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Tuning of Linear Active Disturbance Rejection Controllers Based On Step Response Curves

WENQING CUI¹, WEN TAN¹, (Member, IEEE), DONGHAI LI², AND YUTONG WANG¹

¹School of Control and Computer Engineering, North China Electric Power University, Beijing 102206, China

²State Key Laboratory of Power Systems, Department of Thermal Engineering, Tsinghua University, Beijing 100084, China

Corresponding author: Wenqing Cui (2351964946@qq.com)

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ABSTRACT Most of the controlled plants in the industrial field are complex in structure and the models are difficult to obtain, thus tuning of the controller parameters without a detailed model is critical in practice. This paper proposes a specific parameter tuning formula for second-order linear active disturbance rejection controller (LADRC) based on step response curves of the controlled plant. The proposed tuning formula is only related to the two special points on the step response curve of the plant. Simulation results for a wide range of systems show that the proposed method can achieve satisfactory performance in disturbance rejection and robustness, and the Bode plots show that this tuning method also performs well in noise suppression. The effect is also verified with a practical heater temperature control experiment.

INDEX TERMS Linear active disturbance rejection controllers, parameter tuning, step response curves, the temperature control lab system.

I. INTRODUCTION

Among the practical industrial process control, the most popular control strategy is PID control. The three parameters of PID controller have a very clear connection with the performance of the system, so it can be easily tuned online. However, with the increasing complexity of industrial processes, the control effect of traditional PID controller may not be satisfactory due to its specific structure. Active disturbance rejection control (ADRC) proposed by Han is composed of tracking differentiator (TD), extended state observer(ESO) and nonlinear state error feedback(NLSEF). The main purpose of the tracking differentiator is to arrange the transition process of the closed-loop system by tracking the differential output and the fastest synthesis function of the differentiator, so that the system can realize tracking without overshoot, and solve the contradiction between rapidity and overshoot. ADRC combines the internal uncertainty (constant or time-varying, linear or nonlinear) and external uncertainty into a 'total disturbance'. By constructing an ESO, the 'total disturbance' is estimated and compensated in real time, so strong disturbance rejection ability may be achieved [1]. In the past 20 years, ADRC has been widely used in the fields of

electromechanical system, power generation process, chemical process, etc., [2]–[4].

There are still some problems to be solved in ADRC technology, among which parameter tuning is the most important one. The traditional ADRC structure needs lots of parameters to tune and the tuning procedure is complex. At present, optimization algorithm is the most commonly used tuning method of ADRC [5], [6]. However, the convergence of the optimization algorithm depends on the initial value. How to select the initial value is always a problem, which can not be well applied in engineering application. To simplify the structure of ADRC, Gao proposed the 'linear' version of ADRC(LADRC) that uses the estimated output and its derivatives for linear state feedback, and transformed the design of linear extend state observer (LESO) and state feedback into the selection of two parameters of the observer bandwidth and the controller bandwidth [7], [8]. The structure of LADRC is much simpler, and the parameter tuning is simplified, so LADRC found wide applications in practice. Up to now, there are many tuning methods. Reference [9] proposes a method that takes the desired settling time as the only parameter to tune. References [10], [11] claim that the parameters of LADRC can be obtained from PID parameters. References [12], [13] propose to tune the LADRC parameters using the extra model information. Reference [14] proposes a tuning formula for second-order LADRC for first-order plus

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dead-time (FOPDT) model. The above tuning methods either need specific model information or need to set some known conditions in advance, such as settling time. It is difficult to know the information in practice. Furthermore, [15] proposes a tuning formula for LADRC based on relay feedback experiment without modeling, but the relay feedback experiment needs the system to produce stable constant amplitude oscillation, which is very dangerous in the practical industrial process. Therefore, how to determine the tuning parameters of LADRC without a detailed model is still a problem to be solved, and this tuning method should be meaningful to guide engineering practice.

As a way to improve the automation of control system, PID auto-tuning products has been widely used in the market [16]. The purpose of PID auto-tuning is to quickly get the initial value of the controller for an acceptable performance. The auto-tuning steps are shown as follows [17]: (1) By giving an excitation signal to the system, the output data of the system is obtained by sampling; (2) The dynamic characteristic parameters of the system are obtained by using the output data of the system; (3) The controller is auto-tuned by characteristic parameters. At present, many large companies (ABB, Honeywell) use the step identification or the relay feedback identification strategy for PID auto-tuning products. Thus as a potential replacement of PID control technique, it is required that there is a similar auto-tuning method for LADRC. The contribution of this paper is to propose a tuning formula for second-order LADRC based on step response, which is simple and easy to master. The tuning parameters of LADRC can be quickly obtained by only two time points (t_1, t_2) on the step response curve.

The rest of the paper is arranged as follows: Section 2 introduces the principle of LADRC; Section 3 describes the identification methods using step response curves; Section 4 introduces the relevant derivation process of the proposed tuning formula for LADRC; Simulation results are given in Section 5; A practical temperature control experiment is implemented in Section 6 and the conclusion is described in Section 7.

II. LINEAR ACTIVE DISTURBANCE REJECTION CONTROL

Consider a second order plant [7]:

$$\ddot{y} = bu + g(t, y, \dot{y}, d) \tag{1}$$

where d is the external disturbance, b is the high frequency gain and $g(t, y, \dot{y}, d)$ is the comprehensive characteristics of unknown dynamics and external disturbances of the system. For most processes, the exact value of b is difficult to obtain, so the plant model can be assumed as:

$$\ddot{y} = b_0u + \omega \tag{2}$$

where $\omega = g + (b - b_0)u$ is the total disturbance to be estimated as the extended state.

Let:

$$x_1 = y, x_2 = \dot{y}, x_3 = w(y, u, d) \tag{3}$$

write model (2) in the form of state space expression [7]:

$$\begin{cases} \dot{x} = A_e x + B_e u + E_e \dot{w} \\ y = C_e x \end{cases} \tag{4}$$

where

$$A_e = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, B_e = \begin{bmatrix} 0 \\ b_0 \\ 0 \end{bmatrix}, C_e = [1 \quad 0 \quad 0], E_e = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{5}$$

Therefore, the following Luenberger observer is designed for the system to estimate the total disturbance:

$$\begin{cases} \dot{z} = A_e z + B_e u + L_o(y - \hat{y}) \\ \hat{y} = C_e z \end{cases} \tag{6}$$

where L_o is the observer gain:

$$L_o = [\beta_1 \quad \beta_2 \quad \beta_3]^T \tag{7}$$

The stability and observation ability of ESO are analyzed in [18]. Suppose the total disturbance ω is bounded. When $A_e - L_o C_e$ is asymptotically stable, z_1, z_2 will approach the output y and \dot{y} , and z_3 will approach the disturbance ω . Therefore, z_3 can be used to suppress ω quickly.

The following state feedback control rate is adopted:

$$u_0 = k_1(r - y) + k_2(\dot{r} - \dot{y}) \tag{8}$$

where r is the reference input and the final control law:

$$u = \frac{u_0 - z_3}{b_0} = \frac{k_1(r - z_1) + k_2(\dot{r} - z_2) - z_3}{b_0} = K_o(\hat{r} - z) \tag{9}$$

where

$$\hat{r} = [r \quad \dot{r} \quad 0]^T, z = [z_1 \quad z_2 \quad z_3]^T \tag{10}$$

The controller gain K_o is defined as:

$$K_o = [k_1 \quad k_2 \quad 1] / b_0 \tag{11}$$

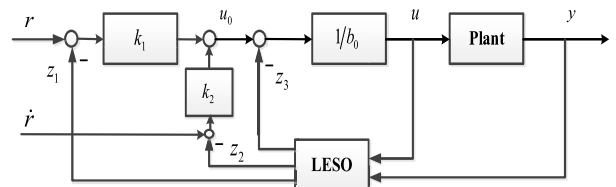


FIGURE 1. The structure of second-order LADRC.

In summary, a second-order LADRC is realized in the following state space and its structure is shown in Figure 1:

$$\begin{cases} \dot{z} = A_e z + B_e u + L_o(y - C_e z) = (A_e - L_o C_e)z + B_e u + L_o y \\ u = K_o(\hat{r} - z) \end{cases} \tag{12}$$

And the feedback loop transfer function from y to u is

$$K_c(s) = \frac{u(s)}{y(s)} = \frac{(\beta_3 + \beta_1 k_1 + \beta_2 k_2)s^2 + (\beta_2 k_1 + \beta_3 k_2)s + \beta_3 k_1}{b_0 s(s^2 + (\beta_1 + k_2)s + (\beta_2 + k_1 + \beta_1 k_2))} \quad (13)$$

According to the above design process, a linear ADRC needs to tune the following three parameters: b_0 , L_o and K_o . [7] proposed the concept of bandwidth and the tuning parameters are transformed into the selection of ω_c and ω_o . Therefore, for the second-order LADRC, the relationship between K_o , L_o and ω_c , ω_o are shown as follows:

$$k_1 = \omega_c^2, k_2 = 2\zeta\omega_c, \beta_1 = 3\omega_o, \beta_2 = 3\omega_o^2, \beta_3 = \omega_o^3 \quad (14)$$

where ζ is a damping ratio to improve the closed-loop performance. Thus the tuning of LADRC becomes tuning the four parameters ω_c , ω_o , b_0 and ζ .

III. PARAMETER IDENTIFICATION BASED ON STEP RESPONSE CURVES

A step signal is easy to generate and has low harm to the controlled process, so the control system identification based on a step test is widely used in the practical industrial field. The step test system identification has the following characteristics: (1) the generation of a step signal is simple; (2) whether the system is open-loop or closed-loop, step test identification can be applied; (3) there is no need to have a detailed understanding of the system structure. Due to these advantages, these classical representative methods based on step test have always been favored by field engineers [19], [20].

A step excitation input is added to a stable and self-balanced plant. It can be roughly divided into two categories by its output response. One is the monotonic rising FOPDT plant without overshoot, and the other is the second-order underdamped plant with oscillation [21]. This paper mainly studies FOPDT model as shown in Eq.(15) and designs auto-tuning formula for it.

$$G(s) = \frac{Ke^{-\tau s}}{Ts + 1} \quad (15)$$

A. TWO-POINT APPROACH [22]

For a FOPDT model, the static gain can be obtained from the following formula according to a step experiment shown in Figure 2:

$$K = \frac{y(\infty) - y(0)}{\Delta u} \quad (16)$$

where Δu is the amplitude of the step input, $y(0)$ and $y(\infty)$ are the initial value and the final value of the step response.

The relationship between the output y and the time t is shown in Table 1 [23].

Let t_1 and t_2 be the time when the step response curve reaches 39.3% and 63.2% of the steady value, from Table 1, we have

$$\begin{cases} \frac{T}{2} + \tau = t_1 \\ T + \tau = t_2 \end{cases} \quad (17)$$

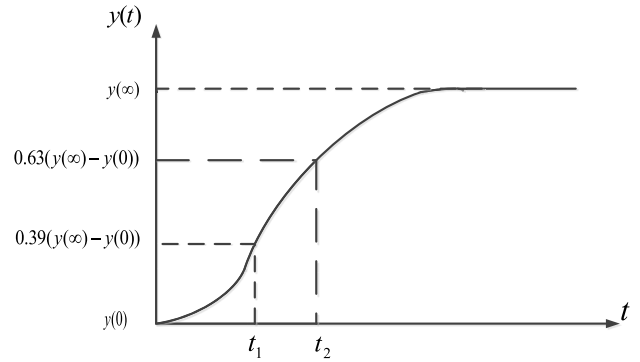


FIGURE 2. Illustration of two point approach.

TABLE 1. The relationship between y and t .

$y(\infty)/\%$	t
28.4	$T/3 + \tau$
39.3	$T/2 + \tau$
55	$0.8T + \tau$
59.3	$0.9T + \tau$
63.2	$T + \tau$
77.7	$1.5T + \tau$
86.5	$2T + \tau$

Solving the equation (17), the value of T and τ can be obtained

$$\begin{cases} T = 2(t_2 - t_1) \\ \tau = 2t_1 - t_2 \end{cases} \quad (18)$$

Using (16) and (18), it is not difficult to identify a FOPDT model from the reaction curve via the two-point approach.

B. FLEXION TANGENT APPROACH [24]

The flexion tangent approach is easy to obtain the FOPDT model of the controlled plant. Different from the two-point approach, this method needs to find the inflection point of the curve, that is, the point with the largest slope of the curve. As shown in Figure 3, the tangent passing through the inflection point P intersects the time axis at N-point and the steady value at M-point. Then the delay time τ and the time constant T are respectively:

$$\tau = ON, \quad T = NQ \quad (19)$$

where NQ is the projection of NM on the time axis.

C. AREA METHOD [16]

Consider the response of a stable self-balancing process under step signal input. As shown in Figure 4, $y(\infty)$ is the steady value and t_r is the rising time.

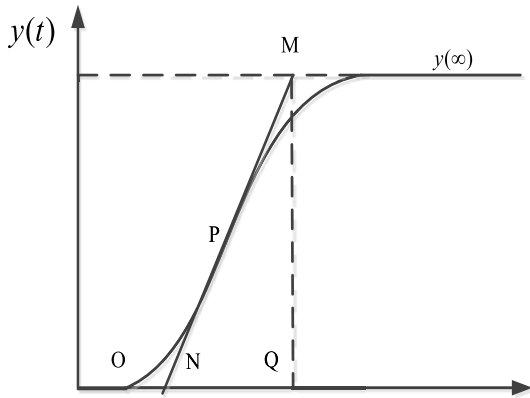


FIGURE 3. Illustration of flexion tangent approach.

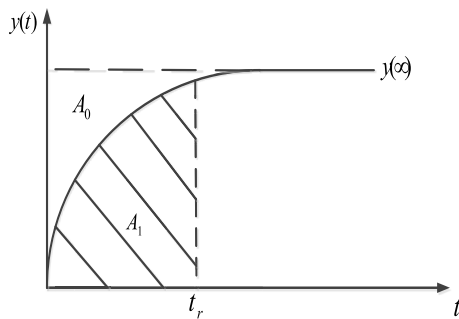


FIGURE 4. Illustration of area method.

According to the area method, the parameters indicated in Figure 4 are calculated as follows [25]:

$$\begin{cases} A_0 = \int_0^{+\infty} (y(\infty) - y(t))dt \\ t_r = \frac{A_0}{y(\infty)} \\ A_1 = \int_0^{t_r} y(t)dt \end{cases} \quad (20)$$

Further, the parameters in FOPDT model can be identified by

$$T = \frac{eA_1}{y(\infty)}, \quad \tau = \frac{A_0}{y(\infty)} - T \quad (21)$$

Based on these commonly used methods to identify model parameters by step response, [26] and [27] further proposed some methods to obtain PID parameters directly through step response, and achieved good control effect. Thus it will be feasible to obtain the parameters of LADRC by step response.

Remark 1: All three methods need step disturbance experiment. In a complex industrial process, the determination of the amplitude of step disturbance is very important. The amplitude should be large enough to reduce the influence of other disturbances on the test results. However, if the amplitude of disturbance is too large, the nonlinear factors of the plant itself will increase. In addition, before the step

disturbance, the system should be operated in steady state for a period of time to ensure the accuracy of the step response data.

Remark 2: From the above analysis of the three identification methods, it is not difficult to see that the area method has high accuracy and is suitable for irregular response curve, but the calculation is complex, it needs to calculate the parameters through integration. In flexion tangent approach, the inflection point of tangent method is difficult to determine, so the tangent at inflection point will be inaccurate, resulting in low identification accuracy. Reference [23] analyses and compares different identification methods and their corresponding identification accuracy, and the identification error as a measurement index is used to evaluate the identification accuracy. From the data and simulation, it can be seen that the identification errors of the area method and the two-point approach are similar for most systems with regular step response curves. It can be predicted that the identification accuracy of area method may be higher for the system with irregular step response curve. The purpose of this paper is to design a simple and practical tuning formula for LADRC. Due to the excellent performance of LADRC, the requirements for identification accuracy are not high. Thus the two-point approach will be used in the subsequent derivation and experiment in this paper.

IV. DERIVATION OF TUNING FORMULA FOR LADRC BASED ON STEP RESPONSE

A. DERIVATION OF TUNING FORMULA

For system (15), there are many PID tuning formulas [28]–[30] and LADRC tuning formulas [14], [31], but they need to know the parameters of FOPDT model. A LADRC tuning formula based on the two point approach of step response is proposed in this paper. The parameter value of LADRC can be obtained directly from the two points of step response. The formula is deduced as follows.

A parallel PID tuning formula with good control effect for large delay plant is proposed [32], which is called MO-PID.

$$\begin{cases} K_p = \frac{1}{4K} (1 + 3.26 \frac{T}{\tau}) \\ T_i = (\frac{1}{3.9\tau} + \frac{1}{T})^{-1} + \frac{1}{3} \tau \\ T_d = (\frac{3.26}{\tau} + \frac{1}{T})^{-1} \end{cases} \quad (22)$$

Consider the system (15) with a normalized delay

$$\bar{G}(s) = \frac{e^{-\bar{\tau}s}}{s + 1} \quad (23)$$

As $\bar{\tau}$ increases from 0.1 to 10 with an appropriate step, we can get the corresponding PID parameters by the formula (22). And then we can calculate the corresponding parameters $\bar{\omega}_c, \bar{\omega}_o, \bar{b}_0, \bar{\zeta}$ of LADRC by the method of [9]. The procedure to get the parameters of a second-order LADRC from the parameters of a PID goes as follows:

- 1) Choose α so that the following equation has a positive solution, take it as the observer bandwidth ω_o .

$$\omega_o^5 - \alpha K_d \omega_o^2 + 3\alpha K_p \omega_o - 6\alpha K_i = 0 \quad (24)$$

- 2) Compute the controller bandwidth ω_c and the damping ratio ζ as:

$$\omega_c = \sqrt{\frac{\alpha K_i}{\beta_3}}, \quad \zeta = \frac{\alpha K_p - \alpha K_i \beta_2 / \beta_3}{2\omega_c \beta_3} \quad (25)$$

- 3) Compute b_0 as

$$b_0 = \frac{\alpha}{\beta_2 + k_1 + \beta_1 k_2} \quad (26)$$

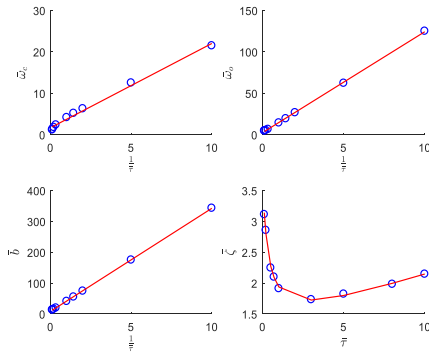


FIGURE 5. The relationship between $\bar{\tau}$ and $\bar{\omega}_c, \bar{\omega}_o, \bar{b}_0, \bar{\zeta}$.

The relationship between $\bar{\tau}$ and $\bar{\omega}_c, \bar{\omega}_o, \bar{b}_0, \bar{\zeta}$ is shown in Figure 5. It is shown that $\bar{\omega}_c, \bar{\omega}_o$ and \bar{b}_0 are all in approximate linear relationship with $1/\bar{\tau}$, which can be fitted by linear function. However, the relationship between $\bar{\tau}$ and $\bar{\zeta}$ cannot be fitted by straight line, so we fit it by Nike function. Thus for the system (23), the following LADRC tuning formula can be obtained

$$\begin{cases} \bar{\omega}_c = 2.0327 \frac{1}{\bar{\tau}} + 1.6910 \\ \bar{\omega}_o = 12.1663 \frac{1}{\bar{\tau}} + 2.5825 \\ \bar{b}_0 = 33.3936 \frac{1}{\bar{\tau}} + 8.4602 \\ \bar{\zeta} = 0.0852(\bar{\tau} + 0.3494) + \frac{0.8632}{\bar{\tau} + 0.3494} + 1.1820 \end{cases} \quad (27)$$

Further considering the tuning formula for system (15), the relationship between the tuning parameters $\omega_c, \omega_o, b_0, \zeta$ for system (15) and the tuning parameters $\bar{\omega}_c, \bar{\omega}_o, \bar{b}_0, \bar{\zeta}$ for system (23) is given as follows [31].

$$\omega_c = \frac{\bar{\omega}_c}{T}, \quad \omega_o = \frac{\bar{\omega}_o}{T}, \quad b = \bar{b} \frac{K}{T^2}, \quad \zeta = \bar{\zeta}, \quad \bar{\tau} = \frac{\tau}{T} \quad (28)$$

By substituting (18) and (27) into equation (28), the tuning formula for system (15) can be obtained

$$\begin{cases} \omega_c = \frac{2.0327}{2t_1 - t_2} + \frac{1.6910}{2(t_2 - t_1)} \\ \omega_o = \frac{12.1663}{2t_1 - t_2} + \frac{2.5825}{2(t_2 - t_1)} \\ b_0 = K \left(\frac{33.3936}{2(t_2 - t_1)(2t_1 - t_2)} + \frac{8.4602}{(2(t_2 - t_1))^2} \right) \\ \zeta = 0.0852 \left(\frac{2t_1 - t_2}{2(t_2 - t_1)} + 0.3494 \right) \\ + \frac{0.8632}{\frac{2t_1 - t_2}{2(t_2 - t_1)} + 0.3494} + 1.1820 \end{cases} \quad (29)$$

where t_1 and t_2 are the time when the step response curve reaches 39.3% and 63.2% of the steady value if the initial value is zero. K is the steady-state gain and can be obtained by Eq.(16). It can be seen from (29) that it is easy to tune the parameters for LADRC by using the tuning formula proposed in this paper. t_1, t_2 and K can be easily obtained by a step response experiment.

B. ROBUSTNESS MEASURE

In order to measure the robustness and performance of the system, robustness measure is a comprehensive index to analyze the robustness and disturbance rejection performance of the system. There are usually two forms of measurement index of robustness.

$$M_s = \|S\|_\infty = \max_\omega \left| \frac{1}{1 + L(j\omega)} \right| \quad (30)$$

$$M_p = \|T\|_\infty = \max_\omega \left| \frac{L(j\omega)}{1 + L(j\omega)} \right| \quad (31)$$

where $L(s) = P(s)K_c(s)$ is the open-loop transfer function of the system. M_s is a good measure of mid- and low frequency uncertainty. The larger M_s , the stronger the disturbance rejection ability is. M_p is a good measure of mid- and high frequency uncertainty, the noise of the system is usually high frequency. The smaller M_p , the stronger the noise suppression ability is and the stronger the robustness stability is. Therefore, for the full frequency uncertainty, it is not appropriate to measure only one of M_s or M_p . It may be more appropriate to consider a combination of M_s and M_p [33].

It is assumed that the plant has the following uncertainties at the same time.

$$P_\Delta = (I - \Delta_1)^{-1} P (I + \Delta_2), \quad \Delta_1, \Delta_2 \in H_\infty \quad (32)$$

Suppose a normalized left coprime factorization of P is $P = \tilde{M}^{-1} \tilde{N}$, we have

$$\begin{aligned} P_\Delta &= (\tilde{M} - \tilde{M} \Delta_1)^{-1} (\tilde{N} + \tilde{N} \Delta_2) \\ &= : (\tilde{M} + \Delta_M)^{-1} (\tilde{N} + \Delta_N) \end{aligned} \quad (33)$$

Compared with the uncertainty of coprime factors [34], the uncertainty of Eq.(33) contains additional structural information ($\Delta_M = -\tilde{M} \Delta_1, \Delta_N = \tilde{N} \Delta_2$), so the analysis of robustness can be simplified.

TABLE 2. Systems for testing.

Plant	Description of the plant	Plant	Description of the plant
$G_1: \frac{1}{Ts+1}e^{-s}, T=0.2,2,10$	standard systems FOPDT	$G_5: \frac{1}{(s+1)(\alpha s+1)(\alpha^2 s+1)(\alpha^3 s+1)}, \alpha=0.1,0.2,0.5$	fourth-order systems with different poles
$G_2: \frac{1}{(Ts+1)^2}e^{-s}, T=0.5,2,5$	second-order non-oscillatory systems	$G_6: \frac{e^{-3s}}{(s^2+10s+1)(s+1)^2}$	high-order systems with large delay time
$G_3: \frac{1}{(s+1)^n}, n=2,3,4,8$	high-order systems	$G_7: \frac{(0.5s+1)e^{-s}}{(s+1)^2(2s+1)}$	high-order systems with a negative zero
$G_4: \frac{1-\alpha s}{(s+1)^3}, \alpha=0.2,0.5,1,2$	high-order systems with a positive zero	$G_8: \frac{e^{-2.2s}}{(4s^2+2.8s+1)(s+1)^2}$	high-order systems with a group of conjugate complex poles

Suppose that the controller K_c is a controller which can make the controlled plant P stable, and then make the following definition.

$$M := \begin{bmatrix} (I + PK_c)^{-1} & (I + PK_c)^{-1}P \\ -K_c(I + PK_c)^{-1} & -K_c(I + PK_c)^{-1}P \end{bmatrix},$$

$$\Delta := \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{bmatrix} \quad (34)$$

Using the small μ theorem [35], the closed-loop system under the controller K_c is robust and stable for all uncertainties $P_\Delta(\|\Delta\|_\infty \leq \gamma)$, if and only if (35) holds

$$\varepsilon := \mu_\Delta(M) < 1/\gamma \quad (35)$$

Therefore, $\mu_\Delta(M)$ can be used as a measure of the robustness of the system. A robustness index measure of single loop system is proposed in [33].

$$\varepsilon = \sup_\omega (\|S\|_\infty + \|T\|_\infty) \quad (36)$$

where ε is the combination of M_s and M_p , which is a comprehensive index to measure the robust stability and disturbance rejection performance of the system. The larger ε is, the stronger disturbance rejection performance of the system is, and the weaker the robust stability is. The smaller ε is, the stronger the robust stability is and the weaker the disturbance rejection ability is. In general, the value of ε is between 3 and 5 is the best. If the system ε is 4, it means that the system can be stable as long as the uncertainty of input and output is within $1/4 \approx 25\%$.

V. SIMULATION

In this section, benchmark systems G_1, G_2, G_3, G_4, G_5 in [36] and high-order systems G_6, G_7, G_8 in [21] shown in Table 2 are used to illustrate the validity of the formula (29). In order to better evaluate the dynamic performance and robustness of the system, we use the settling time ($\pm 2\%$) and the overshoot under a unit step setpoint disturbance at $t=0s$ and a unit step input disturbance at appropriate time to analyze the dynamic

performance of the system, and use the robustness measure to measure the robustness of the system.

Figure 6 shows the response of G_1 under different controllers. Because our tuning formula is designed based on FOPDT system, good performance can be achieved for these three standard FOPDT systems. For the system G_1 with large T/τ , for both tracking and disturbance rejection responses, the overshoot is smaller and the settling time is shorter under the proposed tuning formula (29). To ensure the better disturbance rejection performance, the tuning formula has to sacrifice a little bit of robustness, which is shown by the slightly larger robustness measure in Table 3. The specific controller parameters and performance indices are shown in Table 3. For the smaller T/τ , that is, the delay-dominated system, the effect of the proposed formula is slightly worse than that of [14] and [21]. The main reason is that when using a straight line to fit the data of ω_c , there is errors for large delays. We can reduce ω_c , or increase b_0 to slow down the response speed and reduce the overshoot.

In general, the design of the controller is a process of seeking balance between noise suppression and disturbance suppression. In order to measure the noise suppression performance of the proposed tuning formula, the Bode plots of different controllers for G_1 systems are shown in Figure 7. The low frequency of the Bode plots reflects the integral performance, which indicates the requirements of the controller for the steady state error. The intermediate frequency should make the system have enough bandwidth to ensure the amplitude margin and the phase margin of the system. For the high frequency of Bode plots, we hope that the high frequency attenuation is as fast as possible, and the high frequency gain is as small as possible, so that the noise suppression ability is strong enough. Figure 7 shows that the proposed LADRC and LADRC in [14] have approximate attenuation speed and noise suppression capability.

G_2, G_3, G_4, G_5 are non-oscillatory and can be regarded as FOPDT models approximatively, so it is expected that the proposed tuning formula (29) can achieve good performance,

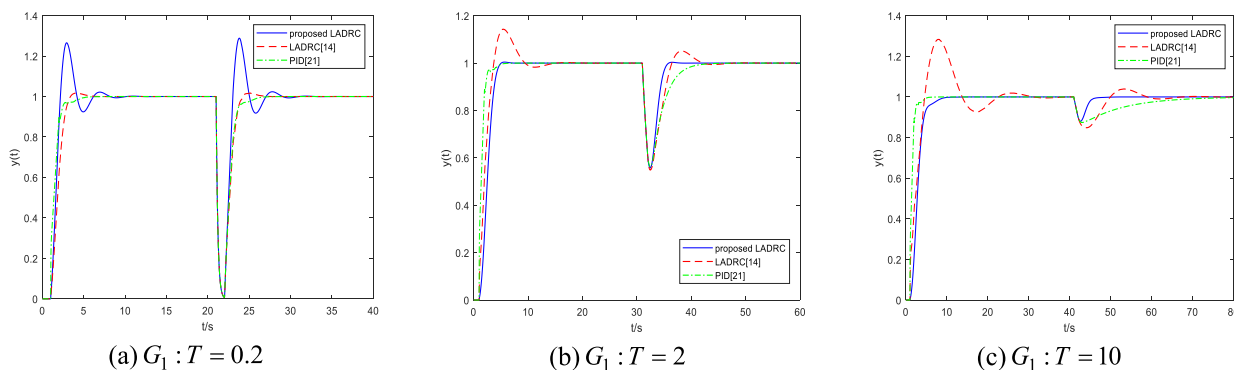


FIGURE 6. Responses of G_1 under different controllers.

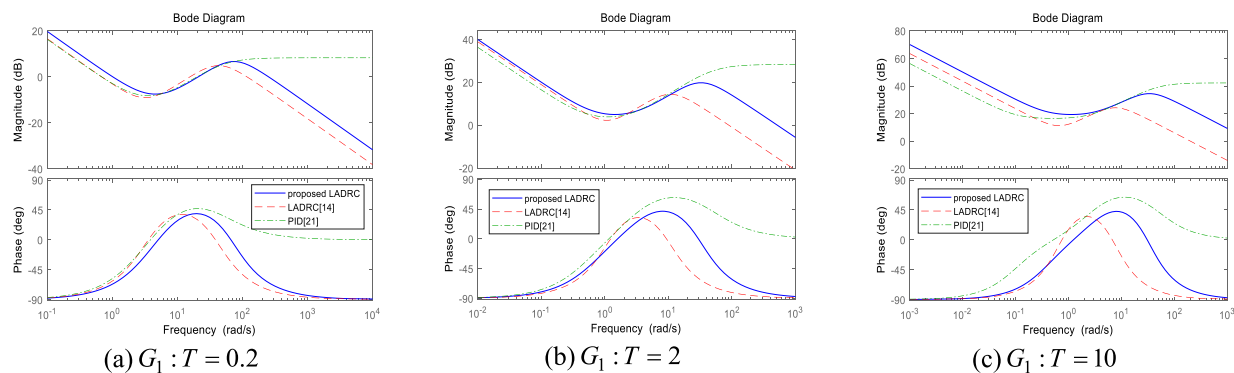


FIGURE 7. Bode plots of different controllers for G_1 .

TABLE 3. Parameters of the controllers and performance indices of the system G_1 .

G_1	Tuning method	step test		Controller parameters				Tracking performance		Disturbance rejection performance		Robustness Measure
		t_1	t_2	b_0	ω_c	ω_o	ξ	T_s/s	$\sigma\%$	T_s/s	$\sigma\%$	
0.2	Proposed	1.11	1.20	443.00	11.39	26.28	1.84	7.18	26.6	8.15	99.30	3.75
	LADRC[14]			179.91	5.33	19.88		3.17	1.75	3.97	99.34	2.33
	PID[21]							3.98	0	5.23	99.30	6.58
2	Proposed	1.99	2.98	19.02	2.89	13.47	2.27	4.52	0.42	5.21	44.26	3.05
	LADRC[14]			4.40	3.99	3.04		8.71	14.3	10.6	45.13	2.34
	PID[21]							3.98	0.01	9.46	43.76	1.64
10	Proposed	5.99	11.05	3.63	2.35	13.34	3.18	7.42	0	5.01	12.00	4.01
	LADRC[14]			0.64	3.87	1.55		21.5	28.5	16.4	15.17	2.68
	PID[21]							3.98	0	22.36	12.59	1.11

which is verified by Figure 8-11. The parameters of the controllers and the performance indices for these systems are shown in Table 5-8 in appendix. For most of the systems in

G_2, G_3, G_4, G_5 , the values of τ/T are between 0.1 and 1.7, which belong to the balanced and time-dominated systems. It can be predicted that our tuning formulas (29) can achieve

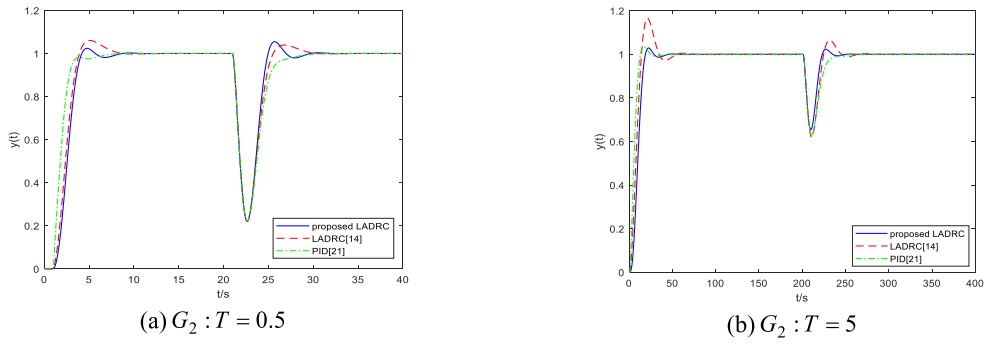


FIGURE 8. Responses of G_2 under different controllers.

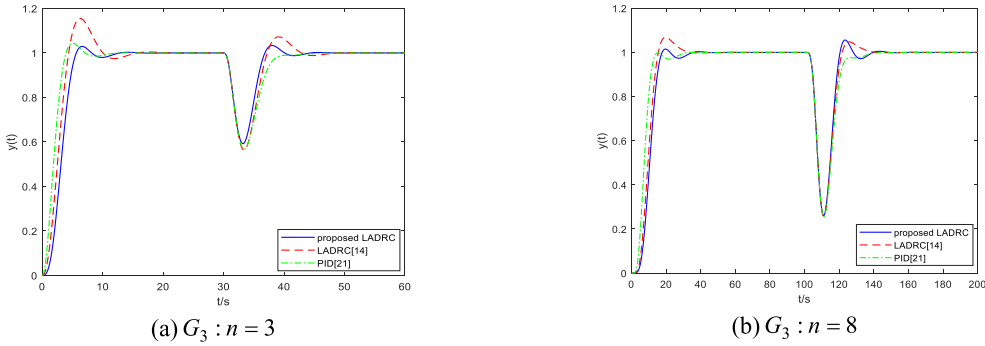


FIGURE 9. Responses of G_3 under different controllers.

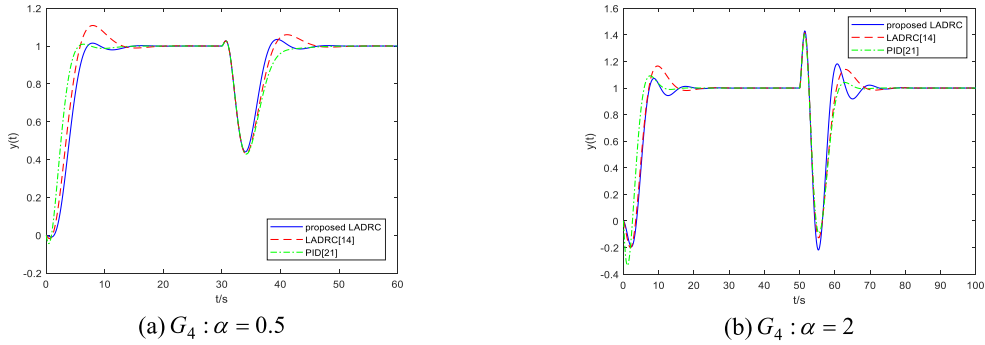


FIGURE 10. Responses of G_4 under different controllers.

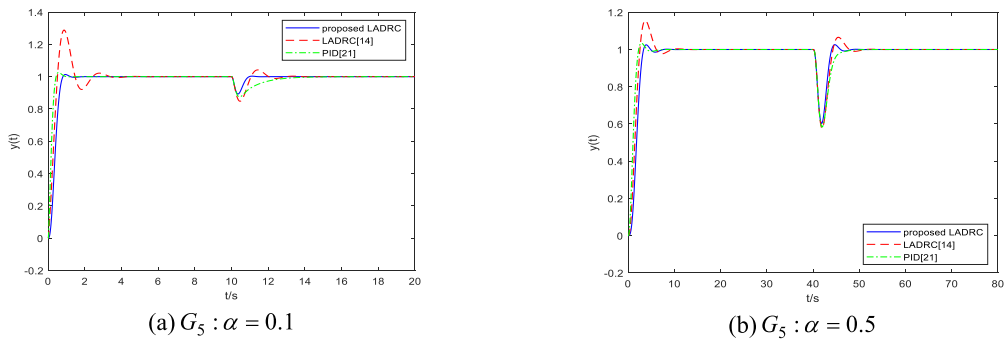


FIGURE 11. Responses of G_5 under different controllers.

good effect, which is also confirmed by the response curve in Figure 8-11 and performance indices in Table 5-8. It should be noted that for the case of $\alpha = 2$ in G_4 , since the τ/T of the

approximate FOPDT model is approximately equal to 2.06, which belongs to the case of larger τ/T , our formula slightly oscillates, but the overall effect is fine.

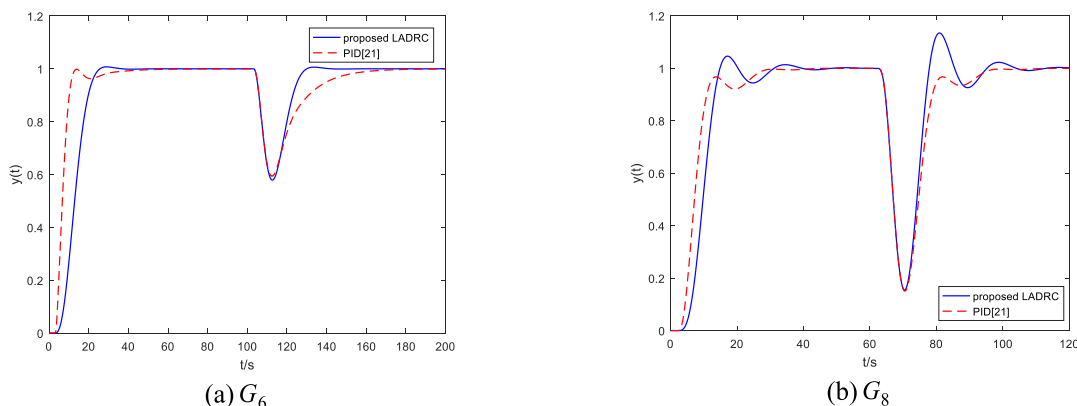
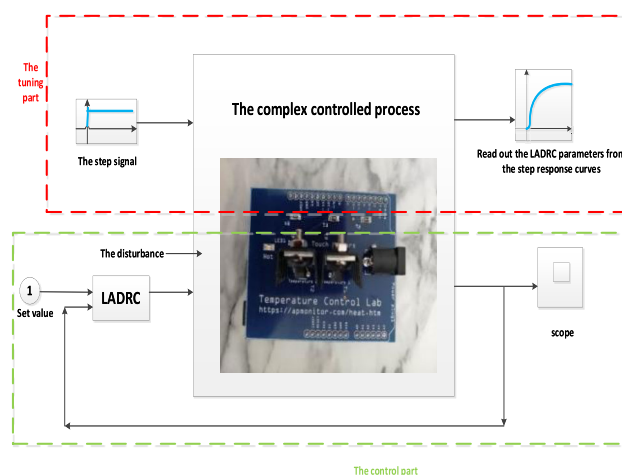


FIGURE 12. Responses of G_6 under different controllers.



(a)



(b)

FIGURE 13. The temperature control lab.

TABLE 4. Comparison of performance indices between LADRC and AMIGO-PID.

	Tracking performance		Disturbance rejection performance	
	T_s / s	$\sigma\%$	T_s / s	$\sigma\%$
LADRC	156	1.0	188	28.3
AMIGO-PID	176	27.9	151	23.5

Further, some complex higher-order systems are studied to verify the applicability of the proposed formula. The responses are shown in Figure 12. The relevant controller parameters and performance indices are shown in Table 9 in appendix. It should be emphasized that, for the slightly oscillatory under-damped system G_8 , the LADRC controller

tuned by the proposed formula can still stabilize it, which shows that the proposed method is highly adaptable.

VI. EXPERIMENTAL VERIFICATION

TCLab(temperature control lab) is a temperature control experimental device with a microcontroller as shown in Figure 13(a) [37]. As a hardware benchmark test device, TCLab is widely used in the teaching of process control [38], [39]. The TCLab device is printed circuit board (PCB) shield, in which there are transistors and thermistors as heaters and temperature sensors, and an Arduino microcontroller is connected for temperature regulation. The control algorithm of the microcontroller can be written by user. After applying a step response to the device, it is found that the temperature response is actually a second-order system with a time constant of about 2.9 minutes and a static gain of $0.9 \frac{^{\circ}C}{\%heater}$. The difficulty of the experiment is that it is easy to be affected by environmental factors, such

TABLE 5. Parameters of the controllers and performance indices of the system G_2 .

G_2	Tuning method	step test		Controller parameters				Tracking performance		Disturbance rejection performance		Robustness Measure
		t_1	t_2	b_0	ω_c	ω_o	ξ	T_s / s	$\sigma\%$	T_s / s	$\sigma\%$	
T												
0.5	Proposed	1.68	2.08	45.83	3.70	12.73	1.79	5.07	2.40	8.13	77.91	2.96
	LADRC[14]			16.76	3.40	5.91		7.14	6.05	8.39	78.12	2.45
	PID[21]							5.69	0	7.62	77.71	2.93
2	Proposed	3.72	5.30	5.79	1.49	6.50	2.11	9.00	1.96	14.22	45.45	2.68
	LADRC[14]			1.67	1.93	1.85		16.0	14.3	19.7	47.5	2.38
	PID[21]							9.81	2.82	15.07	47.22	1.60
5	Proposed	7.83	11.74	1.23	0.73	3.43	2.27	25.26	2.89	29.30	34.62	2.51
	LADRC[14]			0.31	1.05	0.81		47.0	16.5	41.1	38.02	2.34
	PID[21]							21.05	3.46	33	37.03	1.39

TABLE 6. Parameters of the controllers and performance indices of the system G_3 .

G_3	Tuning method	step test		Controller parameters				Tracking performance		Disturbance rejection performance		Robustness Measure
		t_1	t_2	b_0	ω_c	ω_o	ξ	T_s / s	$\sigma\%$	T_s / s	$\sigma\%$	
n												
2	Proposed	1.36	2.15	40.47	4.64	22.98	2.46	4.34	4.06	4.75	25.54	2.41
	LADRC[14]			9.78	7.10	4.63		8.25	19.4	6.91	30.92	2.36
	PID[21]							3.79	4.62	5.69	29.41	1.25
3	Proposed	2.26	3.26	15.37	2.46	10.95	2.15	10.14	2.93	9.07	40.77	2.59
	LADRC[14]			4.32	3.31	2.98		13.1	15.5	11.8	43.59	2.39
	PID[21]							6.29	4.37	8.75	43.31	1.50
4	Proposed	3.18	4.35	8.65	1.73	7.16	2.00	14.43	1.97	12.77	51.31	2.69
	LADRC[14]			2.64	2.13	2.32		13.2	13.2	16.2	52.57	2.43
	PID[21]							8.28	3.63	11.82	52.77	1.78
8	Proposed	6.95	8.64	2.62	0.89	3.08	1.80	30.26	1.51	35.33	73.93	2.87
	LADRC[14]			0.97	0.84	1.42		27.7	6.85	32.5	74.18	2.46
	PID[21]							25.76	0	33.09	74.35	2.76

as ambient temperature, power supply output and airflow (nearby computer fan), etc. The slight disturbance of these factors will make the plant dynamic characteristics have a big fluctuation. In the face of such a “model uncertainty” system, how to quickly get the initial value of the controller will be a very difficult problem.

Figure 13(b) shows the whole idea of using step response method to obtain the parameters of LADRC. The step response data of the system is shown in Figure 14, where the

initial value of the temperature is 24.78°C and a step input change from 0 to 50 (%) at $t=10\text{s}$.

The steady gain can be obtained by the formula (16).

$$K = \frac{y(\infty) - y(0)}{\Delta u} = \frac{66.81 - 24.78}{50} = 0.84 \quad (37)$$

And the corresponding time of 39.3% and 63.2% of $y(\infty) - y(0)$ in the step response data is

$$t_1 = 100; \quad t_2 = 172.5 \quad (38)$$

TABLE 7. Parameters of the controllers and performance indices of the system G_4 .

G_4	Tuning method	step test		Controller parameters				Tracking performance		Disturbance rejection performance		Robustness Measure
		t_1	t_2	b_0	ω_c	ω_o	ξ	T_s/s	$\sigma\%$	T_s/s	$\sigma\%$	
α	Proposed	2.46	3.45	13.63	2.24	9.58	2.07	7.61	2.19	9.80	46.77	2.67
	LADRC[14]			3.95	2.86	2.84		10.4	13.9	12.8	48.49	2.38
	PID[21]							6.46	3.38	9.61	48.40	1.64
0.2	Proposed	2.73	3.71	11.94	2.02	8.27	1.98	11.49	1.52	10.58	55.93	2.85
	LADRC[14]			3.70	2.33	2.75		11.8	10.9	14.3	57.07	2.39
	PID[21]							5.10	0	11.27	57.11	1.85
0.5	Proposed	3.13	4.08	10.41	1.82	6.94	1.89	13.15	1.23	15.08	73.24	3.28
	LADRC[14]			4.35	1.76	3.01		14.5	6.85	17.0	72.96	2.38
	PID[21]							7.90	0	13.6	72.68	2.17
1	Proposed	3.75	4.63	9.34	1.67	5.71	1.79	14.95	7.22	20.4	121.87	5.02
	LADRC[14]			4.93	1.61	3.22		14.5	16.6	17.7	112.7	3.57
	PID[21]							10.50	9.20	14.78	109.27	5.40

TABLE 8. Parameters of the controllers and performance indices of the system G_5 .

G_5	Tuning method	step test		Controller parameters				Tracking performance		Disturbance rejection performance		Robustness Measure
		t_1	t_2	b_0	ω_c	ω_o	ξ	T_s/s	$\sigma\%$	T_s/s	$\sigma\%$	
α	Proposed	0.62	1.12	286.74	18.63	103.97	3.06	0.74	1.33	0.77	10.74	2.45
	LADRC[14]			60.16	36.53	14.78		2.91	28.85	1.78	15.21	2.68
	PID[21]							0.67	2.62	2.31	12.54	1.13
0.1	Proposed	0.76	1.27	139.09	9.79	51.20	2.68	1.88	2.36	1.58	20.02	2.46
	LADRC[14]			30.63	17.28	8.83		3.98	22.65	3.24	24.59	2.46
	PID[21]							1.37	2.94	3.34	21.85	1.22
0.2	Proposed	1.36	1.99	41.63	4.13	18.72	2.19	4.24	2.50	5.14	39.75	2.63
	LADRC[14]			11.05	5.71	4.79		7.86	15.55	7.10	42.30	2.37
	PID[21]							3.47	3.43	5.74	41.64	1.51

Thus the parameters for LADRC is obtained as follows by submitting (37) and (38) into (29).

$$\omega_c = 0.0856; \omega_o = 0.4602; b_0 = 0.0074; \zeta = 2.8292; \tag{39}$$

In order to better test the control effect of our proposed formula, a practical AMIGO-PID is used to compare with our

tuning formula.

$$K_{PID}(s) = K_p(1 + \frac{1}{T_i s} + \frac{T_d s}{N s + 1}) \tag{40}$$

where

$$K_p = 6.6513; T_i = 64.4521; T_d = 6.9414; N = 20$$

TABLE 9. Parameters of the controllers and performance indices of the system G_6, G_7, G_8 .

	Tuning method	step test		Controller parameters				Tracking performance		Disturbance rejection performance		Robustness Measure
		t_1	t_2	b_0	ω_c	ω_o	ξ	T_s / s	$\sigma\%$	T_s / s	$\sigma\%$	
G6	Proposed	10.13	15.11	0.74	0.56	2.62	2.25	23.36	0.74	27.24	42.13	2.85
	PID[21]							28.21	0	51.61	40.64	2.44
G7	Proposed	3.41	4.75	7.20	1.61	6.84	2.05	8.27	1.88	13.20	50.40	2.76
	PID[21]							13.03	2.39	19.25	45.95	2.53
G8	Proposed	6.44	7.77	3.65	1.03	3.35	1.76	29.16	4.61	40.04	84.56	3.32
	PID[21]							26.13	0	33.85	84.99	2.49

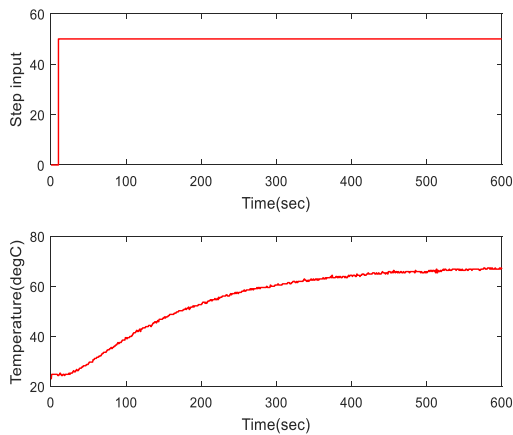


FIGURE 14. Step response of the open-loop system.

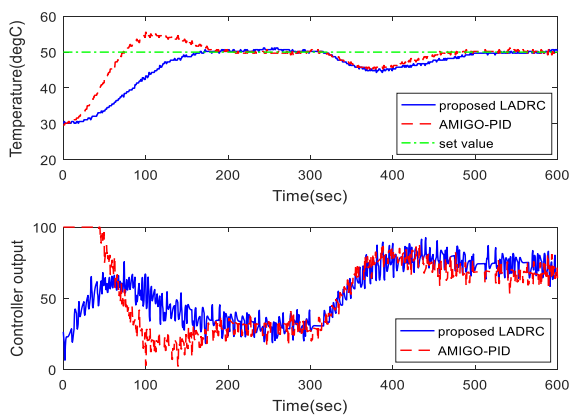


FIGURE 15. The response of the closed-loop system under LADRC (39).

Figure 15 shows the response curves of the plant output y and the controller output u under the parameters of LADRC by Eq.(39) and the parameters of AMIGO-PID by Eq.(40), where a step setpoint is changed from $30^\circ C$ to $50^\circ C$ at $t=0s$, and a step input disturbance with amplitude 40% is inserted at $t=300s$. From Table 4, it can be read that the tracking

overshoot and the tracking settling time ($\pm 5\%$) of the system are 1% and $t_s = 156s$ respectively, the overshoot and the settling time ($\pm 5\%$) under the input disturbance are 28.3% and $t_s = 188s$ respectively. Compared with AMIGO-PID, the tuning formula (29) can achieve better tracking and similar disturbance rejection performance for this system. It can be seen from the output of the controller, the oscillation of LADRC is smaller and smoother.

VII. CONCLUSION

In this paper, a simple and practical method is proposed to tune the parameters for second-order LADRC via step test. The parameters of LADRC can be determined by two points on the step response curves quickly. Simulation results show that the tuning formula is applicable to a wide range of systems, and can achieve good performance for FOPDT systems and high-order non-oscillatory systems. The practical temperature control experiment also illustrates the effectiveness of the tuning formula. The method can be quickly grasped by control engineers and provides initial parameters for second-order LADRC. The parameters can be re-tuned online to achieve better performance if necessary. Since the tuning formula Eq.(29) is not suitable for the oscillation system G_8 in Table 2, the tuning method of LADRC for the system with oscillation will be studied in the future work. Especially, the control of heavy oscillation system is always a difficult problem in engineering control, it is necessary to try to solve it.

APPENDIX

See Tables 5–9.

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WENQING CUI received the B.S. degree from the School of Control and Computer Engineering, North China Electric Power University, Beijing, China, in 2018, where he is currently pursuing the M.S. degree. His research interests include active disturbance rejection control and its application in electric power systems.



WEN TAN (Member, IEEE) received the B.Sc. degree in applied mathematics and the M.Sc. degree in systems science from Xiamen University, Xiamen, China, in 1990 and 1993, respectively, and the Ph.D. degree in automation from the South China University of Technology, Guangzhou, China, in 1996. He joined the Faculty of the Power Engineering Department, North China Electric Power University, China, in 1996. From January 2000 to December 2001, he was a Postdoctoral Fellow with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB, Canada. He is currently a Professor with the School of Control and Computer Engineering, North China Electric Power University. His research interests include modeling, analysis, and control of complex industrial processes.



DONGHAI LI received the Ph.D. degree from Tsinghua University, Beijing, China, in 1994. He is currently an Associate Professor with the Department of Energy and Power Engineering, Tsinghua University. He has published more than 160 articles in control science, chaired or participated in more than 30 research projects. His research interests include PID control, active disturbance rejection control, hydro generator control, and gasifier control.



YUTONG WANG received the B.S. degree from the School of Control and Computer Engineering, North China Electric Power University, Beijing, China, in 2019, where she is currently pursuing the M.S. degree. Her research interests include active disturbance rejection control and its application in electric power systems.

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