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Establishment and Verification of Nonlinear Bolt Head Connection Stiffness Theoretical Model Based on Levenberg-Marquardt Method

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ABSTRACT Although some scholars have established the calculation method of bolt head connection characteristics, the method contains fewer influence factors; the calculation accuracy is not high, and the general applicability is limited. Based on the Levenberg Marquardt optimization algorithms, a more accurate calculation model of bolt head connection stiffness is proposed in this article. The model includes many parameters such as bolt head thickness, equivalent diameter, bolt diameter, thickness of connected parts, elastic modulus of bolt and elastic modulus of connected parts. Through the finite-element verification and comparison with the results of the Fears Alkatan method, the calculation results of the bolt head connection stiffness calculation model in this article are effective and accurate, which provides a universal calculation model for calculating the bolt head connection stiffness of various parameters, and fills the shortage of inaccurate calculation of the current theoretical model of bolt head stiffness.

INDEX TERMS Bolt head, stiffness, finite element, connected parts.

I. INTRODUCTION

Based on the advantages of bolt connection, such as convenient, easy disassembly and easy replacement, bolt connection is one of the common connection methods of mechanical connection. In some precision machinery, the requirement of bolt connection precision is higher. Because of the influence of bolt connection stiffness on the static and dynamic characteristics of equipment connection, especially the high-precision mechanical device, this influence cannot be ignored, so it is necessary to study. When bolted connection is used (see Fig. 1), the bolt head in bolt connection also has important influence on static and dynamic performance of equipment. Therefore, many scholars have carried out research in this field.

Wang *et al.* [1] studied the elastic interaction stiffness and its effect on Bolt Preloading. Zhu *et al.* [2] carried out numerical simulation analysis on the single bolt connection structure of a carbon-fiber square tube. Nassar and Abbound [3] proposed a better model for calculating the stiffness of bolted connections, and compared with the finite-element results.

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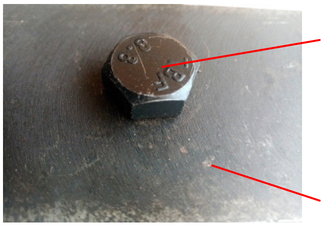


FIGURE 1. Bolt head in bolt connection.

Pedersen and Pedersen [4] have found a simplified expression of member stiffness, including the case that the width of a member is limited, and put forward a practical stiffness calculation formula. In order to obtain the deformation mechanism of bolt members, Yang *et al.* [5] studied the prediction of member stiffness and characterization of interface contact conditions, and constructed a novel and effective finite-element model. Wang *et al.* [6] proposed a three-dimensional axisymmetric analytical model of bolted connections to obtain the joint stiffness of bolted connections. Wei *et al.* [7] systematically studied the influence of surface roughness, strength and stress of contact surface on the natural frequency of bolt structure. Hua *et al.* [8] proposed the finite-element analysis method of bolt connection structure under the optimal preload, and studied the influence of bolt preload, connector thickness and outer diameter and nut parameters on the connection stiffness. Alkatan *et al.* [9] proposed a new method to calculate the axial stiffness of bolts, nuts and fastening plates, and proposed the calculation method of axial stiffness based on an empirical formula, and provided a method of finite-element modeling on bolted connections using to beam elements in the future. Miller *et al.* [10] established the spring model of thread force's analysis, and combined with mathematical theory, analyzed the thread stress. Cai *et al.* [11] carried out an experimental test to elucidate the effect of the joint surface. Wileman *et al.* [12] provided a simple technique for calculating the stiffness of members. Wang and Marshek [13] proposed an improved spring model to analyze the load distribution of thread. The Sopwith [14] method provides a method to calculate the axial force distribution of threaded connections. Yamamoto [15] provides a method of calculating the thread stiffness, which cannot only to calculate the axial force distribution of the thread, but also calculate the stiffness of the threaded connection.

Chen *et al.* [16] established a three-dimensional finite-element parametric model of screw connection based on ANSYS software, analyzed the load distribution law of the screw pair, and analyzed the influence of various parameters of the load distribution law of the screw pair. Chaaban and Jutras [17] studied the thread stress field in the end cover of thick wall high pressure vessels by using the finite-element method. The elastic analysis was carried out for some vessels with standard serrated thread, and a prediction method of load distribution along the thread length direction was proposed. Kenny and Patterson [18] found the load distribution and normalized stress distribution in the thread of ISO metric nuts and bolts through the photoelastic analysis of freezing stress by using the fringe multiplication polarimeter and recording micro optical densitometer. Liao *et al.* [19] considered the influence of material plastic deformation on axial load and stress distribution. A new convergence criterion for nonlinear finite-element analysis of loading and unloading process is proposed. Grewal and Sabbaghian [20] used analytical method and finite-element method to analyze the load distribution in threaded connection, and studied the influence of thread shape, wall thickness of supporting thread, thread spacing, number of engaged threads and boundary conditions on the load distribution in threaded connection Wang and Marshek [13] put forward an improved spring model of threaded connection, which distinguishes between compression part and expansion part, compares the load distribution of elastic threaded connection and yield threaded connection, and analyzes the influence of thread yield on axial load distribution. Chen *et al.* [21] carried out theoretical analysis and numerical simulation on the axial load distribution of screw thread pairs, studied the parameters affecting the axial load distribution, and proposed an analytical method for determining the axial force distribution by using the curve of the relationship between the axial force per unit length and the corresponding deformation of the thread teeth. Chen *et al.* [22] proposed an analytical model for the pre tightening process of bolt connections with non parallel bearing surfaces, revealed the mechanism of pre tightening deviation of bolts on non parallel bearing surfaces, and analyzed the tightening behavior of bolt connections with non parallel supporting surfaces by the parametric finite-element method. Zhang *et al.* [23] proposed a new method to calculate the thread stiffness according to the load distribution in the thread. The accuracy of the proposed model was verified by establishing the finite-element analysis of the three-dimensional model of the threaded connection and a series of tensile tests. Zhang *et al.* [24] proposed a new method to predict the load distribution of threaded connections. Compared with the results of three-dimensional finite-element analysis and Yamamoto's method, the correctness of the calculation results is verified.

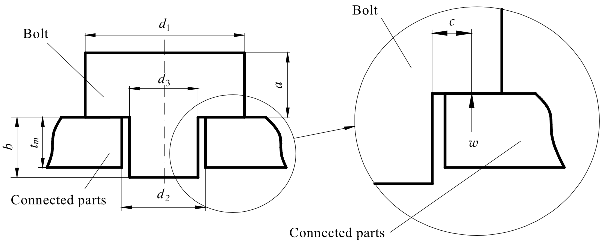


FIGURE 2. Bolt head and member.

As mentioned above, most of the previous studies focused on the connected parts and screw connection stiffness. Except for Alkata *et al.* [9], few studies have involved the connection stiffness of the bolt head. According to the literature, Alkata *et al.* [9] and others have proposed a new method for calculating the axial stiffness of multiple elements of bolts (head and meshing part), nuts and fastening plates. By fitting the finite-element analysis results of the bolt head, an expression is obtained. Although the method has certain applicability, it cannot get the influence of bolt head thickness, equivalent diameter and other factors on the stiffness because of the few influencing factors. Therefore, it is a rough calculation formula for bolt head stiffness, and has great limitations. Although some scholars have established the calculation method of bolt head connection characteristics, it is a rough calculation method of bolt head stiffness characteristics, which contains less influencing factors, and its universal applicability and application are limited. Therefore, it is necessary to study this subject in depth.

II. ESTABLISHMENT OF THEORETICAL MODEL OF BOLT HEADS CONNECTION STIFFNESS

A. DEFORMATION CALCULATION

In bolt connection, the bottom surface of the bolt head contacts with the upper surface of the connected part (see Fig. 2). In practical application, the bolt should be pre tightened, and the lower surface of the bolt head is subjected to upward load. If the load is set as F_b, the bolt head will deform under this load. According to the stiffness formula, if the connection

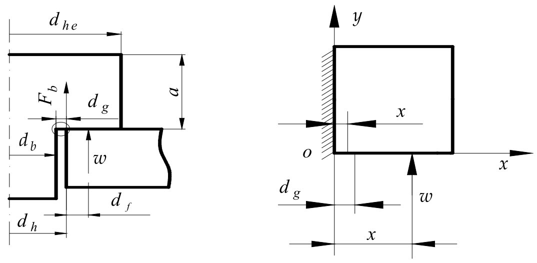


FIGURE 3. **Bolt head connection parameters.**

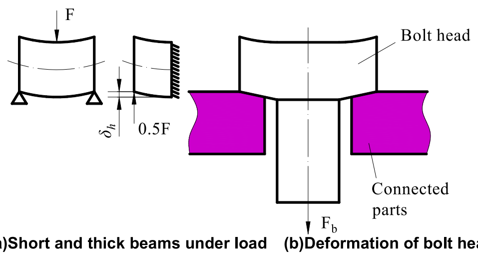


FIGURE 4. **Stiffness transformation diagram of bolt head based on Timoshenko beam theory.**

stiffness of the bolt head is calculated, these deformations should be calculated. For the convenience of calculation, the hexagon bolt head is transformed into a round bolt head, and the circular bolt head is used to replace the hexagon bolt head for finite-element calculation. The conversion method is as follows: let the hexagonal area of the bolt head equal to the area of a circle, and calculate the diameter of the circle, which is the diameter of the converted round bolt head.

According to Timoshenko beam theory, under the action of load, the beam will not only produce deflection under the action of bending moment, but also produce additional deflection under the action of shear force. When the ratio of height and length of the beam is small, the deflection due to shear force is small and can be ignored; When the ratio of beam height to length reaches a certain value (i.e. the beam belongs to short thick beam), the additional deflection caused by shear force cannot be ignored; As shown in Fig. 4, under the action of a unit load F_b, the deflection calculation formula is

$$\frac{d^2y}{dx^2} = -\frac{1}{E_{bolt} I_z(x)}\left[M_{head} + \frac{\alpha E_b I_z(x)}{A_{head}(x)\, G_h} F_b\right] \tag{1}$$

where x is the location of unit load. E_{bolt} is the elastic modulus of bolt head. $I_z(x)$ is the moment of inertia of bolt head section. M_{head} is the bending moment produced by unit load. α is the numerical factor, for rectangular section, $\alpha = 1.5$. G_h is the shear modulus of bolt material. $A_{\mathrm{head}}(x)$ is the sectional area.

Because the bolt head is circular. It can be regarded as a variable cross-section beam, in essence, and its section width and moment of inertia can be expressed as

$$\begin{cases} l(x) = \pi\,(d_b + 2x) \\ I_z(x) = \pi\,(d_b + 2x)\,a^3/12 \end{cases} \tag{2}$$

where $l(x)$ is the section width, $0 < x < (d_{\mathrm{h}} - d_{\mathrm{b}})/2$.

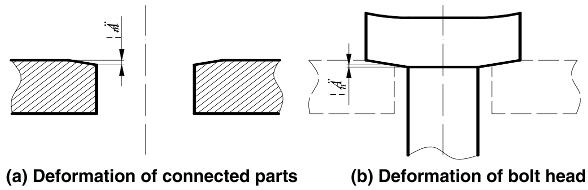


FIGURE 5. **Breakdown diagram of bolt head connection deformation.**

The sectional area and bending moment can be expressed as

$$\begin{cases} A_{head}(x) = \pi a\,(d_b + 2x) \\ M_{head} = F_b\,(d_h - d_b)/2 \end{cases} \tag{3}$$

If formula (2) and (3) are substituted for equation (1), then

$$\frac{d^2y}{dx^2} = -\frac{12}{E_{bolt}\pi\,(d_b+2x)\,a^3}\left(\frac{d_h-d_b}{2} + \frac{\alpha E_b a^2}{12G_h}\right)F_b \tag{4}$$

The first integral of formula (4) can be obtained

$$\frac{dy}{dx} = -\frac{12}{E_{bolt}\pi a^3}\left(\frac{d_h-d_b}{2} + \frac{\alpha E_b a^2}{12G_h}\right)F_b \cdot \frac{1}{2} ln\,(2x+d_b) + C_1 \tag{5}$$

The second integral of formula (4) can be obtained

$$\begin{aligned} y = &-\frac{12x}{E_{bolt}\pi a^3}\left(\frac{d_h-d_b}{2} + \frac{\alpha E_b a^2}{12G_h}\right)F_b \\ &\cdot\frac{1}{2}\left\{[ln\,(2x+d_b)]\,x - x + \frac{d_b}{2} ln\,(2x+d_b)\right\} \\ &+ C_1 x + C_2 \end{aligned} \tag{6}$$

In this article, it is assumed that the angle $dy/dx = 0$ at the root of the bolt head, and the angular deformation is 0 at the root of the bolt heads $x = 0$, which is substituted into equation (6). So, you can get

$$\begin{aligned} y = &-\frac{12x}{E_{bolt}\pi a^3}\left(\frac{d_h-d_b}{2} + \frac{\alpha E_b a^2}{12G_h}\right)F_b \\ &\cdot\frac{1}{2}\left\{[ln\,(2x+d_b)]\,x - x + \frac{d_b}{2} ln\,(2x+d_b)\right\} \\ &+ \frac{6}{E_{bolt}\pi a^3}\left(\frac{d_h-d_b}{2} + \frac{\alpha E_b a^2}{12G_h}\right)F_b \cdot ln\,(d_b)\,x \\ &+ \frac{12x}{E_{bolt}\pi a^3}\left(\frac{d_h-d_b}{2} + \frac{\alpha E_b a^2}{12G_h}\right)F_b \cdot \frac{d_b}{4} ln\,(d_b) \end{aligned} \tag{7}$$

Therefore, when $x = l_h = (d_h - d_b)/2$, the deformation of the bolt head is obtained (as shown in Fig. 5(b)) as

$$\begin{aligned} \delta_h = &-\frac{12}{E_{bolt}\pi a^3}\left(\frac{d_h-d_b}{2} + \frac{\alpha E_b a^2}{12G_h}\right)F_b \\ &\cdot\frac{1}{2}\left\{[ln\,(d_h)]\cdot\frac{d_h-d_b}{2} - \frac{d_h-d_b}{2} + \frac{d_b}{2} ln\,(d_h)\right\} \\ &+ \frac{6}{E_{bolt}\pi a^3}\left(\frac{d_h-d_b}{2} + \frac{\alpha E_b a^2}{12G_h}\right)F_b \cdot ln\,(d_b)\cdot\frac{d_h-d_b}{2} \\ &+ \frac{12}{E_{bolt}\pi a^3}\left(\frac{d_h-d_b}{2} + \frac{\alpha E_b a^2}{12G_h}\right)\cdot\frac{d_h-d_b}{2} F_b \cdot \frac{d_b}{4} ln\,(d_b) \end{aligned} \tag{8}$$

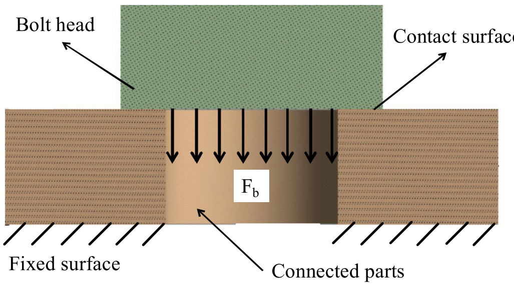


FIGURE 6. **Principle of finite element analysis.**

According to the definition of stiffness, the stiffness of the bolt head can be expressed as

$$k_h = \frac{F_b}{\delta_h} = 1 \Bigg/ \left\{ \begin{array}{l} -\dfrac{12}{E_{bolt}\pi a^3}\left(\dfrac{d_h-d_b}{2}+\dfrac{\alpha E_b a^2}{12G_h}\right) \\ \cdot\dfrac{1}{2}\left\{[ln\,(d_h)]\cdot\dfrac{d_h-d_b}{2}-\dfrac{d_h-d_b}{2}+\dfrac{d_b}{2}ln\,(d_h)\right\} \\ +\dfrac{6}{E_{bolt}\pi a^3}\left(\dfrac{d_h-d_b}{2}+\dfrac{\alpha E_b a^2}{12G_h}\right)\cdot ln\,(d_b)\cdot\dfrac{d_h-d_b}{2} \\ +\dfrac{12}{E_{bolt}\pi a^3}\left(\dfrac{d_h-d_b}{2}+\dfrac{\alpha E_b a^2}{12G_h}\right)\cdot\dfrac{d_h-d_b}{2} \\ \cdot\dfrac{d_b}{4}ln\,(d_b) \end{array} \right\} \tag{9}$$

B. FINITE-ELEMENT CORRECTION OF STIFFNESS FORMULA

When calculating the stiffness of bolt heads based on Timoshenko beam theory, the concentrated load is located at the edge of the threaded hole. Since the contact between the bolt head and the connected part is surface contact, the actual load action position is not concentrated on the edge of the threaded hole. Therefore, the finite-element method is needed to correct formula (9).

The finite element analysis stiffness of bolt head is k_{FEAH}. And order

$$\alpha_{head} = \frac{k_{FEAH}}{k_h} \tag{10}$$

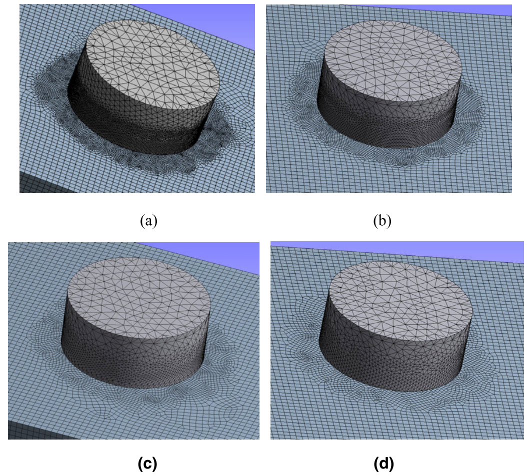
α_{head} is called the stiffness correction factor.

1) FINITE-ELEMENT MODEL AND SINGLE FACTOR ANALYSIS

In order to study the displacement of the bottom surface under the bolt head load, the model is simplified in this article. The length of the polished rod of the bolt head is taken as zero, and the load acts on the bottom surface (as shown in Fig. 6).

2) INFLUENCE OF MESH ON STIFFNESS

Experience shows that the mesh size has an effect on the analysis results. In this article, the mesh density of the bolt heads joint surface and nearby area is higher, and coarse mesh is selected for non joint area. In order to obtain more accurate bolt head stiffness, the bolt head connection stiffness



(a) The overall mesh size is 1 mm, the contact surface is 0.3 mm (b) the overall grid size is 1 mm, the contact surface is 0.4 mm (c) the overall grid size is 1 mm, the contact surface is 0.5 mm (d) the overall grid size is 1 mm, and the contact surface is 0.6 mm

FIGURE 7. **Mesh generation results of the model(M16).**

of various specifications for the mesh size of various bolts joint surface contact areas (part of the model mesh size is shown in Fig. 7) is analyzed, and some analysis results are shown in Fig. 8.

In the case of the influence of mesh size of the analysis results, this article selects the larger value as the stiffness of bolt head when the mesh size is dense and the difference between the two adjacent analysis results is small.

3) EFFECT OF AXIAL LOAD ON THE STIFFNESS OF BOLTS

In the process of using bolt connection, the axial preload must reach a certain value in order to ensure the safety and service life of bolt connection.

The Chinese national standard GB / T 3098.1-2000 specifies the preload to be achieved in the use of different grade bolts. Therefore, it is of practical significance to study the static and dynamic characteristics of bolt connections under such loads.

During the pre tightening process of bolt connection, the axial force changes gradually from small to large, and the load on the bolt head and the connected parts also increase from small to small. Does the stiffness of the bolt head remain unchanged during the gradual increase in the preload? This needs finite element analysis or experimental study to reach a conclusion.

In this article, based on the finite element software, the stiffness analysis of bolt heads with different axial forces is carried out for bolt heads with specifications of M10, M12 and M14. The size and material parameters of the finite element model are shown in TABLE 1.

The finite analysis results of the variation of bolt head stiffness with axial tensile load are shown in Fig. 9.

It can be seen from Fig. 9 that when the bolt head changes within and near the guaranteed load range, its stiffness is not the same under different axial loads. With the increase in load, the stiffness changes in a certain range in a jumping manner. Therefore, in this article, it is considered that within

TABLE 1. Bolt head size, material characteristic parameters.

Model	Bolt head diameter	Bolt head thickness	Bolt diameter	Bolt hole diameter	Elastic modulus of bolt head	Elastic modulus of connected parts	Poisson's ratio	Friction coefficient
	D_{head}(mm)	A(mm)	D_b(mm)	D_h(mm)	E_b(GPa)	E_m(GPa)	ν	μ
M10	16	6.4	10	10.5				
M12	18	7.5	12	13.5	206	206	0.286	0.2
M14	21	8.8	14	15				

the preload range specified in the national standard, the stiffness does not change with the change of axial load, but is a constant value.

4) INFLUENCE OF BOLT HEAD PARAMETERS ON STIFFNESS

In order to understand the influence of various factors on the stiffness of the bolt head, it is necessary to analyze the influence law of each parameter on the stiffness. In the analysis, only the parameters to be analyzed are allowed to change, and other parameters are fixed. For example, take the same bolt head nominal diameter d_b, thickness a, diameter d_h of connected parts, elastic modulus $E_b(E_m)$ of the bolt headed and connected part; when taking the different bolt heads diameter d_{he}, the regularity of the change of parameters on the stiffness of the bolt head is investigated. Part of the finite-element analysis results are shown in Fig. 10~Fig. 13.

It can be seen from Fig. 10 that when the same load $F_b = 45492$N, with the increase of bolt head width, the total axial deformation of bolt head and connected parts gradually decreases, and the maximum deformation decreases from 0.03mm to 0.0133mm. It can be seen from Fig. 11 that under the same load $F_b = 25119$N, with the increase of bolt head thickness, the total axial deformation of bolt head and connected parts gradually decreases, and the maximum deformation decreases from 0.01018mm to 0.00846mm.

It can be seen from Fig. 12 that when the same load $F_b = 29893$N, with the increase of the bolt head holes, the total axial deformation of bolt head and connected parts gradually increases, and the maximum deformation increases from 0.01mm to 0. 0145mm.

It can be seen from Fig. 13 that under the same load $F_b = 29839$N, with the increase of the elastic modulus of connected parts, the total axial deformation of bolt head and connected parts gradually decreases, and the maximum deformation decreases from 0.01mm to 0. 00533mm.

Because the analysis results here shows the total deformation of bolt head and connected parts, the calculation method of bolt head deformation is the total deformation here minus the deformation of connected parts (see Fig. 5(a)).

Based on the finite-element calculation results, the Levenberg Marquardt optimization algorithm is used to conduct nonlinear fitting for the results, and the influence law of the bolt head parameters on the stiffness is studied. The relationship curve between the parameters of the bolt head and the stiffness is shown in Fig. 14.

It can be seen from Fig. 14 (a) that the diameter of the bolt head affects the stiffness of the bolt head. With the increase of diameter of the bolt head, the stiffness increases nonlinearly, which increases rapidly in the early stage, and gradually slows down in the later stage, and finally tends to a fixed value.

It can be seen from Fig. 14 (b) that the thickness of the bolt head also has an impact on the stiffness of the bolt head. With the increase of the thickness of the bolt head, the stiffness increases nonlinearly, which increases rapidly in the early stage, and gradually slows down in the later stage, and finally tends to a certain fixed value.

It can be seen from Fig. 14 (c) that the size of the connected hole also has an impact on the bolt head stiffness. With the increase of the connected hole, the stiffness decreases nonlinearly, which decreases rapidly in the early stage, and gradually slows down with the increase in the later stage.

It can be seen from Fig. 14 (d) that the size of Young's modulus of the connected parts also has an impact on the bolt head stiffness. With the increase of Young's modulus, the stiffness decreases nonlinearly, which decreases rapidly in the early stage, and gradually slows down in the later stage, and finally tends to a certain fixed value.

In order to study the general rule of the influence of the single factors on the stiffness of the bolt head, such as the nominal diameter of bolt head, thickness, diameter of connected part, Young's modulus of bolt head and connected part, the parameters are dimensionless compared with d_b or E_b.

$$\bar{d}_{he} = \frac{d_{he}}{d_b}, \quad \bar{a} = \frac{a}{d_b}, \ \bar{d}_h = \frac{d_h}{d_b}, \ \bar{E}_m = \frac{E_m}{E_b} \tag{11}$$

For each type of bolt head, the stiffness k_h is calculated according to formula (9), and then the stiffness correction coefficient is calculated according to formula (10). According to the calculation results, the curve of stiffness correction coefficient α_{head} with each dimensionless parameter is obtained.

It can be seen from Figure 15(a) that the stiffness correction coefficient α_{head} changes nonlinearly with the increase of the dimensionless parameter $\bar{d}_{he}$. With the increase of the dimensionless parameters $\bar{d}_{he}$, the stiffness correction coefficient increases non-linearly, increasing faster in the early stage, and slowly increasing in the later stage, and finally tends to a fixed value.

It can be seen from Fig. 15(b) that the stiffness correction coefficient α_{head} changes nonlinearly with the increase of dimensionless parameters $\bar{a}$. With the increase of dimensionless parameters $\bar{a}$, the stiffness correction coefficient

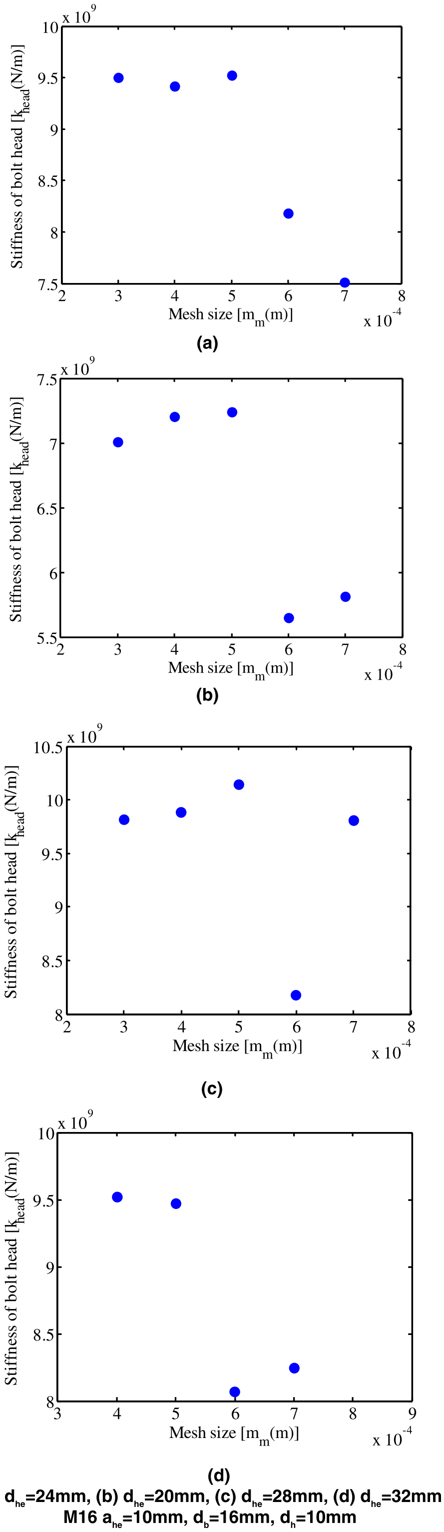


FIGURE 8. Influence of mesh size on analysis results.

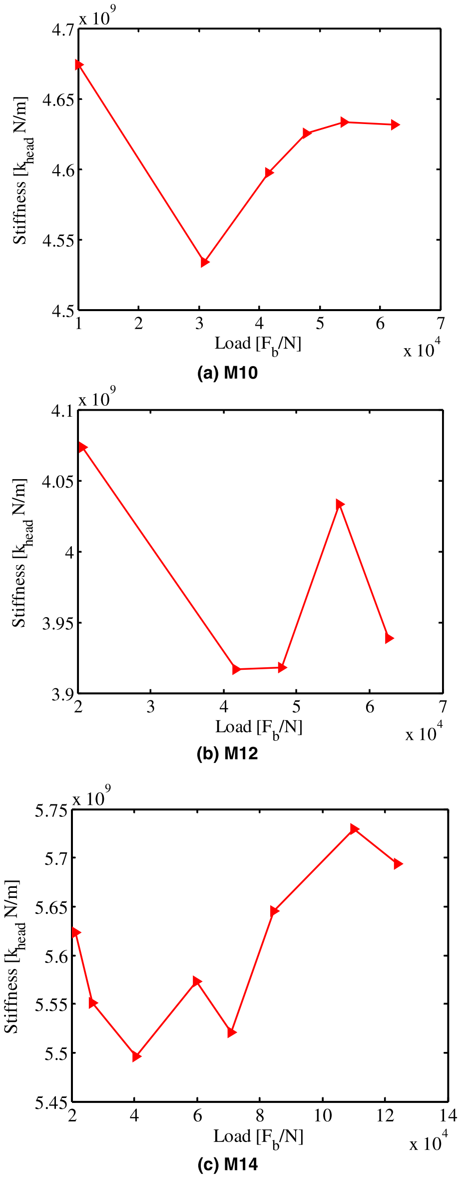


FIGURE 9. Effect of load on stiffness finite element analysis.

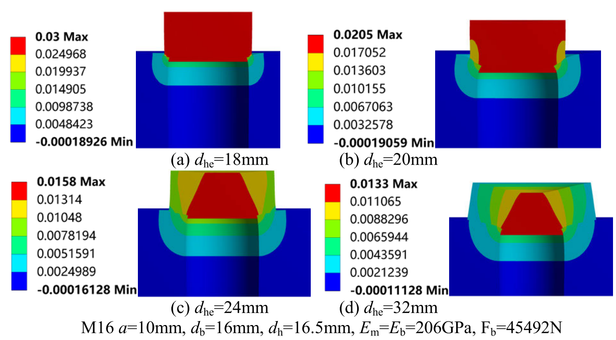


FIGURE 10. Total deformation with different bolt head diameter.

decreases nonlinearly, which decreases rapidly in the early stage and gradually slows down in the later stage, and finally tends to a fixed value.

It can be seen from Fig. 15(c) that the stiffness correction coefficient is nonlinear with the increase of dimensionless parameters $\bar{d}_h$. With the increase of dimensionless parameters $\bar{d}_h$, the stiffness correction coefficient is nonlinear and the growth rate gradually decreases.

It can be seen from Fig. 15(d) that the stiffness correction coefficient changes nonlinearly with the increase of dimensionless parameters $\bar{E}_m$. With the increase of

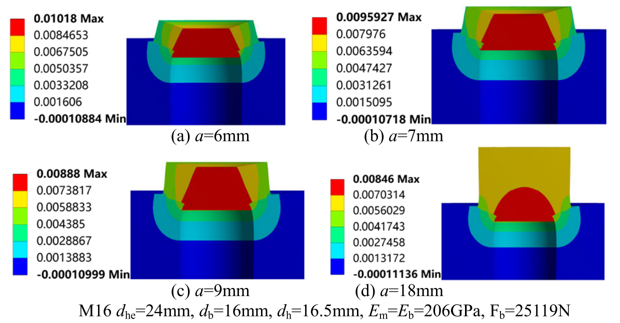


FIGURE 11. Total deformation with different bolt head thickness.

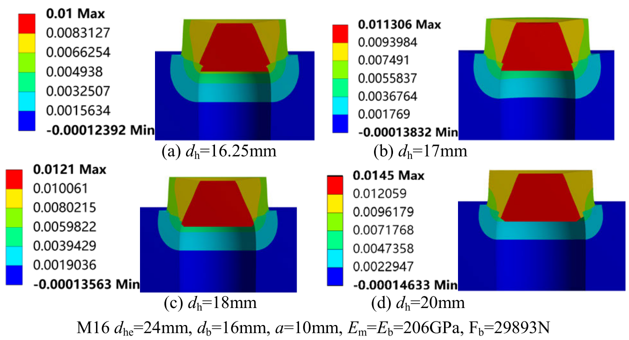


FIGURE 12. Total deformation with different bolt hole diameters.

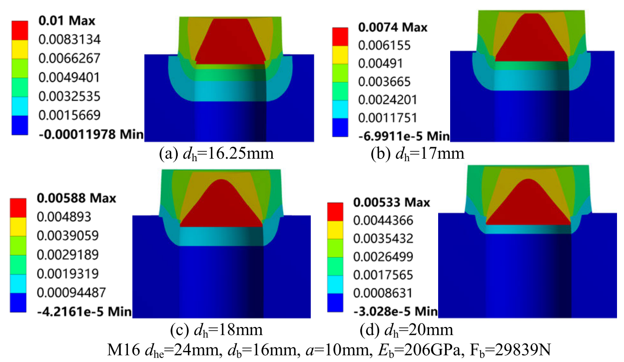


FIGURE 13. Total deformation with different elastic modulus of connected parts.

dimensionless parameters $\bar{E}_m$, the stiffness correction coefficient decreases nonlinearly, with a faster rate of decrease in the early stage and a tendency to 0 in the later stage.

5) CORRECTION COEFFICIENT OF THE MODEL

On the basis of known the variation law of stiffness correction coefficient α_{head} with single factor dimensionless parameters, it is assumed that the variation law of α_{head} and multi factor dimensionless parameters satisfies the following multi parameter and multi variable relationship

$$\alpha_{head}(x_1, x_2, x_3, x_4) = k_1 + \frac{1}{p_1 + p_2/x_1^{p_3}} + \frac{p_4}{1+[(x_2-p_5)/p_6]^{p_7}} + \frac{p_9}{x_3^{p_8}} + p_{10}x_4 \tag{12}$$

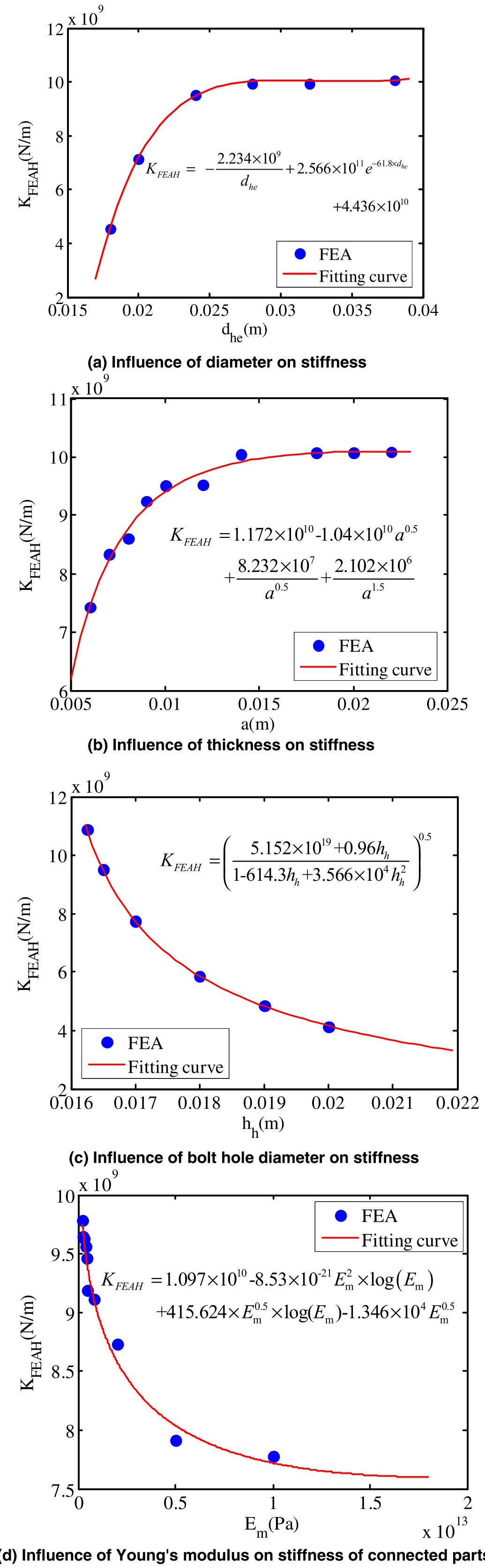
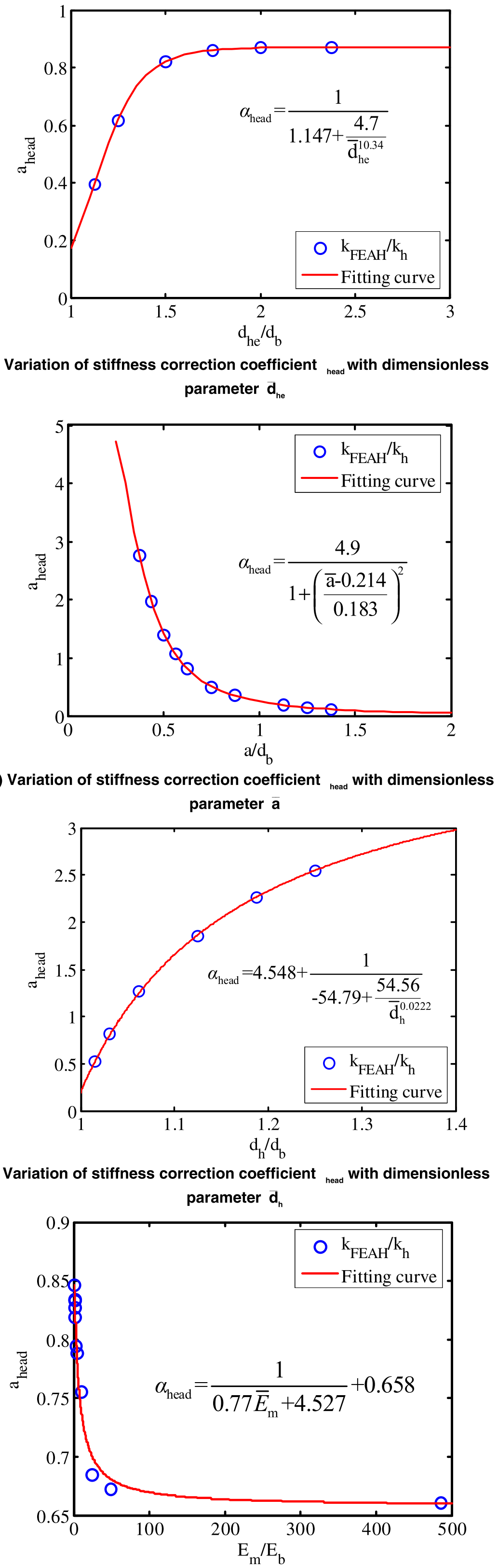


FIGURE 14. The influence of each parameter of M16 bolt connection on the stiffness of bolt head.



(a) Variation of stiffness correction coefficient $_{head}$ with dimensionless parameter $\bar{d}_{he}$

(b) Variation of stiffness correction coefficient $_{head}$ with dimensionless parameter $\bar{a}$

(c) Variation of stiffness correction coefficient $_{head}$ with dimensionless parameter $\bar{d}_h$

(d) Variation of stiffness correction coefficient $_{head}$ with dimensionless parameter $\bar{E}_m$

FIGURE 15. α_{head} **varies with each dimensionless parameter.**

where k_1, p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9 and p_{10} are constants, $x_1 = \bar{d}_{he}$, $x_2 = \bar{a}$, $x_3 = \bar{d}_h$, $x_4 = \bar{E}_m$ are variables.

Taking the correction coefficient α head of bolt head as dependent variable and x_1, x_2, x_3 and x_4 as independent variables, the Levenberg Marquardt optimization algorithm is used to carry out multi parameter and multivariable nonlinear fitting for all known variables, and the fitting correlation coefficient R $\geqslant$ 0.99%. The fitting parameter values are substituted into the above equation (12), then

$$\begin{aligned}\alpha_{head} = 1.522 &+ \frac{1}{1.137 + 4.499/x_1^{10.219}} \\ &+ \frac{5.992}{1 + [(x_2 - 0.174)/0.187]^{2.079}} \\ &- \frac{2.9}{x_3^{6.821}} - 0.0063x_4 \end{aligned} \tag{13}$$

where: $1.125 \leq x_1 \leq 2.375$, $0.375 \leq x_2 \leq 1.375$, $1.009 \leq x_3 \leq 1.375$, $1 \leq x_4 \leq 25$.

In order to verify the accuracy of the fitting equation, the non dimensional single factor validation change verification is carried out for the fitting equation.

a) Taking the diameter of bolt head ranging from 18mm to 28mm, bolt head thickness of 10mm, major diameter of thread of 16mm, bolt hole diameter of 16.5mm, 16.8mm, 17.1mm and 17.4mm, Young's modulus of bolt head is 2.06×10^{11}, and Young's modulus of bolt connected part is 2.06×10^{11}. The dimensionless parameters obtained are $x_1 = 1.125 \sim 1.75$, $x_2 = 0.625$, $x_3 = 1.031$, 1.05, 1.069 and 1.088, $x_4 = 1$, the accuracy of the fitting equation is investigated, as shown in Fig. 16;

b) The diameter of the bolt head is 24mm, the thickness of the bolt head is 7mm to 15mm, and the major diameter of the thread is 16mm. The diameter of the bolt hole of the connected part is 16.5mm, 16.8mm, 17.1mm and 17.4mm respectively. The young's modulus of the bolt head is 2.06×10^{11} and that of the connected part is 2.06×10^{11}. The dimensionless quantities obtained are $x_1 = 1.5$, $x_2 = 0.438 \sim 0.938$, $x_3 = 1.031$, 1.05, 1.069 and 1.088, $x_4 = 1$. Then the accuracy of the fitting equation is investigated, as shown in Fig. 17;

c) The diameter of bolt head is 20 mm, 24 mm, 28 mm and 32 mm, the thickness of bolt head is 10 mm, the major diameter of thread is 16 mm, the diameter of bolt hole is 16.25 mm to 19.2 mm, the young's modulus of bolt head is 2.06×10^{11}, the dimensionless dimension is $x_1 = 1.25$, 1.5, 1.75 and 2, $x_2 = 0.625$, $x_3 = 1.016 \sim 1.2$, $x_4 = 1$, and then the accuracy of the fitting equation is investigated, as shown in Fig. 18;

d) The diameter of the bolt head is 24mm, the thickness of the bolt head is 10mm, the major diameter of the thread is 16mm, the diameter of the bolt hole of the connected part is 16.5mm, 16.8mm, 17.1mm and 17.4mm, and the Young's modulus of the bolt head is 2.06×10^{11}, the Young's modulus of the bolt to be connected is 2.06×10^{11} to 5.15×10^{12},

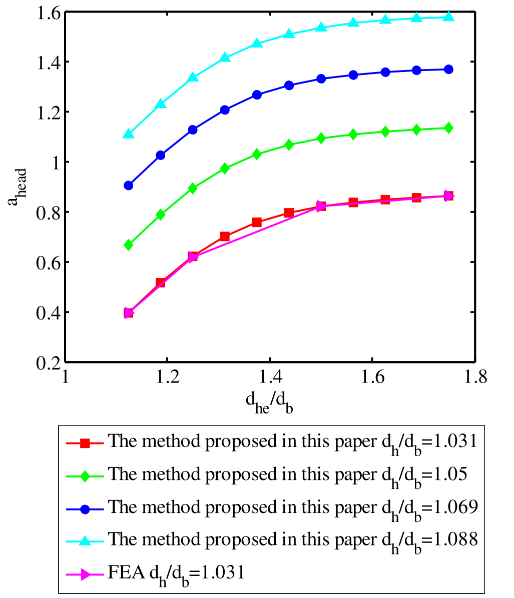


FIGURE 16. **The influence of the ratio of bolt head diameter to bolt diameter on α_{head}.**

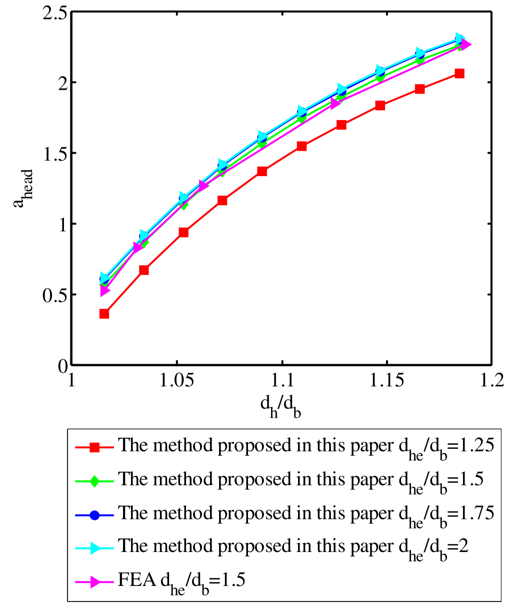


FIGURE 18. **The influence of the ratio of bolt hole to bolt diameter on α_{head}.**

on head

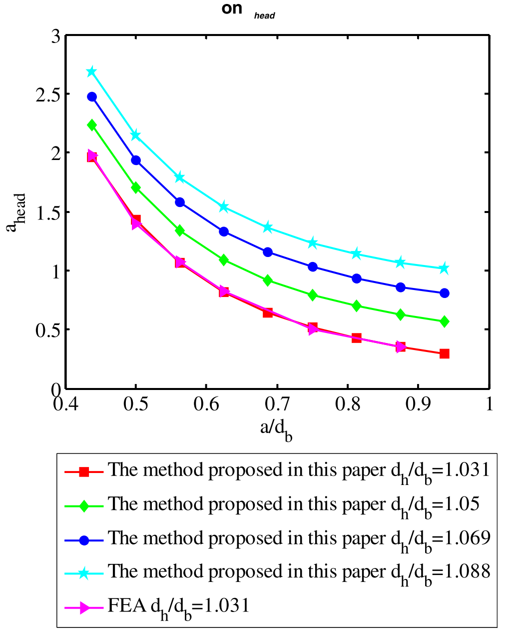


FIGURE 17. **The influence of the ratio of bolt head thickness to bolt diameter on α_{head}.**

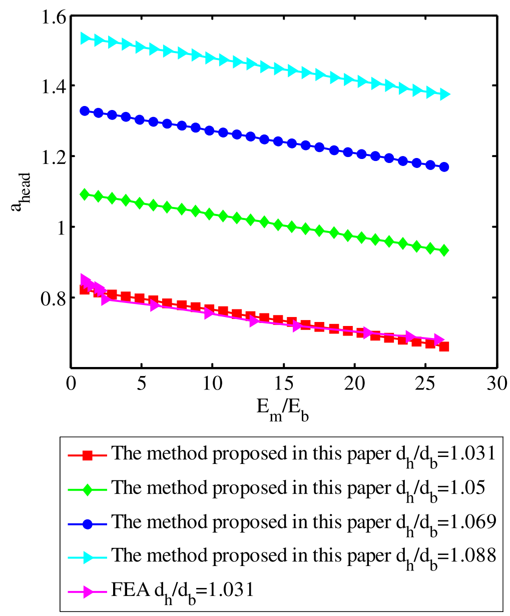


FIGURE 19. **The influence of the ratio of the young's modulus of the connected part to the young's modulus of the bolt on α_{head}.**

the dimensionless quantity obtained is $x_1 = 1.5$, $x_2 = 0.625$, $x_3 = 1.031$, 1.05, 1.069 and 1.088, $x_4 = 1\sim25$, and then the accuracy of the fitting equation is investigated, as shown in Fig. 19;

It can be seen from Fig. 16~ Fig. 19 that the error between the results obtained by the fitting and by the finite-element, analysis is small, and the two results have the same trend as the independent variables. Therefore, it can be concluded that the relationship obtained by the combination is valid and credible.

Therefore, by substituting formula (11) into formula (13), the relationship between the bolt structure parameters E_{bolt}, d_{he}, a_{t}, d_{b}, d_{h}, E_{m}, E_{b} is

$$\alpha_{head} = 1.522 + \frac{1}{1.137+\frac{4.499}{(d_{he}/d_b)^{10.219}}} + \frac{5.992}{1+(\frac{a_t/d_b-0.174}{0.187})^{2.079}} - \frac{2.9}{(d_h/d_b)^{6.821}} - 0.0063E_m/E_b$$
$$\text{其中}:\left(\begin{array}{ll} 1.125 \le \frac{d_{he}}{d_b} \le 2.375, & 0.375 \le \frac{a}{d_b} \le 1.375, \\ 1.009 \le \frac{d_h}{d_b} \le 1.375, & 1 \le \frac{E_m}{E_b} \le 25, \end{array}\right) \tag{14}$$

TABLE 2. **Size parameters and material parameters of bolt head structure.**

No.	1	2	3	4	7	8	9
Specification	Bolt head diameter/(mm)	Bolt hole diameter/(mm)	Bolt diameter/(mm)	Bolt head thickness/(mm)	Young's modulus of bolt head/(GPa)	Young's modulus of connected parts/(GPa)	Poisson's ratio of bolts and connected parts
	d_{he}	d_h	d_b	a_t	E_b	E_m	v_m, v_b
M6	10	6.4	6	4			
M10	16	10.5	10	6.4			
M16	24	16.5	16	10	206	$206 \leq E_m \leq 5150$	0.286
M36	55	39	36	22.5			
M64	95	66	64	40			

TABLE 3. **Finite element analysis of stiffness($E_m = E_b = 2.06 \times 10^{11}$).**

Bolt specification	M6	M10	M16	M36	M64
Calculation results of the presented model(N/m)	2.66×10^{9}	4.99×10^{9}	9.48×10^{9}	1.86×10^{3}	4.23×10^{10}
Results of Feras Alkatan method(N/m)	1.99×10^{9}	3.44×10^{9}	5.73×10^{9}	1.16×10^{10}	2.29×10^{10}
Finite element calculation results(N/m)	2.58×10^{9}	4.68×10^{9}	9.50×10^{9}	1.69×10^{3}	3.85×10^{10}
Error rate of Feras Alkatan method(%)	22.7	26.5	39.68	31.36	40.42
Error rate of this model(%)	3.07	6.54	-3.702	9.89	9.72

Therefore, formula (14) is substituted into formula (10) to obtain the calculation model of bolt head stiffness (without washer) as

$$
\begin{aligned}
k_{he} &= \alpha_{head} k_h \\
&= \left[\begin{array}{l} 1.522 + \dfrac{1}{1.137 + \dfrac{4.499}{\left(\frac{d_{he}}{d_b}\right)^{10.219}}} + \dfrac{5.992}{1 + \left(\dfrac{\frac{a_t}{d_b} - 0.174}{0.187}\right)^{2.079}} \\ - \dfrac{2.9}{\left(\frac{d_h}{d_b}\right)^{6.821}} - 0.0063 \dfrac{E_m}{E_b} \end{array} \right] \\
&\div \left\{ \begin{array}{l} - \dfrac{12}{E_{bolt} \pi a^3} \left(\dfrac{d_h - d_b}{2} + \dfrac{\alpha E_b a^2}{12 G_h} \right) \\ \cdot \dfrac{1}{2} \left\{ \begin{array}{l} [\ln(d_h)] \cdot \dfrac{d_h - d_b}{2} \\ - \dfrac{d_h - d_b}{2} + \dfrac{d_b}{2} \ln(d_h) \end{array} \right\} \\ + \dfrac{6}{E_{bolt} \pi a^3} \left(\dfrac{d_h - d_b}{2} + \dfrac{\alpha E_b a^2}{12 G_h} \right) \cdot \ln(d_b) \cdot \dfrac{d_h - d_b}{2} \\ + \dfrac{12}{E_{bolt} \pi a^3} \left(\dfrac{d_h - d_b}{2} + \dfrac{\alpha E_b a^2}{12 G_h} \right) \cdot \dfrac{d_h - d_b}{2} \cdot \dfrac{d_b}{4} \ln(d_b) \end{array} \right\}
\end{aligned} \tag{15}
$$

Therefore, the root deformation of the bolt head under load can be expressed as

$$
\delta_{he} = \frac{F_b}{k_{he}(d_{he}, a_t, d_b, d_h, E_m, E_b)} \tag{16}
$$

III. VERIFICATION OF THE THEORETICAL MODEL OF BOLT HEAD CONNECTION STIFFNESS

In order to verify the correctness of the theoretical model calculations in this article, a model with bolt head specifications M6, M10, M16, M24, M36 and M64 is established. The elastic modulus of the bolt head material of each specification is $E_b = 206$GPa. According to the Chinese national standard, bolts of each specification are used. The size parameters and material parameters of the head structure and the material parameters of the connected parts are shown in TABLE 2. The comparison of the calculated stiffness results of the finite-element model and the theoretical model in this article is shown in TABLE 3.

According to the comparison in TABLE 3, the maximum error is 9.89%, the minimum absolute error is 3.7%, and the overall error is within 10%. The minimum error of the Feras Alkatan method is 22.7%, and the maximum error is 40.42%. Therefore, it can be seen that the accuracy of this method is relatively high.

IV. CONCLUSION

Based on the influence of the bolt head on the stiffness of the bolt connection, a new calculation model of the bolt head stiffness is proposed from the perspective of bolt head deformation. The model is based on the finite-element simulation analysis of bolt head stiffness with various parameters, and based on the Levenberg-Marquardt optimization algorithms for multi parameter and multi variable nonlinear fitting, the theoretical calculation model of the bolt head structural connection stiffness is obtained. The model includes the thickness of bolt head, equivalent diameter, bolt diameter, thickness of connected parts, elastic modulus of bolt, elastic modulus of connected parts, etc. influence. The validity and accuracy of the calculation results of this model are verified by the finite-element method. It provides a universal calculation model for calculating the stiffness of bolt head connection with various structural parameters.

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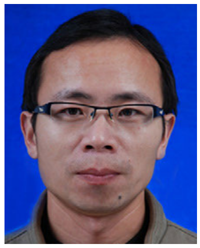

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