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Noniterative DOA Estimation Algorithms of Noncircular Signals in Nonuniform Noise Environment

YU QIAO [1], HAN WANG [2], (Member, IEEE), XIANPENG WANG [1], (Member, IEEE), MENGXING HUANG [1], LIANGTIAN WAN [3], (Member, IEEE), AND DI WU [1]

[1] State Key Laboratory of Marine Resource Utilization in South China Sea, School of Information and Communication Engineering, Hainan University, Haikou 570228, China
[2] Institute of Data Science, City University of Macau, Macau 999078, China
[3] Key Laboratory for Ubiquitous Network and Service Software of Liaoning Province, School of Software, Dalian University of Technology, Dalian 116620, China

Corresponding authors: Xianpeng Wang (wxpeng1986@126.com) and Mengxing Huang (huangmx09@163.com)

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ABSTRACT In this paper, two noniterative direction-of-arrival (DOA) estimation algorithms of noncircular signals in nonuniform noise environment are proposed. Different from the mainstream nonuniform iterative algorithm, the algorithms we proposed in this paper could attain DOA estimation effectively in nonuniform noise environment without iterative and convex optimization processing. In the direct removal of nonuniform noise (DRONN) method, the noise subspace is estimated by using special processing of the array output covariance matrix, moreover, it does not require to estimate the noise covariance matrix. On the other hand, the piecewise estimation of nonuniform noise (PEONN) method first estimates the noise covariance matrix, and the noise subspace used in this process is estimated by using the DRONN method, then the generalized eigendecomposition (GED) is used to estimate the noise covariance matrix. The above two proposed methods are able to suppress the interference of nonuniform noise effectively, and accurately estimate DOA without iterative processing. In addition, the two proposed methods use the reduced-dimensional noncircular multiple signal classification (RD-NC-MUSIC) algorithm to estimate DOA without complex two-dimensional spatial search, and they can effectively reduce the computational complexity. The effectiveness of the two proposed methods are proved via the simulation results.

INDEX TERMS Direction-of-arrival (DOA) estimation, noncircular signal, nonuniform noise, noniterative estimation.

I. INTRODUCTION

Direction-of-arrival (DOA) estimation has become an essential and indispensable branch of array signal processing [1]–[13], which is extensively applied in detections, underwater acoustics, wireless communications, locations, tracking [5] and assistant vehicle localizations [6]. With the rapid development of array signal processing, a number of classical DOA estimation algorithms have been proposed, such as parametric subspace-based method [7], sparse representation-based approaches [8] and beamforming-based algorithm [9]. Subspace-based algorithms are an important milestone of DOA estimation algorithm, which has been deeply studied by many scholars, and many innovative methods have been proposed, such as multiple signal classification (MUSIC) [10], estimation of signal parameters via rotational invariance techniques (ESPRIT) [11], Capon [12] and maximum likelihood (ML) [13]. In recent years, sparse signal recovery (SSR) techniques have developed rapidly, and a lot of SSR-based DOA estimation methods have been proposed. For example, a reweighted regularized sparse recovery algorithm has been proposed in [14], which optimizes the DOA

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estimation in the case of unknown mutual coupling. In addition, a robust weighted subspace fitting DOA estimation algorithm has been proposed in [15], which transforms the DOA estimation problem into a block sparse recovery problem and effectively avoids the problem of mutual coupling. These algorithms mentioned above assume that signal sources are circular signals, in which its corresponding elliptic covariance matrix tends to zero. However, for the noncircular signals, its elliptical covariance matrix o is not zero, which makes it possible to utilize of the information to achieve accurate DOA estimation.

Many complex noncircular signals are applied to practical communication and radar systems, such as BPSK, MSK, and UQPSK signals [16]. With further studying the property of noncircular signal, noncircular signals have occupied a more and more essential position in DOA estimation. The noncircular multiple signal classification (NC-MUSIC) algorithm has been proposed in [17], and it takes advantage of the noncircularity of the signals to expand the array aperture and improve the estimation accuracy. However, large-scale spectral peak search results in extremely high computational complexity of the algorithm. To address the problem, the noncircular root MUSIC (NC-Root-MUSIC) algorithm has been proposed in [18], which does not require a large-scale spectral peak search. As a result, the computational complexity is reduced remarkably. On the other hand. The reduced dimensional NC-MUSIC (RD-NC-MUSIC) algorithm has been proposed in [19], which can estimate DOA without two-dimensional search, thus has lower computational complexity. In recent years, many excellent algorithms have been proposed. An improved noncircular rotational invariance propagator method (NC-RI-PM) algorithm has been proposed in [20], which optimizes the insufficient angle estimation of NC-RI-PM algorithm, and realizes the automatic pairing between the elevation and noncircular phase. However, the algorithm still has high computational complexity. In [21], a noncircular signal angle estimation algorithm based on PM and Euler transform is proposed to further reduce the computational complexity. It uses Euler transform to convert complex numbers into real numbers, and reconstructs the output of the extended array. Then the computational complexity is greatly reduced, and the performance is similar to that of the improved NC-RI-PM algorithm. In addition, in order to address the issue of DOA estimation of noncircular signals in MIMO system, a combined spatial spectrum method has been proposed in [22]. However, all the methods introduced above only consider the uniform noise. In the environment of nonuniform noise, the above subspace-based algorithms cannot correctly separate the noise subspace from the signal subspace. Consequently, the DOA estimation algorithms are invalid.

Nonidentical sensor noise powers will lead to nonuniform noise, and the noise covariance matrix is with different diagonal elements [23], [24]. In recent years, in order to address the problem of DOA estimation in nonuniform noise environment, a variety of methods have been proposed [23]–[28]. A deterministic ML estimator has been proposed in [23], and its implementation is based on iterative process. A stochastic ML estimator has been proposed in [24], which used a similar concept of stepwise concentration and iterative process. It enriched the understanding of nonuniform ML estimator, and expanded the scope of application of the estimator in [23]. However, due to the complexity of the iterative process, the two algorithms are hard to use in hardware implementation. Then in order to reduce the computational complexity, some optimized iterative algorithms are proposed, such as the algorithms proposed in [25], [26], then the computational complexity is further reduced. Based on the principle of least square (LS) minimization of signal subspace and noise covariance matrix, a simple iterative process has been proposed in [25]. It converges in a few iterations, and the closed estimates of signal subspace and noise covariance matrix could be obtained in each iteration. Two iterative subspace estimation methods with low complexity have been proposed in [26], which include iterative maximum likelihood subspace estimation (IMLSE) and iterative least squares subspace estimation (ILSSE). In the procedure of two algorithm, the signal subspace and noise covariance matrix could be calculated in closed form in each iteration. Compared with the traditional iterative algorithms, they greatly reduce the computational complexity through closed form calculation. However, the high computational complexity generated by the iterative process has not been fundamentally solved. Then a unitary matrix completion (UMC) method has been proposed in [27], this method uses convex optimization to estimate DOA. Its computational complexity is better than that of iterative algorithm [26], but it is still on the high side. In order to further reduce the computational complexity, a noniterative subspace-based DOA estimation method has been proposed in [28]. The algorithm removes the nonuniform noise through two stages. The first step is to estimate the noise subspace through the GED of the appropriately designed matrix, and the estimation is completed without knowing the noise covariance matrix. In the next step, we use the result of the first stage to calculate the noise covariance matrix appropriately, then the noise subspace is estimated by GED of the output array covariance matrix and the noise covariance matrix. The algorithm has no iterative processes and greatly reduces the computational complexity. However, all of these algorithms mentioned above have not sufficiently considered the noncircularity of noncircular signal.

In this paper, in order to address the issue of DOA estimation of noncircular signals in nonuniform noise, two noniterative DOA estimation methods are proposed. The proposed methods do not need iterative and convex optimization processes to achieve DOA estimation. In the DRONN method, according to the noncircularity of the signal, we use the received data matrix and its conjugation to extend the received data matrix, and then the noise subspace is estimated by removing the diagonal elements of the covariance matrix. It can remove the effect of nonuniform noise on covariance, but also lose some signal data. The PEONN method applies

GED to the output covariance matrix and noise covariance matrix for estimating the accurate noise covariance matrix, then more accurate noise subspace is achieved. Two proposed methods not only enlarge the virtual array aperture, but also they do not need iterative and convex optimization processes. The simulation results are used to verify that the proposed methods have better performance and lower complexity over the existing methods.

The structure of this paper is shown as follows. The noncircular signal model in nonuniform noise is manifested in Section II. The DRONN method and PEONN method are proposed in Section III. The simulation results are introduced and analyzed in Section IV. The conclusion is manifested in Section V.

Notation: $E\{.\}$ denotes the mathematical expectation. $(.)^T$, $(.)^H$, $(.)^{-1}$ and $(.)^*$ denote the transpose, conjugate-transpose, inverse and conjugate. $diag\{.\}$ denotes the diagonal matrix. $|.|$ denotes the absolute value operator. $\mathbf{I}_K$ denotes a $K \times K$ dimensional unit matrix.

II. SIGNAL MODEL

In this paper, we assume that there are M sensors to receive $L(L < M)$ noncircular signals from the far field, and the span between contiguous sensors is set to be one-half of the wavelength. The signal observed by the array at time t could be expressed as [23]

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{x}(t)$ is the received data vector, $\mathbf{A} = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_L)] \in \mathbb{C}^{M\times L}$ is the manifold matrix, and its column are manifold vector $\mathbf{a}(\theta_i) = [1, e^{j\pi \sin\theta_i}, \cdots, e^{j\pi(M-1)\sin\theta_i}]^T\, i = 1, \cdots, L$, $\mathbf{s}(t) \in \mathbb{C}^{L\times 1}$ is the signal vector and $\mathbf{n}(t) \in \mathbb{C}^{M\times 1}$ is the nonuniform Gaussian noise vector. It is worth mentioning that the noncircular signal vector satisfies with

$$\mathbf{s}(t) = \mathbf{\Phi} \boldsymbol{s}_R(t) \tag{2}$$

where $\mathbf{\Phi} = diag\left\{e^{-j\varphi_1}, e^{-j\varphi_2}, \cdots, e^{-j\varphi_L}\right\} \in \mathbb{C}^{L\times L}$ is the diagonal matrix containing the noncircularity phase $\boldsymbol{\varphi} = [\varphi_1, \varphi_2 \cdots \varphi_L]$, and $\mathbf{s}_R(t) \in \mathbb{R}^{L\times 1}$ is the real part of noncircular signal, then the received data in (1) can be rewritten as

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{N} \tag{3}$$

where $\mathbf{X} = [\mathbf{x}(t_1), \cdots, \mathbf{x}(t_N)] \in \mathbb{C}^{M\times N}$ is the received data matrix, $\mathbf{S} = \mathbf{\Phi}\mathbf{S}_R \in \mathbb{C}^{L\times N}$ is the noncircular signal matrix, where N is the number of snapshots, and $\mathbf{S}_R = [\mathbf{s}_R(t_1), \cdots, \mathbf{s}_R(t_N)] \in \mathbb{C}^{L\times N}$, $\mathbf{N} = [\mathbf{n}(t_1), \cdots, \mathbf{n}(t_N)] \in \mathbb{C}^{M\times N}$ is the nonuniform Gaussian noise matrix. Noncircular signals have real components and their conjugations are equal to themselves. Utilizing this property, we use the received data matrix $\mathbf{X}$ and its conjugation $\mathbf{X}^*$ to reconstruct the received data vector as

$$\mathbf{Z} = \begin{bmatrix} \mathbf{X} \\ \mathbf{X}^* \end{bmatrix} = \begin{bmatrix} \mathbf{A}\mathbf{\Phi} \\ \mathbf{A}^*\mathbf{\Phi}^* \end{bmatrix} \mathbf{S}_R + \begin{bmatrix} \mathbf{N} \\ \mathbf{N}^* \end{bmatrix} \tag{4}$$

The output covariance matrix is

$$\mathbf{R} = E\{\mathbf{Z}\mathbf{Z}^H\} = \begin{bmatrix} \mathbf{A}\mathbf{\Phi} \\ \mathbf{A}^*\mathbf{\Phi}^* \end{bmatrix} \mathbf{P} \begin{bmatrix} \mathbf{A}\mathbf{\Phi} \\ \mathbf{A}^*\mathbf{\Phi}^* \end{bmatrix}^H + \mathbf{Q} \tag{5}$$

where $\mathbf{R} \in \mathbb{C}^{2M\times 2M}$ and $\mathbf{P} = E\{\mathbf{S}_R\mathbf{S}_R^H\} \in \mathbb{C}^{L\times L}$ is the covariance matrix of the real-valued signal $\mathbf{S}_R$. The signal model in this paper uses the property of noncircular signals to expand the data model of array aperture doubling, then the accuracy and performance of DOA could be greatly improved. $\mathbf{Q} = E\left\{\begin{bmatrix} \mathbf{N} \\ \mathbf{N}^* \end{bmatrix}\begin{bmatrix} \mathbf{N} \\ \mathbf{N}^* \end{bmatrix}^H\right\} = diag\left\{\sigma_1^2, \cdots, \sigma_M^2, \sigma_1^2, \cdots, \sigma_M^2\right\} \in \mathbb{C}^{2M\times 2M}$ is the noise covariance matrix, where $\sigma_1^2, \cdots, \sigma_M^2$ are different noise powers of the sensors. Different from the covariance matrix of uniform noise, the diagonal elements of the covariance matrix of nonuniform noise are different, which greatly increases the difficulty of removing noise interference. In order to facilitate calculation, we assume $\mathbf{B} = \begin{bmatrix} \mathbf{A}\mathbf{\Phi} \\ \mathbf{A}^*\mathbf{\Phi}^* \end{bmatrix} \in \mathbb{C}^{\mathbf{2M}\times\mathbf{L}}$, then (5) can be written as

$$\mathbf{R} = \mathbf{B}\mathbf{P}\mathbf{B}^H + \mathbf{Q} \tag{6}$$

III. NEW PROPOSED ALGORITHMS

In this part, the DRONN algorithm is introduced in Section A, then the PEONN algorithm is introduced in Section B. The RD-NC-MUSIC for DOA estimation is introduced in Section C and the summary is given in Section D.

A. THE DRONN ALGORITHM

Due to the interference of nonuniform noise, the signal subspace and noise subspace cannot be separated correctly. If the nonuniform noise interference is removed, the noise subspace can be estimated. Based on the principle of removing the influence of nonuniform noise in [28], we extend it to eliminate the interference of nonuniform noise in noncircular signal model. First, the output data covariance matrix is rewritten as

$$\mathbf{R} = \mathbf{R}_1 + \mathbf{R}_2 \tag{7}$$

where

$$[\mathbf{R}_1]_{i,\mathbf{k}} = \begin{cases} [\mathbf{R}]_{i,\mathbf{k}}, & i \neq \mathbf{k} \\ 0, & i = \mathbf{k} \end{cases} \tag{8}$$

and

$$\begin{aligned} \mathbf{R}_2 &= diag\left\{[\mathbf{R}]_{1,1}, \cdots [\mathbf{R}]_{M,M}, [\mathbf{R}]_{1,1}, \cdots [\mathbf{R}]_{M,M}\right\} \\ &= diag\left\{\sum_{k=1}^{L} s_k + \sigma_1^2, \cdots \sum_{k=1}^{L} s_k + \sigma_M^2, \sum_{k=1}^{L} s_k + \sigma_1^2\right. \\ &\qquad \left., \cdots \sum_{k=1}^{L} s_k + \sigma_M^2\right\} \end{aligned} \tag{9}$$

where s_k is the received power of the k th source, because $\mathbf{B} = \begin{bmatrix} \mathbf{A}\mathbf{\Phi} \\ \mathbf{A}^*\mathbf{\Phi}^* \end{bmatrix}$ is an $2M \times L$ matrix, and it is full rank, and there are $2M-L$ orthonormal vectors $\mathbf{u}_l, l = 1, \cdots, 2M-L$ satisfying the following homogeneous equation

$$\mathbf{B}^H \mathbf{u}_l = \mathbf{0} \tag{10}$$

where $\mathbf{0}$ is a zero vector. Applying this homogeneous equation to (6), and multiplying $\mathbf{u}_l$ to both sides on the right, we can obtain

$$\mathbf{R}\mathbf{u}_l = \mathbf{B}\mathbf{P}\mathbf{B}^H \mathbf{u}_l + \mathbf{Q}\mathbf{u}_l = \mathbf{Q}\mathbf{u}_l \quad l = 1, \cdots, 2M-L \tag{11}$$

substituting (7), (8), (9) and (10) into (11), we have

$$\mathbf{R}_1\mathbf{u}_l = (\mathbf{Q}-\mathbf{R}_2)\mathbf{u}_l = -\left(\sum_{k=1}^{L} s_k\right)\mathbf{u}_l \tag{12}$$

then we assume that there is an $2M \times 1$ vector $\mathbf{d}$ and a constant λ, and it satisfies the following conditions

$$\mathbf{B}\mathbf{P}\mathbf{B}^H \mathbf{d} \neq 0, \mathbf{R}_1\mathbf{d} = \lambda\mathbf{d} \tag{13}$$

then we obtain

$$\begin{aligned} \mathbf{R}\mathbf{d} &= \mathbf{R}_1\mathbf{d} + \mathbf{R}_2\mathbf{d} \\ &= \lambda\mathbf{d} + \mathbf{R}_2\mathbf{d} \end{aligned} \tag{14}$$

then substituting (6) into (14), we could obtain

$$\begin{aligned} \mathbf{B}\mathbf{P}\mathbf{B}^H \mathbf{d} &= \lambda\mathbf{d} + (\mathbf{R}_2 - \mathbf{Q})\,\mathbf{d} \\ &= \left(\lambda + \sum_{k=1}^{L} s_k\right)\mathbf{d} \end{aligned} \tag{15}$$

as shown in (15), $\mathbf{d}$ is proved to be the eigenvector of $\mathbf{B}\mathbf{P}\mathbf{B}^H$, and its corresponding eigenvalues are $\lambda + \sum_{k=1}^{L} s_k$. In order to satisfy the condition (13), the matrix $\mathbf{B}\mathbf{P}\mathbf{B}^H$ has L positive eigenvalues, we obtain

$$\lambda + \sum_{k=1}^{L} s_k > 0 \Rightarrow \lambda > -\sum_{k=1}^{L} s_k \tag{16}$$

which indicates that the eigenvalues corresponding to $\mathbf{u}_l$, $l = 1, \cdots, 2M-L$ are the lower bound on the smallest eigenvalues. By using (12), the vectors $\mathbf{u}_l, l = 1, \cdots, 2M-L$ is composed of the noise subspace. Then the noise subspace is achieved by applying eigendecomposition to $\mathbf{R}_1$, which is without the interferences of nonuniform noise.

B. THE PEONN ALGORITHM

The purpose of DRONN algorithm is to estimate the noise subspace without estimating the noise covariance matrix. Although the interference of nonuniform noise is removed, due to the direct removal of the diagonal elements of $\mathbf{R}$, part of the signal data information are lost. In this part, referring to [28], we extend the idea to estimate noise covariance matrix in noncircular signal model. First, the noise covariance matrix $\mathbf{Q}$ could be rewritten as

$$\mathbf{Q} = \sigma^2\mathbf{I} + \mathbf{Q}_{num} \tag{17}$$

where σ^2 is the common part of sensor noise power. $\mathbf{Q}_{num}$ is a diagonal matrix and the rank of $\mathbf{Q}_{num}$ is $2M-1$, and the position in $\mathbf{Q}_{num}$ corresponding to the smallest diagonal element of $\mathbf{R}$ is 0. Then $\mathbf{Q}_{num}$ is estimated by using the minimum diagonal element of $\mathbf{R}$, we obtain

$$\mathbf{Q}_{num} = diag\left\{[\mathbf{R}]_{1,1} - c, \cdots, [\mathbf{R}]_{M,M} - c\right\} \tag{18}$$

where c is equal to the smallest diagonal element of the output covariance matrix, which is expressed as c^1. The position of c^1 in $\mathbf{R}$ is k. Then we assume there is a $2M \times 1$ unit vector $\mathbf{e}_k$

$$[\mathbf{e}_k]_i = \begin{cases} 0, & i \neq k \\ 1, & i = k \end{cases} \tag{19}$$

where $\mathbf{e}_k^T\mathbf{Q}_{nun} = \mathbf{0}$. Then multiplying $\mathbf{e}_k^T$ to both sides of (11) on the left. By using (17), and expressing $\mathbf{u}_l, l = 1, \cdots, 2M-L$ in the form of a matrix as $\mathbf{U}$, we obtain

$$\begin{aligned} \mathbf{e}_k^T\mathbf{R}\mathbf{U} &= \mathbf{e}_k^T(\sigma^2\mathbf{I} + \mathbf{Q}_{num})\mathbf{U} \\ &= \sigma^2\mathbf{e}_k^T\mathbf{U} \end{aligned} \tag{20}$$

by using the results of the DRONN method, σ^2 can be computed as

$$\sigma^2 = \left|\frac{\mathbf{e}_k^T\mathbf{R}\mathbf{U}\mathbf{U}^H\mathbf{e}_k}{\mathbf{e}_k^T\mathbf{U}\mathbf{U}^H\mathbf{e}_k}\right| \tag{21}$$

According to [25], it has been shown that the noise subspace could be obtained by applying GED to $\mathbf{R}$ and $\mathbf{Q}$. Then using (18), (19) and (21), a more accurate $\mathbf{Q}$ can be estimated.

C. REDUCED-DIMENSIONAL NC-MUSIC

In this part, we use one-dimensional search to estimate the DOA. Different from the NC-MUSIC algorithm and 2D-NC-MUSIC algorithm commonly used in noncircular signal DOA. RD-NC-MUSIC [19] only needs one-dimensional search, and it does not need to estimate the noncircular phase, which makes it achieve lower computational complexity, it can be expressed as

$$P_{RD-NC-MUSIC}(\theta) = \mathbf{e}^H\left(\mathbf{C}^H\mathbf{U}_N\mathbf{U}_N^H\mathbf{C}\right)^{-1}\mathbf{e} \tag{22}$$

where $\mathbf{C} = \begin{bmatrix} \mathbf{a} & \mathbf{0}_{M\times1} \\ \mathbf{0}_{M\times1} & \mathbf{a} \end{bmatrix}$ and $\mathbf{e} = \begin{bmatrix} e^{-j\varphi} \\ e^{j\varphi} \end{bmatrix}$. As shown in [19], RD-NC-MUSIC algorithm has excellent performance.

D. SUMMARY

The DRONN algorithm is summarized in TABLE 1 and the PEONN algorithm is summarized in TABLE 2

IV. SIMULATION RESULTS

In this section, we have carried out all sorts of simulation trails to evaluate the performance of the proposed methods in this paper. The proposed methods are compared with the NC-MUSIC method [17], the UMC method [27], and the method in [28], and referring to [28], we re-derived

TABLE 1. The key steps of the dronn method.

Algorithm: The DRONN method	
1:	Compute the output covariance matrix $\mathbf{R}$.
2:	Splitting $\mathbf{R}$ into $\mathbf{R}_1$ and $\mathbf{R}_2$
3:	Using $\mathbf{R}_1$ instead of $\mathbf{R}$ to calculate the noise subspace
4:	Using RD-NC-MUSIC to estimate the DOA

TABLE 2. The key steps of the peonn method.

Algorithm: The PEONN method	
1:	Splitting $\mathbf{Q}$ into $\sigma^2\mathbf{I}$ and $\mathbf{Q}_{num}$
2:	Construct matrix $\mathbf{e}_k$, using (21) to calculate the σ^2
3:	Using the smallest diagonal element of $\mathbf{R}$ and using (18) to calculate the $\mathbf{Q}_{num}$
4:	Using $\sigma^2\mathbf{I}$ and $\mathbf{Q}_{num}$ to calculate the $\mathbf{Q}$, and applying GED to $\mathbf{R}$ and $\mathbf{Q}$ to calculate the noise subspace
5:	Using RD-NC-MUSIC to estimate the DOA

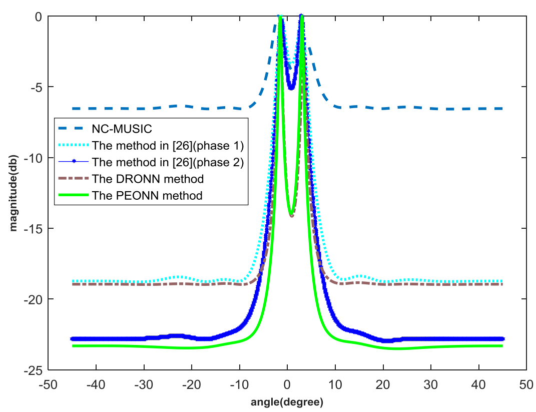

FIGURE 1. The Spatial spectrum of comparative methods and proposed methods ($M = 8, \theta_1 = -3, \theta_2 = 6$, SNR $= -3$, N $= 400$).

the Cramer-Rao bound (CRB) of noncircular sources in nonuniform noise. We assume that sensors number of ULA is $M = 8$, and the desired DOAs are $\theta_1 = -3$ and $\theta_2 = 6$. Then the root mean square error (RMSE) is utilized to evaluate the performance of the algorithm, and in this paper, its form is defined as

$$RMSE = \frac{1}{K}\sum_{k=1}^{K}\sqrt{\frac{1}{100}\sum_{i=1}^{100}\left(\tilde{\theta}_{i,k}-\theta_k\right)^2} \tag{23}$$

where $\tilde{\theta}_{i,k}$ is the ith Monte Carlo trial of the kth signal elevation θ_k estimated value. In these simulations, we choose 100 as the number of the Monte Carlo trials for both comparative methods and proposed methods.

Fig. 1 shows the spatial spectrum of comparative methods and proposed methods. From Fig.1, it can be observed that the reference methods have poor performance in noncircular signal model and nonuniform noise environment, because the

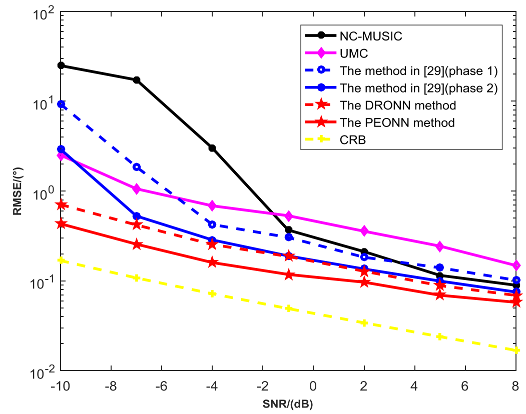

FIGURE 2. The RMSE versus SNRs for comparative methods and proposed methods ($M = 8$, $N = 400$).

algorithm does not take advantage of the noncircularity of the signal. The NC-MUSIC method cannot correctly estimate noise subspace in nonuniform noise environment; thus it is unable to produce two distinct peaks. The proposed methods improve the accuracy of angle estimation by taking advantage of the noncircularity of signal, and solve the problem of subspace estimation in nonuniform noise environment. Thus the proposed methods have much sharper peaks and lower sidelobe than comparative methods.

Fig. 2 shows the RMSE versus SNRs with comparative methods and proposed methods and the CRB, where the number of snapshot N is selected as 400, and the SNR increases from -10dB to 8dB, and each step is 3dB. Fig. 2 depicts that with the increasing of the SNR, the RMSE of all methods decreases and the proposed methods are closer to the CRB in the whole SNR range. The proposed methods have lower RMSE in the whole SNR range than the UMC method, the NC-MUSIC method and the method in [28]. The main reason is that the methods that we proposed exploit the noncircular signal property and remove the interferences of nonuniform noise. In addition, when the SNR>-1dB, the performance of NC-MUSIC method is better than that of the UMC method. When the SNR>4db, the performance of NC-MUSIC method is better than that of the method in [28]. The main reason is that NC-MUSIC method has poor interference suppression to noise. With the increasing of SNR and the weakening of noise interference, the performance of NC-MUSIC method becomes better, UMC method is an ESPRIT based method, and the performance of ESPRIT based method is not as good as that of MUSIC class method in the case of high SNR.

Fig. 3 shows the probability of successful detection (PSD) versus the SNR with comparative methods and proposed methods. The PSD could be expressed by $\left|\theta_k - \hat{\theta}_k\right| < 0.5^\circ$, where θ_k is the expected DOA, and $\hat{\theta}$ is the estimated DOA. Then N is selected as 400, the SNR increases from -10dB to 8dB, and each step is 3dB. Fig. 3 depicts that with the increasing of the SNR, the PSD increases, it indicates that the estimation performance of both comparative methods and proposed methods tend to be ideal. It is proved from Fig. 3

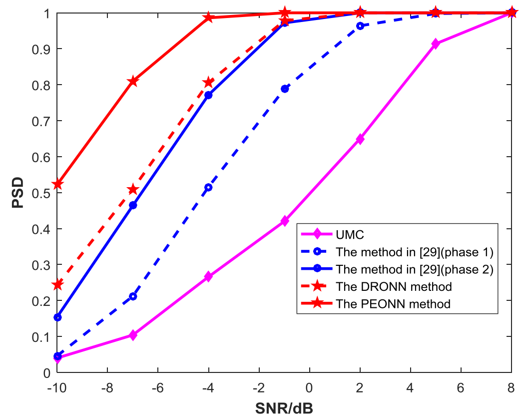

FIGURE 3. The PSD versus SNRs for comparative methods and proposed methods ($N = 400$).

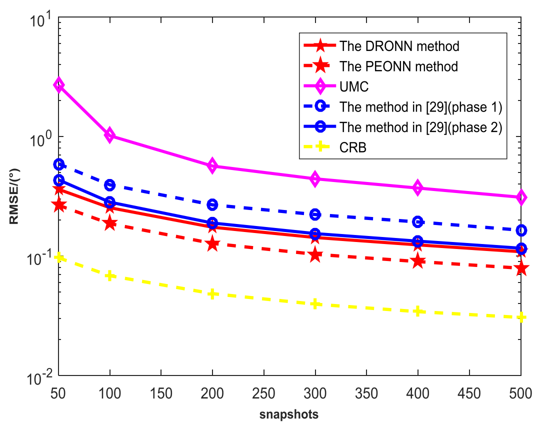

FIGURE 4. The RMSE versus the number of snapshots for comparative methods and proposed methods ($M = 8$, SNR $= -3$dB).

that the proposed methods will obtain 100% PSD at a low SNR, which proves that the proposed methods are excellent in suppressing noise interference.

Fig. 4 draws the RMSE versus the number of snapshots with comparative methods and proposed methods and the CRB. The SNR is selected as -3dB, and N increases from 50 to 1100, and each step is 100. Fig. 4 depicts that with the increasing of the number of snapshots, the RMSE decreases and the proposed methods have lower RMSE in the whole selected snapshots range. It proves that the proposed methods have obvious advantages over other comparison methods in estimation accuracy and performance, whether in small or large number of snapshots, which is caused by exploiting the extended array aperture.

Fig. 5 shows the PSD versus the number of snapshots. The SNR is selected as -3dB, and N increases from 50 to 1100, and each step is 100. Fig. 5 depicts that with the increasing of the number of snapshots, the PSD increases. Since the proposed methods extended array aperture, the PSD of the proposed methods will achieve 100% at a low snapshot.

Fig. 6 draws the RMSE versus WNPRs with comparative methods and proposed methods, where the $WNPR = \sigma_{\max}^2/\sigma_{\min}^2$ is the worst noise power ratios, and $\sigma_{\max}^2$ denotes

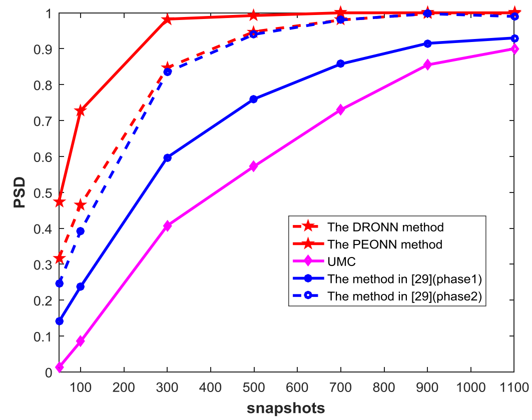

FIGURE 5. The PSD versus the number of snapshots for comparative methods and proposed methods (SNR $= -3$dB).

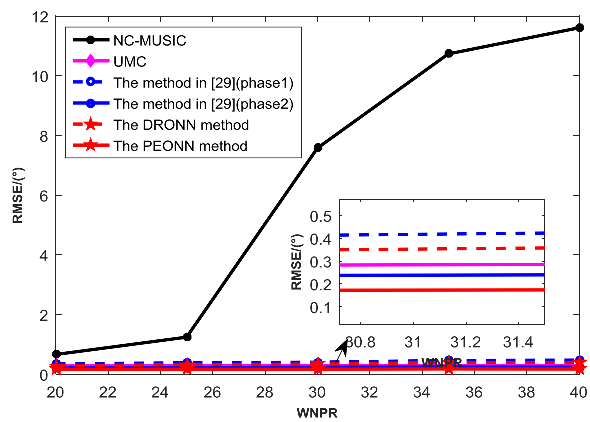

FIGURE 6. The RMSE versus WNPRs for comparative methods and proposed methods (SNR $= -3$dB, N $= 400$).

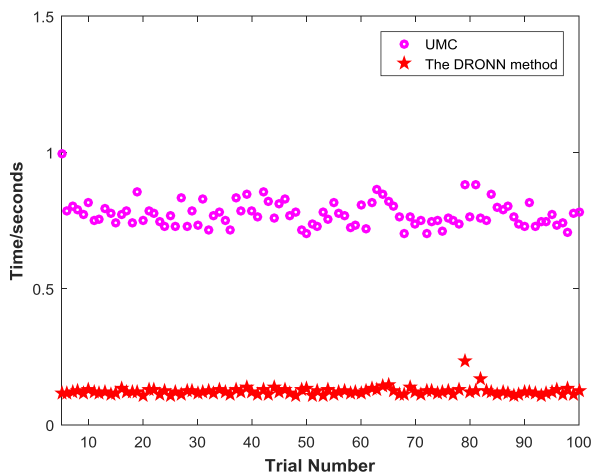

FIGURE 7. Simulation time versus Trial Number comparative methods and proposed methods (SNR $= -3$dB, N $= 400$).

the maximal noise power and $\sigma_{\min}^2$ denotes the minimal noise power. In this part, the WNPR is ranged from 20 to 40, and the influence of nonuniform noise could be mitigated by the UMC method, the method in [28] and the proposed methods. We can see that the performance of the DRONN method is not as good as that of the UMC method and the method in [28].

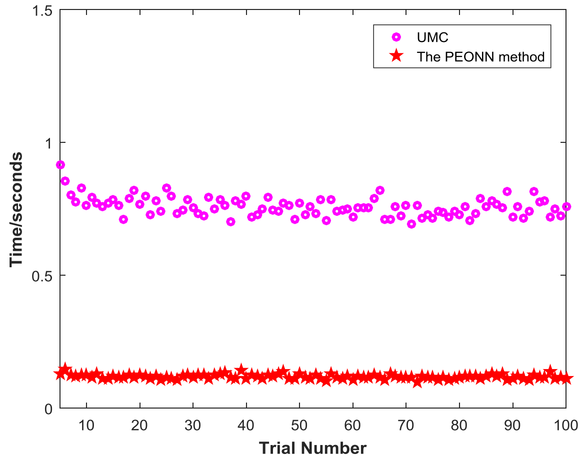

FIGURE 8. Simulation time versus Trial Number for comparative methods and proposed methods (SNR = −3dB, N = 400).

This is because that the DRONN method will lose part of the signal data while removing the interference of nonuniform noise. It is shown that the performance of the PEONN method is better than that of comparison methods, which proves the PEONN method has excellent ability to remove nonuniform noise.

Fig. 7 and Fig. 8 show the simulation time versus Trial Number with the proposed methods and the UMC method. It is seen that the proposed methods have lower computational complexity. In order to expand the array aperture, the proposed methods require a certain computational complexity, but without iterative and convex optimization processes. The proposed methods can achieve DOA estimation in a shorter time.

V. CONCLUSION

In this paper, two DOA algorithms for noncircular signals are proposed in nonuniform noise environment. The DRONN method directly removes the diagonal elements of the data covariance matrix including nonuniform noise interference, and then calculates the noise subspace by GED. This method does not need to estimate the nonuniform covariance matrix. The PEONN method estimates the nonuniform covariance matrix first, and then the noise subspace is solved by GED of the output array covariance matrix and the noise covariance matrix. Both of them are noniterative methods, which lead to lower computational complexity. Simulation results have verified that the methods we proposed have better performance than the existing methods.

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YU QIAO was born in 1995. He received the B.E. degree from Hainan University, Haikou, China, in 2018, where he is currently pursuing the M.S. degree in electronics and communication engineering. His research interests include array signal processing and noncircular sources.

HAN WANG (Member, IEEE) received the B.S. degree in electrical engineering from the Hubei University of Nationalities, China, in 2009, and the M.S. and Ph.D. degrees in information and communication system from Hainan University, Haikou, China, in 2013 and 2017, respectively. He was with China Mobile Jiangxi Branch, as a Network Engineer, for a period of one year. He is currently an Associate Professor with Yichun University. He is also a Postdoctoral Research Fellow with the City University of Macau. He is the author of over 20 papers published in related international conference proceedings and journals. His current research interests include intelligent communication, filter-bank multicarrier communications, and information theory.

XIANPENG WANG (Member, IEEE) was born in 1986. He received the M.S. and Ph.D. degrees from the College of Automation, Harbin Engineering University (HEU), Harbin, China, in 2012 and 2015, respectively. He was a Full-Time Research Fellow with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore, from 2015 to 2016. He is currently a Professor with the School of Information and Communication Engineering, Hainan University. He is the author of over 80 papers published in related journals and international conference proceedings. His major research interests include communication systems, array signal processing, radar signal processing, and compressed sensing and its applications. He served as a Reviewer for over 20 journals.

MENGXING HUANG received the Ph.D. degree from Northwestern Polytechnical University, in 2007. He joined the Research Institute of Information Technology, Tsinghua University, as a Postdoctoral Researcher (Staff). In 2009, he joined Hainan University. He is currently a Professor, a Ph.D. Supervisor of computer science and technology, and the Dean of the School of Information and Communication Engineering. He is also the Executive Vice-President of the Hainan Province Institute of Smart City and the Leader with the Service Science and Technology Team, Hainan University. He has published more than 60 academic articles as the first or corresponding author. He has reported 12 patents of invention and holds three software copyright. He has published two monographs and two translations. He received the Second Class and the Third Class Prizes from the Hainan Provincial Scientific and Technological Progress. His current research interests include signal processing for sensor systems, big data, and intelligent information processing.

LIANGTIAN WAN (Member, IEEE) received the B.S. and Ph.D. degrees from the College of Information and Communication Engineering, Harbin Engineering University, Harbin, China, in 2011 and 2015, respectively. From October 2015 to April 2017, he was a Research Fellow with the School of Electrical and Electrical Engineering, Nanyang Technological University, Singapore. He is currently an Associate Professor with the School of Software, Dalian University of Technology, China. He has published over 40 scientific papers in international journals and conferences. His research interests include social network analysis and mining, big data, array signal processing, wireless sensor networks, and compressive sensing and its application. He serves as an Associate Editor for IEEE ACCESS.

DI WU was born in 1991. He received the M.S. degree in information and communication engineering from Hainan University, Haikou, China. He is currently a Research Assistant with the School of Information and Communication Engineering, Hainan University. He is the author of over ten papers published in related journals and international conference proceedings. His major research interests include artificial intelligence and information processing.