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# Power Gain Pattern Synthesis via Successive Convex Approximation Technique

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**ABSTRACT** Wide beam is necessary for ensuring the main lobe direction in a mobile communication scenario. A small dynamic range ratio (DRR) of excitations is crucial for simple-array and energy-saving design. The power gain pattern synthesis (PGPS) problem is aimed to maximize the minimum power gain in the wide main lobe and solving the PGPS problem can form a wide beam. To control the DRR, an upper bound constraint of DRR is imposed on the PGPS problem. The new problem is concave and can't be solved effectively by the conventional convex optimization method. We convert the DRR constraint into a group of stricter inequalities and transform the concave constraints to be convex with some convex approximations. The general PGPS problem with DRR constraints can be solved by the successive convex approximation (SCA) technique. The weights obtained by the proposed method can converge to the stationary point and we present the convergence proofs. Simulation results show that the proposed algorithm has better performance on increasing the minimum power gain in the main lobe (PGML) and suppressing the sidelobe level (SLL). Meanwhile, the DRR of excitations can be controlled below a given upper bound.

**INDEX TERMS** Adaptive beamforming, array pattern synthesis, successive convex approximation, power gain optimization, wide main lobe beam, dynamic range ratio.

## I. INTRODUCTION

Array antenna has been widely used in many fields such as sonar, radar, speech processing, and navigation to wireless communications because of its flexible beamforming ability [1]–[6]. Pattern synthesis is a process to obtain the required antenna radiation pattern and could improve the performance of the array antenna. Many methods of pattern synthesis have been proposed, which can be classified as global search algorithms [7]–[9], deterministic algorithms [10]–[12], and optimization algorithms [13], [14]. Recently, forming the desired power gain pattern, which is the pattern describing the power gain level in the concerned angular, has been widely concerned [16]. However, in these pattern synthesis problems, the DRR of the optimized weights is uncontrollable and sometimes too large. Reducing the DRR of excitations has practical significance due to the small DRR

of excitations helping to control the mutual coupling between antennas, reduce the output power loss, and simplify the design of the feeding network [13], [17].

Forming a wide beam is a typical application of array pattern synthesis in many scenarios, for example, receiving satellite multimedia signals when antennas are installed on the roof of moving vehicles [18], [19]. In this scenario, considering the vehicles would move when communicating with the satellite, a wide beam can ensure the main lobe pointing at the satellite to receive signals all the time. The power of the signal of interest (SOI) from the satellite would greatly reduce caused by channel fading and multipath effect, therefore the power gain of the receiving array antenna especially in the main lobe region should be as large as possible. The conventional methods to form wide beam are based on shaped beam pattern synthesis (SBPS) [20]–[23], which optimizes the array weights to generate a power gain pattern approximating to the predetermined beam shape. However, these methods based on the SBPS problem can't obtain the

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optimal power gain pattern in the main lobe. Paper [16] proposed an algorithm based on the PGPS problem, which forms the power gain pattern by maximizing the minimum PGML. This method can obtain a larger minimum PGML than the method based on the SBPS problem. But it cannot ensure the acquirement of the optimal solution.

The task of array pattern synthesis with a reduced DRR is to design the excitations to generate a desired power gain pattern while controlling the DRR of the excitations [24]. However, because the DRR constraint is non-convex, it is difficult to obtain good control on DRR for the conventional convex optimization method. A simple approach described in [25], [26] is to eliminate antennas with small weights. But it may not be able to limit the DRR to the desired threshold and may distort the power pattern. Paper [24] focuses on minimizing the DRR of excitations in the array pattern synthesis problem by introducing the ADMM method [27], but it can't guarantee the maximum-minimum PGML if applied in the PGPS problem. Paper [28] extends the semidefinite relaxation method (SDR) to be capable of synthesizing linearly polarized shaped patterns with accurate control of SLL and DRR of the excitation distribution for arbitrary antenna arrays when considering the mutual coupling. But SDR has high computational complexity. Reference [29] discussed the DRR control scheme while adding a maximum desired DRR constraint in the PGPS problem proposed in [16]. But this method only considers the upper bound constraint of excitations and replace smaller excitations by the lower bound expected at each iteration, it will destroy the optimal power gain pattern in the main lobe region. It's worth noting that when the upper bound of DRR is infinity, the method in [16] can be seen as a special case of the method in [29].

In this paper, primary and secondary contributions are presented. The primary contribution solves the general PGPS problem by an iterative method based on the SCA technique [30]. The method can obtain the stationary point of this non-convex problem. While as described in [16] and [29], the existing method cannot ensure the acquirement of the optimal solution or the stationary point. The secondary contribution is that the proposed method can precisely control the DRR of excitations in the PGPS problem. Note that existing methods can only roughly control the DRR and may reduce the efficiency in the practical system. The work presents a novel algorithm, which has better performance on increasing the minimum PGML and suppressing SLL compared with existing methods.

The rest of this paper is organized as follows: The general PGPS problem with DRR constraint is described in Sec.II. The new iterative algorithm based on the SCA technique is presented in Sec.III. Simulation analysis and results are shown in Sec.IV. And in Sec.V, conclusions are presented.

## II. PROBLEM FORMULATION

In this section, we formulate the general PGPS problem with the DRR constraint when the mutual coupling exists.

### A. PGPS PROBLEM

Considering a uniformly linear antenna array with  $M$  isotropic elements. The element spacing is half-wavelength. Considering the mutual coupling influence, we can express the far-field synthesis electric field of the array antenna in direction  $\theta$  as follows:

$$E(\theta) = \omega^H C a(\theta) \quad (1)$$

where  $\omega$  is the complex weights of array antenna,  $C$  is the mutual coupling matrix and  $a(\theta) = [1, e^{jk1d[\sin(\theta)-\sin(\theta_c)]}, \dots, e^{jk(M-1)d[\sin(\theta)-\sin(\theta_c)]}]^T$ . In steering vector  $a(\theta)$ ,  $k$  is the wavenumber of the received electromagnetic wave,  $d$  is the spacing of elements equals to half-wavelength,  $\theta$  is the grazing angle with respect to the normal direction of the array antenna, and  $\theta_c$  is the angle of the central direction in the predetermined main lobe region.

The PGPS problem proposed in [16] is formulated as (2). The desired main lobe region is represented by  $\Theta_{ML}$ . And it can be extended directly to the multidimensional or arbitrarily distributed array antenna.

$$\begin{aligned} & \max_{\omega, G} G \\ & \text{s.t. } \frac{\omega^H A_{\theta} \omega}{\omega^H A \omega} \geq G, \quad \theta \in \Theta_{ML} \\ & \quad \frac{\omega^H A_{\theta} \omega}{\omega^H A \omega} \leq \rho G, \quad \theta \in \Theta_{SL} \end{aligned} \quad (2)$$

where  $A = C \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} a(\theta) a^H(\theta) \cos(\theta) d\theta C^H$  is a constant matrix and  $A_{\theta} = C a(\theta) a^H(\theta) C^H$ .  $\Theta_{SL}$  represents the sidelobe region and  $\rho$  denotes the sidelobe suppression ratio (SLSR).

Several solutions are adopted to improve the isolation between the antenna elements in a two-element antenna array such as the defected ground structures, electromagnetic bandgap structures, polarization converters, and metamaterials [32]. If the mutual coupling could be ignored, the mutual coupling matrix will be a unit matrix  $I$ . While if the mutual coupling couldn't be ignored, we can obtain the coupling matrix from the normalized impedance matrix  $Z$  [31].

$$C = Z^{-1} \quad (3)$$

### B. THE GENERAL PGPS PROBLEM WITH DRR CONSTRAINT

In the feeding network, the magnitude of array weights can be adjusted by the attenuator. The adjustment range of attenuators is limited by the control bit. In the PGPS problem (2), the DRR of array weights is uncontrollable. Low DRR of excitations will simplify the design of the feeding network. Hence, a DRR upper bound constraint is imposed on the problem (2). It can be written as:

$$DRR(\omega) = \frac{\max\{|\omega_1|, \dots, |\omega_M|\}}{\min\{|\omega_1|, \dots, |\omega_M|\}} = \frac{\|\omega\|_{\infty}}{\|\omega\|_{-\infty}} \leq D \quad (4)$$

where  $|\cdot|$  denotes the modulus of a complex number,  $\|\cdot\|_{\infty}$  denotes  $\ell_{\infty}$  and  $\|\cdot\|_{-\infty}$  denotes  $\ell_{-\infty}$ .  $D$  is the predetermined upper bound of DRR.

Introduce the constraint (4) into the PGPS problem (2), the general PGPS problem with DRR constraint can be written as follows:

$$\begin{aligned} \max_{\omega, G} \quad & G \\ \text{s.t.} \quad & \frac{\omega^H \mathbf{A}_\theta \omega}{\omega^H \mathbf{A} \omega} \geq G, \quad \theta \in \Theta_{ML} \\ & \frac{\omega^H \mathbf{A}_\theta \omega}{\omega^H \mathbf{A} \omega} \leq \rho G, \quad \theta \in \Theta_{SL} \\ & \frac{\|\omega\|_\infty}{\|\omega\|_{-\infty}} \leq D \end{aligned} \quad (5)$$

Note that when  $D = \infty$ , the problem (2) can be seen as a special case of problem (5). The new PGPS problem (5) is more practical and general. But it is concave and difficult to be solved by the conventional convex optimization method.

### III. METHOD FOR PGPS PROBLEMS VIA SCA

#### A. SUCCESSIVE CONVEX APPROXIMATION TECHNIQUE

Considering a general non-convex problem as follows:

$$\begin{aligned} \min_{\omega} \quad & g_0(\omega) \\ \text{s.t.} \quad & g_i(\omega) \leq 0, \quad i = 1, 2, \dots, n \end{aligned} \quad (6)$$

where  $g_0(\omega)$  is a convex objective function, and the constraint function  $g_i(\omega)$  could be non-convex. Therefore, it is difficult to solve the problem (6) directly by traditional convex optimization methods. SCA technique solves the problem (6) by replacing the non-convex constraint functions with a series of convex approximation  $g_i(\omega) \approx f_i(\omega, \omega_t)$ , where  $\omega_t$  is the feasible solution of problem (6). Then the approximated problem can be solved easily by the conventional convex optimization method. It turns out that the solutions of this series of approximations converge to a point satisfying the KKT conditions of the original problem if these approximations could satisfy the following three requirements [33]:

- $g_i(\omega) \leq f_i(\omega, \omega_t)$  for all  $\omega$
- $g_i(\omega_t) = f_i(\omega_t, \omega_t)$
- $\frac{\partial g_i(\omega)}{\partial \omega_m} |_{\omega=\omega_t} = \frac{\partial f_i(\omega, \omega_t)}{\partial \omega_m} |_{\omega=\omega_t}, m = 1, \dots, M$

#### B. METHOD FOR GENERAL PGPS PROBLEM WITH DRR CONSTRAINT

The general PGPS problem (5) is concave due to the power gain constraints function and DRR constraint function. In general, a globally optimal solution of a non-convex problem cannot be obtained effectively and efficiently. Obtaining a stationary point is the classic goal for dealing with a non-convex problem [15]. We can reformulate problem (5) in a general way which is to maximize the minimum value of the numerator in the main lobe when restricting the denominator to be a constant  $c$  [16]. According to the equation  $\mathbf{A}_\theta = \mathbf{a}(\theta)\mathbf{a}^H(\theta)$ , and together with the fact that maximizing a variable  $G$  equals to minimizing  $-G$ , the problem (5) can be rewritten as an equivalent

problem.

$$\begin{aligned} \min_{\omega, G} \quad & -\sqrt{G} \\ \text{s.t.} \quad & -|\omega^H \mathbf{C} \mathbf{a}(\theta)| \geq \sqrt{cG}, \theta \in \Theta_{ML} \\ & \omega^H \mathbf{A}_\theta \omega \leq \rho cG, \theta \in \Theta_{SL} \\ & \omega^H \mathbf{A} \omega = c \\ & \frac{\|\omega\|_\infty}{\|\omega\|_{-\infty}} \leq D \end{aligned} \quad (7)$$

According to the DRR definition, it is known that  $D \geq 1$  and the magnitude of excitations are non-negative. Based on these two properties, the DRR constraint can be reformulated as:

$$\|\omega\|_\infty - D\|\omega\|_{-\infty} \leq 0 \quad (8)$$

We could get two non-convex constraint functions:

$$\begin{aligned} g_1(\omega) &= -|\omega^H \mathbf{a}(\theta)| \\ g_2(\omega) &= \begin{bmatrix} \|\omega\|_\infty \\ \|\omega\|_\infty \\ \dots \\ \|\omega\|_\infty \end{bmatrix} - \begin{bmatrix} D|\omega_1| \\ D|\omega_2| \\ \dots \\ D|\omega_M| \end{bmatrix} \end{aligned} \quad (9)$$

By using the Cauchy-Schwarz inequality and introducing the auxiliary variable  $\omega_t$ . The convex approximations of  $g_1(\omega)$  and  $g_2(\omega)$  can be written as:

$$\begin{aligned} f_1(\omega, \omega_t) &= -\frac{\text{Re}\{\omega^H \mathbf{C} \mathbf{a}(\theta) \mathbf{a}^H(\theta) \mathbf{C}^H \omega_t\}}{|\mathbf{a}^H(\theta) \omega_t|} \\ f_2(\omega, \omega_t) &= \begin{bmatrix} \|\omega\|_\infty \\ \|\omega\|_\infty \\ \dots \\ \|\omega\|_\infty \end{bmatrix} - \begin{bmatrix} D \frac{\text{Re}\{\omega_1 \omega_{t,1}\}}{|\omega_{t,1}|} \\ D \frac{\text{Re}\{\omega_2 \omega_{t,2}\}}{|\omega_{t,2}|} \\ \dots \\ D \frac{\text{Re}\{\omega_M \omega_{t,M}\}}{|\omega_{t,M}|} \end{bmatrix} \end{aligned} \quad (10)$$

where  $\text{Re}\{\cdot\}$  denotes the real part of a complex number and  $\omega_m$  denotes the  $m$ -th element in vector  $\omega$ .

Problem (7) can be reformulated as a convex approximate problem (11) by replacing  $g_1(\omega)$  with  $f_1(\omega, \omega_t)$  and  $g_2(\omega)$  with  $f_2(\omega, \omega_t)$ . And we replace the  $\sqrt{G}$  with  $G_0$  and  $\sqrt{c}$  with  $c_0$  in problem (7).

$$\begin{aligned} \min_{\omega, G_0} \quad & -G_0 \\ \text{s.t.} \quad & -\frac{\text{Re}\{\omega^H \mathbf{C} \mathbf{a}(\theta) \mathbf{a}^H(\theta) \mathbf{C}^H \omega_t\}}{|\mathbf{a}^H(\theta) \mathbf{C}^H \omega_t|} \geq c_0 G_0, \theta \in \Theta_{ML} \\ & \omega^H \mathbf{A}_\theta \omega \leq \rho c_0^2 G_0^2, \theta \in \Theta_{SL} \\ & \omega^H \mathbf{A} \omega = c_0^2 \\ & \begin{bmatrix} \|\omega\|_\infty \\ \|\omega\|_\infty \\ \dots \\ \|\omega\|_\infty \end{bmatrix} - \begin{bmatrix} D \frac{\text{Re}\{\omega_1 \omega_{t,1}\}}{|\omega_{t,1}|} \\ D \frac{\text{Re}\{\omega_2 \omega_{t,2}\}}{|\omega_{t,2}|} \\ \dots \\ D \frac{\text{Re}\{\omega_M \omega_{t,M}\}}{|\omega_{t,M}|} \end{bmatrix} \leq 0 \end{aligned} \quad (11)$$

Due to the condition  $\theta \in \Theta_{ML}$ , the problem (11) involves semi-infinite constraints. An effective method to deal with

this issue is approximating these semi-infinite constraints by discretizing the main lobe region  $\Theta_{ML}$  [34]. The more precise the discrete angle is, the more accurate solution will be, but the greater amount of computation. The uniform discrete method is used to reformulate the problem as (12), where  $\Theta_1 = [\Theta_{M_1}, \dots, \Theta_{M_L}]$  and  $\Theta_2 = [\Theta_{S_1}, \dots, \Theta_{S_Q}]$ .  $L = \frac{\Theta_{ML}}{\phi}$  and  $Q = \frac{\Theta_{SL}}{\phi}$ ,  $\phi$  is the stepsize when discretizing  $\Theta_{ML}$  and  $\Theta_{SL}$ .

$$\begin{aligned} & \min_{\omega, G_0} \quad -G_0 \\ & \text{s.t.} \quad -\frac{\text{Re}\{\omega^H \mathbf{C} \mathbf{a}(\theta) \mathbf{a}^H(\theta) \mathbf{C}^H \omega_t\}}{|\mathbf{a}^H(\theta) \mathbf{C}^H \omega_t|} \geq c_0 G_0, \quad \theta \in \Theta_1 \\ & \quad \omega^H \mathbf{A}_\theta \omega \leq \rho c_0^2 G_0^2, \quad \theta \in \Theta_2 \\ & \quad \omega^H \mathbf{A} \omega = c_0^2 \\ & \quad \begin{bmatrix} \|\omega\|_\infty \\ \|\omega\|_\infty \\ \dots \\ \|\omega\|_\infty \end{bmatrix} - \begin{bmatrix} D \frac{\text{Re}\{\omega_1 \omega_{t,1}\}}{|\omega_{t,1}|} \\ D \frac{\text{Re}\{\omega_2 \omega_{t,2}\}}{|\omega_{t,2}|} \\ \dots \\ D \frac{\text{Re}\{\omega_M \omega_{t,M}\}}{|\omega_{t,M}|} \end{bmatrix} \leq \mathbf{0} \end{aligned} \quad (12)$$

Problem (12) is a convex problem and can be solved by the CVX toolbox. The general PGPS problem (5) can be solved by solving the approximated problem (12) iteratively via the SCA technique. The steps of the proposed algorithm which call SCA-DRR is described in **ALGORITHM 1**, where  $\mathbf{p}_d = \mathbf{1} \in \mathbb{C}^L$ ,  $N_i$  is the number of iterations, and  $\mathbf{B} = [\mathbf{C} \mathbf{a}(\Theta_1), \dots, \mathbf{C} \mathbf{a}(\Theta_L)] \in \mathbb{C}^{M \times L}$ .

**Algorithm 1** SCA-DRR

- 1: Initialize  $\omega_t = (\mathbf{B} \mathbf{B}^H)^{-1} \mathbf{B} \mathbf{p}_d$ ,  $N_i$ ,  $D$  and  $c_0 = 1$
- 2: Calculate  $\mathbf{A}$
- 3: **while**  $N_i > 0$  **do**
- 4:   Obtain  $\omega^i$ : solving the problem (12) by CVX toolbox;
- 5:   Update  $\omega_t$ :  $\omega_t = \omega^i$ ;
- 6:   Update  $N_i$ :  $N_i = N_i - 1$ ;
- 7: **end while**

**C. CONVERGENCE ANALYSIS**

If the convex approximations  $f_1(\omega, \omega)$  and  $f_2(\omega, \omega_t)$  satisfy the three requirements aforementioned in the SEC.III.A, the weights obtained by SCA-DRR will converge to a point that satisfying the KKT conditions of the problem (7). KKT is the necessary condition of optimal solutions to non-convex problems. And Problem (7) is the equivalent problem of the original problem (5). Therefore, if we could give a proof that  $f_1(\omega, \omega)$  and  $f_2(\omega, \omega_t)$  satisfy the three requirements, the weights obtained by the proposed algorithm will converge to the stationary point of the original problem (5).

Considering the property of Cauchy-Schwarz inequality, it is easy to prove the following two requirements:

$$\begin{aligned} g_1(\omega) &\leq f_1(\omega, \omega_t), \quad g_1(\omega_t) \leq f_1(\omega_t, \omega_t) \\ g_2(\omega) &\leq f_2(\omega, \omega_t), \quad g_2(\omega_t) \leq f_2(\omega_t, \omega_t) \end{aligned} \quad (13)$$

**TABLE 1.** Performance index ( $C = I$  and  $\rho = \infty$ ).

	Min PGML	SLL	Mean PGML	DRR
SCA-DRR(D= $\infty$ )	9.6376dBi	-6.8300dBi	9.8743dBi	611.9554
ITER-DRR(D= $\infty$ )	9.6185dBi	-6.1459dBi	9.8044dBi	22.6388
SCA-DRR(D=5)	9.4395dBi	-2.5326dBi	9.5585dBi	4.9998
ITER-DRR(D=5)	9.0551dBi	-1.7006dBi	9.5253dBi	2.1768

What's more, if  $\omega_t \neq \mathbf{0}$ ,  $g_2(\omega)$  and  $f_2(\omega, \omega_t)$  will be differentiable. In problem (12), it is obvious that  $\omega_t \neq \mathbf{0}$ . To prove the following equations:

$$\begin{aligned} \frac{\partial g_1(\omega)}{\partial \omega_m} \Big|_{\omega=\omega_t} &= \frac{\partial f_1(\omega, \omega_t)}{\partial \omega_m} \Big|_{\omega=\omega_t} \\ \frac{\partial g_2(\omega)}{\partial \omega_m} \Big|_{\omega=\omega_t} &= \frac{\partial f_2(\omega, \omega_t)}{\partial \omega_m} \Big|_{\omega=\omega_t} \end{aligned} \quad (14)$$

It can be converted to prove the following equation:

$$\frac{\partial |\mathbf{x}^H \mathbf{v}|}{\partial \mathbf{x}} \Big|_{\mathbf{x}=\mathbf{x}_t} = \frac{\partial \frac{\text{Re}\{\mathbf{x}^H \mathbf{v} \mathbf{v}^H \mathbf{x}_t\}}{|\mathbf{v}^H \mathbf{x}_t|}}{\partial \mathbf{x}} \Big|_{\mathbf{x}=\mathbf{x}_t} \quad (15)$$

where  $\mathbf{x}, \mathbf{x}_t, \mathbf{v} \in \mathbb{C}^N$  are  $N$ -dimension vector, both  $\mathbf{x}_t$  and  $\mathbf{v}$  are constant vectors. We can rewrite the  $\mathbf{v}^H \mathbf{x}_t$  as follows:

$$\mathbf{v}^H \mathbf{x}_t = |\mathbf{v}^H \mathbf{x}_t| e^{j\beta} \quad (16)$$

where  $\beta$  is the phase of  $\mathbf{v}^H \mathbf{x}_t$ . It can be derived that:

$$\frac{\partial |\mathbf{x}^H \mathbf{v}|}{\partial \mathbf{x}} \Big|_{\mathbf{x}=\mathbf{x}_t} = \frac{\partial \frac{\text{Re}\{\mathbf{x}^H \mathbf{v} \mathbf{v}^H \mathbf{x}_t\}}{|\mathbf{v}^H \mathbf{x}_t|}}{\partial \mathbf{x}} \Big|_{\mathbf{x}=\mathbf{x}_t} = \mathbf{v} e^{j\beta} \quad (17)$$

Hence, the proposed method could obtain a stationary point of general PGPS problem (5).

**D. COMPUTATIONAL COMPLEXITY**

Regarding the computational complexity,  $\omega^H \mathbf{A} \omega$  can be decomposed by Cholesky factorization as  $\|\mathbf{L}\omega\|^2$ . Problem (12) can be regarded as an SOCP problem. It is shown in [35] that to solve an SOCP problem by the interior point method needs  $\sqrt{N_c}$  iterations, where  $N_c$  is the number of the second-order cone in the problem. The complexity in each iteration of the SOCP problem is  $O(\gamma \sum_i q_i)$  where  $\gamma$  is the dimension of optimization variable and  $q_i$  is the dimension of the  $i$ -th second-order cone. So the computational complexity of **ALGORITHM 1** is  $O(N_i M^3 N_c^{1.5})$  where  $N_i$  is the number of iterations.

**IV. SIMULATION RESULTS AND ANALYSIS**

In this section, we validate the advantage and effectiveness of the proposed algorithm by comparing it with the method proposed in [29] which named ITER-DRR. When  $D = \infty$ , the method described in [16] can be regarded as a special case of ITER-DRR. In all numerical simulations, an uniformly distributed ULA with 22 isotropic elements is considered.

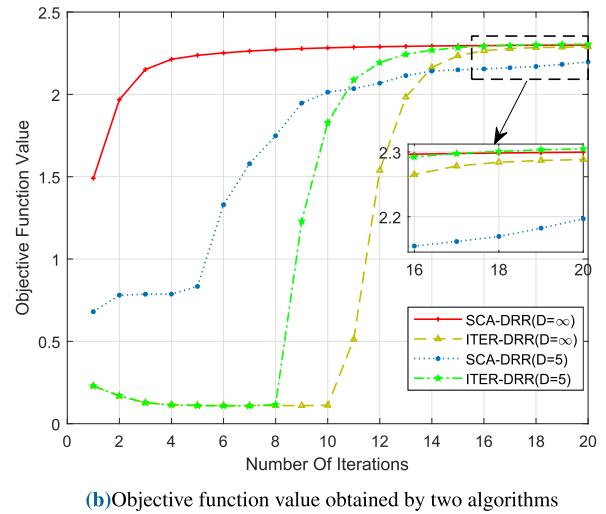
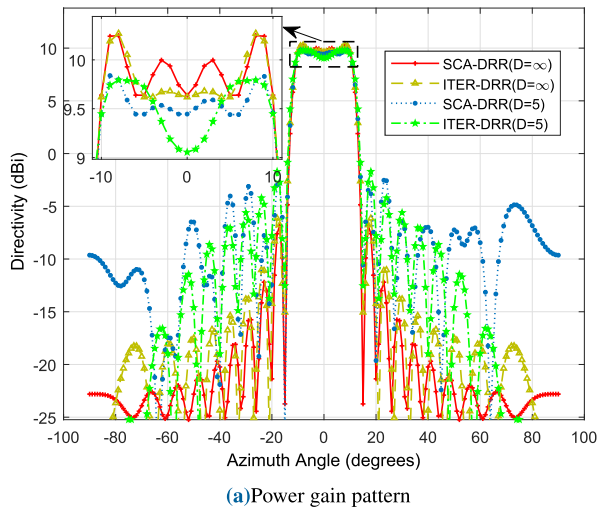


FIGURE 1. Power gain pattern and objective function value obtained by two algorithms with different D. ( $C = I$  and  $\rho = \infty$ ).

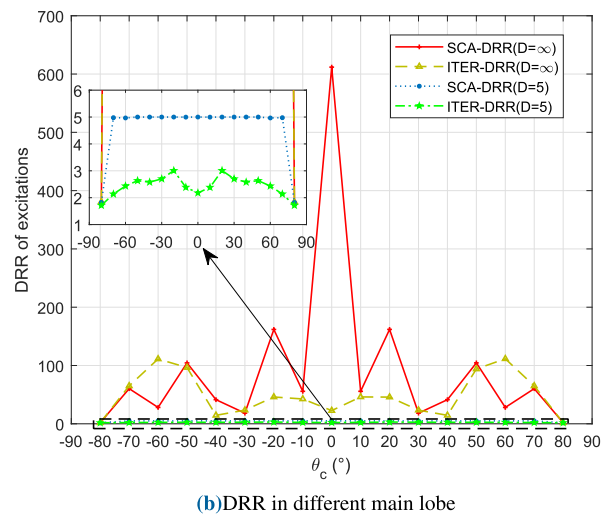
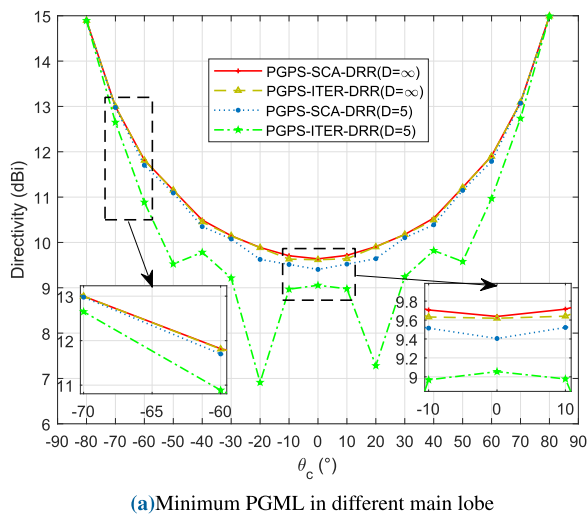


FIGURE 2. Minimum PGML and DRR obtained by two algorithms with different D at various main lobe. ( $C = I$  and  $\rho = \infty$ ).

The specific parameters for ITER-DRR is  $\alpha = 0.2$  and  $\sigma = 0.01$ .

Assuming that mutual coupling is too small and could be ignored, the mutual coupling matrix should be  $C = I$  in simulations.

In Fig.1 and TABLE 1, the simulation results are obtained by SCA-DRR and ITER-DRR with different D. It is seen that the minimum PGML obtained by SCA-DRR is larger than ITER-DRR about 0.02dB when  $D = \infty$  and 0.38dB when  $D = 5$  respectively. The proposed algorithm could obtain a larger minimum PGML. The SLL obtained by SCA-DRR has been reduced 0.68dB and 0.8dB compared with ITER-DRR when  $D = \infty$  and  $D = 5$ . From TABLE 1, the obtained DRR of SCA-DRR approaches to the predefined upper bound, while the obtained DRR of ITER-DRR is at most 3.82 less than the upper bound. Seen from Fig.1.(b), the objective function value of SCA-DRR is larger than ITER-DRR when

$D = \infty$ . It is illustrated that our method could obtain a better solution. When  $D = 5$ , the objective function value of PGPS-ITER-DRR will be larger than the proposed algorithm, but the minimum PGML is worse. It is because that the constraint of DRR in SCA-DRR is stricter than ITER-DRR and replacing the small excitations with the expected lower bound will bring pattern distortion.

In Fig.2, the minimum PGML and the DRR of 200 Monte Carlo experiments which obtained by SCA-DRR and ITER-DRR with different D when scanning the main lobe from  $[-90^\circ, -70^\circ]$  to  $[70^\circ, 90^\circ]$  are plotted. As seen in Fig.3.(a), SCA-DRR has a stable performance on minimum PGML because the difference among the minimum PGML is always less than 0.27dB when D is different, while the ITER-DRR has a serious performance degradation more than 3dB when  $D = 5$  and main lobe =  $[-30^\circ, -10^\circ]$ . SCA-DRR with  $D = \infty$  has the largest minimum PGML in all cases, and



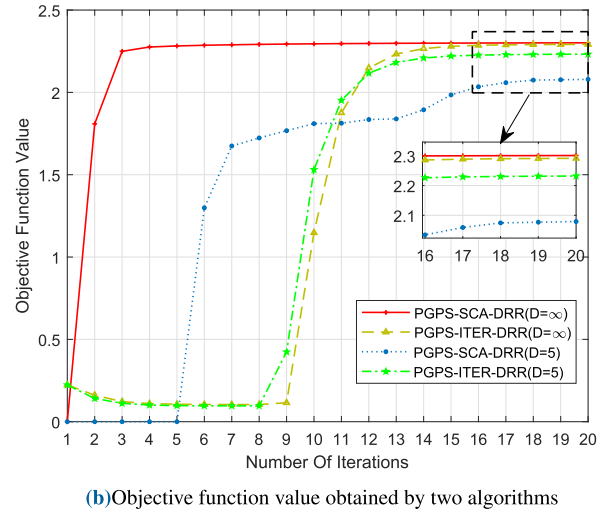
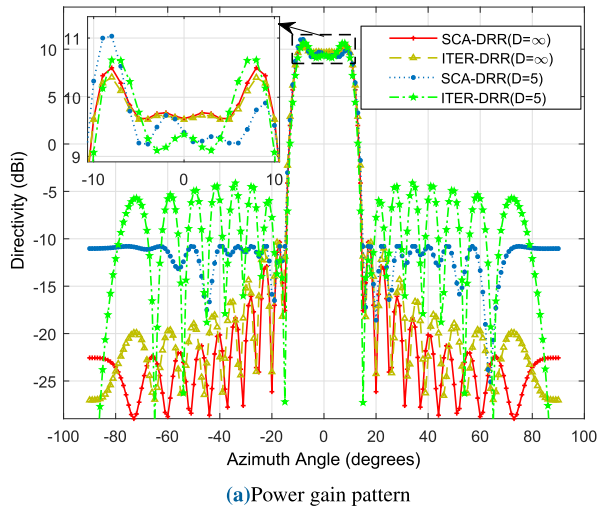


FIGURE 3. Performance index obtained by two algorithm when D is different. ( $C = I$  and  $\rho = 0.01$ ).

TABLE 2. Performance index ( $C = I$  and  $\rho = 0.01$ ).

	Min PGML	SLL	Mean PGML	DRR
SCA-DRR( $D=\infty$ )	9.6403dBi	-10.3597dBi	9.8898dBi	115.5716
ITER-DRR( $D=\infty$ )	9.6234dBi	-10.3775dBi	9.8280dBi	68.2198
SCA-DRR( $D=5$ )	9.2072dBi	-10.8005dBi	9.6919dBi	5.0000
ITER-DRR( $D=5$ )	9.0677dBi	-4.1208dBi	9.7160dBi	2.6449

the ITER-DRR with  $D = 5$  has the worst performance in some main lobe regions such as  $[-80^\circ, -60^\circ]$ . Besides, from Fig.2.(b), the obtained DRR of SCA-DRR can be controlled precisely and approach the given upper bound, while the DRR of ITER-DRR is less than the given upper bound at least 2. In the general PGPS problem, controlling the obtained DRR precisely will improve the performance on minimum PGML.

In order to assess the effectiveness of SCA-DRR when controlling the DRR and SLL together, the sidelobe level constraint is considered in simulations. The simulation results are shown in Fig.3 and TABLE 2. The distance between the main lobe  $\Theta_{ML}$  and the side lobe  $\Theta_{SL}$  is  $5^\circ$  and  $\rho = 0.01$  (SLSR = 20dB). The performance on minimum PGML of SCA-DRR is about 0.017dB better than ITER-DRR when  $D = \infty$  and 0.139dB better when  $D = 5$ . The performance on SLL of SCA-DRR is 1.79dB better than ITER-DRR when  $D = \infty$ . The SLSR obtained by SCA-DRR is 20.0000dB and 20.0077dB when  $D = \infty$  and  $D = 5$  respectively, while the SLSR obtained by ITER-DRR is not achieved to 20dB in the cases of  $D = 5$ . When controlling the DRR and SLL together, SCA-DRR could ensure to obtain the optimal weights which will generate the power gain pattern with the desired SLSR and larger minimum PGML.

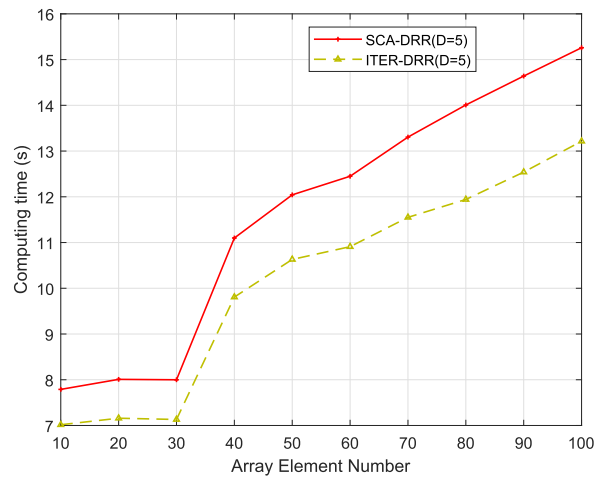


FIGURE 4. Computing time versus array element number. ( $C = I$ ,  $\rho = \infty$  and  $D = 5$ ).

In Fig.4, the computing time of 200 Monte Carlo experiments obtained by SCA-DRR and ITER-DRR with different array element number is plotted. It can be seen that the computational complexity of the two algorithms is on the same order of magnitude. ITER-DRR is at most 2s less than SCA-DRR in all cases while SCA-DRR has better performance. The proposed method is a low-complexity algorithm and can be applied in large-scale array antenna.

When the mutual coupling effect exists, the mutual coupling matrix should be  $C = Z^{-1}$ . We obtain the impedance matrix of the antenna from the High Frequency Structure Simulator (HFSS) software. And HFSS is employed in the following simulations.

The simulation results are shown in Fig.5 and TABLE 3 when considering the mutual coupling effect. The performance of both SCA-DRR and ITER-DRR has been degraded because of the mutual coupling. When  $D = \infty$ , the min PGML of SCA-DRR is smaller than ITER-DRR about 0.05dB and the SLL is larger about 0.9dB. While the

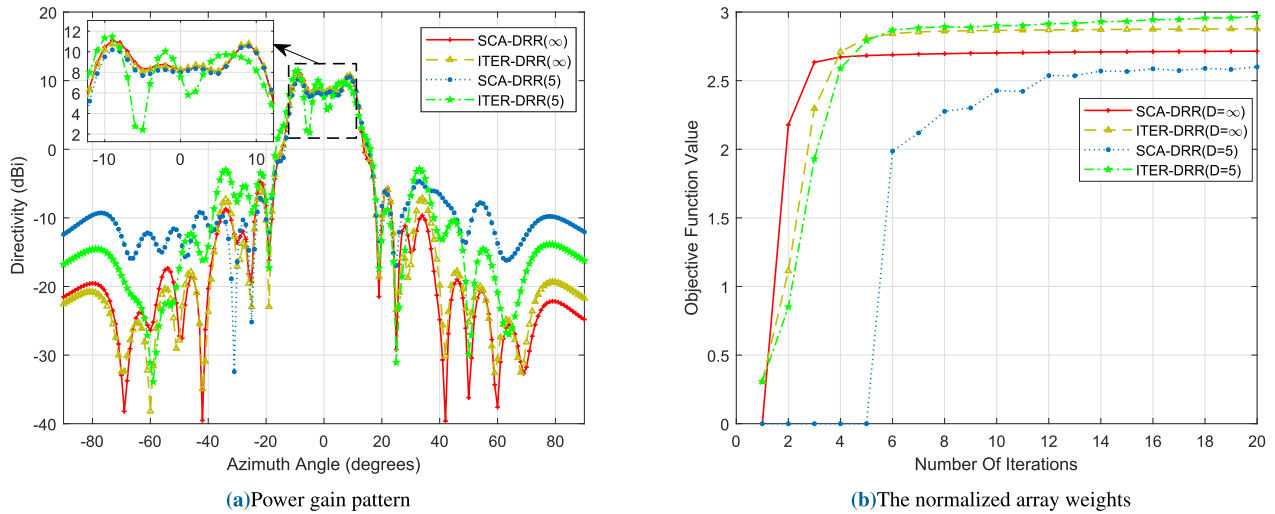


FIGURE 5. Power gain pattern and objective function value obtained by two algorithm with different D. ( $C = Z^{-1}$  and  $\rho = 0.01$ ).

TABLE 3. Performance index ( $C = Z^{-1}$  and  $\rho = 0.01$ ).

	Min PGML	SLL	Mean PGML	DRR
SCA-DRR( $D=\infty$ )	8.0596dBi	-4.7611dBi	9.2040dBi	64.6280
ITER-DRR( $D=\infty$ )	8.1099dBi	-5.6488dBi	9.1409dBi	25.2893
SCA-DRR( $D=5$ )	7.6846dBi	-4.6456dBi	8.8016dBi	4.9956
ITER-DRR( $D=5$ )	2.4165dBi	-2.8672dBi	8.2631dBi	3.3321

mean PGML of SCA-DRR is better than ITER-DRR about 0.06dB. When  $D = 5$ , the performance of ITER-DRR has a serious degradation. The min PGML and SLL of ITER-DRR are 2.4165dBi and  $-2.8672$ dBi respectively, while the min PGML and SLL of SCA-DRR are 7.6846dBi and  $-4.6456$ dBi. But the SLSR obtained by two algorithms doesn't meet the preset requirement which is 20dB.

### V. CONCLUSION

In this paper, we proposed an iterative method based on the SCA technique to solve the general PGPS problem with the DRR constraint. We convert the DRR constraint into a group of stricter inequalities and transform the concave constraints to be convex with some convex approximations. Then the general PGPS problem can be solved iteratively by the proposed algorithm called SCA-DRR. The proposed method could ensure to obtain the stationary point of the original non-convex PGPS problem. Simulations results illustrate the proposed algorithm has a better performance on increasing the minimum PGML and suppressing the SLL compared with the existing method ITER-DRR proposed in [16] and [29]. Meanwhile, the DRR can be controlled precisely below a given upper bound. And the proposed method has low computation complexity. When the mutual coupling exists, the mutual coupling will degrade the algorithm performance.

Our proposed method has a better performance than the existing method ITER-DRR when considering mutual coupling.

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