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Multi-Model Method Decentralized Adaptive Control for a Class of Discrete-Time Multi-Agent Systems

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ABSTRACT This paper studies the decentralized adaptive tracking control problem for a class of discrete-time multi-agent systems with unknown parameters and high-frequency gains using multi-model method. Each agent is strong coupling with its neighbors by the historical outputs. All agents are interacted either directly or indirectly. In the face of uncertainties, the projection algorithm as a normal adaptive method is adopted. In order to improve quality of identification, the multi-model method is taken to identify unknown parameters and high-frequency gains using switching sets of the multiple parameters' and high-frequency gains' estimates, and the index switching functions. Using the certainty equivalence principle, the control law for the hidden leader agent is designed by the desired reference signal; the control law for each follower agent is devised by neighbors' historical outputs. Moreover, the proposed decentralized adaptive control laws can guarantee the following performances of the system: (1) the leader agent tracks the reference trajectory and each follower agent follows the average value of its neighborhood historical outputs; (2) the synchronization of all the follower agents to the leader agent is achieved; (3) all the agents track the reference trajectory, and the closed-loop system eventually achieves strong synchronization. Finally, simulations validate the effectiveness on improving control performance of multi-model adaptive algorithm by comparing with the projection algorithm.

INDEX TERMS Adaptive tracking control, multi-agent system, multi-model method, discrete-time system.

I. INTRODUCTION

During the past decades, the control of multi-agent systems (MASs) has attracted extensive attention due to its potential applications in many fields, such as unmanned ground/air vehicles [1], multiple spacecrafts [2], sensor networks [3] and so on. The tracking control for MASs is a common control problem, which has been widely studied [4]–[10]. Specifically, the adaptive tracking control for MASs has been investigated as one of paradigms to deal with some uncertainties [11]–[14].

Generally speaking, the continuous- or discrete-time multi-agent system under consideration involves structured uncertainty, parametric uncertainty, input uncertainty, environmental uncertainty and so on. A variety of new treatment methods have emerged. In this paper [15], a type of multi-agent system with time-delay, uncertainties and linear feedback is adopted, and the LMI approach is taken to guarantee the robust stability of the sliding surface. The consensus problem of multi-agent systems with uncertainties and randomly occurring nonlinearities is investigated in [16], which designs an effective impulsive control protocol, and obtains sufficient conditions to ensure consensus of multi-agent systems based on the Lyapunov stability theory and hybrid control theory. The work [17] develops a robust control method for formation maneuvers of a leader-follower multi-agent system with unknown bounded uncertainties, and uses the technique of nonlinear disturbance observer to overcome the adverse effects of the uncertainties.

When a multi-agent system independently faces to unknown parameters, the researchers have presented many estimation algorithms such as the gradient algorithm [18], the maximum likelihood algorithm [19], the least-squares algorithm [20], the backstepping approach [21], the observer

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method and sliding mode technique [22] and so on. The studies mentioned above use single-model method. Multi-model method is to track targets by combining different models. To some extent, multi-model method can improve tracking results to adapt to different target maneuvers. At present, there are few research in the control of multi-agent systems using multi-model method.

The strategies for formation tracking control can be classified into centralized strategy and decentralized strategy. They all have their own merits and flaws. For instance, centralized control strategy is easier to realize but have flaws of leader failure. Decentralized control strategy can readuce the communication data and improve the robustness. However, decentralized control strategy is difficult to make mathematical analysis and control design. [13], [23] study the multi-agent systems with uncertainties, and a projection-type algorithm is proposed to identify system, and then the decentralized control strategies are designed. Reference [24] uses I/O data and neural network to identify unknown dynamics, an approximate model is established by the direct data-driven method, the decentralized adaptive control is designed.

Motivated by the above observations, this paper addresses the decentralized adaptive tracking control problem for a class of discrete-time multi-agent systems with unknown parameters and high-frequency gains. Agent dynamics are described by the discrete-time nonlinearly parameterized models. This work extends results presented in [23] using the projection-type algorithm to identify system. This paper is to study the tracking control of the multi-agent system using not only involves the projection-type algorithm but also uses multi-model method to identify unknown parameters and high-frequency gains. The control performance of multi-agent system is improved using multi-model method by comparing with the projection algorithm. Due to the complex dynamics of the system, there are significant challenges involved in the problem on the control of the multi-agent systems with uncertainties in both algorithm design and theoretic analysis. The minimal value theorem, convergence criterion of the positive series, limit and set knowledge and so on. To address such challenges, in this paper, a discrete-time MAS with unknown parameters and high-frequency gains is investigated and the contributions are highlighted as follows: (1) for the discrete-time MAS with the parameter uncertainties and completely unknown high-frequency gains, the unknown internal parameters and unknown high-frequency gains are dealt with by the projection-type parameter estimation algorithm and multi-model method; (2) the leader's output tracks the desired reference trajectory as time goes on, and each follower's output asymptotically tracks the mean value of its neighbors' outputs; (3) each follower's output tends to the hidden leader's output as time goes by; (4) the MAS eventually achieves synchronization in the presence of strong couplings. Although the system without noise is addressed, this note makes the first step to analyze the adaptive tracking problem of multi-agent systems with unknown parameters and high-frequency gains under the assumptions that the interconnection topology is strong connected. Our work will motivate future study of more general cases that are not presently addressed in this note.

TABLE 1. Nomenclature.

f_i	unknown structure function of agent i
$\hat{f}_i$	approximate structure function of agent i
$y_i(k)$	output of agent i at the time k
u_i	control input of agent i
R	real number
R^n	n-dimensional Euclidean space
$\hat{\theta}_i$	estimate of agent i
$\hat{g}_i$	high-frequency estimate of agent i
$\frac{\partial f}{\partial x}$	partial derivative of the function f with respect to the variable x
$[\cdot]^{-1}$	inverse of matrix
$\|\cdot\|$	vector or matrix Euclidian norm
$[\cdot]^T$	vector or matrix transpose

This paper is organized as follows. Problem formulation and some assumptions are introduced in Section II and the projection algorithm and multi-model adaptive method are described in Section III. As the estimates of the parameter and high-frequency gain are known, the decentralized adaptive control laws are designed based on the desired reference trajectory and the mean value of the neighborhood agents in Section IV. Section V proves the performances of the closed-loop system eventually achieves strong synchronization in the presence of strong couplings. A simulation example is considered in Section VI to illustrate the improved control performance of the multi-agent system by multi-model method. Finally, conclusions are drawn in Section VII. And here we give the nomenclature in the following Table 1.

II. PROBLEM FORMULATION AND ASSUMPTIONS

A. ALGEBRAIC GRAPH THEORY

For an MAS consisting of N agents, the topology is expressed by a directed graph $\mathcal{G} = (\mathcal{V}, \varepsilon, \mathcal{A})$, in which, $\mathcal{V} = \{1, 2, \cdots, N\}$ is a set of all agents and i denotes agent i, $\varepsilon = \mathcal{V} \times \mathcal{V}$ is a set of the ordered edges of the form (i, j), representing that agent j has access to the information of agent j, by the way, in this case agent j is called be a neighbor of agent i; the matrix $\mathcal{A}(a_{ij} = 0, 1) \in R^{N\times N}$ is an adjacency matrix, whose entries $a_{ii} = 0$, $a_{ij} = 1$ if $(i, j) \in \varepsilon$, and $a_{ij} = 0$ if $(i, j) \notin \varepsilon$. The set of all neighbors of agent i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} | (i, j) \in \varepsilon\}$.

Definition 1 [11]: An adjacency matrix $\mathcal{A}(a_{ij} = 0, 1)$ is a strongly connected matrix if there exists a directed path such that any two agents are connected.

Definition 2 [13]: The agent in one multi-agent system is called one hidden leader if the agent knows the given signal, while other agents are aware of neither given signal nor the existence of the leader.

B. SYSTEM REPRESENTATION AND ASSUMPTIONS

The MAS is consisted of N agents, and the dynamics of agent i is expressed as follows:

$$y_i(k+1) = f_i(\theta_i, y_i(k), \varphi_i(k)) + g_i u_i(k), \tag{1}$$

where parameter $\theta_i \in R$ and high-frequency gain $g_i \in R \setminus \{0\}$ are unknown; the symbols $y_i(k+1)$ and $u_i(k)$ are the output and input of agent i at the time k, respectively; the vector $\varphi_i(k)$

is consisted of the outputs from the neighbors of agent i at the time k; the nonlinear mapping f_i is a known function, which is first-order continuously differentiable with respect to θ_i, and the derivative is expressed by $\Phi_i(k) = \frac{\partial f_i(\theta, y_i(k), \varphi_i(k))}{\partial \theta}|_{\theta=\hat{\theta}_i(k)}$.

Remark 1: For simplicity, this work that towards full understanding to the decentralized adaptive control is preliminary,, and the ideas may be generalized to more general high-dimensional discrete-time systems using more technical efforts.

In order to analyze multi-model adaptive control problem of this MAS to be discussed, some technical assumptions are introduced as follows.

Assumption A1: The MAS's adjacency matrix is strongly connected.

Assumption A2: The reference signal sequence $\{y^*(k)\}$ is bounded.

Assumption A3: Without loss of generality, the hidden leader is supposed to be the agent 1.

Remark 2: The hidden leader know the reference signal, while the followers are unaware of either the reference signal or who is the leader.

Assumption A4: The function $\Phi_i(\cdot)$ is Lipschitz function with respect to $(y_i(k), \varphi_i(k))$.

III. MULTI-MODEL ADAPTIVE METHOD

A. THE PROJECTION ALGORITHM

An algorithm is chosen to identify the unknown parameter and high-frequency gain for the nonlinear discrete-time dynamics of agent i and this algorithm brings a number of benefits, such as efficiency, good robustness, etc. Consider an identification criterion function as follows.

$$\begin{aligned} J_i(\theta_i, g_i) = {} & [y_i(k) - f_i(\theta_i, y_i(k-1), \varphi_i(k-1)) \\ & - g_i u_i(k-1)]^2 + \mu_i(\theta_i - \hat{\theta}_i(k-1))^2 \\ & + \nu_i \left(g_i - \hat{g}_i(k-1)\right)^2, \end{aligned} \tag{2}$$

where $\hat{\theta}_i(k-1)$ and $\hat{g}_i(k-1)$ are estimates of θ_i and g_i at the instance time k, respectively; two constants μ_i and ν_i are punishment factors of $(\theta_i - \hat{\theta}_i(k-1))$ and $\left(g_i - \hat{g}_i(k-1)\right)$, respectively, and $0 < \max\{\mu_i, \nu_i\} < 2\min\{\mu_i, \nu_i\}$. And define the errors

$$\begin{cases} \tilde{\theta}_i(k) = \hat{\theta}_i(k) - \theta_i \\ \tilde{g}_i(k) = \hat{g}_i(k) - g_i. \end{cases} \tag{3}$$

The estimates of θ_i and g_i are seeking to minimize (2). Denote

$$F_i(\theta_i, g_i) \triangleq f_i(\theta_i, y_i(k-1), \varphi_i(k-1)) + g_i u_i(k-1).$$

There are the partial derivatives of the function F_i with respect to independent θ_i and g_i. According to Taylor expanding formula of dualistic function, for example, $f(x, y)$,

$$\begin{aligned} f(x, y) = {} & f(x_0, y_0) + [(x - x_0)\frac{\partial}{\partial x} + (y - y_0)\frac{\partial}{\partial y}]f(x_0, y_0) \\ & + \frac{1}{2}[(x - x_0)\frac{\partial}{\partial x} + (y - y_0)\frac{\partial}{\partial y}]^2 f(x_0, y_0) + \cdots \end{aligned}$$

where (x_0, y_0) is the centre of Taylor expansion. Thus, using Taylor expansion in $(\hat{\theta}_i(k-1), \hat{g}_i(k-1))$ to form a local cost function, one has

$$\begin{aligned} F_i(\theta_i, g_i) \cong {} & F_i(\hat{\theta}_i(k-1), \hat{g}_i(k-1)) \\ & + \frac{\partial F_i(\theta, \hat{g}_i(k-1))}{\partial \theta}|_{\theta=\hat{\theta}_i(k-1)}(\theta_i - \hat{\theta}_i(k-1)) \\ & + \frac{\partial F(\hat{\theta}_i(k-1), g)}{\partial g}|_{g=\hat{g}_i(k-1)}(g_i - \hat{g}(k-1)) \\ = {} & f(\hat{\theta}_i(k-1), y_i(k-1), \varphi_i(k-1)) \\ & + \hat{g}_i(k-1)u_i(k-1) + \alpha_i(k-1) \\ & \times (\theta_i - \hat{\theta}_i(k-1)) + \beta_i(k-1)(g_i - \hat{g}_i(k-1)), \end{aligned} \tag{4}$$

where

$$\begin{cases} \alpha_i(k-1) = \dfrac{\partial F_i(\theta, \hat{g}_i(k-1))}{\partial \theta}|_{\theta=\hat{\theta}_i(k-1)} \\ \beta_i(k-1) = \dfrac{\partial F_i(\hat{\theta}_i(k-1), g)}{\partial g}|_{g=\hat{g}_i(k-1)}. \end{cases}$$

Applying (4) into (2), we have

$$\begin{aligned} & J_i(\theta_i, g_i) \\ & \cong [y_i(k) - f_i(\hat{\theta}_i(k-1), y_i(k-1), \varphi_i(k-1)) \\ & \quad - \hat{g}_i(k-1)u_i(k-1) - \alpha_i(k-1)(\theta_i - \hat{\theta}_i(k-1)) \\ & \quad - \beta_i(k-1)(g_i - \hat{g}_i(k-1))]^2 + \mu_i(\theta_i - \hat{\theta}_i(k-1))^2 \\ & \quad + \nu_i(g_i - \hat{g}_i(k-1))^2. \end{aligned} \tag{5}$$

Remark 3: In (5), neglected higher order terms have not been ensured to be small, not even be bounded, by the proposed adaptive law and control scheme. When they are not small, the approximation is not good. Here we suppose that the neglected higher order terms are convergent to zero as time goes by.

In order to obtain $\hat{\theta}_i(k)$ and $\hat{g}_i(k)$ to minimize (5), the easy way is to apply the minimal value theorem, one has

$$\begin{cases} \dfrac{\partial J_i(\theta, g_i)}{\partial \theta}|_{\theta=\hat{\theta}_i(k)} = 0 \\ \dfrac{\partial J_i(\theta_i, g)}{\partial g}|_{g=\hat{g}_i(k)} = 0. \end{cases}$$

By (5), we get

$$\begin{cases} [y_i(k) - f_i(\hat{\theta}_i(k-1), y_i(k-1), \varphi_i(k-1)) \\ -\hat{g}_i(k-1)u_i(k-1) - \alpha_i(k-1)(\hat{\theta}_i(k) - \hat{\theta}_i(k-1)) \\ -\beta_i(k-1)(\hat{g}_i(k) - \hat{g}_i(k-1))]\alpha_i(k-1) \\ -\mu_i(\hat{\theta}_i(k) - \hat{\theta}_i(k-1)) = 0 \\ [y_i(k) - f_i(\hat{\theta}_i(k-1), y_i(k-1), \varphi_i(k-1)) \\ -\hat{g}_i(k-1)u_i(k-1) - \alpha_i(k-1)(\hat{\theta}_i(k) - \hat{\theta}_i(k-1)) \\ -\beta_i(k-1)(\hat{g}_i(k) - \hat{g}_i(k-1))]\beta_i(k-1) \\ -\nu_i(\hat{g}(k) - \hat{g}_i(k-1)) = 0. \end{cases}$$

Arranging the above equations, it leads to the update laws

$$\begin{cases} \hat{\theta}_i(k) = \hat{\theta}_i(k-1) + \dfrac{[y_i(k) - \hat{y}_i(k)]\nu_i\alpha_i(k-1)}{\nu_i\alpha_i^2(k-1) + \mu_i\beta_i^2(k-1) + \mu_i\nu_i} \\ \hat{g}_i(k) = \hat{g}_i(k-1) + \dfrac{[y(k)_i - \hat{y}_i(k)]\mu_i\beta_i(k-1)}{\nu_i\alpha_i^2(k-1) + \mu_i\beta_i^2(k-1) + \mu_i\nu_i}, \end{cases} \tag{6}$$

where

$$\hat{y}_i(k) = f_i(\hat{\theta}_i(k-1), y_i(k-1), \varphi_i(k-1)) + \hat{g}_i(k-1)u_i(k-1).$$

Remark 4: Here, we only analyze the case that parameter θ_i is a scalar. Similarly, the multi-parameters' update laws can be obtained

$$\begin{cases} \hat{\theta}_i(k) = \hat{\theta}_i(k-1) + \dfrac{[y_i(k) - \hat{y}_i(k)]\nu_i\alpha_i(k-1)}{\nu_i \|\alpha_i(k-1)\|^2 + \mu_i\beta_i^2(k-1) + \mu_i\nu_i} \\ \hat{g}_i(k) = \hat{g}_i(k-1) + \dfrac{[y_i(k) - \hat{y}_i(k)]\mu_i\beta_i(k-1)}{\nu_i \|\alpha_i(k-1)\|^2 + \mu_i\beta_i^2(k-1) + \mu_i\nu_i}, \end{cases}$$

where $\hat{\theta}_i(\cdot) \in R^{n\times 1}$ and $\alpha_i(\cdot) \in R^{n\times 1}$.

B. MULTIPLE ADAPTIVE PARAMETERS AND MODELS

Suppose that each parameter and high-frequency gain are varying in their respective given convex sets. The following gives a minute description.

Firstly, multiple adaptive parameter and high-frequency gain of agent i are discussed. The unknown model parameter θ_i and unknown high-frequency gain g_i satisfy $\theta_i \in \Omega_i \subset R^{p_i}$, $g_i \in G_i \subset R^{q_i}$, where Ω_i and G_i are two given nonempty convex sets. The sets Ω_i and G_i have the following segmentations:

(1) Given sets Ω_{is_1} and G_{is_2} satisfy that

$$\Omega_{is_1} \subset \Omega_i, \quad G_{is_2} \subset G_i,$$

where

$$s_1 = 1, 2, \cdots, d_{i_1}, s_2 = 1, 2, \cdots, d_{i_2},$$

and

$$\Omega_{is_1}, G_{is_2}, \Omega_i, G_i \neq \varnothing.$$

(2) Sets Ω_{is_1} and G_{is_2} meet the certain conditions:

$$\Omega_i = \bigcup_{s_1=1}^{d_{i_1}} \Omega_{is_1}, \quad G_i = \bigcup_{s_2=1}^{d_{i_2}} G_{is_2};$$

Obviously, d_{i_1} and d_{i_2} are the numbers of objects, which are contained in the sets Ω_i and G_i, respectively.

(3) Let θ_{is_1}, $r_{is_1} \geq 0$ stand the center and radius of Ω_{is_1}, and g_{is_2}, $r_{is_2} \geq 0$ represent the in respective the center and radius of G_{is_2}. The mathematical expressions are denoted

$$\|\theta_i - \theta_{is_1}\| \leq r_{is_1}, \quad \|g_i - g_{is_2}\| \leq r_{is_2},$$

where $\theta_i \in \Omega_{is_1}$ and $g_i \in G_{is_2}$.

Considering (1), for the dynamic model for agent i, some sets of multiple fixed invariant parameters and high-frequency gains respectively are established

$$\hat{\Theta}_i(k) = \{\theta_{is_1}(k), s_1 = 1, 2, \cdots, d_{i_1}\}$$

and

$$\hat{\mathcal{G}}_i(k) = \{g_{is_2}(k), s_2 = 1, 2, \cdots, d_{i_2}\}.$$

For each agent, normal adaptive model is established to the dynamics (1) using the projection parameter update law (6). Denote normal adaptive parameter $\hat{\theta}_{i_1}(k)$ and high-frequency gain $\hat{g}_{i_1}(k)$ as $\hat{\theta}_{i,d_{i_1}+1}(k)$ and $\hat{g}_{i,d_{i_2}+1}(k)$, respectively. That is to say, sets $\hat{\Theta}_i(k)$ and $\hat{\mathcal{G}}_i(k)$ have been extended. They are described with accurate mathematical linguistic forms:

$$\hat{\Theta}_i(k) = \{\theta_{is_1}(k), s_1 = 1, 2, \cdots, d_{i_1}, \hat{\theta}_{i,d_{i_1}+1}(k)\} \tag{7}$$

and

$$\hat{\mathcal{G}}_i(k) = \{g_{is_2}(k), s_2 = 1, 2, \cdots, d_{i_2}, \hat{g}_{i,d_{i_2}+1}(k)\}, \tag{8}$$

where $\hat{\theta}_{i,d_{i_1}+1}(k)$ and $\hat{g}_{i,d_{i_2}+1}(k)$ are obtained from the normal update laws (6).

For improving control performance to accelerate parameter convergence, we draw into another adaptive model parameter $\hat{\theta}_{i,d_{i_1}+2}(k)$ and high-frequency gain $\hat{g}_{i,d_{i_2}+2}(k)$, whose initial values would be dynamically adjusted to the nearest model parameter and high-frequency gain of the dynamics. The set of multiple parameters with $d_{i_1} + 2$ elements is

$$\hat{\Theta}_i(k) = \{\theta_{is_1}(k), s_1 = 1, 2, \cdots, d_{i_1}, \hat{\theta}_{i,d_{i_1}+1}(k), \hat{\theta}_{i,d_{i_1}+2}(k)\}. \tag{9}$$

And the set of multiple high-frequency gains with $d_{i_2} + 2$ elements is

$$\hat{\mathcal{G}}_i(k) = \{g_{is_2}(k), s_2 = 1, 2, \cdots, d_{i_2}, \hat{g}_{i,d_{i_2}+1}(k), \hat{g}_{i,d_{i_2}+2}(k)\}. \tag{10}$$

Thus, Cartesian product of the set $\hat{\Theta}_i(k)$ and the set $\hat{\mathcal{G}}_i(k)$ can be expressed as follows:

$$\hat{\Theta}_i(k) \times \hat{\mathcal{G}}_i(k) = \{(\theta_{is_1}(k), g_{is_2}(k)) | \theta_{is_1}(k) \in \hat{\Theta}_i(k), g_{is_2}(k) \in \hat{\mathcal{G}}_i(k)\}$$

According to (6), (7), (8), (9) and (10), the adaptive multiple models are established:

$$\hat{y}_{is}(k+1) = f_i(\hat{\theta}_{is_1}(k), y_i(k), \varphi_i(k)) + \hat{g}_{is_2}(k)u_i(k),$$

where $\hat{\theta}_{is_1}$ is from $\hat{\Theta}_i(k)$, $\hat{g}_{is_2}$ is from $\hat{\mathcal{G}}_i(k)$, and

$$s = \{1, 2, \cdots, d_{i_1}d_{i_2}, \cdots, (d_{i_1}+1)(d_{i_2}+1), \cdots, (d_{i_1}+2)(d_{i_2}+2)\}.$$

Remark 5: For agent i, $(d_{i_1}+2)(d_{i_2}+2)$ models are established. Thus, for the whole system, the number of all models is $\sum_{i=1}^{N}(d_{i_1}+2)(d_{i_2}+2)$.

Facing to sets $(d_{i_1} + 2)$ parameters and $(d_{i_2} + 2)$ high-frequency gains of agent i, how to choose the optimal parameter and high-frequency gain for fast and accurate tracking their respective true values. The details are as follows.

C. MULTI-MODEL ADAPTIVE OPTIMAL PARAMETER

For agent i, in order to establish one adaptive optimal parameter, two important definitions are given as follows.

Definition 3: Define output error as

$$e_{is}(k) = \left\| \frac{\eta_{is}(k+1)}{[\nu_i\alpha_i^2(k-1) + \mu_i\beta_i^2(k-1) + \mu_i\nu_i]^{1/2}} \right\|,$$

where

$$s = \{1, 2, d_{i_1}d_{i_2}, \cdots, (d_{i_1}+1)(d_{i_2}+1), \cdots, (d_{i_1}+2)(d_{i_2}+2)\},$$

and

$$\begin{aligned}\eta_{is}(k+1) &= y_i(k+1) - \hat{y}_{is}(k+1)\\ &= f_i(\theta_i, y_i(k), \varphi_i(k)) + g_i u_i(k)\\ &\quad - f_i(\hat{\theta}_{is}, y_i(k), \varphi_i(k)) - \hat{g}_i(k)u_i(k).\end{aligned}$$

Definition 4: Define index switching function as

$$J_{is}(k_{i0}, k_{i1}) = \sum_{t=k_{i0}}^{k_{i1}} e_{is}^2(t),$$

where

$$s = \{1, 2, d_{i_1}d_{i_2}, \cdots, (d_{i_1}+1)(d_{i_2}+1), \cdots, (d_{i_1}+2)(d_{i_2}+2)\}.$$

From Definition 4, it is obvious to get

$$J_{is}(k_{i0}, k_{i1}) = J_{is}(k_{i0}, k_{i1}-1) + e_{is}^2(k_{i1}),$$

where

$$s = \{1, 2, d_{i_1}d_{i_2}, \cdots, (d_{i_1}+1)(d_{i_2}+1), \cdots, (d_{i_1}+2)(d_{i_2}+2)\}.$$

From Definitions 3 and 4, using an linearization technique, based on multi-model adaptive control strategy, the optimal parameter estimate and high-frequency gain estimate are designed in detail.

(1) When $k = k_{i0}$, for given enough small one positive real number $\epsilon_i > 0$, let the difference set $I_i(0)$ be that

$$I_i(k) = I_i(0) = s/\{(d_{i_1}+1)(d_{i_2}+1), (d_{i_1}+2)(d_{i_2}+2)\}. \tag{11}$$

(2) When $k > k_{i0}$, we calculate index switching functions

$$\begin{aligned}\hat{I}_i(k) = \{s|\ &\|y_i(k) - \hat{y}_{is}(k)\| \le r_{is_1}\|\alpha_i(k-1)\|\\ &+ g_{is_2}\|\beta_i(k-1)\|, s \in I_i(k-1)\},\end{aligned} \tag{12}$$

where

$$s_1 = 1, 2, \cdots, d_{i_1}, s_2 = 1, 2, \cdots, d_{i_2}, \tag{13}$$

and

$$I_i(k) = \hat{I}_i(k) \bigcap I_i(k-1). \tag{14}$$

We denote the index $s_i(k)$ as

$$s_i(k) = arg \min_{l \in \{I_i(k), (d_{i_1}+1)(d_{i_2}+1), (d_{i_1}+2)(d_{i_2}+2)\}} J_{il}(k_{i0}, k_i). \tag{15}$$

Let

$$\hat{\theta}_{i,d_i+2}(k) = \hat{\theta}_{i,s_i(k)}, \quad \hat{g}_{i,d_i+2}(k) = \hat{g}_{i,s_i(k)} \tag{16}$$

and

$$J_{i,d_i+2}(k_{i0}, k_i) = J_{i,s_i(k)}(k_{i0}, k_i). \tag{17}$$

The time is calculated by

$$k_{i1} = \min\{k_i' | k_i' > k_{i0}, e_{s_i(k_i')} < \epsilon_i\}. \tag{18}$$

If $k < k_{i1}$, then the parameter estimate $\hat{\theta}_i(k)$ and high-frequency gain estimate $\hat{g}_i(k)$ are chosen, and back to step (2) for calculating the next time output $y_i(k+1)$.

If $k \ge k_{i1}$, then adaptive model $(d_{i_1}+2)(d_{i_2}+2)$ will degenerate into normal adaptive identifier or better identifier, that is to say, the worst identifier is

$$\hat{\theta}_i(k) = \hat{\theta}_{d_{i_1}+2}(k), \quad \hat{g}_i(k) = \hat{g}_{d_{i_2}+2}(k).$$

Remark 6: For agent i, the optimal parameter estimate $\hat{\theta}_i(k)$ and optimal high-frequency gain estimate $\hat{g}_i(k)$ have been chosen by two index switching functions.

Remark 7: The error $\sum_{t=k_{i0}}^{k_{i1}} e_{is}^2(t)$ can be obtained by optimal parameter and optimal high-frequency gain is not more than the error that can be the projection algorithm.

Facing to each optimal parameter and optimal high-frequency gain, our objective is to design the decentralized adaptive control laws such that all agents among this system (1) track the given reference signal.

IV. DECENTRALIZED ADAPTIVE CONTROL LAWS

By Assumption $A3$, the first agent is to track the reference trajectory $y^*(k)$. Using the certainty equivalence principle and the reference trajectory, we design the control law for the first agent as follows:

$$u_1(k) = \frac{1}{\hat{g}_1(k)}(-f_1(\hat{\theta}_1(k), y_1(k), \varphi_1(k)) + y^*(k+1)). \tag{19}$$

As for other agents, since each follower agent only knows its own and its neighbors' historical information and is unaware of the existence of the leader. Using the certainty equivalence principle and the mean value of the neighbors' agents, the local control law for each follower agent is designed as follows:

$$u_i(k) = \frac{1}{\hat{g}_i(k)}(-f_i(\hat{\theta}_i(k), y_i(k), \varphi_i(k)) + \frac{1}{d_i}\sum_{l \in \mathcal{N}_i} y_l(k)). \tag{20}$$

In the decentralized adaptive control laws (19) and (20), there exists singularity problem for any $\hat{g}_i(k) = 0$. Thus, we can assume that

$$\hat{g}_i(k) = \begin{cases}\hat{g}_i(k-1), & if\ |g_i(k)| < r_i(k),\\ \hat{g}_i(k), & otherwise,\end{cases}$$

where $r_i(k)$ is a dead zone for gain update and can be a smaller real number.

The error between the output of the leader agent and the reference signal at the time instant $(k+1)$ is denoted by $\tilde{y}_1(k+1)$, which is expressed in mathematical language

$$\tilde{y}_1(k+1) = y_1(k+1) - y^*(k+1). \tag{21}$$

As for the follower agent i, the error between the output and the average values of the outputs of neighbors at the time

$(k+1)$ are denoted by $\tilde{y}_i(k+1), i = 2, 3\cdots, N$. The mathematical expression is

$$\tilde{y}_i(k+1) = y_i(k+1) - z_i(k), \quad i = 2, 3\cdots, N, \tag{22}$$

where

$$z_i(k) = \frac{1}{d_i}\sum_{l\in\mathcal{N}_i} y_l(k), \quad i = 2, 3\cdots, N.$$

Combining (21), (22) with (1), (19), (20), we can get

$$\tilde{y}_i(k+1) = f_i(\theta_i, y_i(k), \varphi_i(k)) - f_i(\hat{\theta}_i(k), y_i(k), \varphi_i(k)) + g_i u_i(k) - \hat{g}_i(k)u_i(k), \tag{23}$$

which together with (4) yields

$$\tilde{y}_i(k+1) \cong -\alpha_i(k)\tilde{\theta}_i(k) - \beta_i(k)\tilde{g}_i(k), \tag{24}$$

where

$$\begin{cases}\tilde{\theta}_i(k) = \hat{\theta}_i(k) - \theta_i \\ \tilde{g}_i(k) = \hat{g}_i(k) - g_i.\end{cases}$$

V. TRACKING PERFORMANCE OF THE MULTI-AGENT SYSTEM

A. SEVERAL AUXILIARY LEMMAS

Lemma 1: The projection algorithm satisfies the following properties:

(1) There is a positive real number M_{i1} and M_{i2} such that

$$\left\|\hat{\theta}_i(k) - \theta_i\right\| \le M_{i1}, \quad \left\|\hat{g}_i(k) - g_i\right\| \le M_{i2}.$$

(2) The series consisting of output errors is convergent, that is

$$\lim_{k\to\infty}\sum_{t=1}^{k}\frac{\|\eta_i(t+1)\|^2}{\nu_i\alpha_i^2(t-1) + \mu_i\beta_i^2(t-1) + \mu_i\nu_i} < \infty.$$

(3) The sequence consisting of output errors is convergent, that is

$$\lim_{k\to\infty}\frac{\|\eta_i(k+1)\|^2}{\nu_i\alpha_i^2(k-1) + \mu_i\beta_i^2(k-1) + \mu_i\nu_i} = 0,$$

where

$$\begin{aligned}\eta_i(k+1) &= y_i(k+1) - \hat{y}_i(k+1) \\ &= f_i(\theta_i, y_i(k), \varphi_i(k)) - f_i(\hat{\theta}_i(k), y_i(k), \varphi_i(k)) \\ &\quad + g_i u_i(k) - \hat{g}_i(k)u_i(k).\end{aligned}$$

Proof:

(1) Consider a Lyapunov function

$$V_i(k) = \tilde{\theta}_i^2(k) + \tilde{g}_i^2(k).$$

The difference $\Delta V_i(k)$ is that

$$\begin{aligned}\Delta V_i(k) &= V_i(k) - V_i(k-1) \\ &= \tilde{\theta}_i^2(k) - \tilde{\theta}_i^2(k-1) + \tilde{g}_i^2(k) - \tilde{g}_i^2(k-1) \\ &= (\tilde{\theta}_i(k) - \tilde{\theta}_i(k-1))^2 + 2\tilde{\theta}_i(k-1)(\tilde{\theta}_i(k) \\ &\quad - \tilde{\theta}_i(k-1)) + (\tilde{g}_i(k) - \tilde{g}_i(k-1))^2 \\ &\quad + 2\tilde{g}_i(k-1)(\tilde{g}_i(k) - \tilde{g}_i(k-1)).\end{aligned} \tag{25}$$

From (25), it is easy to get

$$\begin{cases}\tilde{\theta}_i(k) - \tilde{\theta}_i(k-1) = \hat{\theta}_i(k) - \hat{\theta}_i(k-1) \\ \tilde{g}_i(k) - \tilde{g}_i(k-1) = \hat{g}_i(k) - \hat{g}_i(k-1).\end{cases} \tag{26}$$

Substituting (26) into (25), we have

$$\begin{aligned}\Delta V_i(k) &= (\hat{\theta}_i(k) - \hat{\theta}_i(k-1))^2 + 2\tilde{\theta}_i(k-1)(\hat{\theta}_i(k) \\ &\quad - \hat{\theta}_i(k-1)) + (\hat{g}_i(k) - \hat{g}_i(k-1))^2 \\ &\quad + 2\tilde{g}_i(k-1)(\hat{g}_i(k) - \hat{g}_i(k-1)).\end{aligned} \tag{27}$$

By the update laws in (6), one has

$$\begin{cases}\hat{\theta}_i(k) - \hat{\theta}_i(k-1) = \dfrac{[y_i(k) - \hat{y}_i(k)]\nu_i\alpha_i(k-1)}{\nu_i\alpha_i^2(k-1) + \mu_i\beta_i^2(k-1) + \mu_i\nu_i} \\ \hat{g}_i(k) - \hat{g}_i(k-1) = \dfrac{[y_i(k) - \hat{y}_i(k)]\mu_i\beta_i(k-1)}{\nu_i\alpha_i^2(k-1) + \mu_i\beta_i^2(k-1) + \mu_i\nu_i},\end{cases} \tag{28}$$

where

$$\begin{cases}\hat{y}_i(k) = f_i(\hat{\theta}_i(k-1), y_i(k-1), \varphi_i(k-1)) \\ \qquad + \hat{g}_i(k-1)u_i(k-1) \\ \alpha_i(k-1) = \dfrac{\partial f_i(\theta, y_i(k-1), \varphi_i(k-1))}{\partial\theta}\Big|_{\theta=\hat{\theta}_i(k-1)} \\ \beta_i(k-1) = \dfrac{\partial(gu_i(k-1))}{\partial g}\Big|_{g=\hat{g}_i(k-1)} = u_i(k-1).\end{cases}$$

Putting (23), (28) into (27) yields

$$\begin{aligned}&\Delta V_i(k) \\ &= \frac{\tilde{y}_i^2(k)(\alpha_i^2(k-1)\nu_i^2 + \mu_i^2\beta_i^2(k-1))}{(\nu_i\alpha_i^2(k-1) + \mu_i\beta_i^2(k-1) + \mu_i\nu_i)^2} \\ &\quad + \frac{2\tilde{y}_i(k)[\nu_i\alpha_i(k-1)\tilde{\theta}_i(k-1) + \mu_i\beta_i(k-1)\tilde{g}_i(k-1)]}{\nu_i\alpha_i^2(k-1) + \mu_i\beta_i^2(k-1) + \mu_i\nu_i}.\end{aligned}$$

Our objective in this step is to deduce $\Delta V_i(k) \le 0$. By the above equation, it is clear to get

$$\begin{aligned}&\Delta V_i(k) \\ &\le \frac{\max\{\mu_i, \nu_i\}[\tilde{y}_i^2(k)\nu_i\alpha_i^2(k-1) + \tilde{y}_i^2(k)\mu_i\beta_i^2(k-1)]}{(\nu_i\alpha_i^2(k-1) + \mu_i\beta_i^2(k-1) + \mu_i\nu_i)^2} \\ &\quad + \frac{2\tilde{y}_i(k)[\nu_i\alpha_i(k-1)\tilde{\theta}_i(k-1) + \mu_i\beta_i(k-1)\tilde{g}_i(k-1)]}{\nu_i\alpha_i^2(k-1) + \mu_i\beta_i^2(k-1) + \mu_i\nu_i},\end{aligned}$$

which together with (24), it immediately leads to

$$\begin{aligned}\Delta V_i(k) &\le \frac{\max\{\mu_i, \nu_i\}\tilde{y}_i^2(k)}{\nu_i\alpha_i^2(k-1) + \mu_i\beta_i^2(k-1) + \mu_i\nu_i} \\ &\quad - \frac{2\min\{\mu_i, \nu_i\}\tilde{y}_i^2(k)}{\nu_i\alpha_i^2(k-1) + \mu_i\beta_i^2(k-1) + \mu_i\nu_i} \\ &= -\frac{[2\min\{\mu_i, \nu_i\} - \max\{\mu_i, \nu_i\}]\tilde{y}_i^2(k)}{\nu_i\alpha_i^2(k-1) + \mu_i\beta_i^2(k-1) + \mu_i\nu_i}.\end{aligned} \tag{29}$$

Noticing $0 < \max\{\mu_i, \nu_i\} < 2\min\{\mu_i, \nu_i\}$, it is easy to conclude that $\Delta V_i(k) \le 0$. According to Lyapunov theory, it is clear to get that $V_i(k)$ is bounded. So $\tilde{\theta}_i(k)$ and $\tilde{g}_i(k)$ are bounded.

(2) From (29), we know

$$\Delta V_i(k) \leq -\frac{[2\min\{\mu_i, \nu_i\} - \max\{\mu_i, \nu_i\}]\tilde{y}_i^2(k)}{\nu_i\alpha_i^2(k-1) + \mu_i\beta_i^2(k-1) + \mu_i\nu_i}. \quad (30)$$

Taking summation on both sides of (30), one has

$$\lim_{k\to\infty} V_i(k) - V_i(0) \leq -\sum_{k=1}^{\infty}\frac{[2\min\{\mu_i, \nu_i\} - \max\{\mu_i, \nu_i\}]\tilde{y}_i^2(k)}{\nu_i\alpha_i^2(k-1) + \mu_i\beta_i^2(k-1) + \mu_i\nu_i}.$$

Since $\max\{\mu_i, \nu_i\} \leq 2\min\{\mu_i, \nu_i\}$ and

$$\lim_{k\to\infty} V_i(k) \geq 0,$$

we have

$$\sum_{k=1}^{\infty}\frac{[2\min\{\mu_i, \nu_i\} - \max\{\mu_i, \nu_i\}]\tilde{y}_i^2(k)}{\nu_i\alpha_i^2(k-1) + \mu_i\beta_i^2(k-1) + \mu_i\nu_i} \leq V_i(0).$$

(3) When $2\min\{\mu_i, \nu_i\} > \max\{\mu_i, \nu_i\}$, according to the convergence criterion of the positive series, the positive series

$$\sum_{k=1}^{\infty}\frac{[2\min\{\mu_i, \nu_i\} - \max\{\mu_i, \nu_i\}]\tilde{y}_i^2(k)}{\nu_i\alpha_i^2(k-1) + \mu_i\beta_i^2(k-1) + \mu_i\nu_i}$$

is a convergent. And by a necessary condition for convergence of series, it is easy to get

$$\lim_{k\to\infty}\frac{\eta_i^2(k)}{\nu_i\alpha_i^2(k-1) + \mu_i\beta_i^2(k-1) + \mu_i\nu_i} = 0.$$

Under the decentralized adaptive control, the closed-loop stability is summarized in the following lemma.

Lemma 2 [13]: Suppose that the MAS consisting of N agents satisfies Assumptions $A1$ - $A4$, then under the decentralized adaptive control based on the projection-type parameter estimation algorithm, the closed-loop system has the following properties:

1) The error between the first agent's output and the desired reference signal tends to zero as time goes on. And each follower agent's output can track the mean value of its neighbors' outputs, i.e.,

$$\lim_{k\to\infty}(y_1(k) - y^*(k)) = 0$$

and

$$\lim_{k\to\infty}(y_i(k) - \frac{1}{d_i}\sum_{l\in\mathcal{N}_i} y_l(k)) = 0, \quad i = 2, \cdots, N,$$

where d_i is the number of i^{th} agent's neighbors.

2) At the time k, the error between i^{th} agent's output and the first agent's output is denoted by $e_{i1}(k)$, then the error approaches to zero as time goes by, i.e.,

$$\lim_{k\to\infty} e_{i1}(k) = \lim_{k\to\infty}(y_i(k) - y_1(k)) = 0.$$

3) At the time k, the error between i^{th} agent's output $y_i(k)$ and the reference signal $y^*(k)$ is denoted by $e_i(k)$, then the error converges to zero as time passes, i.e.,

$$\lim_{k\to\infty} e_i(k) = \lim_{k\to\infty}(y_i(k) - y^*(k)) = 0.$$

4) The whole system can achieve synchronization, i.e.,

$$\lim_{k\to\infty}(y_i(k) - y_j(k)) = 0.$$

As space is limited, the detailed proving steps will not be written here. Please see [13].

Lemma 3: For agent i, using the multi-model adaptive parameters (11)-(18) and decentralized adaptive laws (19), (20), there exists the time k_{i4}, when $k > k_{i4}$, the multi-model adaptive controllers are converted into normal single one.

Proof: When $k > k_{i0}$, for fixed models $s \in I_i(k)$, the index switching function has two cases: bounded and divergent. Consider the $(d_{i1}d_{i2}+1)^{th}$ model, according to (3) of Lemma 1, the index switching satisfies

$$\lim_{k\to\infty} J_{d_i+1}(k_{i0}, k) = \lim_{k\to\infty}\sum_{t=1}^{k} e_{is}^2(t) < \infty.$$

Then, for the subset of dynamic system

$$I_i'(k) = \{j | j \in I_i(k) and \lim_{k\to\infty} J_{i,j}(k_{i0}, k) \to \infty\},$$

there is the time k_{i3}, when $k > k_{i3}$, one has

$$J_{i,j}(k_{i0}, k) > J_{i,d_i+1}(k_{i0}, k), \quad j \in I_i'(k),$$

which implies that index functions in the set of dynamic model cannot take part in switching. In other words, when $k > k_{i3}$, multi-model adaptive controllers are in the subset of dynamic models

$$I_i''(k) = \{j | j \in I_i(k) and \lim_{k\to\infty} J_{i,j}(k_{i0}, k) < \infty\}$$

and adaptive models $d_{i1}d_{i2}+1$, $d_{i1}d_{i2}+2$. So, identifying the parameter $\hat{\theta}_i(k)$ is switching in the parameters $\hat{\theta}_{S_i(k)}(S_i(k) = \{I_i''(k), (d_{i1}+1)(d_{i2}+1), (d_{i1}+2)(d_{i2}+2)\})$. According to Definitions 3, 4, (4) of Lemma 1 and (28), one has

$$\lim_{k\to\infty} e_{S_i(k)} = 0.$$

Thus, there exists the time $k_{i4} > k_{i3}$, when $k > k_{i4}$, one has

$$e_{S_i(k)} < \epsilon_i, S_i(k) \in \{I_i''(k), (d_{i1}+1)(d_{i2}+1), (d_{i1}+2)(d_{i2}+2)\}.$$

At this time multi-model adaptive controllers can be converted into normal single one.

Lemma 4: For the whole system, using the multi-model adaptive parameters (11)- (18) and decentralized adaptive laws (19), (20), there exists the time k_4, when $k > k_4$, for each agent the multi-model adaptive controllers are converted into normal single one.

Proof: From Lemma 3, we see that there is the time

$$k_4 = \max(k_{14}, k_{24}, \cdots, k_{n4}),$$

when $k > k_4$, for the whole system, the multi-model adaptive controllers can ultimately be converted into normal adaptive controllers.

Remark 8: For a discrete-time non-linearly parameterized heterogeneous MAS, using the control laws (19) and (20), the multi-model adaptive controllers are established. The system is guaranteed to track the desired signal if each identification parameter satisfies that $\left\|\hat{\theta}_i(k) - \theta_i\right\|$ is bounded. Thus, it is guaranteed to each identification parameter does not jump by multi-model adaptive controllers, which is why that multi-model adaptive controllers can ultimately be converted into normal single one.

B. MAIN THEORETICAL RESULTS

From Remark 8, Lemmas 2, 3, 4, we have the following theorem:

Theorem 1: If a discrete-time non-linearly parameterized heterogeneous MAS (1) satisfies Assumptions **A1-A4**, under the multi-model adaptive parameters (11)- (18) and decentralized adaptive laws (19), (20), then

(1) the hidden leader agent tracks the desired reference trajectory $\lim_{k\to\infty}(y_1(k) - y^*(k)) = 0$, and each follower agent follows the average value of its own neighborhood historical outputs $\lim_{k\to\infty}(y_i(k) - z_i(k-1)) = 0$, $i = 2, 3, \cdots, N$;

(2) the synchronization of all the follower agents to the hidden leader agent is achieved, that is, $\lim_{k\to\infty}(y_i(k) - y_1(k)) = 0$, $i = 2, 3, \cdots, N$;

(3) all the agents track the desired trajectory, that is, $\lim_{k\to\infty}(y_i(k) - y^*(k)) = 0$, $i = 2, 3, \cdots, N$.

Remark 9: Although multi-model adaptive controllers can ultimately be converted into normal single one for each agent, before this, switching mechanism of multi-model adaptive controllers the performance of designed controllers based on the identification parameters $\hat{\theta}_i(k)$ fast approach for one of designed controllers based on the true parameters θ_i. Thus, it extremely improves transient response of the control system.

VI. SIMULATIONS

In this section, a simulation example is provided to verify the illustrate the efficiency and feasibility of the proposed theoretical results. This strategy can be applied to multi-robot system and high-speed electric multiple units and so on. An example is refered to system in general, not to anyone in particular. A group of five discrete-time nonlinear agents with unknown parameters and high-frequency gains is expressed as follows:

$$y_i(k+1) = f_i(\theta_i, \varphi_i(k)) + g_i u_i(k), \tag{31}$$

where

$$\begin{cases} y_1(k+1) = 0.8\theta_1 y_1(k) + 0.2\theta_1 y_2(k) - 0.1\theta_1 y_5(k) \\ \qquad\qquad + g_1 u_1(k) \\ y_2(k+1) = \theta_2 y_2(k) - \sin(\theta_2 y_5(k)) + g_2 u_2(k) \\ y_3(k+1) = 0.9\theta_3 y_3(k) + 0.2\theta_3 y_1(k) - \cos(\theta_3 y_4(k)) \\ \qquad\qquad + g_3 u_3(k) \\ y_4(k+1) = 0.5\theta_4 y_4(k) + 0.5\theta_4 y_1(k) - e^{-|y_3(k)|} \\ \qquad\qquad + g_4 u_4(k) \\ y_5(k+1) = \theta_5 y_5(k) - \theta_5 \sin(y_3(k)) + g_5 u_5(k) \end{cases} \tag{32}$$

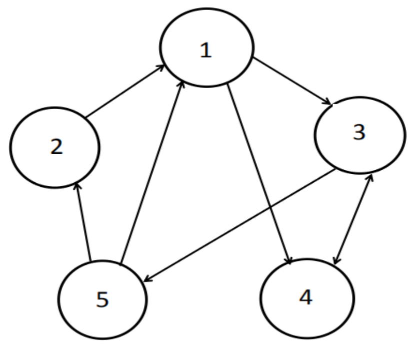

FIGURE 1. Architecture graph.

and

$$\begin{cases} \theta_1 = 3, & g_1 = 5 \\ \theta_2 = 2, & g_2 = 4 \\ \theta_3 = 3.5, & g_3 = 3 \\ \theta_4 = 4, & g_4 = 2 \\ \theta_5 = 2, & g_5 = 3. \end{cases}$$

The adjacency matrix and architecture diagram from (32) are

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

and this system is one leader-follower MAS consists of five agents. At any time, each agent has its own neighbors. The desired objective is to make agents be uniformly distributed on the line. From the dynamics of the system, the unknown parameters and high-frequency gains are estimated. Under the decentralized control laws, the system achieves the performance index. We assume that the first agent is the leader, and the desired signal is $y^*(k) = 15 + \frac{1}{k}$, it is easy to check that Assumptions $A1 - A4$ hold in this MAS.

We will make simulations using the projection algorithm and multi-model method to estimate parameters and high-frequency gains respectively and demonstrate corresponding graphs of inputs and outputs.

A. THE SIMULATION RESULTS BASED ON PROJECTION-TYPE ESTIMATION ALGORITHM

In this subsection, the simulation results according to the projection algorithm are provided.

We use the update laws defined by (6) with $\mu_i = 0.3$, $\nu_i = 0.5$, $i = 1, 2, \cdots, N$ to estimate unknown parameters and unknown high-frequency gains. It is noted that $\max\{\mu_i, \nu_i\} < 2\min\{\mu_i, \nu_i\}$.

From Figs. 2 and 3, based on the projection algorithm each parameter estimate and each high-frequency estimate are convergent, each parameter error and high-frequency error are bounded. The leader agent tracks the desired signal and each following agent follows the mean value of the historical outputs for its neighbors as shown in Fig.5. And the whole

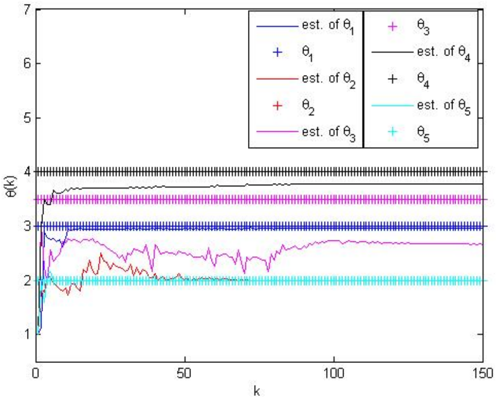

FIGURE 2. $\theta_i, \hat{\theta}_i$ based on projection algorithm.

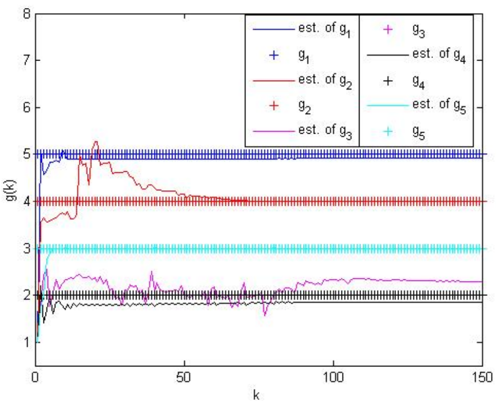

FIGURE 3. $g_i, \hat{g}_i$ based on projection algorithm.

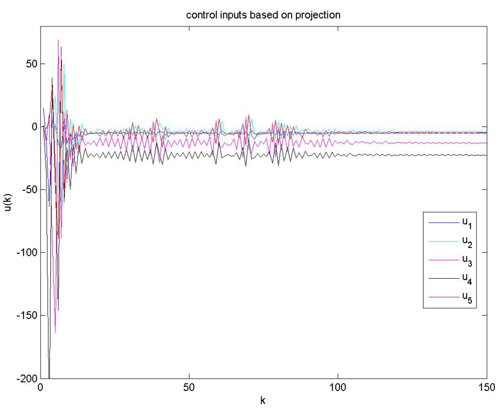

FIGURE 4. u_i based on projection algorithm.

system achieves strong synchronization in the presence of strong couplings.

B. THE SIMULATION RESULTS BASED ON MULTI-MODEL METHOD

In this subsection, the simulation results for the simulation example (31) and (32) using multi-model method based on Section IV are given. Under the same conditions of Subsection VI-A, parameter estimates and high-frequency gain estimates almost tracks the true values from Figs. 6 and 7. Under the different application backgrounds control input

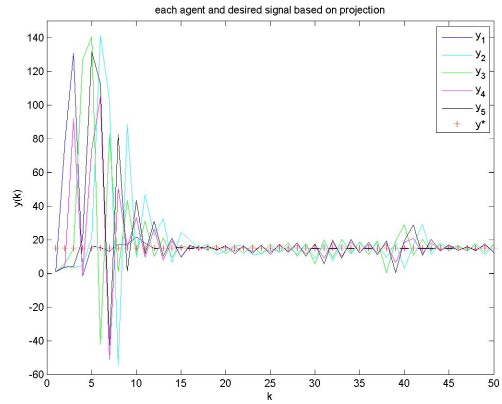

FIGURE 5. y_i, y^* based on projection algorithm.

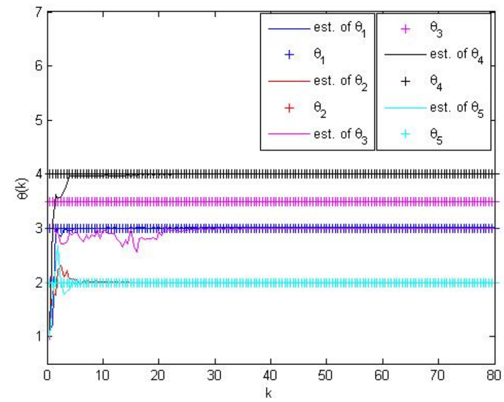

FIGURE 6. $\theta_i, \hat{\theta}_i$ based on multi-model method.

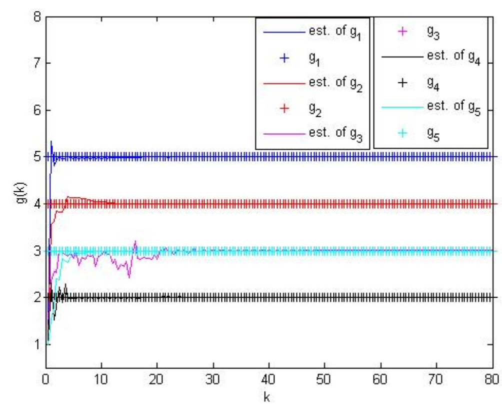

FIGURE 7. $g_i, \hat{g}_i$ based on multi-model method.

range is different. The saturated restriction could be imposed on control input. The each control input range is chosen in $[-50, 50]$ in the simulations, and the range could be adjusted as needed. By Fig. 8, the local control is bounded. And the whole system achieves strong synchronization in the presence of strong couplings by the Fig. 9.

To sum up, under the decentralized adaptive control the whole system achieves strong synchronization using either the projection algorithm or multi-model method to identify parameters and high-frequency gains. It is not difficult to

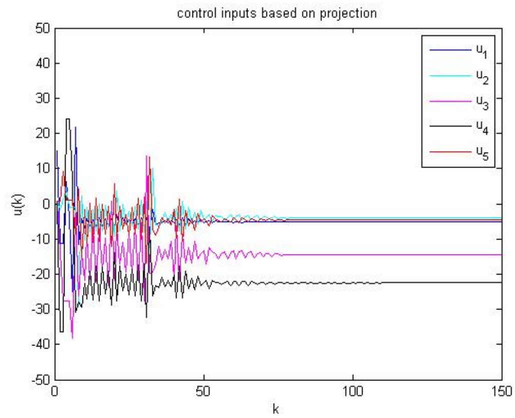

FIGURE 8. u_i **based on multi-model method.**

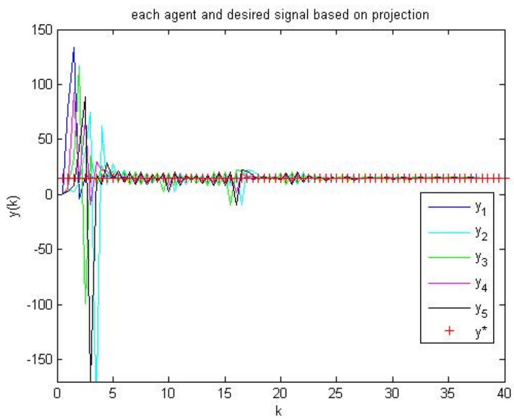

FIGURE 9. y_i, y^* **based on multi-model method.**

see from the simulations that the multi-model adaptive control algorithm is effectiveness on improving control performance on identifying parameters and high-frequency gains by comparing with the normal projection algorithm through simulation results.

VII. CONCLUSION

In this paper, the problem of decentralized adaptive tracking control has been discussed for a class of leader-following MASs, in which each agent is modeled by the discrete-time nonlinear system with unknown parameter and unknown high-frequency gain. The multi-model adaptive method is adopted to identify uncertainties. Each agent can obtain the historical information of its neighbors in the model, the local control law based on the certainty equivalence principle are designed. Through analysis, it has been proven that all the agents track the desired trajectory, and the closed-loop system eventually achieves strong synchronization in the presence of strong couplings. Finally, the effectiveness of the proposed control method is further verified by a simulation example. The study on the asynchronous decentralized tracking control of discrete-time nonlinear MASs can be also extended to more general cases such as noise and switching topologies, which would be some interesting topics for our future work. Furthermore, the autonomous mobile robots moving in the plane is also an interesting topic that needs to be developed.

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