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Remaining Useful Life Prediction Under Imperfect Prior Degradation Information

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ABSTRACT The remaining useful life (RUL) prediction is the core of equipment maintenance and decision-making. Accurate RUL prediction can make effective maintenance before the failure occurs to reduce the probability of equipment failure. However, in industrial practice, we often face the situation that prior information is insufficient or inaccurate, which could influence our prediction of RUL and reduce the prediction accuracy severely. To solve this problem, we study the issue of RUL prediction under imperfect prior degradation data by reasonably fusing the prior information and the field degradation data. Firstly, based on the linear Wiener degradation process, we prove a conclusion that the random parameter estimation results by the joint Bayesian algorithm and the expectation-maximization (EM) algorithm are the same as that by the maximum likelihood estimation (MLE) method. It shows that the joint estimation method completely overcomes the influence of prior information, and the more iterations, the smaller proportion of prior information. Secondly, we also prove that the random parameter joint updating method for the nonlinear Wiener degradation process has the same characteristics. Then, a heuristic algorithm that reasonably fuses the prior information and the field degradation data is proposed, which controls the number of iterations of the joint updating algorithm based on the credibility of the prior information. It is also applied to the nonlinear degradation process by not iterating the nonlinear coefficients. Finally, the correctness of the conclusion of this paper is verified by the simulated degradation data, and the effectiveness of the proposed heuristic updating method is verified by the actual lithium battery degradation data.

INDEX TERMS Imperfect prior information, fused data, adaptive parameter updating, heuristic algorithm, Bayesian method, EM algorithm.

I. INTRODUCTION

In engineering practice, real-time mastering and managing the health status of equipment is an efficient measure to effectively reduce the probability of failure and improve safety [1]–[3]. Prognostics and health management (PHM) technology is a crucial way to grasp the real-time status of equipment and carry out effective management [4], [5], which is mainly divided into two parts. One is the prediction. The prediction is primarily to predict the remaining useful life (RUL) of the equipment. The prediction result is expressed in the form of the mean or the probability density function (PDF) of RUL. Since the PDF can reflect the uncertainty of the RUL prediction results, which contains more information with individual differences during the use of the equipment or the external changing environment, the PDF is generally used to describe the RUL [6]. The other is health management. Based on the prediction, health management can be carried out effectively to reduce failures and maintenance costs [7].

The current RUL estimation methods are mainly divided into two major categories of physical modeling methods and data-driven methods [1], [6], [8]. The principle of physical modeling methods is to model the equipment's degradation mechanism based on its physical characteristics. Its advantage is that the equipment is individually modeled according to the equipment 's specific characteristics, and thus could obtain relatively accurate estimation results. However, this

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TABLE 1. Acronym.

CM	condition monitoring
EM	expectation maximization
MLE	maximum likelihood estimation
MSEs	mean square errors
PHM	prognostics and health management
RUL	remaining useful life
PDFs	probability distribution functions

TABLE 2. Notations.

$f(\cdot)$	Probability density distribution function of RUL
x	Condition monitoring data
λ	Drift coefficient of the Wiener process model
σ_B	Diffusion coefficient of the Wiener process model
σ_0^2	Prior variance of λ
$\sigma_{\lambda,k}^2$	Posterior variance of λ
μ_0	Prior mean of λ
$\mu_{\lambda,k}$	Posterior mean of λ
L	The length of iteration interval
i	The number of iterations.
$\Lambda(t:\theta)$	Nonlinear coefficient of the nonlinear Wiener process
w	The failure threshold
θ	The fixed coefficient of nonlinear Wiener process
b	The nonlinear coefficient of the nonlinear Wiener process
Θ	Prior parameters of the Wiener degradation model
$B(t)$	Brownian motion
$D(\cdot)$	The variance of estimation
$E(\cdot)$	The expectation of estimation
$N(\cdot)$	Normal distribution
$X(t)$	Predicted condition data at time t

kind of methods needs to model the degradation mechanism of each equipment separately, which requires a lot of modeling works. Moreover, with the development of industrial technology, modern equipment systems become more complex. It is often difficult to establish physical models of complex equipment based on physical modeling methods. The data-driven method does not depend on the physical characteristics and working principles of the complex equipment, and the generalization ability is strong. There are mainly artificial intelligence methods and statistical data-driven methods. Among them, artificial intelligence methods include support vector machine regression [9], correlation vector machine [10], neural network [11], [12], ant colony algorithm [13], etc. This type of method is based on a large amount of degradation data. Then, the artificial intelligence method is used to fit the equipment degradation path to achieve the RUL prediction. When the degradation data is insufficient, or the prior information is not accurate enough, it is difficult to achieve desired prediction accuracy. The statistical data-driven method uses condition monitoring (CM) data to construct a statistical model for RUL prediction with calculating the expectation and PDF of the RUL. Statistical data-driven methods have been widely used in RUL prediction due to their good mathematical properties [8]. Among the statistical data-driven methods, the most typical methods are based on stochastic process modeling, which mainly includes: regression model [14], gamma process [15], Wiener process [16]–[18], inverse Gaussian process [19]–[21], Markov chain [22], [23], etc.

The stochastic process modeling based method has the following advantages. First, the degradation model is easy to be established and understood, the mathematical symbols in the degradation model could intuitively describe different degradation characteristics of the degradation process. For example, the mean value of drift coefficient in the Wiener process represents the average degradation speed of the degradation process. The variance of the drift coefficient can describe the difference of the degradation speed between different items, and the diffusion coefficient can express the uncertainty of the degradation process. Second, it is convenient to obtain the life distribution and the distribution of the RUL to describe the uncertainty of the prediction results. For example, the gamma distribution and its deformation can describe the life distribution and the distribution of the RUL of the gamma degradation process in different situations [24]. The life distribution and the RUL distribution of the inverse Gaussian degradation process under different conditions can be described by the inverse Gaussian distribution and its deformation [19]–[21] Third, it is convenient to fuse the prior information based on historical degradation data, failure time data and the field degradation data under Bayesian framework [25]–[35]. In the degradation model based on stochastic process, the connection between the prior information and field data was first established by Gebraeel *et al.* [25] and [26], Gebraeel [27] under the Bayesian framework, which can reasonably fuse the prior information into the RUL prediction based on on-site degradation data. This method first aims at the regression model [25]–[27], and then generalizes to the Wiener process [28], the Wiener process with measurement error [29], the gamma process [30], the Inverse Gaussian process [31], [32], the Inverse Gaussian process considering random effects [33], and the Inverse Gaussian process considering the time-varying effects [34].

However, this kind of RUL prediction method based on Bayesian theory is often affected by imperfect prior information. Imperfect prior information refers to the situation where the prior information is insufficient, inaccurate or even without prior information, which could lead to inaccurate estimation of prior parameters or inability to estimate prior parameters [26], [36], [37]. There are two main ways to solve the imperfect prior information in the existing literature: The first solution is to fuse historical failure time data of congeneric units. Lifetime data reveals the reliability information on the time scale, and degradation data describes the failure process. Wang *et al.* [38] utilized a Bayesian method to

fuse historical failure time data and accelerated degradation data to construct a joint likelihood function to predict the RUL. Wang *et al.* [39] regarded the failure time data as a Bernoulli distribution, and combined the failure time data and the field degradation data under the Bayesian framework. Zhao *et al.* [40] uesd the maximum entropy increase method to convert the failure time data into prior information and utilized the Bayesian method to update the parameters in combination with the field degradation data. Lehmann [41] used the degradation–threshold–shock models and derived a joint likelihood function of failure time data and field degradation data at discrete observation times. Similar works can be found in [42]–[44] and references therein. However, none of the above studies considered the random effects representing the differences among congeneric items. Tang *et al.* [35] fused the failure time data into prior degradation information with considering the random effects and achieved good results. However, this method can only use historical failure time data of congeneric items, which cannot use the degradation data of congeneric items. For high-reliability and long-life products, it is difficult to obtain sufficient failure time data, and there is often only historical degradation data of congeneric items. In this case, this method is no longer applicable.

The second solution is to combine the Bayesian method and the EM algorithm to estimate model parameters, which was first proposed by Wang *et al.* [45] and applied to the Brownian motion model with an adaptive drift. Later, Si *et al.* [6], [28] applied this algorithm to the basic linear Wiener process. Subsequently, this mechanism was extended to the nonlinear Wiener process [46], [47], the linear Wiener process with measurement error [48], the nonlinear Wiener process with measurement error [49], [50] and other random effects models [51]–[54]. This joint parameter updating method can overcome the influence of imperfect prior information. However, how does this method produce a better prediction result? What is the relationship between the prediction results and historical prior information and field degradation information? To answer this question, Tang *et al.* [37] studied this joint parameter updating method based on the linear Wiener process with measurement error. And, it is concluded that this joint estimation method completely overcomes the influence of prior information and only relies on field degradation information. Based on this conclusion, a heuristic algorithm that reasonably fuses the prior information and the field degradation information is proposed. However, the interesting conclusions and heuristic RUL prediction method proposed by Tang *et al.* are only suitable for the hidden degradation process with measurement error, but not for the basic linear Wiener process, especially the nonlinear Wiener process. Both the Gamma process and the inverse Gaussian process are only suitable for modeling monotonic degradation paths. In contrast, the Wiener process is suitable for describing non-monotonic degradation processes with the discontinuous increase or decrease trends and can better represent the randomness of the degradation process. It has been widely used in various types of degradation modeling [16]. Therefore, it is necessary to study the RUL prediction method based on the Wiener degradation process under imperfect prior information.

From the above review over related works, the existing heuristic RUL prediction method under the imperfect prior information needs to be further studied in terms of the Wiener degradation process. There mainly exist the following two issues that need to be solved: 1. Is the interesting conclusion that the joint estimation method only relies on the field degradation information proposed by Tang *et al.* [37] based on the degradation process with measurement error applies to the basic linear Wiener degradation process, especially for the nonlinear Wiener processes? 2. Is the heuristic RUL prediction method equally effective for the basic linear Wiener degradation process and nonlinear Wiener process?

Therefore, this paper attempts to conduct in-depth research on generalizing the heuristic RUL prediction method to the basic linear Wiener degradation process and nonlinear Wiener process. First, it is proved in theory that the conclusions proposed by Tang.et.al equally applies to the basic linear Wiener process and the nonlinear Wiener process. Second, an adaptive parameter updating method that joint combines the Bayesian method and the EM algorithm is proposed to update the drift parameter of the nonlinear Wiener processes. Third, a heuristic RUL prediction method for the nonlinear Wiener process which is different from that proposed by Tang *et al.* [37] is proposed. Finally, the correctness of the conclusions of this paper is verified through simulated degradation data. And the effectiveness of the proposed heuristic RUL prediction method is validated by using an actual lithium battery degradation data.

The remainders of this paper are organized as follows: Section II proves that the interesting conclusion equally applies to the basic linear Wiener process; Section III introduces the basic theory of nonlinear Wiener process for RUL prediction; Section IV proposes a heuristic RUL prediction algorithm for the nonlinear Wiener process; Section V validates the conclusion and the RUL prediction method proposed in this paper; Section VI summarizes the full text.

II. PARAMETER UPDATING FOR THE LINEAR WIENER PROCESS UNDER BAYES FRAMEWORK

A. DEGRADATION MODEL BASED ON LINEAR WIENER PROCESS

The Wiener process is a type of diffusion process with a drift term and a Brownian motion representing the dynamic uncertainty. This process can suitably describe the degradation path of the equipment and quantify the uncertainty of the degradation process. Therefore, it has been widely used in various types of degradation modeling [16]. The linear Wiener process can be expressed as follows

$$X(t) = x_0 + \lambda t + \sigma_B B(t), \tag{1}$$

where x_0 is the initial state, λ is the drift coefficient, which characterizes the degradation rate, σ_B is the diffusion coefficient, $B(t)$ is the standard Brownian motion. Without loss of

generality, it is set that $x_0 = 0$. Considering the individual differences among congeneric items, the drift coefficient λ is generally regarded as a random variable that follows normal distribution, i.e. $\lambda \sim N(\mu_\lambda, \sigma_\lambda^2)$.

B. ONLINE UPDATING THE RANDOM PARAMETER BASED ON THE TRADITIONAL BAYESIAN METHOD

As mentioned above, the random drift coefficient $\lambda$is used to describe the individual differences. In the RUL prediction process, the random parameters need to be updated according to the field degradation data to adapt to the individual characteristics of the evaluated device. The online updating of the random parameters is based on the fixed parameters estimated by the maximum likelihood estimation (MLE) method based on the historical degradation data of congeneric items. The specific estimation method can refer to the literature [55], [56]. For the basic linear Wiener process, the fixed parameters are $\boldsymbol{\Theta} = \{\mu_0, \sigma_0^2, \sigma_B\}$. To make the RUL prediction result adapt to the individual item's characteristic, it is necessary to use the field degradation data of the evaluated item to online update the drift coefficient λ.

The drift coefficients are generally updated under the Bayesian framework. $\boldsymbol{X}_{1:k} = \{x_1, x_2, x_3, \ldots, x_k\}$ is defined as the actual degradation data at CM times $t_1, t_2, t_3, \ldots t_k$. According to the nature of the normal distribution, the posterior distribution of the drift coefficient also follows normal distribution. Then, given the prior distribution of drift coefficients $N(\mu_0, \sigma_0^2)$, we have [57]

$$\lambda_k | \boldsymbol{X}_{1:k} \sim N\left(\mu_{\lambda,k}, \sigma_{\lambda,k}^2\right), \tag{2}$$

where

$$\mu_{\lambda,k} = \frac{x_k\sigma_0^2 + \mu_0\sigma_B^2}{t_k\sigma_0^2 + \sigma_B^2}, \quad \sigma_{\lambda,k}^2 = \frac{\sigma_B^2\sigma_0^2}{t_k\sigma_0^2 + \sigma_B^2} \tag{3}$$

C. ONLINE UPDATING THE RANDOM PARAMETER BASED ON THE JOINT ESTIMATION METHOD

When the prior information is not perfect or even does not exist, the prior unknown parameters need to be estimated principally based on the on-site information. To solve this problem, Wang *et al.* [45] and Si *et al.* [28] proposed a parameter updating method by combining the Bayesian method and the EM algorithms. For the basic linear Wiener process, the specific calculation process can be written as follows [28].

First, it is defined that $\boldsymbol{\Theta}_k = \left[\mu_{\lambda,k}, \sigma_{\lambda,k}^2, \sigma_{B,k}^2\right]$ is the estimated results at time t_k based on field degradation data $\boldsymbol{X}_{1:k}$. The log-likelihood function of $\boldsymbol{X}_{1:k}$ can be written as follows:

$$\begin{aligned}
&\ln p(\boldsymbol{X}_{1:k}, \lambda_k | \Theta_k) \\
&\quad = \ln p(\boldsymbol{X}_{1:k}, \lambda_k | \Theta_k) + \ln p(\lambda_k | \Theta_k) \\
&\quad = -\frac{k+1}{2}\ln 2\pi - \frac{1}{2}\sum_{j=1}^{k}\ln \Delta t_j - \frac{k}{2}\ln\sigma_B^2 - \frac{1}{2}\ln\sigma_{\lambda,k}^2 \\
&\qquad - \sum_{j=1}^{k}\frac{(\Delta x_j - \lambda_k \Delta t_j)^2}{2\sigma_B^2\Delta t_j} - \frac{(\lambda_k - \mu_{\lambda,k})^2}{2\sigma_{\lambda,k}^2}
\end{aligned} \tag{4}$$

where $\Delta t_j = t_j - t_{j-1}$, $\Delta x_j = x_j - x_{j-1}$.

Then, given $\hat{\boldsymbol{\Theta}}_k^{(i)} = \left[\hat{\mu}_{\lambda,k}^{(i)}, \hat{\sigma}_{\lambda,k}^{2(i)}, \hat{\sigma}_{B,k}^{2(i)}\right]$ as the estimated results in the ith step based on $\boldsymbol{X}_{1:k}$, the calculation iteration process of the $(i+1)$th step can be divided into the following two steps.

E-Step: Calculate the expectation of joint the log-likelihood function $L(\boldsymbol{\Theta}_k | \hat{\boldsymbol{\Theta}}_k^{(i)})$.

$$\begin{aligned}
&L\left(\boldsymbol{\Theta}_k | \hat{\boldsymbol{\Theta}}_k^{(i)}\right) \\
&\quad = \mathrm{E}_{\lambda_k | X_{1:k}, \hat{\boldsymbol{\Theta}}_k^{(i)}}\left[\ln p(\boldsymbol{X}_{1:k}, \lambda_k | \Theta_k)\right] \\
&\quad = -\frac{k+1}{2}\ln 2\pi - \frac{1}{2}\sum_{j=1}^{k}\ln \Delta t_j - \frac{k}{2}\ln\sigma_B^2 - \frac{1}{2}\ln\sigma_{\lambda,k}^2 \\
&\qquad - \sum_{j=1}^{k}\frac{(\Delta x_j - M\Delta t_j)^2 + N\Delta t_j^2}{2\sigma_B^2\Delta t_j} - \frac{(M - \mu_{\lambda,k})^2 + N}{2\sigma_{\lambda,k}^2}
\end{aligned} \tag{5}$$

where $M = E(\lambda_k | X_{1:k}, \hat{\Theta}_k^{(i)})$ and $N = var(\lambda_k | X_{1:k}, \hat{\Theta}_k^{(i)})$. The two parameters can be calculated by Eq (2).

M-Step: Maximize $L(\boldsymbol{\Theta}_k | \hat{\boldsymbol{\Theta}}_k^{(i)})$.

$$\hat{\boldsymbol{\Theta}}_k^{(i+1)} = \arg\max_{\boldsymbol{\Theta}} L(\boldsymbol{\Theta}_k | \hat{\boldsymbol{\Theta}}_k^{(i)}) \tag{6}$$

Taking the partial derivative of $L(\boldsymbol{\Theta}_k | \hat{\boldsymbol{\Theta}}_k^{(i)})$ with respect to $\boldsymbol{\Theta}_k = \left[\mu_{\lambda,k}, \sigma_{\lambda,k}^2, \sigma_{B,k}^2\right]$ and setting them to zero gives

$$\hat{\mu}_{\lambda,k}^{(i+1)} = E(\lambda_k | \boldsymbol{X}_{1:k}, \hat{\boldsymbol{\Theta}}_k^{(i)}), \tag{7}$$

$$\hat{\sigma}_{\lambda,k}^{2(i+1)} = var(\lambda_k | \boldsymbol{X}_{1:k}, \hat{\boldsymbol{\Theta}}_k^{(i)}), \tag{8}$$

$$\hat{\sigma}_{B,k}^{2(i+1)} = \frac{1}{k}\sum_{j=1}^{k}\frac{1}{\Delta t_j}\left[(\Delta x_j - M\Delta t_j)^2 + N\Delta t_j^2\right]. \tag{9}$$

D. COMPARISON BETWEEN THE JOINT ESTIMATION METHOD AND THE DIRECT MLE METHOD

It can be concluded that this joint estimation method based on the EM algorithm can overcome the influence of imperfect prior information. However how does this method overcome the influence of prior information? What is the relationship between the prediction results and degradation information (i.e. prior historical information and the on-site degradation information), is still unclear. This section will study the relationship between this joint estimation method and the direct MLE algorithm, which only considers the field degradation data.

For the field degradation data of a single item to be evaluated, the drift coefficient only contains the degradation speed information of a single item without containing the difference information of congeneric items. And thus, it can be regarded as a fixed parameter. Then, by giving the field degradation

data $\boldsymbol{X}_{1:k} = \{x_1, x_2, \ldots, x_k\}$, the likelihood function can be expressed as

$$\ln L(\lambda, \sigma_B^2|\boldsymbol{X}_{1:k}) = -\frac{k}{2}(\ln 2\pi + \ln \sigma_B^2) - \frac{1}{2}\sum_{j=1}^{k} \ln \Delta t_j - \frac{1}{2\sigma_B^2}\sum_{j=1}^{k} \frac{1}{\Delta t_j}(\Delta x_j - \lambda \Delta t_j)^2 \quad (10)$$

Taking the partial derivatives of σ_B^2 and λ for (10) gives

$$\frac{\partial \ln L(\lambda, \sigma_B^2|\boldsymbol{X}_{1:k})}{\partial \lambda} = \frac{1}{\sigma_B^2}\sum_{j=1}^{k} \frac{1}{\Delta t_j}(\Delta x_j \Delta t_j - \lambda \Delta t_j^2) \quad (11)$$

$$\frac{\partial \ln L(\lambda, \sigma_B^2|\boldsymbol{X}_{1:k})}{\partial \sigma_B^2} = -\frac{k}{2\sigma_B^2} + \frac{1}{2\sigma_B^4}\sum_{j=1}^{k} \frac{1}{\Delta t_j}(\Delta x_j - \lambda \Delta t_j)^2 \quad (12)$$

Let their partial derivative be zero, and the estimate of σ_B^2 and λ can be obtained as follows.

$$\hat{\sigma}_B^2 = \frac{1}{k}\sum_{j=1}^{k} \frac{1}{\Delta t_j}(\Delta x_j - \Delta t_j x_k/t_k)^2 \quad (13)$$

$$\hat{\lambda} = \frac{x_k}{t_k} \quad (14)$$

Theorem 1: The final iteration result by the EM algorithm in this section is the same as that by the direct MLE method based on the field observation data.

Proof: Making $\boldsymbol{\Theta}_k = \boldsymbol{\Theta}_k^{(i)} = \boldsymbol{\Theta}_k^{(i+1)}$, gives:

$$\sigma_{B,k}^2 = \frac{1}{k}\sum_{j=1}^{k} \frac{1}{\Delta t_j}\left[(\Delta x_j - \mu_{\lambda,k}\Delta t_j)^2 + \sigma_{\lambda,k}^2 \Delta t_j^2\right] \quad (15)$$

$$\mu_{\lambda,k} = \frac{\sigma_{\lambda,k}^2 x_k + \mu_{\lambda,k}\sigma_{B,k}^2}{\sigma_{\lambda,k}^2 t_k + \sigma_{B,k}^2} \quad (16)$$

$$\sigma_{\lambda,k}^2 = \frac{\sigma_{\lambda,k}^2 \sigma_{B,k}^2}{\sigma_{\lambda,k}^2 t_k + \sigma_{B,k}^2} \quad (17)$$

It can be known from the iterative process of the EM algorithm that the iteration result of the EM algorithm is the solution of the above equation. By solving (7) and (8) respectively, we have $\hat{\mu}_{\lambda,k} = x_k/t_k$ and $\hat{\sigma}_{\lambda,k}^2 = 0$. By substituting $\hat{\sigma}_{\lambda,k}^2$ and $\hat{\mu}_{\lambda,k}$into (15), (13) can be derived (9).

This completes the proof.

Remark 1: Theorem 1 indicates that for the basic linear Wiener degradation process, the parameters estimation results using the joint estimation method of the Bayesian method and the EM algorithm is the same as the parameters estimation results based on the direct MLE algorithm. In theory, this paper proves that the conclusion proposed by Tang *et al.* [37] is also applicable to the basic linear Wiener process. In the following, the nonlinear Wiener process would be studied.

III. RUL PREDICTION BASED ON THE NONLINEAR WIENER PROCESS UNDER BAYESIAN FRAMEWORK

A. DEGRADATION MODEL

In industrial practice, due to the uncertainty of the external environment or the complexity of the equipment, the basic linear Wiener process is sometimes difficult to describe the nonlinear characteristics of the degradation process. In this case, the nonlinear Wiener process is required. This section focuses on the nonlinear Wiener process. The general nonlinear Wiener process can be expressed as follows [16], [46], [47], [58], [59]:

$$X(t) = x_0 + \lambda\Lambda(t;\theta) + \sigma_B B(t) \quad (18)$$

where x_0 is the initial state, without loss of generality, we set $x_0 = 0$. $\Lambda(t;\boldsymbol{\theta})$ is a monotone continuous nonlinear function about $\boldsymbol{\theta}$. The typical nonlinear functions are $\Lambda(t;\boldsymbol{\theta}) = t^{\boldsymbol{\theta}}$ and $\Lambda(t;\boldsymbol{\theta}) = \exp(\boldsymbol{\theta}t) - 1$. Considering the individual differences between different units, the drift coefficient λ is regarded as a random variable that follows normal distribution, i.e. $\lambda \sim N(\mu_\lambda, \sigma_\lambda^2)$.$\sigma_B$ is the diffusion coefficient and $B(t)$ is the standard Brownian motion, which represents the dynamic characteristics of the degradation process.

B. ONLINE UPDATING THE RANDOM PARAMETER BASED ON THE TRADITIONAL BAYESIAN METHOD

For the general nonlinear Wiener process, the unknown fixed parameters are $\boldsymbol{\Theta} = \{\mu_\lambda, \sigma_\lambda^2, \theta, \sigma B\}$. The drift coefficient λ is online updated to reflect the individual characteristic of the evaluated item.

The drift coefficient λ is generally updated under the Bayesian framework. According to the nature of the normal distribution and Bayesian theory, the drift coefficient λ_k also obeys normal distribution. Then, given the prior distribution of drift coefficients $N(\mu_0, \sigma_0^2)$ and field degradation data $\boldsymbol{X}_{1:k}$, we have [57]

$$\lambda_k|\boldsymbol{X}_{1:k} \sim N\left(\mu_{\lambda,k}, \sigma_{\lambda,k}^2\right) \quad (19)$$

where

$$\mu_{\lambda,k} = \frac{B_k\sigma_0^2 + \mu_0\sigma_B^2}{A_k\sigma_0^2 + \sigma_B^2}, \sigma_{\lambda,k}^2 = \frac{\sigma_B^2\sigma_0^2}{A_k\sigma_0^2 + \sigma_B^2} \quad (20)$$

$$A_k = \sum_{j=1}^{k} \frac{(\Lambda(t_j;\boldsymbol{\theta}) - \Lambda(t_{j-1};\boldsymbol{\theta}))^2}{\Delta t_j} \quad (21)$$

$$B_k = \sum_{j=1}^{k} \frac{(\Lambda(t_j;\boldsymbol{\theta}) - \Lambda(t_{j-1};\boldsymbol{\theta}))\Delta x_j}{\Delta t_j} \quad (22)$$

C. RUL PREDICTION

RUL prediction is based on detected field degradation data $\boldsymbol{X}_{1:k}$ and updated random parameter.

After detecting the field degradation data $\boldsymbol{X}_{1:k}$, the degradation process when $t > t_k$ can be written as follow:

$$X(t|\boldsymbol{X}_{1:k}) = x_k + (\lambda_k|\boldsymbol{X}_{1:k})\,(\Lambda(t;\boldsymbol{\theta}) - \Lambda(t_k;\boldsymbol{\theta})) + \sigma_B B(t - t_k) \quad (23)$$

Let $l_k = t - t_k (l_k \geq 0)$, then the degradation process $\{X(t), t \geq t_k\}$ can be transformed into:

$$\begin{aligned} Y(l_k) &= X(l_k + t_k) - X(l_k) \\ &= \lambda_k \left(\Lambda(t; \boldsymbol{\theta}) - \Lambda(t_k; \boldsymbol{\theta})\right) + \sigma_B B(l_k) \end{aligned} \tag{24}$$

where $Y(0) = 0$. Then the RUL at the moment t_k can be transformed into the time when $\{Y(l_k), l_k \geq 0\}$ first crosses the failure threshold $w_k = w - x_k$, where w is the failure threshold. Then, the corresponding RUL can be expressed as follows:

$$L_k = \inf\{l_k : X(l_k + t_k) \geq w | \boldsymbol{X}_{1:k}\} \tag{25}$$

Therefore, the PDF of RUL can be obtained as following [6], [57], [60]:

$$\begin{aligned} & f_{L_k|\boldsymbol{X}_{1:k},w}(l_k|\boldsymbol{X}_{1:k}, w) \\ &\approx \frac{1}{\sqrt{2\pi l_k^2 \left(\sigma_{\lambda,k}^2 \varphi(l_k)^2 + \sigma_B^2 l_k\right)}} \\ &\quad \times \exp\left[-\frac{\left(w - x_k - \mu_{\lambda,k}\varphi(l_k)\right)^2}{2\left(\sigma_{\lambda,k}^2 \varphi(l_k)^2 + \sigma_B^2 l_k\right)}\right] \\ &\quad \times \left[w - x_k - \mu_{\lambda,k}\beta(l_k) - \frac{w - x_k - \mu_{\lambda,k}\varphi(l_k)}{\sigma_{\lambda,k}^2 \varphi(l_k)^2 + \sigma_B^2 l_k}\right. \\ &\quad \left. \times \sigma_{\lambda,k}^2 \beta(l_k)\varphi(l_k)\right] \end{aligned} \tag{26}$$

where:

$$\beta(l_k) = \Lambda(l_k + t_k; \boldsymbol{\theta}) - \Lambda(t_k; \boldsymbol{\theta}) - \Lambda'(l_k + t_k; \boldsymbol{\theta}) l_k \tag{27}$$

$$\varphi(l_k) = \Lambda(l_k + t_k; \boldsymbol{\theta}) - \Lambda(t_k; \boldsymbol{\theta}) \tag{28}$$

IV. HEURISTIC RUL METHOD

A. ONLINE UPDATING THE RANDOM PARAMETER BASED ON THE JOINT ESTIMATION METHOD

When the prior information is imperfect or even does not exist, the unknown parameters need to be estimated primarily based on the field degradation information. Si [46] constructed a nonlinear Wiener process based degradation model with modeling a random walk effect of the drift coefficient and use historical observation data combined with the Kalman filter and the EM algorithm to update the drift coefficient. However, many literatures, such as [33], [34], [47], [61]–[63], directly update the random parameter without modeling the random walk, so the method proposed by Si [46] is no longer applicable. Inspired by [6], [28], [46], this paper focuses on the Bayesian method without modeling the random walk, and we first use the Bayesian method and the EM algorithm to update the drift coefficient directly. The specific calculation process is presented as follows.

First, given that $\boldsymbol{\Theta}_k = \left[\mu_{\lambda,k}, \sigma_{\lambda,k}^2, \sigma_{B,k}^2, \boldsymbol{\theta}_k\right]$ is estimated results based on the field degradation data $\boldsymbol{X}_{1:k}$. Then, the log-likelihood function of $\boldsymbol{X}_{1:k}$ can be obtained as follows:

$$\begin{aligned} & \ln p(X_{1:k}, \lambda_k | \Theta_k) \\ &= \ln p(X_{1:k} | \lambda_k, \Theta_k) + \ln p(\lambda_k | \Theta_k) \\ &= -\frac{k+1}{2} \ln 2\pi - \frac{1}{2}\sum_{j=1}^{k} \ln \Delta t_j - \frac{k}{2} \ln \sigma_{B,k}^2 - \frac{1}{2} \ln \sigma_{\lambda,k}^2 \\ &\quad - \sum_{i=1}^{k} \frac{(\Delta x_j - \lambda_k \alpha(t))^2}{2\sigma_{B,k}^2 \Delta t_j} - \frac{(\lambda_k - \mu_{\lambda,k})^2}{2\sigma_{\lambda,k}^2} \end{aligned} \tag{29}$$

where

$$\alpha(t) = \Lambda(t_j; \theta_k) - \Lambda(t_{j-1}; \theta_k) \tag{30}$$

Given $\hat{\boldsymbol{\Theta}}_k^{(i)} = \left[\hat{\mu}_{\lambda,k}^{(i)}, \hat{\sigma}_{\lambda,k}^{2(i)}, \hat{\sigma}_{B,k}^{2(i)}, \hat{\boldsymbol{\theta}}_k^{(i)}\right]$ as the estimate in the ith step based on $\boldsymbol{X}_{1:k}$, the calculation iteration process of the $(i+1)$th step can be divided into E step and M step.

E-Step: Calculate the expectation of joint the log-likelihood function $L(\boldsymbol{\Theta}_k | \hat{\boldsymbol{\Theta}}_k^{(i)})$.

$$\begin{aligned} L(\boldsymbol{\Theta}_k | \hat{\boldsymbol{\Theta}}_k^{(i)}) &= \mathrm{E}_{\lambda_k | X_{1:k}, \hat{\boldsymbol{\Theta}}_k^{(i)}} \left[\ln p(\boldsymbol{X}_{1:k}, \lambda_k \left| \hat{\boldsymbol{\Theta}}_k^{(i)} \right.)\right] \\ &= -\frac{k+1}{2} \ln 2\pi - \frac{1}{2}\sum_{j=1}^{k} \ln \Delta t_j - \frac{k}{2} \ln \sigma_{B,k}^2 \\ &\quad - \sum_{j=1}^{k} \frac{1}{2\sigma_{B,k}^2 \Delta t_j} \left[(\Delta x_j - M\alpha(t))^2 + N\alpha(t))^2\right] \\ &\quad - \frac{1}{2} \ln \sigma_{\lambda,k}^2 - \frac{(M - \mu_{\lambda,k})^2 + N}{2\sigma_{\lambda,k}^2} \end{aligned} \tag{31}$$

where $M = E(\lambda_k | X_{1:k}, \hat{\Theta}_k^{(i)})$ and $N = var(\lambda_k | X_{1:k}, \hat{\Theta}_k^{(i)})$. There two parameters can be calculated by the (19).

M-Step: Maximize $L(\boldsymbol{\Theta}_k | \hat{\boldsymbol{\Theta}}_k^{(i)})$.

$$\hat{\boldsymbol{\Theta}}_k^{(i+1)} = \arg\max_{\boldsymbol{\Theta}} L(\boldsymbol{\Theta}_k | \hat{\boldsymbol{\Theta}}_k^{(i)}) \tag{32}$$

By taking the partial derivative of $L(\boldsymbol{\Theta}_k | \hat{\boldsymbol{\Theta}}_k^{(i)})$ with respect to $\boldsymbol{\Theta}_k = \left[\mu_{\lambda,k}, \sigma_{\lambda,k}^2, \sigma_{B,k}^2\right]$ and setting them to zero, we have

$$\hat{\sigma}_{B,k}^{2(i+1)}(\hat{\boldsymbol{\theta}}_k) = \frac{1}{k}\sum_{j=1}^{k} \frac{1}{\Delta t_j} \left[(\Delta x_j - M\alpha(t)))^2 \quad + N\alpha(t)^2\right] \tag{33}$$

$$\hat{\mu}_{\lambda,k}^{(i+1)} = E(\lambda_k | \boldsymbol{X}_{1:k}, \hat{\boldsymbol{\Theta}}_k^{(i)}) \tag{34}$$

$$\hat{\sigma}_{\lambda,k}^{2(i+1)} = \mathrm{var}(\lambda_k | \boldsymbol{X}_{1:k}, \hat{\boldsymbol{\Theta}}_k^{(i)}) \tag{35}$$

By substituting the (33)-(35) into L_k, we can obtain the profile likelihood function about $\boldsymbol{\theta}$, as shown in (36).

$$\begin{aligned} & L_k(\boldsymbol{\theta} | \hat{\boldsymbol{\Theta}}_k^{(i)}, \hat{\sigma}_{B,k}^{2(i+1)}(\hat{\boldsymbol{\theta}}_k), \hat{\mu}_{\lambda,k}^{(i+1)}, \hat{\sigma}_{\lambda,k}^{2(i+1)}) \\ &= -\frac{1}{2}\sum_{j=1}^{k} \ln \Delta t_j - \frac{k+1}{2}(\ln 2\pi + 1) - \frac{k}{2} \ln \hat{\sigma}_{B,k}^{2(i+1)} \\ &\quad - \frac{1}{2} \ln \hat{\sigma}_{\lambda,k}^{2(i+1)} \end{aligned} \tag{36}$$

Then, by maximizing $L_k(\boldsymbol{\theta}|\hat{\boldsymbol{\Theta}}_k^{(i)}, \hat{\sigma}_{B,k}^{2(i+1)}(\hat{\boldsymbol{\theta}}_k), \hat{\mu}_{\lambda,k}^{(i+1)}, \hat{\sigma}_{\lambda,k}^{2(i+1)})$, we can get an estimate of $\boldsymbol{\theta}_k$. Finally, bringing $\boldsymbol{\theta}_k$ into (33)-(35) can obtain the estimated results of other parameters in the $(i+1)$th step.

It is noted that since $\hat{\boldsymbol{\theta}}_k^{(i+1)}$ is not affected by $\hat{\sigma}_{\lambda,k}^{2(i+1)}$in the profile likelihood function L_k, only $\hat{\sigma}_{B,k}^{2(i+1)}$ is considered when maximizing the profile likelihood function L_k to estimate $\hat{\boldsymbol{\theta}}_k^{(i+1)}$.

For the calculation of $\hat{\boldsymbol{\theta}}_k^{(i+1)}$, the maximum value can be searched by the ''Fminsearch'' function in Matlab. The above E-step and M-step need to be repeated for iterative calculation until $\left\| \hat{\boldsymbol{\Theta}}^{(i+1)} - \hat{\boldsymbol{\Theta}}^{(i)} \right\|$ is sufficiently small.

B. ADAPTIVE PARAMETER ESTIMATION

For the field degradation data $\boldsymbol{X}_{1:k}$ of a single evaluated item, its drift coefficient λ only represents the degradation rate information of a single item, which can be regarded as a constant. Then given the field degradation data $\boldsymbol{X}_{1:k}$, the likelihood function can be expressed as:

$$\ln L(\lambda, \sigma_B^2|X_{1:k}) = -\frac{k}{2}(\ln 2\pi + \ln \sigma_B^2) - \frac{1}{2}\sum_{j=1}^{k} \ln \Delta t_j - \frac{1}{2\sigma_B^2}\sum_{j=1}^{k} \frac{1}{\Delta t_j}(\Delta x_j - \lambda\alpha(t))^2 \quad (37)$$

By taking the partial derivatives of σ_B^2 and λ for (37), and making their partial derivatives be zero, we have:

$$\hat{\sigma}_B^2(\hat{\boldsymbol{\theta}}) = \frac{1}{k}\sum_{j=1}^{k} \frac{1}{\Delta t_j}(\Delta x_j - \hat{\lambda}\alpha(t))^2 \quad (38)$$

$$\hat{\lambda}(\hat{\boldsymbol{\theta}}) = \frac{\sum_{j=1}^{k} \frac{\Delta x_j \alpha(t)}{\Delta t_j}}{\sum_{j=1}^{k} \frac{\alpha(t)^2}{\Delta t_j}} \quad (39)$$

By taking (38) and (39) into (40), we can get the profile likelihood function about $\boldsymbol{\theta}$, as shown in (40).

$$L(\hat{\boldsymbol{\theta}}|\hat{\sigma}_B^2(\hat{\boldsymbol{\theta}}), \hat{\lambda}(\hat{\boldsymbol{\theta}})) = -\frac{k}{2}(\ln 2\pi + \ln \hat{\sigma}_B^2(\hat{\boldsymbol{\theta}}) + 1) - \frac{1}{2}\sum_{j=1}^{k} \ln \Delta t_j \quad (40)$$

By maximizing $L(\hat{\boldsymbol{\theta}}|\hat{\sigma}_B^2(\hat{\boldsymbol{\theta}}), \hat{\lambda}(\hat{\boldsymbol{\theta}}))$, we can get the estimation of $\hat{\boldsymbol{\theta}}$.

$$\hat{\boldsymbol{\theta}} = \arg\max L(\hat{\boldsymbol{\theta}}|\hat{\sigma}_B^2(\hat{\boldsymbol{\theta}}), \hat{\lambda}(\hat{\boldsymbol{\theta}})) \quad (41)$$

Due to the complexity of the likelihood function, it is not easy to directly calculate its partial derivative. This paper directly uses the ''Fminsearch'' function in the Matlab to search the nonlinear coefficient $\boldsymbol{\theta}$. Then, σ_B^2 and λ can be derived by bringing the nonlinear coefficient $\boldsymbol{\theta}$ into (38)-(39).

Based on the above MLE results, the following conclusions can be drawn.

Theorem 2: For the nonlinear Wiener process, the final iteration results based on the EM algorithm is the same as the results by using the MLE method directly based on the field observation data.

Proof: Setting $\boldsymbol{\Theta}_k = \boldsymbol{\Theta}_k^{(i)} = \boldsymbol{\Theta}_k^{(i+1)}$ gives

$$\sigma_{B,k}^2 = \frac{1}{k}\sum_{j=1}^{k} \frac{1}{\Delta t_j}\left[(\Delta x_j - \mu_{\lambda,k}\alpha(t))^2 + \sigma_{\lambda,k}^2\alpha(t)^2\right] \quad (42)$$

$$\mu_{\lambda,k} = \frac{\sigma_{\lambda,k}^2 B_k + \mu_{\lambda,k}\sigma_{B,k}^2}{\sigma_{\lambda,k}^2 A_k + \sigma_{B,k}^2} \quad (43)$$

$$\sigma_{\lambda,k}^2 = \frac{\sigma_{\lambda,k}^2\sigma_{B,k}^2}{\sigma_{\lambda,k}^2 A_k + \sigma_{B,k}^2} \quad (44)$$

It can be known from the iterative process of the EM algorithm that the iteration result of the expectation EM is the solution of the above equation group. From (43) and (44), we have $\hat{\sigma}_{\lambda,k}^2 = 0$ and

$$\hat{\mu}_{\lambda,k} = B_k/A_k = \frac{\sum_{j=1}^{k} \frac{\Delta x_j \alpha(t)}{\Delta t_j}}{\sum_{j=1}^{k} \frac{\alpha(t)^2}{\Delta t_j}} \quad (45)$$

Bringing $\hat{\mu}_{\lambda,k}$ and $\hat{\sigma}_{\lambda,k}^2$ into (42) can obtain (38).

This completes the proof.

Remark 2: The results of Theorem 2 show that for a single item, the parameters estimation results based on the EM algorithm converges to the results by the direct MLE method. Therefore, compared with the traditional Bayesian method that requires accurate prior information to update the random parameters, it can be observed that the EM algorithm can overcome the impact of imperfect prior information for the RUL prediction. However, this algorithm completely discards the prior information of congeneric items, and only uses the on-site degradation information. In addition, the estimation of the fixed coefficients of the single item, such as the diffusion coefficient or the nonlinear coefficient, is not accurate enough. Therefore, how to effectively combine the prior information and the field degradation information for the case of RUL prediction is a worthy research direction. Therefore, this paper applies a heuristic algorithm proposed by Tang *et al.* [37] that reasonably fuses the prior information and on-site degradation information for the basic nonlinear Wiener process.

C. PARAMETER UPDATING BASED ON A HEURISTIC ALGORITHM

In the RUL prediction process under the Bayesian framework, the field degradation data reflects the individual information of the evaluated equipment. As the field degradation data increases, the credibility of the field degradation information becomes higher. Therefore, the number of iterations of parameters updating can be selected based on the credibility of the prior information and the number of field degradation

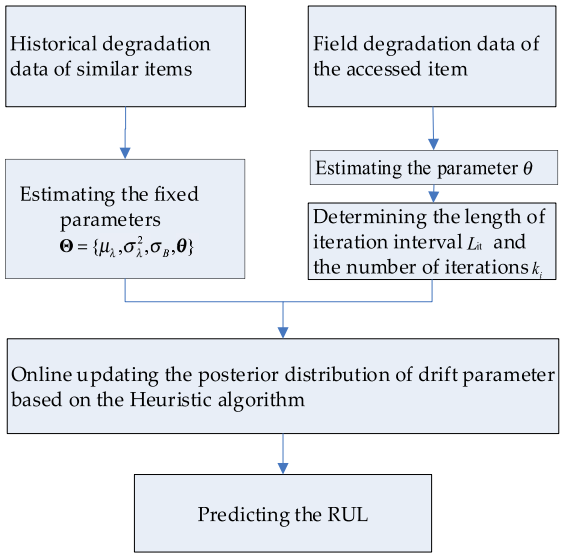

FIGURE 1. Flow chart of the heuristic algorithm for the nonlinear degradation process.

data, which reflects the credibility of the field degradation data [35]. In the later simulation experiments, it is found that the converged results of the EM algorithm are a certain distance away from the MLE, which may be caused by the "Fminsearch" search function in Matlab. However, if the nonlinear coefficient $\boldsymbol{\theta}$ is not updated, the EM algorithm converges to the MLE. Therefore, in the heuristic algorithm proposed in this paper, the nonlinear coefficient $\boldsymbol{\theta}$ does not participate in iteration, and is only estimated by the MLE method. Based on the interesting conclusion in the above subsection, a heuristic RUL prediction method for the nonlinear Wiener process is proposed, as shown in Fig. 1. This method can be divided into the following five steps:

Step 1: Based on the historical degradation data of congeneric items, the MLE method is used to estimate the fixed parameters, i.e. $\boldsymbol{\Theta} = \{\mu_\lambda, \sigma_\lambda^2, \sigma_B, \boldsymbol{\theta}\}$;

Step 2: Given the field degradation data $\boldsymbol{X}_{1:k}$, the nonlinear coefficient $\boldsymbol{\theta}$ is estimated based on the MLE method. The specific calculation process can refer to (37)-(41);

Step 3: According to the accuracy of the prior information, select the length of the iteration interval L_{it};

Step 4: Make the adaptive parameters updating method in section IV.B iterate i times, where i conforms that $(k-1)/L_{\text{it}} \leq i < k/L_{\text{it}}$. Note that, the nonlinear coefficient does not participate in the iterative calculation;

Step 5: Substitute $\boldsymbol{\Theta}_k$ in Step 4 into (26) to predict the RUL.

Since the fixed parameters reflect the common characteristics of congeneric items, the accuracy of the estimated results using historical degradation data of congeneric items' is higher than that of field degradation data using a single item. Therefore, when we use the heuristic algorithm to update the parameters online, only the mean and variance of the drift coefficients which indicate individual differences are updated, while the diffusion coefficient and nonlinear coefficient remain constant.

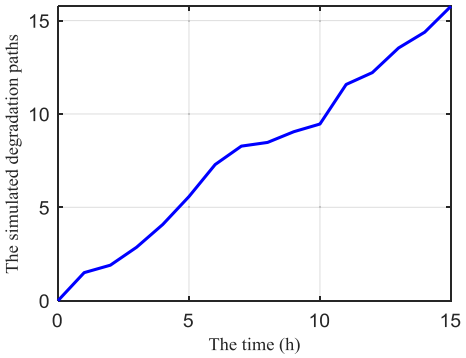

FIGURE 2. Flow simulated linear degradation process.

V. EXPERIMENT

A. NUMERICAL EXAMPLES

This subsection uses simulation data to illustrate that for the linear Wiener process of a single item, the parameter estimation results by the EM algorithm eventually converge to the estimation results by the MLE method. The degradation data of a single item is obtained by the Monte Carlo simulation algorithm. The initial parameters are set to be $\mu_\lambda = 1$, $\sigma_\lambda^2 = 0.0625$ and $\sigma_B^2 = 0.25$. Then, the following 15 specific simulated degradation data are obtained, i.e. 1.50, 1.90, 2.86, 4.08, 5.58, 7.29, 8.28, 8.48, 9.05, 9.46, 11.58, 12.22, 13.53, 14.38, 15.77, as shown in Fig. 2:

The parameters estimation results obtained by using the MLE method are $\mu_\lambda = 1.05$, $\sigma_\lambda^2 = 0$ and $\sigma_B^2 = 0.28$. In this numerical simulation, we take the parameters estimation results based on the MLE method as the reference value. Then, the correct prior information is closer to the MLE parameter estimation value, while the random prior frame information is far away from the MLE parameter estimation value. Therefore, random prior information can be regarded as imperfect prior information. Before adopting the EM algorithm to updating the unknown parameters, we set the correct prior information, and random prior information be $\mu_\lambda = 1$, $\sigma_\lambda^2 = 0.05$, $\sigma_B^2 = 1$ and $\mu_\lambda = 1.2$, $\sigma_\lambda^2 = 0.02$, $\sigma_B^2 = 1.5$ respectively. The results of 150 times iteration by the EM algorithm are shown in Fig. 3. It can be observed that, whether it is based on the correct prior information or random prior information, it all eventually converges to the MLE results. The difference between these two prior information is that the convergence speed of the EM algorithm based on correct prior information is significantly faster than that based on the random prior information. That is to say, for the linear Wiener degradation process of a single item, the correctness of the prior information only affects the convergence speed of the parameter estimation but does not affect the final parameter estimation results. As long as the number of iterations is sufficient, the same parameters estimation results can be finally obtained. This also validates Theorem 1 from an experimental viewpoint.

The conclusion presented in Section IV.B shows that for the nonlinear Wiener degradation process of a single item, the parameter estimation result based on the EM algorithm

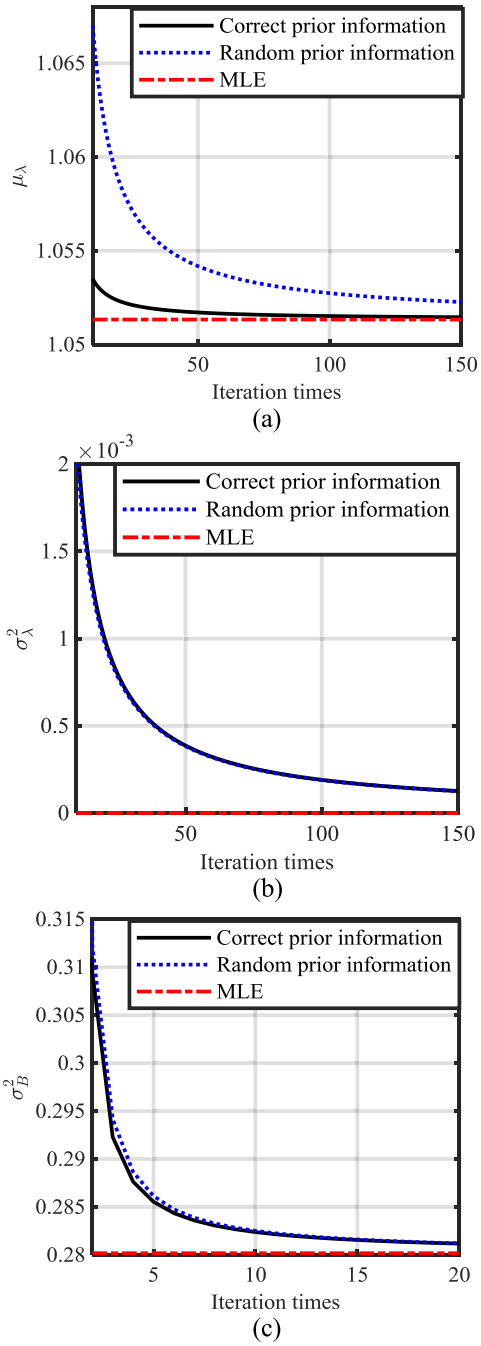

FIGURE 3. The estimated parameters by the EM algorithm of the linear degradation model: (a) μ_λ; (b) σ_λ^2; (c) σ_B^2.

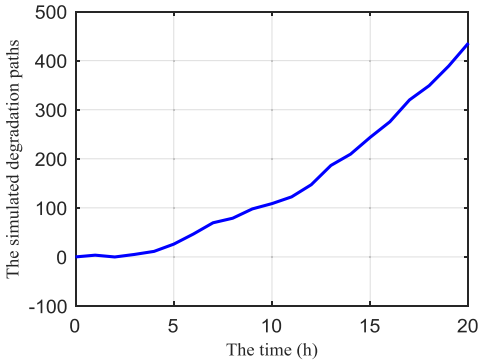

FIGURE 4. Simulated nonlinear degradation process.

also converges to the parameter estimation results based on the MLE method. In the following, we use a simulation to validate this conclusion from an experimental viewpoint. Without loss of generality, we set $\Lambda(t;\boldsymbol{\theta}) = t^b$ in this experimental simulation. The degradation data of a single item is obtained by the Monte Carlo simulation algorithm. The initial parameters are set to be $\mu_\lambda = 1$, $\sigma_\lambda^2 = 0.0625$, $\sigma_B^2 = 64$ and $b = 2$. Then, we obtain 20 degradation simulated data, which are presented as follows: 3.58, −0.097, 5.06, 11.30, 26.04, 46.67, 69.58, 78.8, 97.84, 108.64, 122.41, 147.19, 186.45, 209.45, 243.7, 275.40, 319.9, 349.05, 389.27 and 435.80. We also display the simulated degradation process as shown in Fig. 4.

The parameter estimation result obtained by the MLE algorithm is $\mu_\lambda = 1.05$, $\sigma_B^2 = 41.64$ and $b = 2.01$. Before adopting the EM algorithm to updating the parameters, we set the correct prior information and random prior information be $\mu_\lambda = 0.8$, $\sigma_\lambda^2 = 0.02, \sigma_B^2 = 50$, $b = 1.85$ and $\mu_\lambda = 0.5$, $\sigma_\lambda^2 = 0.1$, $\sigma_B^2 = 60$, $b = 1.2$ respectively. Compared with the correct prior information, the parameters of random prior information are more inaccurate, especially the nonlinear coefficient b. Therefore, random prior information can be regarded as imperfect prior information. The estimated parameters results are shown in Fig. 5, where the EM algorithm is iterated 500 times. They can be seen that the estimation results based on the correct prior information are significantly closer to the MLE than the results based on the random prior information. And compared with the random prior information, the correct prior information has a faster convergence speed. However, it is still different from the MLE. Moreover, with the increase of the number of iterations, the changes of the mean drift coefficient and nonlinear coefficient are not obvious, which may be caused by the local minimum caused by the "Fminsearch" search function in Matlab.

To solve this problem, this paper does not update the nonlinear coefficient in the EM algorithm. First, the nonlinear coefficient b is estimated by the MLE method, which is $b = 2.01$. Other parameters are estimated by the EM algorithm Similar to the linear case, we also set the correct prior information and random prior information be $\mu_\lambda = 0.8$, $\sigma_\lambda^2 = 0.02$, $\sigma_B^2 = 50$ and $\mu_\lambda = 0.5$, $\sigma_\lambda^2 = 0.05$, $\sigma_B^2 = 60$, respectively. The estimated results after 500 iterations are shown in Fig. 6. From the comparison between Fig. 5 and Fig. 6, we can observe that when the nonlinear coefficient b does not participate in the iteration, the parameters estimation results based on correct prior information and random prior information both converge to the MLE results. This indicates that the EM algorithm can overcome the impact of imperfect prior information, and validates the proof of Theorem 2 from an experimental viewpoint.

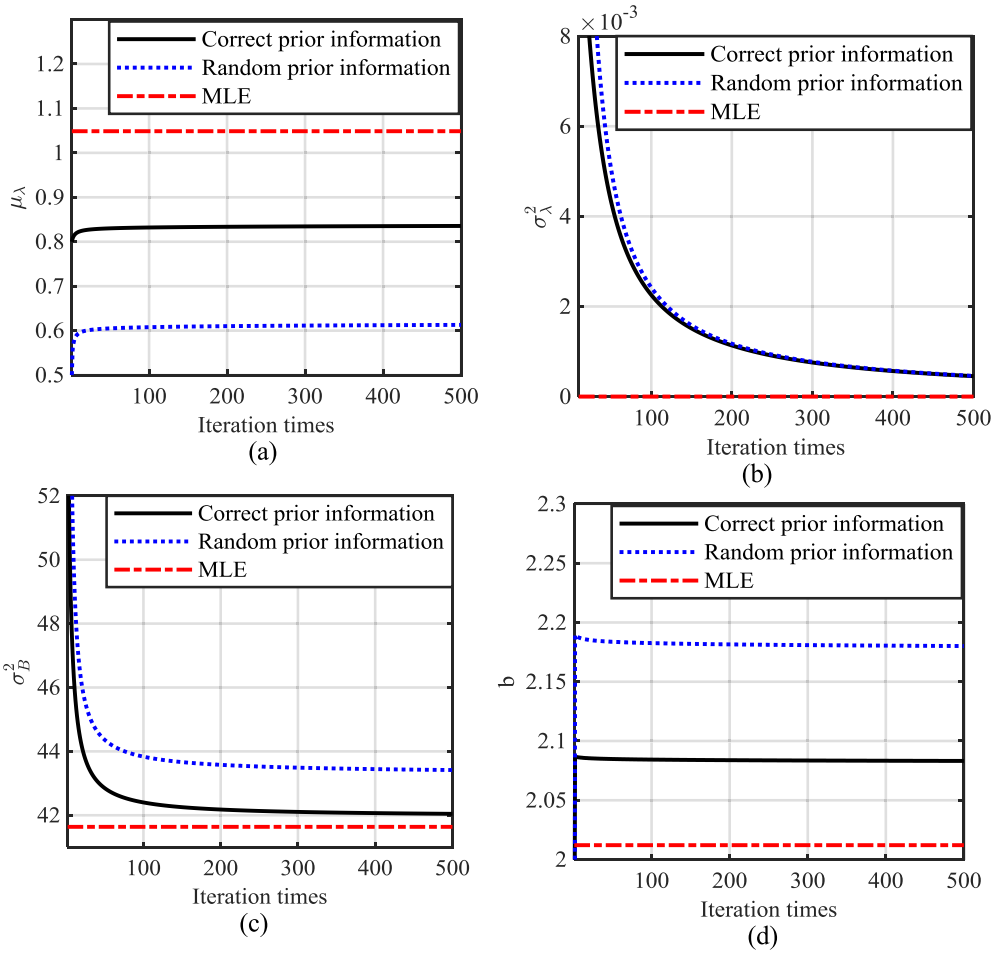

FIGURE 5. The estimated parameters by the EM algorithm of the nonlinear degradation model: (a) μ_λ; (b) σ_λ^2; (c) σ_B^2; (d) b.

B. A PRACTICAL CASE STUDY

Due to the uncertain aging effect of lithium battery, the RUL prediction of lithium battery has a great challenge [64]–[68]. The RUL prediction approaches of lithium battery are mainly divided into three types: particle filter, artificial intelligence and stochastic process modeling. Since the Brownian motion in Wiener process can well describe the uncertainty in the aging process of lithium battery, in this paper the proposed Wiener process based algorithm is verified by lithium battery data.

In this experiment, we use the data collected from the National Aeronautics and Space Administration (NASA) Ames Prognostics Center of Excellence to verify the superiority of the heuristic algorithm. The degradation data includes four lithium batteries (i.e. No.5, No.6, No.7, and No.18) that repeatedly work in three modes of charging, discharging, and impedance measurement at room temperature. And, the lithium batteries data includes two times scales, namely calendar time and cycle number. The original degradation process is displayed in Fig. 7. These degradation data have a relaxation effect, which leads to the recovery of the battery after some rest time. The degradation data after extracting the relaxation effect is shown in Fig. 8. More details regarding the extracting methods can be found in [62].

In this experiment, we use the No. 5 lithium battery as the prediction object. No.6, No.7, and No.18 lithium batteries are regarded as congeneric items for obtaining the prior information. First, the prior parameters estimated by the MLE method are $\mu_\lambda = 0.00831$, $\sigma_\lambda^2 = 8.82 \times 10^{-6}$, $\sigma_B^2 = 2.04 \times 10^{-4}$ and $b = 0.9711$. It can be seen that the degradation data of the No. 6, 7, and 18 lithium batteries is near linear. This is different from the No. 5 battery, which has obvious nonlinear characteristic as shown in Fig. 8. This indicates that the parameters which are estimated by No. 6, 7, and 18 lithium batteries can be regarded as the imperfect prior information compared with the No. 5 battery.

In this paper, we set the length of the interval satisfies that $L_{\text{it}} = 20$. Then, the RUL distribution can be obtained as displayed in Fig. 9. It shows that despite the prior information is inaccurate or scarce, the RUL distribution predicted by the traditional Bayesian method and the heuristic algorithm can both cover the actual RUL. However, the RUL distribution by

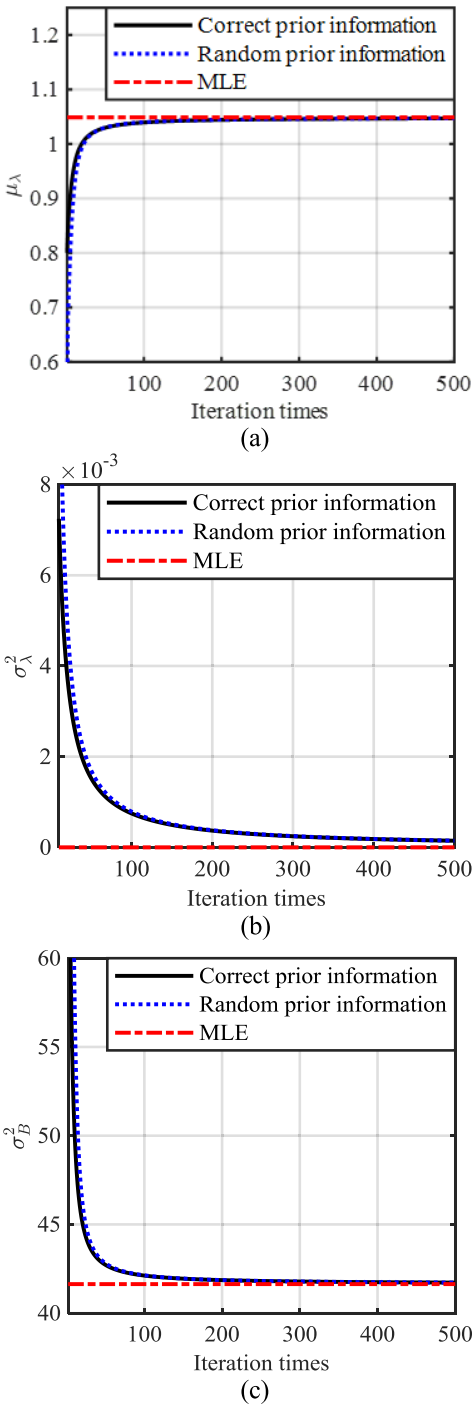

FIGURE 6. The estimated parameters by the EM algorithm of the nonlinear degradation model without updating nonlinear coefficient: (a) μ_λ; (b) σ_λ^2; (c) σ_B^2.

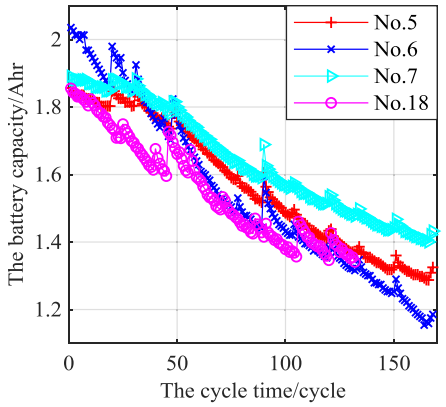

FIGURE 7. Cycle-based lithium battery capacity degradation data.

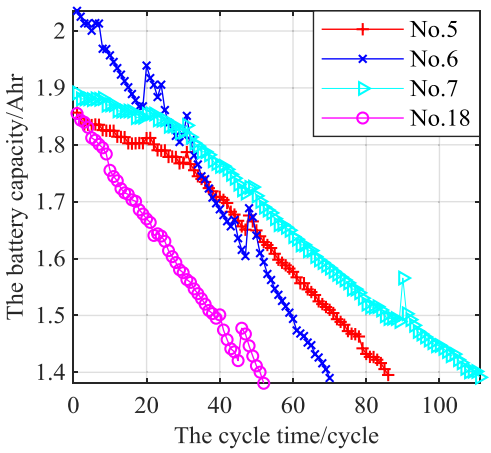

FIGURE 8. Lithium battery degradation data after elimination.

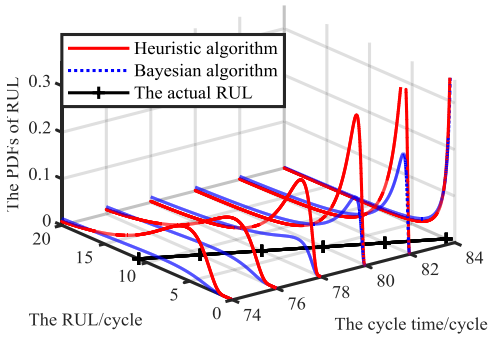

FIGURE 9. Prediction of RUL based on random prior information.

the heuristic algorithm is narrower and more concentrated in the around of real RUL, which indicates a higher prediction accuracy. And the Bayesian method has a significant error, especially when the RUL is relatively large. Then, this paper uses the mean square error (MSE) to measure the forecasting effect. The MSE is a statistic value used to measure the deviation between the predicted RUL and the actual RUL. The mathematical expression can be written as follows.

$$MSE_k = \int_0^{+\infty} (l_k + t_k - T)^2 f_{L_k|\mathbf{Y}_{1:k}}(l_k|\mathbf{Y}_{1:k})\mathrm{d}l_k \quad (46)$$

The MSEs by different methods is shown in Fig. 10. It shows that the MSE of the heuristic algorithm is much smaller than that of the traditional Bayesian algorithm, especially when the RUL is large. And the prediction accuracy of the traditional Bayesian method has a large deviation, and the heuristic algorithm can cover the actual RUL better. The

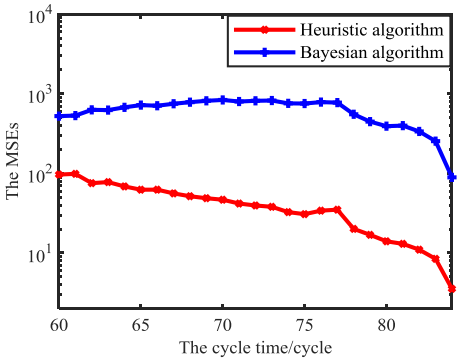

FIGURE 10. Mean square error based on random prior information.

reason is that when the number of cycles is small, the field information is not enough. When the prior information is not accurate enough, the traditional Bayesian method could cause a large deviation. The heuristic algorithm in this paper fuses the field information and prior information through reasonable iteration, and thus achieves an ideal prediction accuracy. This indicates that the prediction accuracy of the heuristic algorithm is high, which is consistent with result implied by Fig. 9. In general, the heuristic RUL prediction method proposed in this paper can effectively overcome imperfect prior information.

VI. CONCLUSION

RUL prediction has always been the core content of PHM. This paper proposes a heuristic RUL prediction algorithm to control the number of iterations of the EM algorithm based on the credibility of the prior information and the number of on-site information detection for the situation where the prior information is insufficient or inaccurate. Practical experiments show that the heuristic algorithm has higher residual life prediction accuracy than the traditional Bayesian algorithm when there is imperfect prior information.

The main innovations of this article are summarized as follows:

(1) It is proved that for the basic linear and nonlinear Wiener degradation process, the parameters estimation results by the EM algorithm based on the field degradation data converge to the MLE results, which is also verified by the simulation experiments.

(2) Based on the joint EM and Bayesian algorithm, this paper proposes a heuristic algorithm for parameters adaptively updating. The heuristic algorithm is mainly used to update the drift coefficient's expectation and variance by using the field degradation data, which is suitable for both the linear and nonlinear Wiener processes. In this paper, we do not update the nonlinear coefficient and diffusion coefficient and only update the mean and variance of the drift coefficient. When the prior information is not accurate or insufficient, the heuristic algorithm can significantly improve the RUL prediction accuracy with comparing the traditional Bayesian method.

Although the usefulness and superiority of the modified the heuristic algorithm are demonstrated by a numerical example and a case study in this paper. The following questions are still not answered. E.g. How do the heuristic algorithms predict RUL when the drift coefficient follows other distributions? How to solve the RUL prediction problem under imperfect prior information by AI approach? How to handle the measurement noise and shift noise under imperfect prior information? How to evaluate the level of the prior information? That is to say, which prior information is correct or imperfect?

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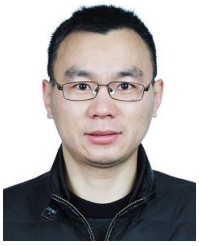

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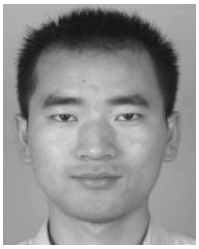

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