IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received September 13, 2020, accepted October 12, 2020, date of publication October 15, 2020, date of current version October 28, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3031328

A Robust Predefined-Time Stable Tracking Control for Uncertain Robot Manipulators

NANSHENG ZHANG[1], SHANSHAN WANG[1], YINLONG HOU[2], AND LIYIN ZHANG[2]
[1]Beijing Key Laboratory for Precision Optoelectronic Measurement Instrument and Technology, School of Optics and Photonics, Beijing Institute of Technology, Beijing 100081, China
[2]Xi'an Key Laboratory of Advanced Control and Intelligent Process, School of Automation, Xi'an University of Posts and Telecommunications, Xi'an 710121, China

Corresponding author: Shanshan Wang (wang33_0921@126.com)

This work was supported by the Scientific Research Program Funded by the Shaanxi Provincial Education Department under Program 20JK0916.

ABSTRACT In this paper, a novel robust predefined-time approach is proposed for global predefined-time tracking control of uncertain robot manipulators. With the sufficient consideration of the effects of uncertainties and external disturbances (UED) on the trajectory tracking performance of robot manipulators, a singularity-free robust control with an auxiliary nonlinear vector is constructed for the predefined-time tracking. The global predefined-time stable tracking by using Lyapunov stability theory has been accomplished for ensuring that both the position and velocity tracking errors arrive at the origin within a predefined time. The proposed approach has the following advantages: (i) the proposed approach with a simple structure is easy to implement with the global robust predefined-time tracking control; (ii) the convergence time of the proposed approach independently of the initial states can be given as an exact controller parameter in advance; (iii) the proposed approach is easy to apply into uncertain robot manipulators with time-constraints tracking control. Extensive simulation and experimental results have been accomplished to show the effectiveness and improved performances of the proposed approach.

INDEX TERMS Robust control, predefined-time stability, robot manipulators, robot control.

I. INTRODUCTION

Tracking control has been paid much more attention in both academic and industrial applications since the increasing demands on accuracy, convergence rate and robustness [1]. It is still a challenge to develop a simple robust control with an improved tracking performance and transient respond for robot manipulators in the research community [2].

In general, the tracking performance of robot systems is mainly affected by the uncertain dynamics and matched/mismatched disturbances. To this end, several robust tracking controls have been developed to cover with this issues. In the initial approaches, several approaches are proposed such as PID control [3], intelligent and learning control [4]–[6], optimal control [7], robust controls [8]–[12], etc. As a general robust technique, sliding mode control (SMC) has widely been applied into trajectory tracking of nonlinear system since its strong robustness and disturbances rejection capability [13]–[21]. Due to the convergence time of these robust controls cannot calculated from control parameters in advance and its tracking performance is always affected by the initial conditions of closed-loop system, in general these approaches are commonly called as the finite-time controls (FTC).

However, these finite-time robust controls have a minor shortage, i.e., it has a slower convergence than exponential stable systems if the tracking errors between system states and equilibrium point are far away from the origin [22], [23]. This unexpected result is mainly caused by the fact that the FTC systems have an convergence time related to the initial states. As a result, the closed-loop system with different initial states has different convergence performance. In view of above analysis and shortages, several fixed-time stable controls have been proposed in [24]–[26]. Different from these FTC systems, the settling time of fixed-time controls independently of the initial states can be calculated from control parameters in advance [27]. Recognizing these advantages of fixed-time theory, a continuous approach scheme with

The associate editor coordinating the review of this manuscript and approving it for publication was Zhiguang Feng.

fixed-time convergence is proposed in [28] for the double integrator system, where the bi-limit homogeneous technique is applied in the formulation of controller and observer; while a terminal SMC (TSMC) with fixed-time stability has been developed in [29] for multi-agent consensus tracking systems. By utilizing the theory of fixed-time stability, several fixed-time SMCs have been developed in [11], [30] for robot manipulators with uncertainties and external disturbances. However, the convergence time of these fixed-time controls is the upper bound instead of the least upper bound of time function. As a result, they cannot show the direct relationship between the convergence time and tracking performance, which also cannot be used in a nonlinear system with time-constraints tracking control.

With this purpose, Fragucla *et al.* [31] proposes a predefined-time stable control (PSC) by selecting the values of the tuning parameters. Based on seminal works, several PSC systems have been developed in [32], [33] for which the convergence time can be given as an exact parameter of closed-loop system in advance. A control approach in [34] is addressed to obtain the optimal predefined-time stabilization. In particular, a control problem with the optimal predefined-time stabilization has been accomplished for a given system by using the sufficient conditions. In [35], a predefined-time stable control with nonconservative settling time is developed for a class of nonlinear systems. A continuous controller is proposed in [36] to achieve into a vicinity of the origin. These mentioned predefined-time approaches have greatly promoted for the development of time-constraints tracking control. However, the traditional robust controls still have one or more of following shortages and limitations that affect its tracking performance of uncertain robot manipulators [22]: i) the settling time cannot chosen as an exact control parameter; ii) it lacks the ability to deal with the fast variation of uncertainties and external disturbance (UED); and iii) its control input has high computational complexity. Consequently, it is looking forward to develop a simple robust predefined-time tracking control featuring with simplicity and robustness to uncertain dynamics and external disturbances.

Although the above predefined-time stable controls have made a great development in recent years, but there still exist several deficiencies which need to be further conquered for tracking control of uncertain robot manipulators. They are as follows: the existing predefined-time controls cannot directly applied into robot manipulators with uncertain dynamics and external disturbances owing to the existence of the singularity [13] and the algebraic loop problem [37]; while for the existing robust controls of robot manipulators with finite/fixed-time convergence, the convergence time cannot given as an exact control parameters, and also is the upper bound of the convergence function. By considering the UEDs adequately and inspired by the works in [32], [35], thus in this paper we proposed a robust predefined-time control for global tracking of robot manipulators. The contributions of this paper are as follows:

i) Compared with the predefined-time controls [31]–[35], the proposed robust predefined-time tracking control (RPTC) for uncertain robot manipulators removes the assumption that the lumped uncertainty involving the acceleration of joints are bounded by a constant, thus which can conquer the algebraic loop problem [37];
ii) Different from finite/fixed-time tracking controls [10], [13]–[16], [19], [26], [45], [46], the convergence time of the proposed approach independently of the initial conditions can be given as an exact controller parameter, which is easy to implement with the time-constraints tracking control of robot manipulators subject uncertain dynamics and external disturbances;
iii) In comparison with the existing robust tracking controls [7]–[9], [22], the proposed RPTC has more higher steady-state tracking precision and simpler control structure in both position and velocity trajectory tracking for robot manipulators with uncertainties and external disturbances.
iv) The uncertain dynamics and external disturbances are eliminated by an system transformation with an auxiliary nonlinear vector; while the predefined-time stability of robot tracking system is guaranteed by utilizing a control term with a predefined convergence time.

The global predefined-time stable tracking of robot manipulators by using Lyapunov stability theory has been guaranteed for ensuring that both position and velocity tracking errors arrive at the origin within a predefined time. Extensive simulation and experimental results are accomplished on 2-DOFs robots to declare the improved tracking performance of the proposed approach in both the position and velocity tracking.

This paper is organized as follows. Some preliminaries are introduced in Section II such as the properties and model of robot manipulators and the predefined-time stable theory. The control design and stability analysis are accomplished in Section III. Simulation and experimental comparisons have been performed in Sections IV and V, respectively. Finally, a conclusion and further outlook are represented in section VI.

II. PRELIMINARIES

A. ROBOT MANIPULATOR MODEL AND PROPERTIES

Considering n-joint robot manipulators are [38]

$$\bar{M}(q_r)\ddot{q}_r + \bar{C}(q_r, \dot{q}_r)\dot{q}_r + \bar{g}(q_r) = u_r + d_r \tag{1}$$

where $q_r, \dot{q}_r, \ddot{q}_r \in \Re^n$ denote the position, velocity and acceleration of joints, respectively, $\bar{M}(q_r) \in \Re^{n\times n}$ and $\bar{C}(q_r, \dot{q}_r) \in \Re^{n\times n}$ represent the symmetric positive definite inertia and centrifugal-Coriolis matrix, respectively, $\bar{g}(q_r) \in \Re^n$ stands for the vector of gravitational torque, and $d_r \in \Re^n$ and $u_r \in \Re^n$ are the bounded external disturbances and control input, respectively.

Remark 1: Without loss of generality, the following assumptions have been established [13], [38]: (i) the external disturbances d_r described by system (1) are bounded by $\|d_r\| \leq d_{rm}$ where d_{rm} denotes a known positive constant; (ii) q_r and $\dot{q}_r$ are available; (iii)the desired trajectory $q_d \in \Re^n$ be C^2 for the robotic system and bounded by some positive constants.

Additionally, the following facts are explained for further discussion [13], [38].

Assumption 1: The system model of robot manipulators given by (1) can be modified as [13]

$$\begin{aligned}\bar{M}(q_r) &= \bar{M}_0(q_r) + \Delta\bar{M}(q_r)\\ \bar{C}(q_r,\ \dot{q}_r) &= \bar{C}_0(q_r,\ \dot{q}_r) + \Delta\bar{C}(q_r,\ \dot{q}_r)\\ \bar{g}(q_r) &= \bar{g}_0(q_r) + \Delta\bar{g}(q_r)\end{aligned} \tag{2}$$

where $\bar{M}_0(q_r)$, $\bar{C}_0(q_r, \dot{q}_r)$ and $\bar{g}_0(q_r)$ denote the nominal parts, and $\Delta\bar{M}(q_r)$, $\Delta\bar{C}(q_r, \dot{q}_r)$ and $\Delta\bar{g}(q_r)$ stand for the uncertain parts.

To facilitate the following formulation of the proposed controller, we have defined the vector $\mathrm{Sgn}(\xi) \in \Re^n$ and $\mathrm{Sig}^r(\xi) \in \Re^n$ as follows

$$\mathrm{Sgn}(\xi) = \left[\mathrm{sign}(\xi_1),\ \ldots,\ \mathrm{sign}(\xi_n)\right]^T \tag{3}$$

$$\mathrm{Sig}^r(\xi) = \left[\mathrm{sig}^r(\xi_1),\ \ldots,\ \mathrm{sig}^r(\xi_n)\right]^T \tag{4}$$

where $\mathrm{sig}^r(\xi_i) = |\xi_i|^r \mathrm{sign}(\xi_i)$ with $\xi = [\xi_1, \ldots, \xi_n]^T \in \Re^n$, and $\mathrm{sign}(\cdot)$ stands for the standard signum function.

In this paper, we have developed a novel robust control for global predefined-time tracking of robot manipulators with uncertain dynamics and external disturbances. As an objective, the position and velocity tracking errors converge globally to the origin with a predefined time.

To facilitate the future design and analysis, firstly we have defined the position and velocity tracking errors are as follows

$$e = q_r - q_d, \quad \dot{e} = \dot{q}_r - \dot{q}_d \tag{5}$$

where q_d and $\dot{q}_d$ represent the desired position and velocity trajectories.

B. FUNDAMENTAL FACTS

To accomplish the subsequent design and analysis, the following fundamental facts are introduced.

Definition 1 (Predefined-Time Stability): Consider the following system [39]

$$\dot{z} = g(t,\ z), \quad z(0) = z_0 \tag{6}$$

where $z \in \Re^n$ is the state variables, $g : \Re^+ \times \Re^n \to \Re^n$ denotes a continuous/discontinuous nonlinear function. If there exists a predefined time constant $T_c \in \Re^+$ satisfied $T \leq T_c$ and $z(t) = 0$ for all $t \geq T$ and $z_0 \in \Re^n$, then system (6) is predefined-time stability.

Lemma 1 (A Lyapunov Predefined-Time Stable Systems): A class of nonlinear system is described as [39]

$$\dot{z} = f(z;\ \rho), \quad z(0) = z_0 \tag{7}$$

where $z \in \Re^n$ and $\rho \in \Re^b$ stand for the state variable and the constant system parameters, respectively, $f : \Re^n \times \Re^b \to \Re^n$ denotes a class of nonlinear function, and then the origin $z = 0$ is an equilibrium point of system (7) such that $f(0;\ \rho) = 0$.

Given a radially unbounded Lyapunov function $V : \Re^n \to \Re$, for any solution $z(t;\ z_0)$ of system (7), if there exists

$$\dot{V}(z) \leq -\frac{1}{\beta T_c} \exp\left(V^{\beta}(z)\right) V^{1-\beta}(z), \quad \forall z \in \Re^n \backslash \{0\} \tag{8}$$

where $0 < \beta \leq 1$ denotes a positive constant.

Then, the system (7) exists an predefined-time stable equilibrium point, and the *strong* predefined time can be given as an exact system parameters T_c.

Lemma 2: For $\beta \in \Re^+$, $z \in \Re$, the following equality holds [12]

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}|z|^{\beta+1} &= (\beta+1)|z|^{\beta}\dot{z}\ \mathrm{sign}(z)\\ \frac{\mathrm{d}}{\mathrm{d}t}\left(|z|^{\beta+1}\mathrm{sign}(z)\right) &= (\beta+1)|z|^{\beta}\dot{z}\end{aligned} \tag{9}$$

where $\mathrm{sign}(\cdot)$ is defined by (4).

III. CONTROL DESIGN AND STABILITY ANALYSIS

In this section, we have focused on the development of a novel robust control design with a predefined-time convergence, thereafter stability analysis has been accomplished on the tracking control of robot manipulators. First, we have developed the open-loop system to facilitate the following discussion.

A. OPEN-LOOP SYSTEM DEVELOPMENT

In light of Assumption 1 and (1), we have obtained the following modified system model

$$\bar{M}_0(q_r)\ddot{q}_r + \bar{C}_0(q_r, \dot{q}_r)\dot{q}_r + \bar{g}_0(q_r) = u_r + \rho_r \tag{10}$$

where $\rho_r \in \Re^n$ denotes the uncertainties and represented as

$$\rho_r = -\Delta\bar{M}(q_r)\ddot{q}_r - \Delta\bar{C}(q_r, \dot{q}_r)\dot{q}_r - \Delta\bar{g}(q_r) + d_r \tag{11}$$

Combining system (1) and Assumption 1, we have obtained

$$\Delta\bar{M}(q_r)\ddot{q}_r = \bar{E}\left(u_r - \bar{C}(q_r, \dot{q}_r)\dot{q}_r - \bar{g}(q_r) + d_r\right) \tag{12}$$

where $\bar{E} \in \Re^{n\times n}$ is given by [37] and represented as

$$\bar{E} = I_n - \bar{M}_0(q_r)\bar{M}^{-1}(q_r) \tag{13}$$

with I_n is the identity symmetric matrix.

Inspired by [37], $\bar{M}_0(q_r)$ can be defined as

$$\bar{M}_0 = \frac{2}{\bar{\gamma}_1 + \bar{\gamma}_2} I_n \tag{14}$$

where $\bar{\gamma}_1$ and $\bar{\gamma}_2$ are some given positive constants and satisfies

$$\bar{\gamma}_1 \leq \left\|\bar{M}^{-1}(q_r)\right\| \leq \bar{\gamma}_2 \tag{15}$$

then the following equality for $\bar{E}$ holds [37]

$$\left\|\bar{E}\right\| \leq \bar{\sigma} \tag{16}$$

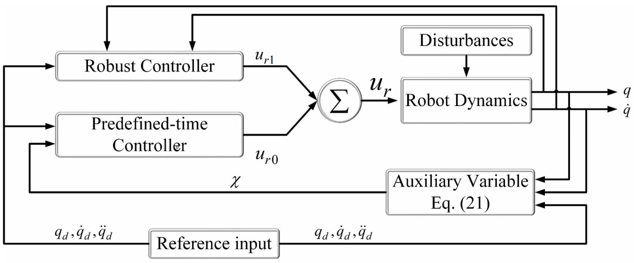


FIGURE 1. Block diagram of the proposed RPTC control strategy.

with $\bar{\sigma}$ stands for a known positive constant and given by

$$\bar{\sigma} = \frac{\bar{\gamma}_2 - \bar{\gamma}_1}{\bar{\gamma}_1 + \bar{\gamma}_2} \tag{17}$$

In virtue of the definition of constant matrix (14), in this paper $\bar{M}_0(q_r)$ can be rewritten as $\bar{M}_0$ in the subsequent development. Additionally, the uncertainties ρ_r given by (11) satisfies [19], [37]

$$\|\rho_r\| \le \bar{a}_0 + \bar{a}_1 \|\dot{q}_r\|^2 + \bar{\sigma}\, \|u_r\| \tag{18}$$

where $\bar{a}_i > 0,\ i = 0, 1$ stand for some given constants related to the robotic system, and $\bar{\sigma}$ is given by (17).

B. CONTROL FORMULATION

Upon substituting (5) into (10), it follows that

$$\bar{M}_0 \ddot{e} = u_r + \rho_r + \eta_r \tag{19}$$

where $\bar{M}_0$ is a constant matrix given by (14), $\rho_r \in \Re^n$ given by (11) represents the lumped uncertainties and $\eta_r \in \Re^n$ denotes the nominal parts and can be defined as

$$\eta_r = -\bar{C}_0(q_r, \dot{q}_r)\dot{q}_r - \bar{g}_0(q_r) - \bar{M}_0 \ddot{q}_d \tag{20}$$

To facilitate the subsequent control design, for robot tracking system (19) an auxiliary vector is defined as

$$\chi = \dot{e} + \lambda \mathrm{Sig}^{\alpha}(e) \tag{21}$$

where $\lambda > 0$ and $\alpha > 1$ denote some positive constants, $\mathrm{Sig}^{\alpha}(e)$ is given by (4), and e and $\dot{e}$ are defined by (5).

For system (19) and the auxiliary vector (21), then, a robust predefined-time tracking control (RPTC) is represented as

$$u_r = -\eta_r + u_{r0} + u_{r1} \tag{22}$$

where $\eta_r \in \Re^n$ is given by (20) and then

$$\begin{aligned} u_{r0} = & -\lambda\alpha \bar{M}_0 \mathrm{diag}\left\{|e_i|^{\alpha-1}\right\} \dot{e} - \bar{M}_0 \mathrm{Sgn}(\chi) \dot{e}^T e \\ & - \bar{M}_0 \mathrm{Sgn}(\chi) \frac{1}{(r-1)T_c} \\ & \times \exp\left(\left(\sum_{i=1}^{n} \left(|\chi_i| + \frac{1}{2}|e_i|^2 \right) \right)^{r-1} \right) \end{aligned} \tag{23}$$

$$u_{r1} = -\frac{\bar{\gamma}_1 + \bar{\gamma}_2}{2} \bar{M}_0 \mathrm{Sgn}(\chi) w \tag{24}$$

with $r > 1$ denote some positive constants, α and λ are defined by (21), T_c is a predefined time constant, χ and $\mathrm{Sgn}(\chi)$ are given by (21) and (4), respectively, and

$$w = \frac{1}{1-\bar{\sigma}} \left(\bar{a}_0 + \bar{a}_1 \|\dot{q}_r\|^2 + \bar{\sigma}\, \|u_{r0} - \eta_r\| \right) \tag{25}$$

where $\bar{\sigma}$, $\bar{a}_0$ and $\bar{a}_1$ are represented by (17) and (18), respectively.

Upon combining (22) into (19), it follows that

$$\bar{M}_0 \ddot{e} = u_{r0} + u_{r1} + \rho_r \tag{26}$$

Remark 2: Observed by (22)-(25), the formulation of control component u_{r1} does not include its upper bounds of $\|u_{r1}\|$. Then, our design given by (22)-(25) may overcomes the algebraic loop problem [37] completely. Compared with the existing robust controls, the uncertain dynamics and external disturbances will be considered adequately in the formulation of the proposed RPTC given by (22)-(25), which also preserve a simple control structure and appropriate controller gains to implement the trajectory tracking of robot manipulators.

C. STABILITY ANALYSIS

Given a system (26),the following statement have been accomplished.

Theorem 1: For the robot system (1), the proposed RPTC given by (22)-(25) ensures that the predefined-time stability of both the position and velocity tracking errors can be guaranteed within a predefined time T_c given by (23).

Proof: For system (26), there exists a Lyapunov function candidate as follows

$$V = \left(\sum_{i=1}^{n} \left(|\chi_i| + \frac{1}{2}|e_i|^2 \right) \right)^r \tag{27}$$

The first time differential of V along with system (26) yields

$$\begin{aligned} \dot{V} &= r\left(\sum_{i=1}^{n} \left(|\chi_i| + \frac{1}{2}|e_i|^2 \right) \right)^{r-1} \left(\dot{\chi}^T \mathrm{Sgn}(\chi) + e^T \dot{e} \right) \\ &= r\left(\sum_{i=1}^{n} \left(|\chi_i| + \frac{1}{2}|e_i|^2 \right) \right)^{r-1} \\ &\quad \times \left(\left(\ddot{e} + \lambda\alpha \mathrm{diag}\left\{|e_i|^{\alpha-1}\right\} \dot{e} \right)^T \mathrm{Sgn}(\chi) + e^T \dot{e} \right) \end{aligned} \tag{28}$$

After substituting (26) into (28) yields

$$\begin{aligned} \dot{V} = & r\left(\sum_{i=1}^{n} \left(|\chi_i| + \frac{1}{2}|e_i|^2 \right) \right)^{r-1} \\ & \times \left(\left(\bar{M}_0^{-1}(u_{r0} + u_{r1} + \rho_r) \right)^T \mathrm{Sgn}(\chi) + e^T \dot{e} \right) \\ & + r\left(\sum_{i=1}^{n} \left(|\chi_i| + \frac{1}{2}|e_i|^2 \right) \right)^{r-1} \\ & \times \left(\lambda\alpha \mathrm{diag}\left\{|e_i|^{\alpha-1}\right\} \dot{e} \right)^T \mathrm{Sgn}(\chi) \end{aligned}$$

$$
\begin{aligned}
&\leq r\left(\sum_{i=1}^{n}\left(|\chi_i|+\frac{1}{2}|e_i|^2\right)\right)^{r-1} \\
&\quad\times\left(\left(\bar{M}_0^{-1}(u_{r0}+u_{r1})\right)^T \mathrm{Sgn}(\chi)+\left\|\bar{M}_0^{-1}\right\|\|\rho_r\|\right) \\
&\quad+r\left(\sum_{i=1}^{n}\left(|\chi_i|+\frac{1}{2}|e_i|^2\right)\right)^{r-1} \\
&\quad\times\left(\lambda\alpha\dot{e}^T \mathrm{diag}\left\{|e_i|^{\alpha-1}\right\}\mathrm{Sgn}(\chi)+e^T\dot{e}\right) \qquad (29)
\end{aligned}
$$

Then, substituting (23) and (24) into (29), we have

$$
\begin{aligned}
\dot{V} &\leq r\left(\sum_{i=1}^{n}\left(|\chi_i|+\frac{1}{2}|e_i|^2\right)\right)^{r-1} \\
&\quad\times\Bigg(-\frac{\bar{\gamma}_1+\bar{\gamma}_2}{2}w+\left\|\bar{M}_0^{-1}\right\|\|\rho_r\| \\
&\quad-\frac{1}{(r-1)T_c}\exp\left(\left(\sum_{i=1}^{n}\left(|\chi_i|+\frac{1}{2}|e_i|^2\right)\right)^{r-1}\right)\Bigg) \\
&= r\left(\sum_{i=1}^{n}\left(|\chi_i|+\frac{1}{2}|e_i|^2\right)\right)^{r-1} \\
&\quad\times\Bigg(\frac{\bar{\gamma}_1+\bar{\gamma}_2}{2}(-w+\|\rho_r\|) \\
&\quad-\frac{1}{(r-1)T_c}\exp\left(\left(\sum_{i=1}^{n}\left(|\chi_i|+\frac{1}{2}|e_i|^2\right)\right)^{r-1}\right)\Bigg) \qquad (30)
\end{aligned}
$$

where the fact $\left\|\bar{M}_0^{-1}\right\|=(\bar{\gamma}_1+\bar{\gamma}_2)/2$ is used from (14).

By utilizing (22), (25) and ρ_r given by (18), it follows that

$$
\begin{aligned}
-w+\|\rho_r\| &= -w+\left(\bar{a}_0+\bar{a}_1\|\dot{q}_r\|^2+\bar{\sigma}\|u_r\|\right) \\
&\leq -(1-\bar{\sigma})w-\bar{\sigma}w \\
&\quad+\left(\bar{a}_0+\bar{a}_1\|\dot{q}\|^2+\bar{\sigma}\|u_{r0}-\eta_r\|\right)+\bar{\sigma}\|u_{r1}\| \\
&= -\bar{\sigma}w+\bar{\sigma}\|u_{r1}\| \\
&= 0 \qquad (31)
\end{aligned}
$$

where the fact $\|u_r\|\leq\|u_{r0}-\eta_r\|+\|u_{r1}\|$ and $\|u_{r1}\|=w$ with $w>0$ are obtained from (22) and (25), respectively.

Upon substituting (27) and (31) into (30), we have

$$
\begin{aligned}
\dot{V} &\leq -r\left(\sum_{i=1}^{n}\left(|\chi_i|+\frac{1}{2}|e_i|^2\right)\right)^{r-1} \\
&\quad\times\frac{1}{(r-1)T_c}\exp\left(\left(\sum_{i=1}^{n}\left(|\chi_i|+\frac{1}{2}|e_i|^2\right)\right)^{r-1}\right) \\
&= -\frac{1}{\beta T_c}V^{\beta}\exp(V^{\beta}) \qquad (32)
\end{aligned}
$$

where $\beta=(r-1)/r$ with $r>1$.

According to (32) and the facts $0<\beta<1$ and $r>1$, we can conclude that $y=V$ is the solution to system (8) if $V\neq 0$. As a result, $V=0$ is an equilibrium point and then $e=0$ and $\chi=0$ from (27) and $\dot{e}=0$ from (21). According to Lemma 1 and 'Comparison Principle' of differential equations [40], consequently, the predefined-time stability of both the position and velocity tracking errors have been guaranteed within a predefined time T_c given by (23).

Then, the proof of Theorem 1 have been completed.

Remark 3: Different from the existing robust stable controls [4], [7], [9], [10], the convergence time of the proposed RPTC can be given as an exact controller parameter in advance. In particular, the proposed RPTC has the least convergence time, which provides an exact relationship between the control parameters and the transient tracking performance.

Remark 4: In Ref. [26], we have developed a novel sliding mode control approach with fixed-time convergence for robot manipulators with UEDs. The convergence time of the work [26] is the upper bound of time function and can be calculated from control parameters in advance. However, the fixed-time stable control cannot be applied for a given time-constraints tracking system. Accordingly, in comparison with our previous work [26], the proposed approach given by (22)-(25) is to design a robust predefined-time stable control instead of one with fixed-time convergence for global tracking of robot manipulator. The settling time of the proposed approach is the least upper bound instead of upper bound of time function. The proposed approach is easy to apply into a nonlinear system with time-constraints tracking control.

Remark 5: The robust predefined-time stable control (22)-(25) proposed in this paper has some discontinuous components that can cause the chattering situation. In this paper, we have used the following function to eliminate this chattering situation [41].

$$
\mathrm{sign}(\chi_i)=\frac{\exp(\kappa\chi_i)-1}{\exp(\kappa\chi_i)+1} \qquad (33)
$$

where κ denotes a given constant.

IV. SIMULATION COMPARISONS

In this section, the following two-DOFs robot model [13] is employed to prove the improved tracking performance in comparison with the existing robust controls.

$$
\bar{M}(q_r)=\begin{bmatrix} H_1+2H_2\cos(q_{r2}) & H_3+H_2\cos(q_{r2}) \\ H_3+H_2\cos(q_{r2}) & H_4 \end{bmatrix} \qquad (34)
$$

$$
\bar{C}(q_r,\dot{q}_r)=\begin{bmatrix} -H_2\sin(q_{r2})\dot{q}_{r1} & -2H_2\sin(q_{r2})\dot{q}_{r1} \\ 0 & H_2\sin(q_{r2})\dot{q}_{r2} \end{bmatrix} \qquad (35)
$$

$$
\bar{g}(q_r)=\begin{bmatrix} H_5\cos(q_{r1})+H_6\cos(q_{r1}+q_{r2}) \\ H_6\cos(q_{r1}+q_{r2}) \end{bmatrix}^T \qquad (36)
$$

with

$$
\begin{aligned}
H_1 &= (m_{r1}+m_{r2})r_1^2+m_{r2}r_2^2+J_{r1},\ H_2=m_{r2}r_1r_2 \\
H_3 &= m_{r2}r_2^2,\ H_4=H_3+J_{r2} \\
H_5 &= (m_{r1}+m_{r2})r_1g_1,\ H_6=m_{r2}r_2g_1 \qquad (37)
\end{aligned}
$$

The parameters of robot manipulators are as follows: $J_{r1}=J_{r2}=5.0$ kg·m, $m_{r1}=0.5$ kg, $m_{r2}=1.5$ kg,

$r_1 = 1.0$ m, $r_2 = 1.0$ m and $g_1 = 9.8$ m/s^2. The nominal value of m_{r1} and m_{r2} are $\hat{m}_{r1} = 0.4$ kg and $\hat{m}_{r2} = 1.2$ kg. In this simulation comparisons, the sampling period is 1 ms.

In this section, the effectiveness of the proposed approach can be verified in the following two aspects: (i) upon the sufficient consideration of uncertainties and disturbances, we have focused on the effectiveness of the proposed RPTC in both position and velocity steady-state tracking precision; (ii) it is emphasised on the advantage of the proposed RPTC in transient convergence performance, in which the settling time independently of the initial states can be given as an exact controller parameters in advance.

A. TRACKING PERFORMANCE WITH UNCERTAIN DYNAMICS

By considering uncertain dynamics (2), firstly, we have involved the tracking performance of the proposed RPTC compared with the typical robust controls (Predefined-time sliding mode control (PSMC) [47], Predefined-time robust stabilization control (PRSC) [36]).

In virtue of the definition of sliding surface and controller given by (7) of Ref. [47], for robot manipulators, the predefined-time sliding surface and control (PSMC) is given as

$$S = \dot{e} + \mathrm{Sig}^{1/2}(z) \tag{38}$$

$$\begin{aligned} u_r = & -\gamma_1^2\left(q_1 + p_1 \mathrm{diag}\left\{|e_i|^{p_1}\right\}\right)\mathrm{diag}\left\{|e_i|^{q_1-1}\right\} \\ & \times \mathrm{diag}\left\{\exp\left(|e_i|^{p_1}\right)\right\}\mathrm{Sgn}(S) - k\mathrm{Sgn}(S) \\ & -\gamma_2 \mathrm{diag}\left\{\exp\left(\alpha_2|S_i|^{p_2}\right)\right\}\mathrm{Sig}^{\beta_2 q_2}(S) \end{aligned} \tag{39}$$

with $T_{c1} > 0$, $T_{c2} > 0$, $p_1 > 0$, $1 \le q_1 < 2$, $\alpha_2 > 0$, $\beta_2 > 0$, $p_2 > 0$ and $q_2 > 0$ are some positive constants such that $\beta_2 q_2 < 1$, diag$\{\cdot\}$ denotes the diagonal matrix, Sgn(S) and Sig$^{1/2}(z)$ are defined by (3) and (4), respectively, and

$$z = \mathrm{Sig}^2(\dot{e}) + 2\gamma_1^2 \mathrm{diag}\left\{\exp\left(|e_i|^{p_1}\right)\right\}\mathrm{Sig}^{q_1}(e) \tag{40}$$

$$k = b_0 + b_1\|q_r\| + b_2\|\dot{q}_r\|^2 \tag{41}$$

with b_0, b_1 and b_2 denote some positive constants which depend on robotic system, and

$$\gamma_1 = 2^{\frac{1-q_1/2}{p_1}}\Gamma\left(\frac{1-q_1/2}{p_1}\right)\Big/ p_1 T_{c1} \tag{42}$$

$$\gamma_2 = \alpha_2^{\frac{\beta_2 q_2 - 1}{p_2}}\Gamma\left(\frac{1-\beta_2 q_2}{p_2}\right) p_2 T_{c2} \tag{43}$$

where $\Gamma(\cdot)$ stands for gamma function [34].

Remark 6: In Ref. [47], the control gain k satisfies the fact $k > \delta$ with $|\Delta| \le \delta$ where Δ denotes the lumped uncertainties and external disturbances. Based on Ref. [13] and (12), consequently, for uncertian robot manipulators, the control gain k given by (39) can be modified as (41).

While for the PRSC [36], it can be designed as

$$S = \dot{q}_r - \dot{q}_m \tag{44}$$

$$\dot{q}_m = \dot{q}_d - \alpha(q_r - q_d) \tag{45}$$

$$u_r = u_{r1} + u_{r2} + \hat{Y}_r \tag{46}$$

with

$$u_{r1} = \begin{cases} u_{r0}, & \|S\| \ge \delta \\ -\dfrac{\pi\left(\gamma^{1-\eta/2}\delta^{-\eta} + \gamma^{1+\eta/2}\delta^{\eta}\right)}{\eta T_c}S, & \|S\| < \delta \end{cases} \tag{47}$$

$$u_{r2} = \begin{cases} -k\dfrac{S}{\|S\|}, & \|S\| \ge \delta \\ -\dfrac{k}{\delta}S, & \|S\| < \delta \end{cases} \tag{48}$$

$$\hat{Y}_r = \hat{\bar{M}}(q_r)\ddot{q}_m + \hat{\bar{C}}(q_r, \dot{q}_r)\dot{q}_m + \hat{\bar{g}}(q_r) \tag{49}$$

and

$$u_{r0} = -\frac{\pi}{\eta T_c}\left(\gamma^{1-\eta/2}\|S\|^{-\eta} + \gamma^{1+\eta/2}\|S\|^{\eta}\right)S \tag{50}$$

where $T_c > 0$, $\eta > 0$, $\gamma > 0$, $\alpha > 0$, $k > 0$ and $\delta > 0$ denote some positive constants.

For the PSMC, PRSC and the proposed RPTC, in this part the initial conditions are as follows

$$\left[q_r(0)^T,\ \dot{q}_r(0)^T\right] = [-2.0,\ 2.5,\ 0,\ 0]^T \tag{51}$$

The desired trajectories are

$$q_d = \begin{bmatrix} 1.25 - 7/5\exp(-t) + 7/20\exp(-4t) \\ 1.25 + \exp(-t) - 1/4\exp(-4t) \end{bmatrix} \tag{52}$$

For above controllers, by using (34) and the above given system parameters, we have obtained $\bar{\gamma}_1 = 0.09$ and $\bar{\gamma}_2 = 0.2$ from (15), and hence $\bar{M}_0 = 6.89I_n$ and $\bar{\sigma} = 0.38$ given by (14) and (17). Furthermore, $\bar{C}_0(q_r, \dot{q}_r)$ and $\bar{g}_0(q_r)$ will be constructed by substituting the nominal ones $\hat{m}_{r1}$ and $\hat{m}_{r2}$ into (35)-(37) instead of m_{r1} and m_{r2}. The parameters of the PSMC, PRSC and the proposed RPTC are summarized in Table 1.

TABLE 1. The parameters of PSMC, PRSC and RPTC.

Controllers	Parameters
PSMC	$q_1 = 1.2,\ p_1 = 1,\ T_{c1} = 1,\ q_2 = 0.5$ $p_2 = 1,\ T_{c2} = 1,\ \alpha_2 = 0.001,\ \beta_2 = 1$ $b_0 = 12,\ b_1 = 2.2,\ b_2 = 2.8$
PRSC	$\alpha = 5,\ \gamma = 0.03,\ \eta = 0.5$ $T_c = 1,\ k = 150,\ \delta = 0.2$
RPTC	α=1.1 $r = 1.2,\ T_c = 1.8,\ \bar{a}_0 = 12,\ \bar{a}_1 = 2.8$ $\kappa = 100,\ \lambda = \mathrm{diag}\{5,\ 5\},\ \bar{\gamma}_1 = 0.09,\ \bar{\gamma}_2 = 0.2$

Firstly, we have completed the simulation comparisons with the typical predefined-time controllers (PSMC (38)-(43) and PRSC (44)-(50)). Fig. 2 shows the position tracking of the proposed RPTC. The position and velocity tracking errors of PRSC, PSMC and the proposed RPTC with its zoomed plots have been depicted in Figs. 3 and 4, respectively; while Fig. 5 depicts the control inputs. Obviously, the proposed RPTC shows better transient-state tracking precision than PRSC and PSMC subject to uncertainties and external disturbances as shown in Figs. 3 and 4. No matter whether the system is affected by uncertainties and external disturbances, consequently, we can conclude that the proposed RPTC always provides higher steady-state tracking precision in both

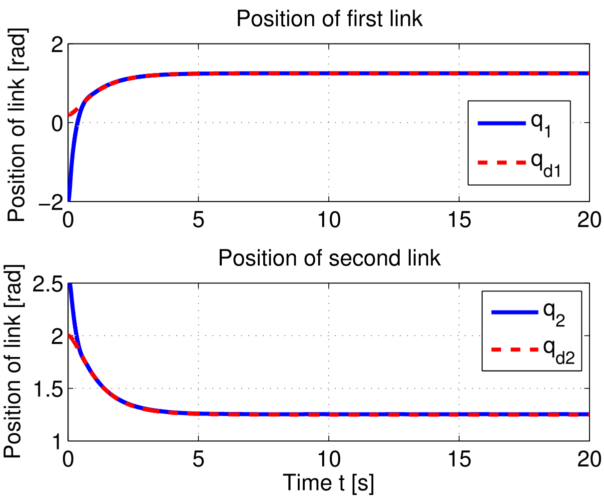


FIGURE 2. Position tracking of the proposed RPTC.

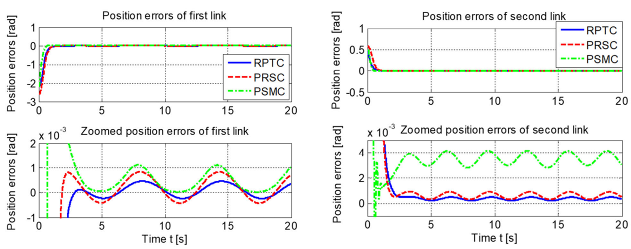


FIGURE 3. Position tracking error of RPTC, PRSC and PSMC.

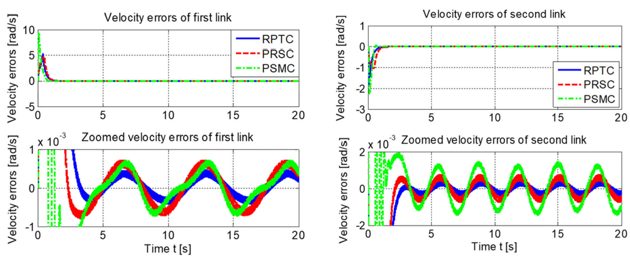


FIGURE 4. Velocity tracking error of RPTC, PRSC and PSMC.

position and velocity trajectory tracking system. Moreover, these superior tracking performances of the proposed RPTC have been accomplished without using the excessive input torque than other controls observed by Fig. 5.

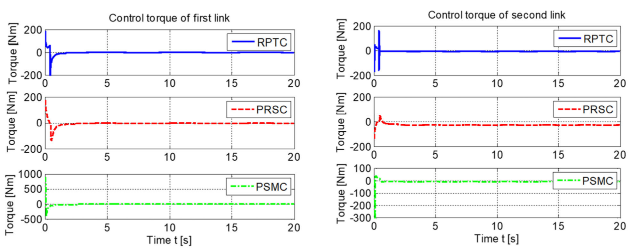


FIGURE 5. Input toques of RPTC, PRSC and PSMC.

After that, the effectiveness of the proposed RPTC has been accomplished in comparison with the typical sliding mode controls with sinusoidal desired trajectories. Then, the non-singular fast TSMC (FTSMC) [44] is selected for comparison. The FTSMC controller and sliding surface are selected as

$$S_{ft} = e + \mathrm{Sig}^{\bar{\Gamma}_1}(e) + \mathrm{Sig}^{\bar{\Gamma}_2}(\dot{e}) \tag{53}$$

$$u_{ft} = -\bar{M}_0(q_r)\Big[\bar{M}_2 S_{ft} + (\bar{\zeta} + \bar{M}_1) b(S_{ft}) + \bar{F}_2 + \bar{\Gamma}_2^{-1}\left(I_2 + \bar{\Gamma}_1 \mathrm{diag}\left\{|\dot{e}|^{\bar{\Gamma}_1 - I_2}\right\}\right)\mathrm{Sig}^{2I_2 - \bar{\Gamma}_2}(\dot{e})\Big] \tag{54}$$

where

$$\bar{\zeta} = \left\|\bar{M}_0^{-1}(q_r)\right\|\left(\bar{c}_0 + \bar{c}_1\|q_r\| + \bar{c}_2\|\dot{q}_r\|^2\right) \tag{55}$$

$$\bar{F}_2 = -\bar{M}_0^{-1}(q_r)\left(\bar{C}_0(q_r, \dot{q}_r)\dot{q}_r + \bar{g}_0(q_r)\right) - \ddot{q}_d \tag{56}$$

$$b(S_{ft}) = \begin{cases} \dfrac{S_{ft}}{\|S_{ft}\|}, & \|S_{ft}\| \neq 0 \\ 0, & \|S_{ft}\| = 0 \end{cases} \tag{57}$$

with $\bar{M}_1 > 0$, $\bar{M}_2 > 0$ and $\bar{c}_i > 0$, $i = 0, 1, 2$ are some constants, and $\bar{\Gamma}_1$ and $\bar{\Gamma}_2$ denote the positive definite diagonal matrices.

Similar to Ref. [44], the external disturbances have been considered in this simulation comparisons and given as $d_r = [2\sin(t) + 0.5\sin(200\pi t),\ \cos(2t) + 0.5\sin(200\pi t)]^T$. The initial conditions are $\left[q_r(0)^T,\ \dot{q}_r(0)^T\right]^T = [0.5, -1.5, 0, 0]^T$. For more variation of the tracking, the desired trajectories are

$$q_d = \begin{bmatrix} 1.25 + (\pi/3)\sin(2t + \pi/4) \\ 0.5\cos(2t + \pi/4) \end{bmatrix} \tag{58}$$

The FTSMC has the same $\bar{C}_0(q_r, \dot{q}_r)$ and $\bar{g}_0(q_r)$ as the proposed RPTC; while $\bar{M}_0(q_r)$ of the FTSMC is chosen by substituting $\hat{m}_{r1}$ and $\hat{m}_{r2}$ into (34) instead of m_{r1} and m_{r2}. The RPTC have the same parameters as Table 1; while the parameters of the FTSMC are $\bar{\Gamma}_1 = \mathrm{diag}\{2, 2\}$,, $\bar{\Gamma}_2 = \mathrm{diag}\{5/3, 5/3\}$, $\bar{M}_1 = \bar{M}_2 = 2$, $\bar{b}_0 = 12$, $\bar{b}_1 = 2.2$ and $\bar{b}_2 = 2.8$.

Secondly, we have accomplished the simulation comparisons with the typical finite-time sliding mode controllers (FTSMC (53)-(57)). Figs. 6 and 7 depict the position and velocity tracking errors of the FTSMC and the proposed RPTC with its zoomed plots, respectively; while Fig. 8 depicts their control inputs. As a result, we have obtained the conclusion from Figs. 6-8 as follows: (i) the proposed RPTC provides an improved transient and steady-state tracking performance than the FTSMC no matter what form interferences the robot manipulators receives; (ii) both the position and velocity tracking performances have been enhanced by the proposed RPTC; (iii) no excessive control input (See Fig. 8) is used in the proposed RPTC in comparison with FTSMC.

After that, in this part we have accomplished a simulation comparisons to show the effects of control parameters of the

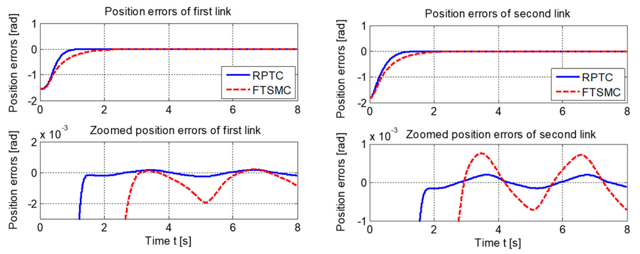


FIGURE 6. Position tracking errors of RPTC and FTSMC.

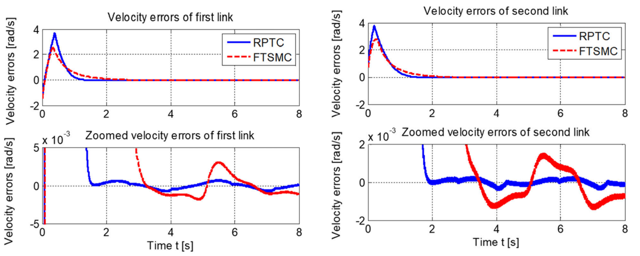


FIGURE 7. Velocity tracking errors of RPTC and FTSMC.

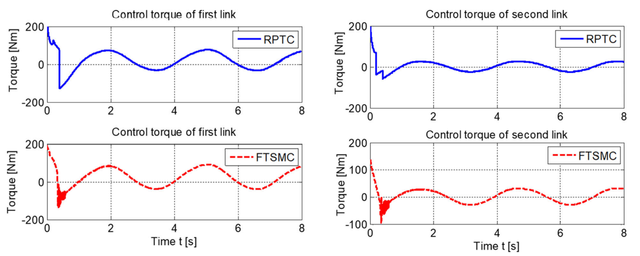


FIGURE 8. Input toques of RPTC and FTSMC.

proposed RPTC on the tracking performance. To this end, the different control parameters are follows

$$\begin{cases} \mathrm{P1}: \lambda = \mathrm{diag}\{5, 5\}, \ \alpha = 1.1, \ r = 1.2 \\ \mathrm{P2}: \lambda = \mathrm{diag}\{1, 1\}, \ \alpha = 1.1, \ r = 1.2 \\ \mathrm{P3}: \lambda = \mathrm{diag}\{5, 5\}, \ \alpha = 1.1, \ r = 3 \\ \mathrm{P4}: \lambda = \mathrm{diag}\{5, 5\}, \ \alpha = 3, \ r = 1.2 \end{cases} \tag{59}$$

In this comparison, the same initial conditions and desired trajectories given by (51) and (52) are adopted. In addition to λ, α and r, the other parameters have the same values as the above comparisons depicted by Figs. 2-5 in Table 1. Figs. 9 and 10 represent the position errors and the control torque input with their zoomed plots. By analysing the different control parameters (59) and Figs. 9 and 10, we have obtained the following conclusions: i)the proposed RPTC have better tracking performance with the increased λ observed by P1 and P2, but the control torque will be increased with the large λ; ii) the increased α and r cannot significantly improve the tracking performance as shown in Figs. 9 and 10, however, these increased parameters may enlarge the control torque input from Fig. 10. In virtue of the above analysis, consequently, we can conclude that the

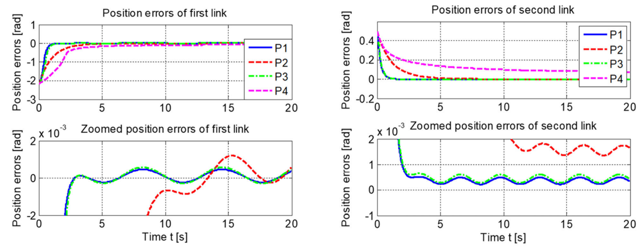


FIGURE 9. Position tracking errors of the proposed RPTC with different controller parameters.

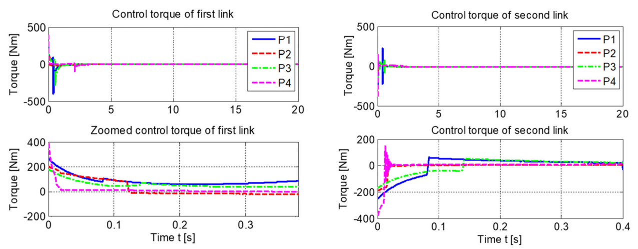


FIGURE 10. Input torques of the proposed RPTC with different controller parameters.

proposed RPTC selects the parameters on a trade-off between the tracking performance and control inputs.

B. TRACKING CONTROL WITH DIFFERENT INITIAL CONDITIONS

Another advantage of the proposed RPTC is that its convergence time independently of the initial conditions can be given as an exact controller parameters. It means that the position and velocity tracking errors started from anywhere of the state space always converge globally to the origin within a predefined time T_c given by (23). Consequently, in this part we have focused on the effects of initial states and controller parameters on the position and velocity tracking performance.

Thus, the following initial states will be used in the proposed RPTC given by (22)-(25), where the same controller parameters are adopted in Table 1. It follows that

$$\text{Type I} \begin{cases} \text{Case 1}: \ [q_{r1}(0), \ q_{r2}(0)]^T = [1.5, \ 1.8]^T \\ \text{Case 2}: \ [q_{r1}(0), \ q_{r2}(0)]^T = [1.0, \ 1.5]^T \\ \text{Case 3}: [q_{r1}(0), \ q_{r2}(0)]^T = [-0.5, \ 0.5]^T \\ \text{Case 4}: \ [q_{r1}(0), \ q_{r2}(0)]^T = [-1.0, \ -1.5]^T \\ \text{Case 5}: \ [q_{r1}(0), \ q_{r2}(0)]^T = [-2.0, \ -2.5]^T \end{cases} \tag{60}$$

By utilizing different initial states to the proposed RPTC, Fig.11 depict the position and velocity tracking errors, respectively. According to Theorem 1, e and $\dot{e}$ converge always to the origin within a predefined time T_c given by (23). The simulation results of Fig. 11 are to further verify Theorem 1 in which the convergence time of the proposed RPTC is defined as an exact control parameters instead of the initial states of close-loop system. The simulation comparisons of

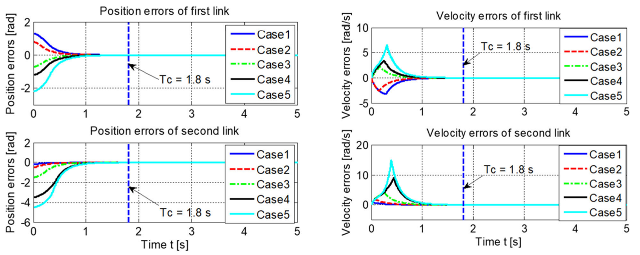


FIGURE 11. Position and velocity tracking errors of RPTC in Type I.

Fig. 11 will further verified the effectiveness of the proposed RPTC for uncertain robot manipulators in both position and velocity trajectory tracking.

Accordingly, upon the simulation analysis of subsections A and B, no matter the systemic states start from any positions in state space, the proposed RPTC with a simple structure always has an improved tracking performance than other robust controllers in both position and velocity trajectory tracking.

V. EXPERIMENTAL RESULTS

In this section, several experimental results have been accomplished on the SCARA robot system as shown in Fig. 12 to further verify the effectiveness of the proposed RPTC. The SCARA robot system consists mainly of three parts such as servo driver, high-performance computer (HC) and SCARA robot. The position of joint is obtained from a relative encoder. Then, this position information is transmitted into an high-performance computer by the interface board GT400-CV of Googol from the motor encoder; while the control torque is transmitted into the motor with its driver through the interface board GT400-CV of Googol from the HC. For joint 1, the maximum output torque of motor 1 is 0.64 Nm, then the maximum output torque of joint 1 is 51.2 Nm because the joint consists of a motor and harmonic reducer (harmonic reduction ratio:1 : 80); while for joint 2, the maximum output torque of motor 2 is 0.32 Nm, then the maximum output torque of joint 2 is 16 Nm (harmonic reduction ratio:1 : 50). The program is written with the simulink of Matlab 2012b.

In this experiments, the sample period is $T = 2$ ms. Both two joints start from zero. According to the proposed RPTC given by (22)-(25), the parameters are selects: $\lambda = 1$, $\alpha = 1.1$, $r = 1.2$, $T_c = 1.8$, $\bar{a}_0 = 5$ and $\bar{a}_1 = 2.2$.

Fig. 13 shows the position and control torque input of the proposed RPTC; while Fig. 14 depict the tracking errors with its zoomed plots of the proposed RPTC. Obviously, the real positions of joint can track the desired trajectories with fast transient state speed and high-precision steady-state tracking performance shown in Fig. 13. Observed by Fig. 14, the proposed RPTC can obtains the high-precision steady-state tracking performance (± 0.4 [degree]). The experimental results further verify that the proposed controller achieves an

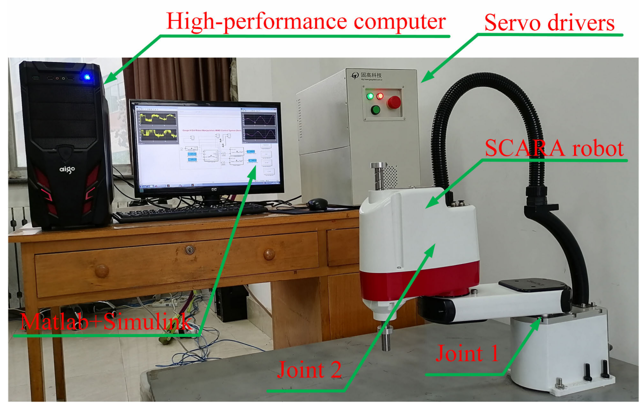


FIGURE 12. The experimental robot setup.

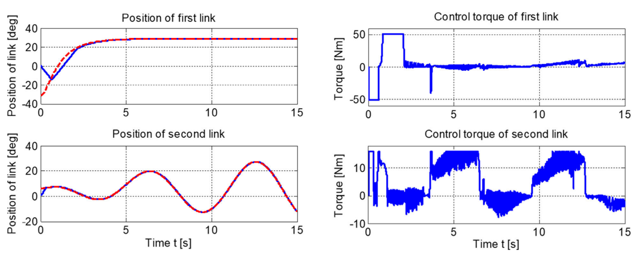


FIGURE 13. Positions of the proposed RPTC with its control torque input.

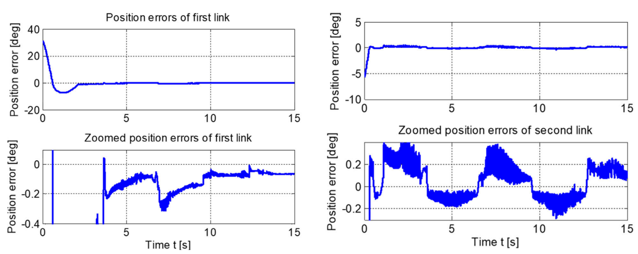


FIGURE 14. Position tracking errors of the proposed RPTC with its zommed plots.

improved tracking performance such as faster transient and smaller steady-state tracking error.

VI. CONCLUSION

In this paper, we have developed a novel robust predefined-time control for global predefined-time tracking of robot manipulators with uncertainties and external disturbances. Numerical simulations demonstrate the enhanced tracking performance of the proposed approach in comparison with the traditional robust controls in both position and velocity tracking. As a result, the proposed approach gets higher steady-state tracking precision with a predefined time than other robust controllers. Meanwhile, the developed approach provides higher robustness subject to uncertainties and external disturbances. Future efforts will focus on finding a robust predefined-time tracking control with actuator constraints and continuity for robot manipulators with uncertain dynamics and external disturbances.

REFERENCES

[1] H. Yu, J.-H. Shen, K. M. Joos, and N. Simaan, "Calibration and integration of B-Mode optical coherence tomography for assistive control in robotic microsurgery," *IEEE/ASME Trans. Mechatronics*, vol. 21, no. 6, pp. 2613–2623, Dec. 2016.

[2] S. Islam and X. P. Liu, "Robust sliding mode control for robot manipulators," *IEEE Trans. Ind. Electron.*, vol. 58, no. 6, pp. 2444–2453, Jun. 2011.

[3] W. Yu and J. Rosen, "Neural PID control of robot manipulators with application to an upper limb exoskeleton," *IEEE Trans. Cybern.*, vol. 43, no. 2, pp. 673–684, Apr. 2013.

[4] A. Tayebi, S. Abdul, M. B. Zaremba, and Y. Ye, "Robust iterative learning control design: Application to a robot manipulator," *IEEE/ASME Trans. Mechatronics*, vol. 13, no. 5, pp. 608–613, Oct. 2008.

[5] M. Wang and A. Yang, "Dynamic learning from adaptive neural control of robot manipulators with prescribed performance," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no. 8, pp. 2244–2255, Aug. 2017.

[6] Q. Yan, J. Cai, Y. Ma, and Y. Yu, "Robust learning control for robot manipulators with random initial errors and iteration-varying reference trajectories," *IEEE Access*, vol. 7, pp. 32628–32643, 2019.

[7] F. Lin and R. D. Brandt, "An optimal control approach to robust control of robot manipulators," *IEEE Trans. Robot. Autom.*, vol. 14, no. 1, pp. 69–77, Feb. 1998.

[8] A. Ferrara and G. P. Incremona, "Design of an integral suboptimal second-order sliding mode controller for the robust motion control of robot manipulators," *IEEE Trans. Control Syst. Technol.*, vol. 23, no. 6, pp. 2316–2325, Nov. 2015.

[9] M. Jin, S. H. Kang, P. H. Chang, and J. Lee, "Robust control of robot manipulators using inclusive and enhanced time delay control," *IEEE/ASME Trans. Mechatronics*, vol. 22, no. 5, pp. 2141–2152, Oct. 2017.

[10] D. Zhao, F. Gao, S. Li, and Q. Zhu, "Robust finite-time control approach for robotic manipulators," *IET Control Theory Appl.*, vol. 4, no. 1, pp. 1–15, Jan. 2010.

[11] A. T. Vo and H.-J. Kang, "A chattering-free, adaptive, robust tracking control scheme for nonlinear systems with uncertain dynamics," *IEEE Access*, vol. 7, pp. 10457–10466, 2019.

[12] S. Mobayen and J. Ma, "Robust finite-time composite nonlinear feedback control for synchronization of uncertain chaotic systems with nonlinearity and time-delay," *Chaos, Solitons Fractals*, vol. 114, pp. 46–54, Sep. 2018.

[13] Y. Feng, X. Yu, and Z. Man, "Non-singular terminal sliding mode control of rigid manipulators," *Automatica*, vol. 38, no. 12, pp. 2159–2167, Dec. 2002.

[14] S. H. Yu, X. H. Yu, B. Shirinzadeh, and Z. H. Man, "Continuous finite-time control for robotic manipulators with terminal sliding mode," *Automatica*, vol. 41, no. 11, pp. 1957–1964, 2005.

[15] L. Wang, T. Chai, and L. Zhai, "Neural-network-based terminal sliding-mode control of robotic manipulators including actuator dynamics," *IEEE Trans. Ind. Electron.*, vol. 56, no. 9, pp. 3296–3304, Sep. 2009.

[16] M. Jin, J. Lee, P. Hun Chang, and C. Choi, "Practical nonsingular terminal sliding-mode control of robot manipulators for high-accuracy tracking control," *IEEE Trans. Ind. Electron.*, vol. 56, no. 9, pp. 3593–3601, Sep. 2009.

[17] S. Mobayen and F. Tchier, "Nonsingular fast terminal sliding-mode stabilizer for a class of uncertain nonlinear systems based on disturbance observer," *Scientia Iranica*, vol. 24, no. 3, pp. 1410–1418, Jun. 2017.

[18] J. Baek, M. Jin, and S. Han, "A new adaptive sliding-mode control scheme for application to robot manipulators," *IEEE Trans. Ind. Electron.*, vol. 63, no. 6, pp. 3628–3637, Jun. 2016.

[19] L. Zhang, L. Liu, Z. Wang, and Y. Xia, "Continuous finite-time control for uncertain robot manipulators with integral sliding mode," *IET Control Theory Appl.*, vol. 12, no. 11, pp. 1621–1627, Jul. 2018.

[20] S. Baek, J. Baek, and S. Han, "An adaptive sliding mode control with effective switching gain tuning near the sliding surface," *IEEE Access*, vol. 7, pp. 15563–15572, 2019.

[21] S. Mobayen, "Design of LMI-based sliding mode controller with an exponential policy for a class of underactuated systems," *Complexity*, vol. 21, no. 5, pp. 117–124, May 2016.

[22] M. Van, M. Mavrovouniotis, and S. S. Ge, "An adaptive backstepping nonsingular fast terminal sliding mode control for robust fault tolerant control of robot manipulators," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 49, no. 7, pp. 1448–1458, Jul. 2019.

[23] Z. Chen, X. Yang, X. Zhang, and P. X. Liu, "Finite-time trajectory tracking control for rigid 3-DOF manipulators with disturbances," *IEEE Access*, vol. 6, pp. 45974–45982, 2018.

[24] V. Andrieu, L. Praly, and A. Astolfi, "Homogeneous approximation, recursive observer design, and output feedback," *SIAM J. Control Optim.*, vol. 47, no. 4, pp. 1814–1850, Jan. 2008.

[25] A. Polyakov, "Nonlinear feedback design for fixed-time stabilization of linear control systems," *IEEE Trans. Autom. Control*, vol. 57, no. 8, pp. 2106–2110, Aug. 2012.

[26] L. Zhang, Y. Wang, Y. Hou, and H. Li, "Fixed-time sliding mode control for uncertain robot manipulators," *IEEE Access*, vol. 7, pp. 149750–149763, 2019.

[27] A. Polyakov, D. Efimov, and W. Perruquetti, "Robust stabilization of MIMO systems in finite/fixed time," *Int. J. Robust Nonlinear Control*, vol. 26, no. 1, pp. 69–90, Jan. 2016.

[28] B. Tian, Z. Zuo, X. Yan, and H. Wang, "A fixed-time output feedback control scheme for double integrator systems," *Automatica*, vol. 80, pp. 17–24, Jun. 2017.

[29] Z. Zuo, "Nonsingular fixed-time consensus tracking for second-order multi-agent networks," *Automatica*, vol. 54, pp. 305–309, Apr. 2015.

[30] Y. Su, C. Zheng, and P. Mercorelli, "Robust approximate fixed-time tracking control for uncertain robot manipulators," *Mech. Syst. Signal Process.*, vol. 135, Jan. 2020, Art. no. 106379.

[31] L. Fraguela, M. T. Angulo, J. A. Moreno, and L. Fridman, "Design of a prescribed convergence time uniform robust exact observer in the presence of measurement noise," in *Proc. IEEE 51st IEEE Conf. Decis. Control (CDC)*, Dec. 2012, pp. 6615–6620.

[32] J. D. Sanchez-Torres, E. N. Sanchez, and A. G. Loukianov, "A discontinuous recurrent neural network with predefined time convergence for solution of linear programming," in *Proc. IEEE Symp. Swarm Intell. (SIS)*, Dec. 2014, pp. 1–5.

[33] J. D. Sánchez-Torres, D. Gómez-Gutiérrez, E. López, and A. G. Loukianov, "A class of predefined-time stable dynamical systems," *IMA J. Math. Control Inf.*, vol. 35, no. 1, pp. i1–i29, Mar. 2018.

[34] E. Jiménez-Rodríguez, J. D. Sánchez-Torres, and A. G. Loukianov, "On optimal predefined-time stabilization," *Int. J. Robust Nonlinear Control*, vol. 27, no. 17, pp. 3620–3624, 2017.

[35] R. Aldana-López, D. Gómez-Gutiérrez, E. Jiménez-Rodríguez, J. D. Sánchez-Torres, and M. Defoort, "Enhancing the settling time estimation of a class of fixed-time stable systems," *Int. J. Robust Nonlinear Control*, vol. 29, no. 12, pp. 4135–4148, Aug. 2019.

[36] A. J. Munoz-Vazquez, J. D. Sanchez-Torres, E. Jimenez-Rodriguez, and A. G. Loukianov, "Predefined-time robust stabilization of robotic manipulators," *IEEE/ASME Trans. Mechatronics*, vol. 24, no. 3, pp. 1033–1040, Jun. 2019.

[37] W. H. Zhu, "Comments on 'Robust tracking control for rigid robotic manipulators,"' *IEEE Trans. Autom. Control*, vol. 45, no. 8, pp. 1577–1580, Jul. 2000.

[38] W. H. Zhu, *Modelling and Control of Robot Manipulators*, 2nd ed. London, U.K.: Springer-Verlag, 2012.

[39] J. D. Sancheztorres, M. Defoort, and A. J. Munozvazquez, "Predefined-time stabilisation of a class of nonholonomic systems," *Int. J. Control*, pp. 1–8, Jan. 2019.

[40] K. H. Khalil, *Nonlinear Systems*, 3rd ed. Upper Saddle River, NJ, USA: Prentice-Hall, 2005.

[41] H. Li and Y. Cai, "On SFTSM control with fixed-time convergence," *IET Control Theory Appl.*, vol. 11, no. 6, pp. 766–773, Apr. 2017.

[42] M. Van, H.-J. Kang, and Y.-S. Suh, "A novel neural second-order sliding mode observer for robust fault diagnosis in robot manipulators," *Int. J. Precis. Eng. Manuf.*, vol. 14, no. 3, pp. 397–406, Mar. 2013.

[43] Y. Su, P. C. Muller, and C. Zheng, "Global asymptotic saturated PID control for robot manipulators," *IEEE Trans. Control Syst. Technol.*, vol. 18, no. 6, pp. 1280–1288, Nov. 2010.

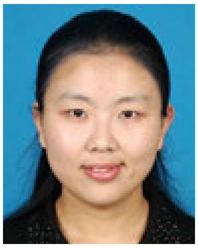
[44] L. Yang and J. Yang, "Nonsingular fast terminal sliding-mode control for nonlinear dynamical systems," *Int. J. Robust Nonlinear Control*, vol. 21, no. 16, pp. 1865–1879, Nov. 2011.

[45] L. Zhang, Y. Su, and Z. Wang, "A simple non-singular terminal sliding mode control for uncertain robot manipulators," *Proc. Inst. Mech. Eng., I, J. Syst. Control Eng.*, vol. 233, no. 6, pp. 666–676, Jul. 2019.

[46] L. Y. Zhang, Y. X. Su, and H. H. Wang, "A nonsingular fast terminal sliding mode control with an exponential reaching law for robot manipulators," *Proc. Inst. Mech. Eng. C, J. Mech. Eng. Sci.*, vol. 233, no. 5, pp. 1575–1587, 2019.

[47] J. D. Sanchez-Torres, M. Defoort, and A. J. Munoz-Vazquez, "A second order sliding mode controller with predefined-time convergence," in *Proc. 15th Int. Conf. Electr. Eng., Comput. Sci. Autom. Control (CCE)*, Sep. 2018, pp. 1–4.

NANSHENG ZHANG received the B.E. degree in measurement and control technology and instruments from the Beijing Institute of Technology, Beijing, China, in 2016, where he is currently pursuing the master's degree with the School of Optics and Photonics. His research interests include optical measurement and ultra-precision motion control.



SHANSHAN WANG received the Ph.D. degree in instrument science and technology from the Beijing Institute of Technology, China, in 2009. She is currently an Associate Professor with the School of Optics and Photonics, Beijing Institute of Technology. Her research interests include interferometric measurement, solar collector inspection, and intelligent control.

YINLONG HOU received the M.S. and Ph.D. degrees in instrument science and technology from the Beijing Institute of Technology, China, in 2013 and 2018, respectively. He is currently a Lecturer with the School of Automation, Xi'an University of Posts and Telecommunications. His research interests include optical measurement, adaptive optics, and intelligent control.

LIYIN ZHANG received the B.S. degree in communication engineering from Ningxia University, Ningxia, China, in 2008, and the M.S. degree in circuits and systems and the Ph.D. degree in mechanical manufacture and automation from Xidian University, Xi'an, China, in 2014 and 2018, respectively. He is currently a Lecturer with the School of Automation, Xi'an University of Posts and Telecommunications. His current interests include sliding mode control, feedback control, and fault tolerant control for nonlinear system with matched and mismatched disturbances.