

Economic and Energetic Performance Evaluation of Unreliable Production Lines: An Integrated Analytical Approach

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ABSTRACT The current context is increasingly driving researchers and industry to focus not only on the economic but also on the energetic performance of manufacturing systems. However, considerable work on the enhancement of energetic performance of complex production systems is still needed. This paper addresses a novel integrated analytical method to evaluate simultaneously the economic and energetic performances of a serial production line composed of unreliable machines and intermediate buffers. This approach is based on a discrete Markov chain formulation of machines states transitions and a birth-death Markov process for buffers states evaluation. It introduces throughput, energy consumption and energy efficiency as key performance indicators for assessing economic and energetic performances. Structural characteristics of the problem are analyzed to establish and evaluate the impact of buffers size, reliability parameters, and production rates of the machines on the energetic performance of the production line. A large experimental study, based on different instances inspired by the literature, is carried out to analyze the behavior and the complex trade-off between throughput and energy efficiency performances.

INDEX TERMS Energy efficiency, throughput, performance evaluation, production lines, unreliable machines, Markov chain.

I. INTRODUCTION

For decades, economies around the world have been able to grow rapidly thanks to the almost unlimited availability of energy and cheap resources. However, starting in the 1970s, a great deal of research has highlighted the consequences of limited resources. Since then, many economies have competed vigorously for access to resources. In addition, energy prices have risen sharply and energy costs have become consequent and human-induced climate change is directly related to global energy consumption. In particular, the industrial sector, represents more than 31% of total world energy consumption [19] (Fig. 1).

Thus, improving energy efficiency in manufacturing is becoming an unavoidable necessity for energy conservation, emission reduction, and sustainability. Motivated by financial pressures from increasing energy prices, legislative measures such as eco-design standards for industrial machines, as well as signs of induced climate change, the industrial sector

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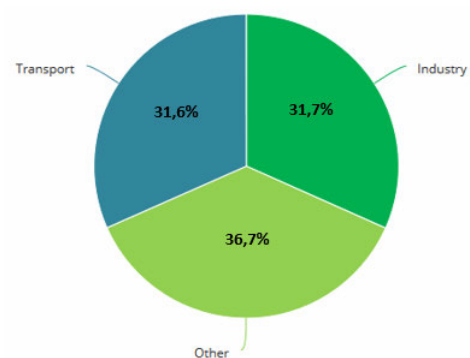


FIGURE 1. Total world energy consumption [19].

is increasingly interested in the issue of energy efficiency. Unfortunately, little work has been done when it comes to studying the combination of energetic and economic efficiency of industrial production lines. In fact, many research results have been reached separately in each field, especially the field of improving energy efficiency for machine tools.

However, a significant lack of work is noticed in the case of complex production systems. Hence, this study, to the best of our knowledge, aims to establish the first integrated approach to evaluate and analyze the complex trade-off between the economic and energetic efficiency of a complex production system. The main contribution of this paper can be summarized as follows:

- To develop an integrated analytical and computational method to evaluate both economic and energetic performance of a serial production line composed of unreliable machines and intermediate buffers.
- To formulate throughput, energy consumption, and energy efficiency as key performance indicators for assessing economic and energetic performance.
- To propose a non-linear programming approach implemented on Lingo solver for solving the problem.
- To carry out a large experimental study based on well-established instances from the literature, to evaluate and analyze the impact of buffers size, reliability parameters, and machine production rates on the energy efficiency of the system.
- To propose an important critical analysis of the trade-off between both performances.

The remainder of this paper is organized as follows: section II presents a literature review of economic performance evaluation for serial production lines, as well as energy evaluation for industrial systems. Section III introduces the problem formulation and assumptions. In section IV and V the economic and energetic evaluation methods are respectively presented to obtain, eventually, throughput, energy consumption, and energy efficiency formulations for a serial production line. Numerical experiments are presented, discussed, and analyzed in section VI. Finally, section VII summarizes the guidelines of our contribution, and highlights future work possibilities.

II. LITERATURE REVIEW

The issue of economic efficiency of serial production lines has been largely treated in the literature. In fact, the main approaches for performance evaluation of flow lines can be summarized in three categories as described in [36]: Markov chain analysis, decomposition and aggregation methods, and simulation. Results with Markov chain analysis being usually difficult to obtain for larger production lines, the approximate methods based on decomposition or aggregation, remain the most used [13]. Many methods are derived from these basic approaches, whereas simulation methods are flexible but time-consuming tools, generally adapted to complex systems.

Hence, for small systems with two machines and a single buffer, exact solutions were derived [12], [25]. For larger systems approximate solutions were proposed, generally based on aggregation [20], [21], [26], [27], decomposition approaches [9]–[11], [15]–[17] or other derived analytical methods such as the Equivalent Machine Method [31]. The latter evaluates the system throughput of a buffered serial production line of unreliable machines with exponentially

distributed parameters. The proposed method that based on the analysis of the different states of each buffer using birth-death Markov processes, reduces significantly the state space cardinality of the Markov chain representation of the system and consequently the computational times.

However, nowadays, it has become fundamental to not only optimize production lines from an economic perspective but also from an energetic one. Yet little work has been done when it comes to evaluating energy efficiency for serial production lines, according to our literature review.

Bernoulli serial production lines were studied by [33], who developed an integrated model for improving energy efficiency for lines composed of two machines. The energy consumption of the line was formulated as shown in equation 1. W_i is the total electrical power consumed, W_{O_i} represents the set-up power needed for the machine to reach 'ready' status, and $k_i\rho_i$ the additional power needed to process parts, where k_i is a constant (energy coefficient associated to machine M_i), and ρ_i the processing rate of machine M_i .

$$W_i = \sum_{i=1}^2 W_{O_i} + \sum_{i=1}^2 k_i\rho_i \quad (1)$$

Later, [34] extended the study for larger systems with unreliable machines and finite buffers. An integrated model for minimizing energy consumption under a desired production rate was presented (Eq 2), where ρ_i is the machine's processing rate and PR_d is the desired production rate. This study only considers the processing energy required for production.

$$\begin{cases} \min E = \sum_{i=1}^K k_i\rho_i \\ \text{s.t. } PR \geq PR_d \end{cases} \quad (2)$$

Reference [39] also studied energy consumption optimization in two-machine Bernoulli serial lines. It formulated the problem of minimizing the energy consumed by machines in two-machine Bernoulli lines while maintaining the required productivity. An effective algorithm based on binary search method is developed to solve the problem. Based on results from this study as well as the aggregation method, [40] treated the problem for three-machine Bernoulli serial lines. An effective method and an algorithm were designed to solve the energy consumption optimization problem in the three-machine Bernoulli serial line.

On the other hand, [5] developed new energy savings opportunity strategies to maximize energy savings for a serial production line. The ESO (energy saving opportunity) is an opportunity window calculated from on-line production data. It allows certain machines to be turned off for energy saving, without negatively affecting throughput. New energy efficiency performance indicators were presented. The latter used real-time production data to identify the least energy-efficient machine on the line. The energy savings opportunity strategy utilizes the Energy Efficiency Performance Indicators EEPI (Eq. 3 where $W_{i,1}$ is the static part of the

energy consumption for machine i , and $W_{i,2}$ the dynamic part attributed to random disruptions on the line). It takes the opportunity window for the least energy-efficient machine at opportune times, allowing for improvements to be made for this machine to increase the energy efficiency of the line. Later, [6] expanding upon previous research, incorporated the warm-up time of each machine in the analysis.

$$EEPI_i = \frac{W_{i,1}}{W_{i,1} + W_{i,2}} \quad (3)$$

Reference [2], with a similar perspective, established performance indices for measuring energy efficiency and also productivity, using available sensor data from the production line. The energy structure and effects of downtime events on the production line were studied. Methods for measuring efficiency and highlighting the areas of inefficiency in the system were also proposed. Moreover, transient performance analysis of serial production lines with geometric machines was also studied by [7].

Solutions for the improvement of serial production lines performances were also proposed. Reference [8] studied feedback control of machine start-up for energy consumption reduction in two machine Bernoulli serial line. The authors considered buffer depletion at the end of each shift and used transient analysis. Reference [22] extended the previous study for larger systems considering warm-up time in both transient and steady state. Further solutions for energy efficiency improvement in manufacturing systems were proposed by [41]. The authors developed a Gaussian mixture model to predict machine idle periods duration for manufacturing systems. They suggested optimal actions for energy saving under throughput constraint.

In a more general perspective, [18] used a thermodynamic framework to characterize the material and energy resources used in manufacturing processes. Various processes were analyzed, the relevance of thermodynamics was illustrated for the analysis of manufacturing processes, and exergy analysis was used to identify where resources are lost in these processes. The aim was the design of efficient processes (see the energy consumption formulation for a manufacturing system in Eq. 4, where \dot{W} is total power used by the process equipment, \dot{W}_0 the “idle” power for the equipment in the ready position, \dot{m} the rate of material processing in (mass/time), and k a constant (J/mass)).

$$\dot{W} = \dot{W}_0 + k\dot{m} \quad (4)$$

Reference [14] proposed a novel generic method to model the energy consumption behavior of machines and plants based on a statistical discrete event formulation.

Other research work considered particular fields to study the issue. For instance, [24] evaluated energy consumption performance and energy-saving potentials of the ceramic production chain using a first-order hybrid Petri net model. The authors proposed a multi-objective linear programming model for solving the problem as well as sensitivity analysis.

The latter aims to find the optimal specific energy consumption of the production chain. Reference [29] developed a calculation model to forecast the energy consumption of ball mills in ceramic industry during the grinding process based on power feature deployment. An integrated framework for energy savings in sheet metal forming was developed by [38]. Energy consumption modeling, energy performance evaluation, and production optimization were studied.

Energy efficiency was widely studied for machine tools, considering models based on machine components or machine tools globally. Reference [43] presented a comprehensive literature review about energy consumption models and energy efficiency of machine tools. They discussed the connotation of energy efficiency of machine tools, their design, scheduling, optimization, and assessment based on energy efficiency. They also presented several perspectives for the energy consumption modelling and decomposition for machine tools. Later, [42] presented a systematic overview of the classification and prediction methods of energy consumption, together with strategies for energy consumption reduction in machining processes. Reference [32] developed the total energy efficiency index. This is a metric that quantifies the design of machine tools regarding energy efficiency based on the respective assembly of components. Reference [35] focused on the design and selection of machine tools. They developed key performance indicators referred to as “inherent energy performance” (IEP) indexes. This is a systematic method which consists of simplified measurement of basic data and the calculation of the indexes from the data.

Reference [30] studied the green performance of CNC (Computer Numerical Control) machine tools. In this work, a model of energy efficiency, carbon efficiency, and green degree is established. Energy efficiency η was defined by the ratio of the energy required for the cutting process (material removal energy) E_{cut} and the total energy consumed in the machining process $E_{process}$ (Eq. 5). The latter is decomposed into start-up state energy E_{start} , standby state energy $E_{standby}$, no-load state energy $E_{no-load}$, and load state energy E_{load} . Each state energy is defined as the corresponding state power (P_{start} , $P_{standby}$, $P_{no-load}$, and P_{load}), balanced by each state’s duration (t_{start} , $t_{standby}$, $t_{no-load}$, and t_{load}) for start-up, standby, no-load, and load states respectively (Eq. 6). Whereas the cutting energy E_{cut} is obtained based on cutting power P_{cut} and load state time t_{load} (Eq. 7).

$$\eta = \frac{E_{cutting}}{E_{process}} \quad (5)$$

$$\begin{aligned} E_{process} &= E_{start} + E_{standby} + E_{no-load} + E_{load} \\ &= P_{start} \times t_{start} + P_{standby} \times t_{standby} \\ &\quad + P_{no-load} \times t_{no-load} + P_{load} \times t_{load} \end{aligned} \quad (6)$$

$$E_{cutting} = P_{cut} \times t_{load} \quad (7)$$

The energy efficiency is therefore defined, and its evaluation depends on machine and process parameters. Reference [23] proposed an energy consumption model for a mechanical machining process based on

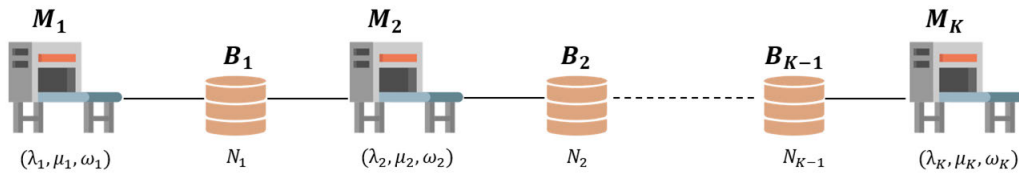


FIGURE 2. Serial production line.

empirical modeling. This work studied the relationship between processing parameters (spindle speed, feed rate, and depth of cut) and energy consumption. Reference [28] focused on machine tools selection. A novel method, named Potential Efficiency (PE) Method, was established for energy efficiency evaluation. In this novel method, machining tasks were also considered along with energy efficiency to provide efficient machine tool selection.

It is clear from the above illustrations and to the best of our knowledge that present literature lacks sufficient insight into a system where both productivity and energy efficiency of serial production lines are integrated. Although the issue has been widely studied and considerable results have been obtained in the case of machine tools, more effort is still required for the study, evaluation, and optimization of energy efficiency for serial production lines. This paper aims to present an analytical method that allows the evaluation of energy consumption and efficiency for serial production lines taking into account their economic performance. Therefore, an integrated method for evaluating both the economic and energetic performance of unreliable serial production lines is developed. Results from numerical experiments are used to investigate the effect of buffer size and machine reliability parameters on the energy consumption and efficiency of the system. The trade-off between both performances is also analyzed in this study.

III. PROBLEM FORMULATION

The objective of this study is to develop an analytical and computational method that allows the evaluation of both economic and energetic performance of a serial production line. The system under consideration consists of K machines connected by intermediate storage areas ($K - 1$) buffers (Fig. 2).

The system is subject to non-availabilities due to the limited capacity of the buffers and/or the failure and repair rates of the machines. The following assumptions are frequently used in the literature. They address the machines, the buffers, the energy consumption and their mutual interactions:

- The failure state of the machines depends on the operations. A machine cannot fail if it is starved or blocked.
- The first machine cannot be starved and the last machine cannot be blocked.
- The failure and repair times are independent and distributed according to an exponential law.

- Buffer B_i has a finite capacity N and cannot be down. The transition times between machines and buffers are zero.
- Energy consumption of each buffer B_i is neglected.

We introduce the following notations:

K	Number of machines
$K - 1$	Number of buffers
ω_i	Processing rate of machine M_i
λ_i	Failure rate of machine M_i
μ_i	Repair rate of machine M_i
$N_j, j \in \{1, \dots, K - 1\}$	Capacity of buffer B_j
$\rho_i, i \in \{1, \dots, K\}$	Production rate of machine M_i
ψ	Production line throughput

IV. ECONOMIC EVALUATION APPROACH

The economic evaluation (throughput evaluation) is based on the Equivalent Machine Method developed by [31]. This method evaluates the system throughput of a buffered serial production line of unreliable machines with exponentially distributed parameters. Based on the analysis of the different states of each buffer using birth-death Markov processes, each original machine is replaced by an equivalent one taking into account the probabilities of blockage and starvation.

The simplest system of two machines separated by one buffer is used as a building block for the general model. To analyze the steady states of the buffer, a birth-death Markov process with $(N + 1)$ states is considered. N is the capacity of the intermediate buffer, ω_1 and ω_2 respectively the birth and death transition rates, and α the processing rates ratio related to the buffer B . The differential equations for the probability that the system is in state j at time t are:

$$\begin{cases} \frac{\partial p_0(t)}{\partial t} = -\omega_1 \times p_0(t) + \omega_2 \times p_1(t) \\ \frac{\partial p_j(t)}{\partial t} = \omega_1 \times p_{j-1}(t) - (\omega_1 + \omega_2) \times p_j(t) + \omega_2 \times p_{j+1}(t) \\ \frac{\partial p_N(t)}{\partial t} = \omega_1 \times p_{N-1}(t) - \omega_2 \times p_N(t) \end{cases} \quad (8)$$

Considering all the differential terms equal to zero at the steady state, and after simplifying the system above and considering the normalization equation: $\sum_{j=0}^N P_j = 1$, the steady probabilities of each buffer state are obtained (Eq. 9). Further

formulation details can be found in [31].

$$p_j = \begin{cases} \frac{\alpha_j \times (1 - \alpha)}{1 - \alpha^{N+1}} & \text{if } \alpha \neq 1 \\ \frac{1}{N + 1} & \text{if } \alpha = 1 \end{cases} \quad (9)$$

The probabilities of empty and full buffer states are given by the following equations:

$$p_0 = \begin{cases} \frac{1 - \alpha}{1 - \alpha^{N+1}} & \text{if } \alpha \neq 1 \\ \frac{1}{N + 1} & \text{if } \alpha = 1 \end{cases} \quad (10)$$

$$p_N = \begin{cases} \frac{\alpha^N \times (1 - \alpha)}{1 - \alpha^{N+1}} & \text{if } \alpha \neq 1 \\ \frac{1}{N + 1} & \text{if } \alpha = 1 \end{cases} \quad (11)$$

The effective production rate of each machine M_i is therefore defined as follows:

$$\rho_i = \omega_i \times \frac{\mu_i \times \xi_i}{\mu_i + \xi_i \times \lambda_i} \quad (12)$$

Such that: $\xi_1 = 1 - p_N$ and $\xi_2 = 1 - p_0$.

However, in the general case of K machines and $(K - 1)$ intermediate stocks, machines M_i and M_{i+1} , $i \in \{1, \dots, K\}$ related to the buffer B_j , $j \in \{1, \dots, K - 1\}$ are subject to starvation and blockage and their effective processing rates are affected by the availabilities of the buffers B_{j-1} and B_{j+1} . The first buffer and the last are particular cases since the first machine cannot be starved and the last machine cannot be blocked. The different states of the $(K - 1)$ buffers are modeled by $(K - 1)$ related birth-death Markov processes. Each stochastic process is defined by its processing rates ratio α_j defined as follows:

$$\alpha_j = \frac{\min_{i=1, \dots, j} \rho_i}{\min_{i=j+1, \dots, K} \rho_i} \quad (13)$$

Based on the previous analysis for the simple system composed of two machines and one buffer, the probabilities of empty and full states of each buffer B_j are given by Eq. 14 and Eq. 15 respectively.

$$p_0^j = \begin{cases} \frac{1 - \alpha_j}{1 - \alpha_j^{N_j+1}} & \text{if } \alpha_j \neq 1 \\ \frac{1}{N_j + 1} & \text{if } \alpha_j = 1 \end{cases} \quad (14)$$

$$p_{N_j}^j = \begin{cases} \frac{\alpha_j^{N_j} (1 - \alpha_j)}{1 - \alpha_j^{N_j+1}} & \text{if } \alpha_j \neq 1 \\ \frac{1}{N_j + 1} & \text{if } \alpha_j = 1 \end{cases} \quad (15)$$

The effective production rates of each machine considering the previous results are given by the following equation:

$$\forall i = 1, \dots, K; \rho_i = \omega_i \times \frac{\mu_i \times \xi_i}{\mu_i + \xi_i \times \lambda_i} \quad (16)$$

where:

$$\begin{cases} \xi_1 = 1 - p_1^{N_1} \\ \xi_K = 1 - p_{K-1}^0 \\ \forall i = 2, \dots, K - 1, \xi_i = (1 - p_{j-1}^0) \times (1 - p_j^{N_j}), j = i \end{cases}$$

Finally, the throughput of the line is defined as the bottleneck between the effective production rates of all the machines (Eq. 17).

$$\psi = \min_{i=1, \dots, K} \{\rho_i\} \quad (17)$$

The relevance of this method has been established in the literature. In fact, it allows the reduction of the state space cardinality of the system through analyzing only full and empty buffer states. Therefore, the computational times are significantly reduced with very accurate results when compared to other existing methods in the literature [1], [31].

V. ENERGY EVALUATION APPROACH

The literature review has allowed us to develop an accurate approach for energy evaluation for the specificities of the problem and the research needs.

The approach to evaluating energy consumption is to assess it per part. The energy consumption is usually evaluated per part in the literature. Several segmentation methods have been introduced by [43]. The latter are summarized in Fig. 3. Depending on the needs of the study, the energy consumption of a machine can be evaluated according to several axes. For the conducted study, we have chosen to assess energy consumption according to the machine states, since we aim at developing an evaluation method valid for all types of serial production lines. Therefore, we could not develop a model based on components or details specific to a certain machine type.

For this study, energy consumption is evaluated according to machine states. We will therefore have a specific energy consumption for each state in which a machine M_i could be. The states: Operating, Down, Starved, Blocked, as well as Starved and Blocked at the same time are the states in which a machine M_i of the line could be found. First, the state probabilities for each machine M_i are calculated by modeling the system using a Markov chain with discrete time and states.

A. MARKOVIAN REPRESENTATION OF MACHINE STATES

A Markov chain with discrete time and states is considered. The set of states S_i consists of the different states of each machine: Operating, Down, Starved, Blocked, and Starved&Blocked. For presentation concerns, we refer to these states as 1, 2, 3, 4, and 5 respectively. The set of states S_i is therefore given as follows:

$$S_i = \{1, 2, 3, 4, 5\}, \quad \forall i = 2 \dots K - 1 \quad (18)$$

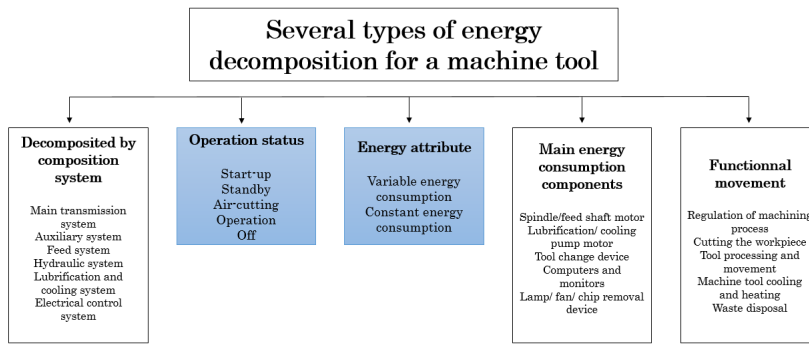


FIGURE 3. Energy consumption decomposition for machine tools [43].

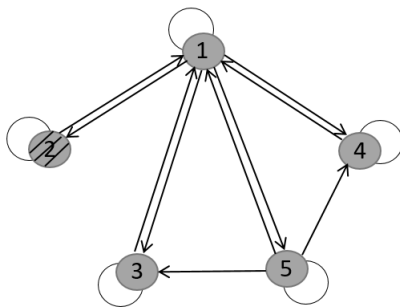


FIGURE 4. Transition graph (general case).

We consider the graph of states (Fig. 4) which presents the different transitions between each state of the machine. The transition probabilities matrix A_i is also given for each machine M_i , $i = 2 \dots K - 1$ (equation 19), as shown at the bottom of the page.

The probabilities of empty and full buffer states are calculated using the formulations presented in the previous section in the context of the throughput evaluation. However, there are two special cases, the case of the first machine ($i = 1$) that cannot be starved, and the case of the last machine ($i = K$) that cannot be blocked according to the assumptions developed previously.

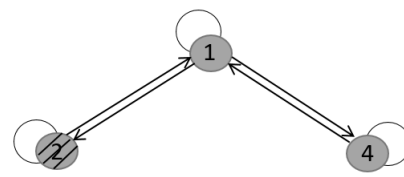


FIGURE 5. Transition graph (Case $i = 1$).

Case $i = 1$: The set of states S consists of: Operating, Down, Blocked ($S_i = \{1, 2, 4\}$). The first machine of the line cannot be starved.

The graph of states is illustrated by Fig. 5 and the transition probabilities matrix is given as follows:

$$A_1 = \begin{matrix} & \begin{matrix} 1 & 2 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 4 \end{matrix} & \begin{bmatrix} 1 & \frac{2}{\lambda_1 + \mu_1} & \frac{4}{\lambda_1 + \mu_1} \\ \frac{\mu_1}{\lambda_1 + \mu_1}(1 - p_1^{N_1}) & \frac{\lambda_1}{\lambda_1 + \mu_1} & \frac{\mu_1}{\lambda_1 + \mu_1}p_1^{N_1} \\ \frac{\mu_1}{\lambda_1 + \mu_1} & \frac{\lambda_1}{\lambda_1 + \mu_1} & 0 \\ 1 - p_1^{N_1} & 0 & p_1^{N_1} \end{bmatrix} \end{matrix}$$

Case $i = K$: The set of states S consists of: Operating, Down, Starved ($S_i = \{1, 2, 3\}$). The last machine of the line cannot be blocked.

$$A_i = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} \frac{\mu_i}{\lambda_i + \mu_i}(1 - p_{i-1}^0)(1 - p_i^{N_i}) & \frac{2}{\lambda_i + \mu_i} & \frac{3}{\lambda_i + \mu_i} & \frac{4}{\lambda_i + \mu_i} & \frac{5}{\lambda_i + \mu_i} \\ \frac{\mu_i}{\lambda_i + \mu_i} & \frac{\lambda_i}{\lambda_i + \mu_i} & 0 & 0 & 0 \\ 1 - p_{i-1}^0 & 0 & p_{i-1}^0 & 0 & 0 \\ 1 - p_i^{N_i} & 0 & 0 & p_i^{N_i} & 0 \\ (1 - p_{i-1}^0)(1 - p_i^{N_i}) & 0 & p_{i-1}^0(1 - p_i^{N_i}) & (1 - p_{i-1}^0)p_i^{N_i} & p_{i-1}^0 p_i^{N_i} \end{bmatrix} \end{matrix} \quad (19)$$

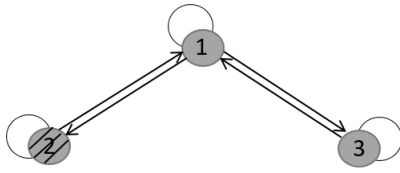


FIGURE 6. Transition graph (Case $i = K$).

The graph of states is illustrated by Fig. 6 and the transition probabilities matrix is given as follows:

$$A_K = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} \frac{\mu_K}{\lambda_K + \mu_K}(1 - p_{K-1}^0) & \frac{\lambda_K}{\lambda_K + \mu_K} & \frac{\mu_K}{\lambda_K + \mu_K} p_{K-1}^0 \\ \frac{\mu_K}{\lambda_K + \mu_K} & \frac{\lambda_K}{\lambda_K + \mu_K} & 0 \\ 1 - p_{K-1}^0 & 0 & p_{K-1}^0 \end{bmatrix} \end{matrix}$$

B. STEADY STATE SYSTEM STUDY

The following step is to calculate the steady probabilities for each machine M_i to be in each of the possible states. The stationary state is defined by the solution to the system:

$$\begin{cases} \pi_i = \pi_i \times A_i \\ \sum_{j \in S_i} P_{i,j} = 1, \quad \forall i = 1 \dots K \end{cases} \quad (20)$$

where π_i and A_i are respectively the stationary probabilities vector and the transition probabilities matrix presented earlier for each machine M_i , $i = 1 \dots K$.

The following results are obtained (21)–(25), as shown at the bottom of the page.

C. ENERGY CONSUMPTION FORMULATION

The energy evaluation is considered over a horizon, in which, a machine M_i is:

- Down with a probability $P_{2,i}$; thus, consuming a constant amount of energy that is represented by $E_{down,i}$.
- Running without load, i.e. starved, blocked or starved and blocked with a probability equal to $(P_{3,i} + P_{4,i} + P_{5,i})$; thus, consuming a constant amount of energy noted $E_{no-load,i}$.
- Operating/processing parts, and thus consuming a constant amount of energy noted $E_{cload,i}$ with a probability $P_{1,i}$ and a variable part of energy which depends on the number of parts operated. This variable energy is equal to the energy $e_{op,i}$ required to operate one part, times the number of parts actually operated by the machine M_i .

The constant energies $E_{down,i}$, $E_{no-load,i}$, and $E_{cload,i}$ are evaluated in units of energy. While $e_{op,i}$, which is the specific operation energy, is evaluated in energy units per part manufactured.

After calculating the steady probabilities, the energy consumption E_i of a machine M_i is obtained. The latter is evaluated for the general case $i = \{2, \dots, K - 1\}$ as well as for the case of the first machine ($i = 1$) and the last machine ($i = K$) as follows:

$$P_{1,i} = \begin{cases} \frac{1}{1 + \frac{\lambda_i}{\mu_i} + \frac{\mu_i}{\lambda_i + \mu_i} \frac{P_i^{N_i}}{1 - p_i^{N_i}}} & \text{if } i = 1 \\ \frac{1}{1 + \frac{\lambda_i}{\mu_i} + \frac{\mu_i}{\lambda_i + \mu_i} \frac{P_{i-1}^0}{1 - p_{i-1}^0}} & \text{if } i = K \\ \frac{1}{1 + \frac{\lambda_i}{\mu_i} + \frac{\mu_i}{\lambda_i + \mu_i} \frac{P_i^{N_i}(1 - p_{i-1}^0)}{(1 - p_{i-1}^0 p_i^{N_i})(1 - p_i^{N_i})} + \frac{\mu_i}{\lambda_i + \mu_i} \frac{P_{i-1}^0(1 - p_i^{N_i})}{(1 - p_{i-1}^0 p_i^{N_i})(1 - p_{i-1}^0)} + \frac{\mu_i}{\lambda_i + \mu_i} \frac{P_{i-1}^{N_i}}{1 - p_{i-1}^0 p_i^{N_i}}} & \text{otherwise.} \end{cases} \quad (21)$$

$$P_{2,i} = \frac{\lambda_i}{\mu_i} P_{1,i} \quad \forall i = \{1, \dots, K\} \quad (22)$$

$$P_{3,i} = \begin{cases} \frac{\mu_i}{\lambda_i + \mu_i} \frac{P_{i-1}^0(1 - p_i^{N_i})}{(1 - p_{i-1}^0 p_i^{N_i})(1 - p_{i-1}^0)} P_{1,i} & \text{if } i = \{2, \dots, K - 1\} \\ \frac{\mu_i}{\lambda_i + \mu_i} \frac{P_{i-1}^0}{1 - p_{i-1}^0} P_{1,i} & \text{if } i = K \end{cases} \quad (23)$$

$$P_{4,i} = \begin{cases} \frac{\mu_i}{\lambda_i + \mu_i} \frac{P_i^{N_i}(1 - p_{i-1}^0)}{(1 - p_{i-1}^0 p_i^{N_i})(1 - p_i^{N_i})} P_{1,i} & \text{if } i = \{2, \dots, K - 1\} \\ \frac{\mu_i}{\lambda_i + \mu_i} \frac{P_i^{N_i}}{1 - p_i^{N_i}} P_{1,i} & \text{if } i = 1 \end{cases} \quad (24)$$

$$P_{5,i} = \frac{\mu_i}{\lambda_i + \mu_i} \frac{P_{i-1}^{N_i} P_{i-1}^0}{(1 - p_{i-1}^0 p_i^{N_i})} P_{1,i} \quad \text{if } i = \{2, \dots, K - 1\} \quad (25)$$

Algorithm 1 Economic and Energetic Performance Evaluation Algorithm

Input data:

K	Number of machines
$K - 1$	Number of buffers
N_j	Capacity of buffer B_j
ω_i	Processing rate of machine M_i
λ_i	Failure rate of machine M_i
μ_i	Repair rate of machine M_i
$E_{down,i}$	Energy consumed by machine M_i while it is down
$E_{no-load,i}$	Energy consumed by machine M_i while working without load
$E_{cload,i}$	Constant part of the energy consumed by machine M_i while operating
$e_{op,i}$	Energy consumed by machine M_i to process a part/piece

for each buffer B_j do

for each machine M_i do

- $p_j^0 \leftarrow$ Calculate the steady probability that buffer B_j is empty
- $p_j^{N_j} \leftarrow$ Calculate the steady probability that buffer B_j is full
- $\alpha_j \leftarrow$ Calculate the processing rate ratio related to buffer B_j
- $\rho_i \leftarrow$ Calculate the equivalent throughput of machine M_i
- $(P_{1,i}, P_{2,i}, P_{3,i}, P_{4,i}, P_{5,i}) \leftarrow$ Calculate steady state probabilities for machine M_i
- $E_i \leftarrow$ Calculate the energy consumed by machine M_i

end for

end for

return $(\psi = \min_{i=1}^K \rho_i; E = \sum_{i=1}^K E_i; \eta = \frac{\sum_{i=1}^K (E_{cload,i} * P_{1,i} + e_{op,i} * \rho_i)}{\sum_{i=1}^K E_i})$

General case $i = \{2, \dots, K - 1\}$:

$$E_i = P_{2,i}E_{down,i} + (P_{3,i} + P_{4,i} + P_{5,i})E_{no-load,i} + P_{1,i}E_{cload,i} + e_{op,i} * \rho_i \quad (26)$$

Case $i = 1$:

$$E_i = P_{2,i}E_{down,i} + P_{4,i}E_{no-load,i} + P_{1,i}E_{cload,i} + e_{op,i} * \rho_i \quad (27)$$

Case $i = K$:

$$E_i = P_{2,i}E_{down,i} + P_{3,i}E_{no-load,i} + P_{1,i}E_{cload,i} + e_{op,i} * \rho_i \quad (28)$$

The energy consumption of a production line made of K machines and $(K - 1)$ buffers is the sum of energy consumption of the K machines of the serial line (Eq. 29).

$$E = \sum_{i=1}^K E_i \quad (29)$$

However, the energy consumption does not completely describe the energy efficiency. The key is the proportion of the energy consumed in effectively producing products. The energy efficiency η of the production line is therefore defined by the ratio of the energy consumed while operating effectively (production) E_{Op} and the total energy consumed by the production line E (Eq. 30).

$$\eta = \frac{E_{Op}}{E} \quad (30)$$

where:

$$E_{Op} = \sum_{i=1}^K E_{Op,i} = \sum_{i=1}^K (E_{cload,i}P_{1,i} + e_{op,i} * \rho_i)$$

A specific energy efficiency indicator is introduced, which is the energy efficiency of each machine M_i :

$$\eta_i = \frac{E_{Op,i}}{E_i} = \frac{E_{cload,i}P_{1,i} + e_{op,i} * \rho_i}{E_i} \quad (31)$$

D. INTEGRATED EVALUATION METHOD IMPLEMENTATION

In addition to being a novel integrated method for evaluating both economic and energetic performance of unreliable production lines, this method presents a substantial originality. The computational complexity is lower than the approaches based on other methods, such as decomposition or aggregation. In fact, the computational complexity of the proposed method in this paper is independent of the buffers sizes.

Considering a production line of K machines and $K - 1$ buffers, we need to solve:

- $2(K - 1)$ non-linear equations to obtain empty and full buffer state probabilities (P_j^0 and $P_j^{N_j}, j = 1 \dots K - 1$).
- $(K - 1)$ non-linear equations to obtain the processing rate ratio α_j for each buffer $B_j, j = 1 \dots K - 1$.
- K non-linear equations to obtain the equivalent throughput ρ_i for each machine $M_i, i = 1 \dots K$.
- $5K$ non-linear equations to obtain the steady state probabilities for each machine M_i .
- K linear equations for calculating the energy consumption of each machine $M_i, i = 1 \dots K$.

- 3 equations for calculating the throughput ψ , energy consumption E and energy efficiency η of the production line.

Therefore, this problem requires solving a system of $10K$ equations including $9K - 1$ non-linear equations. The procedure of the proposed analytical method for evaluating both economic and energetic performances is described by Algorithm 1. For its resolution, this mixed integer non-linear program has been implemented on a LINGO 18.0 solver.

VI. NUMERICAL EXPERIMENTS

In order to analyze the behavior and the trade-off between the economic and the energetic performance of serial production lines, we tested our evaluation model on a LINGO 18.0 solver using existing instances in the literature. We assume, as in the literature [33], [34], and without loss of generality, that an operating machine consumes the same amount of energy while running without load $E_{no-load,i}$ in addition to the energy required to manufacture each part $e_{op,i} * \rho_i$. We therefore have: $E_{load,i} = E_{no-load,i}$. The energy consumption E_i for each machine M_i becomes:

$$E_i = E_{no-load,i} + P_{2,i}(E_{down,i} - E_{no-load,i}) + e_{op,i} * \rho_i \quad (32)$$

A simplified version of the evaluation model is then obtained. Moreover, for all tests, we have chosen to keep energy parameters constant for all machines. Due to the complexity of the procedure and the unavailability of energy benchmarks (instances and data), their influence will be studied in future work.

The choice of these parameters was inspired from the literature. Reference [2] in the numerical experiments section, chose to evaluate the idling power as a fraction of the operating power, the ratio is approximately of 50%. For our study, the same ratio is maintained. The fraction of the no-load energy and the operating energy is approximately equal to 50% for an isolated system. E_{down} usually neglected in the literature, is considered in our study, equal to 10% of the no-load energy. The energy parameters are therefore fixed as follows:

$$E_{down,i} = 1 \text{ Unit of energy.}$$

$$E_{no-load,i} = 10 \text{ Units of energy.}$$

$$e_{op,i} = 8 \text{ Units of energy/part manufactured.}$$

Thereafter, the performance of the evaluation model is studied. The economic evaluation has already been validated [1], [31] and its relevance has been proven. However, it is not possible to assess performance in terms of energy evaluation, due to the unavailability of a benchmark in the literature.

Moreover, several tests are conducted in order to analyze the behavior of serial production lines regarding their economic and energetic performance. We start by analyzing the simplest system with two-machines and one buffer, for which we study correlations with energy consumption and energy

efficiency. Afterwards, larger systems with K machines and $(K - 1)$ buffers are analyzed. Test procedures and results are described in the following section.

A. TWO MACHINES-ONE BUFFER SYSTEM ANALYSIS

In order to study certain behaviors and correlations, the simplest system with two machines and one buffer is first analyzed. Interesting results have been formulated in this section.

First, we consider a system composed of two machines separated by an intermediate stock. We study the correlation between the buffer size, the energy consumption E and energy efficiency η respectively. The buffer size N is therefore varied from 1 to 100 and tests on [16] instances are performed (The parameters of each configuration are presented in Table 4). The first configuration represents a homogeneous line with identical machines, the second represents a homogeneous line with non-identical machines, while the last one deals with the general case of non-identical machines in a non-homogeneous line.

Then, to establish the relationship between the reliability parameters of the two machines of the production line and respectively the line's energy consumption and energy efficiency, tests are conducted on a homogeneous line and then on a non-homogeneous line (the parameters of each configuration are presented in Table 5). For each of the two lines, four scenarios are tested by varying the size of the buffer (1, 10, 25, and 100).

Additional tests are conducted on [37] instances. The latter consider several configurations varying the reliability parameters of the machines, their production capacities, and the size of the buffer. The parameters of each configuration are presented in Table 6.

The parameters of the first machine are kept constant, whereas for the second machine, they are chosen in such a way that for each configuration, the machines have the same isolated production rate as indicated in the Table 6. Each configuration has three different instances varying the failure and repair rates and production capacities of the second machine. The size of the intermediate stock varies from 1 to 100 for each instance. For each configuration, the first two instances consider the case of a homogeneous line, whereas the third considers the non-homogeneous case. Subsequently, the impact of the positioning of the machines in a line with two machines and an intermediate stock on energy consumption and efficiency is studied. For this purpose, evaluation tests are carried on the two configurations represented in Table 7. For each configuration two instances are tested, in which the positions of the two machines are inverted. The energy consumption and energy efficiency of the line are then evaluated by varying the size of the buffer from 1 to 100. The parameters of each configuration are presented in Table 7.

RESULTS ANALYSIS

The significant findings obtained from the experimental study are highlighted and summarized in Table 1.

TABLE 1. Two machines one-buffer system analysis results.

Two Machines-one buffer system		
Correlation studied	Results	Instance
Energy consumption and buffer size	Positive correlation towards stationarity starting from a size N^* (Fig. 7).	4 [16]
Energy efficiency and buffer size	Positive correlation towards stationarity starting from a size N^* .	4 [16]
Energy consumption and machines reliability parameters	Negative correlation: the energy consumption of the line decreases with the increase of the ratio $\frac{\lambda_i}{\mu_i}$ (Fig. 8)	5
Energy efficiency and machines reliability parameters	Negative correlation: the energy efficiency of the line decreases with the increase of the ratio $\frac{\lambda_i}{\mu_i}$.	5
Machines' positions impact on energy consumption	No impact either on machines' or on line's energy consumption (Fig. 13a and Fig. 13b)	7
Machines' positions impact on energy efficiency	No impact either on machines' or on line's energy efficiency	7
Additional analysis	For the same isolated production rates: - Changing the reliability parameters has no impact either on the energy consumption or the energy efficiency. - However, increasing machine's capacity leads to less energy consumption and better energy efficiency (Fig. 9a - Fig. 12b).	6

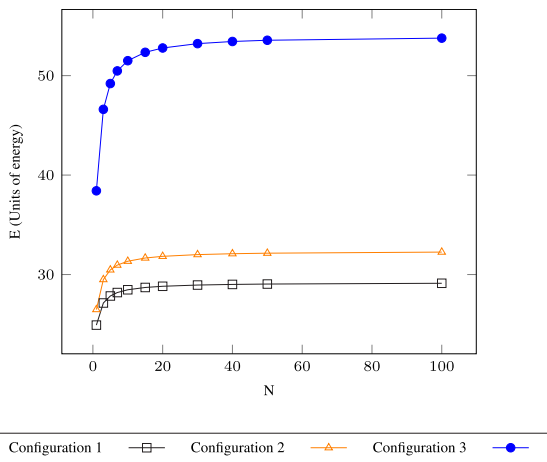


FIGURE 7. Energy consumption and buffer size correlation.

Buffer Size Correlation: the increase in the buffer size N leads to greater energy consumption and better energy efficiency for all configurations (Fig. 7). For energy consumption, we notice an increase of 16.88%, 21.82%, and 39.96% from a buffer size of 1 to 100, for the first, second, and last configuration respectively. The increase ratio of energy consumption is more significant for greater machine availabilities (e.g. configuration 3).

From the point of view of energy efficiency, the increase is of 45.06%, 49.70%, and 39.96% from a buffer size of 1 to 100 for the respective configurations.

Machine Reliability Parameters Correlation: we notice a decrease of 23.9% in energy consumption and 1.58% in energy efficiency between a value of 0.01 and a value of 1

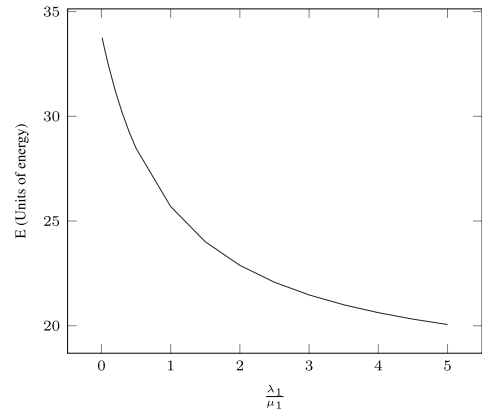
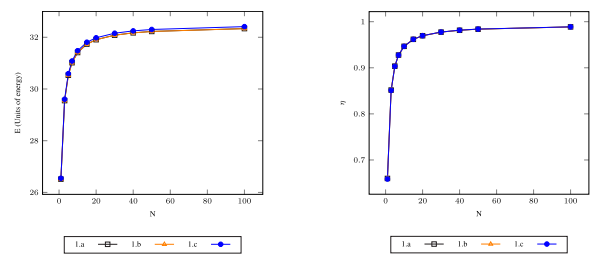
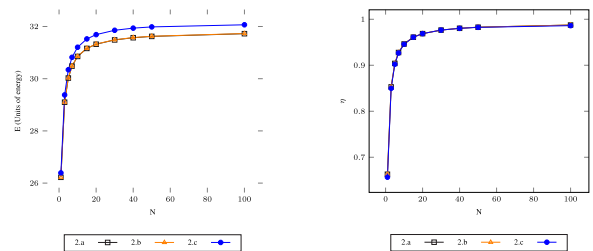


FIGURE 8. Energy consumption and machine reliability parameters correlation.



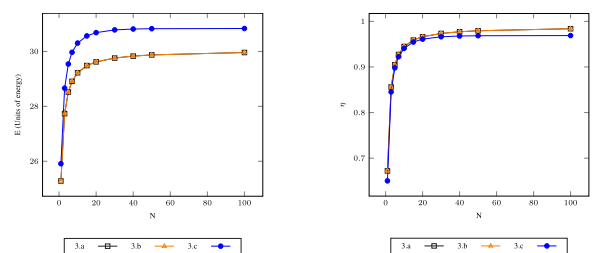
(a) Energy consumption of the line (b) Energy efficiency of the line

FIGURE 9. Energy consumption and energy efficiency for configuration 1.



(a) Energy consumption of the line (b) Energy efficiency of the line

FIGURE 10. Energy consumption and energy efficiency for configuration 2.

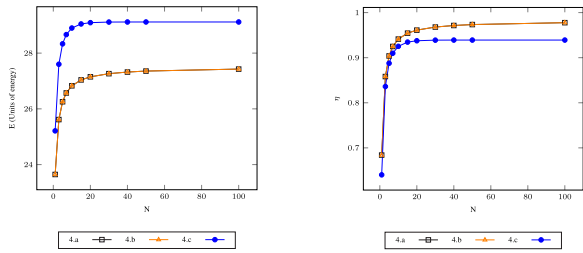


(a) Energy consumption of the line (b) Energy efficiency of the line

FIGURE 11. Energy consumption and energy efficiency for configuration 3.

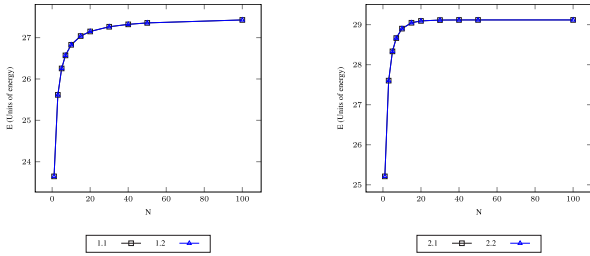
of the ratio $\frac{\lambda_i}{\mu_i}$. This supports the result in Table 1 for the negative correlation of the ratio $\frac{\lambda_i}{\mu_i}$ with the energy consumption (Fig. 8) and energy efficiency of the line respectively.

Additional Analysis: for all configurations, the first and second instances consume the same amount of energy and



(a) Energy consumption of the line (b) Energy efficiency of the line

FIGURE 12. Energy consumption and energy efficiency for configuration 4.



(a) Energy consumption (configuration 1) (b) Energy consumption (configuration 2)

FIGURE 13. Energy consumption and energy efficiency for the production line (machines positions impact study).

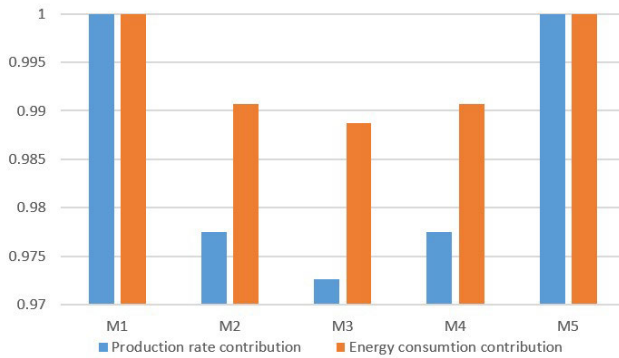


FIGURE 14. Homogeneous line with identical machines and buffers.

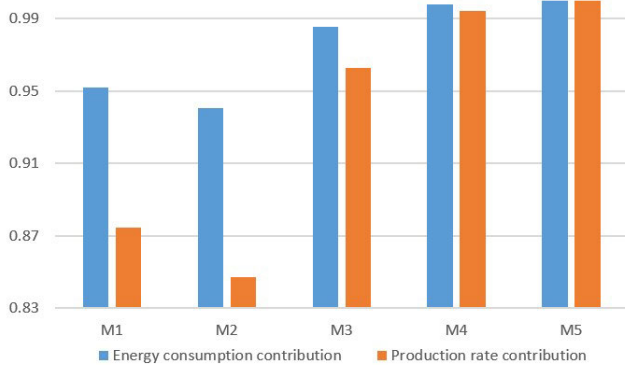


FIGURE 15. Homogeneous line with identical machines and non-identical buffers.

are characterized by an identical energy efficiency. However, the third instance of each configuration is distinguishable.

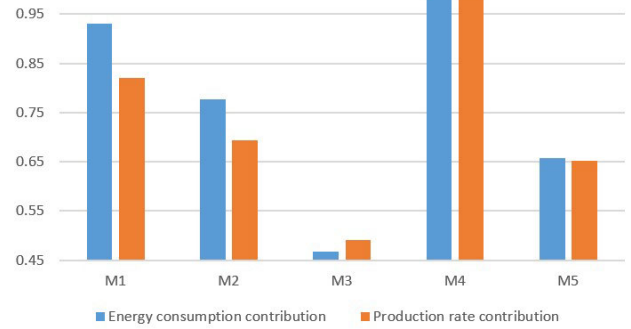


FIGURE 16. Homogeneous line with non-identical machines and buffers.



FIGURE 17. Ranking for homogeneous line with non-identical machines and buffers.

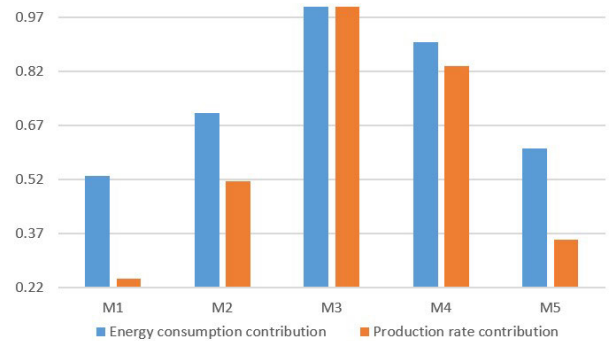


FIGURE 18. Non-homogeneous line with non-identical machines and buffers and $\lambda_j = \lambda$ and $\mu_j = \mu$.

TABLE 2. Numerical results.

Configuration	ψ	E	$\eta(\%)$
3 Machines: Case A	0.06	25.23	66.06
3 Machines: Case B	0.1198	25.55	78.10
5 Machines: Case A	0.1351	54.23	75.06
5 Machines: Case B	0.1424	52.95	77.16
7 Machines: Case A	0.1355	72.72	77.48
7 Machines: Case B	0.1359	74.13	76.37
15 Machines: Case A	0.8604	259.39	93.97
15 Machines: Case B	1.3896	454.37	92.31
20 Machines	0.0006	194.79	4.95

Configuration 1 demonstrated almost no gap for energy consumption and efficiency of the three instances, it is of 0.24% and 0.01% between instance 3 and 2 for energy consumption and energy efficiency respectively (Fig. 9a and Fig. 9b).

The gap beginning with the second configuration, where it is 1.05% and 0.14% for energy consumption and energy efficiency respectively (Fig. 10a and Fig. 10b). It continues to increase attaining 6.16% for energy consumption

TABLE 3. K machines and (K – 1) buffers system analysis results.

K machines and (K-1) buffers system		
System configuration	Results	Instances
Identical machines and buffers	The bottleneck machine (minimal production rate) is positioned in the middle of the line and it is characterized by minimal energy consumption. The energy consumption and production rate increase gradually from the bottleneck as a V shape (Fig. 14).	13
Identical machines and non-identical buffers	The production bottleneck is characterized by minimal energy consumption. The energy consumption and production rate increase gradually from the bottleneck as a V shape (Fig. 15).	14
Homogeneous line with non-identical machines and buffers	The minimal production corresponds to the minimum energy consumption (same result is noticed for the maximum). The ranking of machines by increasing production rate is the same as by energy consumption (Fig. 16 and Fig. 17).	15
Non-homogeneous line with non-identical buffers and machines having identical reliability parameters	The minimal production corresponds to the minimum energy consumption (same result is noticed for the maximum). The ranking of machines by increasing production rate is the same as by energy consumption (Fig. 18 and Fig. 19).	16
General case	Producing more does not necessarily involve consuming more energy (Fig. 20).	17

TABLE 4. Parameters of the tested configurations ([16]).

Configuration	λ_1	μ_1	ω_1	$\omega_1 \frac{\mu_1}{\lambda_1 + \mu_1}$	λ_2	μ_2	ω_2	$\omega_2 \frac{\mu_2}{\lambda_2 + \mu_2}$
Config 1	0.1	0.4	1	0.800	0.1	0.4	1	0.800
Config 2	0.1	0.8	1	0.889	0.08	0.7	1	0.897
Config 3	0.08	0.3	3	2.368	0.06	0.55	2.5	2.254

TABLE 5. Tested configurations parameters.

Configuration	λ_1	μ_1	ω_1	λ_2	μ_2	ω_2
Homogeneous line	varies	0.2	1	0.0025	0.025	1
Non-homogeneous line	varies	0.2	1	0.0025	0.025	1.5



FIGURE 19. Ranking for non-homogeneous line with non-identical machines and buffers and $\lambda_j = \lambda$ and $\mu_j = \mu$.

and 3.94% for energy efficiency in configuration 4 (Fig. 12a and Fig. 12b).

This illustrates the result summarized in Table 1, claiming that a change in the reliability parameters has no impact either on the energy consumption or the energy efficiency (corresponding graphs of instances 1 and 2 overlap for all four configurations). However, an increase in machine capacity leads to less energy consumption and better energy efficiency.

B. ANALYSIS OF A SYSTEM WITH K MACHINES AND (K-1) INTERMEDIATE BUFFERS

In this part, instances of [9] for homogeneous and non-homogeneous production lines are considered.

TABLE 6. Tested configurations parameters ([37]).

instance	λ_2	μ_2	ω_2	$\omega_2 \frac{\mu_2}{\lambda_2 + \mu_2}$
1.a	0.011	0.09	1	0.891
1.b	0.01	0.0817	1	0.891
1.c	0.01	0.09	0.99	0.891
2.a	0.0153	0.09	1	0.855
2.b	0.01	0.059	1	0.855
2.c	0.01	0.09	0.95	0.855
3.a	0.03	0.09	1	0.750
3.b	0.01	0.03	1	0.750
3.c	0.01	0.09	0.83	0.750
4.a	0.06	0.09	1	0.600
4.b	0.01	0.015	1	0.600
4.c	0.01	0.09	0.67	0.600

These examples consider production lines with 3, 5, and 7 different machines respectively. For each example, two different versions are studied, varying the reliability parameters and production capacities of the different machines. The reliability parameters, as well as the intermediate stock capacities of the different instances, are summarized in Tables 8), 9, and 10 for the respective configurations.

TABLE 7. Tested configurations parameters.

instance	λ_1	μ_1	ω_1	$\omega_1 \frac{\mu_1}{\lambda_1 + \mu_1}$	λ_2	μ_2	ω_2	$\omega_2 \frac{\mu_2}{\lambda_2 + \mu_2}$
1.1	0.01	0.09	1	0.9	0.06	0.09	1	0.6
1.2	0.06	0.09	1	0.6	0.01	0.09	1	0.9
2.1	0.01	0.09	1	0.9	0.01	0.09	0.67	0.6
2.2	0.01	0.09	0.67	0.6	0.01	0.09	1	0.9

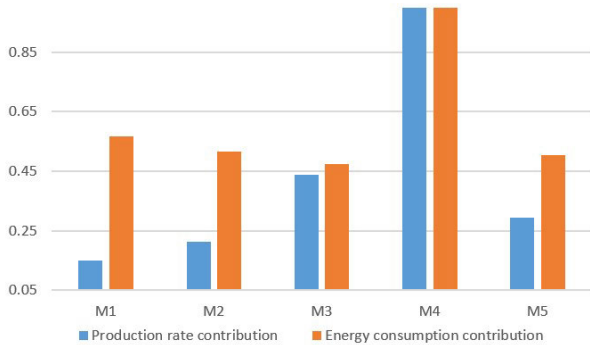


FIGURE 20. Other cases.

TABLE 8. Configuration 1 ([9]).

Case	M_i	λ_i	μ_i	ω_i	N_i
A	1	0.03	0.05	0.5	10
	2	0.04	0.06	0.1	10
	3	0.03	0.05	0.5	-
B	1	0.03	0.05	0.5	10
	2	0.04	0.06	0.2	10
	3	0.03	0.05	0.5	-

Instances studied by [4] for homogeneous lines and by [3] for non-homogeneous lines are also used. However, we only use one failure mode. The examples consist of 2 configurations with production lines composed of 15 and 20 machines respectively. The first configuration considers the case of a homogeneous line and the case of a non-homogeneous line, while the last one only considers a non-homogeneous line. The different parameters of these application examples are shown in Tables 11 and 12. Numerical results are presented in Table 2.

We notice from the previous numerical results that producing more does not necessarily lead to greater energy consumption. This is illustrated by the five machines configuration, where the homogeneous line (case A) produces less than the non-homogeneous line (case B) but consumes more than the later. We conduct further analysis on several cases of serial production lines in support of this result as well as to highlight other findings. These tests' results are summarized in Table 3.

It is important to note that these results (for both cases with two machines and with K machines) were presented after a large variety of conducted tests on instances from the literature as well as appropriately generated instances.

VII. CONCLUSION

In this paper, a new integrated analytical and computational method to evaluate both economic and energetic performance of unreliable serial production lines was developed.

TABLE 9. Configuration 2 ([9]).

Case	M_i	λ_i	μ_i	ω_i	N_i
A	1	0.01	0.1	0.25	4
	2	0.02	0.3	0.2	4
	3	0.04	0.5	0.3	4
	4	0.02	0.5	0.2	4
	5	0.04	0.5	0.3	-
B	1	0.02	0.3	0.2	4
	2	0.05	0.4	0.23	4
	3	0.01	0.1	0.3	4
	4	0.07	0.4	0.26	4
	5	0.02	0.3	0.21	-

TABLE 10. Configuration 3 ([9]).

Case	M_i	λ_i	μ_i	ω_i	N_i
A	1	0.02	0.3	0.20	4
	2	0.05	0.4	0.23	4
	3	0.01	0.1	0.3	6
	4	0.07	0.4	0.26	4
	5	0.03	0.3	0.21	4
	6	0.03	0.1	0.27	6
	7	0.06	0.4	0.26	-
B	1	0.02	0.3	0.2	4
	2	0.05	0.4	0.23	4
	3	0.01	0.1	0.3	5
	4	0.07	0.4	0.26	4
	5	0.02	0.3	0.21	4
	6	0.06	0.5	0.23	5
	7	0.01	0.1	0.27	-

The economic performance is characterized by the throughput, while for the energy performance, energy consumption and energy efficiency indicators are introduced. This evaluation method is based on a decomposition of energy consumption according to machine states. A Markov formulation is used to obtain stationary probabilities for each machine in each of the possible states and then to formulate the energy consumption and energy efficiency for each machine M_i , as well as for the serial production line. The proposed analytical model with its two components, economic and energetic, is formulated as a non-linear programming method implemented on LINGO 18.0 solver.

Thereafter, numerical tests were conducted using instances taken from the literature, for the elementary system with two machines and one buffer as well as for the general case of K machines and $(K - 1)$ buffers. Results were used to analyze the behavior of the economic and energetic performance of serial production lines as well as the trade-off between the two performances. The impact of buffers size, reliability parameters and machines production rates on the energy consumption and efficiency was analyzed.

The study of the trade-off between economic and energetic performance has demonstrated that they are characterized by a very complex inter-dependency. Numerical results have shown that the correlation is far from obvious. Indeed,

TABLE 11. Configuration 1 [4].

Case	Machine	M1	M2	M3	M4	M5	M6	M7	M8
Homogeneous line	λ_i	0.012	0.01	0.1	0.01	0.1	0.0141	0.0011	0.0143
	μ_i	0.22	0.04	0.2	0.087	0.25	0.2323	0.0348	0.135
	ω_i	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
	N_i	55	40	40	55	50	50	65	55
	Machine	M9	M10	M11	M12	M13	M14	M15	-
	λ_i	0.0008	0.0133	0.0296	0.0074	0.0429	0.0037	0.008	-
	μ_i	0.0349	0.1709	0.2323	0.0696	0.2024	0.0698	0.4273	-
	ω_i	1.3	1.3	1.3	1.3	1.3	1.3	1.3	-
	N_i	35	40	55	40	50	55	-	-
Case	Machine	M1	M2	M3	M4	M5	M6	M7	M8
Non-homogeneous line	λ_i	0.0943	0.0038	0.0234	0.0035	0.0206	0.0090	0.0017	0.0134
	μ_i	0.4968	0.0525	0.3051	0.0547	0.3189	0.2876	0.0552	0.1874
	ω_i	2.7256	3.7054	3.5206	3.0724	3.6719	3.6520	4.2079	3.6295
	N_i	70	50	45	60	67	65	75	65
	Machine	M9	M10	M11	M12	M13	M14	M15	-
	λ_i	0.0007	0.0055	0.0042	0.0005	0.1271	0.0003	0.0053	-
	μ_i	0.0414	0.1666	0.2021	0.0824	0.2401	0.0276	0.2106	-
	ω_i	4.0060	4.1955	3.9554	4.2889	2.1252	4.1519	4.0276	-
	N_i	40	45	60	67	65	65	-	-

TABLE 12. Configuration 2 [3].

Machine	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10
λ_i	0.0120	0.0100	0.1000	0.0100	0.1000	0.0141	0.0011	0.0143	0.0008	1.2000
μ_i	0.2200	0.0400	0.2000	0.0870	0.2500	0.2323	0.0348	0.1350	0.0349	0.0133
ω_i	1	1.2	1.1	1.4	1.2	1	1.2	1.1	1.4	0.0546
N_i	55	40	40	55	50	50	65	55	35	40
Machine	M11	M120	M13	M14	M15	M16	M17	M18	M19	M20
λ_i	0.0296	0.0074	0.0429	0.0037	0.0080	0.0071	0.0022	0.0071	0.0032	0.0200
μ_i	0.2323	0.0696	0.2024	0.0698	0.4273	0.2323	0.0696	0.2024	0.0698	0.4273
ω_i	1	1.2	1.1	1.4	1.2	1.03	1.103	1.178	1.041	0.956
N_i	55	40	55	55	65	55	35	40	55	-

TABLE 13. Homogeneous line with identical machines and buffers.

K	λ_i	μ_i	ω_i	N_i
5	0.02	0.5	1	20

TABLE 14. Homogeneous line with identical machines and non-identical buffers.

K	λ_i	μ_i	ω_i	N_i
5	0.02	0.5	1	(3, 8, 16, 50)

TABLE 15. Homogeneous line with non-identical machines and buffers.

Machine	λ_i	μ_i	ω_i	N_i
M1	0.01	0.45	1	12
M2	0.2	0.55	1	3
M3	0.5	0.3	1	20
M4	0.002	0.22	1	7
M5	0.3	0.4	1	//

TABLE 16. Non-homogeneous line with $\lambda_i = \lambda$ and $\mu_i = \mu$.

Machine	λ_i	μ_i	ω_i	N_i
M1	0.02	0.2	0.5	7
M2	0.02	0.2	1.5	15
M3	0.02	0.2	3	5
M4	0.02	0.2	2.5	20
M5	0.02	0.2	1	//

tests demonstrated that producing more does not necessarily involve consuming more energy. Both performances are also strongly influenced by buffers size, machine reliability parameters, and production line structure.

We believe that this work makes a major contribution to the field of production systems engineering and has the potential to be the reference method for the evaluation of both economic and energetic performances of serial production

TABLE 17. Other cases.

Machine	λ_i	μ_i	ω_i	N_i
M1	0.01	0.45	0.2	5
M2	0.2	0.55	0.5	20
M3	0.5	0.3	2	6
M4	0.4	0.22	2.5	8
M5	0.3	0.4	1	//

lines. Moreover, since the proposed approach is derived for the unreliable serial line model, researchers and practitioners can easily adapt and extend it to more complex systems (e.g. assembly lines, series-parallel systems, re-entrant systems, etc.). This work introduces several future research opportunities:

- The proposed method could be adapted for other production systems configurations such as assembly lines or series-parallel systems.
- The study could be extended for lines with non-exponential machines.
- Future analysis could include energy parameters influence on energy performances.
- The method could be applied on practical case studies for more accurate energy analysis.

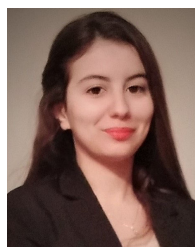
APPENDIX
NUMERICAL EXPERIMENTS DATA

See Tables 4–17.

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