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Consensus Control of Position-Constrained Multi-Agent Systems Without the Velocity Information of Neighbors

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ABSTRACT Currently, most consensus control approaches need each agent to access all its neighbors' states directly. The distributed consensus issue of second-order multi-agent systems subject to position constraints is studied without such requirements. The only condition for the communication topology is to include a directed spanning tree. An innovative reference position is provided to deal with the position constraints while eliminating the need for neighbors' velocity variables. An adaptive control method is designed by constructing a sliding-mode-esque variable so that each agent's transformed position can converge towards the reference position. This new method can guarantee uniform boundedness of each closed-loop signal as well as asymptotic consensus, and the requirements to meet the position constraints are satisfied at all times. Numerical simulation verifies the correctness of the theoretical results.

INDEX TERMS Consensus, adaptive control, uncertain dynamics, directed graphs.

I. INTRODUCTION

Distributed control of multi-agent systems is a relatively new research field that is of great interest because of its wide range of potential applications, such as aerial systems, robotic systems, and collaborative surveillance. One of the critical requirements is the distributed consensus of a multi-agent system, where all agents attempt to arrive at the same position utilizing only limited information [1]–[5]. The initial approach focuses on the consensus of simple multi-agents, without considering the agent's uncertainty. These methods have then been generalized to the consensus literature of complex uncertain multi-agent systems. With the aid of neural network approximation and robust control technology, an adaptive distributed controller is designed for the first-order nonlinear multi-agent system under undirected communication topologies [6]. Reference [7] has studied the consensus control of first-order and second-order linearly parameterized multi-agent systems with unknown identical control directions. By constructing an auxiliary signal using the consensus value estimation, the leaderless consensus control issue of the uncertain nonlinear multi-agent system with different control directions has been addressed in our previous work [8]. A limiting assumption that is usually made on second- and higher-order multi-agent setups is that each neighbors' state must be available for the realization of each agent's controller [7], [8], which poses a considerable challenge when only the neighbors' position-like state can be measured.

For a second-order multi-agent system, a distributed consensus algorithm on a general directed graph has been proposed without requiring velocity states from neighboring agents in [9]. Furthermore, a consensus algorithm has been presented in [10] with a simple static compensator structure to realize consensus, using only neighbors' position-like states, for a higher-order multi-agent system. However, these works [9], [10] do not consider the position restrictions of each agent. In reality, most of the physical system are subject to environmental regulations, saturation or performance and safety specifications, and their position should be limited within a specific range. Therefore, it is of paramount importance to take position constraints into account when designing distributed control laws. In recent years, a variety of solutions have been provided to deal with output constraints of single nonlinear systems [11]–[13]. For the first-order nonlinear multi-agent system in the presence of state constraints,

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a consensus controller is designed by using Laplacian matrix symmetric positive semi-definiteness on the undirected connected graph and state transformation technology in [14]. In addition, the results have been extended to higher-order multi-agents with output limitations and unknown control directions under an undirected graph [15]. With the further development of the directed graph in [16], a desired output is presented using a transformation strategy, and a consensus control law is obtained in the backstepping framework.

This article considers the leaderless consensus issue of uncertain second-order nonlinear agents, including position constraints. The communication topology is a directed graph including a spanning tree. A novel reference position is constructed for every agent, which plays a vital role in handling the position limitations while eliminating the requirement of neighbors' velocity variables. Further, the control problem of the multi-agent system becomes a single agents' regulation control issue. We derive an adaptive control input by constructing a sliding-mode-esque variable so that each agent's transformed position converges towards the reference position. This new method can guarantee uniform boundedness of each closed-loop signal and asymptotic consensus, and the requirements to meet the position constraints are always satisfied. Compared with state-of-the-art results, the novelty and additive value of the present paper are:

(1) We construct a new dynamic signal to deal with the position constraints, and at the same time, to relax the limitations of the above relevant research by removing the requirement of neighbors' velocity measurements, profiting from which the intricate leaderless consensus issue of the position-constrained multi-agent system under directed communication topologies can be interpreted into a single agents' regulation control issue.

(2) Compared with the result of nonlinear multi-agent systems [6]–[8], the considered multi-agent model, including position constraints, is more practical and incorporates a more comprehensive application. All agent positions' asymptotic consensus of the current controller can be accomplished, contrary to the uniformly ultimate boundedness result [6].

(3) Different from the position-constrained consensus controller under the undirected graph [14], [15], this innovative distributed method is able to address the consensus issue and the position-constrained issue under the directed topology condition. In addition, neighbors' velocity measurement required in [16] is unnecessary.

The problem formulation and the basic graph theory are described in Section II. Section III covers the novel reference position, controller design, and stability and convergence properties. Extensive simulation results and conclusion are, respectively, provided in Sections IV and V.

Notation: For a vector function $\eta(t)$, we say $\eta \in \mathcal{L}_\infty[0, t_f)$ if $\sup_{0\leq t<t_f} \|\eta(t)\| < \infty$, and $\eta \in \mathcal{L}_p[0, t_f)$ if $(\int_0^{t_f} \|\eta(t)\|^p dt)^{1/p} < \infty, p = 1, 2$. 1_ℓ and 0_ℓ are the $\ell-$vector of all ones and all zeros, respectively. I_ℓ is the $\ell \times \ell$ identity matrix. $\mathrm{diag}\{k_1, \ldots, k_\ell\}$ is the diagonal matrix with diagonal entries k_1 to k_ℓ.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. GRAPH THEORY

$\mathcal{G} = \{\mathcal{V}, \mathcal{E}, A_n\}$ denotes a directed graph modeling the topology among the N agents, in which $\mathcal{V} = \{1, \ldots, N\}$ represents the set of nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of directed edges, and $A_n = [a_{ij}] \in R^{N\times N}$ is called the adjacency matrix. An edge $(i, j) \in \mathcal{E}$ suggests that node j can get data from node i, and node i is a neighbor of node j. The set of all neighbors of node i is denoted by $\mathcal{N}_i$. A directed path from node i_1 to node i_p is a sequence of directed edges in the form of (i_m, i_{m+1}), $m = 1, \ldots, p-1$. A directed graph is said to contain a directed spanning tree if there exists at least a node from which there is a directed path to each other node in $\mathcal{G}$. $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The Laplacian matrix $L_n = [l_{ij}] \in R^{N\times N}$ associated with $\mathcal{G}$ is defined as $l_{ii} = \sum_{j\in\mathcal{N}_i} a_{ij}$ and $l_{ij} = -a_{ij}, i \neq j$. $D_n = \mathrm{diag}\{d_1, \ldots, d_n\}$ represents the in-degree matrix with $d_i = l_{ii}$ being the in-degree of node i.

B. PROBLEM FORMULATION

A multi-agent group comprised of $n(n \geq 2)$ agents, labeled as agents 1 to n, is considered under a directed communication topology. The dynamics of the ith, $i = 1, \ldots, n$, agent can be represented by

$$\begin{aligned} \dot{p}_i &= v_i \\ \dot{v}_i &= u_i + \varphi_i(p_i, v_i)^T \theta_i + \tau_i(p_i, v_i, t) \end{aligned} \tag{1}$$

in which $p_i \in R$, $v_i \in R$, and $u_i \in R$ are, respectively, the position state, the velocity state, and the control input, $\theta_i \in R^{m_i}$ is an unknown constant vector, $\varphi_i : R^2 \to R^{m_i}$ is a known smooth vector-valued function, $\tau_i \in R$ represents the unknown piecewise continuous system uncertainty. In this study, each agent's position p_i should be restricted to an open set, namely, $L < p_i < U$, where L and U (satisfying $L < U$) are known constants.

The problem to be solved in this work is to develop a distributed controller u_i for each agent (1) so that (i) each closed-loop signal remains bounded, (ii) the agent positions achieve asymptotic consensus, i.e., $\lim_{t\to\infty}(p_i(t)-p_j(t)) = 0$ and $\lim_{t\to\infty} v_i(t) = 0$ for all $i, j = 1, \ldots, n$, and (iii) the requirements to meet the position constraints are satisfied at all times, namely,

$$L < p_i(t) < U, \quad \forall t \geq 0. \tag{2}$$

In order to solve the above-mentioned multi-agent control problem, we make the following assumptions about the agent (1).

Assumption 1: The functions $\tau_i(p_i, v_i, t)$, $i = 1, \ldots, n$ are bounded by $|\tau_i(p_i, v_i, t)| \leq \tau_i^*$, $\forall[p_i, v_i, t]^T \in R^2 \times [0, \infty)$, where τ_i^* are unknown positive constants.

Assumption 2: The initial conditions $p_i(0)$ are inside the constraint bounds, i.e., $L < p_i(0) < U$ for all $i = 1, \ldots, n$.

III. MAIN RESULT

This section introduces an adaptive control approach that enables the agent position to achieve asymptotic consensus without neighbor velocity variables. Before starting the design, $\mathcal{V}$ is partitioned into two subsets as $\mathcal{V}_1$ and $\mathcal{V}_2$, in which $\mathcal{V}_1 = \{i \in \mathcal{V}|d_i \neq 0\}$ and $\mathcal{V}_2 = \{i \in \mathcal{V}|d_i = 0\}$. To deal with the position constraints while eliminating the need for the velocity variables of its neighbors, the new dynamic policy to produce a reference position $x_{i,1}$ for the ith ($i \in \mathcal{V}_1$) agent is designed as

$$\begin{aligned}\dot{x}_{i,1} &= x_{i,2}\\ \dot{x}_{i,2} &= \gamma_i \sum_{j=1}^{n} a_{ij}\xi_j - \lambda_{i,1}x_{i,1} - \lambda_{i,2}x_{i,2}\end{aligned} \tag{3}$$

where $\xi_j = \ln((p_j - L)/(U - p_j))$ is intentionally designed to deal with the position constraints, $\lambda_{i,1} > 0$ and $\lambda_{i,2} > 0$ are constants chosen to make the roots of $s^2 + \lambda_{i,2}s^{m-1} + \lambda_{i,1} = 0$ negative real numbers, and γ_i is set to $\gamma_i = \lambda_{i,1}/d_i$. Note that no data of other agents can be obtained by the agents in $\mathcal{V}_2$. The reference position of the agent in $\mathcal{V}_2$ is designed as $x_{i,1} = \gamma_i$ with $\dot{x}_{i,1} = x_{i,2} = 0$, where γ_i is a constant.

The distributed control strategy for the ith, $i = 1, \ldots, n$, agent (1) is selected as

$$u_i = -\hat{\theta}_i^T \varphi_i(p_i, v_i) - \triangle_i - \frac{k_i z_i + \eta_i v_i - x_{i,2} + \dot{\eta}_i v_i - \dot{x}_{i,2}}{\eta_i} \tag{4}$$

in which $z_i = e_i + \dot{e}_i$ is a sliding-mode-esque signal with $e_i = \xi_i - x_{i,1}$, $k_i > 0$ is a parameter representing the control gain, $\eta_i = \frac{U-L}{(p_i-L)(U-p_i)}$, $\triangle_i(t) = \hat{\omega}_i \tanh(z_i\eta_i/\varepsilon_i(t))$ denotes a robust term. $\varepsilon_i(t)$ represents a positive smooth function meeting $\int_0^\infty \varepsilon_i(t)dt \leq \bar{\varepsilon}_i$, in which $\bar{\varepsilon}_i > 0$ is a finite constant. $\hat{\theta}_i$ and $\hat{\omega}_i$ are the estimates of unknown parameters θ_i and $(\tau_i^* + (U - L)/4)$, respectively. The update strategies for $\hat{\theta}_i$ and $\hat{\omega}_i$ are proposed as

$$\dot{\hat{\theta}}_i = \Gamma_i \eta_i z_i \varphi_i(p_i, v_i),\ \dot{\hat{\omega}}_i = \mu_i \eta_i z_i \tanh(\eta_i z_i/\varepsilon_i(t)) \tag{5}$$

where $\Gamma_i \in R^{m_i \times m_i}$ denotes a positive definite adaptive gain matrix and μ_i denotes a positive parameter.

Having developed the control method and reference position, we are ready to state our main consensus results.

Theorem 1: Assume that the graph $\mathcal{G}$ includes a directed spanning tree. A second-order nonlinear multi-agent system consisting of n agents (1) is considered. Let Assumptions 1-2 hold. The proposed distributed control strategy (4) with the reference position (3) and parameter update laws (5) guarantees that: (i) the agent positions achieve asymptotic consensus, namely, $\lim_{t\to\infty}(p_i(t)-p_j(t)) = 0$ and $\lim_{t\to\infty} v_i(t) = 0$ for all $i, j = 1, \ldots, n$, (ii) each closed-loop signal remains bounded, and (iii) the requirements to meet the position constraints are satisfied at all times, namely, $L < p_i(t) < U$, $\forall t \geq 0$.

Proof: To begin with, we write the agents (1) with control strategies (4) and adaptive parameters (5) in vector form. Towards this direction, let us define generalized states $\zeta = [p^T, v^T, x_1^T, x_2^T, \hat{\theta}^T, \hat{\omega}^T]^T \in R^{5n+m}$, where $m = m_1 + \cdots + m_n$, $p = [p_1, \ldots, p_n]^T \in R^n$, $v = [v_1, \ldots, v_n]^T \in R^n$, $x_1 = [x_{1,1}, \ldots, x_{n,1}]^T \in R^n$, $x_2 = [x_{1,2}, \ldots, x_{n,2}]^T \in R^n$, $\hat{\theta} = [\hat{\theta}_1^T, \ldots, \hat{\theta}_n^T]^T \in R^m$, and $\hat{\omega} = [\hat{\omega}_1, \ldots, \hat{\omega}_n]^T \in R^n$. Then the closed-loop dynamical system of ζ takes the form $\dot{\zeta} = f(\zeta, t)$. The nonempty and open set $\Omega = \{\zeta \in R^{5n+m}|L < p_i < U, i = 1, \ldots, n\}$ is defined. Because of the piecewise continuity and boundedness of $\tau_i(p_i, v_i, t)$ as well as smoothness of nonlinearities $\varphi_i(p_i, v_i)$, it follows from (3)-(5) that $f(\zeta, t)$ is piecewise continuous and locally Lipschitz. That means a unique continuous solution ζ over the set Ω exists as shown in [17, Th. 54]. Let the maximum interval of existence be $[0, t_f)$.

In view of the definition of ξ_i, we obtain $\dot{\xi}_i = \eta_i v_i$ and $\ddot{\xi}_i = \dot{\eta}_i v_i + \eta_i \dot{v}_i$. Noting (3) and $z_i = e_i + \dot{e}_i$, we get

$$\begin{aligned}\dot{z}_i &= (\dot{\xi}_i - x_{i,2}) + (\ddot{\xi}_i - \dot{x}_{i,2})\\ &= \eta_i(u_i + \varphi_i^T\theta_i + \tau_i) + \dot{\eta}_i v_i - \dot{x}_{i,2} + \eta_i v_i - x_{i,2}.\end{aligned} \tag{6}$$

We now consider the Lyapunov function for the ith, $i = 1, \ldots, n$, agent as

$$V_i = \frac{1}{2}z_i^2 + \frac{1}{2}\tilde{\theta}_i^T\Gamma_i^{-1}\tilde{\theta}_i + \frac{1}{2\mu_i}(\tau_i^* + \frac{U-L}{4} - \hat{\omega}_i)^2 \tag{7}$$

with $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ being the parameter estimation error. Taking the derivative of V_i along (6) results in

$$\begin{aligned}\dot{V}_i &= z_i\dot{z}_i + \tilde{\theta}_i^T\Gamma_i^{-1}(-\dot{\hat{\theta}}_i) + \frac{1}{\mu_i}(\tau_i^* + \frac{U-L}{4} - \hat{\omega}_i)(-\dot{\hat{\omega}}_i)\\ &= z_i(\eta_i(u_i + \varphi_i^T\theta_i + \tau_i) + \dot{\eta}_i v_i - \dot{x}_{i,2} + \eta_i v_i - x_{i,2})\\ &\quad + \tilde{\theta}_i^T\Gamma_i^{-1}(-\dot{\hat{\theta}}_i) + \frac{1}{\mu_i}(\tau_i^* + \frac{U-L}{4} - \hat{\omega}_i)(-\dot{\hat{\omega}}_i).\end{aligned} \tag{8}$$

Substituting the control input (4) into (8), we have

$$\begin{aligned}\dot{V}_i &= -k_i z_i^2 + \eta_i z_i \varphi_i^T\tilde{\theta}_i + \eta_i z_i \tau_i - \eta_i z_i \triangle_i(t)\\ &\quad + \tilde{\theta}_i^T\Gamma_i^{-1}(-\dot{\hat{\theta}}_i) + \frac{1}{\mu_i}(\tau_i^* + \frac{U-L}{4} - \hat{\omega}_i)(-\dot{\hat{\omega}}_i).\end{aligned} \tag{9}$$

The substitution of (5) into (9) shows

$$\begin{aligned}\dot{V}_i &= -k_i z_i^2 - \eta_i z_i(\tau_i^* + (U-L)/4)\tanh(\eta_i z_i/\varepsilon_i(t)) + \eta_i z_i \tau_i\\ &\leq -k_i z_i^2 - \eta_i z_i(\tau_i^* + (U-L)/4)\tanh(\eta_i z_i/\varepsilon_i(t)) + \eta_i|z_i|\tau_i^*\\ &\leq \kappa_i\varepsilon_i - k_i z_i^2 - \eta_i z_i((U-L)/4)\tanh(\eta_i z_i/\varepsilon_i(t))\end{aligned}$$

where $\eta_i|z_i|\tau_i^* - \eta_i z_i \tau_i^* \tanh(\eta_i z_i/\varepsilon_i) \leq \kappa_i\varepsilon_i$ with $\kappa_i = 0.2785\tau_i^*$ is used [18]. Upon integration we arrive at

$$\begin{aligned}V_i(t) &\leq V_i(0) + \int_0^t \kappa_i\varepsilon_i(\sigma)d\sigma - \int_0^t k_i z_i^2(\sigma)d\sigma\\ &\quad - \int_0^t \frac{U-L}{4}\eta_i(\sigma)z_i(\sigma)\tanh(\frac{\eta_i(\sigma)z_i(\sigma)}{\varepsilon_i(\sigma)})d\sigma\\ &\leq V_i(0) + \kappa_i\bar{\varepsilon}_i.\end{aligned} \tag{10}$$

which leads us to the conclusion that $V_i, z_i, \tilde{\theta}_i, (\tau_i^* + (U-L)/4 - \hat{\omega}_i), \int_0^t \eta_i(\sigma)z_i(\sigma)\tanh(z_i(\sigma)\eta_i(\sigma)/\varepsilon_i(\sigma))d\sigma \in \mathcal{L}_\infty[0, t_f)$ and $z_i \in \mathcal{L}_2[0, t_f)$. The conclusion that $\hat{\theta}_i(t)$ and $\hat{\omega}_i(t)$ are bounded on $[0, t_f)$ then follows from that θ_i and τ_i^*

are constants. Viewing that $\eta_i(t) \geq 4/(U-L)$, $\forall t \in [0, t_f)$, it is easy to deduce

$$\begin{aligned} 0 \leq \int_0^t |z_i(\sigma)|d\sigma &\leq \int_0^t \frac{U-L}{4}\eta_i(\sigma)|z_i(\sigma)|d\sigma \\ &\leq \int_0^t \frac{U-L}{4}\eta_i(\sigma)z_i(\sigma)\tanh(\frac{\eta_i(\sigma)z_i(\sigma)}{\varepsilon_i(\sigma)})d\sigma \\ &+ \int_0^t \frac{U-L}{4}\kappa_i^*\varepsilon_i(\sigma)d\sigma \end{aligned}$$

which together with $\varepsilon_i \in \mathcal{L}_1[0,\infty)$ suggests that $z_i \in \mathcal{L}_1[0, t_f)$. Using the definition of z_i, we can also conclude $e_i, \dot{e}_i \in \mathcal{L}_1[0, t_f) \cap \mathcal{L}_\infty[0, t_f)$.

Observing (3), we rewrite the dynamics of the reference position as $\dot{\bar{x}}_i = A_i\gamma_i \sum_{j=1}^n a_{ij}\xi_i + B_i\bar{x}_i$, where $i \in \mathcal{V}_1$, $\bar{x}_i = [x_{i,1}, x_{i,2}]^T$, $A_i = [0, 1]^T$, and

$$B_i = \begin{bmatrix} 0 & 1 \\ -\lambda_{i,1} & -\lambda_{i,2} \end{bmatrix}.$$

The eigenvalues of B_i is represented by $c_{i,1}$ and $c_{i,2}$ in a non-specific order. For analytical purposes, a transformation matrix T_i is employed for the ith agent as

$$T_i = \begin{bmatrix} 1 & 0 \\ 1 & -1/c_{i,1} \end{bmatrix}.$$

Then, it holds that $T_iB_i = F_iT_i$, where

$$F_i = \begin{bmatrix} c_{i,1} & -c_{i,1} \\ 0 & c_{i,2} \end{bmatrix}.$$

Taking the state transformation $\bar{q}_i = [q_{i,1}, q_{i,2}]^T = T_i\bar{x}_i$ for $i \in \mathcal{V}_1$ and using $\xi_i = e_i + x_{i,1}$, we get

$$\begin{aligned} \dot{q}_{i,1} &= c_{i,1}q_{i,1} - c_{i,1}q_{i,2} \\ \dot{q}_{i,2} &= c_{i,2}q_{i,2} - (c_{i,2}/d_i)\sum_{j=1}^n a_{ij}(q_{j,1} + e_j) \end{aligned} \quad (11)$$

where $c_{i,1} = \lambda_{i,1}/c_{i,2}$ and $q_{i,1} = x_{i,1}$ are used.

There exists at most one agent with no neighbors since the communication topology involves a directed spanning tree. Two cases are considered: (*C1*) every agent can get data from at least one other agent, i.e., $\mathcal{V}_1 = \mathcal{V}$ and (*C2*) there is an agent that cannot obtain any data from any other agent.

C1: We start by defining the column vectors $\delta = [0_n^T, \delta_1, \ldots, \delta_n]^T$ and $q = [q_{1,1}, \ldots, q_{n,1}, q_{1,2}, \ldots, q_{n,2}]^T$, where $\delta_i = -(c_{i,2}/d_i)\sum_{j=1}^n a_{ij}e_j$, $i = 1, \ldots, n$. It follows from (11) that

$$\dot{q}(t) = -\bar{L}q(t) + \delta(t) \quad (12)$$

where

$$\bar{L} = \begin{bmatrix} -c_1 & c_1 \\ c_2\bar{A}_n & -c_2 \end{bmatrix} \quad (13)$$

$c_1 = \text{diag}\{c_{1,1}, \ldots, c_{n,1}\}$, $c_2 = \text{diag}\{c_{1,2}, \ldots, c_{n,2}\}$, and $\bar{A}_n = D_n^{-1}A_n$. Since $\bar{L}1_{2n} = 0_{2n}$ and each off-diagonal entry of $\bar{L}$ is a non-positive number, $\bar{L}$ has the form of a Laplacian matrix. Consequently, the system (12) can be seen as a multi-agent group comprised of $2n$ agents which are connected under the augmented directed graph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$. Here $\bar{\mathcal{V}} = \{1, \ldots, 2n\}$, $\bar{L}$ is the related Laplacian matrix, and the edge set $\bar{\mathcal{E}}$ is able to be obtained by (13). Noting that $rank(c_2) = n$ and $rank(c_2(I_n - \bar{A}_n)) = rank(L_n) = n-1$, it is readily verified that $rank(\bar{L}) = 2n-1$. We get from [1] that $\bar{L}$ has a single zero eigenvalue and all other eigenvalues have positive real parts. That implies there is a finite constant $\bar{m}$ enabling $\|e^{-\bar{L}t}\| \leq \bar{m}$ for all $t \geq 0$ [19, p. 138]. Integrating (12) over the interval $[0, t]$ yields $q(t) = e^{-\bar{L}t}q(0) + \int_0^t e^{-\bar{L}(t-\sigma)}\delta(\sigma)d\sigma$. Employing $\|e^{-\bar{L}t}\| \leq \bar{m}$ and $e_i \in \mathcal{L}_1[0, t_f)$, we have $q \in \mathcal{L}_\infty[0, t_f)$. The statement $\bar{x}_i \in \mathcal{L}_\infty[0, t_f)$ then follows from that T_i is a nonsingular matrix. Noting $e_i = \xi_i - x_{i,1}$ and $e_i, \dot{e}_i \in \mathcal{L}_\infty[0, t_f)$, it can be concluded that $\xi_i, \dot{\xi}_i \in \mathcal{L}_\infty[0, t_f)$. Thus, there is a constant $\xi_i^* > 0$ so that $|\xi_i(t)| \leq \xi_i^*$ for all $t \in [0, t_f)$. Performing the inverse logarithmic operation on ξ_i results in

$$L < L_i \leq p_i(t) \leq U_i < U, \quad \forall t \in [0, t_f) \quad (14)$$

where $L_i = (Ue^{-\xi_i^*} + L)/(e^{-\xi_i^*} + 1)$ and $U_i = (Ue^{\xi_i^*} + L)/(e^{\xi_i^*} + 1)$. Noting $\dot{\xi}_i = \eta_i v_i$ and $\eta_i(t) \geq 4/(U-L)$ for all $t \in [0, t_f)$, we have $v_i \in \mathcal{L}_\infty[0, t_f)$. Note from $v_i, \bar{x}_i, \hat{\theta}_i, \hat{\omega}_i \in \mathcal{L}_\infty[0, t_f)$ and $L < L_i \leq p_i(t) \leq U_i < U$ for all $t \in [0, t_f)$ that $\zeta(t) \in \Omega^*$ for all $t \in [0, t_f)$, in which Ω^* is a nonempty and compact subset of Ω. As a result, no finite-time escape phenomenon may occur. Hence, $t_f = \infty$. From (4) and (6), we have $\dot{z}_i \in \mathcal{L}_\infty[0, \infty)$ for $i = 1, \ldots, n$. Combining this with $z_i \in \mathcal{L}_\infty[0, \infty) \cap \mathcal{L}_2[0, \infty)$, it follows from Barbalat's lemma that $\lim_{t\to\infty} z_i(t) = 0$, which means, in particular, that $\lim_{t\to\infty} e_i(t) = 0$ and $\lim_{t\to\infty} \dot{e}_i(t) = 0$.

Next, it will show the agent positions achieve asymptotic consensus. For this reason, the relative error vectors $\tilde{q} = [q_1 - q_2, \ldots, q_{2n-1} - q_{2n}]^T \in R^{2n-1}$ and $\tilde{\delta} = [\underline{\delta}_1 - \underline{\delta}_2, \ldots, \underline{\delta}_{2n-1} - \underline{\delta}_{2n}]^T \in R^{2n-1}$ are introduced, where q_ℓ and $\underline{\delta}_\ell$ are, respectively, the ℓth element of q and δ for $\ell = 1, \ldots, 2n$. The dynamics of $\tilde{q}$ can be deduced from (12) as $\dot{\tilde{q}} = -\Omega\tilde{q} + \tilde{\delta}$, where $\Omega \in R^{(2n-1)\times(2n-1)}$ is a constant matrix. We get from $rank(\bar{L}) = 2n-1$ and [1] that $\bar{\mathcal{G}}$ involves a spanning tree. By [1, Th. 2.14], it can be obtained that the system $\dot{\tilde{q}} = -\Omega\tilde{q}$ is asymptotically stable. Note from [20, Th. 4.14] that if a linear time-invariant system is asymptotically stable, then it is also exponentially stable. Combining this with $e_i \in \mathcal{L}_1[0, \infty)$ can have that $\lim_{t\to\infty} \tilde{q}(t) = 0_{2n-1}$. Noting that $x_{i,1} = q_{i,1}$, we have $\lim_{t\to\infty}(x_{i,1}(t) - x_{j,1}(t)) = 0$, which together with $\lim_{t\to\infty}(\xi_i - x_{i,1}(t)) = 0$ yields that $\lim_{t\to\infty}(\xi_i(t) - \xi_j(t)) = 0$ for all $i, j = 1, \ldots, n$. According to the definition of ξ_i, we have over the set Ω^* that $\lim_{t\to\infty}(p_i(t) - L)(U - p_j(t)) - (U - p_i(t))(p_j(t) - L) = \lim_{t\to\infty}(U - L)(p_i(t) - p_j(t)) = 0, \forall 1 \leq i \neq j \leq N$. This together with the fact $U - L > 0$ yields $\lim_{t\to\infty}(p_i(t) - p_j(t)) = 0$. Since $\lim_{t\to\infty}(\gamma_i \sum_{j=1}^n a_{ij}\xi_j(t) - \lambda_{i,1}x_{i,1}(t)) = 0$, we can infer from (3) that $\lim_{t\to\infty} x_{i,2}(t) = 0$, which, with $\lim_{t\to\infty}(\dot{\xi}_i(t) - x_{i,2}(t)) = 0$, gives that $\lim_{t\to\infty} v_i(t) = 0$.

C2: In such a situation, we suppose that the agent with index 1 is the agent without neighbors. $\underline{\mathcal{G}}$ with the node set $\underline{\mathcal{V}} = \{2, \ldots, n\}$ and the edge set $\underline{\mathcal{E}} \subseteq \underline{\mathcal{V}} \times \underline{\mathcal{V}}$ is used to model the topology between the agents 2 to n. A_{n-1}, D_{n-1}, and L_{n-1} are, respectively, the adjacency matrix, the in-degree matrix, and the Laplacian matrix related to $\mathcal{G}_{n-1}$. Thus, L_n related to $\mathcal{G}$ can be separated as

$$L_n = \begin{bmatrix} 0 & 0_{n-1}^T \\ h & L_{n-1} \end{bmatrix},$$

where $h = [a_{21}, \ldots, a_{n1}]^T \in R^{n-1}$. Since $\mathcal{G}$ involves a directed spanning tree, we get from [1] that $rank(L) = n-1$. This means that $rank(L_{n-1}) = n-1$.

Let us define the vectors $q_1 = [\xi_{1,1}, q_{2,1}, \ldots, q_{n,1}]^T$, $q_2 = [q_{2,2}, \ldots, q_{n,2}]^T$, $\delta = [0_{2n-1}^T, \delta_2, \ldots, \delta_n]^T$, and $q = [q_1^T, q_2^T]^T$ with $i = 2, \ldots, n$ and $\delta_i = -(c_{i,2}/d_i)\sum_{j=1}^n a_{ij} z_{j,1}$. Viewing (11) and the fact that $x_{1,1}$ is a constant, we have

$$\dot{q}(t) = -\bar{L}q(t) + \delta(t) \tag{15}$$

where

$$\bar{L} = \begin{bmatrix} 0 & 0_{(n-1)m}^T \\ \hbar & L_{(n-1)m} \end{bmatrix} \tag{16}$$

with

$$\hbar = [0_{n-1}^T, h^T]^T \in R^{2(n-1)}, \quad L_{(n-1)m} = \begin{bmatrix} -c_1 & c_1 \\ c_2\bar{A}_{n-1} & -c_2 \end{bmatrix},$$

$c_1 = \mathrm{diag}\{c_{2,1}, \ldots, c_{n,1}\}$, $c_2 = \mathrm{diag}\{c_{2,2}, \ldots, c_{n,2}\}$, and $\bar{A}_{n-1} = D_{n-1}^{-1} A_{n-1}$. It can be seen that the matrix $\bar{L}$ has the form of a Laplacian matrix, as $\bar{L}1_{2(n-1)+1} = 0_{2(n-1)+1}$ and each off-diagonal entry of $\bar{L}$ is a non-positive number. The system (15) can be seen as a multi-agent group comprised of $2n-1$ agents which are connected under the augmented directed graph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$, where $\bar{\mathcal{V}} = \{1, \ldots, 2(n-1)+1\}$, $\bar{L}$ is the related Laplacian matrix, and the edge set $\bar{\mathcal{E}}$ can be deduced from (16). Since $rank(c_1) = n-1$ and $rank(c_2(I_{n-1} - \bar{A}_{n-1})) = rank(L_{n-1}) = n-1$, we obtain $rank(L_{2(n-1)}) = 2(n-1)$. By (16), it can be obtained $rank(\bar{L}) = 2(n-1)$. Proceeding in a fashion similar to *C1*, it can be concluded that each closed-loop signal remains bounded, $\lim_{t\to\infty}(p_i(t) - p_j(t)) = 0$, and $L < p_i(t) < U$ for all $t \geq 0$, for all $i, j = 1, \ldots, n$. Besides, as $x_{i,1}$ is a constant for $i \in \mathcal{V}_2$ in this situation, we have that $\lim_{t\to\infty} p_j(t) = \gamma_i^*$ for all $j = 1, \ldots, n$, where $\gamma_i^* = (U + Le^{-\gamma_i})/(1 + e^{-\gamma_i})$. The proof is complete.

Remark 1: Because of unknown technical challenges, there exist still some unresolved points that deserve further study. For example, this work does not consider actuator failure. Due to the aging of components, actuator failures are often encountered in practice. Following sliding mode control methods proposed in [21]–[23], future work will solve the consensus issue with actuator faults and model uncertainties.

Remark 2: Although our new method can guarantee uniform boundedness of each closed-loop signal, it is unclear how the parameters affect the convergence speed. Intuitively, increasing the control gain k_i is able to speed up the convergence of consensus. A rigorous analysis of that situation requires further study. The designed control algorithm has a clear structure, and there are not many requirements for its parameters. Therefore, it is easy to implement in practical applications.

Remark 3: Note that the proposed distributed consensus algorithm requires all states of each agent. Constructing a velocity observer for each agent is a promising way to eliminate such requirements.

Remark 4: In recent work [24], the leaderless consensus problem under directed communication topologies was studied by using a dynamic output design method. However, it does not actually take into account the position constraint requirement during operation, which is an essential consideration in practice. We propose a new reference position to deal with this problem, including position constraints and nonlinear transformations. To the best of our knowledge, there has been no research so far to achieve asymptotic consensus without requiring the velocity variables of neighboring agents in the constrained control literature.

IV. SIMULATION STUDY

A multi-agent system consisting of four single-link robots is considered. Each robot can be described by

$$J_i\ddot{q}_i + B_i\dot{q}_i + M_i\sin(q_i) = \mathrm{g}_i + \tau_{di}(t), \quad i = 1, 2, 3, 4 \tag{17}$$

in which J_i, B_i, and M_i denote system parameters that can be found on [25, p. 190], q_i is the angle of the link of the ith robot, g_i is the voltage input, and τ_{di} represents the uncertain disturbance. The simulation parameters are set to $J_i = 1.71 - 0.0i$, $B_i = 0.45 + 0.01i$, $M_i = 0.82 + 0.01i$, and $\tau_{di}(t) = 0.1\cos(t)$. The angle limitation of each robot is -1 (rad) $< q_i(t) <$ 1.4 (rad) for all $t \geq 0$. The directed graph is shown in Fig. 1. The initial configurations of the robots satisfying Assumption 2 are $q_1(0) = 1.25$ (rad), $q_2(0) = 0.31$ (rad), $q_3(0) = -0.52$ (rad), and $q_4(0) = -0.93$ (rad). The initial angular velocities are zero. Let $p_i = q_i$, $v_i = \dot{q}_i$, and $u_i = \mathrm{g}_i/J_i$, $i = 1, 2, 3, 4$. By defining $\varphi_i(p_i, v_i) = [-v_i, -\sin(p_i)]^T$, $\theta_i = [B_i/J_i, M_i/J_i]^T$, and $\tau_i(t) = \tau_{di}(t)/J_i$, model (17) can be transformed to (1). The design parameters are set to $\lambda_{i,1} = 1$, $\lambda_{i,2} = 2$, $k_i = 1.5$, $\Gamma_i = 5I_2$, $\mu_i = 5$, and $\varepsilon_i = e^{-0.05t}$.

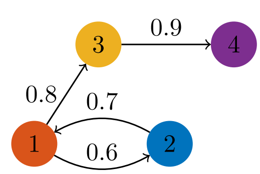

FIGURE 1. Directed communication topology.

Figs. 2-5 show the validity of our proposed approach. The angle and angular velocity profile of each robot is shown in Figs. 2-3, from which we can observe that all robots have reached a consensus and have always met the requirements for robot angle constraints. Figs. 4-5 exhibit the evolution of the reference positions. In the light of the simulation results, regardless of the existence of angle constraints and uncertain dynamic characteristics, our proposed control strategy is able to complete the consensus task under the condition of a directed topology and holds satisfactory closed-loop performance.

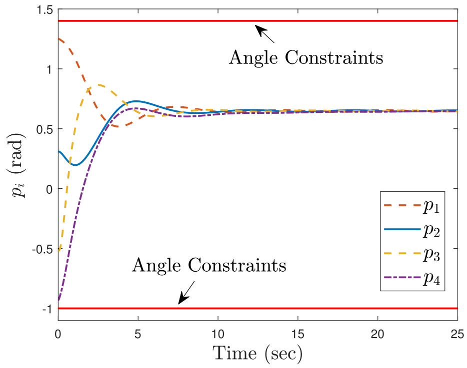

FIGURE 2. The angles p_i for $1 \leq i \leq 4$.

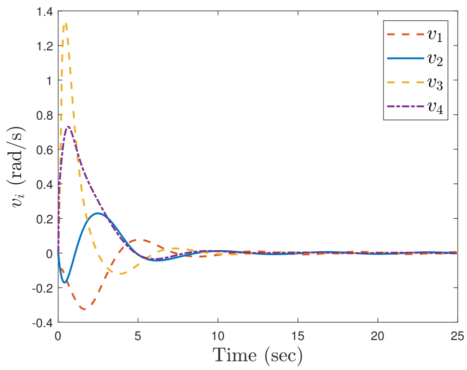

FIGURE 3. The angular velocities v_i for $1 \leq i \leq 4$.

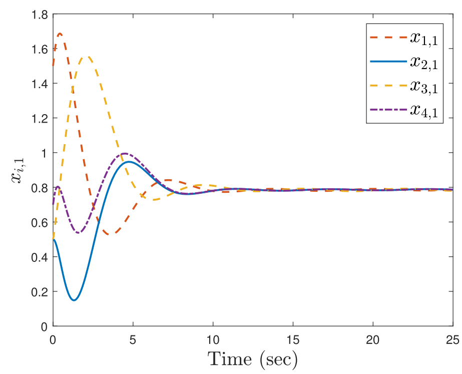

FIGURE 4. Trajectories of $x_{i,1}$ for $1 \leq i \leq 4$.

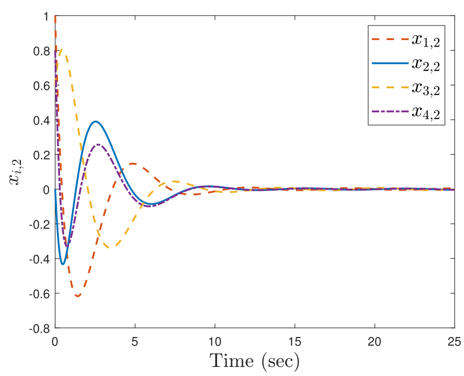

FIGURE 5. Trajectories of $x_{i,2}$ for $1 \leq i \leq 4$.

V. CONCLUSION

The consensus issue of second-order multi-agent systems has been carefully handled. A new reference position has been designed for each agent to address the position constraints while eliminating the requirement of neighbor velocity variables, and an adaptive control scheme has been developed on this basis. It has been shown that the proposed control strategy guarantees not only the convergence of the consensus error to zero but also the boundedness of all closed-loop signals. Simulations on four single-link robots validated the theoretical findings. Following the benchmark method designed in this article, future work includes extending the results to multi-agent systems including position constraints under switching communication topologies.

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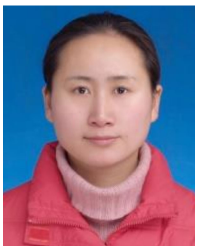

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