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Hull Deformation Measurement With Large Angles Based on Inertial Sensors

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ABSTRACT The partial reference is used in large ships to measure the hull deformation, which is normally realized through the transfer alignment between the master inertial navigation system and the slave inertial navigation system. In recent years, the matching methods based on several inertial measurement units has become the development trend to avoid the initial alignment of the slave system. However, these methods always require that the deformation angle is small and the system model can be regarded as a linear model. The purpose of this article is to solve the large deformation problem through coarse estimation based on the optimized attitude determination and propose an improved transfer alignment method based on the attitude matching in the inertial frame. The coarse estimation does not rely on the deformation model and can decrease the deformation error to a small angle in a short time. Then the system model and the measurement model are built to carry out the attitude matching in the inertial frame, which is stable and can avoid initial alignment. The swing table test show that the proposed method can estimate the large deformation between the two IMUs rapidly and precisely.

INDEX TERMS Inertial navigation, deformation measurement, Kalman filter, attitude determination, attitude matching.

I. INTRODUCTION

Modern large ships are equipped with different kinds of systems and devices whose attitude and position should be achieved to a high degree of accuracy. The high precision master inertial navigation system (MINS) which is installed in the center of the ship is used to provide a unified attitude reference for shipboard equipment [1], [2]. However, the structure of the hull is influenced by a variety of external factors which will cause different degree of dynamic deformation. Even though the attitude information provided by the MINS is relatively high, it will be submerged in the error of hull deformation. As a result, the navigation information of the different fighting points which are all provided by the MINS is not accurate enough [3]–[5]. Therefore, measuring the hull deformation and eliminating the effect of deformation has been researched a lot in recent years.

Various deformation measurement methods have been proposed to address this issue. In one class, some of the

methods use the optical measurement. Using collimating lenses or camera arrays, the hull deformations can be measured accurately and reliably [6]–[8]. But these methods use the instruments which require an inter-vision environment. It is inconvenient as the obstacles are not allowed to exist between each measurement point [9]. Besides, the deformation can also be measured with the Global Navigation Satellite System (GNSS). However, this method needs to be used in the open-air environment and cannot measure the deformation inside the ship [10]. Additionally, slave inertial navigation system (SINS) can be installed to different fighting points to measure the attitude information. Normally, the inertial measurement unit (IMU) of SINS has a lower degree of accuracy but with smaller volume and lower cost than MINS.

Transfer alignment technology based on Kalman filter is normally used to measure the deformation between MINS and SINS [11]–[14]. The accuracy of transfer alignment can be improved by the low-precision SINS calibration with high-precision MINS [2]. Transfer alignment matching algorithms can be divided into two categories, namely the computational

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parameter matching and measurement parameter matching [15]. Normally, the former uses the calculated attitude and velocity of MINS to modify SINS and estimate the relative attitude angle and pay much attention to the circumstance with a master navigation system, which is considered as an error-free system. These methods will estimate the error model of SINS and need initial alignment to calculate the attitude in the navigation frame. While measurement parameter matching methods utilize angular velocity and acceleration to calculate the deformation between different IMUs. The deformation can be calculated directly between every two IMUs in a short time and a single high-grade inertial system like MINS is not needed [14].

Russia scholar at the university of St. Petersburg established the angular velocity matching method with two three-axis laser gyro units [16]. Based on this, a lot of researches have been conducted [1], [9], [17]–[19]. The deformation angle is generally considered as a small angle in these researches. And linear model is built to estimate the deformation with Kalman filter (KF). However, due to the limited installation location and the complicated motion conditions, there can be a large angle between the two sets of IMUs. The linear model is not suitable and non-linear filter techniques such as Unscented Kalman Filter (UKF), Cubature Kalman Filter (CKF) and model-free techniques have been researched in [2], [5], [20], [21]. These methods need much more calculation or need precisely measured data to train the system. In fact, the deformation estimating can also be considered as an attitude determination process. The attitude matrix can be decomposed into several parts and can be derived by using solutions to Wahba’s problem based on the constructed vector observations [22], [23].

In this article, the deformation is divided into a large angle and a small angle. The system model and the measurement model are firstly introduced, and the attitude matching in the inertial frame is adopted in Section II. In order to address the problem caused by the large deformation, a coarse estimation method based on optimized attitude determination is conducted to estimate the large part in a short time in Section III. The output of one IMU can be transformed by the direct cosine matrix (DCM), which is calculated from the coarse estimation. Then the error variance will be minimized optimally and the deformation angle will be calculated effectively by the attitude matching in the inertial frame. The swing table test is done in Section IV. Finally, conclusions are drawn in Section V.

II. HULL DEFORMATION MODEL

Assumed that two IMUs are situated on points p and q of the ship. The coordinate system definition of IMU is as follows:

X axis is parallel to the deck, pointing to the right of the ship;

Y axis is parallel to the deck and orthogonal to the X axis, pointing to the front of the ship;

Z axis is orthogonal to the X and Y axes, forming a righthanded coordinate system.

A. SYSTEM STATE EQUATION

The deformation angles are mainly comprised of two parts, which are the static angle $\psi = [\psi_x \ \psi_y \ \psi_z]^T$ and dynamic angle $\theta = [\theta_x \ \theta_y \ \theta_z]^T$. $\xi = \psi + \theta$, stand for the whole deformation angles. Figure 1 shows two dimensions of the deformation between the two IMUs at point p and point q of the ship. Figure 1-(a) shows the static deformation and Figure 1-(b) shows the dynamic deformation.

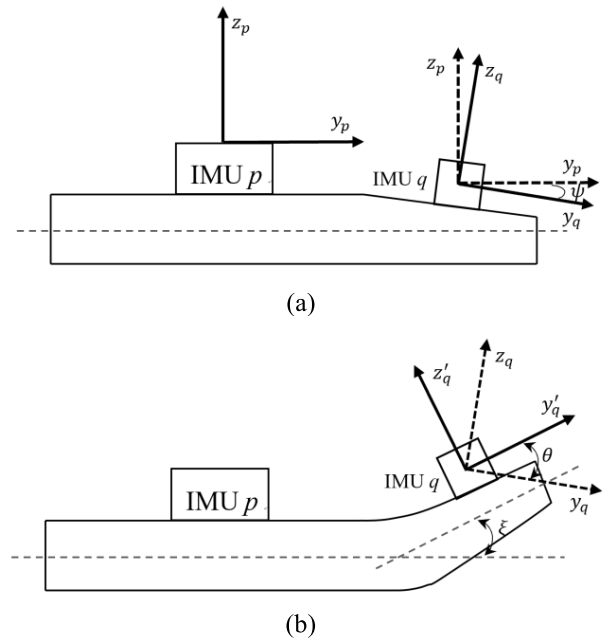


FIGURE 1. Deformation between the IMU p and q .

Generally, the mathematical model of ship dynamic deformation is considered as the second-order Markov process. Dynamic deformation angle $\theta = [\theta_x \ \theta_y \ \theta_z]$ and dynamic deformation angular-rate $\dot{\theta}$ can be described as follows

$$\ddot{\theta}_i + 2\eta_i\dot{\theta}_i + b_i^2\theta_i = 2b_i\sqrt{D_i\eta_i}w_i(t) \quad (1)$$

$$b_i^2 = \eta_i^2 + \lambda_i^2 \quad (2)$$

where $i = x, y, z$. λ_i is the dynamic deformation angle frequency, η_i is the irregular factor, $w_i(t)$ is white noise and D_i is the constant coefficient [5].

The rotation transformation relationship between n-frame and b-frame can be described by the DCM C_n^b . However, there are several kinds of measurement errors and disturbances in the actual navigation system, so rotational errors usually exist between the calculated navigation coordinate frame (n' -frame) and the ideal navigation frame (n-frame). ϕ is used to describe the misalignment angle between n' -frame and n-frame.

From [24], [25], the attitude and velocity error equations of SINS on swaying base can be obtained as follows:

$$\begin{aligned} \dot{\phi} &= \phi \times \omega_{in}^n - \delta\omega_{in}^n - \epsilon^n \\ \delta\dot{V}^n &= -\phi \times C_n^b f^b + \delta V^n \times (2\omega_{ie}^n + \omega_{en}^n) + \nabla^n + V^n \end{aligned} \quad (3)$$

$$\times (2\delta\omega_{ie}^n + \delta\omega_{en}^n) \quad (4)$$

$$\omega_{ie}^n = [0 \ \omega_{ie} \ \cos L \ \omega_{ie} \sin L]^T \quad (5)$$

where ω_{in}^n is the rotating rate of n -frame with respect to the i -frame, which is projected onto the n -frame, and $\epsilon^n = [\epsilon_E \ \epsilon_N \ \epsilon_U]^T = C_b^n \epsilon^b$, ϵ^b is the gyro bias. δV^n is the velocity error, f^b is the accelerometer measurement, L is the latitude, ω_{ie} is the magnitude of the earth rotation rate, ω_{en}^n is the angular rate of n -frame with respect to the e -frame, which is projected onto the n -frame. $\nabla^n = C_b^n \nabla^b$ and ∇^b is the accelerometer bias.

Different from the n -frame, the inertial frame (i -frame) is irrelevant to the earth rotation and ship motion. ϕ^i is used to describe the misalignment angles in the inertial frame. Rewrite (3) in the inertial frame as follows:

$$\dot{\phi}^i = -C_b^i \epsilon^b \quad (6)$$

$$\epsilon^b = \epsilon_c^b + \epsilon_r^b \quad (7)$$

where ϵ_c^b is the constant bias and the ϵ_r^b is the random error.

The state vector X is defined as follows

$$X = [\phi_p^i \ \phi_q^i \ \psi \ \theta \ \dot{\theta} \ \epsilon_c^p \ \epsilon_c^q]^T \quad (8)$$

where the superscript p and q are used to distinguish the IMUs on points p and q , which are stand for the body frame of the IMUs.

The state equation can be established as follows:

$$\dot{X} = FX + W \quad (9)$$

Based on (1) and (6), the transfer matrix of the system of (9) is as follows:

$$F = \begin{bmatrix} \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & -C_p^i & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & -C_q^i \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & I_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & F_1 & F_2 & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \end{bmatrix}$$

$$F_1 = \text{diag}(-b_x^2 \quad -b_y^2 \quad -b_z^2)$$

$$F_2 = \text{diag}(-2\eta_x \quad -2\eta_y \quad -2\eta_z)$$

And the system noise matrix W is as follows:

$$W = [\epsilon_r^b \ \epsilon_r^q \ \mathbf{0}_{1 \times 3} \ \mathbf{0}_{1 \times 3} W_1 \ \mathbf{0}_{1 \times 3} \ \mathbf{0}_{1 \times 3}]^T$$

$$W_1 = [2b_x \sqrt{D_x \eta_x} w_x(t) \ 2b_y \sqrt{D_y \eta_y} w_y(t) \ 2b_z \sqrt{D_z \eta_z} w_z(t)]$$

B. SYSTEM MEASUREMENT EQUATION

The attitude matching method is widely used in the transfer alignment to estimate the deformation between MINS and SINS. This method needs to calculate the attitude in the n -frame and need the initial alignment. And normally only the error model of SINS is measured while MINS is treated as the reference. But in fact, the deformation between the two IMUs could be calculated directly from the output of the gyros without the initial alignment. The attitude matching in

the inertial frame is used to avoid the initial alignment and based on the error model of both the IMUs.

Set the body coordinate frames of the two IMUs are p -frame and q -frame. p_0 -frame and q_0 -frame are the body inertial coordinate frames, which are formed by fixing the p -frame and q -frame respectively at the beginning of the process. The DCM $C_p^{p_0}$ and $C_q^{q_0}$ could be updated from the gyro output:

$$\dot{C}_p^{p_0} = C_p^{p_0} [\omega_{ip}^p \times] \quad (10)$$

$$\dot{C}_q^{q_0} = C_q^{q_0} [\omega_{iq}^q \times] \quad (11)$$

where ω_{ip}^p and ω_{iq}^q are the angular velocity at point p and q , and $[\cdot \times]$ is the skew symmetric matrix of the vector [26].

As there are measurement errors, the calculated body frames are not accurate, which are set as p' -frame and q' -frame. The rotation transformation relationship between p' -frame and p -frame is $C_p^{p'}$. Generally speaking, the misalignment angle here is small. Therefore $C_p^{p'}$ can be approximated as

$$C_p^{p'} \approx I - [\phi_p^i \times] \quad (12)$$

where I is the 3×3 identity matrix. And similarly, we can get

$$C_q^{q'} \approx I - [\phi_q^i \times] \quad (13)$$

The deformation between the p -frame and q -frame could be described with the matrix C_p^q . If the deformation angle is small, $C_p^{p_0}$ and $C_p^{q_0}$ can also be approximated as follows:

$$C_p^q \approx I - [\xi \times] \quad (14)$$

$$C_{p_0}^{q_0} \approx I - [\psi \times] \quad (15)$$

Construct matrix $M = C_{p_0}^{p'} C_q^{q_0}$, based on multiplication chain rules we have

$$M = C_{p_0}^{p'} C_q^{q_0} = C_p^{p'} C_{p_0}^p C_{p_0}^{q_0} C_p^{p_0} C_q^p C_q^{q'} \quad (16)$$

Substitute (12) to (15) into (16) and neglect second-order small quantities to form as (17), shown at the bottom of the next page, where $C_{p_0}^p \psi = [\psi'_x \ \psi'_y \ \psi'_z]$.

The measurement variable Z is set as

$$Z = [M(2, 3) \ M(3, 1) \ M(1, 2)]^T \quad (18)$$

Then the measurement equation can be established as follows:

$$Z = HX + N \quad (19)$$

where N is the measurement noise matrix. Based on (17), the measurement matrix of (19) is

$$H = [I \ -I \ -I + C_{p_0}^p \ -I \ \mathbf{0}_{3 \times 3} \ \mathbf{0}_{3 \times 3} \ \mathbf{0}_{3 \times 3}] \quad (20)$$

Based on (9) and (19), KF can be used to estimate the deformation angle between the two IMUs.

III. COARSE ESTIMATE OF THE FULL DEFORMATION

In Section II, the system measurement equation is based on the small deformation. Thus, the system model can be approximated as a linear model. But due to the limitation of installation location of the IMU, there may be a large installation error which will be calculated as a part of the deformation angle. Besides, after a long term of the deformation, the dynamic angle can also change a lot. It is irrational if considering deformation as small angle, and the measurement model should not be considered as linear model simply.

In general, we could divide the deformation into big angle and small angle. Before estimating the deformation based on the attitude matching in the inertial frame, the big angle, which is mainly caused by the position of installation and the ship motion, should be estimated firstly. Supposed that the large part of the deformation angle is ξ_0 . If ξ_0 can be calculated, then the output of gyro at point p can transform to point \tilde{p} as follows:

$$\omega_{\tilde{p}}^{\tilde{p}} = C_{\tilde{p}}^p \omega_{ip}^p \quad (21)$$

where $C_{\tilde{p}}^p$ is the DCM between the p-frame and the \tilde{p} -frame, which is calculated from ξ_0 . Then $\omega_{\tilde{p}}^{\tilde{p}}$ is used instead of ω_{ip}^p in the attitude matching method in the inertial frame. At last, the calculated deformation from the attitude matching is added to ξ_0 through the DCM to form the whole deformation.

The coarse estimate of ξ_0 is based on the observation vectors, which are derived from the output of gyros and accelerometers. Figure 2 shows the location relationship of the three coordinate systems: the inertial frame, p-frame and q-frame.

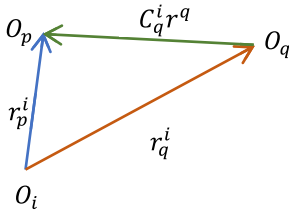


FIGURE 2. Location relationship of the IMUs.

r_p^i represents the location of the IMU p in the i -frame, r_q^i represents the location of the IMU q in the i -frame, and r^q represents the lever arm between p and q in the q -frame. The relationship between them is as follows:

$$r_p^i = r_q^i + C_q^i r^q \quad (22)$$

The derivative of the (22) is

$$\dot{r}_p^i = \dot{r}_q^i + \dot{C}_q^i r^q + C_q^i \dot{r}^q = \dot{r}_q^i + C_q^i [\omega_{iq}^q \times] r^q + C_q^i \dot{r}^q \quad (23)$$

And the derivative of the (23) is

$$\ddot{r}_p^i = \ddot{r}_q^i + C_q^i (\omega_{iq}^q \times \omega_{iq}^q \times r^q + \dot{\omega}_{iq}^q \times r^q + 2\omega_{iq}^q \times \dot{r}^q + \ddot{r}^q) \quad (24)$$

where \ddot{r}_p^i and \ddot{r}_q^i are the accelerations of p and q with respect to the inertial frame respectively. As the outputs of the accelerometers are in the body frame and the gravitational accelerations of p and q are approximative, we can get the relationship between the outputs of the accelerometers at the two points as follows:

$$C_{p0}^i f^p = C_{q0}^i f^q + C_{q0}^i R \quad (25)$$

where $R = (\omega_{iq}^q \times \omega_{iq}^q \times r^q + \dot{\omega}_{iq}^q \times r^q + 2\omega_{iq}^q \times \dot{r}^q + \ddot{r}^q)$, is the term caused by the lever arm r^q . It should be noticed that, during the short period of the coarse estimation of the deformation, r^q can be considered as a constant variable, which means $\dot{r}^q \approx 0$ and $\ddot{r}^q \approx 0$.

Multiplying both sides of (25) from left by C_{i0}^{q0} yields

$$C_{p0}^{q0} C_{p0}^i f^p = C_{q0}^{q0} f^q + C_{q0}^{q0} R \quad (26)$$

The decomposition method of the DCM based on multiplication chain rules is as follows:

$$C_{q0}^p = C_{p0}^p C_{q0}^{p0} C_{q0}^{q0} \quad (27)$$

where C_{p0}^p and C_{q0}^{q0} could be calculated from the gyro output directly from (10) and (11). Therefore, the crucial issue in coarse estimation is how to obtain the DCM C_{q0}^{p0} precisely and rapidly. The DCM could also be expressed by the quaternion form. From [27] the corresponding transformation between quaternion $Q = [Q_0 \ Q_1 \ Q_2 \ Q_3]$ and DCM C can be expressed as (28), shown at the bottom of the next page.

By the transformation manipulation with quaternions, (26) can be rearranged as follows:

$$\beta = Q_{p0}^{q0} \otimes \alpha \otimes (Q_{p0}^{q0})^* \quad (29)$$

where

$$\beta = Q_{p0}^{p0} \otimes f^p \otimes (Q_{p0}^{p0})^* \quad (30)$$

$$\alpha = Q_{q0}^{q0} \otimes (f^q + R) \otimes (Q_{q0}^{q0})^* \quad (31)$$

$$\begin{aligned} M &= (I - [\phi_p^i \times]) C_{p0}^p (I + [\psi \times]) C_{p0}^{p0} (I - [\xi \times]) (I + [\phi_q^i \times]) \\ &= (I - [\phi_p^i \times]) (I + [C_{p0}^p \psi \times]) (I - [\xi \times]) (I + [\phi_q^i \times]) \\ &\approx I - [\phi_p^i \times] + [\phi_q^i \times] + [C_{p0}^p \psi \times] - [\xi \times] \\ &= \begin{bmatrix} 1 & \phi_{pz}^i - \phi_{qz}^i - \psi_z' + \xi_z & \phi_{qy}^i - \phi_{py}^i + \psi_y' - \xi_y \\ \phi_{qz}^i - \phi_{pz}^i + \psi_z' - \xi_z & 1 & \phi_{px}^i - \phi_{qx}^i - \psi_x' + \xi_x \\ \phi_{py}^i - \phi_{qy}^i - \psi_y' + \xi_y & \phi_{qx}^i - \phi_{px}^i + \psi_x' - \xi_x & 1 \end{bmatrix} \end{aligned} \quad (17)$$

Following the optimization-based method for the attitude determination [22], this problem could be posed as a minimization procedure of the constructed $K(t)$, which is given by

$$K(t) = \int_0^t ([\beta \otimes] - [\alpha \odot])^2 dt \quad (32)$$

where

$$[\beta \otimes] = \begin{bmatrix} 0 & -\beta^T \\ \beta & [\beta \times] \end{bmatrix} \quad (33)$$

$$[\alpha \odot] = \begin{bmatrix} 0 & -\alpha^T \\ \alpha & -[\alpha \times] \end{bmatrix} \quad (34)$$

Then, the discrete form of (32) can be calculated as follows:

$$K_k = K_{k-1} + \Delta K \quad (35)$$

$$\Delta K = ([\beta \otimes] - [\alpha \odot])^T ([\beta \otimes] - [\alpha \odot]) \Delta t \quad (36)$$

where Δt is the sampling rate. Therefore, optimal quaternion $Q_q^{q_0}$ is the normalized eigenvector corresponding to the smallest eigenvalue of K_k [28], [29]. Due to the sensors error and the level arm, the relationship between the p -frame the q -frame calculated in this way is not accurate, which is used as a coarse estimation. The estimation result is considered as C_p^p , then ξ_0 can be calculated from C_p^p . Based on (21), the output of IMU p is transformed to $\omega_{ip}^{\tilde{p}}$, then $\omega_{ip}^{\tilde{p}}$ and ω_{iq}^q are used in the attitude matching method to calculate C_p^q . Finally, we can get the whole deformation DCM through $C_p^q = C_p^q C_p^{\tilde{p}}$.

IV. SIMULATION AND RESULT ANALYSIS

A. PROCESS OF THE DEFORMATION MEASUREMENT

For more clarity, the process of the proposed algorithm with coarse estimation and attitude matching in the inertial frame is listed in Table 1.

TABLE 1. Process of the deformation measurement.

Step	Description
Step1	Coarse estimation of the deformation and get ξ_0 based on Section III.
Step2	Transform the output of gyro at point p by (21).
Step3	Compute Deformation by attitude matching in the inertial frame with KF based on (9) and (19)
Step 4	Form the whole deformation with the result of Step 1 and Step 3.

B. SIMULATION CONDITIONS

The experiments are implemented through semi-reality data and semi-simulation data. In order to simulate the condition of ship on the sea, the IMU is installed on the swing table, which is shown in Figure 3. The precision parameters of sensors are listed in Table 2. The swing amplitude and frequency

of the swing table are set as Table 3. One of the IMU on the swing table is set as the IMU p , and the output is ω_{ip}^p and f^p .

TABLE 2. Sensor errors.

Axis	Gyro		Accelerometer	
	Bias (°/h)	Random walk (°/√h)	Bias (μg)	Random walk (μg/√Hz)
X	<0.01	<0.007	<100	<20
Y	<0.01	<0.007	<100	<20
Z	<0.01	<0.007	<100	<20

TABLE 3. Swing parameters.

	Pitch swing	Roll swing	Heaving motion
Amplitude	≤ 0.5°	≤ 2°	0.2m±0.1m
Period	5s	10s	6s

However, the swing table cannot form typical dynamic deformation like hull of the ship, and it is difficult to measure the valid deformation as the reference. Thus, deformation angle is formed by simulation, then the data of IMU p is transformed into the q -frame by the deformation angle velocity ω_{pq}^q and the DCM C_p^q which is calculated by the deformation angle [2]. Then the sensor errors are added to form the output the IMU q . The simulation deformation contains static deformation and dynamic deformation. The static deformation is set as constants and the dynamic deformation is set following (1). The deformation parameters are set as Table 4. And the curves of the simulated deformation are shown in Figure 4. And the estimation errors of the level arm are set as [0.5m 0.35m 0.3m]. Figure 5 shows the diagram of the data simulation process.

TABLE 4. Dynamic parameters.

	x-axis	y-axis	z-axis
η_i	0.02	0.03	0.015
λ_i	0.01	0.009	0.012
D_i	300	420	380

C. EXPERIMENTAL RESULTS

The first simulation begins with small static deformation angle, which is set as [1° 1° 2°] and the simulation last for 600 s. The proposed method is called Method A and the coarse estimation lasts for 100 s. Traditional hull deformation measurement method with angular velocity matching is called Method B, which is used as a comparison. The curves of the deformation error, which include the static and

$$C = \begin{bmatrix} Q_0^2 + Q_1^2 - Q_2^2 - Q_3^2 & 2(Q_1Q_2 - Q_0Q_3) & 2(Q_1Q_3 + Q_0Q_2) \\ 2(Q_1Q_2 + Q_0Q_3) & Q_0^2 - Q_1^2 + Q_2^2 - Q_3^2 & 2(Q_2Q_3 - Q_0Q_1) \\ 2(Q_1Q_3 - Q_0Q_2) & 2(Q_2Q_3 + Q_0Q_1) & Q_0^2 - Q_1^2 - Q_2^2 + Q_3^2 \end{bmatrix} \quad (28)$$



FIGURE 3. Swing table and IMU.

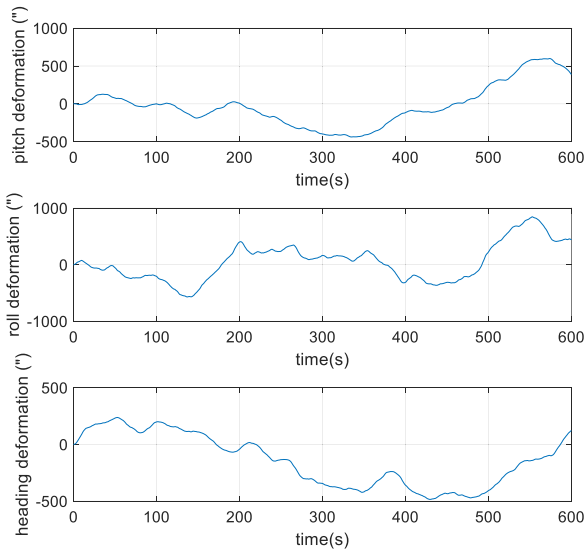


FIGURE 4. Dynamic deformation curves.

TABLE 5. Deformation error angle of simulation 1.

	Pitch	Roll	Heading
Method A	3.67"	1.54"	1.33"
Method B	8.14"	15.71"	6.53"

dynamic deformation, are shown in Figure 6 and the root mean square (RMS) calculated from 590 s to 600 s of the errors is shown in Table 5. The second simulation begins with large deformation angle, which is set as $[10^\circ \ 5^\circ \ 45^\circ]$. The deformation measurement method based on UKF is also used as a comparison, which is called Method C. The simulation last for 600 s and the curves of the deformation measurement error are shown in Figure 7 and the statistic results are shown in Table 6.

D. EXPERIMENTAL ANALYSIS

In the first simulation, when the deformation angle is small, both Method A and Method B can effectively estimate the full deformation angle, as shown in Figure 6. The coarse estimation can reduce the level deformation angles error to

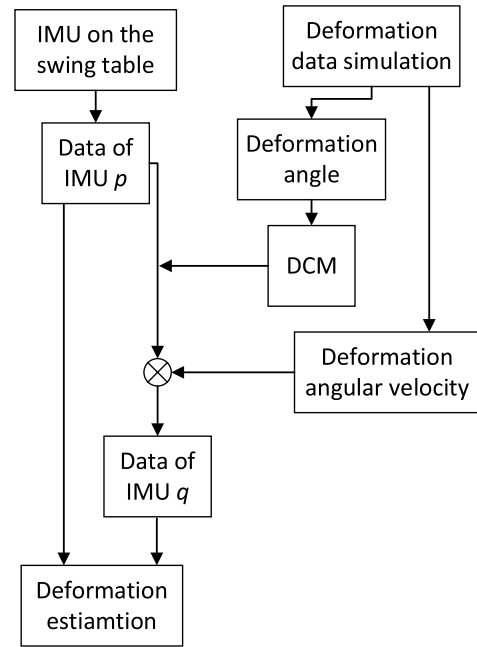


FIGURE 5. Diagram of the data simulation.

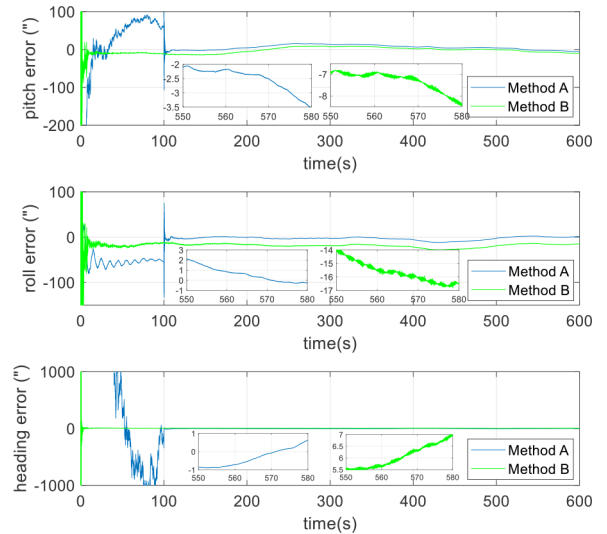


FIGURE 6. Deformation curves of simulation 1.

TABLE 6. Deformation error angle of simulation 2.

	Pitch	Roll	Heading
Method A	9.19"	1.25"	4.79"
Method B	10945"	5382"	14967"
Method C	102"	644"	314"

about 100" and the heading deformation angle error to about 1000" in 100 s. Even though Method B has a better result than the coarse estimation during the first 100 s, Method A converge in a short time and finish with higher accuracy than Method B. As the attitude is the integration of the angular velocity, using attitude matching will have smoother curves

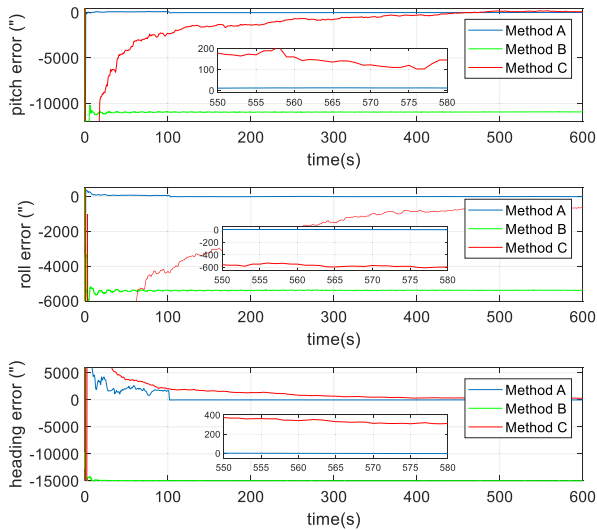


FIGURE 7. Deformation curves of simulation 2.

than angular velocity matching. Method B is easier to be influenced by the noise of the gyro.

In the second simulation from Figure 7, it could be found that Method B is failed to estimate the full deformation with high accuracy due to the large static deformation angle. The linear model is not appropriate for this situation. When Method B has an inaccurate static deformation model, it is not able to effectively estimate the deformation. Using UKF, Method C has a better result than Method B. But the curves of Method C are not smooth and need a longer time to converge. Compared to the first simulation, the accuracy of Method A is not decreased. The coarse estimation can reduce the deformation angle error to less than 1° , then the attitude matching algorithm in the inertial frame can get a high degree of accuracy.

From the two simulations, it can be found that the traditional velocity matching method is fast, but it becomes invalid when the deformation is large. It is also easy to be influenced by the noise of the gyro. Even though UKF is surely improved the effect with large deformation, the accuracy is reduced and more time is needed. The coarse estimation of the proposed method can get approximate deformation angles in about 1 minute and the system model can be simplified to a linear model, then the attitude matching in the inertial frame is used to calculate the deformation precisely.

V. CONCLUSION

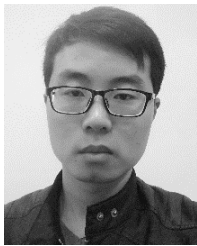
Hull deformation measurement based on inertial sensors is studied in this article. The filter model is established based on the inertial attitude matching, which is a linear model to estimate small deformation. Different from traditional transfer alignment, attitude matching in the inertial frame can be done without regarding one IMU as MINS and does not require initial alignment. And compared to the angular velocity matching, attitude matching is more stable. In order to deal with the large deformation situation, the coarse estimation based on the optimized attitude determination is employed. With

the coarse estimation in a short time, the system model can be considered as linear model, and the attitude matching can converge with high accuracy. The simulation indicates that the proposed method can fulfill the deformation measurement effectively. The following work will focus on the ship model and the time delay of the two IMUs.

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