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Review of Methods for EEG Signal Classification and Development of New Fuzzy Classification-Based Approach

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ABSTRACT The analysis of EEG signal is a relevant problem in health informatics, and its development can help in detection of epileptic's seizures. The diagnosis is based on classification of EEG signal. Different methods and algorithms for classification of EEG signals with an accepted level of reliability and accuracy have been developed over years. All these methods have two steps that are signal preprocessing and classification. The goal of the preprocessing step is removing noise and reduction of the initial signal dimensionality. The signal dimensionality reduction is required by classification methods, but its result is a loss of small information before the classification. In this paper, an approach for EEG signal classification that takes this loss of information into account is considered. The novelty of the considered approach is usage of fuzzy classifier in the classification step. This classifier allows taking uncertainty of initial data into account, which is caused by loss of some information during dimensionality reduction of initial signal. However, application of fuzzy classifier needs modification of the preprocessing step because it requires data in fuzzy form. Therefore, fuzzification procedure is added to the preprocessing step. In this paper, Fuzzy Decision Tree (FDT) is used as the fuzzy classifier for the epileptic's seizure detection. Its application allows achieving 99.5% accuracy of the classification of epileptic's seizure. The comparison with other studies shows that FDT is very effective for task of epileptic's seizure detection.

INDEX TERMS Classification algorithms, decision tree, encephalography, electroencephalogram signal fuzzy classifier, fuzzy decision tree, fuzzy logic, signal analysis, signal classification.

I. INTRODUCTION

Applications of Electroencephalogram (EEG) signal in different areas have been intensively developed in last time. The most dynamically developed areas are human-computer interaction studied in [1], [2] and medicine [3] – [8]. In medicine, the EEG signal analysis is used in epilepsy diagnosis [3], [6], [7], depression [4], stress [5], and other diagnoses [8]. The diagnostics of epilepsy is based mostly on analysis of EEG signal.

Epilepsy is considered as one of the most common chronic neurological disorders [9]. The observable epileptic symptom is recurrent unprovoked seizures which usually occur without warning and that can affect any part of the body [10]. These seizures are consequences of the brain activity that can be characterized by the unexpected and sudden electrical disturbance of brain and excessive neuronal discharge. This activity is recorded using the EEG [11], therefore, the analysis of EEG signal is one of the most important tools in neurology diagnostics [3], [7], [12].

The electrical activity of the brain is recorded using electrodes placed on the top of the head in the form of EEG signals [10], [11]. The analysis of EEG signal is implemented based on curves that are graphical interpretation of captured signals. The visual evaluation of these curves allows doctors to diagnose neural disorder [12]. This visual evaluation can be insufficient, especially in case of not-trained physicians. So, the development of algorithms for automatic and supported

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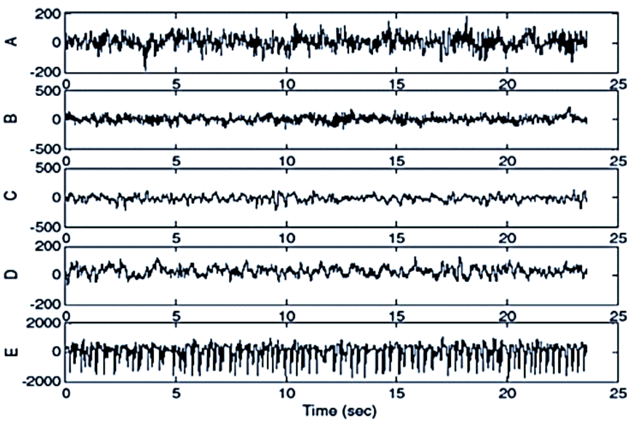

FIGURE 1. **Randomly selected signals from each subset of dataset [12].**

analysis of EEG signals is a relevant problem. These algorithms have to include extraction and analysis of information from the recorded EEG signal that is used in prediction or classification of brain state. In this paper, the EEG signal analysis in detection of epileptic seizures is considered as one of problems in EEG signal analysis.

The development of classification methods is based often on data mining approaches. Data mining-based methods for signal classification are developed based on real collected data. Therefore, data captured by EEG recording devices should be used in such a research. In this paper, we use one of the well-known and often used datasets introduced in [12]. The dataset consists of 500 samples (records) of EEG signals. Each sample represents record of EEG signal with duration of 23.6 seconds. The samples are divided into five subsets (A, B, C, D, and E) of the same size, which means that each of these subsets contains 100 samples of records of EEG signals. Subsets A and B contain measurements recorded from persons who do not suffer from epilepsy. In case of subset A, patients had eyes open during recording while, in case of subset B, patients had closed eyes. Whether the eyes are open or closed affects the electrical activity of the brain according to [12]. Samples in subsets C, D, and E were recorded from persons suffered from epilepsy. EEG of patients in subsets C and D were recorded during seizure-free intervals, while EEG signals of patients in subset E were measured during seizure activity only. Samples in subset D were recorded from within the epileptogenic zone, and those in subset C were obtained from the hippocampal formation of the opposite hemisphere of the brain. Examples of samples from these subsets are shown in Fig. 1.

It can be observed from Fig. 1 that signals in A and E are different from signals in B, C, and D. Nevertheless, only visual inspection can be insufficient to recognize healthy (A, B) and epileptic EEG segments (C, D, E). The most significant similarity between this group is oblivious in case of samples from B, C, and D. The samples are very similar to each other and the use of visual inspection only can lead to a failure in diagnostic (patient suffering from epilepsy is diagnosed as healthy). This example shows that the development of approach for automatic EEG signal analysis can be useful in diagnostics because it allows deciding whether a person has epilepsy without the occurrence of epileptic seizure. Investigation of automatic EEG signal is thus useful in the development of decision support systems for early diagnosis of epilepsy [11], [13], [14].

Data mining methods for signal classification, and EEG signal in particular, include two steps [14]. The first of them is signal preprocessing and the second is classification itself. Typical classifiers used in the second step need numerical attributes for the classification, but EEG signal is represented as a function of time which cannot be directly classified. Therefore, this signal should be transformed into samples of numerical attributes in the step of signal preprocessing [15]–[17], which is also known as the step of the preliminary transformation. EEG signal is preprocessed to remove noise and extract useful information needed for the next step of classification. The signal preprocessing consists of two procedures that are feature extraction and dimensionality reduction. According to investigations in [7], [17], and [18], the preliminary transformation or signal preprocessing step has a significant influence on the result of the classification.

The feature extraction of signal is usually implemented by wavelet [19], [20] or spectral transforms [21], [22]. The result of the feature extraction is a matrix of the signal features, which has typically large dimensionality. This a reason for the second procedure of the preprocessing, which is dimensionality reduction. The dimensionality reduction usually transforms data matrix to a matrix of significantly smaller dimension. Comparison of three basic techniques of dimensionality reduction in EEG signal preprocessing can be found in [23]. These techniques are Linear Discriminant Analysis (LDA), Principal Component Analysis (PCA) and Independent Component Analysis (ICA).

The output of the first step is a set of numerical attributes that are used in the classification step. Existing studies of EEG signal classification focus on various classifiers, such as Support Vector Machine (SVM) [23], [24], k-nearest neighbor (kNN) [25], decision tree [15], [21], or neural networks [14], [26]. Special approaches based on evolution methods and clustering analysis have been considered for EEG signal classification in [25], [27]. However, the researches in [21]–[27] have shown that the accuracy of EEG signal classification depends not only on the used type of classifier but also on the procedures used in the step of the signal preprocessing.

According to investigation performed in [28] and [29], procedures of the feature extraction and dimensionality reduction result in a loss of some information in the step of the preliminary transformation. From point of view of data mining methods, this information loss can be interpreted as uncertainty of the data used for classification. The efficient classification of uncertain data requires application of fuzzy classifiers [15], [30]. These classifiers consider membership of instance to all possible classes. In [31], [32], and [33], it has been shown that the application of fuzzy data allows increasing the accuracy of classification for uncertain data.

Several approaches of EEG signal classification in which the feature extraction, feature selection or dimensionality reduction procedures are developed based on fuzzy logic background have been proposed. The fuzzy entropy-based procedures have been used in the signal preprocessing step in researches [7], [34], and [35]. In other investigations, such as [36], [37], and [38], combined or join classification have been proposed. In these studies, fuzzy-based classifier and classifier of crisp data are used together. The modification of the classification step in these studies is related to the investigated data and decided problem. Therefore the change of procedures of signal preprocessing influences the classification accuracy. The application of proposed methods in [36], [37], and [38] can be for considered problem only. This restricts their application in other similar problems. The surveys of EEG signal classification in [39], [40], and [41] have not highlighted formalized approach of EEG signal classification based on fuzzy classifiers. Therefore, in this paper, we want to develop and propose new approaches for EEG signal classification with the use of fuzzy classifiers. This work is influenced our previous work [15], in which we have investigated fuzzy-based approach for classification of specific signals. In this paper, we propose adaptation of that approach for EEG signal classification and consider the application of fuzzy classifier for EEG signal analysis in epileptic's seizures detection. Application of fuzzy classifier calls for the transformation of the data after the signal preprocessing into fuzzy data. Therefore, the signal preprocessing step should be modified and one more procedure should be added into it. This procedure is fuzzification. So, the main novelty of this paper lies in the usage of fuzzy data to reduce the influence of uncertainty of the data obtained after the signal preprocessing and in the use of fuzzy classifier. In particular, we consider Fuzzy Decision Tree (FDT), which is inducted based on Cumulative Mutual Information (CMI) [42], [43].

This paper is structured as follows. Section II discusses the specifics of the proposed approach that includes two steps. These steps are signal preprocessing and fuzzy classification based on FDT. The principal steps of the signal preprocessing in the proposed approach are described in section III, which deals with the feature extraction for improving the quality of the signal classification, with the dimensionality reduction of the data resulting in a set of attributes used in the classification, and with fuzzification of the attributes. The detailed process of FDT induction for the classification, including basic rules and mathematical background of this process, is presented in section IV. The examination and comparison of the proposed approach for prediction of epileptic's seizures is shown in section V.

II. DESIGN OF APPROACH

The analysis of EEG signal in diagnosis of neural disorder is implemented based on the procedure of classification of this type of signals [3], [7], [42]. Typically, signal classification includes two steps [18], [44]: preprocessing of initial signal and classification of data formed after the preprocessing. The step of preprocessing consists of two procedures (the feature extraction and dimensionality reduction) that transform initial signal into a set of data, which is interpreted as attributes of samples in the step of classification.

The feature extraction in signal classification is needed to remove noise and extract significant features as useful information for signal analysis in the next step. According to surveys of procedures of feature extraction performed in [21] and [39], the most used ones are spectral transforms, such as the Fourier, Wavelet and Welch transforms. Results of these transforms are specific properties (attributes) of the analyzed signal. However, dimension of these attributes is large for direct application of classification and has to be reduced by special methods as PCA, ICA, and LDA [23], [45]. So, the final output of EEG signal preprocessing step is a set of numerical data. Elements of this set are interpreted as classification attributes and can be used for the classification.

The classification itself is implemented in the second step of EEG signal analysis [40]. For signal classification, in particular EEG signal classification, the most commonly used classifiers are SVM [23], [24], [34], kNN [25], [46], Neural Networks [7], [14], [26], [47], Decision Trees [15], [21] and Naïve Bayes Classifier [48].

According to reviews in [21] and [40], accuracy of signal classification does not result only from classifier types but depends also on the data used in the classification, which are obtained as a result of the preprocessing step. Specific of this data is a loss of some information stored in the signal, which is caused by selection of some attributes in the spectral transform and ignoring the less informative attributes within the procedure of dimensionality reduction [28], [29]. This information loss leads into the uncertainty of data formed after the preprocessing step and used as the input for the step of classification. This uncertainty should be considered and incorporated into the methods and approaches for signal classification. Authors of [40] have shown that the accuracy of signal classification can be improved using fuzzy classifiers. However, the data obtained after the preliminary transformation is crisp regardless of application of fuzzy-based procedures in feature extraction [35] or dimensionality reduction [49]. In [7], [34], and [35], the preprocessing of EEG signal has been implemented based on fuzzy entropy. Other fuzzy-based techniques for EEG signal analysis have been described in [50] and [51]. Results of all considered transforms of EEG signal are sets of crisp data, which requires use of crisp classifiers in the classification. There are also several methods for EEG signal classification in which the classification has been implemented based on fuzzy classifier in combination with crisp classifier. Their examples can be found in [36], [37], or [38], but the formalized fuzzy-based approach for EEG signal classification has not been proposed. If the data (classification attributes) obtained after the preprocessing step are fuzzy, then fuzzy classifier can be

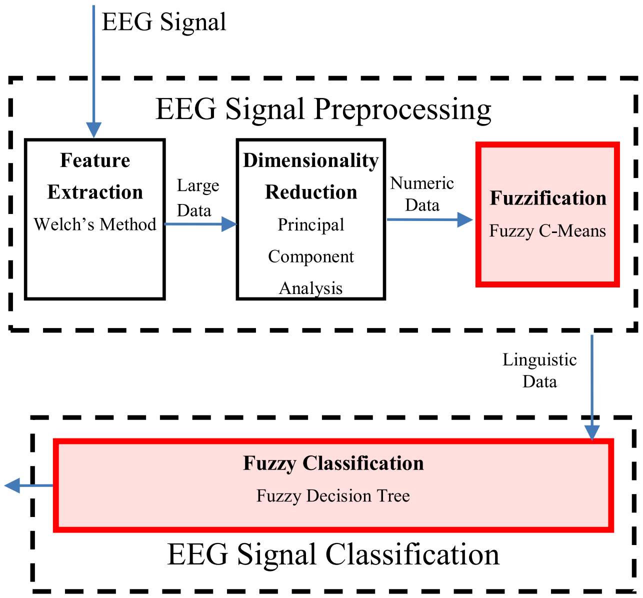

FIGURE 2. The principal steps of the proposed algorithm for EEG signal classification.

used without any complications. One of the possible ways to transform the numeric and crisp data after the preliminary transformation of EEG signal to fuzzy data is introduction of fuzzification as an additional procedure in the preprocessing step. Such a modification used in signal classification method allows increasing the accuracy of classification [42].

In this paper, we propose to modify the approach for EEG signal classification (Fig. 2) by (a) the use of fuzzy classifier in the step of signal classification and by (b) the introduction of new procedure of the fuzzification in the step of signal preprocessing. This modification allows developing fuzzy-based approach for EEG signal classification. This approach can be used not only in detection of epileptic's seizures but also in other EEG-based classification problems [1] – [8].

The approach depicted in Fig. 2 assumes the application of Fuzzy C-Means (FCM) algorithm [52], [53] as the additional procedure in the step of EEG signal preprocessing and Fuzzy Decision Tree (FDT) as the fuzzy classifier used in the step of EEG Signal Classification. An FDT is a tool used for decision support represented by a set of decision choices in the graphical form of a tree. In the task of classification, its purpose is to predict the value of a target variable (the output) from the value of several input attributes. In case of the FDT, the result of classification is usually determined by multiple leaves. It is common that an FDT is transformed into fuzzy classification rules to perform classification effectively. In what follows, we focus on individual procedures of the two steps that the approach depicted in Fig. 2 consists of.

III. SIGNAL PREPROCESSING

A. FEATURE EXTRACTION

The feature extraction is an important procedure in signal preprocessing. Generally, the raw measurement of EEG signals contains noise, artifacts and other defects. These can be caused by eyes blinking, muscular activity or other physiological processes in a human body [3] – [8]. Moreover, measured EEG signals are functions of time. This kind of representation of signals is not usable for classification. Hence, special transformations have to be applied to signals. These transformations can reduce impact of mentioned undesired effect and should extract information from signal that can be used for classification. The purpose of feature extraction is to enlarge distinguishability between classes of EEG signals by Power Spectral Density (PSD) estimation to selectively represent the instances of EEG signals [54].

To select a good method for feature extraction, we have created a comparison of spectral transforms used in field of EEG signals classification (Table 1). Most often spectral transforms used for this problem are Discrete Wavelet Transform (DWT) [20], Discrete Fourier Transform (DFT) [55], and Fast Fourier Transform (FFT) [55]. There are also investigations to develop new methods for the feature extraction based on these transformations for specifics of EEG signals. For example, new wavelet-based transforms for EEG signal analysis have been proposed in [19] and [56]. Authors of [57] have developed a new hybrid method for the feature extraction of EEG signal based on Fourier and wavelet transforms using fuzzy entropy. In addition to spectral transforms, other methods can be used to eliminate signal noise and reveal its specific features. In this case, some studies for feature extraction based on fuzzy logic should also be noted. Authors of [7], [34], [35], [57], and [58] have developed new fuzzy-based procedures for EEG signal feature extraction. In [7], a new method for epileptic seizure detection based on Permutation Fuzzy Entropy (PFEN) has been proposed. Similar to study in [7], fuzzy entropy has been used in [34] and [35]. The special filter based on an artifact rejected multiclass extension of common spatial pattern by using joint approximate diagonalization has been proposed for feature extraction in [58]. These fuzzy-based transformations allow decreasing the influence of the information loss in the step of preprocessing, but their results are crisp data that is not acceptable for fuzzy classifiers.

From common spectral transforms, authors of [21] have shown an advantage of the DFT compared to DWT for classification method of decision tree that is inducted based on method C4.5. Welch's method has been used in [22] and [59] for feature extraction of EEG signal. These studies have shown that power spectrum density estimation by Welch's method provides strong attributes for good representation of EEG signal.

Regarding the previous analysis, we have decided to implement the feature extraction of EEG signal by Fourier transform, in particular DFT [21]. It is a simple spectral transform that has good result in EEG signal classification according to research in [21], [60], and [61]. The common property of spectral transforms is that they sample a signal over time and divide it into frequency components. The frequency components are represented by a single sinusoidal oscillation at

TABLE 1. The analysis of methods for feature extraction in EEG signal classification.

Papers	Method of trans-formation	Field of application	Comment
K.Polat, S.Güneş[21]	DWT, DFT	Epileptic seizure detection	Better performance of DFT in combination with method C4.5 for decision tree induction.
K.Polat, S.Güneş [22] M.Naderi [57]	FFT	Epileptic seizure detection	Spectrum density estimation by FFT-based method provides strong features which represent EEG signals well.
R.Sharma et al [20] Y.Kumar at al [34]	DWT	Epileptic seizure detection	Analysis of impact of sub-band frequencies in DWT
D. P. Subha [39]	Methods for frequency domain and time-frequency analysis	Survey of EEG signal analysis	The frequency methods are considered as not very effective in the analysis of the physiological signals.
A. Alkan, K. M. Kiymik [60]	Methods for spectral transformation	Comparison of signal processing	The results of this study indicate better performance of the covariance methods over autoregressive and FFT methods.
A. Al-Fahoum, A. Al-Fraihat [61]	Methods based on linear analysis in frequency and time-frequency domains	EEG signal analysis	Frequency domain methods may not provide the best performance for specific EEG signals. Time-frequency methods may not provide detailed information on EEG signal analysis as much as frequency domain methods. The most suitable methods can vary depending on analyzed signals.
W. Hussain et al [7]	Method based on Permutation Fuzzy Entropy (PFEN)	Epileptic seizure detection	The study put forwards a new entropy index PFEN, which may delineate between ictal and interictal state of epileptic seizure using different machine learning classifiers.
A. Bhattacharyya, R.B.Pachori [19]	Empirical wavelet transformation	Epileptic seizure detection	The empirical wavelet transform is able to analyze multivariate nonstationary EEG signals
A. Bhattacharyya et al. [56]	The method combines spatial filtering with tunable-Q wavelet transform	Stereo electroencephalogram analysis	The proposed method can be applied for suppressing both low and high frequency of some types of specified signals' artifacts.

distinct frequencies, and each of these oscillations has its own amplitude and phase [21].

After applying the Fourier transform to the EEG signals from [12], we get a matrix of elements with 128 columns and 500 rows. Each row of the matrix represents one EEG signal in the frequency domain. The obtained number of features (128) is too large to create an accurate classifier (we have too many features in proportion to the number of data instances). To solve this problem, we use a dimension reduction technique called PCA to decrease the number of features of the matrix [62].

B. DIMENSIONALITY REDUCTION

The dimensionality reduction is a next procedure in EEG signal preprocessing. In some researches, this procedure is named as feature transformation [40] or feature selection [1]. This procedure is required because the output of feature extraction is a large matrix. If we reduce the dimension of this matrix, the process of classification can be more effective. Typically, PCA, ICA, and LDA are used for feature selection or dimensionality reduction in EEG signal classification. These methods accomplish a linear mapping of a high-dimensional input vector into a low-dimensional vector whose components are uncorrelated [49]. The comparison of PCA, ICA and LDA shows that LDA acquired the best performance. Nevertheless, it has been shown in [45] that PCA could outperform LDA. This result is caused by the classifier used in the second step of EEG signal analysis [15], [18].

Table 2 contains the most popular dimensionality reduction methods used for EEG signal preprocessing. There are also researches to develop other types of procedures for dimensionality reduction for the purposes of EEG signal classification. For example, in [35], [63], and [64], fuzzy-based procedures for transformation of feature matrix have been proposed. These fuzzy transformations used for the purposes of the dimensionality reduction allows increasing accuracy of EEG signal classification, but they form a set of crisp data for classification.

In our study, we use PCA to decrease dimension of the feature matrix obtained after the spectral transformation. Some studies claim that LDA gives better results, but as we said before, it was shown in [45] that PCA can outperform LDA, especially, in cases when the number of training samples per class is quite small. Moreover, LDA assumes Gaussian distribution of data, and in situations when data has not this distribution, LDA can fail. According to [28], PCA can reflect dynamics of original signal. The first principal components with larger variances represent the signal dynamics. The last principal components with smaller variances are dominated by noise. If the data is composed of an information-carrying signal and a Gaussian noise, the PCA is optimal for dimensionality reduction from the information-theoretic point of view [28].

The PCA transforms a feature matrix with n features $Y = (Y_1, \ldots, Y_n)$ into a new matrix where features (columns) are called "principal components" [62]. In order to reduce the dimension of the matrix, we have to select some of the most important principal components. The importance of the principal components can be expressed by variance. The variance of component indicates variability in the data.

TABLE 2. The analysis of methods for dimensionality reduction in EEG signal classification.

Papers	Method	Field of application	Comment
A.Subasi, M.I.Gursoy [23]	ICA, PCA, LDA	Epileptic seizure detection	Comparison of three methods. LDA shows the best performance.
M. Ânez, A.M. Kak [45]	LDA, PCA	Comparison of dimensionality reduction methods	PCA can outperform LDA, especially in cases when the number of training samples per class is small.
K.Delac, M.Grgic [65]	PCA, ICA, LDA	Face recognition	Comparison by multiple metrics. The conclusion is that no algorithm can be considered as the best and the choice of appropriate algorithm is usually made for a specific task.
L.J. Cao, K.S. Chua [66]	PCA, KPCA, ICA	Forecasting and classification	Kernel PCA (KPCA) performs non-linear transformation. The results are good, but the computation time is much more expansive as in PCA and ICA.
R.N.Khushaba et al. [63]	Uncorrelated Fuzzy Neighborhood Preserving Analysis	Drowsiness detection EEG, Electrooculogram Electrocardiogram (ECG) signals	It is about a method that is utilized to derive the discriminant information relevant to the loss of attention that results from drowsiness.
J.H.Abawajy et al [67]	Clustering	ECG data classification	Multistage clustering is used for feature selection and dimensionality reduction.

After transformation, the components are sorted in descending order of their variance. Then, we have to select just a few first components. One of the most commonly used criteria to select an appropriate number of principal components is the eigenvalue-one criterion, known also as the Kaiser criterion [68]. This criterion selects a principal component if the component has variance bigger than 1.00. It is useful to note that the variance of the principal component corresponds to eigenvalue of this component.

After applying PCA on data from [12], we obtain 8 principal components, which describe the reduced feature matrix of origin EEG signals. Hence, each EEG signal is represented by 8 principal components. In the text bellow, these principal components are noted as numerical input attributes X_i ($i = 1,\ldots, 8$) of EEG signals.

A loss of some information is possible in the step of the signal preprocessing according to [28]. This can be shown by evaluation of distributions of principal components. For this purpose, let us test the obtained principal components based on typical distributions by minimum squared error fit. This test calculates sum of squared discrepancies between histogram frequencies and fitted-distribution frequencies. It is assumed that the data has a distribution with minimal squared error. We implemented this test by Arena Input Analyzer (https://www.arenasimulation.com/) for each principal component. The results are shown in Table 3. At final, we calculate sum of squared error fit for each distribution of all principal components. These results are shown in the last column of Table 3. The smallest value of this squared error has normal distribution. Therefore, we assume that the data is distributed normally.

TABLE 3. Sums of squared discrepancies between histogram frequencies and fitted-distribution frequencies.

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	$\sum$
Normal	0.116	**0.138**	**0.082**	**0.183**	**0.209**	**0.206**	0.025	**0.211**	**1.170**
Beta	0.093	0.141	0.098	0.184	0.250	0.208	**0.021**	0.214	1.209
Lognormal	**0.032**	0.174	0.128	0.241	0.257	0.277	0.040	0.299	1.448
Erlang	0.065	0.176	0.119	0.249	0.254	0.299	0.037	0.313	1.512
Gamma	0.066	0.176	0.119	0.249	0.254	0.299	0.037	0.313	1.513
Triangular	0.171	0.222	0.200	0.319	0.355	0.359	0.048	0.390	2.064
Uniform	0.191	0.256	0.220	0.347	0.391	0.393	0.078	0.427	2.303
Exponential	0.065	0.269	0.230	0.358	0.397	0.405	0.093	0.438	2.255

C. FUZZIFICATION

Fuzzification transforms each numeric attribute (principal component) X_i obtained after dimensionality reduction into fuzzy attribute A_i ($i = 1,\ldots, n$). Numerical attribute X_i is defined by a vector of real values $(x_1, x_2, \ldots, x_k, \ldots, x_K)$, where K is the number of samples. Each fuzzy attribute A_i consists of m_i ($m_i \geq 2$) linguistic terms. The j-th linguistic term of A_i is represented by fuzzy set $A_{i,j}$ ($j = 1, \ldots, m_i$). Fuzzy set $A_{i,j}$ with respect to X_i is defined by a membership function $\mu_{A_{i,j}}(x) : X_i \to \langle 0, 1\rangle$. The membership function gives a membership degree $\mu_{A_{i,j}}(x)$ for each x ($x \in X_i$), which defines how strongly element x is the member of fuzzy set $A_{i,j}$. Formally, fuzzy set $A_{i,j}$ is defined as an ordered set of pairs $A_{i,j} = \left\{\left(x, \mu_{A_{i,j}}(x)\right), x \in X_i\right\}$, where:

(a) $\mu_{A_{i,j}}(x) = 0$ if and only if x is not the member of set $A_{i,j}$,

(b) $0 < \mu_{A_{i,j}}(x) < 1$ if and only if x is not the full member of set $A_{i,j}$,

(c) $\mu_{A_{i,j}}(x) = 1$ if and only if x is the full member of set $A_{i,j}$.

Fuzzification can be done using various methods. One of them is Fuzzy C-Means (FCM) clustering [53]. The FCM is an extension of K-means algorithm, and it is based on use of fuzzy membership function. The K-means algorithm assigns each instance to one cluster. In case of FCM, an instance can be assigned to more clusters with some partition degrees. The FCM tries to minimize the objective function defined as:

$$minimize \sum_{j=1}^{m_i} \sum_{k=1}^{K} \left(u_{k,j}\right)^r d\left(x_k, c_j\right)^2, \tag{1}$$

where r is the parameter of cluster fuzziness, $d\left(x_k, c_j\right)$ is a distance between scalar value x_k and center c_j of the j-th cluster. Increasing of r causes smaller values of partition degrees $u_{k,j}$. At the beginning, the instances are randomly assigned to the clusters with some partition degree. Then, the partition degree of the k-th instance into the j-th cluster and the new centers are computed based on the following formulae:

$$u_{k,j} = \frac{1/d\left(x_k, c_j\right)^2}{\sum_{t=1}^{m_i} 1/d\left(x_k, c_t\right)^2} \quad \text{and } c_j = \frac{\sum_{k=1}^{K}\left(u_{k,j}\right)^2 x_k}{\sum_{k=1}^{K}\left(u_{k,j}\right)^2}. \quad (2)$$

This process is repeated until centers do not change. As a result, we found partition degree $u_{k,j}$, which defines the membership degree of scalar value x_k to the j-th fuzzy set (fuzzy term) $A_{i,j}$.

After fuzzification of results of the procedure of the dimensionality reduction applied on signals form dataset [12], we obtain data described by 8 fuzzy input attributes A_i ($i = 1,\ldots, 8$) and one output attribute B. Based on this data, we are able to induct an FDT that can make decision about presence of epileptic's seizure. For the purposes of the FDT induction, these attributes are stored in a form of three-dimensional matrix $A_{i,j,k}$ where i is the index of the attribute, j is the index of the attribute term and k is the index of the row (which corresponds to one signal from the dataset).

IV. FUZZY CLASSIFICATION

A. FUZZY CLASSIFIERS

In most studies of EEG signal classification, the fuzzy classifiers have not been used. More often the fuzzy-based methods have been implemented only in the preprocessing step. Such methods have been developed for the feature extraction in [55], [56], [7], [34], and for the feature selection or dimensionality reduction in [63], [64], and [35]. Based on surveys [18], [39], [40], [41], and [69], we can state that fuzzy classifiers have not been used often in investigations of EEG signal classification. According to last reviews [18] and [69], the most often used classifiers have been SVM [23], kNN [25], neural networks [7], [14], decision tree, and random forest [34], [71], [66]. Hybrid classifiers based on combination of typical classifiers have also been used [70], [72], [73].

Fuzzy classifiers for EEG signal classification have been used in some initiative studies (Table 4). In [36] and [74], fuzzy kNN classifier has been used in combination with other classifiers. Authors of [38] have proposed fuzzy-based neural network for EEG signal classification. The fuzzy-rule based approach has been developed in [75].

It is important to note that the research for the development of fuzzy-based methods in EEG signal classification has shown the efficiency of fuzzy classifiers, but these and other investigations have not been formalized. Therefore, the step of the signal preprocessing has not been formalized too, and fuzzy data for classification is formed by the different way in these investigations.

TABLE 4. The analysis of fuzzy classifiers in EEG signal classification.

Papers	Method	Field of application	Comment
N. Singh and S. Dehuri, [36]	Singular value decomposition fuzzy kNN classifier	Seizure detection	DWT-based singular value decomposition fuzzy k-nearest neighbor classifier technique is proposed.
A.I. Saleh et.al. [37]	Fuzzy-based classification strategy (FBCS)	Brain–computer interface	FBCS minimizes the classification time by perfectly extracting the effective features of the produced EEG signals. FBCS uses feature reduction and electrode selection techniques to reduce the dimensionality of data to be classified, which also improves the classification accuracy.
R. Krishnamurthi and M. Goyal [38]	Fuzzy logic and neural network	Prediction of state of mind of a disabled person	The method is based on PPCA analysis which is a feature extraction technique and the hybrid technique, i.e., a combination of two classifying techniques – fuzzy logic and neural network.
Y. You [74]	Fuzzy kNN	Automatic focal EEG detection algorithm	New method for the feature extraction is proposed and evaluated with application of some classifiers, including fuzzy kNN.
H. Yu et al. [76]	Hidden-mapping ridge regression	The identification of Alzheimer's disease	The combination of network and fuzzy learning is used in Takagi-Sugeno-Kang fuzzy model.
Y. Zhang et al. [77]	Transfer Scenario Construction fuzzy classifier	Seizure Classification	The result of the preliminary transformation is crisp data used for the fuzzy classification.

B. CLASSIFICATION BASED ON FUZZY DECISION TREE

Decision trees are composed of nodes and leaves. Each node is associated with one input attribute (splitting attribute). The set of all possible linguistic values of this input attribute determines outgoing edges of the node. Each value from this set is associated with one outgoing edge. The classification of an unknown instance begins at the root of the tree. Subsequently, the instance travels down the tree. The direction of the instance is determined by the splitting attributes associated with the tree nodes. When the instance enters into a tree leaf, the output class can be determined. In a case of the FDT, the instance traverses through multiple branches. Therefore, the classification result is determined on the basis of a set of leaves.

There are different methods for induction of FDTs [43], [78], [79]. In this paper, the algorithm for FDT induction based on CMI is used [43]. The CMI in output attribute B based on knowledge of attribute A_{i_q} and the sequence of values U_{q-1} has been introduced in [43] as follows:

$$\mathrm{I}\left(B; U_{q-1}, A_{i_q}\right) = \sum_{j_q=1}^{m_{i_q}} \sum_{j=1}^{m_b} \left(\begin{array}{l} M\left(B_j \times U_{q-1} \times A_{i_q,j_q}\right) \\ \times \left(\begin{array}{l} \log_2 M(B_j \times U_{q-1} \times A_{i_q,j_q}) \\ + \log_2 M(U_{q-1}) - \log_2 M(B_j \times U_{q-1}) \\ - \log_2 M(U_{q-1} \times A_{i_q,j_q}) \end{array} \right) \end{array} \right)$$

where $U_{q-1} = \{A_{i_1,j_1} \times \ldots \times A_{i_{q-1},j_{q-1}}\}$ is the fuzzy set defined by the sequence of fuzzy terms $A_{i_1,j_1}, \ldots, A_{i_{q-1},j_{q-1}}$ of selected attributes $A_{i_1}, \ldots, A_{i_{q-1}}$ from the root to the q-th node, $M(B_j \times U_{q-1} \times A_{i_q,j_q})$ is a measure of cardinality of fuzzy set $B_j \times U_{q-1} \times A_{i_q,j_q}$.

The splitting criterion that selects splitting attribute for node of the tree is defined as follows:

$$i_q = \arg\max \left(\mathrm{I}\left(B; U_{q-1}, A_{i_q}\right) / \mathrm{H}\left(A_{i_q} | U_{q-1}\right)\right), \quad (3)$$

where function argmax returns attribute index i_q with the maximal value of CMI, and $\mathrm{H}(A_{i_q}|U_{q-1})$ is the cumulative conditional entropy. This entropy is defined between fuzzy attribute A_{i_q} and the sequence of selected attribute terms U_{q-1} as follows:

$$\mathrm{H}\left(A_{i_q}|U_{q-1}\right) = \sum_{j=1}^{m_{iq}} M\left(A_{i_q,j}, U_{q-1}\right). \times \left(\log_2\left(M\left(U_{q-1}\right)\right) - \log_2 M\left(A_{i_q,j} \times U_{q-1}\right)\right) \quad (4)$$

Dividing CMI by this entropy can eliminate drawback of CMI. CMI tends to prefer attribute with a large set of linguistic values. Division of CMI solves this problem by taking into account the number of branches that would be created before the split. To avoid overfitting of initial data, the FDT induction is stopped in two cases:

a) if the confidence degree b_j of the analyzed node is bigger than manually chosen parameter β. This confidence degree b_j reflects the confidence of the decision that the output attribute belongs to the j-th class. This degree can be calculated as:

$$b_j = \frac{M\left(B_j \times A_{i_1,j_1} \times \ldots \times A_{i_{q-1},j_{q-1}} \times A_{i_q,j_q}\right)}{M\left(A_{i_1,j_1} \times \ldots \times A_{i_{q-1},j_{q-1}} \times A_{i_q,j_q}\right)}. \quad (5)$$

b) if frequency $f(U_q)$ of branch defined by sequence of fuzzy terms $\{A_{i_1,j_1}, \ldots, A_{i_q,j_q}\} = U_q$ is less or equal to manually chosen parameter α. Frequency $f(U_q)$ can be calculated as follows:

$$f\left(U_q\right) = M\left(A_{i_1,j_1} \times \ldots \times A_{i_{q-1},j_{q-1}} \times A_{i_q,j_q}\right)/K, \quad (6)$$

where K denotes the number of data (signals) in the dataset.

The used algorithm for tree induction uses two pre-pruning parameters α and β. According to these parameters, the algorithm can stop tree induction in a branch. If the frequency of branch is less than the value of α, tree induction is stopped. The second parameter β expresses sufficient confidence degree to the some of the classes in the node. If at least one of the confidences b_j to j-th class in the node is bigger than parameter β, then the tree induction in this node is terminated and the node becomes a leaf.

Values of α and β affect the size of the tree, which is defined by the number of nodes in the tree. It is usually required that the tree should be as accurate as possible. However, if the tree induction with different α and β can create trees with the same accuracy, then it is desirable to use a smaller tree. Use of a smaller tree reduces the time required to classify an unknown instance because fewer tests are performed during classification. Trees inducted with bigger value of α have smaller size. On the other hand, trees inducted with bigger value of β have bigger size. The special case is when the value of α is 0 and β is 1. Trees with these values of α and β are unpruned. The second special case is when α is 1 and β is 0. This setting of parameter will produce a tree with single node (the root node). This tree is known as a decision stump. In our experiments, we tries to find values of α and β by the experimental investigation to minimize the classification error of an FDT [15], [43].

The detailed steps and examples about construction of and FDT using the above algorithm can be found in [15], [42], and [43]. The FDT in Fig. 3 has two types of nodes that are leaves (marked by grey) and decision nodes. Each node includes three rows of text. In case of a decision node, the first line informs about input attribute associated with the node. The attribute naming convention is as follows. The attribute name begins with letters "PC" followed by a number that determines the fuzzified principal component: PC1, PC2,..., PC8. In case of leaves, the first line informs about the dominant output class: seizure or seizure free. The next two lines are the same for both types of nodes. The second line informs about the frequency of branch which comes to the node. The third line informs about membership degrees to the output classes (the first number agrees with "Seizure" while the second one with "Seizure free"). The numbers associated with branches of the tree are determined by the values of centroids obtained after fuzzification by using FCM.

The obtained FDT for task of EEG signals classification in case of 8 fuzzy input attributes is shown in Fig. 3. It consists of 25 leaves that allow representing the classification of the signals by 25 fuzzy decision rules. The obtained FDT can also be simply transformed into fuzzy decision rules [43].

The classification rule that corresponds to single leaf on Level 3 (the green path in Fig. 3) is equal to:

IF PC1 is $\mathrm{PC1}_3$ *and* PC2 is $\mathrm{PC2}_2$
THEN $B = [0.952; 0.048]$ with frequency 0.045.

The antecedent condition describes the situation with selection of branches $\mathrm{PC1}_3$ *and* $\mathrm{PC2}_2$ for attributes PC1 *and* PC2, respectively. The result describes the possible value of the output attribute B. The decision "Seizure" should be accepted with confidence 0.952 and decision "Seizure free" in this case has confidence 0.048. The frequency of this decision in this leaf is 0.045.

V. ACCURACY OF THE PROPOSED ALGORITHM

Important modifications of the proposed fuzzy-based approach for EEG signal classification are additional procedure of fuzzification in the first step of the preliminary transformation and application of fuzzy classifier for EEG signal classification in the second step (Fig. 2).

The efficiency of the proposed approach can be shown by the comparison with similar studies implemented for the

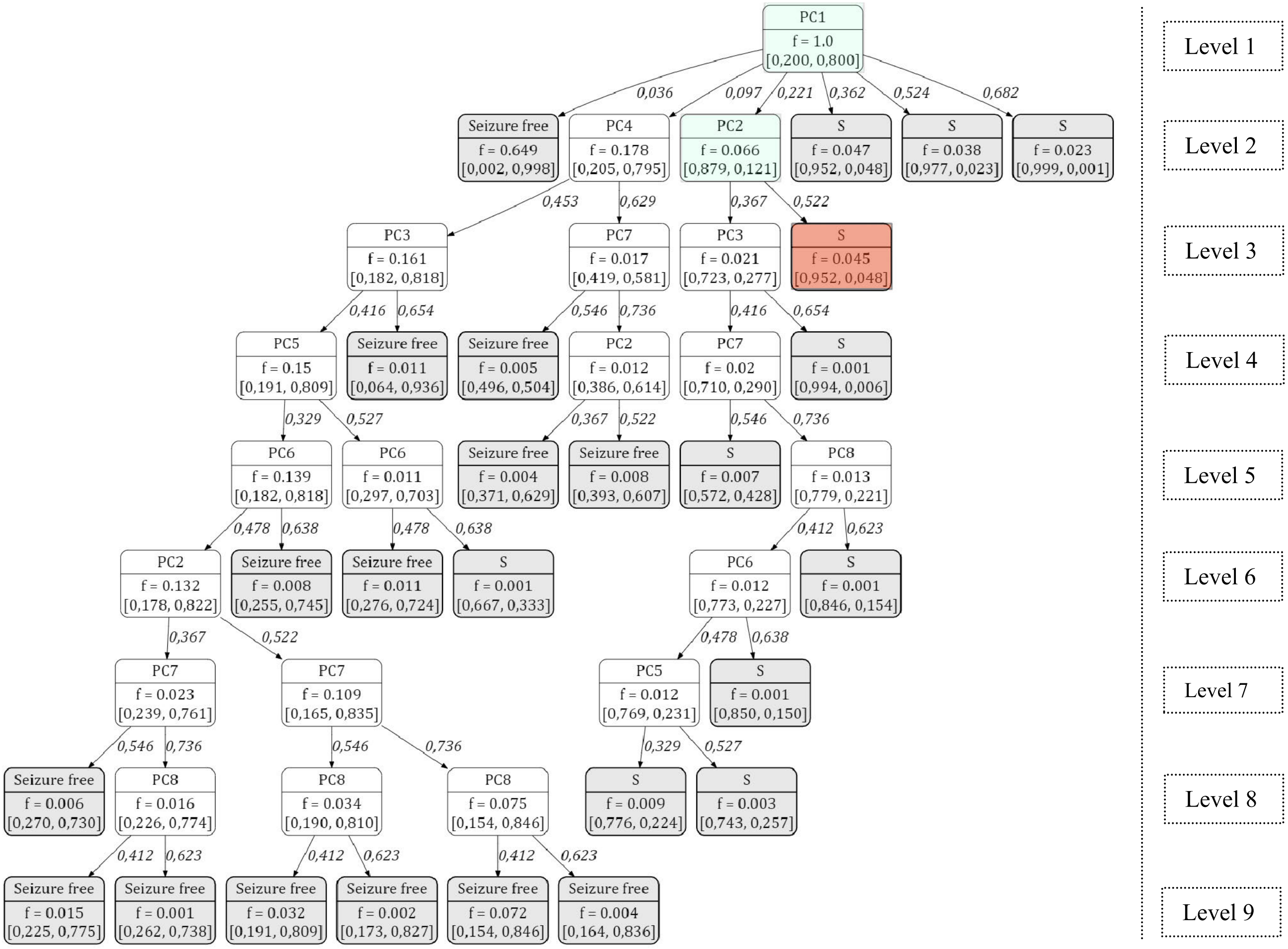

FIGURE 3. The FDT for EEG signal classification. The threshold parameters α and β are defined as $\alpha = 0.011$ and $\beta = 0.882$.

same data. In our investigation, we use the Fourier transform (in particular FFT) for feature extraction of EEG signal, PCA for the dimensionality reduction, and FCM for the data fuzzification in the step of the preprocessing, and FDT as the fuzzy classifier in the step of the EEG signal classification. The similar procedures in the preprocessing step have been used in experimental investigations (for crisp-based classifiers, the Fourier transform and PCA have been used only).

Because the application of fuzzy classifier is a principal modification of the proposed approach, the FDT evaluation is provided to show the efficiency of the fuzzy-based approach for EEG signal classification. The evaluation of the FDT is based on usage of binary classification metrics.

There are many metrics that can be used to measure the performance of a binary classifier. The most common ones are accuracy, sensitivity, and specificity. Accuracy is often considered as the most valuable estimation that is defined as a proportion of correctly classified instances. But this estimation should be supplemented by specificity and sensitivity, when the data are not uniformly distributed into classes because the accuracy can be misleading. Sensitivity measures the proportion of actual positives that are correctly identified as positive. For example, sensitivity measures percentage of people with epileptic seizure who are correctly classified as having the seizure. Specificity measures the proportion of actual negatives that are correctly identified as such. For example, the percentage of seizure-free people who are correctly identified as not have the seizure. Therefore, the classification of instance can be as true (True positive (TP) or True negative (TN)) or false (False positive (FP) or False negative (FN)). According to these four cases, the accuracy, sensitivity, and specificity are defined in Table 5 [16], [17]. In experimental analysis, we also consider other metrics summarized in Table 5.

The comparative analysis is implemented based on dataset introduced in [12] for identification of patients during epileptic's seizure. Hence, classes (A, B, C, D, and E) of the original dataset were merged into two classes. We aim to separate persons during seizure activity and persons in seizure-free interval, therefore, we have two classes (ABCD, E), where class E determine seizure activity and class ABCD stands for seizure free interval. The initial data consists of 500 records of signals (each of duration 23.6 seconds) that are cut into smaller signals (each with 2.95 second duration). In this way,

TABLE 5. The evaluation criteria of classification.

Evaluation criteria	Evaluation criteria definition	Description
Accuracy	$\frac{TP+TN}{TP+TN+FP+FN}$	The accuracy expresses proportion between correctly classified instances and the number of all instances.
Sensitivity	$\frac{TP}{TP+FN}$	The sensitivity agrees with number of positives that are correctly classified as positives.
Specificity	$\frac{TN}{TN+FP}$	The specificity of the test is the ratio of true negative results to the sum of true negative and false positives.
Precision	$\frac{TP}{TP+FP}$	The precision is the proportions of positive and negative results in statistics and diagnostic tests that are true positive and true negative results.
DOR	$\frac{TP/FN}{FP/TN}$	Diagnostic odds ratio (DOR) is a measure of the effectiveness of a diagnostic test. It is defined as the ratio of the odds of the test being positive if the subject has a disease relative to the odds of the test being positive if the subject does not have the disease. The diagnostic odds ratio ranges from zero to infinity, although for useful tests it is greater than one, and higher diagnostic odds ratios are indicative of better test performance.
F1 Score	$\frac{2TP}{2TP+FP+FN}$	The F1 Score is the harmonic mean of the precision and sensitivity.
MCC	$\frac{TP \cdot TN - FP \cdot FN}{\sqrt{(TP+FP)(TP+FN)(TN+FP)(TN+FN)}}$	The Matthews correlation coefficient (MCC) is in essence a correlation coefficient between the observed and predicted binary classifications; it returns a value between −1 and +1. A coefficient of +1 represents a perfect prediction, 0 no better than random prediction and −1 indicates total disagreement between prediction and observation.
Youden's index	Specificity + Sensitivity - 1	Value of Youden's index ranges from 0 to 1. Value 0 is obtained when a diagnostic test gives the same proportion of positive results for groups with and without the disease, i.e., the test is useless. A value of 1 indicates that there are no false positives or false negatives, i.e., the test is perfect.
Jaccard index	$\frac{TP}{TP+FP+FN}$	Jaccard index estimates a likelihood of an element being positive if it is not correctly classified negative element.

we obtain 4,000 signals. These signals are transformed by FFT, PCA and FCM into the set of 8 fuzzy attributes in the step of the preliminary transformation (section III). The dataset of fuzzy-based samples is then split into two subsets: one subset is used for the FDT induction and another one for evaluation of FDT by using criteria defined in Table 5. Splitting data into training and testing sets is done randomly in a 70 to 30 ratio [42], [59]. The FDT is inducted using the method considered in section IV based on the training data subset. The inducted FDT is evaluated by testing set. We repeat this splitting and classification of testing subset 103 times to eliminate influence of the random data split.

We experimentally establish values α and β. This estimation is based on multiple FDT inductions. For each FDT, classification accuracy is computed and then the FDT with the best accuracy is selected. We analyze α from 0.0 to 0.2 (by step 0.001) and β from 0.75 to 1.0 (by step 0.001). The influence of values α and β on the classification accuracy is shown in Fig. 4. We created surfaces (Fig. 5) that show the impact of α and β parameters on the classification accuracy. These surfaces were obtained by transformation of three-dimensional scatter chart to the surface using MATLAB. These surfaces have three axes: the x axis represents value of α, the y axis shows value of β, and the z axis agrees with value of the classification accuracy of the FDT inducted with the corresponding α and β values. This estimation shows that the best accuracy is obtained for $\alpha \approx 0.230$ (the accuracy has maximal value for values of this parameter from range 0.182 to 0.273) and $\beta = 0.864$. The FDT inducted with the indicated pruning parameters has optimized size and maximal value of the accuracy. So, it can be used as the classifier for EEG signal classification (Table 6).

Other classifiers are also applied and evaluated in the problem of the identification of patients during epileptic's seizure for the fuzzy-based dataset formed in the step of preprocessing. We make comparison of classification results of the proposed approach and other approaches with fuzzy classifiers for the same initial data. Four classifiers are involved in this comparison: Fuzzy Naïve Bayes classifier [80], fuzzy classification rules according to the algorithm in [81] and [82], and neural network (more specifically, Fuzzy Multi-Layer Perceptron) [83]. We implement all these classifiers in MATLAB. In the step of the preprocessing the Fourier transform, PCA and FCM procedures are used. According to the presented evaluations in Table 6 the FDT achieves the best result of the classification for the considered data.

We also provide comparison with non-fuzzy approaches. In this experiment, the fuzzification is not used in the EEG

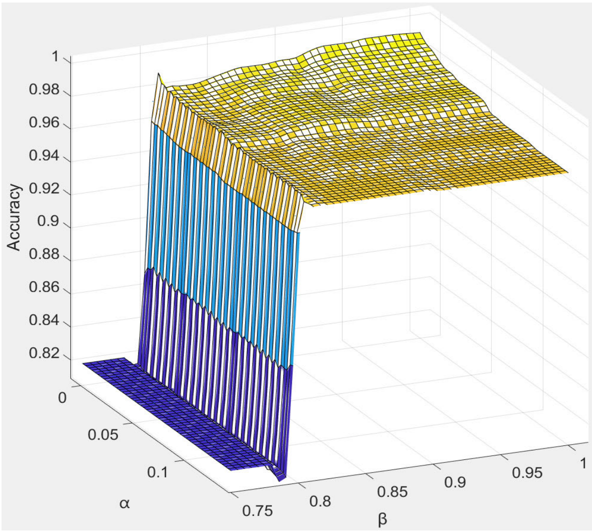

FIGURE 4. The dependency of classification accuracy on values of pruning parameters α and β.

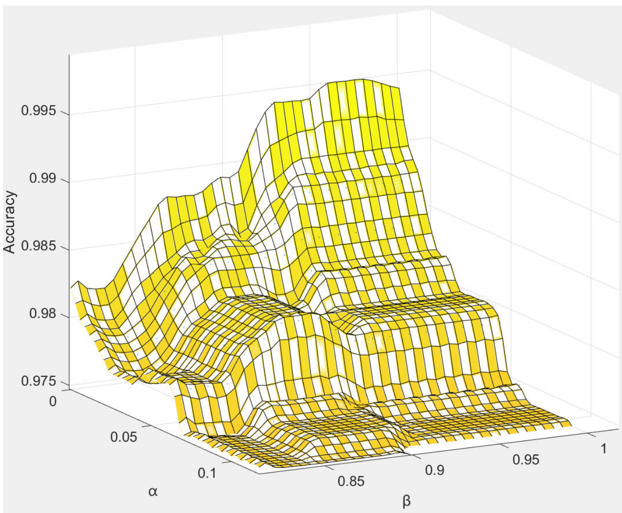

FIGURE 5. The dependency of classification accuracy on values of pruning parameters α and β. This figure shows a cropped part with the best accuracy from Fig. 4.

TABLE 6. The evaluation and comparison of the proposed algorithms based on FDT with existing ones.

	Fuzzy Naïve Bayes	Fuzzy Classification Rules	Fuzzy Multi-Layer Perceptron	FDT
Accuracy	0.944	0.909	0.949	0.995
Specificity	0.916	0.888	0.889	0.993
Sensitivity	0.998	0.912	0.997	0.996
Precision	0.862	0.976	0.919	0.981
DOR	3,116.942	421.455	3,770.547	12,856.263
F1 Score	0.924	0.942	0.956	0.989
MCC	0.887	0.722	0.900	0.985
Youden's index	0.913	0.801	0.885	0.990
Jaccard index	0.860	0.892	0.917	0.978

TABLE 7. The evaluation and comparison of non-fuzzy classification-based algorithms.

	kNN	Multi-Layer Perceptron	Naïve Bayes	SVM	C4.5
Accuracy	0.924	0.942	0.962	0.971	0.981
Specificity	0.892	0.881	0.928	0.952	0.968
Sensitivity	0.946	0.991	0.987	0.976	0.992
Precision	0.923	0.913	0.949	0.987	0.978
DOR	144.690	815.192	978.564	842.105	3,751.000
F1 Score	0.934	0.950	0.968	0.982	0.984
MCC	0.843	0.886	0.923	0.914	0.963
Youden's index	0.839	0.872	0.916	0.929	0.960
Jaccard index	0.876	0.906	0.937	0.965	0.970

signal preprocessing. We use Fourier transform (FFT) for the feature extraction and PCA for the dimensionality reduction in the step of EEG signal preliminary transformation. Five classifiers are examined: k-Nearest Neighbor (kNN), Multi-Layer Perceptron, Decision tree inducted based on method C4.5, SVM, and Naïve Bayes. The experiments are implemented in MATLAB again. We try to find the best values of parameters of mentioned classifiers. Therefore, these classifiers are obtained by multiple runs with different parameters by similar approach as α and β for FDT. The most accurate classifiers for every type are chosen and shown with their evaluation in Table 7.

To compare results of our investigation with similar studies, we have created the comparison shown in Table 8. This table includes only studies which have used the same data as we use. The classification of the selected studies has been targeted on prediction of occurrences of the epileptic's seizures. The selected studies are different in methods of data preprocessing (for the signal feature extraction and dimensionality reduction) or classification. The comparison is presented in Table 8. The second column in this table describes used methods in preliminary data transformation, the third column defines the used classifier, and the last column shows acquired classification accuracy according to the considered publications.

The implemented analysis and evaluations show the efficiency of the fuzzy-based approach for classification of EEG signals in detection of the epileptic's seizures. This result is illustrated by the comparison of fuzzy classifier (Table 6) and

TABLE 8. Table of results from other studies for epileptic's seizure detection.

Study	Preprocessing methods	Classifier	Accuracy
K. Polat and S. Güneş [21]	DWT	Decision Tree (C4.5)	0.987
L. Guo, D. Rivero et al. [84]	DWT	Multilayer Perceptron Neural Network	0.977
M. A. Naderi, H. Mahdavi-Nasab [59]	FFT, PCA	Multilayer Perceptron Neural Network	1.000
U. Orhan, M. Hekim and M. Ozer [25]	DWT	K-means and Multilayer Perceptron Neural Network	0.967
N. Nicolaou and J.G. Kios [24]	Permutation Entropy	Support Vector Machine	0.944
K.Polat and S.Güneş[22]	FFT, PCA	Artificial Immune Recognition System (ARIS)-based Classifier	0.998
J. B. Jian, B. Goparaju et al. [71]	Complete Ensemble Empirical Mode Decomposition (CEEMD)	Random Forest	0.980
This study	**FFT, PCA, Fuzzification**	**FDT**	**0.995**

non-fuzzy classifier (Table 7), which are based on similar methods and are used for identical data for classification. The fuzzy classifiers have comparable accuracy and, for example, the classification accuracy based on FDT is the best in comparison with the decision tree inducted based on C4.5 method. At the same time, the comparison of the proposed approach based on FDT with other researches is acceptable. The classification accuracy of FDT-based method is slightly less with comparison to two researches only. The classification accuracy in [59] has been achieved by specific implementation of the classifier induction, where the data set has not been divided into training and testing subsets and all samples used in training have been involved in testing too. The classifier used in [22] is a hybrid automated identification system based on Artificial Immune Recognition System (AIRS) with fuzzy resource allocation mechanism. This special fuzzy-based mechanism allows taking data uncertainty in preliminary transformation into account.

So, the implemented investigation proves the efficiency of the fuzzy classification in EEG signal analysis which results from considering the uncertainty formed in the step of initial EEG signal preprocessing.

VI. CONCLUSION

The new fuzzy-based approach for classification of EEG signals for purposes of detection of the epileptic's seizures was developed in this paper. This approach is based on typical approach for signal classification which consists of two steps that are the preliminary data transformation (or preprocessing) and classification itself [17]. In the step of preprocessing, noise is removed and useful information needed for the next step of classification is extracted. At the same time, the signal is transformed into a set of samples that can be classified. The data after the preprocessing is a set of numerical crisp attributes. According to investigations in [28] and [29], the data preprocessing causes the loss of some information and arising of the data uncertainty. It results into the decrease of the classification accuracy. Therefore, this data uncertainty should be considered in the step of classification, which is possible by the application of a fuzzy classifier. The change of the classifier type requires other type of data, which should be fuzzy. The fuzzification of data after the preliminary transformation should be implemented. Therefore, the new type of classifier is used and the new procedure of the fuzzification is added in the data preprocessing in the new proposed fuzzy-based approach for classification of EEG signal for the purposes of detection of the epileptic's seizures (Fig. 2).

In this paper, we considered and evaluated the proposed approach for FDT based classifier, which is inducted through application of CMI [42], [43]. We developed the algorithm for EEG signal classification, which uses procedures of Fourier transform for feature extraction, PCA for dimensionality reduction, and FCM for fuzzification of data in the step of the preprocessing and FDT in the step of classification. We used a well-known public dataset from [12]. This dataset originally consists of 500 samples, but we divided each EEG records into smaller pieces in our experiments. In this way, we obtained 4,000 instances divided into two classes (seizure and seizure free). The procedures of the signal preprocessing (feature extraction, dimensionality reduction, and fuzzification) allowed representing each sample by 8 fuzzy attributes, which were used for FDT induction.

We evaluated the classification efficiency of the developed approach for epileptic's seizures detection and compared it with some other. In particular, the proposed FDT-based approach was compared with other fuzzy-based approaches (Table 6) and non-fuzzy-based classifiers that use crisp data (Table 7). In all approaches, procedures of the feature extraction and dimensionality reduction were implemented same. The additional transformation based on FCM was implemented in algorithms with fuzzy-based classifiers. The comparison of these approaches showed that the fuzzy-based classifiers give the best results for this data. Furthermore, the classification based on FDT allows achieving the accuracy of 0.995 that is the best result for evaluated fuzzy-based classifiers (Table 6) and non-fuzzy-based classifiers (Table 7). The best result for non-fuzzy-based classifier is obtained for decision tree inducted based on C4.5 algorithm. The developed FDT-based algorithm for classification of EEG signals was also compared with existing studies (Table 8). Results of the comparisons show efficiency of the developed approach.

In this paper the epileptic seizure detection based on EEG signal classification was considered as one of the basic problems in EEG signal analysis. This problem was chosen to evaluate the efficiency of the fuzzy-based classification approach in EEG signal analysis. The presented results allow us to develop and use this approach for similar problem in EEG signal analysis. The obtained results were achieved for data from [12]. In case of other real data, the indicated accuracy can be less than 0.995, but the expansion of such data samples for the proposed approach allows achieving the indicated accuracy. In future work, we will investigate impact of alternative procedures in the signal preprocessing. The first of the investigated procedures will be fuzzification, i.e., different algorithms of fuzzification will be studied for the purposes of signal classification.

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