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Generalized Imperfect D2D Associations in Spectrum-Shared Cellular Networks Under Transmit Power and Interference Constraints

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ABSTRACT This paper studies imperfect underlay device-to-device (D2D) association in spectrum-shared cellular networks. It addresses important system and design interference constraints, processing load limitations and transmit power constraints at D2D terminals. The paper proposes decentralized schemes for D2D communication between D2D terminals when downlink channel resources can be reused in the D2D network. The D2D transmitters are characterized considering their processing load limitation and allocated transmit power constraints. Moreover, the downlink channels that can be reused in the D2D network are quantified while meeting interference constraints imposed by the primary cellular network. Two schemes to identify reusable channels, which vary in terms of their efficiency, communication overhead requirement and implementation complexity, are described. Moreover, two D2D association schemes, namely the simultaneous and sequential D2D associations, are proposed and both aim to concurrently maximize the desired link quality and minimize the effect of interference effect at D2D receivers. Generalized analytical results that are applicable for various imperfect association scenarios are presented. The findings are applicable for any D2D channel models and performance metrics. They provide insights into various imperfect underlay D2D association scenarios under the practical system and design constraints.

INDEX TERMS D2D communication, cellular networks, imperfect D2D association, spectrum sharing, processing load, interference constraint, power constraint, association schemes, performance analysis, SINR statistics.

I. INTRODUCTION

Emerging cellular systems, such as Long Term Evolution-Advanced (LTE-A), have addressed the potential of device-to-device (D2D) communication in ultra-dense cellular networks [1]–[3]. This technology can utilize the existing cellular resources to realize an underlay D2D network. It can further improve spatial coverage and spectral efficiency of existing systems without the need for new infrastructure [1]–[4]. Moreover, D2D technology is expected to enhance the quality control of existing cellular base stations (BSs) because some cellular traffic can be offloaded to the underlay D2D network.

The successful deployment of D2D communication in cellular networks is however demanding innovative solutions for many technical challenges. These challenges include improved schemes for resource allocation and distribution, transmit power control, peer discovery and mode

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selection between BS or D2D modes of service, and interference mitigation and management. From the various possible approaches to tackle these challenges, the formulation of optimization algorithms for resource allocation [5]–[8], stochastic modeling [9]–[12], and geometry-based solutions [1]–[4], [13], power control algorithms [14]–[17], rate maximization algorithms [18]–[20], graph-based solutions [21]–[23], uplink/downlink decoupling with interference mitigation and resource allocation optimization [24], [25], and cooperative schemes [26]–[30] have been considered. The preceding works have focused on certain analysis for specific D2D communication scenarios. However, they do not provide detailed analysis for the identities of potential D2D transmitters, the conditions of cellular channels, and the D2D associations when downlink resources can be utilized simultaneously in the D2D network while meeting interference constraints and transmit power limitations of active devices. Moreover, they can not be used to model and analyze many practical imperfect D2D association scenarios under concurrent maximization of the desired power and minimization of interference at each D2D receiver in the D2D network.

This paper adds new enhancements to previous contributions. It considers a macro-cellular system that contains licensed devices, which can be potentially involved in D2D communication, and preassigned downlink cellular channels that can be reused in the underlay D2D network. The paper tackles different case study and methodology of analysis than the previous works mentioned above, which are based either on exhaustive processing, complicated algorithms for optimization, or schemes that demand centralized processing with extensive communication overhead. Particularly, it targets more realistic and comprehensive network scenario, and proposes generalized low-complexity D2D association schemes. These schemes are combined with effective decentralized approaches to identifying active devices as well as downlink channels to form successful D2D communication.

The paper also presents detailed modeling and analysis of devices and downlink channel conditions, which include their individual operation constraints, such as processing load, power limitation, and interference constraint. Moreover, it explains association schemes that aim to meet an improved quality of the desired link and a minimized effect of interference concurrently at a D2D receiver. With these two service objectives in mind, various imperfect association scenarios may arise. Specifically, an imperfect D2D association scenario can take place when a D2D receiver falls short in allocating the best possible serving partner (i.e., D2D transmitter), which can concurrently meet the service objectives. This will lead to a degraded quality of service, and hence a decrease in performance at the D2D receiver, regardless of the accuracy in allocating the reusable downlink channel for this D2D association. Moreover, another imperfect association can define the scenario when a D2D receiver can successfully know the D2D transmitter, but it falls short in identifying the best possible reusable downlink channel from the known D2D transmitter. The potential drawback herein will be a different degraded quality of service and a decrease in performance at the D2D receiver than those in the scenario above. Such imperfect association scenarios can occur separately or at the same time, which lead to observing dynamic impacts on the expected performance at D2D receivers. Therefore, the analyses of the two proposed D2D association schemes herein are presented in generalized forms that can explicitly treat any potential imperfect D2D association scenario.

The main contributions of this paper capitalizes on some parts of [31]–[34], which have addressed, in the context of underlay D2D communication or D2D small-cellular networks, dynamic allocation of downlink resources for improved D2D commutation, interference-free channel allocation in coordinated open-access cells, the impact of user identities and access conditions in closed cells, and adaptive interference-aware multichannel assignment for shared access points with limited feedback.

As an extensive expansion of the initial results reported in the conference version in [35], the main contributions of this expanded paper can be summarized as follows:

- Detailed modeling and analysis for forming underlay D2D associations are presented. The main objective of the presented association schemes is to concurrently maximize the quality of the desired link and minimize the effect of interference at each D2D receiver.
- Decentralized approaches for identifying D2D transmitters that can serve other devices in the D2D network as well as their associated downlink channels that can be reused during the D2D service mode are presented. The presented approaches consider the conditions that a cellular device be active and ready to serve others via D2D communication, its a priori limitation on the portion of transmit power that can be utilized to serve others in the D2D network, and its limitation on processing load, which is reflected by the number of D2D receivers in the D2D service mode it can serve simultaneously.
- Two interference-limited methods to classify downlink channels according to their reusability as seen by D2D transceivers in the D2D network are explained in details. The first method addresses that interference limits on downlink channels are set individually for each pair of potential D2D transceivers in the D2D network. On the other hand, the second method treats a universal scenario wherein interference limits on downlink channels are set a priori for any potential D2D association in the D2D network. The aforementioned methods differ in terms of their complexity and processing load, and they lead to different interference mitigation levels on the downlink channels.
- Two D2D association schemes, which vary in terms of their performance and complexity, are presented. These association schemes are performed by each potential D2D receiver in the D2D network, and they target simultaneous and sequential allocation of the suitable D2D transmitter and its associated downlink channel to

serve a D2D receiver in the D2D service mode. More importantly, the two schemes are thoroughly analyzed and compared for practical imperfect D2D association scenarios, from which perfect D2D association for each scheme can be deduced as a limiting case.
- Detailed analytical formulations for the statistics of the instantaneous signal-to-interference plus noise ratio (SINR) at a D2D receiver considering the two D2D association schemes are presented. More importantly, these formulations are applicable to study various imperfect association scenarios. They are key enablers to develop in-depth understanding and assessment for the effect of the various design and network constraints, which are addressed herein, on the efficiency of the proposed D2D association schemes. Moreover, they are applicable for any channels models and performance metrics, and can be used to develop analytical results for various performance measures.

The rest of the paper is organized as follows. Section II presents a preliminary discussion on the system and network models as well as the proposed D2D association schemes. Section III details the modeling and synthesis of the D2D transmitters and their associated reusable downlink channels. Sections IV and V present the generalized imperfect scenarios of the two D2D association schemes under consideration, from which perfect association scenarios are deduced as limiting cases. Section VI discusses selected numerical and simulation results, and Section VII presents the main conclusions of this work.

II. PRELIMINARY DISCUSSIONS

This Section contains two parts. The first part highlights the system and network models under consideration, whereas the second part introduces the proposed D2D association schemes.

A. SYSTEM AND NETWORK MODELS

The system and network models under consideration assume a macro-cellular network with an enabled underlay D2D communication. The cellular devices that can be involved in D2D communication are conditionally permitted to reuse the same downlink spectrum resources of the cellular network. These devices are first partitioned into D2D transmitters and receivers. Each pair of devices, which consists of a D2D transmitter and a receiver, attempts to establish a decentralized point-to-point communication link without intervention from the macrocell BS.

The macro-cell coverage area is assumed to contain a number of M licensed cellular devices. Each of these devices is equipped with a transceiver that is specifically dedicated for D2D communication. Note that potential D2D receivers represent the population of cellular devices that do not receive BS service, and therefore they are not in cellular service mode. These potential D2D receivers subsequently declare their readiness to enter D2D service mode. On the other hand, D2D transmitters can be receiving service from BS in cellular service mode, but may have the potential, under certain conditions, to utilize their D2D transceiver capabilities to serve D2D receivers through a concurrent D2D service mode. However, among the total number of potential D2D transmitters, the devices that can be part of underlay D2D communication are not fixed, but they can vary due to the randomness in their active periods, power and processing load limitations, and their abilities to support D2D service mode. Specifically, a device can only utilize a specific portion of its power to serve others via D2D communication. Moreover, a device can be limited by its permitted processing load, which is related to the number of concurrent D2D services it can support at a time. Therefore, a potential D2D transmitter can support the D2D service mode as long as its operation constraints are not violated, as will be detailed in subsection III-A.

The downlink spectrum resources in the macro-cell of interest are partitioned into N orthogonal physical channels, which may be modeled as sub-carriers of an orthogonal frequency division multiplexing (OFDM) system. These channels are accessible by any device at any time. They can be utilized to either serve devices during the cellular service mode when that service is offered by the macro-cell base station or serve other devices through D2D communication during the D2D service mode while meeting certain interference constraints.

The D2D coverage range is dependent on the maximum amount of transmit power potential D2D transmitter s may dedicate for D2D communication. Within that D2D coverage range, the D2D receiver attempts to establish a D2D association by identifying the most suitable D2D transmitter and its most suitable reusable channel. These steps aim to concurrently achieve a maximized quality of the desired link and a minimized effect of interference at the D2D receiver. Moreover, the D2D transmitter in D2D service mode may exploit the availability of multiple service requests for D2D communication on a given downlink channel, which may be placed by many potential D2D receivers, to further enhance the quality of service at a selected D2D receiver by utilizing a multiuser scheduling scheme.

B. D2D ASSOCIATION SCHEMES

The two D2D association schemes under consideration are explained in Fig. 1, which are referred to as simultaneous and sequential D2D association schemes, respectively. The key difference between these two association schemes is related to the mechanisms of labeling devices and reusable downlink channels within a D2D coverage range. They play a prominent role in their implementation complexity and expected performance under both perfect and imperfect association scenarios. In this regard, the devices within a D2D coverage range are partitioned into D2D transmitters and receivers according to their operation conditions, as highlighted in the previous part. Moreover, to control the amount of intra-cell interference (generated by the D2D network) and inter-cell interference (generated by the macrocell network) on each

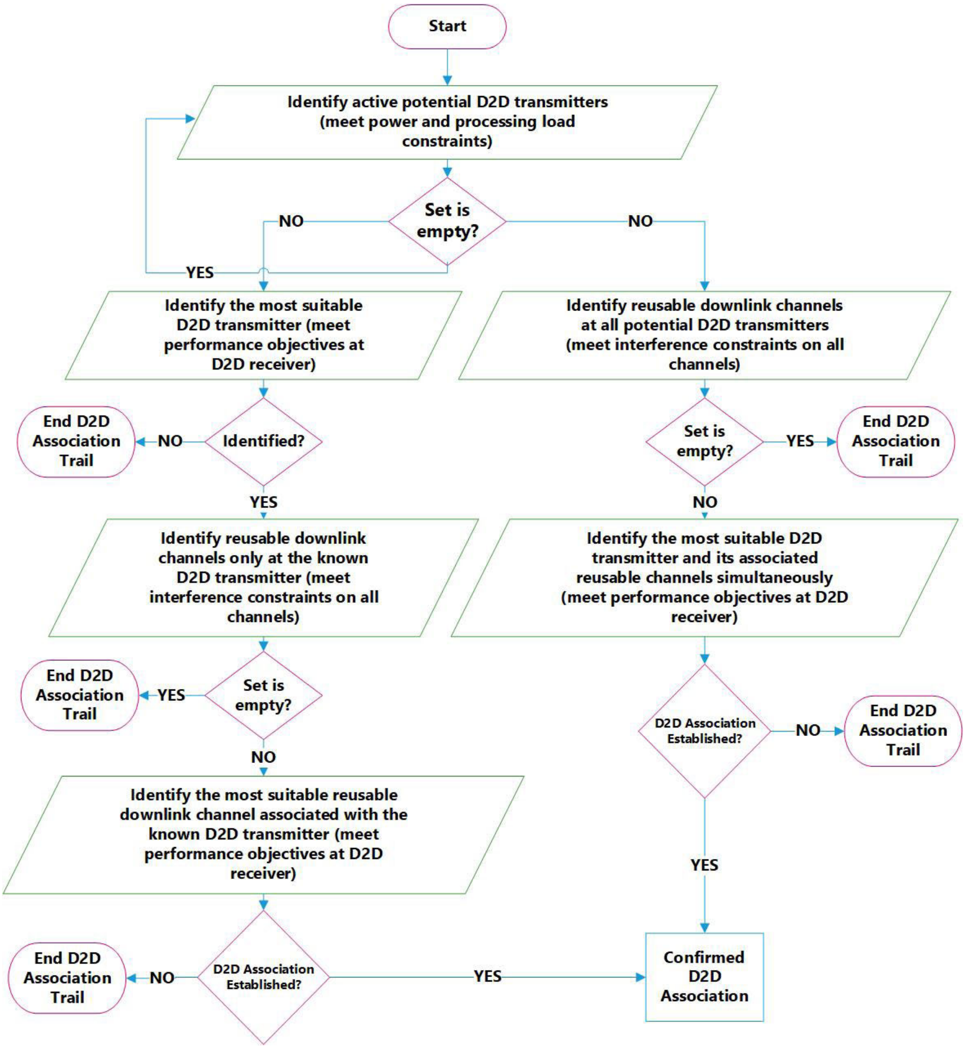

FIGURE 1. Descriptions of the simultaneous (right) and sequential (left) D2D association schemes.

downlink channel due to the co-exitance of an underly D2D network, an interference threshold is imposed by the primary cellular network on each of these channels. Therefore, the reusable channels in the D2D network are classified based on their associated aggregate interference levels. Since the devices that can serve others via D2D communication are identified a priori, irrespective to their associated channels conditions, there are two feasible approaches to determine the set of reusable channels for each potential D2D association.

Overhead signaling is an essential requirement to establish successful D2D associations among D2D transmitters and receivers. It regularly contains additional information to enhance the quality of data communication in the D2D network. Such overhead signaling include common and dedicated pilots as well as feedback signaling between a potential D2D transmitter and a receiver. However, if the amount of overhead signaling is significantly large, which effectively reduces the size of time frame that is dedicated for data transmission, the quality of data symbol communication may be enhanced, but the overall system performance may be degraded, particularly when the overhead signaling exceeds certain threshold. Considering the proposed schemes herein, the first approach counts for aggregate interference levels on available channels as observed by each pair of D2D transmitter and receiver separately. Therefore, a limited communication overhead between these two devices is needed. On the other hand, the second approach demands that aggregate interference levels on available channels are predicated universally by all D2D transmitters and a D2D receiver within the D2D coverage range, and therefore substantial communication overhead among devices is needed. The preceding approaches differ in terms of their complexity and resulting performance. More details are presented in subsection III-B.

Based on Fig. 1, the simultaneous association between a D2D receiver and a D2D transmitter is restricted through one reusable channel at a time. It requires that the D2D receiver should have universal channel state information (CSI) about all potential D2D transmitters within the D2D coverage range. This CSI is utilized by the D2D receiver to allocate the most suitable D2D transmitters and one of its associated channels that can meet the desired performance (i.e., concurrently maximize the desired link to the D2D receiver and minimize the effect of interference it encounters). Note that the universal knowledge of CSI demands extensive communication overhead and processing load. On the other hand, the sequential association can reduce processing load requirement at the D2D receiver because it does not demand a priori knowledge of all downlink channels of potential D2D transmitters if none of them is found suitable to meet the desired performance. Moreover, unlike the simultaneous association scheme, this sequential association scheme can be implemented with reduced communication overhead when the CSI of reusable channels are needed only at the potential D2D transmitter that has been known a priori. However, the main drawback is that the primary cellular network constraints for controlling interference levels on reusable downlink channels may not be maintained due to lack of universal monitoring of interference levels in the underlay D2D network. More details regarding simultaneous and sequential D2D association schemes are presented in Sections IV–V.

III. D2D TRANSMITTERS AND REUSABLE CHANNELS

This Section contains two main parts. The first part discusses the process by which D2D transmitters become known, whereas the second part explains the proposed approaches to know the downlink channels that can be utilized for D2D communication.

A. D2D TRANSMITTERS

The subsection explains the process by which the identities of D2D transmitters within the D2D coverage range can be declared and be available to other devices that anticipate to receive service via D2D association.

The process starts with individual devices by examining whether each of which has an allocated remaining transmit power to serve others via D2D communication. Following this, each device attempts to meet its processing load limit by not altering the maximum number of D2D receivers it can serve at a time. Now, if a device finds that it has remaining transmit power to serve others in the D2D service mode and its processing load allows serving more devices, it declares its identity as a potential D2D transmitter within its D2D coverage range by broadcasting a control message. This broadcast is expected to be collected by other devices that anticipate D2D service. The latter devices, after correctly decoding as many of control messages as possible, attempt to individually generate their own list of potential D2D transmitters. This process paves the way for establishing point-to-point communication links between the declared D2D transmitters and D2D receivers. However, additional constraints will have to be met in order to secure successful D2D associations, as will be explained in the subsequent parts.

The attention now is to quantify the set of potential D2D transmitters as seen by an arbitrary D2D receiver, which is referred to as jth D2D receiver of interest. Due to the differences in the likelihoods that devices be active and be able to serve others via D2D communication, the identities of potential D2D transmitters are subject to vary randomly. For devices whose D2D coverage range extends to the jth D2D receiver, there can be M_j devices that are in active cellular service mode (i.e., are being served by the macrocell BS), where M_j takes on values from $\{0, 1, 2, , \ldots, M\}$. Let p_m refers to the likelihood that the mth device be active, for $m = 1, 2, \ldots, M$. Based on [36], the probability of the event that $M_j = m_0$ can be expressed as

$$\begin{aligned}\Pr\{M_j = m_0\} &= \sum_{\mathcal{S}\in\mathcal{K}_{m_0}} \prod_{m''\in\mathcal{S}} p_{m''} \prod_{m'\in\mathcal{S}^c} (1-p_{m'}),\\ &\simeq \frac{e^{-\lambda}\lambda^{m_0}}{m_0!}, \qquad (1)\end{aligned}$$

where $\Pr\{\cdot\}$ refers to the probability of the random quantity between brackets, $\mathcal{K}_{m_0}$ is the set of all subsets of m_0 integers that can be selected from $\{1, 2, 3, \ldots, M\}$ and it contains $\binom{M}{m_0}$ elements, $\mathcal{S}^c$ is the complement of $\mathcal{S}$. The second equality in (1) provides a tight bound when $M \gg 1$ and $p_m \ll 1$, for $m = 1, 2, \ldots, M$, wherein $\lambda \triangleq \sum_{m=1}^{M} p_m$. Note that, for the limiting case when all devices have equal likelihood to be active (i.e., $p_i = p_m$, for $i = 1, 2, \ldots, M$ and $i \neq m$), the result in (1) can be further reduced to

$$\begin{aligned}\Pr\{M_j = m_0\} &= \binom{M}{m_0}(p_m)^{m_0}(1-p_m)^{M-m_0}\\ &\simeq \frac{e^{-Mp_m}(Mp_m)^{m_0}}{m_0!}. \qquad (2)\end{aligned}$$

From these M_j devices, some of them can not operate as D2D transmitters in the D2D service mode. This is solely related to the each device constraints on the processing load and its transmit power. Since the potential D2D transmitter has its processing load and transmit power limitations be directly related to the number of D2D receiver it can serve at a time, let $\mathcal{M}_{\mathrm{co},j}$ be a set that contains the identities of active devices that can be declared as potential transmitters to serve the jth device. The cardinality of $\mathcal{M}_{\mathrm{co},j}$, which is denoted by $|\mathcal{M}_{\mathrm{co},j}|$, takes on values from the set $\{0, 1, 2, \ldots, m_0\}$, for given $M_j = m_0$. Furthermore, define $p_{m_{\mathrm{co}}}$ as the probability that the m_{co}th active device declares its ability to serve others via point-to-point D2D links. Then, the conditional distribution of $|\mathcal{M}_{\mathrm{co},j}|$ can be expressed as

$$\begin{aligned}&\Pr\left\{|\mathcal{M}_{\mathrm{co},j}| = m_{\mathrm{co},0}|M_j = m_0\right\}\\ &\quad= \sum_{\mathcal{S}_{\mathrm{co}}\in\mathcal{K}_{m_{\mathrm{co},0}}} \prod_{m''_{\mathrm{co}}\in\mathcal{S}_{\mathrm{co}}} p_{m''_{\mathrm{co}}} \prod_{m'_{\mathrm{co}}\in\mathcal{S}^c{}_{\mathrm{co}}} \left(1-p_{m'_{\mathrm{co}}}\right)\\ &\quad\simeq \frac{e^{-\beta}\beta^{m_{\mathrm{co},0}}}{m_{\mathrm{co},0}!}, \qquad (3)\end{aligned}$$

where $\mathcal{K}_{m_{\mathrm{co},0}}$ is the set of all subsets of $m_{\mathrm{co},0}$ integers that can be selected from $\{1, 2, 3, \ldots, m_0\}$ and it contains

$\binom{m_0}{m_{\mathrm{co},0}}$ elements, and ${\mathcal{S}^c}_{\mathrm{co}}$ is the complement of $\mathcal{S}_{\mathrm{co}}$, and $\beta \triangleq \sum_{m_{\mathrm{co}}=1}^{m_0} p_{m_{\mathrm{co}}}$. The unconditional statistics of $|\mathcal{M}_{\mathrm{co},j}|$ can be then obtained, by using (1) and (3), as

$$\begin{aligned}
&\Pr\left\{|\mathcal{M}_{\mathrm{co},j}| = m_{\mathrm{co},0}\right\} \\
&= \sum_{m_0=m_{\mathrm{co},0}}^{M} \sum_{\mathcal{S}_{\mathrm{co}}\in\mathcal{K}_{m_{\mathrm{co},0}}} \left(\prod_{m_{\mathrm{co}}\in\mathcal{S}_{\mathrm{co}}} p_{m_{\mathrm{co}}} \prod_{m'_{\mathrm{co}}\in{\mathcal{S}^c}_{\mathrm{co}}} \left(1-p_{m'_{\mathrm{co},0}}\right)\right) \\
&\quad \times \Pr\{M_j = m_0\} \\
&= \sum_{m_0=m_{\mathrm{co},0}}^{M} \sum_{\mathcal{S}\in\mathcal{K}_{m_0}} \sum_{\mathcal{S}_{\mathrm{co}}\in\mathcal{K}_{m_{\mathrm{co},0}}} \left(\prod_{m_{\mathrm{co}}\in\mathcal{S}_{\mathrm{co}}} p_{m_{\mathrm{co}}}\right. \\
&\quad \left.\times \prod_{m'_{\mathrm{co}}\in{\mathcal{S}^c}_{\mathrm{co}}} \left(1-p_{m'_{\mathrm{co},0}}\right)\right) \left(\prod_{m\in\mathcal{S}} p_m \prod_{m'\in\mathcal{S}^c} (1-p_{m'})\right). \\
&\simeq \sum_{m_0=m_{\mathrm{co},0}}^{M} \frac{e^{-\beta}\beta^{m_{\mathrm{co},0}}}{m_{\mathrm{co},0}!} \frac{e^{-\lambda}\lambda^{m_0}}{m_0!}.
\end{aligned} \tag{4}$$

It is now required to quantify the term $p_{m_{\mathrm{co}}}$, which represents that the m_{co}th active device declares its ability to serve others in D2D service mode, as defined above. To this end, define $Q_{m_{\mathrm{co}}}$ be the number of different D2D services the m_{co}th device can meet at a time. The term $Q_{m_{\mathrm{co}}}$ takes on values from $\{0, 1, 2, \ldots, Q_{\max,m_{\mathrm{co}}}\}$, where $Q_{\max,m_{\mathrm{co}}}$ is the maximum value $Q_{m_{\mathrm{co}}}$, which is set by the m_{co}th device according to its permitted processing load. Based on the results presented in Appendix A for the distribution of $Q_{m_{\mathrm{co}}}$, it can be written that

$$p_{m_{\mathrm{co}}} = \sum_{c_{m_{\mathrm{co}}}=0}^{Q_{\max,m_{\mathrm{co}}}-1} \Pr\left\{Q_{m_{\mathrm{co}}} = c_{m_{\mathrm{co}}}\right\}, \tag{5}$$

which gives $p_{m_{\mathrm{co}}}$ that is needed in (4). Note that $p_{m_{\mathrm{co}}}$ is expected to decrease with the increase in $\mathbb{E}\{Q_{m_{\mathrm{co}}}\}$.

B. REUSABLE CHANNELS

This subsection details the approaches that identify reusable downlink channels for potential D2D associations, which have been highlighted in subsection II-B. The discussions herein are partitioned into three parts. The first part presents a generic analytical results that can quantify reusable downlink channels at an arbitrary device. The second part discusses the first approach discusses the case when reusable channels become known only between pair of transmit/D2D receiver that have the potential for D2D association. And the last part presents the second approach that relies on universal communication overhead among all potential D2D transmitters and a D2D receiver to quantify reusable downlink channels.

1) GENERIC QUANTIFICATION OF REUSABLE CHANNELS

With the objective to balance service load among downlink channels, the reusable channels for D2D communication have to maintain the aggregate interference on each of them below a certain threshold, This threshold is set by the primary cellular network. It is denoted by $s_{\mathrm{I,th}}$, and it can be specified relative to the background noise floor. The primary objective of this interference threshold is to avoid uncontrolled amplification of interference on downlink channels due to the co-existence of the underly D2D network. Therefore, a device will need to examine the interference levels on downlink channels against $s_{\mathrm{I,th}}$ to identify the ones which can be reused by that device to leverage any anticipated D2D association.

For an arbitrary device, define $\mathcal{N}$ as the set that contains indexes of downlink channels that meet the imposed aggregate interference threshold. It follows that $|\mathcal{N}| \in \{0, 1, 2, \ldots, N\}$, where N is the total number of downlink channels, as defined above. Moreover, define $s_{\mathrm{I},n}$ as the aggregate interference power on the nth downlink channel. The term $s_{\mathrm{I},n}$ represents the aggregation of the co-tier D2D network interference and cross-tier macrocell network interference sources. The probability that the nth downlink channel separately satisfies $n \in \mathcal{N}$ or $n \notin \mathcal{N}$ can be obtained as

$$\Pr\{n \in \mathcal{N}\} = \Pr\{s_{\mathrm{I},n} < s_{\mathrm{I,th}}\}, \tag{6}$$

$$\Pr\{n \notin \mathcal{N}\} = 1 - \Pr\{s_{\mathrm{I},n} < s_{\mathrm{I,th}}\}. \tag{7}$$

Based on the preceding results, the limiting case that $\mathcal{N} = \emptyset$ (or $|\mathcal{N}| = 0$), where $\emptyset$ denotes the empty set, can be written as

$$\Pr\{|\mathcal{N}| = 0\} = \prod_{n=1}^{N} \left[1 - \Pr\{s_{\mathrm{I},n} < s_{\mathrm{I,th}}\}\right], \tag{8}$$

which utilizes the fact that aggregate interference power levels on orthogonal channels are uncorrelated. The preceding result represents the case when the device can not be involved in D2D association. Particularly, it can not receive (due to overloaded channels) or support (to prevent further amplification of interference on shared channels) D2D service. On the other hand, the likelihood that all downlink channels are found suitable for D2D communication (i.e., the case of $|\mathcal{N}| = N$) can be written as

$$\Pr\{|\mathcal{N}| = N\} = \prod_{n=1}^{N} \Pr\{s_{\mathrm{I},n} < s_{\mathrm{I,th}}\}. \tag{9}$$

Apart from the aforementioned limiting cases, and due to the possibility that the interference powers on downlink channels undergo non-identical statistical properties, the distribution of $|\mathcal{N}|$ can be generally expressed as

$$\begin{aligned}
\Pr\{|\mathcal{N}| = n\} = &\sum_{\mathcal{S}\in\mathcal{K}_n} \prod_{n''\in\mathcal{S}} \left[\Pr\{s_{\mathrm{I},n''} < s_{\mathrm{I,th}}\}\right] \\
&\times \prod_{n'\in\mathcal{S}^c} \left(1 - \left[\Pr\{s_{\mathrm{I},n'} < s_{\mathrm{I,th}}\}\right]\right),
\end{aligned} \tag{10}$$

where $\mathcal{K}_n$ is the set of all subsets of n integers that can be selected from $\{1, 2, 3, \ldots, N\}$ and it contains $\binom{N}{n}$ elements, and $\mathcal{S}^c$ is the complement of $\mathcal{S}$.

Note that the reuse of a specific downlink channel to support a D2D association requires that both the D2D transmitter and receiver find that specific channel reusable. In this regard, for the m_{co}th device that can operate as D2D transmitter in the

D2D service mode, the set $\mathcal{N}_{m_{\mathrm{co}}}$ will contain indexes of downlink channels that are found reusable by that device, where $|\mathcal{N}_{m_{\mathrm{co}}}| \in \{0, 1, 2, \ldots, N\}$. Hence, $\Pr\{|\mathcal{N}_{m_{\mathrm{co}}}| = n_{m_{\mathrm{co}}}\}$ is similar to (10), after adjusting the associated indexes therein.

2) REUSABLE CHANNELS AT D2D TRANSCEIVER (First Approach)

The downlink channels that can be used for D2D communication between an arbitrary jth D2D receiver of interest and the m_{co}th potential D2D transmitter can be identified from the common intersection between the sets $\mathcal{N}_j$ and $\mathcal{N}_{m_{\mathrm{co}}}$. This approach ensures to meet the interference constraint at both devices simultaneously. Specifically, these devices, which are known to each other a priori, examine the reusability of downlink channels separately following the generic quantification approach described in the preceding part. The resulting reusable downlink channels at the two devices will form the sets $\mathcal{N}_j$ and $\mathcal{N}_{m_{\mathrm{co}}}$, respectively.

The m_{co}th potential D2D transmitter shares the indexes of reusable channels from the set $\mathcal{N}_{m_{\mathrm{co}}}$ with the jth D2D receiver. The later searches for common channels between its own set of reusable channels in $\mathcal{N}_j$ and $\mathcal{N}_{m_{\mathrm{co}}}$ to identify channels that can be reusable for D2D association. Following this, jth D2D receiver informs the m_{co}th D2D transmitter about the resulting common reusable channels. The later device examines these common findings to find the best one that can meet the performance requirement at the jth D2D receiver, which has been explained in subsection II-B.

The resulting set of reusable channels, which is defined as $\mathcal{N}_{j,m_{\mathrm{co}}} = \mathcal{N}_j \cap \mathcal{N}_{m_{\mathrm{co}}}$, can take on values from $\{0, 1, 2, \ldots, N\}$. Consequently, the likelihood that the nth channel has $n \in \mathcal{N}_{j,m_{\mathrm{co}}}$ is given as

$$\Pr\{n \in \mathcal{N}_{j,m_{\mathrm{co}}}\} = \Pr\{s_{\mathrm{I},n,j} < s_{\mathrm{I,th}}\}\Pr\{s_{\mathrm{I},n,m_{\mathrm{co}}} < s_{\mathrm{I,th}}\}, \qquad (11)$$

where the spatial independence between interference powers at devices has been a key enabler to the preceding result. Capitalizing on (11), the distribution of the cardinality of $\mathcal{N}_{j,m_{\mathrm{co}}}$, which is denoted by $|\mathcal{N}_{j,m_{\mathrm{co}}}|$, can be written as

$$\begin{aligned}&\Pr\{|\mathcal{N}_{j,m_{\mathrm{co}}}| = n_{j,m_{\mathrm{co}}}\}\\ &= \sum_{\mathcal{S}_{j,m_{\mathrm{co}}} \in \mathcal{K}_{n_{j,m_{\mathrm{co}}}}} \prod_{n \in \mathcal{S}_{j,m_{\mathrm{co}}}} \left[\Pr\{s_{\mathrm{I},n,j} < s_{\mathrm{I,th}}\}\Pr\{s_{\mathrm{I},n,m_{\mathrm{co}}} < s_{\mathrm{I,th}}\}\right]\\ &\quad \times \prod_{n' \in \mathcal{S}^c_{j,m_{\mathrm{co}}}} \left(1 - \left[\Pr\{s_{\mathrm{I},n',j} < s_{\mathrm{I,th}}\}\Pr\{s_{\mathrm{I},n',m_{\mathrm{co}}} < s_{\mathrm{I,th}}\}\right]\right),\end{aligned} \qquad (12)$$

where $\mathcal{K}_{n_{j,m_{\mathrm{co}}}}$, $\mathcal{S}^c_{j,m_{\mathrm{co}}}$, and $\mathcal{S}_{j,m_{\mathrm{co}}}$ have same definitions as $\mathcal{K}_{n_j}$, $\mathcal{S}^c_j$, and $\mathcal{S}_j$ above, respectively. For the special case when $\{s_{\mathrm{I},n,j}\}_{n=1}^N$ and $\{s_{\mathrm{I},n,m_{\mathrm{co}}}\}_{n=1}^N$ are identically distributed, (12) can be simplified to

$$\begin{aligned}&\Pr\{|\mathcal{N}_{j,m_{\mathrm{co}}}| = n_{j,m_{\mathrm{co}}}\}\\ &\qquad \simeq e^{-[\Pr\{s_{\mathrm{I},n} < s_{\mathrm{I,th}}\}]^2} \frac{\left(\Pr\{s_{\mathrm{I},n} < s_{\mathrm{I,th}}\}\right)^{2n_{j,m_{\mathrm{co}}}}}{n_{j,m_{\mathrm{co}}}!}.\end{aligned} \qquad (13)$$

The reusable channels herein consider the search outcomes at the two devices that have a potential D2D communication between them. Therefore, it may not provide universal control of interference levels in the D2D network. However, this approach incurs some performance degradation when compared to that presented in subsection III-B1, but also has some advantages when compared with the second approach presented in the following part. These advantages are increasing the possibility to find suitable reusable channels since two devices are only involved as well as reducing the communication overhead among devices within the D2D coverage range.

Illustration Example: To quantify the performance degradation associated with this approach as compared with that in subsection III-B1, the result in (13) can be expressed in terms of (10), for given number of reusable channels $n = n_{j,m_{\mathrm{co}}}$, as

$$\frac{\Pr\{|\mathcal{N}_{j,m_{\mathrm{co}}}| = n\}}{\Pr\{|\mathcal{N}| = n\}} \simeq e^{-[\Pr\{s_{\mathrm{I},n} < s_{\mathrm{I,th}}\}]} \left(\Pr\{s_{\mathrm{I},n} < s_{\mathrm{I,th}}\}\right)^n, \qquad (14)$$

which shows a degraded likelihood of observing the same n reusable channels by the two devices having a potential D2D association as compared with their individual observations of the same reusable channels. This degradation takes place for all possible values of $n > 0$, excluding $n = 0$ (i.e., none of downlink channels is found reusable). To this end, The likelihoods of the two important limiting cases of $|\mathcal{N}_{j,m_{\mathrm{co}}}| = 0$ and $|\mathcal{N}_{j,m_{\mathrm{co}}}| = N$ can be drawn from (12) as

$$\begin{aligned}&\Pr\{|\mathcal{N}_{j,m_{\mathrm{co}}}| = 0\}\\ &\quad = \prod_{n=1}^{N}[1 - \Pr\{s_{\mathrm{I},n,j} < s_{\mathrm{I,th}}\}\Pr\{s_{\mathrm{I},n,m_{\mathrm{co}}} < s_{\mathrm{I,th}}\}],\end{aligned} \qquad (15)$$

$$\begin{aligned}&\Pr\{|\mathcal{N}_{j,m_{\mathrm{co}}}| = N\}\\ &\quad = \prod_{n=1}^{N}[\Pr\{s_{\mathrm{I},n,j} < s_{\mathrm{I,th}}\}\Pr\{s_{\mathrm{I},n,m_{\mathrm{co}}} < s_{\mathrm{I,th}}\}].\end{aligned} \qquad (16)$$

Note that the event that $|\mathcal{N}_{j,m_{\mathrm{co}}}| = 0$ refers to the case when the identified sets of reusable channels by the two devices under consideration are disjoint, but this does not require any of these sets to be an empty set. On the other hand, the event that $|\mathcal{N}_{j,m_{\mathrm{co}}}| = N$ is only feasible when both the jth D2D receiver and the m_{co}th potential D2D transmitter predict all downlink channels as reusable for D2D communication. When these results are compared with the corresponding ones for individual devices in the preceding part, they show an increased likelihood of the undesired event $|\mathcal{N}_{j,m_{\mathrm{co}}}| = 0$ and a decreased likelihood of the desirable event $|\mathcal{N}_{j,m_{\mathrm{co}}}| = N$.

3) UNIVERSAL REUSABLE CHANNELS (Second Approach)

This approach requires that all potential D2D transmitters be involved in determining the set of reusable downlink channels for any potential D2D association. Specifically, each D2D receiver (e.g., the jth D2D receiver of interest) and all $|\mathcal{M}_{\mathrm{co},j}|$ potential D2D transmitters perform the search for reusable channels independently following the procedure described in subsection III-B1. The resulting sets of reusable channels will be then tabulated in the sets $\mathcal{N}_j$ and $\{\mathcal{N}_{m_{\text{co}}}\}_{m_{\text{co}}=1}^{m_{\text{co},0}}$, for given $|\mathcal{M}_{\text{co},j}| = m_{\text{co},0}$, respectively. The D2D transmitters share their $\{\mathcal{N}_{m_{\text{co}}}\}_{m_{\text{co}}=1}^{m_{\text{co},0}}$ with the jth D2D receiver. The later then determines the common reusable channels among $\mathcal{N}_j$ and $\{\mathcal{N}_{m_{\text{co}}}\}_{m_{\text{co}}=1}^{m_{\text{co},0}}$ to identify the universal and common reusable channels. The indexes of these channels are then shared with potential D2D transmitters, and one of them will pave the way for serving the jth D2D receiver by a potential D2D transmitter.

Based on the preceding description of this approach, the resulting set of reusable channels can be expressed as $\widetilde{\mathcal{N}}_j = \mathcal{N}_j \cap \mathcal{N}_1 \cap \cdots \cap \mathcal{N}_{m_{\text{co},0}}$. Therefore, the probability that the nth channel has its index in $\widetilde{\mathcal{N}}_j$ can be written as

$$\begin{aligned}&\Pr\left\{n \in \widetilde{\mathcal{N}}_j \big| |\mathcal{M}_{\text{co},j}| = m_{\text{co},0}\right\}\\ &\quad = \Pr\{s_{\text{I},n,j} < s_{\text{I,th}}\} \prod_{k=1}^{m_{\text{co},0}} \Pr\{s_{\text{I},n,k} < s_{\text{I,th}}\}. \qquad (17)\end{aligned}$$

Based on (17), the distribution of $|\widetilde{\mathcal{N}}_j|$, for given $|\mathcal{M}_{\text{co},j}| = m_{\text{co},0}$, can be expressed as

$$\begin{aligned}&\Pr\left\{|\widetilde{\mathcal{N}}_j| = \widetilde{n}_j \big| |\mathcal{M}_{\text{co},j}| = m_{\text{co},0}\right\}\\ &= \sum_{\mathcal{S}_j \in \mathcal{K}_{\widetilde{n}_j}} \prod_{n \in \mathcal{S}_j} \left[\Pr\{s_{\text{I},n,j} < s_{\text{I,th}}\} \prod_{k=1}^{m_{\text{co},0}} \Pr\{s_{\text{I},n,k} < s_{\text{I,th}}\}\right]\\ &\quad \times \prod_{n' \in \mathcal{S}_j^c} \left(1 - \left[\Pr\{s_{\text{I},n',j} < s_{\text{I,th}}\} \prod_{k=1}^{m_{\text{co},0}} \Pr\{s_{\text{I},n',k} < s_{\text{I,th}}\}\right]\right), \qquad (18)\end{aligned}$$

where $\mathcal{K}_{\widetilde{n}_j}$ has a similar definition of that for $\mathcal{K}_{n_j}$ above. For the special case when $\{s_{\text{I},n,j}\}_{n=1}^{N}$ and $\{s_{\text{I},n,k}\}_{n=1}^{N}$, $\forall\, k$, are identically distributed, the result in (18) can be simplified to

$$\begin{aligned}&\Pr\left\{|\widetilde{\mathcal{N}}_j| = \widetilde{n}_j \big| |\mathcal{M}_{\text{co},j}| = m_{\text{co},0}\right\}\\ &\quad \simeq e^{-[\Pr\{s_{\text{I},n} < s_{\text{I,th}}\}]^{(m_{\text{co},0}+1)}} \frac{\left(\Pr\{s_{\text{I},n} < s_{\text{I,th}}\}\right)^{(m_{\text{co},0}+1)\widetilde{n}_j}}{\widetilde{n}_j!}. \qquad (19)\end{aligned}$$

For any number of D2D transmitters, the unconditional statistics of $|\widetilde{\mathcal{N}}_j|$ can be obtained from (4) and (18) (or (19)) as shown in (20), at the bottom of the next page.

This approach may be suitable to control interference levels on all downlink channels within D2D network. It is because all D2D transmitters are involved in determining reusable channels with the same objective to keep the interference levels on them below the imposed threshold by the primary cellular network. However, it reduces the possibility of having more reusable channels for D2D association and does require more communication overhead among devices within a D2D coverage range, as compared with the first approach treated in subsection III-B2.

Illustration Example: To quantify the amount of performance degradation associated with this approach as compared with the findings in subsections III-B1 and III-B2, the result in (19) can be expressed in terms of (10), for given $n = \widetilde{n}_j$, as

$$\begin{aligned}&\frac{\Pr\{|\widetilde{\mathcal{N}}_j| = n \big| |\mathcal{M}_{\text{co},j}| = m_{\text{co},0}\}}{\Pr\{|\mathcal{N}| = n\}}\\ &\quad \simeq e^{-[\Pr\{s_{\text{I},n} < s_{\text{I,th}}\}]^{m_{\text{co},0}}} \left(\Pr\{s_{\text{I},n} < s_{\text{I,th}}\}\right)^{n m_{\text{co},0}}, \quad n > 0, \qquad (21)\end{aligned}$$

which reduces to (14) when $m_{\text{co},0} = 1$ (i.e., only one a priori known D2D transmitter is being involved), as expected. The preceding result shows the amplification in the performance loss of observing n reusable channels when all potential D2D transmitters are involved, as compared to the cases in (10) and (14), respectively. This loss extends for all $n > 0$.

The probability of the two limiting cases that $|\widetilde{\mathcal{N}}_j| = 0$ and $|\widetilde{\mathcal{N}}_j| = N$, for given $|\mathcal{M}_{\text{co},j}| = m_{\text{co},0}$, can be obtained from (18) as

$$\begin{aligned}&\Pr\left\{|\widetilde{\mathcal{N}}_j| = 0 \big| |\mathcal{M}_{\text{co},j}| = m_{\text{co},0}\right\}\\ &= \prod_{n=1}^{N} \left[1 - \Pr\{s_{\text{I},n,j} < s_{\text{I,th}}\} \prod_{k=1}^{m_{\text{co},0}} \Pr\{s_{\text{I},n,k} < s_{\text{I,th}}\}\right], \qquad (22)\\ &\Pr\left\{|\widetilde{\mathcal{N}}_j| = N \big| |\mathcal{M}_{\text{co},j}| = m_{\text{co},0}\right\}\\ &= \prod_{n=1}^{N} \left[\Pr\{s_{\text{I},n,j} < s_{\text{I,th}}\} \prod_{k=1}^{m_{\text{co},0}} \Pr\{s_{\text{I},n,k} < s_{\text{I,th}}\}\right]. \qquad (23)\end{aligned}$$

They indicate that the search for the reusable channels for D2D communication becomes a rather involved process. Also, the undesirable event of finding no reusable channels for D2D communication becomes more likely to happen, and the desirable event that all downlink channels are found reusable becomes less likely as compared with the cases described in subsections III-B1 and III-B2.

The following two Sections exploit the findings above to describe the proposed D2D association schemes, namely the simultaneous and sequential D2D associations. The developed analytical results in the coming two sections are generic and applicable for various operating conditions, including many imperfect association scenarios. As stated before, the two schemes are implemented to meet the same aim of achieving maximized desired link and minimized effect of interference at any D2D receiver. The similarities and differences between these association schemes are also detailed.

IV. SIMULTANEOUS D2D ASSOCIATION SCHEME

This Section is divided into three parts. The first part describes the simultaneous D2D association scheme. The second part develops generalized results for the statistics of the instantaneous SINR at a D2D receiver under the generalized imperfect association scenario. Finally, the third part depicts some special cases, including the perfect simultaneous D2D association.

A. DESCRIPTION OF THE SIMULTANEOUS D2D ASSOCIATION

The simultaneous D2D association scheme aims to identify the D2D transmitter and its associated reusable channel simultaneously to meet the performance requirements at a D2D receiver. In this regard, it requires from the D2D receiver (as the jth device for instance) to perform an extensive search over the sets of potential D2D transmitters and the set of reusable channels concurrently.

The search for the most suitable D2D transmitter will be performed over the set $\mathcal{M}_{\text{co},j}$, which includes the indexes of potential D2D transmitters. On the other hand, and due to the fact that the simultaneous association does not permit the D2D receiver to know the identity of its D2D transmitter a priori, the D2D receiver will have then to search for the most suitable reusable channel from the set $\widetilde{\mathcal{N}}_j$, which is formed by involving all D2D transmitters as explained in subsection III-B3.

With the objective to maximize the desired link quality while at the same time minimizing the effect of interference at the jth D2D receiver of interest, the likelihood that the simultaneous D2D association will result in a D2D transmitter and a reusable channel of indexes $m \in \mathcal{M}_{\text{co},j}$ and $n \in \widetilde{\mathcal{N}}_j$, respectively, can be written as

$$\begin{aligned}&\Pr\{(m,n)\ \text{Identified}|\mathcal{M}_{\text{co},j},\widetilde{\mathcal{N}}_j\}\\&\quad=\underbrace{\prod_{m'\in\mathcal{M}_{\text{co},j}}\prod_{n'\in\widetilde{\mathcal{N}}_j}}_{(m',n')\neq(m,n)}\\&\qquad\times\Pr\{s_{\text{D},m,n,j}>s_{\text{D},m',n',j}\}\Pr\{s_{\text{I},m,n,j}<s_{\text{I},m',n',j}\},\qquad(24)\end{aligned}$$

where $s_{\text{D},m,n,j}$ and $s_{\text{I},m,n,j}$ are the received desired power and the aggregate interference power, respectively, that are observed at the jth device served by (m,n)-labeled D2D association. Clearly, this scheme results in a successful D2D association only if the indexes of the D2D transmitter and its associated reusable channel that achieve the best desired link gain are identical to those that provide the lowest possible interference effect. Therefore, the aforementioned strict conditions reduce the likelihood of D2D association. However, the increase in $|\mathcal{M}_{\text{co},j}|$ and/or $|\widetilde{\mathcal{N}}_j|$ can improve the performance at the jth D2D receiver.

B. ANALYSIS OF GENERALIZED IMPERFECT ASSOCIATION

Consider that the jth D2D receiver under simultaneous D2D association scheme concurrently identifies the D2D transmitter and its associated reusable channel of indexes (m,n), where $m \in \mathcal{M}_{\text{co},j}$ and $n \in \widetilde{\mathcal{N}}_j$. To simplify the notations, let $r \equiv (m,n)$ be a specific combination of m and n. Then r can take on values from the set $r \in \{1,2,\ldots,|\mathcal{M}_{\text{co},j}||\widetilde{\mathcal{N}}_j|\}$.

Now, define $s_{\text{D},(r^\star),j}$, for $1 \le r^\star \le |\mathcal{M}_{\text{co},j}||\widetilde{\mathcal{N}}_j|$, as the received desired power when the $r^\star$th D2D transmitter and its associated reusable channel be identified to serve the jth D2D receiver. From the order statistics

$$s_{\text{D},(1),j} < s_{\text{D},(2),j} < \ldots < s_{\text{D},(|\mathcal{M}_{\text{co},j}||\widetilde{\mathcal{N}}_j|),j},$$

which is obtained by arranging $\{s_{\text{D},r,j}\}_{r=1}^{|\mathcal{M}_{\text{co},j}||\widetilde{\mathcal{N}}_j|}$ in increasing order of magnitudes, the $r^\star$th D2D association link in this order statistics may be identified to serve the jth device, for $1 \le r^\star \le |\mathcal{M}_{\text{co},j}||\widetilde{\mathcal{N}}_j|$. The best scenario is when $r^\star = |\mathcal{M}_{\text{co},j}||\widetilde{\mathcal{N}}_j|$, which gives

$$s_{\text{D},(r^\star),j} = \max_r\{s_{\text{D},r,j}\}_{r=1}^{|\mathcal{M}_{\text{co},j}||\widetilde{\mathcal{N}}_j|}, .$$

The worst scenario occurs when $r^\star = 1$, which corresponds to

$$s_{\text{D},(r^\star),j} = \min_r\{s_{\text{D},r,j}\}_{r=1}^{|\mathcal{M}_{\text{co},j}||\widetilde{\mathcal{N}}_j|}.$$

Consider the aggregate interference power that is observed at the jth device when it is served through a D2D association link of indexes $u \equiv (m,n)$ with $u \in \{1,2,\ldots,|\mathcal{M}_{\text{co},j}||\widetilde{\mathcal{N}}_j|\}$, where u is generally different than r. Define $s_{\text{I},(u^\star),j}$, for $1 \le u^\star \le |\mathcal{M}_{\text{co},j}||\widetilde{\mathcal{N}}_j|$, as the experienced interference power level when the $u^\star$th D2D association link is identified. Then

$$s_{\text{I},(1),j} < s_{\text{I},(2),j} < \ldots < s_{\text{I},(|\mathcal{M}_{\text{co},j}||\widetilde{\mathcal{N}}_j|),j}$$

is obtained by arranging $\{s_{\text{I},u,j}\}_{u=1}^{|\mathcal{M}_{\text{co},j}||\widetilde{\mathcal{N}}_j|}$ in increasing order of magnitudes, the $u^\star$th link in this order statistics may be identified based on minimizing the interference effect, for $1 \le u^\star \le |\mathcal{M}_{\text{co},j}||\widetilde{\mathcal{N}}_j|$, wherein the best scenario takes place when $u^\star = 1$ at which

$$s_{\text{I},(u^\star),j} = \min_u\{s_{\text{I},u,j}\}_{u=1}^{|\mathcal{M}_{\text{co},j}||\widetilde{\mathcal{N}}_j|},$$

whereas the worst scenario occurs when $u^\star = |\mathcal{M}_{\text{co},j}||\widetilde{\mathcal{N}}_j|$, which corresponds to

$$s_{\text{I},(u^\star),j} = \max_u\{s_{\text{I},u,j}\}_{u=1}^{|\mathcal{M}_{\text{co},j}||\widetilde{\mathcal{N}}_j|}.$$

Under the generalized imperfect simultaneous D2D association scheme presented above, the received SINR at the jth device can be generally expressed as

$$\gamma_{\text{SINR},j} = \frac{s_{\text{D},(r^\star),j}}{s_{\text{I},(u^\star),j}+\sigma^2},\qquad(25)$$

$$\begin{aligned}&\Pr\left\{|\widetilde{\mathcal{N}}_j| = \widetilde{n}_j\right\}\\&\quad=\sum_{m_0=m_{\text{co},0}}^{M}\sum_{\mathcal{S}\in\mathcal{K}_{m_0}}\sum_{\mathcal{S}\in\mathcal{K}_{m_{\text{co},0}}}\sum_{\mathcal{S}_j\in\mathcal{K}_{\widetilde{n}_j}}\prod_{n\in\mathcal{S}_j}\left[\Pr\{s_{\text{I},n,j}<s_{\text{I,th}}\}\times\prod_{k=1}^{m_{\text{co},0}}\Pr\{s_{\text{I},n,k}<s_{\text{I,th}}\}\right]\\&\qquad\times\prod_{n'\in\mathcal{S}_j^c}\left(1-\left[\Pr\{s_{\text{I},n',j}<s_{\text{I,th}}\}\prod_{k=1}^{m_{\text{co},0}}\Pr\{s_{\text{I},n',k}<s_{\text{I,th}}\}\right]\right)\left(\prod_{m_{\text{co}}\in\mathcal{S}}p_{m_{\text{co}}}\prod_{m'_{\text{co}}\in\mathcal{S}^c}(1-p_{m'_{\text{co},0}})\right)\prod_{m\in\mathcal{S}}p_m\prod_{m'\in\mathcal{S}^c}(1-p_{m'}).\qquad(20)\end{aligned}$$

where σ^2 represents the average power of the background white noise. Appendix B provides detailed analysis for the conditional statistics of $\widetilde{\gamma}_{\mathrm{SINR},j}$, conditioned on the values of $|\mathcal{M}_{\mathrm{co},j}|$ and $|\widetilde{\mathcal{N}}_j|$, which results in (26), as shown at the bottom of the page.

Using the results in Appendix B and the findings in Section III and (26), The unconditional statistics of $\widetilde{\gamma}_{\mathrm{SINR},j}$ can be expressed as

$$\begin{aligned}
&\Pr\left\{\widetilde{\gamma}_{\mathrm{SINR},j} < x\right\}\\
&\quad = \eta \sum_{m_{\mathrm{co},0}=1}^{M} \frac{1}{1-\Pr\{|\widetilde{\mathcal{N}}_j| = 0 | |\mathcal{M}_{\mathrm{co},j}| = m_{\mathrm{co},0}\}}\\
&\qquad \times \sum_{\widetilde{n}_j=1}^{N} \Pr\left\{\widetilde{\gamma}_{\mathrm{SINR},j} < x | |\mathcal{M}_{\mathrm{co},j}| = m_{\mathrm{co},0}, |\widetilde{\mathcal{N}}_j| = \widetilde{n}_j\right\}\\
&\qquad \times \Pr\{|\widetilde{\mathcal{N}}_j| = \widetilde{n}_j | |\mathcal{M}_{\mathrm{co},j}| = m_{\mathrm{co},0}\} \Pr\left\{|\mathcal{M}_{\mathrm{co},j}| = m_{\mathrm{co},0}\right\},
\end{aligned} \tag{27}$$

where $\eta \triangleq \left(1-\Pr\left\{|\mathcal{M}_{\mathrm{co},j}| = 0\right\}\right)^{-1}$, $\Pr\left\{|\mathcal{M}_{\mathrm{co},j}| = m_{\mathrm{co},0}\right\}$ and $\Pr\{|\widetilde{\mathcal{N}}_j| = \widetilde{n}_j | |\mathcal{M}_{\mathrm{co},j}| = m_{\mathrm{co},0}\}$ are given in (4) and (18), respectively. Note that the events $|\mathcal{M}_{\mathrm{co},j}| = 0$ (i.e., no potential D2D transmitters) and $|\widetilde{\mathcal{N}}_j| = 0$ for given $|\mathcal{M}_{\mathrm{co},j}| > 0$ (i.e., none of downlink channels can be reused for D2D communication) have been excluded.

The result in (27) can be utilized to study various performance measures, such as outage performance, average throughput, or error rate. Moreover, they can be used to deduce many special cases, such as the perfect association scenario. They are useful to explain various imperfect association scenarios that can be due to imperfect D2D transmitter and reusable channel association, imperfect D2D transmitter association alone, or imperfect reusable channel association alone.

C. SPECIAL CASES

1) PERFECT SIMULTANEOUS D2D ASSOCIATION

This part presents the simultaneous D2D association scheme under perfect implementation. It is shown herein as a special case of the generalized results developed in the previous part.

The perfect association scenario aims to provides the best desired link quality, which can be achieved when $r^\star = |\mathcal{M}_{\mathrm{co},j}||\widetilde{\mathcal{N}}_j|$, at which

$$s_{\mathrm{D},(r^\star),j} = \max_{r}\{s_{\mathrm{D},r,j}\}_{r=1}^{|\mathcal{M}_{\mathrm{co},j}||\widetilde{\mathcal{N}}_j|}. \tag{28}$$

Moreover, it guarantees the best interference immunity, which becomes feasible when $u^\star = 1$, at which

$$s_{\mathrm{I},(u^\star),j} = \min_{u}\{s_{\mathrm{I},u,j}\}_{u=1}^{|\mathcal{M}_{\mathrm{co},j}||\widetilde{\mathcal{N}}_j|}. \tag{29}$$

Iff the indexes $m \in \mathcal{M}_{\mathrm{co},j}$ and $n \in \widetilde{\mathcal{N}}_j$ concurrently meet the aforementioned objectives of $s_{\mathrm{I},(1),j}$ and $s_{\mathrm{D},(|\mathcal{M}_{\mathrm{co},j}||\widetilde{\mathcal{N}}_j|),j}$ are identical, then the received SINR at the jth device under perfect simultaneous D2D association can be expressed as

$$\gamma_{\mathrm{SINR},j} = \frac{s_{\mathrm{D},(|\mathcal{M}_{\mathrm{co},j}||\widetilde{\mathcal{N}}_j|),j}}{s_{\mathrm{I},(1),j}+\sigma^2}. \tag{30}$$

Appendix C shows the analysis for the conditional statistics of $\widetilde{\gamma}_{\mathrm{SINR},j}$, conditioned on the values of $|\mathcal{M}_{\mathrm{co},j}|$ and $|\widetilde{\mathcal{N}}_j|$, which results in (31), as shown at the bottom of the next page, and its special case in (47).

Now, using the findings in Section III, the unconditional statistics of $\gamma_{\mathrm{SINR},j}$ can be expressed in a similar form to (27), but with the conditional statistics of $\gamma_{\mathrm{SINR},j}$ in (31) replace that of $\widetilde{\gamma}_{\mathrm{SINR},j}$ therein.

2) SCENARIOS OF IMPERFECT SIMULTANEOUS D2D ASSOCIATION

Further special cases that may have practical presence can be deduced from the generalized results in subsection IV-B, for given $\mathcal{M}_{\mathrm{co},j}$ and $\widetilde{\mathcal{N}}_j$. Three scenarios are highlighted below:

- The best possible desired link quality with the worst interference power effect at the jth device can be directly deduced from the findings in subsection IV-B using the parameters $r^\star = u^\star = |\mathcal{M}_{\mathrm{co},j}||\widetilde{\mathcal{N}}_j|$.
- The worst desired link quality coupled with the best possible interference mitigation can be deduced using the parameters $r^\star = u^\star = 1$.

$$\begin{aligned}
&\Pr\left\{\widetilde{\gamma}_{\mathrm{SINR},j} < x | |\mathcal{M}_{\mathrm{co},j}| = m_{\mathrm{co},0}, |\widetilde{\mathcal{N}}_j| = \widetilde{n}_j\right\}\\
&\quad = \mathbb{E}_{s_{\mathrm{I},(u^\star),,j}}\left\{\Pr\left\{s_{\mathrm{D},(r^\star),j} < ((s_{\mathrm{I},(u^\star),j}+\sigma^2)x)\right\}\right\}\\
&\quad = \int_0^{s_{\mathrm{I,th}}} \left[\sum_{\ell=r^\star}^{m_{\mathrm{co},0}\widetilde{n}_j} \sum_{S_\ell} \prod_{p=1}^{\ell} \Pr\left\{s_{\mathrm{D},r_p,j} < (t+\sigma^2)x\right\} \prod_{p=\ell+1}^{m_{\mathrm{co},0}\widetilde{n}_j} (1-\Pr\left\{s_{\mathrm{D},r_p,j} < (t+\sigma^2)x\right\})\right]\\
&\qquad \times \sum_{\ell_1=u^\star}^{m_{\mathrm{co},0}\widetilde{n}_j} \sum_{S_{\ell_1}} \left[\left(\sum_{p'=1}^{\ell_1}\left[\frac{d}{dt}\Pr\{s_{\mathrm{I},u_{p'},j} < t\}\right]\frac{1}{\Pr\{s_{\mathrm{I},u_{p'},j} < t\}}\right) + \left(\sum_{p'=\ell_1+1}^{m_{\mathrm{co},0}\widetilde{n}_j}\left[\frac{d}{dt}(1-\Pr\{s_{\mathrm{I},u_{p'},j} < t\})\right]\frac{1}{1-\Pr\{s_{\mathrm{I},u_{p'},j} < t\}}\right)\right]\\
&\qquad \times \prod_{p=1}^{\ell_1}\Pr\{s_{\mathrm{I},u_p,j} < t\} \prod_{p=\ell_1+1}^{m_{\mathrm{co},0}\widetilde{n}_j}(1-\Pr\{s_{\mathrm{I},u_p,j} < t\})dt.
\end{aligned} \tag{26}$$

- The worst desired link quality and the worst interference effect can be readily obtained when $r^{\star} = 1$ and $u^{\star} = |\mathcal{M}_{\text{co},j}||\widetilde{\mathcal{N}}_j|$.

Each of these special cases leads to different statistical characteristics of the instantaneous received SINR, and therefore, difference performance outcomes at the jth D2D receiver.

The performance degradation of a specific imperfect simultaneous D2D association scenario as compared with the perfect association case presented in subsection IV-C1 vary with many parameters. They include the processing load and transmit power constraints at potential D2D transmitters, the interference constraints imposed on reusing downlink channels in the D2D network, the average values of desired and interference powers, the fading severity on the D2D association link, and the cardinalities of $\mathcal{M}_{\text{co},j}$ and $\widetilde{\mathcal{N}}_j$.

V. SEQUENTIAL D2D ASSOCIATION SCHEME

Following the same procedure used in the previous Section, this Section is divided into three parts. The first part describes the sequential D2D association scheme. The second part develops generalized results for the statistics of the instantaneous received SINR at a D2D receiver under a generalized imperfect association scenario. Finally, the third part shows some special cases of practical interest, including the scenario of perfect sequential D2D association.

A. DESCRIPTION OF THE SEQUENTIAL ASSOCIATION

The sequential D2D association aims to meet the interference effect minimization and desired link maximization. Herein, the D2D transmitter that meets the aforementioned objectives is first identified by, for instance, the jth D2D receiver based on $\mathcal{M}_{\text{co},j}$. The probability that the mth D2D transmitter, where $m \in \mathcal{M}_{\text{co},j}$, satisfies the aforementioned performance objectives can be written as

$$\begin{aligned}\Pr\left\{m \text{ Device Id.}|\mathcal{M}_{\text{co},j}\right\} &= \prod_{m'\in\mathcal{M}_{\text{co},j}, m'\neq m} \times\Pr\{s_{\text{D},m,n,j} > s_{\text{D},m',n,j}\}\\ &\quad\times\Pr\{s_{\text{I},m,n,j} < s_{\text{I},m',n,j}\}.\end{aligned} \tag{32}$$

The following step is to identify the reusable downlink channel that can also meet the aforementioned performance objectives. There can be two possibilities to determine the most suitable reusable channel, as explained in subsections III-B2 and III-B3.

1) REUSABLE CHANNELS BASED ON FIRST APPROACH

Based on the findings in subsection III-B2, the a priori association of the most suitable D2D transmitter can be utilized to reduce the search load for the most suitable downlink channel based on $\mathcal{N}_{j,m}$, where $m \in \mathcal{M}_{\text{co},j}$ is the index of the known D2D transmitter. Conditioned on the association with the D2D transmitter, the probability that the nth channel, for $n \in \mathcal{N}_{j,m}$, meets the intended performance objectives at the jth D2D receiver can be expressed as

$$\begin{aligned}&\Pr\left\{n \text{ Channel Id.}|\mathcal{N}_{j,m}, m\right\}\\ &\quad= \prod_{n'\in\mathcal{N}_{j,m}, n'\neq n}\\ &\qquad\times\Pr\{s_{\text{D},m,n,j} > s_{\text{D},m,n',j}\}\Pr\{s_{\text{I},m,n,j} < s_{\text{I},m,n',j}\}.\end{aligned} \tag{33}$$

Now, combining the two results in (32) and (33) gives the same form of (24) but with $\widetilde{\mathcal{N}}_j$ is now replaced by $\mathcal{N}_{j,m}$.

2) REUSABLE CHANNELS FROM SECOND APPROACH

Based on the findings in subsection III-B3, in which the association of the reusable channel is performed over the set $\widetilde{\mathcal{N}}_j$, the likelihood that the jth D2D receiver will identify the D2D communication link of indexes (m, n), where $m \in \mathcal{M}_{\text{co},j}$ and $n \in \widetilde{\mathcal{N}}_j$, to meet its performance objectives has an identical form to that given in (24). However, this observation is only valid under perfect sequential D2D association scenario.

A direct advantage of the sequential D2D association is that it can reduce the search load at the D2D receiver since the search for the reusable channel can be terminated if a suitable D2D transmitter can not be first identified. This is not applicable for the simultaneous D2D association, in which the D2D receiver has to perform an extensive search for the most suitable D2D transmitter and its associated reusable channel concurrently.

B. ANALYSIS OF GENERALIZED IMPERFECT ASSOCIATION

This subsection develops generalized results for imperfect sequential D2D association. It first treats the imperfect D2D transmitter association, and thereafter treats the imperfect reusable channel association considering the approaches presented in subsections III-B2 and III-B3.

$$\begin{aligned}\Pr\left\{\gamma_{\text{SINR},j} < x\,\middle|\,|\mathcal{M}_{\text{co},j}| = m_{\text{co},0}, |\widetilde{\mathcal{N}}_j| = \tilde{n}_j\right\} &= \int_0^{s_{\text{I,th}}}\left[\prod_{m'=1}^{m_{\text{co},0}}\prod_{n'=1}^{\tilde{n}_j}\Pr\{s_{\text{D},m',n',j} < (t+\sigma^2)x\}\right]\\ &\quad\times\left(\sum_{m''=1}^{m_{\text{co},0}}\sum_{n''=1}^{\tilde{n}_j}\left[\frac{d}{dt}\Pr\{s_{\text{I},m'',n'',j} < t\}\right]\underbrace{\prod_{m'=1}^{m_{\text{co},0}}\prod_{n'=1}^{\tilde{n}_j}}_{(m',n')\neq(m'',n'')}\left[1-\Pr\{s_{\text{I},m',n',j} < t\}\right]\right)dt.\end{aligned} \tag{31}$$

It is worth noting that the analysis for generalized imperfect sequential D2D association is more involved than that for simultaneous D2D association. This is because the sequential association performs the identification of the most suitable D2D transmitter and its associated reusable channel in subsequent steps, wherein each processing step is subject to introduce imperfect desired link quality maximization and/or imperfect interference effect minimization at a D2D receiver. Moreover, because of the subsequent processing, the sequential association can be handled using the approaches presented in subsections III-B2 and III-B3, which is not the case for the simultaneous D2D association scheme presented in the previous section.

1) IMPERFECT ASSOCIATION WITH D2D TRANSMITTER

Considering D2D transmitter association, and from

$$s_{\mathrm{D},(1),n,j} < s_{\mathrm{D},(2),n,j} < \ldots < s_{\mathrm{D},(|\mathcal{M}_{\mathrm{co},j}|),n,j}$$

that is obtained by arranging $\{s_{\mathrm{D},m,n,j}\}_{m=1}^{|\mathcal{M}_{\mathrm{co},j}|}$ in increasing order of magnitudes, the $m^\star$th D2D transmitter from order statistics may be identified to serve the jth D2D receiver, for $1 \leq m^\star \leq |\mathcal{M}_{\mathrm{co},j}|$.

The conditional statistics of the received desired power for an arbitrary value of $m^\star$ can be expressed as

$$\begin{aligned}\Pr&\left\{s_{\mathrm{D},(m^\star),n,j} < x \big| |\mathcal{M}_{\mathrm{co},j}|\right\}\\ &= \sum_{\ell_1=m^\star}^{|\mathcal{M}_{\mathrm{co},j}|} \sum_{S_{\ell_1}} \prod_{p=1}^{\ell_1} \Pr\{s_{\mathrm{D},m_p,n,j} < x\}\\ &\quad\times \prod_{p=\ell_1+1}^{|\mathcal{M}_{\mathrm{co},j}|} (1-\Pr\{s_{\mathrm{D},m_p,n,j} < x\}).\end{aligned} \tag{34}$$

The sum S_{ℓ_1} extends over all permutations $(m_1, m_2, \ldots, m_{|\mathcal{M}_{\mathrm{co},j}|})$ of $1, \ldots, |\mathcal{M}_{\mathrm{co},j}|$, for which $m_1 < m_2 < \cdots < m_{\ell_1}$ and $m_{\ell_1+1} < \cdots < m_{|\mathcal{M}_{\mathrm{co},j}|}$.

On the other hand, to quantify the effect of D2D transmitter association on the requirement to minimizing the interference level, define

$$s_{\mathrm{I},(1),n,j} < s_{\mathrm{I},(2),n,j} < \cdots < s_{\mathrm{I},(|\mathcal{M}_{\mathrm{co},j}|),n,j},$$

which is obtained by arranging $\{s_{\mathrm{I},g,n,j}\}_{g=1}^{|\mathcal{M}_{\mathrm{co},j}|}$ in an increasing order of magnitudes. Therefore, the conditional statistics of the observed interference power at the jth D2D receiver following the D2D transmitter association, and for an arbitrary value of $g^\star$, can be expressed as

$$\begin{aligned}\Pr&\left\{s_{\mathrm{I},(g^\star),n,j} < x \big| |\mathcal{M}_{\mathrm{co},j}|\right\}\\ &= \sum_{\ell_3=g^\star}^{|\mathcal{M}_{\mathrm{co},j}|} \sum_{S_{\ell_3}} \prod_{p=1}^{\ell_3} \Pr\{s_{\mathrm{I},g_p,n,j} < x\}\\ &\quad\times \prod_{p=\ell_3+1}^{|\mathcal{M}_{\mathrm{co},j}|} (1-\Pr\{s_{\mathrm{I},g_p,n,j} < x\}).\end{aligned} \tag{35}$$

The sum S_{ℓ_3} extends over all permutations $(g_1, g_2, \ldots, g_{|\mathcal{M}_{\mathrm{co},j}|})$ of $1, \ldots, |\mathcal{M}_{\mathrm{co},j}|$, for which $g_1 < g_2 < \cdots < g_{\ell_3}$ and $g_{\ell_3+1} < \cdots < g_{|\mathcal{M}_{\mathrm{co},j}|}$.

Note that the preceding results count for all possible cases that can be observed due to the imperfect association with the D2D transmitter on the received desired power and/or the interference power at the jth D2D receiver. For instance, the most suitable D2D transmitter that maximizes the desired power (i.e., the case when $m^\star = |\mathcal{M}_{\mathrm{co},j}|$) may not be the best possible choice to provide the lowest possible interference effect (i.e., the case when $g^\star = 1$ for given $|\mathcal{M}_{\mathrm{co},j}|$) and vice versa.

2) IMPERFECT ASSOCIATION WITH REUSABLE CHANNEL

For suitable channel association, the following discussion considers the two possibilities by which reusable channels become known, as explained in subsections III-B2 and III-B3.

a: REUSABLE CHANNELS BASED ON THE FIRST APPROACH

Again, consider the association link between the jth D2D receiver and its identified D2D transmitter, where the later may lead to imperfect desired power maximization (modeled through $m^\star$) and/or imperfect interference effect minimization (modeled through $g^\star$) as explained in the preceding subsection. To complete the association under consideration, the reusable channels will be examined by the D2D receiver based on $\mathcal{N}_{j,[(m^\star),(g^\star)]}$ as detailed in subsection III-B2).

Consider that the $m^\star$th D2D transmitter is identified from the desired link quality maximization perspective, for $1 \leq m^\star \leq |\mathcal{M}_{\mathrm{co},j}|$, define

$$s_{\mathrm{D},(m^\star),(1),j} < s_{\mathrm{D},(m^\star),(2),j} < \cdots < s_{\mathrm{D},(m^\star),(|\mathcal{N}_{j,[(m^\star),(g^\star)]}|),j}$$

that is obtained by arranging $\{s_{\mathrm{D},(m^\star),n,j}\}_{n=1}^{|\mathcal{N}_{j,[(m^\star),(g^\star)]}|}$ in an increasing order of magnitudes. Then, the $n^\star$th reusable channel from this order statistics may be used to serve the jth D2D receiver, for $1 \leq n^\star \leq |\mathcal{N}_{j,[(m^\star),(g^\star)]}|$.

Regardless of the accuracy of the D2D transmitter to maximize desired link quality, the best scenario for the reusable channel from the D2D receiver perspective is when $n^\star = |\mathcal{N}_{j,[(m^\star),(g^\star)]}|$.

Now, consider that the D2D transmitter has been identified from the interference power minimization perspective, for $1 \leq g^\star \leq |\mathcal{M}_{\mathrm{co},j}|$, and define

$$s_{\mathrm{I},(g^\star),(1),j} < s_{\mathrm{I},(g^\star),(2),j} < \cdots < s_{\mathrm{I},(g^\star),(z^\star),j},$$

which is obtained by arranging $\{s_{\mathrm{I},(g^\star),z,j}\}_{z=1}^{|\mathcal{N}_{j,[(m^\star),(g^\star)]}|}$ in an increasing order of magnitudes. The $z^\star$th reusable channel from this order statistics may be identified to serve the jth D2D receiver, for $1 \leq z^\star \leq |\mathcal{N}_{j,[(m^\star),(g^\star)]}|$. The worst scenario is when $z^\star = |\mathcal{N}_{j,[(m^\star),(g^\star)]}|$.

Based on the preceding descriptions, the received SINR at the jth device can now be expressed as

$$\tilde{\gamma}^{[1]}_{\mathrm{SINR},j} = \frac{s_{\mathrm{D},(m^\star),(n^\star),j}}{s_{\mathrm{I},(g^\star),(z^\star),j} + \sigma^2}. \tag{36}$$

Appendix D details the analysis of the conditional statistics of $s_{\mathrm{D},(m^\star),(n^\star),j}$ and $s_{\mathrm{I},(g^\star),(z^\star),j}$, conditioned on $|\mathcal{M}_{\mathrm{co},j}|$ and $|\mathcal{N}_{j,[(m^\star),(g^\star)]}|$, which are given in (49) and (51), respectively. Moreover, it shows the conditional statistics of $\widetilde{\gamma}^{[1]}_{\mathrm{SINR},j}$ can be expressed in generic form as

$$\begin{aligned}\Pr&\left\{\widetilde{\gamma}^{[1]}_{\mathrm{SINR},j}<x\big||\mathcal{M}_{\mathrm{co},j}|=m_{\mathrm{co},0},\,|\mathcal{N}_{j,[(m^\star),(g^\star)]}|=n_{j,[(m^\star),(g^\star)]}\right\}\\&=\int_0^{s_{\mathrm{I,th}}}\Pr\left\{s_{\mathrm{D},(m^\star),(n^\star),j}<(t+\sigma^2)x|m_{\mathrm{co},0},\,n_{j,[(m^\star),(g^\star)]}\right\}\\&\quad\times\left[\frac{d}{dt}\Pr\left\{s_{\mathrm{I},(g^\star),(z^\star),j}<x|m_{\mathrm{co},0},\,n_{j,[(m^\star),(g^\star)]}\right\}\right]dt,\qquad(37)\end{aligned}$$

where $\Pr\left\{s_{\mathrm{D},(m^\star),(n^\star),j}<(t+\sigma^2)x||\mathcal{M}_{\mathrm{co},j}|,\,|\mathcal{N}_{j,[(m^\star),(g^\star)]}|\right\}$ is given in (49).

The unconditional statistics of $\widetilde{\gamma}^{[1]}_{\mathrm{SINR},j}$ can be then obtained following (27), but using the conditional statistics of $\widetilde{\gamma}^{[1]}_{\mathrm{SINR},j}$ in (37), $\mathcal{N}_{j,[(m^\star),(g^\star)]}$, and $n_{j,[(m^\star),(g^\star)]}$ replace $\gamma_{\mathrm{SINR},j}$, $\widetilde{\mathcal{N}}_j$, and $\widetilde{n}_j$ therein, respectively.

The preceding analysis shows that the imperfectness in the D2D transmitter association affects the quality the reusable channel association due to their sequential association. In this regard, the perfect D2D transmitter association or reusable channel association from desired link quality maximization perspective may not provide the desired interference effect minimization through any or both steps of sequential association scheme.

b: SUITABLE CHANNELS BASED ON THE SECOND APPROACH

The analysis for this case can be conducted by following the same footsteps of that described in the preceding part above, but with the set from which the reusable channel can be identified be $\widetilde{\mathcal{N}}_j$. This set, $\widetilde{\mathcal{N}}_j$, will replace $\mathcal{N}_{j,[(m^\star),(g^\star)]}$ in all analytical results presented above. Note that $\widetilde{\mathcal{N}}_j$ is independent of the identity of the D2D transmitter because it is constructed based on the reusable channels that are found common among all potential D2D transmitters.

C. SPECIAL CASES

There are many interesting special cases that can be deduced from the preceding results. However, due to space limitation, the discussion below focuses on the characteristics of the received SINR under the perfect sequential D2D association scenario. It is presented for the two approaches from which reusable channels can be known.

1) REUSABLE CHANNELS BASED ON FIRST APPROACH

Herein, the perfect D2D transmitter association has to result in $m^\star=|\mathcal{M}_{\mathrm{co},j}|$ to satisfy the desired link quality maximization and $g^\star=1$ to satisfy the interference effect minimization. Accordingly, the reusable channels whose indexes belong to $\mathcal{N}_{j,[(|\mathcal{M}_{\mathrm{co},j}|),(1)]}$ are considered in the subsequent search for the most suitable reusable channel, where $\mathcal{N}_{j,[(|\mathcal{M}_{\mathrm{co},j}|),(1)]}$ refers to the set of reusable channels as identified by the jth D2D receiver and the already identified D2D transmitter that satisfied $m^\star=|\mathcal{M}_{\mathrm{co},j}|$ and $g^\star=1$ (referred to in the index of the set as $[(|\mathcal{M}_{\mathrm{co},j}|),(1)]$). Then, the perfect reusable channel association should result in $n^\star=|\mathcal{N}_{j,[(|\mathcal{M}_{\mathrm{co},j}|),(1)]}|$ and $z^\star=1$ in order to meet the aforementioned performance objectives at the jth D2D receiver.

The received SINR at the jth device under the perfect sequential D2D association scenario can now be expressed as a limiting case of that in (36) as

$$\gamma^{[1]}_{\mathrm{SINR},j}=\frac{s^{[1]}_{\mathrm{D},(|\mathcal{M}_{\mathrm{co},j}|),(|\mathcal{N}_{j,[(|\mathcal{M}_{\mathrm{co},j}|),(1)]}|),j}}{s^{[1]}_{\mathrm{I},(1),(1),j}+\sigma^2}.\qquad(38)$$

It can be shown that the statistics of $s^{[1]}_{\mathrm{D},(m_{\mathrm{co},0}),(n_{j,[(m_{\mathrm{co},0}),(1)]}),j}$ and $s^{[1]}_{\mathrm{I},(1),(1),j}$ have the same forms to those in (45) and (46), respectively, but with $|\mathcal{N}_{j,[(m_{\mathrm{co},0}),(1)]}|=n_{j,[(m_{\mathrm{co},0}),(1)]}$ for given $|\mathcal{M}_{\mathrm{co},j}|=m_{\mathrm{co},0}$ herein replaces $|\widetilde{\mathcal{N}}_j|=\widetilde{n}_j$ therein. Therefore, the conditional statistics of $\gamma^{[1]}_{\mathrm{SINR},j}$ can be obtained in a similar form to that in (31). Moreover, the unconditional statistics of $\gamma^{[1]}_{\mathrm{SINR},j}$ can be then written in a similar form to that given in (27) with $\gamma^{[1]}_{\mathrm{SINR},j}$, $\mathcal{N}_{j,[(m_{\mathrm{co},0}),(1)]}$, and $n_{j,[(m_{\mathrm{co},0}),(1)]}$ replace $\gamma_{\mathrm{SINR},j}$, $\widetilde{\mathcal{N}}_j$, and $\widetilde{n}_j$ therein, respectively.

2) REUSABLE CHANNELS BASED ON SECOND APPROACH

Under the perfect D2D transmitter association, which gives $m^\star=|\mathcal{M}_{\mathrm{co},j}|$ $g^\star=1$, and the perfect reusable channel association, which is determined based on $\widetilde{\mathcal{N}}_j$ and gives $m^\star=|\mathcal{M}_{\mathrm{co},j}|$ and $g^\star=1$, it can be shown the received SINR at the jth D2D receiver follows exactly the findings in subsection IV-C1. This leads to the conclusion that the perfect sequential D2D association scheme that is performed when all D2D transmitters are involved in determining reusable downlink channels provides the same metric of the perfect simultaneous D2D association scheme.

Note that these highlighted similarities between simultaneous and sequential D2D association schemes are only applicable under their perfect association scenarios. However, in practice, imperfect associations of the D2D transmitter and/or reusable downlink channel are likely to occur. Herein, the two schemes will result into different properties of the received SINR, and hence, different performance. With the developed generalized analytical results in the preceding Sections, further analytical results for specific channel and fading models, specific distributions of D2D transmitters and reusable channels for the two association schemes, and certain distributions of desired power levels and/or interference power levels for various imperfect associations are omitted due to space limitation.

VI. RESULTS AND DISCUSSIONS

This Section presents some results and discussions based on the findings from preceding Sections. The adopted channel models of the desired links and interference links, which are used to generate the case studies in the following figures, are assumed to follow complex-valued Gaussian processes.

These models are needed for the developed analytical formulations in the preceding Sections. For an (m, n) D2D association link, the statistics of the desired power and the coherent aggregation of interference power can be expressed as

$$\begin{aligned}\Pr\{s_{\mathrm{D},m,n,j} < x\} &= 1 - e^{x/\bar{s}_{\mathrm{D},m,n,j}},\\ \Pr\{s_{\mathrm{I},m,n,j} < x\} &= 1 - e^{x/\bar{s}_{\mathrm{I},m,n,j}},\end{aligned}$$

where $\bar{s}_{\mathrm{D},m,n,j}$ is the desired average power and $\bar{s}_{\mathrm{I},m,n,j}$ is the aggregate average interference power from interference sources on the (m, n) association link. Moreover, the results are presented for various cases of number of devices, number of downlink channels for both the perfect and selected imperfect simultaneous and sequential D2D association scenarios. Moreover, each figures shows additional associated parameters that are listed in legends. For all cases, average power quantities are given in μW, and the fading powers and background noise variance are normalized, unless stated otherwise. The numerical results have been verified by simulations, which have been generated by depicting the proposed association schemes and the methods by which devices and reusable channels are partitioned and analyzed as detailed in preceding Sections.

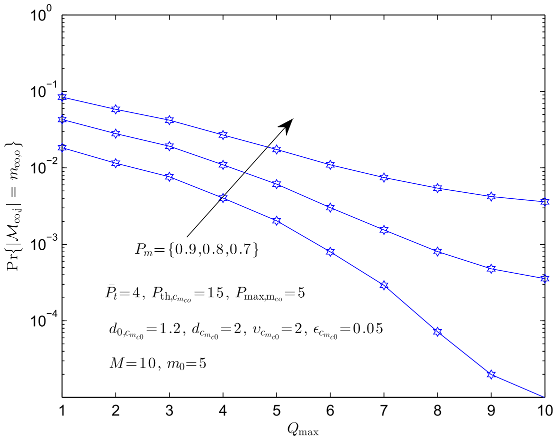

FIGURE 2. **Distribution of the cardinality of set of potential D2D transmitters to be half of the number of available devices *M* versus the number concurrent D2D services a D2D transmitter can meet at a time for different values of the probability that the *m*th device is active, P_m.**

Fig. 2 shows the distribution of the cardinality of set of D2D transmitters to be half of the number of available devices M versus the maximum number of concurrent D2D services a D2D transmitter can meet at a time for different values of the probability that the mth device is active, P_m. The other parameters that are associated with this case study have been given fixed quantities, and they are listed in the figure legends. A general trend of the results show that the increase in the likelihood that a device be active results in a higher possibility to improve the cardinality of the set of potential D2D transmitters. However, as the number of concurrent D2D service requests a D2D transmitter can meet increases under fixed allowed transmit power that is specifically allocated by that device to serve others via D2D association, the chance that more devices will be classified as potential D2D transmitters decreases. This is because the device can not meet more D2D service demands due to its transmit power limitation.

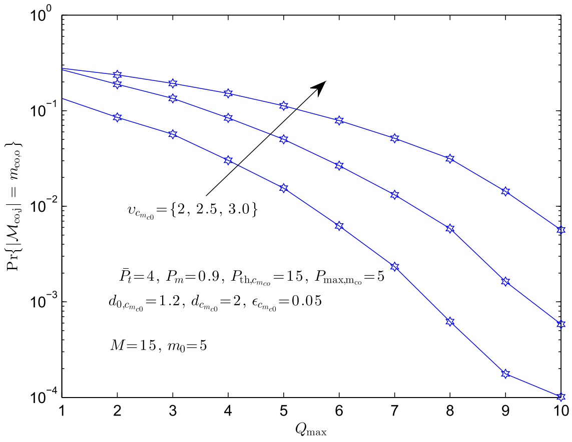

FIGURE 3. **Distribution of that the cardinality of set of potential D2D transmitters to be one-third of the number of available devices *M* versus the maximum number D2D services a D2D transmitter can meet at a time for different values of the D2D path loss exponent, denoted by $\upsilon_{c_{m_{co}}}$.**

Fig. 3 examines the impact of the D2D path loss exponent, which is inversely proportionally to the average desired power on D2D association link (See Appendix A), on the likelihood that the cardinality of set of potential D2D transmitters be one-third of the number of available devices M versus the maximum number of D2D services a D2D transmitter can meet at a time. The results show that lower values of path loss exponent improve the possibility to have more potential D2D transmitter s since lower transmit power per each potential D2D association will be required. This permits more concurrent D2D services by each potential D2D transmitter. However, the general trend of the impact of the maximum number of D2D services is still observed under a fixed transmit power allocated by potential D2D transmitters to serve others via D2D communication.

Fig. 4 studies the probability of observing different cardinalities of the set of reusable channels for D2D communication versus the aggregate interference threshold per channel (in dBμ). The results are shown for the second approach of determining reusable downlink channels, in which all potential D2D transmitters are involved in identifying these channels for different values of available channels N. They are shown when the cardinality of D2D transmitters is half of the number of available devices M. A general observation herein is that the decrease in aggregate interference threshold per each channel reduces the chance to have more reusable channels as observed by all D2D transmitters, regardless of the number of available channels or the cardinality of set of available channels. For a given interference threshold and a given number of available channels, the chance to have more reusable channels tends to decrease sharply. However, having more available channels improves the chance of determining more reusable channels for D2D communication. The likelihood to have full cardinality of set of reusable channels

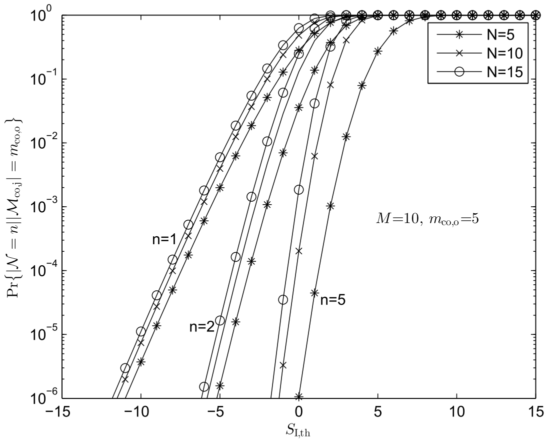

FIGURE 4. **Probability of observing different cardinalities of the set of reusable channels for D2D communication versus the aggregate interference threshold per channel (in dBμ) when all potential D2D transmitters are involved (second approach) for different number of available channels *N* and given cardinality of D2D transmitter s as half of available devices.**

approaches unity when the interference threshold is set to a relatively high value (herein above 10 dBμ) since all available channels will be found meeting the interference constraint even when all potential D2D transmitters are involved.

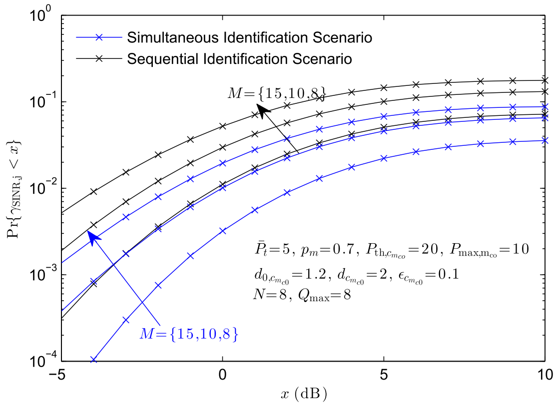

FIGURE 5. **Outage performance of the *j*th D2D receiver versus the received SINR threshold (*x* in dB) for the simultaneous D2D association scheme (all potential D2D transmitters are involved in determining reusable channels(second approach)) and the sequential D2D association scheme (only identified D2D transmitter is involved in determining reusable channels (first approach)) under the perfect D2D association scenario. The results are shown for different number of available devices *M*.**

Fig. 5 compares the outage performance of the jth D2D receiver as a function of the received SINR threshold (x in dB) for the simultaneous and sequential D2D association schemes under the perfect association scenario, considering different number of available devices M. The sequential D2D association scheme considers that the reusable channels are identified based on the first approach wherein the D2D receiver and the a priori identified D2D transmitter are only involved. On the other hand, the simultaneous D2D association scheme requires that all potential D2D transmitters and the D2D receiver be involved to determine the reusable channels, which represent the second approach therein. It is seen that, for both schemes, the increase in M improves the outage performance of the D2D receiver. Moreover, the increase in the outage threshold improves the outage performance. Due to the variation between the approaches for identifying reusable channels for the two association schemes, there is a noticeable performance gap between the simultaneous and the sequential association schemes even under their perfect scenarios, regardless of M.

Fig. 6 extends the same studies depicted in Fig. 5 for a given number of available devices and different likelihoods that a D2D transmitter be active. The increase in the probability that a device be active incurs noticeable gain on the outage performance of the D2D receiver, where more devices will be likely to serve others in the D2D service mode. This observation is applicable for both the simultaneous and the sequential D2D association schemes since they both utilize potential D2D transmitters to meet the concurrent objectives of maximizing the desired link quality and minimizing the effect of interference. Under perfect D2D transmitter association, both schemes provide the same outage performance gain but they differ in terms of their approaches to quantify the set of reusable channels, which are further utilized to meet the aforementioned performance objectives.

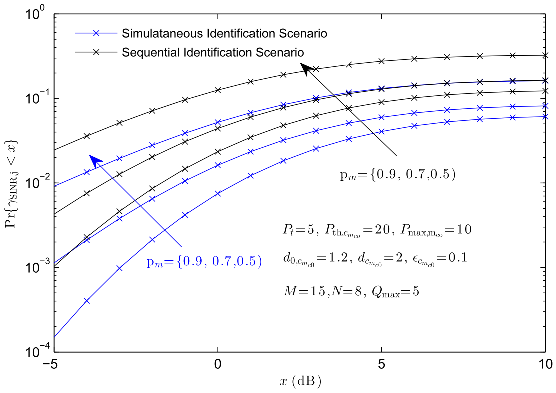

FIGURE 6. **Outage performance of the *j*th D2D receiver as function of the received SINR threshold (*x* in dB) for the simultaneous D2D association scheme (based on using the second approach for identifying reusable channels) and the sequential D2D association scheme (based on using the first approach for identifying reusable channels) under their perfect association scenarios. The results are shown for different values of probability that a device be active for given number of available devices.**

Fig. 7 shows the outage performance of the jth D2D receiver as a function of the received SINR threshold (x in dB). It presents comparisons between the perfect and various imperfect association scenarios of the simultaneous D2D association scheme based on the second approach for identifying reusable channels and the sequential D2D association scheme based on the first approach for identifying reusable channels. As compared to the perfect association scenarios, the results clearly show the importance to model and analyze generic imperfect D2D association scenarios. This imperfectness can lead to substantial

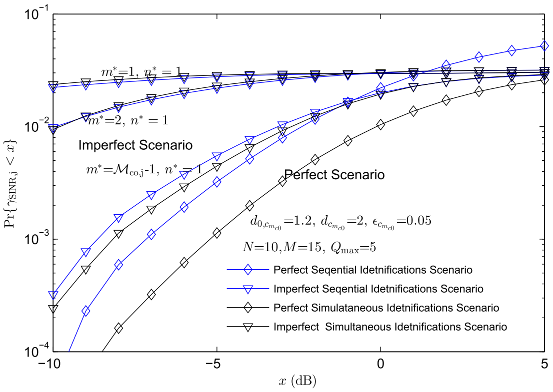

FIGURE 7. **Comparisons for perfect and various imperfect scenarios of the simultaneous D2D association scheme (based on using the second approach for identifying reusable channels) and the sequential D2D association scheme (based on using the first approach for identifying reusable channels). The results are for the outage performance of the jth D2D receiver as function of the received SINR threshold (x in dB).**

performance degradation, regardless of the associated complexity of the undertaken association scheme. For instance, the performance loss due to the considered imperfect association with the D2D transmitter tends to saturate the outage performance of the D2D receiver for the two D2D association schemes when the worst possible D2D transmitter or even the second worst D2D transmitter are allocated. These observations reveal a potential marginal performance advantage of complicated D2D association schemes if imperfect association scenarios are not precisely addressed and tolerated.

VII. CONCLUSION

The paper has proposed generalized and comprehensive modeling and analysis of two D2D association schemes in downlink cellular networks. The analysis covered the important issue of observing imperfect association, while addressing the effect processing load limitation and transmit power constraint at potential D2D transmitters as well as interference constraint on reusable downlink channels imposed by the primary cellular network. New decentralized schemes that pave the way for a successful pairing of D2D transmitters and receivers when downlink channels are reusable in the underlay D2D network have been presented. The downlink channels that can be reused by D2D transmitters within their D2D coverage range have been also characterized via two different approaches, where each of which aims to maintain controlled levels of interference. The first approach requires communication overhead only between pair of D2D transmitter and receiver that have a potential D2D association, whereas the second approach demands extensive communication overhead that involves all potential D2D transmitters. Two D2D association schemes (i.e., simultaneous and sequential associations), which aim to concurrently maximize the desired link quality and minimize the effect of interference at a D2D receiver have been explained in details. For these two association schemes, detailed analytical formulations for the statistics of the instantaneous received SINR at an arbitrary D2D receiver under generalized imperfect association scenarios have been developed. These findings have been used to deduce many special cases of practical interest, such as the perfect association scenario. Results have been also presented to further explain the usefulness and limitations of the proposed association schemes, their related system and network design constraints, and to compare between the two association schemes under various association situations. Among the main observations, it has been noted that the imperfect association of D2D communication can hinder the anticipated performance gains of the formation of underlay D2D networks even when cellular resources are reusable. The developed results in this paper addressed generic and comprehensive analysis for various imperfect D2D association scenarios considering the practical system and network constraints.

APPENDIX A

This Appendix quantifies the distribution of $Q_{m_{\mathrm{co}}}$, which is needed in (5). To this end, let $P_{c_{m_{\mathrm{co}}}}$ be the average transmit power that the m_{co}th device can allocate to the $c_{m_{\mathrm{co}}}$th D2D communication link. This quantity can be quantified from a truncated channel inversion model as

$$P_{c_{m_{\mathrm{co}}}} = \begin{cases} \dfrac{P_{\mathrm{th},c_{m_{\mathrm{co}}}}}{\rho |h_{c_{m_{\mathrm{co}}}}|^2}, & |h_{c_{m_{\mathrm{co}}}}|^2 \geq \epsilon/\rho \\ \overline{P}_t, & |h_{c_{m_{\mathrm{co}}}}|^2 < \epsilon/\rho, \end{cases} \tag{39}$$

where $\rho \triangleq (d_{0,c_{m_{\mathrm{co}}}}/d_{c_{m_{\mathrm{co}}}})^{v_{c_{m_{\mathrm{co}}}}}$, $P_{\mathrm{th},c_{m_{\mathrm{co}}}}$ is the power threshold required to establish the $c_{m_{\mathrm{co}}}$th D2D association link, $|h_{c_{m_{\mathrm{co}}}}|^2$ is the fading channel power gain, $\epsilon \geq 0$ is a design parameter that represents the truncation threshold to avoid deep fading instants, $d_{0,c_{m_{\mathrm{co}}}}$ is the far-field distance of the m_{co}th device, $d_{c_{m_{\mathrm{co}}}} > d_{0,c_{m_{\mathrm{co}}}}$ is the association link separation distance, $v_{c_{m_{\mathrm{co}}}}$ is the link loss exponent, and $\overline{P}_t$ represents a threshold on transmit power a device can use for D2D communication link that falls under deep fading instants.

Now, the aggregate transmit power that is needed to establish as many D2D association links as possible should not exceed the maximum allocated transmit power at the m_{co}th device, which is referred to as $\widetilde{P}_{\max,m_{\mathrm{co}}}$. Therefore, it can be written that

$$\Pr\left\{Q_{m_{\mathrm{co}}} = 0\right\} = \Pr\left\{P_{1_{m_{\mathrm{co}}}} > \widetilde{P}_{\max,m_{\mathrm{co}}}\right\}. \tag{40}$$

The result in (40) reveals that the m_{co}th device will be unable to support any D2D communication since the required transmit power for any D2D association exceeds the maximum allocated power for D2D communication at that device. Moreover

$$\begin{aligned} &\Pr\left\{Q_{m_{\mathrm{co}}} = 1\right\} \\ &\quad = \Pr\{P_{1_{m_{\mathrm{co}}}} < \widetilde{P}_{\max,m_{\mathrm{co}}}, P_{1_{m_{\mathrm{co}}}} + P_{2_{m_{\mathrm{co}}}} > \widetilde{P}_{\max,m_{\mathrm{co}}}\} \\ &\quad = \mathbb{E}\left\{\Pr\left\{P_{2_{m_{\mathrm{co}}}} > \left(\widetilde{P}_{\max,m_{\mathrm{co}}} - P_{1_{m_{\mathrm{co}}}}\right)\right\} \middle| P_{1_{m_{\mathrm{co}}}} < \widetilde{P}_{\max,m_{\mathrm{co}}}\right\}, \end{aligned} \tag{41}$$

where $\mathbb{E}\{\cdot|x\}$ refers to the statistical expectation taken over the conditional random quantity x. The preceding result shows that only one D2D association can be supported if the sum of the required transmit powers to support two D2D associations concurrently exceeds the maximum allocated transmit power limit but the first D2D association can be supported. In general, the event that $Q_{m_{\mathrm{co}}} = c_{m_{\mathrm{co}}}$, for $1 < c_{m_{\mathrm{co}}} < Q_{\max,m_{\mathrm{co}}}$, has a probability of occurrence

$$\begin{aligned}
&\Pr\left\{Q_{m_{\mathrm{co}}} = c_{m_{\mathrm{co}}}\right\} \\
&\quad = \Pr\left\{\sum_{k=1_{m_{\mathrm{co}}}}^{c_{m_{\mathrm{co}}}} P_k < \widetilde{P}_{\max,m_{\mathrm{co}}}, \sum_{k=1_{m_{\mathrm{co}}}}^{(c+1)_{m_{\mathrm{co}}}} P_k > \widetilde{P}_{\max,m_{\mathrm{co}}}\right\} \\
&\quad = \mathbb{E}\Big\{\Pr\left\{P_{(c+1)_{m_{\mathrm{co}}}} > \big(\widetilde{P}_{\max,m_{\mathrm{co}}} - \beta_{m_{\mathrm{co}}} < \widetilde{P}_{\max,m_{\mathrm{co}}}\big)\right\} \\
&\qquad \times \Big|\beta_{m_{\mathrm{co}}} < \widetilde{P}_{\max,m_{\mathrm{co}}}\Big\}, \qquad (42)
\end{aligned}$$

where $\beta_{m_{\mathrm{co}}} \triangleq \sum_{k=1_{m_{\mathrm{co}}}}^{c_{m_{\mathrm{co}}}} P_k$. The preceding results characterize the distribution of $Q_{m_{\mathrm{co}}}$, which is needed in (5).

APPENDIX B

This Appendix details the derivation for the conditional statistics of the received SINR at the jth D2D receiver, which is defined in (25), under the generalized imperfect simultaneous D2D association scheme. From [37, Ch. 5], the statistics of the received desired power at the jth D2D receiver for an arbitrary $r^\star$ can be given as

$$\begin{aligned}
&\Pr\left\{s_{\mathrm{D},(r^\star),j} < x \big| |\mathcal{M}_{\mathrm{co},j}|, |\widetilde{\mathcal{N}}_j|\right\} \\
&\quad = \sum_{\ell=r^\star}^{|\mathcal{M}_{\mathrm{co},j}||\widetilde{\mathcal{N}}_j|} \sum_{S_\ell} \prod_{p=1}^{\ell} \Pr\{s_{\mathrm{D},r_p,j} < x\} \\
&\qquad \times \prod_{p=\ell+1}^{|\mathcal{M}_{\mathrm{co},j}||\widetilde{\mathcal{N}}_j|} (1 - \Pr\{s_{\mathrm{D},r_p,j} < x\}). \qquad (43)
\end{aligned}$$

The sum S_ℓ extends over all permutations $(r_1, r_2, \ldots, r_{|\mathcal{M}_{\mathrm{co},j}||\widetilde{\mathcal{N}}_j|})$ of $1, \ldots, |\mathcal{M}_{\mathrm{co},j}||\widetilde{\mathcal{N}}_j|$, for which $r_1 < r_2 < \cdots < r_\ell$ and $r_{\ell+1} < \cdots < r_{|\mathcal{M}_{\mathrm{co},j}||\widetilde{\mathcal{N}}_j|}$.

On the other hand, the statistics of the observed interference power at the jth D2D receiver for an arbitrary $u^\star$ can be written as

$$\begin{aligned}
&\Pr\left\{s_{\mathrm{I},(u^\star),j} < x \big| |\mathcal{M}_{\mathrm{co},j}|, |\widetilde{\mathcal{N}}_j|\right\} \\
&\quad = \sum_{\ell_1=u^\star}^{|\mathcal{M}_{\mathrm{co},j}||\widetilde{\mathcal{N}}_j|} \sum_{S_{\ell_1}} \prod_{p=1}^{\ell_1} \Pr\{s_{\mathrm{I},u_p,j} < x\} \\
&\qquad \times \prod_{p=\ell_1+1}^{|\mathcal{M}_{\mathrm{co},j}||\widetilde{\mathcal{N}}_j|} (1 - \Pr\{s_{\mathrm{I},u_p,j} < x\}). \qquad (44)
\end{aligned}$$

The sum S_{ℓ_1} extends over all permutations $(u_1, u_2, \ldots, u_{|\mathcal{M}_{\mathrm{co},j}||\widetilde{\mathcal{N}}_j|})$ of $1, \ldots, |\mathcal{M}_{\mathrm{co},j}||\widetilde{\mathcal{N}}_j|$, with $u_1 < u_2 < \cdots < u_{\ell_1}$ and $u_{\ell_1+1} < \cdots < u_{|\mathcal{M}_{\mathrm{co},j}||\widetilde{\mathcal{N}}_j|}$.

Based on the expression of $\widetilde{\gamma}_{\mathrm{SINR},j}$ in (25), the conditional statistics of $\widetilde{\gamma}_{\mathrm{SINR},j}$ can now be obtained, using (43) and (44), as shown in (26).

APPENDIX C

This Appendix discusses the special case of perfect simultaneous D2D association scenario. The perfect association requires that $r^\star = |\mathcal{M}_{\mathrm{co},j}||\widetilde{\mathcal{N}}_j|$ and $u^\star = 1$ be satisfied concurrently. Therefore, the statistics of the desired power at the jth D2D receiver can be deduced as a special case from (43) as

$$\begin{aligned}
&\Pr\left\{s_{\mathrm{D},(|\mathcal{M}_{\mathrm{co},j}||\widetilde{\mathcal{N}}_j|),j} < x \big| |\mathcal{M}_{\mathrm{co},j}| = m_{\mathrm{co},0}, |\widetilde{\mathcal{N}}_j| = \widetilde{n}_j\right\} \\
&\quad = \prod_{m'=1}^{m_{\mathrm{co},0}} \prod_{n'=1}^{\widetilde{n}_j} \Pr\{s_{\mathrm{D},m',n',j} < x\}. \qquad (45)
\end{aligned}$$

On the other hand, the statistics of the interference power observed at the jth D2D receiver for $u^\star = 1$ can be deduced as a special case from (44), and the result becomes

$$\begin{aligned}
&\Pr\left\{s_{\mathrm{I},(1),j} < x \big| |\mathcal{M}_{\mathrm{co},j}| = m_{\mathrm{co},0}, |\widetilde{\mathcal{N}}_j| = \widetilde{n}_j\right\} \\
&\quad = 1 - \prod_{m'=1}^{m_{\mathrm{co},0}} \prod_{n'=1}^{\widetilde{n}_j} \left[1 - \Pr\left\{s_{\mathrm{I},m',n',j} < x\right\}\right]. \qquad (46)
\end{aligned}$$

Using the results in (45) and (46), the conditional statistics of $\gamma_{\mathrm{SINR},j}$, which is defined in (30), can now be obtained as shown in (31).

For the special case when the desired power levels $\{s_{\mathrm{D},m,n,j}\}$ and the aggregate interference power levels $\{s_{\mathrm{I},m,n,j}\}$ experience identically distributed fading processes, respectively, (31) reduces to

$$\begin{aligned}
&\Pr\left\{\gamma_{\mathrm{SINR},j} < x \big| |\mathcal{M}_{\mathrm{co},j}| = m_{\mathrm{co},0}, |\widetilde{\mathcal{N}}_j| = \widetilde{n}_j\right\} \\
&\quad = m_0\widetilde{n}_j \int_0^{s_{\mathrm{I,th}}} \Big[\Pr\{s_{\mathrm{D},m,n,j} < (t + \sigma^2)x\}\Big]^{m_0\widetilde{n}_j} \\
&\qquad \times \left[1 - \Pr\{s_{\mathrm{I},m,n,j} < t\}\right]^{m_0\widetilde{n}_j - 1} \left[\frac{d}{dt}\Pr\{s_{\mathrm{I},m,n,j} < t\}\right] dt. \qquad (47)
\end{aligned}$$

APPENDIX D

When the $m^\star$th D2D transmitter is identified based on maximizing the desired link quality, for $1 \le m^\star \le |\mathcal{M}_{\mathrm{co},j}|$, the resulting statistics of the received desired power at the jth D2D receiver, for an arbitrary value of $n^\star$, can be written as

$$\begin{aligned}
&\Pr\left\{s_{\mathrm{D},(m^\star),(n^\star),j} < x \big| |\mathcal{M}_{\mathrm{co},j}|, |\mathcal{N}_{j,[(m^\star),(g^\star)]}|\right\} \\
&\quad = \sum_{\ell_2=n^\star}^{|\mathcal{N}_{j,[(m^\star),(g^\star)]}|} \sum_{S_{\ell_2}} \prod_{p=1}^{\ell_2} \Pr\{s_{\mathrm{D},(m^\star),n_p,j} < x \big| |\mathcal{M}_{\mathrm{co},j}|\} \\
&\qquad \times \prod_{p=\ell_2+1}^{|\mathcal{N}_{j,[(m^\star),(g^\star)]}|} (1 - \Pr\{s_{\mathrm{D},(m^\star),n_p,j} < x \big| |\mathcal{M}_{\mathrm{co},j}|\}). \qquad (48)
\end{aligned}$$

The sum S_{ℓ_2} extends over all permutations $(n_1, n_2, \ldots, n_{|\mathcal{N}_{j,[(m^\star),(g^\star)]}|})$ of $1, \ldots, |\mathcal{N}_{j,[(m^\star),(g^\star)]}|$, for which $n_1 < n_2 < \cdots < n_{\ell_2}$ and $n_{\ell_2+1} < \cdots < n_{|\mathcal{N}_{j,[(m^\star),(g^\star)]}|}$.

The substitution of (34) for $\Pr\{s_{\mathrm{D},(m^\star),n,j} < x \big| |\mathcal{M}_{\mathrm{co},j}|\}$ into (48) give the final form of the statistics of $s_{\mathrm{D},(m^\star),(n^\star),j}$, for given $|\mathcal{M}_{\mathrm{co},j}|$ and $|\mathcal{N}_{j,[(m^\star),(g^\star)]}|$, as

$$
\begin{aligned}
&\Pr\left\{s_{\mathrm{D},(m^\star),(n^\star),j} < x \big| |\mathcal{M}_{\mathrm{co},j}|, |\mathcal{N}_{j,(m^\star)}|\right\}\\
&= \sum_{\ell_2=n^\star}^{|\mathcal{N}_{j,(m^\star)}|} \sum_{S_{\ell_2}} \prod_{p_2=1}^{\ell_2} \Bigg(\sum_{\ell_1=m^\star}^{|\mathcal{M}_{\mathrm{co},j}|} \sum_{S_{\ell_1}} \prod_{p_1=1}^{\ell_1} \Pr\{s_{\mathrm{D},m_{p_1},n_{p_2},j} < x\}\\
&\quad \times \prod_{p_1=\ell_1+1}^{|\mathcal{M}_{\mathrm{co},j}|} (1-\Pr\{s_{\mathrm{D},m_{p_1},n_{p_2},j} < x\}) \Bigg)\\
&\quad \times \prod_{p_2=\ell_2+1}^{|\mathcal{N}_{j,(m^\star)}|} \Bigg(1 - \Bigg[\sum_{\ell_1=m^\star}^{|\mathcal{M}_{\mathrm{co},j}|} \sum_{S_{\ell_1}} \prod_{p_1=1}^{\ell_1} \Pr\{s_{\mathrm{D},m_{p_1},n_{p_2},j} < x\}\\
&\quad \times \prod_{p_1=\ell_1+1}^{|\mathcal{M}_{\mathrm{co},j}|} (1-\Pr\{s_{\mathrm{D},m_{p_1},n_{p_2},j} < x\}) \Bigg]\Bigg). \qquad (49)
\end{aligned}
$$

The preceding result can be readily used to obtain various imperfect D2D transmitter association, imperfect reusable channel association, or concurrently imperfect D2D transmitter and reusable channel association scenarios under the objective of maximizing the desired link quality at the jth device.

Now, consider the transmit device be identified based on the interference power minimization perspective, for $1 \leq g^\star \leq |\mathcal{M}_{\mathrm{co},j}|$. The resulting statistics of the observed interference power at the jth D2D receiver, for an arbitrary value of $z^\star$, can be expressed as

$$
\begin{aligned}
&\Pr\left\{s_{\mathrm{I},(g^\star),(z^\star),j} < x \big| |\mathcal{M}_{\mathrm{co},j}|, |\mathcal{N}_{j,[(m^\star),(g^\star)]}|\right\}\\
&= \sum_{\ell_4=z^\star}^{|\mathcal{N}_{j,[(m^\star),(g^\star)]}|} \sum_{S_{\ell_4}} \prod_{p=1}^{\ell_4} \Pr\{s_{\mathrm{I},(g^\star),z_p,j} < x \big| |\mathcal{M}_{\mathrm{co},j}|\}\\
&\quad \times \prod_{p=\ell_4+1}^{|\mathcal{N}_{j,[(m^\star),(g^\star)]}|} (1-\Pr\{s_{\mathrm{I},(g^\star),z_p,j} < x \big| |\mathcal{M}_{\mathrm{co},j}|\}). \qquad (50)
\end{aligned}
$$

The sum S_{ℓ_4} extends over all permutations $(z_1, z_2, \ldots, z_{|\mathcal{N}_{j,[(m^\star),(g^\star)]}|})$ of $1, \ldots, |\mathcal{N}_{j,[(m^\star),(g^\star)]}|$, for which $z_1 < z_2 < \cdots < z_{\ell_2}$ and $z_{\ell_2+1} < \cdots < z_{|\mathcal{N}_{j,[(m^\star),(g^\star)]}|}$.

The substitution of (35) into (50) gives the statistics of $s_{\mathrm{I},(g^\star),(z^\star),j}$, for given $|\mathcal{M}_{\mathrm{co},j}|$ and $|\mathcal{N}_{j,[(m^\star),(g^\star)]}|$, as

$$
\begin{aligned}
&\Pr\left\{s_{\mathrm{I},(g^\star),(z^\star),j} < x \big| |\mathcal{M}_{\mathrm{co},j}|, |\mathcal{N}_{j,(g^\star)}|\right\}\\
&= \sum_{\ell_4=z^\star}^{|\mathcal{N}_{j,(g^\star)}|} \sum_{S_{\ell_4}} \prod_{p_4=1}^{\ell_4} \Bigg(\sum_{\ell_3=g^\star}^{|\mathcal{M}_{\mathrm{co},j}|} \sum_{S_{\ell_3}} \prod_{p_3=1}^{\ell_3} \Pr\{s_{\mathrm{I},g_{p_3},z_{p_4},j} < x\}\\
&\quad \times \prod_{p_3=\ell_3+1}^{|\mathcal{M}_{\mathrm{co},j}|} (1-\Pr\{s_{\mathrm{I},g_{p_3},z_{p_4},j} < x\}) \Bigg)\\
&\quad \times \prod_{p_4=\ell_4+1}^{|\mathcal{N}_{j,(g^\star)}|} \Bigg(1 - \sum_{\ell_3=g^\star}^{|\mathcal{M}_{\mathrm{co},j}|} \sum_{S_{\ell_3}} \prod_{p_3=1}^{\ell_3} \Pr\{s_{\mathrm{I},g_{p_3},z_{p_4},j} < x\}\\
&\quad \times \prod_{p_3=\ell_3+1}^{|\mathcal{M}_{\mathrm{co},j}|} (1-\Pr\{s_{\mathrm{I},g_{p_3},z_{p_4},j} < x\}) \Bigg). \qquad (51)
\end{aligned}
$$

The preceding result can be used to study various imperfect D2D transmitter association, reusable channel association, or concurrently imperfect D2D transmitter and reusable channel associations based on minimizing the effect of interference at the jth D2D receiver.

The received SINR at the jth D2D receiver in this case can be expressed as shown in (36), where the statistics of $s_{\mathrm{D},(m^\star),(n^\star),j}$ and $s_{\mathrm{I},(g^\star),(z^\star),j}$ for given $|\mathcal{M}_{\mathrm{co},j}|$ and $|\mathcal{N}_{j,[(m^\star),(g^\star)]}|$ can be found from (49) and (51), respectively. Using these results, the conditional statistics of $\widetilde{\gamma}^{[1]}_{\mathrm{SINR},j}$ can now be expressed as shown in (37).

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