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Stabilization and Event-Triggered Control of Stochastic Delay Systems With Markovian Jump Parameters

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ABSTRACT This article is concerned with the problem of delay-dependent stabilization for a class of stochastic Markov systems with event-triggered feedback control. An event-triggered mechanism (ETM) is proposed with the purpose of effectively reducing the transmissions of redundant massages, in which the generation of sensor sampling and control actuation is not periodic but only when some event-driven conditions are satisfied. In the meanwhile, a novel Lyapunov-Krasovskii functional (LKF) is applied to the closed-loop systems to establish the criterion of practically exponential mean-square stability. And a positive lower bound on the inter-execution times is guaranteed, that is, the Zeno behavior will not happen under this ETM. Furthermore, the event-triggered feedback controller can be constructed by solving the relevant linear matrix inequalities (LMIs). In the end, a numerical example displays the feasibility of our results.

INDEX TERMS Delay-dependent stability, event-triggered control, Markovian jump parameters.

I. INTRODUCTION

Stochastic time delay systems (STDSs) have been extensively investigated since they can exactly describe the dynamic processes that are influenced by the stochastic perturbations and time delays. In effect, the above affect factors often give rise to oscillation, poor performance and even instability. As a hot issue in the study of STDSs, stability analysis has aroused wide concerns among many investigators (see, e.g., [1]–[5]), and it is the primary task for researchers to discuss how to construct a controller to stabilize an unstable controlled system (see, e.g., [6]–[8]).

In traditional control theories, the researches on controller design with continuous-time state feedback control were well studied(see, e.g., [9], [10]), but it is hard, and even unrealistic in practical applications since the systems need to achieve stable and reliable real-time signal transmissions and controller computations at all times. Compared with the continuous-time state feedback case, the strategies of discrete-time state feedback control are easy to implement the desired stability and performance properties (see, e.g., [11], [12]). Nonetheless, one can not deny that the above-mentioned time-triggered strategies may cause redundant transmissions and unnecessary waste of limited resources, especially for systems with presented communication constraints. Thus event-triggered control (ETC) mechanisms came into existence as strategies where the samplings and the updates of the control inputs are aperiodic and only with the occurrence of the certain events (see, e.g., [13]–[17]), in this manner, the utilization of communication resources was effectively improved while the required performance of systems can be satisfied. For example, in [18], an event-triggered real-time scheduling strategy was investigated for embedded systems to decide when the control task would be executed while the asymptotical stability was guaranteed. More importantly, to avoid unlimited data transfers in finite time, namely Zeno behavior, a positive lower bound on inter-execution times should be guaranteed, which ultimately ensure the feasibility of presented event-triggered mechanism (ETM). Thus, by using model transformation method, [19] remodeled ETC embedded systems as hybrid systems to establish the Lyapunov stability theorems for these

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systems, and the lower bound of inter-execution times be further enlarged. Nevertheless, one can discover the above researches in [13]–[19] all concentrated on the ETC issues for deterministic systems.

In regard to stochastic systems, the random changes in response to external effects bring great difficulty to the researches of systems. In [20], the design mechanisms about ETC for stochastic systems with state-dependent noise were discussed, periodic ETC and continuous ETC were studied in both static and dynamic cases. And the analysis of delay-independent stabilization for STDSs was presented in [21] with a given ETM. In general, delay-independent criterion is likely to be more conservative than delay-dependent criterion, especially for small delays. Moreover, it should be pointed out that despite there are many practical applications for stochastic Markov systems in financial investments and ecology (see, e.g., [22]–[24]), but few researchers have taken advantage of ETM on such systems. Consequently, this article will analyze the problem of stabilization for STDSs with Markovian jump parameters in the case of an ETM, where the delay-dependent conditions of practically exponential mean-square stabilization are given in terms of linear matrix inequalities (LMIs). Specifically, by resorting to an appropriate Lyapunov-Krasovskii functional (LKF), we first derive LMI-based delay-dependent conditions to assure the exponential stability of the closed-loop systems with the ETM. Meanwhile, the existence of a positive lower bound on inter-execution times is guaranteed, which means Zeno behavior can be avoid under our ETM. Finally the conclusions of stabilization for stochastic Markov systems are obtained, and the desired event-triggered feedback gain matrix is given by solving LMIs.

The content of this note is arranged as follows: Section II contains the necessary symbolism and theoretical knowledge. In Section III, stabilization conditions presented in terms of LMIs for STDS are deeply discussed, and the event-triggered feedback controller can be constructed by solving the aforementioned LMIs. A numerical case and its simulation support the practicability and validity of our conclusions in Section IV. In the end, we present a recapitulation of this note in Section V.

II. PROBLEM DESCRIPTION AND PRELIMINARIES

Notation: Throughout this article, we let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. it is right continuous and $\{\mathcal{F}_0\}$ contains all $\mathcal{P}$-null sets); Let $\mathbb{R}^+ = [0, +\infty)$ and $\mathbb{N} = \{0, 1, 2, \ldots\}$; $\mathbb{R}^n$ is the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ denotes the space of $n \times m$ real matrices, $|\cdot|$ stands for the Euclidean norm operator of a vector and $\|P\| = \sup_{x\in\mathbb{R}^n,\ |x|=1} |Px|$; Given integers b and d with $b < d$, let $\overline{b, d} = \{b, b+1, \cdots, d\}$; For $p, q \in \mathbb{R}$, $p \vee q = \max\{p, q\}$, and $p \wedge q = \min\{p, q\}$; The transpose of $P \in \mathbb{R}^{n\times n}$ is denoted by P^T, and its inverse is P^{-1}; For $P \in \mathbb{R}^{n\times n}$, $\lambda_{min}(P)$ is the smallest eigenvalue of P, $trace(P)$ is the trace of $P = (p_{ij})_{n\times n}$; $P < (\leq, >, \geq)\ 0$ means P is a negative definite (negative semi-definite, positive, positive semi-definite) symmetric matrix; $\mathbb{S}_+^{n\times n}$ is the set of matrices $P \in \mathbb{R}^{n\times n}$ with $P > 0$; Let $\mathcal{C}([-\tau, 0]; \mathbb{R}^n)$ denotes the family of continuous functions ϕ from $[-\tau, 0]$ to $\mathbb{R}^n$ with the norm $\|\phi\| = \sup_{-\tau\leq\theta\leq 0} |\phi(\theta)|$, where $\tau > 0$; Let $\mathcal{L}^2_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$ be the family of all $\mathcal{F}_0$-measurable $\mathcal{C}([-\tau, 0]; \mathbb{R}^n)$-valued stochastic variables $\phi = \{\phi(\theta) : -\tau \leq \theta \leq 0\}$ such that $\sup_{-\tau\leq\theta\leq 0} \mathbb{E}|\phi(\theta)|^2 < \infty$, where $\mathbb{E}(\cdot)$ means the mathematical expectation operator.

Let $\{r(t),\ t \in \mathbb{R}^+\}$ be a right-continuous Markovian chain in $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ taking values in a finite set $\mathcal{S} = \{1, 2, \ldots, \mathcal{N}\}$ with generator $\Gamma = (\pi_{ij})_{\mathcal{N}\times\mathcal{N}}$ given by

$$\mathbb{P}\{r(s+h) = j | r(s) = i\} = \begin{cases} \pi_{ij} h + o(h), & i \neq j, \\ 1 + \pi_{ij} h + o(h), & i = j, \end{cases}$$

where $h > 0$, $\lim_{h\to 0} \frac{o(h)}{h} = 0$. Here $\pi_{ij} \geq 0\ (i \neq j)$ is transition rate from i to j while

$$\pi_{ii} = -\sum_{j\neq i} \pi_{ij}.$$

In this article, we aim to consider the following STDS with Markovian jump parameters:

$$dx(t) = [A(r(t))x(t) + B(r(t))x(t-\tau) + D(r(t))u(t)]dt + \sigma(t, x(t), x(t-\tau), r(t))dw(t) \quad (1)$$

with initial condition $\phi \in \mathcal{L}^2_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$, $x(t) \in \mathbb{R}^n$ is the state vector, and $u(t) \in \mathbb{R}^m$ is the control input. $w(t)$ is a scalar Brownian motion defined on the complete probability $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$. In this article, we assume that the Markov chain $r(\cdot)$ is independent of $w(t)$. For convenience, for each $r(t) = i \in \mathcal{S}$, we let $A(r(t)) = A_i \in \mathbb{R}^{n\times n}$, $B(r(t)) = B_i \in \mathbb{R}^{n\times n}$, $D(r(t)) = D_i \in \mathbb{R}^{n\times m}$, where A_i, B_i, D_i are known constant matrices for the fixed mode.

Moreover, throughout this article, the following assumption is required to guarantee the existence of a unique solution of (1).

Assumption 1: The noise intensity function $\sigma(\cdot, \cdot, \cdot, \cdot) : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathcal{S} \to \mathbb{R}^n$ is assumed to satisfy the local Lipschitz condition and the linear growth condition, and for each $r(t) = i \in \mathcal{S}$, there exist $\gamma_{i1} \in \mathbb{R}^+$ and $\gamma_{i2} \in \mathbb{R}^+$ such that

$$trace[\sigma^T(t, x(t), x(t-\tau), r(t))\sigma(t, x(t), x(t-\tau), r(t))] \leq \gamma_{i1}|x(t)|^2 + \gamma_{i2}|x(t-\tau)|^2, \quad \sigma^T(t, 0, 0, r(t)) \equiv 0. \quad (2)$$

Remark 1: According to Theorem 7.3 of [25], for any initial condition $\phi \in \mathcal{L}^2_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$, the existence and uniqueness of the system state of (1) can be guaranteed by the local Lipschitz condition and the linear growth condition.

Considering the following ETM. For $\alpha_1 > 0$, $\alpha_2 > 0$, let sampling time sequence $\{t_j : j \in \mathbb{N}\}$ meets conditions $t_0 = 0$ and

$$t_{j+1} = inf\{t | t > t_j,\ |\varepsilon(t)|^2 - \alpha_1|x(t_j)|^2 - \alpha_2 > 0\}, \quad (3)$$

where $\varepsilon(t)$ is the error between current state and sampled state, that is, $\varepsilon(t) = x(t_j) - x(t)$, $t \in [t_j, t_{j+1})$. Next, we take the control input $u(t)$ in the sample-and-hold sense into account, which is given as follows:

$$u(t) = K_i x(t_j), \quad t \in [t_j, t_{j+1}),\ j \in \mathbb{N}, \tag{4}$$

for every mode $r(t) = i \in \mathcal{S}$, the feedback gain matrix $K_i \in \mathbb{R}^{m\times n}$ will be decided later.

From the above description, only at event-triggered instant t_j the controller $u(t)$ can receive the data of system state $x(t_j)$, and the data will be held by the Zero-Order-Hold (ZOH) until the next event is triggered at t_{k+1}. Next, substituting (3) and (4) into (1) leads to the following closed-loop system with initial condition $\phi \in \mathcal{L}^2_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$:

$$\begin{cases} dx(t) = [(A_i + D_iK_i)x(t) + B_ix(t-\tau) + D_iK_i\varepsilon(t)]dt \\ \qquad\quad +\sigma(t, x(t), x(t-\tau), r(t))dw(t), \\ \varepsilon(t) = x(t_j) - x(t),\ for\ t \in [t_j, t_{j+1}), \\ t_{j+1} = inf\{t | t > t_j,\ |\varepsilon(t)|^2 - \alpha_1|x(t_j)|^2 - \alpha_2 > 0\}. \end{cases} \tag{5}$$

To express our results more precisely, we need to introduce the following definitions and lemmas.

Definition 1: ([26]) The trivial solution of (5) is called practically exponentially mean-square stable (PEMS), if for any initial condition $\phi \in \mathcal{L}^2_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$, there exist constants $C > 0$, $\gamma > 0$ and $e \geq 0$ such that

$$\mathbb{E}|x(t;\phi)|^2 \leq C\mathbb{E}\|\phi\|^2 e^{-\gamma t} + e, \quad t \geq 0. \tag{6}$$

Definition 2: The trivial solution of (1) is called practically exponentially mean-square stabilizable if there exist feedback gain matrix $K_i \in \mathbb{R}^{m\times n}$, $r(t) = i \in \mathcal{S}$ such that (5) is PEMS.

Definition 3: ([25]) Consider the stochastic differential delay equations $dx(t) = f(t, x(t), x(t-\tau), r(t))dt + g(t, x(t), x(t-\tau), r(t))dw(t)$ with initial condition $\phi \in \mathcal{L}^2_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$. Let $x_t = x(t+\theta)$, $\theta \in [-\tau, 0]$, $\mathcal{C}^{1,2}([t_0 - \tau, +\infty) \times \mathbb{R}^n \times \mathcal{S}; \mathbb{R}^+)$ is the set of all nonnegative functions $V(t, x_t, r(t))$ that are continuously once differentiable in t and twice in x_t. Define an operator $\mathcal{L}V$ from $[t_0 - \tau, +\infty) \times \mathbb{R}^n \times \mathbb{R}^n \times \mathcal{S}$ to $\mathbb{R}$ by

$$\begin{aligned} &\mathcal{L}V(t, x, y, i) \\ &\quad = V_t(t, x, i) + V_x(t, x, i)f(t, x, y, i) + \sum_{j=1}^{\mathcal{N}} \pi_{ij}V(t, x, j) \\ &\qquad + \frac{1}{2}trace[g^T(t, x, y, i)V_{xx}(t, x, i)g(t, x, y, i)], \end{aligned}$$

where

$$V_t(t, x, i) = \frac{\partial V(t, x, i)}{\partial t},$$
$$V_x(t, x, i) = (\frac{\partial V(t, x, i)}{\partial x_1}, \cdots, \frac{\partial V(t, x, i)}{\partial x_n}),$$
$$V_{xx}(t, x, i) = (\frac{\partial^2 V(t, x, i)}{\partial x_i \partial x_j})_{n\times n}.$$

Lemma 1: For vectors $x, y \in \mathbb{R}^n$, let $\mathcal{H}$ be any real matrix with an appropriate dimension satisfying $\mathcal{H} > 0$, then $2x^Ty \leq x^T\mathcal{H}^{-1}x + y^T\mathcal{H}y$.

Lemma 2: ([27])(Jensen's Inequality) For $G \in \mathbb{S}^{n\times n}_+$, if there exists function $h(t) : [0, \delta] \to \mathbb{R}^n$ ($\delta \in \mathbb{R}^+$) such that the following integrals exist, then

$$[\int_0^\delta h(t)dt]^T G[\int_0^\delta h(t)dt] \leq \delta \int_0^\delta h^T(t)Gh(t)dt.$$

Lemma 3: ([28]) The following LMI:

$$\begin{pmatrix} \mathcal{Q} & \mathcal{W} \\ \mathcal{W}^T & -\mathcal{F} \end{pmatrix} < 0$$

is equivalent to $\mathcal{F} > 0$, $\mathcal{Q} + \mathcal{W}\mathcal{F}^{-1}\mathcal{W}^T < 0$, where $\mathcal{Q} = \mathcal{Q}^T$.

III. MAIN RESULTS

This section establishes the practically exponential mean-square stability conditions for (5) by using the aforementioned ETM while the existence of a positive lower bound on inter-execution times can be guaranteed, which means the Zeno behavior will not occur under our conditions. Then we can get the stabilization conditions for (1) by designing the event-triggered feedback controller and K_i.

Theorem 1: Assume that there exist matrices $P_i \in \mathbb{S}^{n\times n}_+$, $Q \in \mathbb{S}^{n\times n}_+$, $R_j \in \mathbb{S}^{n\times n}_+$ and $F_{ij} \in \mathbb{R}^{n\times n}$, $i \in \mathcal{S}$, positive constants $\theta > 0$, $\beta_j > 0$, $\alpha_j > 0$, $j \in \overline{1, 2}$, the scalars satisfy $\alpha_1 < \theta$ and $\theta < 1$, if the following LMIs hold for all $i \in \mathcal{S}$:

$$P_i < \beta_1^{-1} \cdot I, \tag{7}$$
$$R_2 < \beta_2^{-1} \cdot I, \tag{8}$$

$$\begin{pmatrix} \Psi^i_{11} & \Psi^i_{12} & P_iD_iK_i & F^T_{i1} & F^T_{i1} & \Psi^i_{16} \\ * & \Psi^i_{22} & 0 & F^T_{i2} & F^T_{i2} & \tau B^T_i \\ * & * & \Psi^i_{33} & 0 & 0 & \tau K^T_i D^T_i \\ * & * & * & -R_1 & 0 & 0 \\ * & * & * & * & -R_2 & 0 \\ * & * & * & * & * & -R^{-1}_1 \end{pmatrix} < 0, \tag{9}$$

where "$*$" denotes the transpose of the corresponding part above diagonal, and

$$\begin{aligned} \Psi^i_{11} &= P_i(A_i + D_iK_i) + (A_i + D_iK_i)^TP_i + \sum_{s=1}^{\mathcal{N}} \pi_{is}P_s \\ &\quad + Q + (\beta_1^{-1}\gamma_{i1} + \tau\beta_2^{-1}\gamma_{i1} + \frac{\alpha_1}{1-\theta}) \cdot I + F^T_{i1} + F_{i1}, \\ \Psi^i_{12} &= P_iB_i - F^T_{i1} + F_{i2}, \\ \Psi^i_{22} &= -Q - F^T_{i2} - F_{i2} + (\beta_1^{-1}\gamma_{i2} + \tau\beta_2^{-1}\gamma_{i2}) \cdot I, \\ \Psi^i_{33} &= (\frac{\alpha_1}{\theta} - 1) \cdot I, \quad \Psi^i_{16} = \tau(A^T_i + K^T_i D^T_i), \end{aligned}$$

then the trivial solution of (5) is PEMS. Furthermore, for each $j \in \mathbb{N}$, there exists a positive constant T^* such that the inter-execution time $T_j = t_{j+1} - t_j \geq T^*$ holds.

Proof: First, by Lemma 3, LMI (9) is equivalent to

$$\Pi_i = \begin{pmatrix} \Psi^i_{11} & \Psi^i_{12} & P_iD_iK_i \\ * & \Psi^i_{22} & 0 \\ * & * & \Psi^i_{33} \end{pmatrix} + F^T_i R^{-1}_1 F_i$$

$$+F_i^TR_2^{-1}F_i+\tau^2\Gamma_i^TR_1\Gamma_i<0, \quad (10)$$

where

$$F_i=\begin{pmatrix}F_{i1} & F_{i2} & 0\end{pmatrix},$$
$$\Gamma_i=\begin{pmatrix}A_i+D_iK_i & B_i & D_iK_i\end{pmatrix}.$$

Set $g(t)=\sigma(t,x(t),x(t-\tau),r(t))$ and

$$f(t)=(A_i+D_iK_i)x(t)+B_ix(t-\tau)+D_iK_i\varepsilon(t), \quad (11)$$

then (5) becomes

$$dx(t)=f(t)dt+g(t)dw(t). \quad (12)$$

It follows from (12) that

$$x(t-\tau)=x(t)-\int_{t-\tau}^{t}f(s)ds-\int_{t-\tau}^{t}g(s)dw(s). \quad (13)$$

Now, for each $r(t)=i\in\mathcal{S}$, let's consider the following Lyapunov-Krasovshii functional:

$$\begin{aligned}V(t,x_t,i)&=x^T(t)P_ix(t)+\int_{t-\tau}^{t}x^T(s)Qx(s)ds\\&+\tau\int_{-\tau}^{0}\int_{t+\theta}^{t}f^T(s)R_1f(s)dsd\theta\\&+\int_{-\tau}^{0}\int_{t+\theta}^{t}trace[g^T(s)R_2g(s)]dsd\theta, \quad (14)\end{aligned}$$

where $P_i\in\mathbb{S}_+^{n\times n}$, $Q\in\mathbb{S}_+^{n\times n}$, $R_1\in\mathbb{S}_+^{n\times n}$, $R_2\in\mathbb{S}_+^{n\times n}$ will be determined.

According to Itô's formula, the weak infinitesimal generator $\mathcal{L}V(t,x_t,i)$ is formulated as follows:

$$\begin{aligned}&\mathcal{L}V(t,x_t,i)\\&=2x^T(t)P_i[(A_i+D_iK_i)x(t)+B_ix(t-\tau)+D_iK_i\varepsilon(t)]\\&\quad+trace[g^T(t)P_ig(t)]+x^T(t)[\sum_{s=1}^{\mathcal{N}}\pi_{is}P_s]x(t)\\&\quad+x^T(t)Qx(t)-x^T(t-\tau)Qx(t-\tau)+\tau^2f^T(t)R_1f(t)\\&\quad-\tau\int_{t-\tau}^{t}f^T(s)R_1f(s)ds+\tau\cdot trace[g^T(t)R_2g(t)]\\&\quad-\int_{t-\tau}^{t}trace[g^T(s)R_2g(s)]ds. \quad (15)\end{aligned}$$

Let $\eta(t)=col(x(t),x(t-\tau),\varepsilon(t))$, from (13), for $t\in\mathbb{R}^+$, the following equation holds:

$$\begin{aligned}0&=\mathbb{E}\{2\eta^T(t)F_i^T[x(t)-x(t-\tau)-\int_{t-\tau}^{t}f(s)ds\\&\quad-\int_{t-\tau}^{t}g(s)dw(s)]\}\\&=\mathbb{E}\{x^T(t)[F_{i1}^T+F_{i1}]x(t)+2x^T(t)[-F_{i1}^T+F_{i2}]x(t-\tau)\\&\quad+x^T(t-\tau)[-F_{i2}^T-F_{i2}]x(t-\tau)\\&\quad-2\eta^T(t)F_i^T\int_{t-\tau}^{t}f(s)ds-2\eta^T(t)F_i^T\int_{t-\tau}^{t}g(s)dw(s)\}.\end{aligned} \quad (16)$$

From Lemma 1 and Lemma 2, for $i\in\mathcal{S}$, one can get

$$\begin{aligned}&-2\eta^T(t)F_i^T\int_{t-\tau}^{t}f(s)ds\\&\le\eta^T(t)F_i^TR_1^{-1}F_i\eta(t)+[\int_{t-\tau}^{t}f(s)ds]^TR_1[\int_{t-\tau}^{t}f(s)ds]\\&\le\eta^T(t)F_i^TR_1^{-1}F_i\eta(t)+\tau\int_{t-\tau}^{t}f^T(s)R_1f(s)ds \quad (17)\end{aligned}$$

and

$$\begin{aligned}&-2\eta^T(t)F_i^T\int_{t-\tau}^{t}g(s)dw(s)\\&\le[\int_{t-\tau}^{t}g(s)dw(s)]^TR_2[\int_{t-\tau}^{t}g(s)dw(s)]\\&\quad+\eta^T(t)F_i^TR_2^{-1}F_i\eta(t), \quad (18)\end{aligned}$$

by Itô isometry,

$$\begin{aligned}&\mathbb{E}\{[\int_{t-\tau}^{t}g(s)dw(s)]^TR_2[\int_{t-\tau}^{t}g(s)dw(s)]\}\\&\qquad=\mathbb{E}\{\int_{t-\tau}^{t}trace[g^T(s)R_2g(s)]ds\}. \quad (19)\end{aligned}$$

On the other hand, according to (3), for $t\in[t_j,t_{j+1})$, the following inequality holds:

$$0\le\frac{\alpha_1}{1-\theta}|x(t)|^2+(\frac{\alpha_1}{\theta}-1)|\varepsilon(t)|^2+\alpha_2. \quad (20)$$

Applying (16)-(20) to (15), we can obtain

$$\begin{aligned}&\mathbb{E}\mathcal{L}V(t,x_t,i)\\&\le\mathbb{E}\{x^T(t)[P_i(A_i+D_iK_i)+(A_i+D_iK_i)^TP_i+Q\\&\quad+F_{i1}^T+F_{i1}+(\beta_1^{-1}\gamma_{i1}+\tau\beta_2^{-1}\gamma_{i1}+\frac{\alpha_1}{1-\theta})\cdot I\\&\quad+\sum_{s=1}^{\mathcal{N}}\pi_{is}P_s]x(t)+(\frac{\alpha_1}{\theta}-1)\varepsilon^T(t)\varepsilon(t)\\&\quad+2x^T(t)(P_iB_i-F_{i1}^T+F_{i2})x(t-\tau)\\&\quad+2x^T(t)P_iD_iK_i\varepsilon(t)+x^T(t-\tau)[-Q-F_{i2}^T\\&\quad-F_{i2}+(\beta_1^{-1}\gamma_{i2}+\tau\beta_2^{-1}\gamma_{i2})\cdot I]x(t-\tau)\\&\quad+\eta^T(t)F_i^TR_1^{-1}F_i\eta(t)+\eta^T(t)F_i^TR_2^{-1}F_i\eta(t)\\&\quad+\tau^2\eta^T(t)\Gamma_i^TR_1\Gamma_i\eta(t)+\alpha_2\}. \quad (21)\end{aligned}$$

It then follows from Lemma 3 and (10), (21) yields that

$$\begin{aligned}\mathbb{E}\mathcal{L}V(t,x_t,i)&\le\mathbb{E}\{\eta^T(t)\Pi_i\eta(t)\}+\alpha_2\\&\le-\lambda\mathbb{E}|x(t)|^2+\alpha_2, \quad (22)\end{aligned}$$

where $\lambda=\min_{i\in\mathcal{N}}\{\lambda_{min}(-\Pi_i)\}>0$.

By the definition of $V(t,x_t,i)$, there exist $\xi_j>0,\ j\in\overline{1,3}$ such that

$$\xi_1|x(t)|^2\le V(t,x_t,i)\le\xi_2|x(t)|^2+\xi_3\int_{t-2\tau}^{t}|x(s)|^2ds. \quad (23)$$

Next, we choose $\gamma>0$ such that

$$2\gamma\xi_3\tau e^{2\tau\gamma}+\gamma\xi_2\le\lambda. \quad (24)$$

According to Dynkin formula, for $t \geq \tau$, from (22) and (23), we obtain that

$$
\begin{aligned}
&e^{\gamma t}\mathbb{E}V(t,x_t,i) - e^{\gamma\tau}\mathbb{E}V(\tau,x_\tau,r(\tau)) \\
&= \mathbb{E}\int_\tau^t [\gamma e^{\gamma s}V(s,x_s,r(s)) + e^{\gamma s}\mathcal{L}V(s,x_s,r(s))]ds \\
&\leq (\gamma\xi_2 - \lambda)\mathbb{E}\int_\tau^t e^{\gamma s}|x(s)|^2 ds + \alpha_2\mathbb{E}\int_\tau^t e^{\gamma s}ds \\
&\quad + \gamma\xi_3\mathbb{E}\int_\tau^t e^{\gamma s}\int_{s-2\tau}^s |x(u)|^2 duds. \quad (25)
\end{aligned}
$$

From Fubini theorem, we arrive at

$$
\begin{aligned}
&\mathbb{E}\int_\tau^t e^{\gamma s}\int_{s-2\tau}^s |x(u)|^2 duds \\
&= \mathbb{E}\int_{-\tau}^t |x(u)|^2 \int_{\tau\vee u}^{t\wedge(u+2\tau)} e^{\gamma s}dsdu \\
&\leq 2\tau e^{2\tau\gamma}\mathbb{E}\int_{-\tau}^t e^{\gamma s}|x(s)|^2 ds. \quad (26)
\end{aligned}
$$

Hence, from (24)-(26), the following equation holds:

$$
\begin{aligned}
&e^{\gamma t}\mathbb{E}V(t,x_t,i) \\
&\leq e^{\gamma\tau}\mathbb{E}V(\tau,x_\tau,r(\tau)) + (\gamma\xi_2 - \lambda)\mathbb{E}\int_\tau^t e^{\gamma s}|x(s)|^2 ds \\
&\quad + 2\gamma\xi_3\tau e^{2\gamma\tau}\mathbb{E}\int_{-\tau}^t e^{\gamma s}|x(s)|^2 ds + \alpha_2\mathbb{E}\int_\tau^t e^{\gamma s}ds \\
&\leq e^{\gamma\tau}[\xi_2\mathbb{E}|x(\tau)|^2 + \xi_3\mathbb{E}\int_{-\tau}^\tau |x(s)|^2 ds] \\
&\quad + 2\gamma\xi_3\tau e^{2\gamma\tau}\mathbb{E}\int_{-\tau}^\tau e^{\gamma s}|x(s)|^2 ds + \frac{\alpha_2}{\gamma}(e^{\gamma t} - e^{\gamma\tau}). \quad (27)
\end{aligned}
$$

From Theorem 7.3 of [25], there exists $C_0 > 0$ such that $\mathbb{E}(\sup_{-\tau\leq s\leq\tau}|x(s)|^2) \leq C_0\mathbb{E}\|\phi\|^2$. Therefore, by (23) and (27), we have

$$
\mathbb{E}|x(t)|^2 \leq C\mathbb{E}\|\phi\|^2 e^{-\gamma t} + \frac{\alpha_2}{\xi_1\gamma}, \quad (28)
$$

where $C = \xi_1^{-1}(\xi_2 + 2\tau\xi_3 + 4\tau^2\gamma\xi_3 e^{2\gamma\tau})e^{\gamma\tau}C_0$, this means that (5) is PEMS.

Next, we take attention to the existence of the lower bound on inter-execution times. From the definition of $\varepsilon(t)$ and (5), one sees that

$$
d\varepsilon(t) = -f(t)dt - g(t)dw(t), \ t \in [t_j, t_{j+1}).
$$

For $t \in [t_j, t_{j+1})$, by Itô's formula conjunction with Lemma 1, formula for $\mathcal{L}|\varepsilon(t)|^2$ is deduced as follows:

$$
\begin{aligned}
&\mathcal{L}|\varepsilon(t)|^2 \\
&= -2\varepsilon^T(t)(A_i + D_iK_i)x(t_j) + 2\varepsilon^T(t)A_i\varepsilon(t) \\
&\quad - 2\varepsilon^T(t)B_ix(t-\tau) + trace[\sigma^T(t)\sigma(t)] \\
&\leq (2||A_i|| + 2 + 2\gamma_{i1})|\varepsilon(t)|^2 + (||B_i||^2 + \gamma_{i2})|x(t-\tau)|^2 \\
&\quad + (||A_i + D_iK_i||^2 + 2\gamma_{i1})|x(t_j)|^2.
\end{aligned}
$$

Let

$$
\begin{aligned}
h_1 &= \max_{i\in\mathcal{S}}\{2||A_i|| + 2 + 2\gamma_{i1}\}, \\
h_2 &= \max_{i\in\mathcal{S}}\{||A_i + D_iK_i||^2 + 2\gamma_{i1}\}, \\
M &= \max_{i\in\mathcal{S}}\{||B_i||^2 + \gamma_{i2}\} \cdot (C\mathbb{E}\|\phi\|^2 e^{\gamma\tau} + \frac{\alpha_2}{\xi_1\gamma}),
\end{aligned}
$$

note from (28) that

$$
\mathbb{E}\mathcal{L}|\varepsilon(t)|^2 \leq h_1\mathbb{E}|\varepsilon(t)|^2 + h_2\mathbb{E}|x(t_j)|^2 + M. \quad (29)
$$

Now set $Z(t) = |\varepsilon(t)|^2$, for any $\Delta t > 0$ and $t \in [t_j, t_{j+1})$ such that $t + \Delta t \in (t_j, t_{j+1})$, since $\mathbb{E}Z(t)$ is continuous on $t \in [t_j, t_{j+1})$, then

$$
\mathbb{E}Z(t + \Delta t) = \mathbb{E}Z(t) + \int_t^{t+\Delta t} \mathbb{E}\mathcal{L}|\varepsilon(s)|^2 ds.
$$

Therefore, for $t \in [t_j, t_{j+1})$,

$$
D^+\mathbb{E}Z(t) = \limsup_{\Delta t\to 0^+}\frac{\mathbb{E}Z(t+\Delta t) - \mathbb{E}Z(t)}{\Delta t} = \mathbb{E}\mathcal{L}|\varepsilon(t)|^2,
$$

the above equation combine with (29) implies that

$$
D^+(\mathbb{E}|\varepsilon(t)|^2) \leq h_1\mathbb{E}|\varepsilon(t)|^2 + h_2\mathbb{E}|x(t_j)|^2 + M.
$$

One can see

$$
\begin{aligned}
&D^+[\frac{\mathbb{E}|\varepsilon(t)|^2}{\alpha_1\mathbb{E}|x(t_j)|^2 + \alpha_2}] \\
&\leq \frac{h_1\mathbb{E}|\varepsilon(t)|^2 + \frac{h_2}{\alpha_1}[\alpha_1\mathbb{E}|x(t_j)|^2 + \alpha_2] - \frac{h_2}{\alpha_1}\alpha_2 + M}{\alpha_1\mathbb{E}|x(t_j)|^2 + \alpha_2} \\
&\leq h_1\frac{\mathbb{E}|\varepsilon(t)|^2}{\alpha_1\mathbb{E}|x(t_j)|^2 + \alpha_2} + L, \ t \in [t_j, t_{j+1}),
\end{aligned}
$$

where $L = \frac{h_2}{\alpha_1} + \frac{M}{\alpha_2}$. Then we can deduce that

$$
\frac{\mathbb{E}|\varepsilon(t)|^2}{\alpha_1\mathbb{E}|x(t_j)|^2 + \alpha_2} \leq q(t - t_j), \quad t \in [t_j, t_{j+1}),
$$

where $q(t - t_j)$ is the solution of $\dot{q}(t - t_j) = h_1 q(t - t_j) + L$, that is, $q(t - t_j) = \frac{L}{h_1}[e^{h_1(t-t_j)} - 1]$, $t \in [t_j, t_{j+1})$. Thus the inter-execution times are bounded by the time it takes for $q(t - t_j)$ to evolve from 0 to 1, that is, the inter-execution times are lower bounded by the solution $T^* \in \mathbb{R}^+$ of $q(T^*) = 1$. From $\frac{L}{h_1}[e^{h_1T^*} - 1] = 1$, one can see $T^* = \frac{1}{h_1}ln(\frac{h_1}{L} + 1)$, to more specific, we exclude the sampling Zeno phenomenon under our ETM. ■

Remark 2: One can find that Theorem 1 takes the size of delay into consideration, that is, we establish the delay-dependent conditions of practically exponential mean-square stability for the closed-loop system, which is less conservative than delay-independent stability criterion.

Remark 3: To avoid Zeno behavior, for deterministic event-triggered systems, the positive lower bounds on inter-execution times were generally guaranteed by computing $\frac{d}{dt}\frac{|\varepsilon(t)|}{|x(t)|}$ to give the bounds of fraction $\frac{|\varepsilon(t)|}{|x(t)|}$ (see, e.g., [18]). However, it is well known that the solution of a stochastic differential equation does not have a derivative, which is different from the deterministic case. Therefore, it does not work for stochastic systems to use above computing technique which depends on the state derivative. In this note, on the base of the our ETM, Theorem 1 gave a lower bound of inter-execution times, and that means the Zeno behavior will not happen under ETM (3).

Next, we will establish the criterion of practically exponential mean-square stabilization for (1) and design the event-triggered feedback gain matrix K_i.

Theorem 2: Assume that there exist matrices $X_i \in \mathbb{S}_+^{n\times n}$, $Y_i \in \mathbb{R}^{n\times n}$, $\bar{Q} \in \mathbb{S}_+^{n\times n}$, $\bar{R}_j \in \mathbb{S}_+^{n\times n}$, $\bar{F}_{ij} \in \mathbb{R}^{n\times n}$, $i \in \mathcal{S}$, and positive constants $\theta > 0$, $\beta_j > 0$, $\alpha_j > 0$, $j \in \overline{1,2}$, the scalars satisfy $\beta_2 < 2\beta_1$, $\alpha_1 < \theta$ and $\theta < 1$, if the following LMIs hold for all $i \in \mathcal{S}$, then (1) is practically exponentially mean-square stabilizable:

$$\bar{R}_2 < (2\beta_1 - \beta_2)\cdot I, \tag{30}$$

$$\beta_1 \cdot I < X_i, \tag{31}$$

$$\begin{pmatrix} \bar{\Psi}_{11}^i & \bar{\Psi}_{12}^i & D_iY_i & \bar{F}_{i1}^T & \bar{F}_{i1}^T & \bar{\Psi}_{16}^i & \bar{\Psi}_{17}^i & 0 & L_i \\ * & \bar{\Psi}_{22}^i & 0 & \bar{F}_{i2}^T & \bar{F}_{i2}^T & \tau X_iB_i^T & 0 & \bar{\Psi}_{28}^i & 0 \\ * & * & \bar{\Psi}_{33}^i & 0 & 0 & \tau Y_i^TD_i^T & 0 & 0 & 0 \\ * & * & * & \bar{R}_1 - 2X_i & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\bar{R}_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\bar{R}_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\bar{\Psi}_{77} & 0 & 0 \\ * & * & * & * & * & * & * & -\bar{\Psi}_{88} & 0 \\ * & * & * & * & * & * & * & * & -\mathcal{X}_i \end{pmatrix} < 0, \tag{32}$$

where

$$\begin{aligned} \bar{\Psi}_{11}^i &= (A_iX_i + D_iY_i) + (A_iX_i + D_iY_i)^T + \bar{Q} + \bar{F}_{i1}^T \\ &\quad + \bar{F}_{i1} + \pi_{ii}X_i, \\ \bar{\Psi}_{12}^i &= B_iX_i - \bar{F}_{i1}^T + \bar{F}_{i2}, \quad \bar{\Psi}_{22}^i = -\bar{Q} - \bar{F}_{i2}^T - \bar{F}_{i2}, \\ \bar{\Psi}_{33}^i &= (\frac{\alpha_1}{\theta} - 1)\beta_1^2 \cdot I, \quad \bar{\Psi}_{16}^i = \tau(X_iA_i^T + Y_i^TD_i^T), \\ \bar{\Psi}_{17}^i &= (\sqrt{\gamma_{i1}}X_i, \sqrt{\tau\gamma_{i1}}X_i, X_i), \\ \bar{\Psi}_{77} &= diag(\beta_1 \cdot I, \beta_2 \cdot I, \frac{1-\theta}{\alpha_1} \cdot I), \\ \bar{\Psi}_{28}^i &= (\sqrt{\gamma_{i2}}X_i, \sqrt{\tau\gamma_{i2}}X_i), \quad \bar{\Psi}_{88} = diag(\beta_1 \cdot I, \beta_2 \cdot I), \\ \mathcal{X}_i &= diag(X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_{\mathcal{N}}), \\ L_i &= (\sqrt{\pi_{i,1}}X_i, \ldots, \sqrt{\pi_{i,i-1}}X_i, \sqrt{\pi_{i,i+1}}X_i, \ldots, \sqrt{\pi_{i,\mathcal{N}}}X_i). \end{aligned}$$

And the feedback gain matrix K_i of the desired controller (4) is provided by

$$K_i = Y_iX_i^{-1}.$$

Proof: First, according to Lemma 3, LMI (32) is equivalent to

$$\begin{pmatrix} \Phi_{11}^i & \bar{\Psi}_{12}^i & D_iY_i & \bar{F}_{i1}^T & \bar{F}_{i1}^T & \bar{\Psi}_{16}^i \\ * & \Phi_{22}^i & 0 & \bar{F}_{i2}^T & \bar{F}_{i2}^T & \tau X_iB_i^T \\ * & * & \bar{\Psi}_{33}^i & 0 & 0 & \tau Y_i^TD_i^T \\ * & * & * & \bar{R}_1 - 2X_i & 0 & 0 \\ * & * & * & * & -\bar{R}_2 & 0 \\ * & * & * & * & * & -\bar{R}_1 \end{pmatrix} < 0, \tag{33}$$

where

$$\begin{aligned} \Phi_{11}^i =& (A_iX_i + D_iY_i) + (A_iX_i + D_iY_i)^T + \bar{Q} + \bar{F}_{i1}^T + \bar{F}_{i1} \\ & + [\beta_1^{-1}\gamma_{i1} + \tau\beta_2^{-1}\gamma_{i1} + \frac{\alpha_1}{1-\theta}]X_iX_i \\ & + X_i\sum_{s=1}^{\mathcal{N}}\pi_{is}X_s^{-1}X_i, \\ \Phi_{22}^i =& \bar{\Psi}_{22}^i + (\beta_1^{-1}\gamma_{i2} + \tau\beta_2^{-1}\gamma_{i2})X_iX_i. \end{aligned}$$

Let $P_i = X_i^{-1}$, $Q = P_i\bar{Q}P_i$, $R_1 = \bar{R}_1^{-1}$, $R_2 = P_i\bar{R}_2P_i$, $Y_i = K_iX_i$, $F_{ij} = P_i\bar{F}_{ij}P_i$, $j \in \overline{1,2}$. According to the relationship

$$R_2^{-1} + X_iR_2X_i - 2X_i = (X_i - R_2^{-1})^TR_2(X_i - R_2^{-1}) \geq 0,$$

together with (30) and (31), we have $R_2^{-1} \geq 2X_i - \bar{R}_2 > \beta_2 \cdot I$, that is, (8) holds. Next, pre- and post-multiplying (33) by $diag(P_i, P_i, P_i, P_i, P_i, I)$, we can arrive at the following LMI:

$$\begin{pmatrix} \Psi_{11}^i & \Psi_{12}^i & P_iD_iK_i & F_{i1}^T & F_{i1}^T & \Psi_{16}^i \\ * & \Psi_{22}^i & 0 & F_{i2}^T & F_{i2}^T & \tau B_i^T \\ * & * & \bar{\Psi}_{33}^iP_i^2 & 0 & 0 & \tau K_i^TD_i^T \\ * & * & * & \bar{\Psi}_{44}^i & 0 & 0 \\ * & * & * & * & -R_2 & 0 \\ * & * & * & * & * & -R_1^{-1} \end{pmatrix} < 0, \tag{34}$$

where $\bar{\Psi}_{44}^i = P_iR_1^{-1}P_i - 2P_i$.

Clearly, considering (31) together with $\alpha_1 < \theta$, we get (7), and $(\frac{\alpha_1}{\theta} - 1)\beta_1^2P_i^2 > (\frac{\alpha_1}{\theta} - 1)\cdot I$. On the other hand, according to the relationship

$$P_iR_1^{-1}P_i - 2P_i + R_1 = (P_i - R_1)^TR_1^{-1}(P_i - R_1) \geq 0,$$

$P_iR_1^{-1}P_i - 2P_i \geq -R_1$ is set up. Therefore, it follows from LMI (34) that LMI (9) holds for (5).

Hence, for (5), LMIs (7)-(9) hold. Consequently, Theorem 1 is available to support the fact that (1) is practically exponential mean-square stabilizable. ■

Remark 4: Theorem 2 presents sufficient conditions of practically exponential mean-square stabilization for (1), furthermore, the gain matrix of feedback controller (4) is provided by $K_i = Y_iX_i^{-1}$.

IV. AN EXAMPLE

In this section, the efficiency of our results which have been derived in the preceding section will be verified by the following numerical example. Specifically, a feasible solution

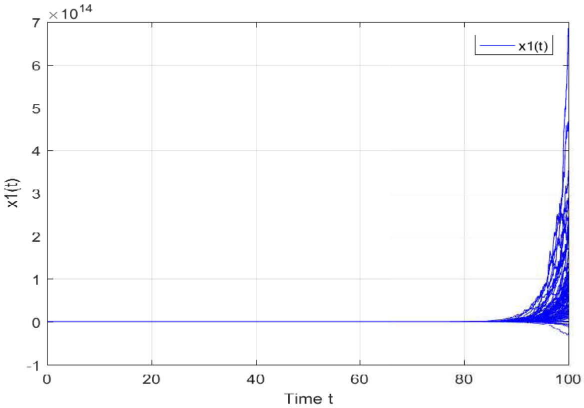

FIGURE 1. 100 sample path trajectories of $x_1(t)$ with $u(t) = 0$.

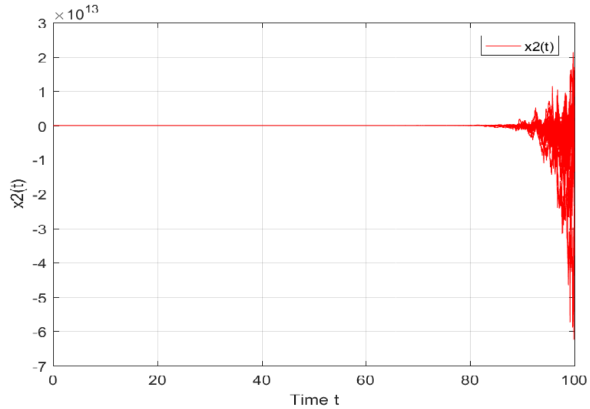

FIGURE 2. 100 sample path trajectories of $x_2(t)$ with $u(t) = 0$.

of Theorem 2 for (1) can be solved, and the corresponding simulations confirm the validity of our conclusions, both of their realization are presented by MATLAB.

Example 1: Consider (1) with $\tau = 0.1$ and with initial condition $x_1(\theta) = 5$, $x_2(\theta) = -1$, $\theta \in [-0.1, 0]$. The parameters of (1) are given as follows:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -0.3 & -1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.2 & -0.2 \\ 0.5 & -0.1 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} -0.1 & 0.1 \\ 0.2 & -0.1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.6 & -1 \\ 0.2 & -1.2 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} -0.6 & 0.2 \\ 1 & 0.8 \end{bmatrix}, \quad D_2 = \begin{bmatrix} -0.2 & 0.1 \\ -0.1 & -0.1 \end{bmatrix},$$
$$\sigma_1 = \begin{bmatrix} 0.1x_2(t-\tau)sin(x_1(t)) \\ 0.1x_2(t-\tau)cos(x_1(t)) \end{bmatrix},$$
$$\sigma_2 = \begin{bmatrix} 0.2x_1(t)cos(x_2(t-\tau)) \\ 0.2x_1(t)sin(x_2(t-\tau)) \end{bmatrix}, \quad \Gamma = \begin{bmatrix} -5 & 5 \\ 4 & -4 \end{bmatrix}.$$

First, we consider (1) with $u(t) = 0$, that is,

$$dx(t) = [A(r(t))x(t) + B(r(t))x(t-\tau)]dt + \sigma(t, x(t), x(t-\tau), r(t))dw(t). \quad (35)$$

We apply Euler-Maruyama method on (35) over [0, 100) with a stepsize $\delta t = 0.01$, the simulations of 100 sample path trajectories are drawn in Fig. 1 and Fig. 2. From Fig. 1 and Fig. 2, we can find that the system states of (35) are not quite in line with what we expect for practical stability.

Next, we take $\beta_1 = 1$, $\theta = 0.5$, $\alpha_1 = 0.1$ and $\alpha_2 = 0.01$. From $trace[\sigma^T(t, x(t), x(t-\tau), 1)\sigma(t, x(t), x(t-\tau), 1)] \leq 0.01|x(t-\tau)|^2$, and $trace[\sigma^T(t, x(t), x(t-\tau), 2)\sigma(t, x(t), x(t-\tau), 2)] \leq 0.04|x(t)|^2$, we get that $\gamma_{11} = 0$, $\gamma_{12} = 0.01$, $\gamma_{21} = 0.04$, $\gamma_{22} = 0$. According to Theorem 2, by using the Matlab toolbox, we can get the feasible solution:

$$X_1 = \begin{bmatrix} 1.0480 & -0.1826 \\ -0.1826 & 1.7089 \end{bmatrix}, \quad Y_1 = \begin{bmatrix} -7.4106 & -8.1549 \\ -15.3995 & -8.1207 \end{bmatrix},$$
$$X_2 = \begin{bmatrix} 1.0190 & 0.0849 \\ 0.0849 & 1.3892 \end{bmatrix}, \quad Y_2 = \begin{bmatrix} 2.8400 & 2.5949 \\ -2.2274 & 5.2257 \end{bmatrix},$$

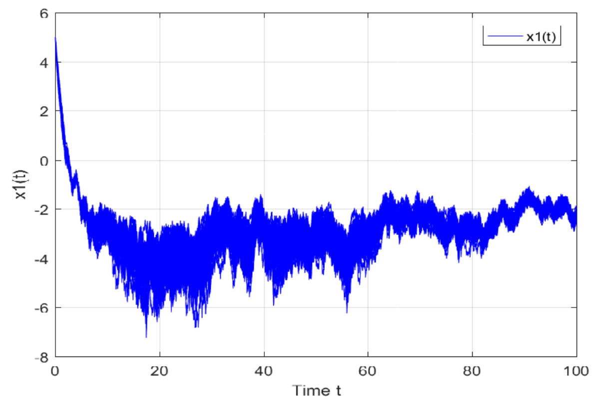

FIGURE 3. 100 sample path trajectories of $x_1(t)$ under ETC.

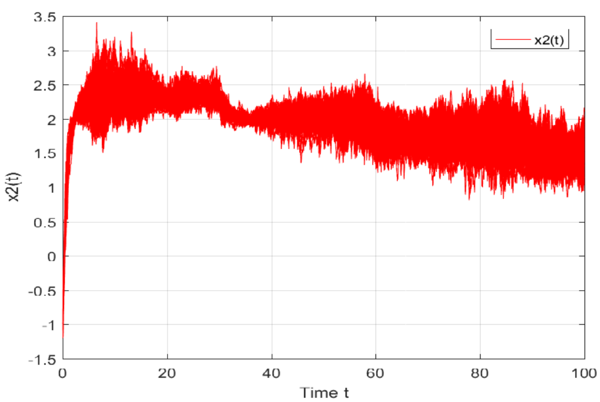

FIGURE 4. 100 sample path trajectories of $x_2(t)$ under ETC.

$$Q = \begin{bmatrix} 0.1752 & -0.3064 \\ -0.3064 & 1.3825 \end{bmatrix}, \quad R_1 = \begin{bmatrix} 0.2810 & 0.1323 \\ 0.1323 & 1.0133 \end{bmatrix},$$
$$R_2 = \begin{bmatrix} 1.7747 & -0.0071 \\ -0.0071 & 1.75420 \end{bmatrix}, \quad F_{11} = \begin{bmatrix} -0.8829 & 0.1079 \\ 0.1077 & -1.0031 \end{bmatrix},$$
$$F_{12} = \begin{bmatrix} 0.8829 & -0.1084 \\ -0.1076 & 1.0046 \end{bmatrix}, \quad F_{21} = \begin{bmatrix} -0.8827 & -0.0076 \\ -0.0076 & -0.8796 \end{bmatrix},$$
$$F_{22} = \begin{bmatrix} 0.8827 & 0.0076 \\ 0.0076 & 0.8796 \end{bmatrix}, \quad \beta_2 = 0.2195.$$

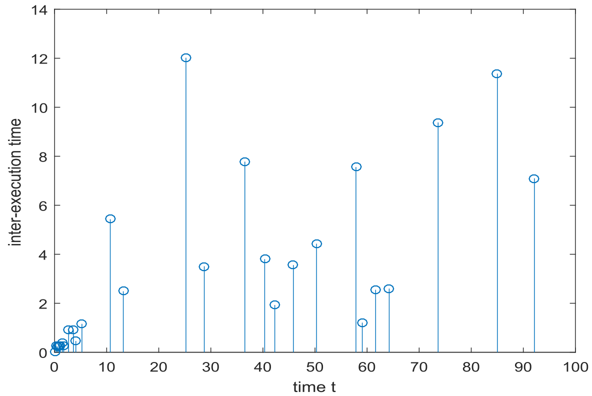

FIGURE 5. **Inter-execution times corresponding to the trajectories in Fig. 3 and Fig. 4.**

And the feedback gain matrices are as follows:

$$K_1 = Y_1 X_1^{-1} = \begin{bmatrix} -8.0530 & -5.6325 \\ -15.8174 & -6.4422 \end{bmatrix},$$

$$K_2 = Y_2 X_2^{-1} = \begin{bmatrix} 2.6450 & 1.7063 \\ -2.5120 & 3.9151 \end{bmatrix}.$$

Then we take the controller $u(t)$ into consideration and use Euler-Maruyama method, the 100 sample path trajectories are plotted in Fig. 3 and Fig. 4. Fig. 5 plots the inter-execution times corresponding to the trajectories in Fig. 3 and Fig. 4. We can find that the system states are well controlled, that is, the system is practically exponential stabilizable in mean square.

V. CONCLUSION

This article considered the stabilization for a class of STDSs with Markovian jump parameters. Based on introducing the related definitions of practically exponential stability, we utilize the stochastic version free-weighting matrix technique and a static event-triggered strategy to the closed-loop systems, then delay-dependent stability criterion has been set up in mean square sense by using a novel LKF, and the conditions about stabilization for STDSs have been well derived in terms of LMIs. In the sequel, considering the advantage of dynamic ETM may efficiently generate larger inter-execution times than static ETM (see, e.g., [20]), we expect to investigate the problems on the dynamic ETM of stochastic systems in the following studies.

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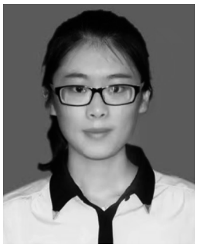

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