270	0.782	2.084
FDI	0.923	0.914	1.227	1.535	**0.865**	1.927

TABLE 5. Mean square forecasting error relative to Bayesian-AR(4), $h = 2$.

Variable	BCVAR	TVP-BCVAR	BVAR-MINN	BFAVAR	BDFM	BVAR-OLS
			$h = 2$			
Interest Rate	1.737	**0.487**	2.816	1.258	11.866	2.851
M1	1.215	0.883	**0.543**	0.969	1.337	1.037
Inflation	**1.181**	1.193	1.358	1.954	1.613	2.247
Unemployment	0.842	0.976	**0.578**	0.588	0.662	1.202
FDI	0.906	**0.879**	1.220	2.057	0.839	1.183

TABLE 6. Mean square forecasting error relative to Bayesian-AR(4), $h = 3$.

Variable	BCVAR	TVP-BCVAR	BVAR-MINN	BFAVAR	BDFM	BVAR-OLS
			$h = 3$			
Interest Rate	1.951	**0.417**	2.547	2.505	10.130	2.880
M1	1.267	0.967	**0.644**	1.123	1.076	1.133
Inflation	**1.128**	1.163	1.166	2.209	1.367	2.003
Unemployment	0.857	0.973	**0.428**	1.147	0.658	0.571
FDI	1.004	0.989	1.123	1.099	**0.978**	1.667

TABLE 7. Mean square forecasting error relative to Bayesian-AR(4), $h = 4$.

Variable	BCVAR	TVP-BCVAR	BVAR-MINN	BFAVAR	BDFM	BVAR-OLS
			$h = 4$			
Interest Rate	2.442	**0.454**	2.585	1.864	10.228	1.820
M1	1.071	0.847	**0.589**	0.723	0.843	0.918
Inflation	1.190	**1.186**	1.255	2.000	1.362	1.890
Unemployment	0.858	0.958	**0.404**	0.927	0.659	0.587
FDI	0.970	**0.965**	1.099	1.472	0.982	1.378

TABLE 8. Mean square forecasting error relative to Bayesian-AR(4), $h = 5$.

Variable	BCVAR	TVP-BCVAR	BVAR-MINN	BFAVAR	BDFM	BVAR-OLS
			$h = 5$			
Interest Rate	2.405	**0.376**	2.128	1.735	8.684	1.169
M1	1.013	0.819	**0.535**	0.764	0.782	0.599
Inflation	1.103	**1.060**	1.313	1.784	1.437	1.104
Unemployment	0.895	0.979	**0.418**	0.915	0.634	0.566
FDI	**0.946**	0.957	0.984	1.322	0.992	0.950

TABLE 9. Mean square forecasting error relative to Bayesian-AR(4), $h = 6$.

Variable	BCVAR	TVP-BCVAR	BVAR-MINN	BFAVAR	BDFM	BVAR-OLS
			$h = 6$			
Interest Rate	2.291	**0.325**	1.711	2.138	7.195	0.880
M1	0.973	0.795	**0.483**	0.573	0.755	0.491
Inflation	1.058	**1.004**	1.281	1.607	1.360	1.057
Unemployment	0.889	0.948	**0.488**	1.014	0.650	0.742
FDI	**0.959**	0.982	0.937	1.728	0.972	1.135

TABLE 10. Mean square forecasting error relative to Bayesian-AR(4), $h = 7$.

Variable	BCVAR	TVP-BCVAR	BVAR-MINN	BFAVAR	BDFM	BVAR-OLS
			$h = 7$			
Interest Rate	2.094	**0.279**	1.401	1.814	5.957	0.804
M1	0.937	0.770	**0.450**	0.510	0.713	0.453
Inflation	1.069	**1.018**	1.228	2.775	1.348	1.201
Unemployment	0.876	0.942	0.439	1.600	0.648	**0.628**
FDI	0.975	0.984	0.998	2.274	0.963	**0.915**

BVAR-Minnesota prior provides the best performance after $h = 3$ to $h = 12$ as presented in the tables.

For Thailand inflation forecasting exercise, the best predictors seem to be its own lag where B-AR model performs quite well in Large-sized VAR especially for $h = 1, 2, 3, 4$. The longer horizon, however, Bayesian VAR using OLS method prediction delivers quite close values to the actual ones.

BVAR with Minnesota prior is surprisingly performing well on forecasting Thailand unemployment rate for every size of VAR and every horizontal predicting. Minnesota prior is of a popular one among Bayesian researchers where the prior is shrunk to follow its own first lag only, and the prior

TABLE 11. Mean square forecasting error relative to Bayesian-AR(4), $h = 8$.

Variable	BCVAR	TVP-BCVAR	BVAR-MINN	BFAVAR	BDFM	BVAR-OLS
			$h = 8$			
Interest Rate	1.891	**0.236**	1.188	2.077	5.030	0.884
M1	0.934	0.770	0.415	**0.392**	0.715	0.497
Inflation	1.036	0.998	1.136	1.397	1.300	**0.921**
Unemployment	0.905	0.968	**0.502**	1.057	0.611	0.667
FDI	0.975	0.981	0.996	2.879	0.981	**0.961**

TABLE 12. Mean square forecasting error relative to Bayesian-AR(4), $h = 9$.

Variable	BCVAR	TVP-BCVAR	BVAR-MINN	BFAVAR	BDFM	BVAR-OLS
			$h = 9$			
Interest Rate	1.702	**0.202**	1.032	2.011	4.292	0.699
M1	0.936	0.774	**0.392**	0.368	0.709	0.508
Inflation	0.992	0.964	1.091	2.389	1.263	**0.942**
Unemployment	0.891	0.937	**0.474**	0.724	0.609	0.566
FDI	**0.967**	0.973	0.971	5.710	0.975	1.061

TABLE 13. Mean square forecasting error relative to Bayesian-AR(4), $h = 10$.

Variable	BCVAR	TVP-BCVAR	BVAR-MINN	BFAVAR	BDFM	BVAR-OLS
			$h = 10$			
Interest Rate	1.532	**0.173**	0.933	1.679	3.708	0.638
M1	0.947	0.777	**0.369**	0.547	0.707	0.557
Inflation	0.983	**0.970**	1.034	3.461	1.201	1.228
Unemployment	0.895	0.938	**0.411**	0.535	0.625	0.541
FDI	0.950	0.963	**0.929**	6.472	0.974	1.127

TABLE 14. Mean square forecasting error relative to Bayesian-AR(4), $h = 11$.

Variable	BCVAR	TVP-BCVAR	BVAR-MINN	BFAVAR	BDFM	BVAR-OLS
			$h = 11$			
Interest Rate	1.737	**0.487**	2.816	1.258	11.866	2.851
M1	1.215	0.883	**0.543**	0.969	1.337	1.037
Inflation	**1.181**	1.193	1.358	1.954	1.613	2.247
Unemployment	0.842	0.976	**0.578**	0.588	0.662	1.202
FDI	0.906	0.879	1.220	2.057	**0.839**	1.183

TABLE 15. Mean square forecasting error relative to Bayesian-AR(4), $h = 12$.

Variable	BCVAR	TVP-BCVAR	BVAR-MINN	BFAVAR	BDFM	BVAR-OLS
			$h = 12$			
Interest Rate	1.252	**0.132**	0.829	1.518	2.849	0.608
M1	0.979	0.810	**0.378**	0.958	0.749	0.534
Inflation	1.020	1.020	1.191	12.254	1.127	**0.945**
Unemployment	0.878	0.910	**0.350**	2.464	0.667	0.518
FDI	**0.971**	0.981	1.007	1.464	0.998	1.280

for covariance matrix is also theoretically shrunk to contain a smaller size.

Finally, what we can conclude about forecasting Thailand foreign direct investment (FDI) are as follows. For **Small-VAR**, the best model is BCVAR without the time-varying parameters for first three horizons $h = 1, 2, 3$. For $h = 6, 8$ the best model goes for BVAR-MINN, and $h = 12$ is BDFM. Despite to that empirical results, the difference of predictive performances is not large relative to other alternative models. For **Medium-VAR**, TVP-BCVAR is slightly better than fixed coefficient BCVAR model for $h = 1, 2, 3$. For the longer horizon where $h = 6, 8, 12$ the MSFE is found to be similar to that of **Small-VAR**. For **Large-VAR**'s predictive performance, the BVAR-MINN offers better predictive performance than the others for most cases. For $h = 6, 8$, BVAR-MINN delivers the lowest MSFEs for 3 variables.

In comparison to benchmark model (B-AR), with the time-varying characteristic in TVP-BCVAR model, we found a huge advantage in forecasting interest rate. In addition to this the longer horizontal forecasting the more accuracy interest rate prediction, these evidence can be seen in the table for more insight detail. There is also

TABLE 16. Data Appendix.

Variables	Name	Transformation Code	**Small-VAR**	**Medium-VAR**	**Large-VAR**
	Dependent Variables				
1	Thailand Interest Rates	1	×	×	×
2	Money Supply (M1)	4	×	×	×
3	Inflation Rate	2	×	×	×
4	Unemployment Rate	1	×	×	×
5	Foreign Direct Investment	1	×	×	×
	Financial Factors				
6	USD-THB	2	×	×	×
7	Gold	5	×	×	×
8	Thailand Stock Exchange Market	5	×	×	×
9	Government Bond 10Y	2	×	×	×
10	BTC/USD		×	×	×
	Business Factors				
11	Thailand Business Confidence (Index Point)	2		×	×
12	Thailand Capacity Utilization (Index Point)	2		×	×
13	Car Production	2		×	×
14	Cement Production (Thousands of Ton)	4		×	×
15	Coincident Index	2		×	×
16	Industrial Production (%)	2		×	×
17	Leading Economic Index	2		×	×
18	New Car Registrations	2		×	×
19	Total Vehicle Sales	2		×	×
	Consumer Sector				
20	Prime Leading Rate	1		×	×
21	Consumer Confidence (Index Point)	2		×	×
22	Personal Spendings	1		×	×
23	Retail Sales (YoY)	2		×	×
	Government Sector				
24	Fiscal Expenditure	4		×	×
25	Government Revenues	4		×	×
26	Government Budget Value	1			×
	Labour Sector				
27	House Price Index	2			×
28	Housing Starts	1			×
29	Employment Persons (Thousand)	4			×
30	Job Vacancies	4			×
31	Unemployed Persons (Thousand)	4			×
	Money Sector				
32	Central Bank Balance Sheet (Million Bahts)	2			×
33	Foreign Exchange Reserves (Million USD)	2			×
34	Loans Private Sector (Million Baht)	5			×
35	Money Supply M0 (Million Baht)	4			×
36	Money Supply M3 (Billion Baht)	4			×
	Trade Sector				
37	Balance of Trade	2			×
38	Crude Oil Production	2			×
39	Current Account	1			×
40	Exports	5			×
41	Imports	5			×
42	Remittances	2			×
43	Terms of Trade	2			×
44	Tourist Arrivals	1			×
	Price Sector				
45	Consumer Price Index (CPI)	2			×
46	Core Consumer Prices	2			×
47	CPI Housing	2			×
48	CPI Transportation	2			×
49	Export Prices	2			×
50	Food Inflation	1			×
51	Import Prices	2			×
52	Inflation Rate (MoM)	1			×
53	Producer Prices	2			×

statistical evidence that the predictive performance gain is quite large relative to BCVAR. Overall speaking, we found that the BCVAR, TVP-BCVAR and other alternative models perform better as compared to typical algorithm of B-AR(4) model, the detail of comparative predictive performance are shown in Tables 4-15.

V. FUTURE RESEARCH DIRECTIONS

According to our proposed method, some interesting topics for this problem can be further explored. One possible extension of the proposed approach is to utilize a more sophisticated sequential Bayesian filtering, a particle filter, to specific. Particle filter (PF) relaxes the assumption of linearity of the system as well as the Gaussian prior of the perturbation and measurement noises [61]–[64]. Therefore, by using PF, the probability distributions of the states can be more precise, capturing the stochastic nature of financial and economic parameters more effectively. Besides, looking into continuous-time positive hidden Markov jump systems framework can be another promising technique for macroeconomic forecasting [65], [66]. In addition, parameter variation analysis should be further investigated extensively to see how the parameter uncertainty affects the distribution of the forecasting parameters [67]–[69].

VI. CONCLUSION

In this work, a macroeconomic forecasting based on time-varying Bayesian compressed VAR framework has been presented. We implemented random projection matrix into VAR model to randomly pick the number of variables in VAR. Moreover, we performed true out-of-sample forecasting where the forecasting results were averaged all across models by using the Bayesian model averaging (BMA). In empirical evaluation, we illustrated via forecasting most Thailand core macroeconomics variables including interest rate, money supply (M1), inflation, unemployment rate and foreign direct investment (FDI). According to the forecasting results, we found that the mean square forecasting error is substantially reduced relative to the benchmark model Bayesian AR(4), especially in predicting interest rate, money supply unemployment rate and foreign direct investment. Both BCVAR and TVP-BCVAR were found to meet parsimonious characteristics by allowing parameters to be time-varying and computational friendly, especially when the number of variables in VAR is large. Moreover, in predicting the selected macroeconomic variables, BCVAR and TVP-BCVAR performed mostly better than traditional methods such as Bayesian VAR with Minnesota prior, FAVAR, Dynamic Factor Model. According to this study, BCVAR approach can be used to forecast the macroeconomic effectively in terms of both accuracy and computational aspects.

VII. DATA APPENDIX

See Table 16.

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