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Research on a Differential Geometric Guidance Law Based on Fractional-Order Theory

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ABSTRACT In this paper, fractional-order calculus theory is used to investigate the geometric law for intercepting an agile target. In order to overcome the challenges presented by the divergence of line of sight rate (LOSR) of proportional navigation (PN), the fractional LOSR is used as a compensated term in the proposed fractional differential geometric guidance law (FDGGL). By adjusting the navigation gain of the FDGGL, the new proposed guidance law can be transformed into the traditional differential geometric guidance law (DGGL) and PN. The average overload and ballistic stability of the FDGGL are analyzed based on the fractional and control theory. Some analytical results about energy consumption and trajectory variation of the FDGGL were obtained. The simulation results shows that, compared with PN and DGGL, the FDGGL has better guidance performance when intercepting different maneuvering targets.

INDEX TERMS Missile guidance, differential geometric, fractional, relative motion, command.

I. INTRODUCTION

With advances in science and technology, high speed maneuvering targets have become a real threat, and intercepting maneuvering targets is a challenging task [1], [2]. It is difficult to meet guidance accuracy requirements with the traditional guidance law, and so it is necessary to study new guidance methods in order to deal with current threats from the air. The proportional guidance method is widely used in weapons systems because of its simple form and easy implementation in engineering [3]–[5]. However, at the end of interception, the proportional guidance method is prone to overload saturation caused by the divergence of the line of sight rate, which leads to full deflection of the actuator. A number of researchers have improved this guidance method by adjusting the proportion coefficient and compensating for the acceleration of the target, which greatly improves guidance performance [6]–[8]. Considering that the trajectories of interceptors and targets are both curves when they are engaged in space, and that differential geometry theory is just the classical mathematical theory for studying curves and surfaces, research on interceptor trajectories by using differential geometry theory provides a new perspective for guidance law design [9]–[12]. In reference [9], differential geometry theory is introduced into the guidance field for the first time.

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By establishing a line of sight coordinate system, it is beneficial to the trajectory equation of the interceptor described by curvature and torsion. Thus, the differential geometry guidance law in three-dimensional space is derived and the acquisition conditions of the differential geometry guidance law are analyzed. However, the method of transforming the guidance instructions from the arc domain to the time domain is not given in this paper. In reference [12], based on the theory of the involute in differential geometry, the involute of the interceptor is deduced by supposing the virtual intercept point, and then the trajectory of interceptor is obtained by using the involute of target. The research mainly focuses on non-maneuvering targets, and so can not be applied to a maneuvering target. The capture conditions of design guidance law are not further studied. Based on his research, combined with Frey's internal standard frame, Li *et al.* [13] designed curvature guidance instructions and torsion guidance instructions in a time domain, transformed the guidance instructions into a control of attack angle and sideslip angle of the interceptor, and analyzed the acquisition conditions of the designed guidance instruction. In order to overcome the loss of guidance information caused by three-dimensional space decoupling, Peng *et al.* [14] designed a novel differential geometry guidance law based on second-order sliding mode control and differential geometry theory. The transformation method of guidance instructions from arc domain to time domain is given, and the acquisition characteristics of the

guidance law are analyzed. The successful application of differential geometry theory in the field of guidance has opened new research perspectives for the research of guidance law. The combination of differential geometry theory and modern control theory has completed the transformation of guidance instructions from the arc length domain to the time domain, which has laid a theoretical foundation for the application of differential geometry theory.

When we consider the memory effect of fractional calculus, which is related to the state value of the system at a past time, a fractional derivative has global characteristics. The fractional derivative of the function shows the history and dependent process of the function change of the control system, while the conventional integral order only has the local property, and the integral derivative is only related to the state near to the derivative time [15], [16]. The differential geometry guidance law designed in reference [17] contains the first derivative of the guidance parameters, such as the relative velocity of missile and target, the angular velocity of the line of sight, etc. According to fractional calculus theory, the integral differential or integral of the guidance parameters only indicate the relative motion information of missile and target at the current time. The above guidance laws do not adopt historical information in the process of missile and target rendezvous. If the integral calculus of guidance parameters is extended to fractional calculus, the designed guidance law will contain more comprehensive rendezvous information, which will directly affect the final guidance effect. Reference [18] proposed a modified proportional guidance law based on fractional calculus for the problem of missile maneuvering target tracking. By selecting a Lyapunov-like function, it was theoretically shown that the designed fractional order guidance law can hit a maneuvering target with time-varying normal acceleration. In order to improve the robustness of intercepting the incoming target, Bin Feng Pan designed a PID type fractional order guidance law by combining finite time convergence theory and fractional order theory, and verified the guidance performance of the designed fractional order guidance law with a six degree of freedom simulation [19]. However, this method involves the calculation of the remaining flight time of interceptor, which will directly affect the final guidance accuracy. Reference [20] combines fractional order theory with the traditional PN shape, derives a fractional order extended proportional guidance law in three-dimensional space, and analyzes the average overload, trajectory straightness, and robustness of the guidance law. The simulation results show that the new fractional order guidance law can effectively overcome the problem of terminal overload divergence of the traditional guidance law, and has better performance guidance accuracy. The above fractional order guidance law, described in [18]–[20], is mainly used in the time domain.

The fractional order theory has never been applied to the arc length domain. If differential geometry theory is combined with fractional order theory, the traditional fractional calculus theory is extended to the differential geometry

guidance law, and the integral calculus of the guidance parameters in the differential geometry guidance law is extended to the fractional calculus, so as to make full use of historical information in the course of the engagement. This will significantly improve the guidance performance of the differential geometry guidance law.

Based on differential geometry theory, this paper analyzes the geometric relationship between the interceptor and target engagement with the help of Frenet. It designs the velocity direction expression of the interceptor when directly against the maneuvering target. Considering that the first derivative of the LOS angle is included in the designed guidance law, the fractional subdivision of the LOS angle is introduced to modify the designed guidance algorithm. Moreover, the difference between the designed guidance law and the traditional proportional guidance law is analyzed in terms of average overload and ballistic stability.

II. PRELIMINARY KNOWLEDGE

A. FRACTIONAL CALCULUS

The fractional order differential operators are defined as follows:

$$\alpha D_t^\lambda = \begin{cases} \frac{d^\lambda}{dt^\lambda} & \text{Re}(\lambda) > 0 \\ 1 & \text{Re}(\lambda) = 0 \\ \int_0^t (dt)^{-\lambda} & \text{Re}(\lambda) = 0 \end{cases} \quad (1)$$

where λ is the order of fractional calculus, $\text{Re}(\lambda)$ denotes the real component of λ , α represents the initial time, ${}_\alpha D_t^\lambda$ express the fractional order differential operation.

Definition 1: If the function can be derivative of n order in the domain of definition, here $m - 1 \leq \lambda \leq m$, and n is a positive integer, the Caputo fractional calculus is defined as:

$${}_\alpha D_t^\lambda f(t) = \frac{1}{\Gamma(m - \lambda)} \int_0^t \frac{f^m(\tau)}{(t - \tau)^{\lambda - m + 1}} d\tau \quad (2)$$

where $\Gamma(x)$ represents the gamma function, α is the initial time, and the general initial time in practical problems is 0, Therefore, formula 2 is abbreviated as $D^\lambda f(t)$.

Definition 2: The fractional calculus of Grunwald-Letnikov can be defined as follows:

$${}_\alpha D_t^\lambda f(t) = \lim_{h \rightarrow 0} \frac{1}{h^\lambda} \sum_{j=0}^{[(t-\alpha)/h]} \omega_j^\lambda f(t - jh) \quad (3)$$

where

$$\omega_j^\lambda = \frac{(-1)^j \Gamma(\lambda + 1)}{\Gamma(j + 1) \Gamma(\lambda - j + 1)} \quad (4)$$

$[x]$ represents the largest integer no greater than x . The calculation of ω_j^λ can be realized by recursion, i.e.

$$\omega_0^\lambda = 1, \omega_j^\lambda = \left[1 - \frac{1}{j} (\alpha + 1) \right] \omega_{j-1}^\lambda, j \in N. \quad (5)$$

The above two definitions are completely equivalent in a mathematical sense. However, definition 1 involves

high-order derivative and integral operations, which are usually used in theoretical analysis. Definition 2 uses the limit sum to define the fractional calculus. If the appropriate calculation step is selected, the fractional calculus of the function can be easily calculated. Numerical calculation of fractional order derivatives include Fourier series method, direct calculation method and frequency-domain filtering method. The direct calculation method is used in this paper.

B. THEORY OF DIFFERENTIAL GEOMETRY

The theory of differential geometry is used to study change in curve and surface in space. The flight path of the interceptor is described by using knowledge of differential geometry, which provides new ideas for the research of guidance methods. Curvature and torsion are commonly used to describe the degree of bending and the reverse of space curves in differential geometry theory. The relations among the tangent vector, the normal vector, and the sub normal vector of point P on the curve meet the following formula [17]:

$$\begin{cases} \mathbf{t}' = k\mathbf{n} \\ \mathbf{n}' = -k\mathbf{t} + \tau\mathbf{b} \\ \mathbf{b}' = -\tau\mathbf{n} \end{cases} \quad (6)$$

where \mathbf{t} is the tangent vector, \mathbf{n} is the normal vector, and \mathbf{b} is the binormal vector. This set of formulas is also known as the Frenet formulas. This group of formulas form the basic formula of the space curve. Its characteristic is that the derivative of basic vector \mathbf{t} , \mathbf{n} and \mathbf{b} to arc length s can be expressed by linear combination of \mathbf{t} , \mathbf{n} and \mathbf{b} . The coefficients of Frenet formula coefficients form an antisymmetric matrix which can be expressed as follows:

$$\begin{pmatrix} 0 & k & 0 \\ -k & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix}.$$

The above basic knowledge is the basis of the fractional differential geometry guidance law.

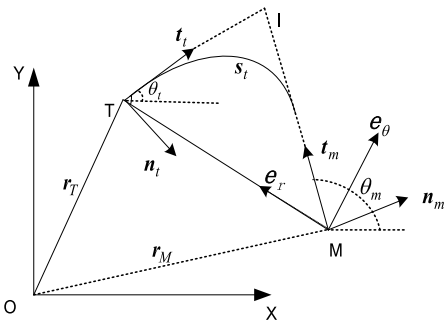


FIGURE 1. The engagement relation of the missile and the target.

III. DESIGN OF THE FRACTIONAL ORDER DIFFERENTIAL GEOMETRIC LAW

A. THE DIFFERENTIAL GEOMETRY MODEL OF RELATIVE MOTION BETWEEN MISSILE AND TARGET

The engagement of the missile and target is shown in Fig. 1. Where M represents the missile, T represents the target,

$\mathbf{t}_t, \mathbf{n}_t$ represents the unit tangent vector and the unit normal vector of target motion, θ_t is the angle between the target tangent vector and the OX axis. $\mathbf{t}_m, \mathbf{n}_m$ represents the unit tangent vector and the unit normal vector of missile motion, θ_m is the angle between the missile tangent vector and the OX axis, $\mathbf{e}_r, \mathbf{e}_\theta$ represent the unit vector along the line of sight direction and perpendicular to the line of sight direction. s_t, s_m are the arc length of the target and the missile respectively, and the arc length s of the missile motion is taken as the natural parameter.

In the course of missile motion, $\mathbf{t}_m, \mathbf{n}_m$ constitute the Frenet frame of missile motion. Assuming the velocities of missile and target are constant, the curve arc length s_m of missile motion is proportional to the curve arc length s_t of target motion, that is

$$s_t = ms_m \quad (7)$$

$$m = V_t/V_m \quad (8)$$

where m is the ratio of the target and missile velocities.

According to the geometric relationship between the missile and the target in Fig. 1, we have the following formula:

$$\mathbf{r}_m = \mathbf{r}_t - \mathbf{r}. \quad (9)$$

According to the trajectory of the missile and target, the velocity of the missile and target can be expressed as

$$V_m = \frac{ds_m}{dt}, V_t = \frac{ds_t}{dt}. \quad (10)$$

According to the Frenet frame, differential equation (9) on both sides can be obtained as follows:

$$V_m \mathbf{t}_m = V_t \mathbf{t}_t - V_m r' \mathbf{e}_r - r V_m \theta' \mathbf{e}_\theta \quad (11)$$

$$\mathbf{t}_m = m \mathbf{t}_t - r' \mathbf{e}_r - r \theta' \mathbf{e}_\theta. \quad (12)$$

The components of relative velocity along the \mathbf{e}_r direction and perpendicular to the \mathbf{e}_r direction can be expressed as

$$r' = m \cos \theta_t - \cos \theta_m \quad (13)$$

$$r \theta' = m \sin \theta_t - \sin \theta_m. \quad (14)$$

Differential equation (12) on both sides can be obtained as,

$$k_m \mathbf{n}_m = m^2 k_t \mathbf{n}_t - (r'' - r \theta'^2) \mathbf{e}_r - (r \theta'' + 2r' \theta') \mathbf{e}_\theta. \quad (15)$$

Formula (15) can be obtained by decomposing along the line of sight \mathbf{e}_r direction and perpendicular to the line of sight \mathbf{e}_r direction as follows:

$$r'' - r \theta'^2 = m^2 k_t (\mathbf{n}_t \cdot \mathbf{e}_r) - k_m (\mathbf{n}_m \cdot \mathbf{e}_r) \quad (16)$$

$$r \theta'' + 2r' \theta' = m^2 k_t (\mathbf{n}_t \cdot \mathbf{e}_\theta) - k_m (\mathbf{n}_m \cdot \mathbf{e}_\theta) \quad (17)$$

where k_m, k_t represent the curvature of missile and target.

B. DESIGN OF THE DIFFERENTIAL GEOMETRY GUIDANCE LAW

The differential geometric guidance law is proposed stemming from the idea of zeroing the rate of LOS. If there is a virtual pointing velocity V_{mp} , when the missile flies to the target along the virtual pointing velocity V_{mp} (the angle between the direction of V_{mp} and the X axis is θ_{mp}), the LOS rate can be guaranteed to be zero [9]. Then, according to the position relationship between the missile and the target in space, we can know that

$$m \sin \theta_t - \sin \theta_{mp} = 0. \tag{18}$$

When the LOS rate θ' is zero, Eq. (15) can be denoted as

$$k_{mp} = m^2 k_t \cos(\theta_t - \theta_{mp}) - r'' \sin \theta_{mp}. \tag{19}$$

From Eq. (16), we have

$$r'' = m^2 k_t \sin \theta_t - k_m \sin \theta_{mp}. \tag{20}$$

By substituting Eq. (13) and Eq. (20) into Eq. (19), we can find the following expression:

$$k_{mp} = m^2 k_t \left(1 + \frac{r'}{\cos \theta_{mp}} \right). \tag{21}$$

The differential of Eq. (18) gives us the following expression:

$$\frac{\theta'_{mp}}{\theta'_t} = \frac{m \cos \theta_t}{\cos \theta_{mp}}. \tag{22}$$

According to $r'_{mp} = m \cos \theta_t - \cos \theta_{mp}$, Eq. (22) can be expressed as

$$\frac{\theta'_{mp}}{\theta'_t} = \frac{r'_{mp}}{\cos \theta_{mp}} + 1. \tag{23}$$

The relation of the angle of the missile and the target can be denoted as

$$\theta'_{mp} = -\theta' - k_{mp} \tag{24}$$

$$\theta'_t = -\theta' - m k_t. \tag{25}$$

According to Eq. (21), Eq. (22), Eq. (23), Eq. (24), and Eq. (25), we have

$$k_{mp} = \frac{r'_{mp} \theta'}{\cos \theta_{mp}} + m k_t \left(\frac{r'_{mp}}{\cos \theta_{mp}} + 1 \right). \tag{26}$$

According to Eq. (26), We can see that the guidance commands contain two items. The first is related to the relative velocity and the LOSR, the second is directly related to the maneuverability of target, and both are related to the virtual pointing angle of the target. If there is a proportionality coefficient A in the first term, the guidance instruction can be described using the following expression:

$$k_m = \frac{A r'_m \theta'}{\cos \theta_m} + m k_t \left(\frac{r'_m}{\cos \theta_m} + 1 \right). \tag{27}$$

The new differential geometry guidance law can be regarded as an extension of the traditional proportional guidance law.

The curvature command tries to adjust the velocity direction of the missile in each step, so that it can meet the zero-effort intercept triangle state as much as possible and minimize the rotation rate of the line of sight. The guidance performance of the DGGL has been deeply analyzed and simulated in reference [9], and will not be discussed here. However, the first item contains the integral order information of the LOSR. It mainly uses the guidance information at the current time, but not the historical guidance information. In order to give full value to the guidance information in the whole interception process, the fractional compensation term of LOSR is added to the traditional DGGL to further improve the guidance performance of the proposed guidance law.

Considering the advantages of fractional calculus, the new fractional differential geometry guidance law (FDGGL) can be expressed as follows:

$$k_m = \frac{A r'_m}{\cos \theta_m} (\theta' + {}_\alpha D_t^\lambda \theta) + m k_t \left(\frac{r'_m}{\cos \theta_m} + 1 \right). \tag{28}$$

The new guidance law includes the first order integral derivative term of the LOSR and fractional order term of the LOSR ${}_\alpha D_t^\lambda \theta$, which contains the history information of the LOSR in the whole interception process. If λ equals zero, the FDGGL can be transformed into the DGGL.

IV. PERFORMANCE ANALYSIS OF THE fdggl

A. QUALITATIVE ANALYSIS OF THE AVERAGE OVERLOAD PERFORMANCE OF THE FDGGL

The average overload of the guidance law reflects energy consumption in the interception process and in a sense represents the capability demand for the actuator. The FDGGL contains the interceptor's velocity and angle information, the target's angle information, velocity, maneuverability, and other information. According to the relationship between the trajectory curvature and the overload in differential geometry, the average overload is the integral of the overload to time in the time domain, while the average curvature command is the integral of curvature instruction to arc length in differential geometry. Next, we analyze the characteristics of the mean curvature command.

In order to simplify the analysis process, the guidance curvature command is expressed as

$$k_m = N_1 \theta'_L + N_2 D_t^\lambda \theta + F(k_t) \tag{29}$$

where $N_1 = \frac{A r'_m}{\cos \theta_m}$, $N_2 = \frac{A \lambda r'_m}{\cos \theta_m}$, $F = m k_t \left(\frac{r'_m}{\cos \theta_m} + 1 \right)$.

According to the definition of average overload, the average overload of the designed guidance law can be expressed as

$$\bar{u}_{FDGGL} = \frac{1}{s_f} \int_0^{s_f} k_m ds = \frac{1}{s_f} \int_0^{s_f} (N_1 \theta' + N_2 D_t^\lambda \theta + F(k_t)) ds \tag{30}$$

For convenience, the average curvature instruction can be expressed as follows:

$$\begin{aligned} u_1 &= \frac{1}{s_f} \int_0^{s_f} N_1 \theta' ds \\ u_2 &= \frac{1}{s_f} \int_0^{s_f} N_2 D_t^\lambda \theta ds. \\ u_3 &= \frac{1}{s_f} \int_0^{s_f} F(k_t) ds \end{aligned} \quad (31)$$

The subscripts O and F in Eq. (30) and Eq. (31) represent the initial arc length and end arc length of the integral. After integrating u_1 and u_2 , according to the Eq. (3), we can get the following results:

$$u_1 = \frac{1}{s_f} \int_0^{s_f} N_1 \theta' ds = \frac{N_1(\theta_f - \theta_0)}{s_f} \quad (32)$$

$$\begin{aligned} u_2 &= \frac{1}{s_f} \int_0^{s_f} N_2 D_t^\lambda \theta ds \\ &= \lim_{h \rightarrow 0} \frac{N_2}{s_f} \int_0^{s_f} h^{-\lambda} \sum_{j=0}^{[s/h]} \omega_j^\lambda \theta(s - jh) ds \\ &= \lim_{h \rightarrow 0} \frac{N_2}{s_f} h^{-\lambda} \sum_{j=0}^{[s/h]} \omega_j^\lambda \int_0^{s_f} \theta(s - jh) ds \\ &\approx \frac{N_2 h^{-\lambda}}{s_f} \sum_{j=0}^{[s/h]} \omega_j^\lambda (\theta_f - \theta_0) \end{aligned} \quad (33)$$

The term u_2 of the FDGGL can be expressed as

$$u_2 \approx \frac{\bar{N}_2}{s_f} (\theta_f - \theta_0). \quad (34)$$

where $\bar{N}_2 \approx \frac{N_2 h^{-\lambda}}{s_f} \sum_{j=0}^{[s/h]} \omega_j^\lambda$, the mean curvature can be written as

$$\bar{u} = (N_1 + \bar{N}_2) \frac{(\theta_f - \theta_0)}{s_f} + \frac{1}{s_f} \int_0^{s_f} F(k_t) ds. \quad (35)$$

According to the same method, the mean curvature of the DGGL and PN can be calculated as [20]

$$\bar{u}_{DGGL} = N_1 \frac{(\theta_f - \theta_0)}{s_f} + \frac{1}{s_f} \int_0^{s_f} F(k_t) ds \quad (36)$$

$$\bar{u}_{PN} = N_{PN} \frac{(\theta_f - \theta_0)}{s_f} \quad (37)$$

where N_{PN} is the proportional coefficient of the traditional PN guidance law. According to Eq. (35), Eq. (36), and Eq. (37), it can be found that the average overload of the newly designed FDGGL is closely related to the change of line of sight angle in the whole interception process. When the

line of sight angle becomes larger or smaller, the direct average overload will also become larger or smaller. By adjusting the ratio coefficients N_1 and N_2 , the FDGGL can be transformed into DGG and PN, and a reasonable ratio N_2 can be set to adjust the average overload of interceptors, so as to ensure that the average overload changes more smoothly in the whole interception process.

B. ANALYSIS OF BALLISTIC STABILITY OF THE FDGGL

Another index to evaluate the performance of the guidance law is ballistic stability. The change in ballistic angle in missile flight reflects the fluctuation of the trajectory, so it can represent the quality of the designed guidance law. The ballistic front angle is the angle between the missile velocity direction and the line of sight, which can be expressed as

$$\eta_m = \theta - \theta_m. \quad (38)$$

In differential Eq. (38), we have

$$\eta'_m = \theta' - \theta'_m. \quad (39)$$

According to the previous analysis, we know that the change of missile velocity angle can be expressed by curvature expression. We have

$$\theta_m = \int_0^{s_f} k_m ds. \quad (40)$$

The expression of ballistic angle can be obtained by analyzing Eq. (39) as follows:

$$\begin{aligned} \eta_{mf} &= \eta_{m0} + (1 - N_1)(\theta_f - \theta_0) - N_2 \int_0^{s_f} D_t^\lambda \theta ds - \int_0^{s_f} F(k_t) ds \\ &= \eta_{m0} + (1 - N_1)(\theta_f - \theta_0) - N_2 h^{-\lambda} \\ &\quad \times \sum_{j=0}^{[s/h]} \omega_j^\lambda (\theta_f - \theta_0) - \int_0^{s_f} F(k_t) ds \end{aligned} \quad (41)$$

Let $\tilde{N}_2 = N_2 h^{-\lambda} \sum_{j=0}^{[s/h]} \omega_j^\lambda$. The angle between the missile velocity direction and the line of sight can be denoted as

$$\eta_{mf} = \eta_{m0} + (1 - N_1 - \tilde{N}_2)(\theta_f - \theta_0) - \int_0^{s_f} F(k_t) ds \quad (42)$$

Thus the change of ballistic angle can be expressed as

$$\theta_f - \theta_0|_{FDGG} = \frac{\eta_{mf} - \eta_{m0} + \int_0^{s_f} F(k_t) ds}{1 - N_1 - \tilde{N}_2}. \quad (43)$$

From the traditional DGGL, we know that the change of ballistic angle is

$$\theta_f - \theta_0|_{DGG} = \frac{\eta_{mf} - \eta_{m0} + \int_0^{s_f} F(k_t) ds}{1 - N_1}. \quad (44)$$

As we know the proportional coefficient in traditional guidance is more than 1, comparing Eq. (43) and Eq. (44) we know

$$|\theta_f - \theta_0|_{DGG} > |\theta_f - \theta_0|_{FDGGL} \quad (45)$$

According Eq. (45), we know that the change of ballistic angle using the DGGL is bigger than that using the FDGGL. This means that the trajectory of the FDGGL is flatter than that of DGGL. The major factor is the fractional order term modifying the trajectory and reducing the change of ballistic angle. And so, the designed FDGGL law will consume less energy and the actuators can be easier to control.

V. NUMERICAL SIMULATION

In this section, some simulation results are presented to prove the effectiveness of the FDGGL. In order to illustrate the guidance effect of the designed guidance law in detail, four cases of target maneuvering are used for simulation and comparison. The simulation results are presented in this section together with a comparison of the traditional DGGL and PN. The initial parameters of the missile and target are listed in Table 1. According to the method of ergodic optimization [20], the order of fractional calculus is 1.5. Here, the proportional coefficient is 3. In the present context, we assume that the rate of line of sight is contaminated with uncertainty during the terminal guidance phase. The test error of the seeker is $0.01 \text{ }^\circ/\text{s}$.

TABLE 1. Initial states of missile and target.

Parameters of missile and target	
(X_{m_0}, Y_{m_0})	$(0, 0)\text{km}$
(X_{t_0}, Y_{t_0})	$(20,20)\text{km}$
θ_t	175°
θ_m	30°
V_m	1000
V_t	400

The four cases are respectively as follows:

Case 1, Assuming the target makes an S-shaped escape maneuver, the acceleration of the target is

$$a_t = -10\text{sign}\left(\sin\frac{\pi t}{12}\right)m/s^2.$$

Case 2. Assuming the target makes an S-shaped escape maneuver, the acceleration of the target is

$$a_t = -20\text{sign}\left(\sin\frac{\pi t}{12}\right)m/s^2.$$

Case 3. Assuming the target makes a circular escape maneuver, the acceleration of the target is 10m/s^2 .

Case 4. Assuming the target makes a circular escape maneuver, the acceleration of the target is 20m/s^2 .

In order to compare the performance of the FDGGL, the DGGL and PN more intuitively in this paper, the guidance

parameters of the interceptor are transformed from the arc length domain to the time domain. According to the relationship between the curvature command and overload in reference [17], the curvature command of the interceptor is transformed into the overload. By the definition of a relative velocity coordinate system in reference [21], the relative trajectory performance of different guidance methods is compared and analyzed as well.

The comparison of the interception performance between the FDGGL and the DGGL is presented in a Monte Carlo sense and is listed in Table 2, where Miss is the minimum distance between the interceptor and the target. Mean_Am denotes the mean miss distance, T is the intercept time, and Max_Am denotes the maximum absolute load.

TABLE 2. Comparison of interception performance.

	Guidance Law	T/s	Miss/m	Max_Am/m /s ²	Mean_Am/m/s ²
Case 1	FDGGL	25.18	0.568	103.693	32.438
	DGGL	25.23	2.689	95.225	33.034
	PN	25.30	1.450	400.00	34.451
Case 2	FDGGL	26.36	1.058	142.783	31.842
	DGGL	26.57	4.019	101.886	32.846
	PN	26.69	1.850	400.00	34.467
Case 3	FDGGL	32.11	1.679	142.378	29.104
	DGGL	32.39	2.997	101.878	29.005
	PN	32.58	3.009	290.586	33.414
Case 4	FDGGL	27.05	0.177	135.725	29.560
	DGGL	27.35	0.951	95.025	31.848
	PN	27.45	2.911	85.553	32.937

As indicated in Table 1, regardless of the type of target, all the guidance schemes perform well. The interception time of the four cases with the FDGGL, the DGGL and PN are close. However, the FDGGL performs better than the DGGL and PN. Just as in case 3, the interception time of the FDGGL is 32.11 seconds, compared with 32.39 seconds for the traditional PN guidance law and 32.58 seconds for the DGGL. The interception time of the guidance law designed in this paper is significantly shorter, which means that the missile can destroy the target earlier. It is also interesting to note that the miss of the FDGGL is nearly 100% better than traditional DGGL and PN guidance, which indicates that the FDGGL greatly improves guidance accuracy. We can observe that the FDGGL has superior performance to the DGGL and PN in mean acceleration, which demonstrates that the FDGGL demands less control effort. The maximum acceleration of the FDGGL is larger than that of the DGGL, and smaller than that of PN, which means that the FDGGL needs to adjust

the state of the interceptor more quickly than the DGGL. Moreover, the FDGGL overcomes the acceleration saturation of traditional PN guidance.

The simulation diagrams for case 2 and case 3 are shown respectively from Fig. 2 to Fig. 11.

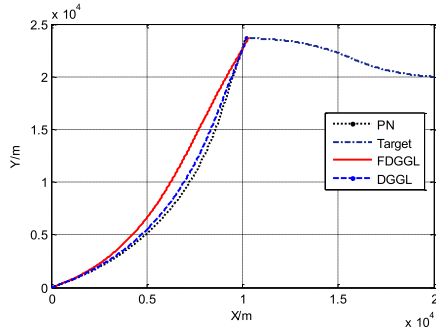


FIGURE 2. The three dimensional engagement.

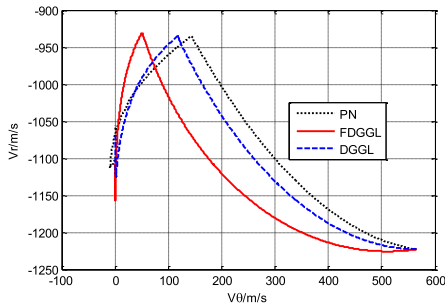


FIGURE 3. Relative motion trajectory.

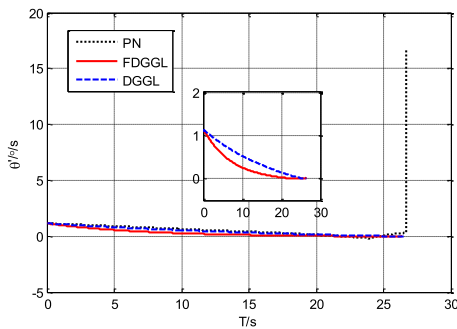


FIGURE 4. Time history of the LOSR.

The trajectories of interceptors guided by the FDGGL, DGGL, and PN are shown in Fig. 2 and Fig. 7, when the target makes a circle or S maneuver. The trajectory of the FDGGL is smoother than that of DGGL and PN guidance. Referring to Fig. 3 and Fig. 8 in the relative motion diagram, the FDGGL has better ballistic performance than the other two guidance laws. It can be seen from the LOSR diagram in Fig. 4 and Fig. 9 that the FDGGL can make the line of sight rate approach zero in a limited time, which is similar to the change in the line of sight rate of the DGG, and overcomes

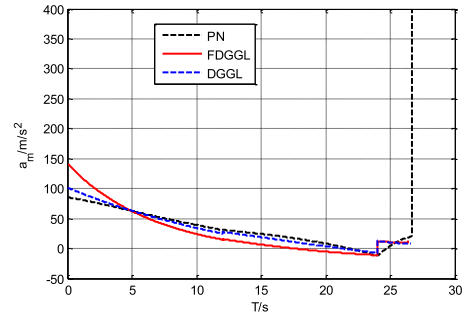


FIGURE 5. Time history of missile acceleration.

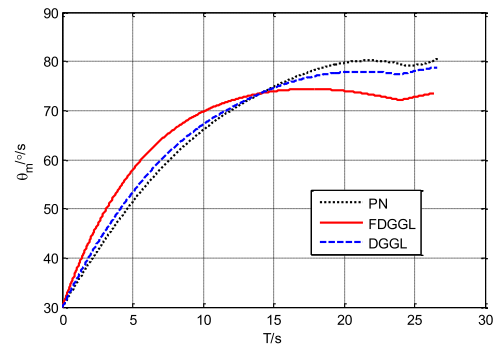


FIGURE 6. Time history of ballistic angle.

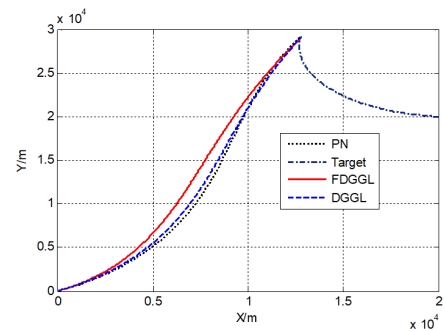


FIGURE 7. The two dimensional engagement.

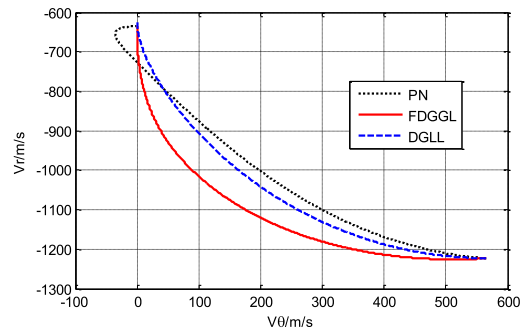


FIGURE 8. Relative motion trajectory.

the disadvantage of the divergence of the line of sight rate at the end of PN. This means that the FDGGL consumes less energy at the end of the interception, and greatly reduces the design requirements for the actuator of the missile.

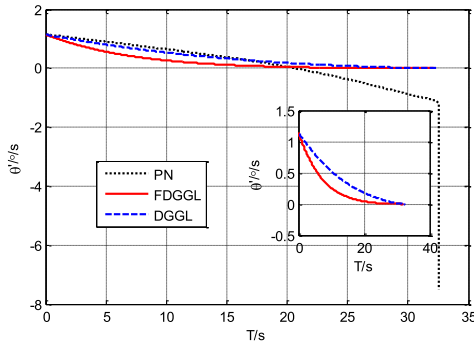


FIGURE 9. Time history of the LOSR.

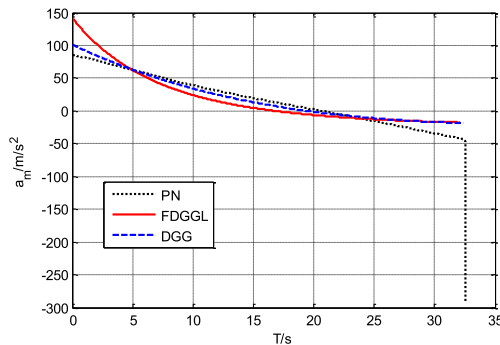


FIGURE 10. Time history of missile acceleration.

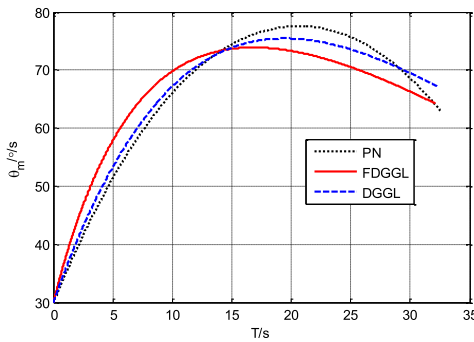


FIGURE 11. Time history of ballistic angle.

As illustrated in Fig. 5 and Fig. 10, the acceleration of the FDGGL is larger than that of the DGGL and PN at the early stage of interception, and the acceleration of the FDGGL drops rapidly as the interception goes on. The main reason for this is that the fractional order item of the FDGGL plays the role of filter, which improves the control precision and stability of the LOSR. The FDGGL enables the missile to adjust its attitude with a large overload at the early stage of interception, which ensures that the flight trajectory is smooth and avoids the surge of the overload at the moment of the encounter, thus greatly cutting down the design difficulty of the actuator. This phenomenon is identical to the idea of the designed guidance law in this paper.

As can be seen from Fig. 6 and Fig. 11, the FDGGL has the slowest change of ballistic angle among the three guidance laws. It shows that the trajectory of the FDGGL is straighter.

This is consistent with the theoretical analysis of ballistic stationarity in this paper.

It is obvious that, compared with the PN and DGGL, the FDGGL designed in this paper has better guidance performance. The main reason is that the FDGGL solves the problem of LOSR estimation by a fractional differential term. The fractional differential term contains some information about the previous LOS angle rate, which is more conducive to LOS control accuracy and the stability of the guidance system in the interception process. In short, the FDGGL keeps the advantages of the DGGL and avoids the disadvantages of the traditional PN guidance law in some sense.

VI. CONCLUSION

In this paper, a novel differential geometric guidance law is presented that aims to improve guidance performance based on the fractional order theory. The interception model of missile and target is established based on the differential geometry theory. The guidance performance of the FDGGL, such as curvature command and ballistic stability, are theoretically analyzed. Compared with the DGGL and PN guidance, the proposed FDGGL algorithm is more effective in controlling the line of sight, and has a shorter intercept time and smoother trajectory. The FDGGL adjusts the missile's altitude at the beginning of the intercepting trajectory, so that its overload saturation is avoided during the encounter. Some simulation results demonstrated that the designed new guidance law has satisfactory guidance characteristics when utilized against maneuvering targets.

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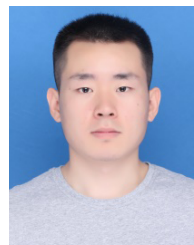
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