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Adaptive Synchronization of Reaction Diffusion Neural Networks With Infinite Distributed Delays and Stochastic Disturbance

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ABSTRACT In this paper, the diffusion effect, distributed delays and stochastic disturbance are involved in constructing the model of neural networks. Then, the global exponential synchronization problem is investigated for a class of reaction diffusion neural networks (RDNNs) with infinite distributed delays and stochastic disturbance. By employing the stochastic analysis method and Lyapunov functional theory, an adaptive controller is designed to guarantee the exponential synchronization of the drive and response RDNNs. The derived synchronization conditions are simple and the theoretical results can be directly extended to other RDNNs with or without distributed delays and stochastic disturbance. Finally, one example is provided to verify the effectiveness of the theoretical results and adaptive control approach.

INDEX TERMS Adaptive synchronization, neural networks, reaction diffusion, infinite distributed delays, stochastic disturbance.

I. INTRODUCTION

During the past several decades, various models of neural networks (NNs) have been put forward and applied in different fields, such as the optimized calculation, associative memory and image processing [1]–[4]. These successful applications are heavily dependent on the dynamical behaviors of NNs, which the main cases are the stability and synchronization. In addition, as a typical cluster discharge activity of neurons, the synchronous behavior is closely related to the neural information processing function [5]–[8]. Thus, it is critical and necessary to study the dynamics, especially the synchronization of NNs.

The diffusion effects with spatial dynamic characteristics should be considered in circuit design of NNs [9]. The main reason is that the magnetic field environment of the electronic circuit simulating the NNs is often non-uniform, which leads to the phenomenon of spatial diffusion of its electrons [10]–[12]. Then, a new kind of reaction diffusion NNs (RDNNs) whose neuron states vary with time and space simultaneously, is proposed [13], [14]. Correspondingly, to sufficiently reflect this time-space feature, the mathematical model of RDNNs can be described by a partial differential equation. Therefore, dynamical analysis of RDNNs is necessary and difficult, deserving to be further investigated. Recently, great efforts have been devoted to analyze dynamical behaviors of various RDNNs, especially the synchronization problem via different control approaches [15]–[19].

Time delays are inevitable in light of the finite switching limits of neuron amplifiers and the signal transmission [20], [21]. Generally, they are unavoidably existed in circuits of NNs and may result in undesirable dynamics, such as instability and chaotic behavior [22]–[24]. Furthermore, owing to the existence of massive parallel pathways with extensive axon sizes and lengths, the delays are not sufficiently described in the discrete form but in the distributed form [25]. In other words, the distributed delays are more

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general and should be considered in models of RDNNs. In [26], [27], the periodic solution and its stability were discussed for RDNNs with continuous distributed delays. In [28], [29], the stability was analyzed for RDNNs with S-type distributed delays. In [30], [31], the stabilization problem was addressed for memristor-based RDNNs with finite distributed delays via state feedback and intermittent control methods. As is well known, the current behaviors of a neuron relate to its entire past states, which gives rise to the existence of the infinite distributed delays [32]. Thus, it is more reasonable to consider the infinite distributed delays in model of RDNNs.

On the other hand, in practical applications, the stochastic disturbance is always encountered because of the complicated environmental noise and interference [33]–[36]. Regarding the RDNNs with stochastic disturbance, lots of research works have been reported [37]–[41]. Moreover, by taking both the distributed delays and stochastic disturbance into consideration, authors in [42]–[44] studied the stability and synchronization of RDNNs via the stochastic analysis method. It is noted that the distributed delays in above papers [42]–[44] are restricted to be finite and bounded. Up to now, there is little work on the dynamics of RDNNs with infinite distributed delays and stochastic disturbance, which deserves further investigation and motivates the synchronization study of this paper.

Considering the RDNNs with infinite distributed delays and stochastic disturbance, the model is represented as a class of partial differential stochastic systems with mixed delays. Thus, it brings great difficulties to realize the synchronization of this kind of systems. In this paper, we aim to investigate the drive-response synchronization for RDNNs with infinite distributed delays and stochastic disturbance. The contributions are listed as follows.

1) This paper addresses the synchronization problem for a class of stochastic RDNNs with infinite distributed delays. Since the model studied involves the diffusion effect, stochastic disturbance and infinite distributed delays, it is generalized compared to the stochastic RDNNs with finite distributed delays in [42]–[44].

2) By employing the theory of partial differential system and stochastic analysis method, the simple and easily verified synchronization criteria are derived via a designed adaptive controller. The comparisons over existing work are provided in Remarks 4-6, which shows that the synchronization results effectually complement or improve the existing results of RDNNs with or without stochastic disturbance. Moreover, the superiority of the adaptive control approach compared with the linear one is also presented in Corollary 1.

The rest of this paper is organized as follows. The model and problem descriptions are given in Section II. The main synchronization results of RDNNs via the adaptive controller are given in Section III. Sections IV and V show the numerical simulations and conclusions, respectively.

II. PRELIMINARIES

In this paper, $\mathcal{R}^n$, $\mathcal{R}$ and $\mathcal{R}_+$ denote the set of n-dimensional Euclidean space, real numbers and nonnegative numbers, respectively. For a constant n, let $n^\sharp = \{1, 2, \ldots, n\}$. $(\Lambda, \mathcal{F}, \mathcal{P})$ is a completed probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual condition. $\mathcal{E}$ is the mathematical expectation operator concerning the probability $\mathcal{P}$. $\Pi = \{(u_1, u_2, \ldots, u_I)^T \,||u_i| < r_i, i \in I^\sharp\}$ is bounded compact set with smooth boundary $\partial\Pi$ and measure $\mathrm{mes}\Pi > 0$.

Consider the following RDNNs model with infinite distributed delays

$$\begin{aligned} \mathrm{d}x_s(u,t) = \Bigg[& \sum_{i=1}^{I} \frac{\partial}{\partial u_i}\left(\alpha_{si}\frac{\partial x_s(u,t)}{\partial u_i}\right) - d_s x_s(u,t) \\ & + \sum_{k=1}^{n} a_{sk} f_{1k}(x_k(u,t)) \\ & + \sum_{k=1}^{n} b_{sk} f_{2k}(x_k(u,t-\lambda_k(t))) \\ & + \sum_{k=1}^{n} c_{sk}\int_{-\infty}^{t} K_{sk}(t-\omega) f_{3k}(x_k(u,\omega))\mathrm{d}\omega \Bigg]\mathrm{d}t \end{aligned} \tag{1}$$

where $s, k \in n^\sharp$, $i \in I^\sharp = \{1, 2, \ldots, I\}$. $x_s(u,t)$ is the state at space u and time t, and $u = (u_1, u_2, \ldots, u_I)^T \in \Pi \subset \mathcal{R}^I$. $\alpha_{si} \geq 0$ is the transmission diffusion parameter. $d_s > 0$ is the self feedback connection weight, the parameters a_{sk}, b_{sk} and c_{sk} are the connection weight coefficients, where b_{sk} and c_{sk} correspond to the delayed and distributed delayed ones. Discrete delay $\lambda_k(t)$ is bounded and the delay kernel $K_{sk} \in \mathcal{R}_+$ is a continuous function. The activations $f_{jk} \in \mathcal{R}(j = 1, 2, 3)$ are continuous functions.

Remark 1: Compared with the stochastic RDNNs model with finite distributed delays in [42]–[44], the distributed delays in our model are infinite, which implies wider range of applications in practice.

The following assumptions are given for the activation functions and delays in system (1).

H1: For each $k \in n^\sharp$, there exist positive constants F_{1k}, F_{2k}, F_{3k} such that

$$\begin{aligned} |f_{1k}(y) - f_{1k}(x)| &\leq F_{1k}|y-x|, \\ |f_{2k}(y) - f_{2k}(x)| &\leq F_{2k}|y-x|, \\ |f_{3k}(y) - f_{3k}(x)| &\leq F_{3k}|y-x|, \end{aligned} \tag{2}$$

for all $x, y \in \mathcal{R}$.

H2: For each $k \in n^\sharp$, there exist positive constants λ_1 and λ_2 such that

$$0 \leq \lambda_k(t) \leq \lambda_1, \dot{\lambda}_k(t) \leq \lambda_2 < 1. \tag{3}$$

H3: For each $s, k \in n^\sharp$, there exists positive constant ϑ_{sk} such that

$$\int_0^{+\infty} K_{sk}(\omega)\mathrm{d}\omega = 1, \quad \int_0^{+\infty} e^{\vartheta_{sk}\omega}K_{sk}(\omega)\mathrm{d}\omega < \infty. \tag{4}$$

Consider system (1) as the drive system, and involve the stochastic disturbance, then the corresponding response system is

$$\begin{aligned} \mathrm{d}y_s(u,t) = \Bigg[&\sum_{i=1}^{I} \frac{\partial}{\partial u_i}\left(\alpha_{si}\frac{\partial y_s(u,t)}{\partial u_i}\right) - d_s y_s(u,t) \\ &+ \sum_{k=1}^{n} a_{sk} f_{1k}(y_k(u,t)) + \Delta_s(u,t) \\ &+ \sum_{k=1}^{n} b_{sk} f_{2k}(y_k(u,t-\lambda_k(t))) \\ &+ \sum_{k=1}^{n} c_{sk}\int_{-\infty}^{t} K_{sk}(t-\omega) f_{3k}(y_k(u,\omega))\mathrm{d}\omega \Bigg]\mathrm{d}t \\ &+ \sum_{k=1}^{n} \varrho_{sk}(v_k(u,t), v_k(u,t-\lambda_k(t)))\mathrm{d}B_k(t) \end{aligned} \quad (5)$$

where $y_s(u,t)$ is the state variable, ϱ_{sk} is the noise intensity function, $v_k(u,\cdot) = y_k(u,\cdot) - x_k(u,\cdot)$, and the stochastic disturbance $B(t) = (B_1(t), B_2(t), \ldots, B_n(t))^T \in \mathcal{R}^n$ is a Brownian motion defined on the complete probability space $(\Lambda, \digamma, \mathcal{P})$, with $\mathcal{E}\{\mathrm{d}B(t)\} = 0$ and $\mathcal{E}\{\mathrm{d}B^2(t)\} = \mathrm{d}t$. $\Delta_s(u,t)$ is the external control input designed as

$$\Delta_s(u,t) = \beta_s(u,t)v_s(u,t) \quad (6)$$

with

$$\frac{\partial\beta_s(u,t)}{\partial t} = -\gamma_s \mathbf{e}^{\pi t}|v_s(u,t)|^{\mathbb{p}} \quad (7)$$

where $\mathbf{e}$ is the base of the natural logarithm, π, γ_s, $\mathbb{p} \geq 2$ are positive constants for $s \in n^\sharp$.

Then the following assumption is presented for the noise intensity functions.

H4: For each $s, k \in n^\sharp$, there exist $\tau_{sk} > 0$, $\eta_{sk} > 0$ such that

$$|\varrho_{sk}(\hat{e}_1, \check{e}_1) - \varrho_{sk}(\hat{e}_2, \check{e}_2)|^2 \leq \tau_{sk}|\hat{e}_1 - \hat{e}_2|^2 + \eta_{sk}|\check{e}_1 - \check{e}_2|^2 \quad (8)$$

for all $\hat{e}_1, \check{e}_1, \hat{e}_2, \check{e}_2 \in \mathcal{R}$.

Remark 2: The noise intensity functions $\varrho_{sk}(s,k \in n^\sharp)$ are of multiplicative case. The stochastic Brownian noise with these functions can be regarded as a result from the occurrence of the internal error of NNs circuits and random fluctuation. Therefore, the noise intensity functions $\varrho_{sk}(s,k \in n^\sharp)$ also rely on the state variables $x_k(u,t), y_k(u,t), x_k(u,t-\lambda_k(t)), y_k(u,t-\lambda_k(t))$ of systems (1) and (5).

Remark 3: The control gains of the adaptive controller can be adjusted to be optimized, which shows the superiority compared with the linear state feedback control whose feedback control strength is fixed.

Then the following error system is derived from systems (1) and (5).

$$\begin{aligned} \mathrm{d}v_s(u,t) = \Bigg[&\sum_{i=1}^{I} \frac{\partial}{\partial u_i}\left(\alpha_{si}\frac{\partial v_s(u,t)}{\partial u_i}\right) - d_s v_s(u,t) \\ &+ \sum_{k=1}^{n} a_{sk} h_{1k}(v_k(u,t)) + \Delta_s(u,t) \\ &+ \sum_{k=1}^{n} b_{sk} h_{2k}(v_k(u,t-\lambda_k(t))) \\ &+ \sum_{k=1}^{n} c_{sk}\int_{-\infty}^{t} K_{sk}(t-\omega) h_{3k}(v_k(u,\omega))\mathrm{d}\omega \Bigg]\mathrm{d}t \\ &+ \sum_{k=1}^{n} \varrho_{sk}(v_k(u,t), v_k(u,t-\lambda_k(t)))\mathrm{d}B_k(t) \end{aligned} \quad (9)$$

where $h_{jk}(v_k(u,\cdot)) = f_{jk}(y_k(u,\cdot)) - f_{jk}(x_k(u,\cdot)), j = 1,2,3$.

The boundary conditions of systems (1) and (5) are $x_s(u,t) = 0$ and $y_s(u,t) = 0$ for $(u,t) \in \partial\Pi \times (-\infty, +\infty)$. The initial conditions of systems (1) and (5) are $x_s(u,t) = \varphi_s(u,\omega)$ and $y_s(u,t) = \psi_s(u,\omega)$ for $(u,\omega) \in \Pi \times (-\infty, 0]$. For $v(u,t) = (v_1(u,t), v_2(u,t), \ldots, v_n(u,t))^T$, define

$$\|v(u,t)\| = (\int_{\Pi}\sum_{s=1}^{n}|v_s(u,t)|^{\mathbb{p}}\mathrm{d}u)^{1/\mathbb{p}}. \quad (10)$$

$\mathcal{C}(\Pi \times (-\infty, 0], \mathcal{R}^n)$ denotes the Banach space of continuous functions, and for any $\phi(u,\omega) \in \mathcal{C}$, define the norm

$$\|\phi(u,\omega)\| = (\int_{\Pi}\sup_{-\infty<\omega\leq 0}\sum_{s=1}^{n}|\phi_s(u,\omega)|^{\mathbb{p}}\mathrm{d}u)^{1/\mathbb{p}} \quad (11)$$

with $\mathbb{p} \geq 2$, $\phi(u,\omega) = (\phi_1(u,\omega), \phi_2(u,\omega), \ldots, \phi_n(u,\omega))^T$ and for $s \in n^\sharp$, $\phi_s(u,\omega) = \psi_s(u,\omega) - \varphi_s(u,\omega)$.

Based on (10) and (11), the following basic definition and useful lemma are given.

Definition 1: The drive and response systems (1) and (5) are said to be $\mathbb{p}$th moment globally exponentially synchronized, if there exist positive constants $\mu > 0$ and $\delta \geq 1$ such that for any $\psi, \varphi \in \mathcal{C}$, $t \in \mathcal{R}_+$

$$\mathcal{E}\|y(u,t) - x(u,t)\|^{\mathbb{p}} \leq \delta(\mathcal{E}\|\psi - \varphi\|^{\mathbb{p}})\mathbf{e}^{-\mu t}. \quad (12)$$

Lemma 1 [10]: Give $\Pi = \{u = (u_1, u_2, \ldots, u_I)^T \,||u_i| < r_i, i \in I^\sharp\}$ with smooth boundary $\partial\Pi$, constant $\mathbb{p} \geq 2$ and function $h(u) \in \mathcal{C}^1(\Pi)$ with $h(u)|_{\partial\Pi} = 0$, then for $i \in I^\sharp$

$$\int_{\Pi}|h(u)|^{\mathbb{p}}\mathrm{d}u \leq \frac{\mathbb{p}^2 r_i^2}{4}\int_{\Pi}|h(u)|^{\mathbb{p}-2}|\frac{\partial h}{\partial u_i}|^2\mathrm{d}u. \quad (13)$$

III. MAIN RESULTS

Theorem 1: Suppose that assumptions H1-H4 hold, then systems (1) and (5) are globally exponentially synchronized via the adaptive controller (6).

Proof: Construct the Lyapunov-Krasovskii functional

$$\begin{aligned}
&V(v(u,t),t)\\
&=\int_{\Pi}\sum_{s=1}^{n}\varepsilon_s\Big[g_s(t)+\frac{\mathbb{p}}{2\gamma_s}(\beta_s(u,t)+\tilde{\beta}_s)^2\\
&\quad+\sum_{k=1}^{n}\Big(\frac{|b_{sk}|F_{2k}+(\mathbb{p}-1)\eta_{sk}}{1-\lambda_2}\int_{t-\lambda_k(t)}^{t}\mathbf{e}^{\pi\lambda_1}g_k(\omega)\mathrm{d}\omega\\
&\quad+|c_{sk}|F_{3k}\int_{-\infty}^{0}\int_{t+\theta}^{t}\mathbf{e}^{-\pi\theta}K_{sk}(-\theta)g_k(\omega)\mathrm{d}\omega\mathrm{d}\theta\Big)\Big]\mathrm{d}u
\end{aligned}\tag{14}$$

where $\varepsilon_s>0$ is a constant, $g_s(t)=\mathbf{e}^{\pi t}|v_s(u,t)|^{\mathbb{p}}$, and $\tilde{\beta}_s$ is a positive constant determined later.

By employing the Itô formula [35], the derivation of $V(t)$ is

$$\begin{aligned}
&\mathrm{d}V(v(u,t),t)\\
&\quad=\pounds V(v(u,t),t)\mathrm{d}t\\
&\qquad+V_v(v(u,t),t)\varrho(v(u,t),v(u,t-\lambda(t)))\mathrm{d}B(t)
\end{aligned}\tag{15}$$

where

$$\begin{aligned}
\pounds V(v(u,t),t)&=V_t(v(u,t),t)+V_v(v(u,t),t)\Omega\\
&\quad+\frac{1}{2}\mathrm{trace}[\varrho^T V_{vv}(v(u,t),t)\varrho],\\
V_t(v(u,t),t)&=\frac{\partial V(v(u,t),t)}{\partial t},\\
V_v(v(u,t),t)&=\Big(\frac{\partial V(v(u,t),t)}{\partial v_1},\frac{\partial V(v(u,t),t)}{\partial v_2},\\
&\quad\ldots,\frac{\partial V(v(u,t),t)}{\partial v_n}\Big),\\
V_{vv}(v(u,t),t)&=\Big(\frac{\partial^2 V(v(u,t),t)}{\partial v_s\partial v_k}\Big)_{n\times n},\\
\Omega&=(\Omega_1,\Omega_2,\ldots,\Omega_n)^T,\\
\Omega_s&=\sum_{i=1}^{I}\frac{\partial}{\partial u_i}\Big(\alpha_{si}\frac{\partial v_s(u,t)}{\partial u_i}\Big)-d_s v_s(u,t)\\
&\quad+\sum_{k=1}^{n}a_{sk}h_{1k}(v_k(u,t))+\Delta_s(u,t)\\
&\quad+\sum_{k=1}^{n}b_{sk}h_{2k}(v_k(u,t-\lambda_k(t)))\\
&\quad+\sum_{k=1}^{n}c_{sk}\int_{-\infty}^{t}K_{sk}(t-\omega)h_{3k}(v_k(u,\omega))\mathrm{d}\omega.
\end{aligned}$$

It follows

$$\begin{aligned}
&\pounds V(v(u,t),t)\\
&\le\int_{\Pi}\sum_{s=1}^{n}\varepsilon_s\Big\{\mathbf{e}^{\pi t}\Big[\frac{\mathbb{p}}{2}|v_s(u,t)|^{\mathbb{p}-2}\Big(\frac{\partial v_s^2(u,t)}{\partial t}+(\mathbb{p}-1)\\
&\quad\times\sum_{k=1}^{n}\varrho_{sk}^2(v_k(u,t),v_k(u,t-\lambda_k(t)))\Big)+\pi|v_s(u,t)|^{\mathbb{p}}\Big]\\
&\quad-\mathbb{p}(\beta_s(u,t)+\tilde{\beta}_s)g_s(t)\\
&\quad+\sum_{k=1}^{n}\Big[\mathbf{e}^{\pi\lambda_1}(|b_{sk}|F_{2k}+(\mathbb{p}-1)\eta_{sk})\\
&\quad\times\Big(\frac{g_k(t)}{1-\lambda_2}-g_k(t-\lambda_k(t))\Big)\\
&\quad+|c_{sk}|F_{3k}\Big(\int_{-\infty}^{0}\mathbf{e}^{-\pi\theta}K_{sk}(-\theta)g_k(t)\mathrm{d}\theta\\
&\quad-\int_{-\infty}^{0}\mathbf{e}^{-\pi\theta}K_{sk}(-\theta)g_k(t+\theta)\mathrm{d}\theta\Big)\Big]\Big\}\mathrm{d}u\\
&\le\int_{\Pi}\sum_{s=1}^{n}\varepsilon_s\mathbf{e}^{\pi t}\Big\{\mathbb{p}|v_s(u,t)|^{\mathbb{p}-2}v_s(u,t)\\
&\quad\times\sum_{i=1}^{I}\frac{\partial}{\partial u_i}\Big(\alpha_{si}\frac{\partial v_s(u,t)}{\partial u_i}\Big)-\mathbb{p}d_s|v_s(u,t)|^{\mathbb{p}}\\
&\quad+\sum_{k=1}^{n}\mathbb{p}|a_{sk}||v_s(u,t)|^{\mathbb{p}-1}|h_{1k}(v_k(u,t))|\\
&\quad+\sum_{k=1}^{n}\mathbb{p}|b_{sk}||v_s(u,t)|^{\mathbb{p}-1}|h_{2k}(v_k(u,t-\lambda_k(t)))|\\
&\quad+\sum_{k=1}^{n}\mathbb{p}|c_{sk}||v_s(u,t)|^{\mathbb{p}-1}\\
&\quad\times\int_{-\infty}^{t}K_{sk}(t-\omega)|h_{3k}(v_k(u,\omega))|\mathrm{d}\omega\\
&\quad+\mathbb{p}\beta_s(u,t)|v_s(u,t)|^{\mathbb{p}}+\frac{\mathbb{p}(\mathbb{p}-1)}{2}|v_s(u,t)|^{\mathbb{p}-2}\\
&\quad\times\sum_{k=1}^{n}\varrho_{sk}^2(v_k(u,t),v_k(u,t-\lambda_k(t)))\\
&\quad+\pi|v_s(u,t)|^{\mathbb{p}}-\mathbb{p}(\beta_s(u,t)+\tilde{\beta}_s)|v_s(u,t)|^{\mathbb{p}}\\
&\quad+\sum_{k=1}^{n}\Big[(|b_{sk}|F_{2k}+(\mathbb{p}-1)\eta_{sk})\\
&\quad\times\Big(\frac{\mathbf{e}^{\pi\lambda_1}|v_k(u,t)|^{\mathbb{p}}}{1-\lambda_2}-|v_k(u,t-\lambda_k(t))|^{\mathbb{p}}\Big)\\
&\quad+|c_{sk}|F_{3k}\Big(\int_{0}^{+\infty}\mathbf{e}^{\pi\omega}K_{sk}(\omega)|v_k(u,t)|^{\mathbb{p}}\mathrm{d}\omega\\
&\quad-\int_{-\infty}^{t}K_{sk}(t-\omega)|v_k(u,\omega)|^{\mathbb{p}}\mathrm{d}\omega\Big)\Big]\Big\}\mathrm{d}u.
\end{aligned}\tag{16}$$

Based on H1 and H3, and the Young's inequality $yx\le 1/\sigma_1\, y^{\sigma_1}+1/\sigma_2\, x^{\sigma_2}$ for $y>0, x>0$ with constants $\sigma_1>0, \sigma_2>0$ satisfying $1/\sigma_1+1/\sigma_2=1$, one can obtain

$$\begin{aligned}
&\sum_{k=1}^{n}\mathbb{p}|a_{sk}||v_s(u,t)|^{\mathbb{p}-1}|f_{1k}(v_k(u,t))|\\
&\le\sum_{k=1}^{n}|a_{sk}|F_{1k}((\mathbb{p}-1)|v_s(u,t)|^{\mathbb{p}}+|v_k(u,t)|^{\mathbb{p}}),
\end{aligned}\tag{17}$$

$$\begin{aligned}
&\sum_{k=1}^{n}\mathbb{p}|b_{sk}||v_s(u,t)|^{\mathbb{p}-1}|f_{2k}(v_k(u,t-\lambda_k(t)))|\\
&\le\sum_{k=1}^{n}|b_{sk}|F_{2k}((\mathbb{p}-1)|v_s(u,t)|^{\mathbb{p}}+|v_k(u,t-\lambda_k(t))|^{\mathbb{p}}),
\end{aligned}\tag{18}$$

$$\sum_{k=1}^{n}\mathbb{p}|c_{sk}||v_s(u,t)|^{\mathbb{p}-1}\times\int_{-\infty}^{t}K_{sk}(t-\omega)|f_{3k}(v_k(u,\omega))|\mathrm{d}\omega\leq\sum_{k=1}^{n}|c_{sk}|F_{3k}((\mathbb{p}-1)|v_s(u,t)|^{\mathbb{p}}+\int_{-\infty}^{t}K_{sk}(t-\omega)|v_k(u,\omega)|^{\mathbb{p}}\mathrm{d}\omega),\tag{19}$$

$$\frac{\mathbb{p}(\mathbb{p}-1)}{2}|v_s(u,t)|^{\mathbb{p}-2}\sum_{k=1}^{n}\varrho_{sk}^2(v_k(u,t),v_k(u,t-\lambda_k(t)))\leq\frac{\mathbb{p}-1}{2}\sum_{k=1}^{n}[\tau_{sk}((\mathbb{p}-2)|v_s(u,t)|^{\mathbb{p}}+2|v_k(u,t)|^{\mathbb{p}})+\eta_{sk}((\mathbb{p}-2)|v_s(u,t)|^{\mathbb{p}}+2|v_k(u,t-\lambda_k(t))|^{\mathbb{p}})].\tag{20}$$

By using the Green formula [45] and Lemma 1 in [10], it follows that

$$\int_{\Pi}\mathbb{p}|v_s(u,t)|^{\mathbb{p}-2}v_s(u,t)\sum_{i=1}^{I}\frac{\partial}{\partial u_i}\left(\alpha_{si}\frac{\partial v_s(u,t)}{\partial u_i}\right)\mathrm{d}u\leq-\sum_{i=1}^{I}\frac{4(\mathbb{p}-1)\alpha_{si}}{\mathbb{p}r_i^2}\int_{\Pi}|v_s(u,t)|^{\mathbb{p}}\mathrm{d}u.\tag{21}$$

Thus, it follows from (16)-(21) that

$$\begin{aligned}&\pounds V(v(u,t),t)\\&\leq\int_{\Pi}\sum_{s=1}^{n}\varepsilon_s\mathbf{e}^{\pi t}\Bigg[\Bigg(-\sum_{i=1}^{I}\frac{4(\mathbb{p}-1)\alpha_{si}}{\mathbb{p}r_i^2}+\pi-\mathbb{p}d_s-\mathbb{p}\tilde{\beta}_s\\&\quad+\sum_{k=1}^{n}(\mathbb{p}-1)(|a_{sk}|F_{1k}+|b_{sk}|F_{2k}+|c_{sk}|F_{3k})\\&\quad+\sum_{k=1}^{n}\frac{(\mathbb{p}-1)(\mathbb{p}-2)}{2}(\tau_{sk}+\eta_{sk})\Bigg)|v_s(u,t)|^{\mathbb{p}}\\&\quad+\sum_{k=1}^{n}(\mathbb{p}-1)\tau_{sk}|v_k(u,t)|^{\mathbb{p}}+\sum_{k=1}^{n}|a_{sk}|F_{1k}|v_k(u,t)|^{\mathbb{p}}\\&\quad+\sum_{k=1}^{n}(\mathbb{p}-1)\eta_{sk}|v_k(u,t-\lambda_k(t))|^{\mathbb{p}}\\&\quad+\sum_{k=1}^{n}|b_{sk}|F_{2k}|v_k(u,t-\lambda_k(t))|^{\mathbb{p}}\\&\quad+\sum_{k=1}^{n}|c_{sk}|F_{3k}\int_{-\infty}^{t}K_{sk}(t-\omega)|v_k(u,\omega)|^{\mathbb{p}}\mathrm{d}\omega\\&\quad+\sum_{k=1}^{n}(|b_{sk}|F_{2k}+(\mathbb{p}-1)\eta_{sk})\\&\quad\times\left(\frac{\mathbf{e}^{\pi\lambda_1}|v_k(u,t)|^{\mathbb{p}}}{1-\lambda_2}-|v_k(u,t-\lambda_k(t))|^{\mathbb{p}}\right)\\&\quad+\sum_{k=1}^{n}|c_{sk}|F_{3k}\Bigg(\int_{0}^{+\infty}\mathbf{e}^{\pi\omega}K_{sk}(\omega)|v_k(u,t)|^{\mathbb{p}}\mathrm{d}\omega\\&\quad-\int_{-\infty}^{t}K_{sk}(t-\omega)|v_k(u,\omega)|^{\mathbb{p}}\mathrm{d}\omega\Bigg)\Bigg]\mathrm{d}u\\&\leq\int_{\Pi}\sum_{s=1}^{n}\mathbf{e}^{\pi t}\Bigg[\varepsilon_s\Bigg(-\sum_{i=1}^{I}\frac{4(\mathbb{p}-1)\alpha_{si}}{\mathbb{p}r_i^2}+\pi-\mathbb{p}d_s-\mathbb{p}\tilde{\beta}_s\\&\quad+\sum_{k=1}^{n}(\mathbb{p}-1)(|a_{sk}|F_{1k}+|b_{sk}|F_{2k}+|c_{sk}|F_{3k})\\&\quad+\sum_{k=1}^{n}\frac{(\mathbb{p}-1)(\mathbb{p}-2)}{2}(\tau_{sk}+\eta_{sk})\Bigg)\\&\quad+\sum_{k=1}^{n}\varepsilon_k\Bigg((\mathbb{p}-1)\tau_{ks}+|a_{ks}|F_{1s}\\&\quad+\frac{(|b_{ks}|F_{2s}+(\mathbb{p}-1)\eta_{ks})\mathbf{e}^{\pi\lambda_1}}{1-\lambda_2}\\&\quad+|c_{ks}|F_{3s}\int_{0}^{+\infty}\mathbf{e}^{\pi\omega}K_{ks}(\omega)\mathrm{d}\omega\Bigg)\Bigg]|v_s(u,t)|^{\mathbb{p}}\mathrm{d}u.\end{aligned}\tag{22}$$

Then, choose $\tilde{\beta}_s=(\varsigma_s-\sum_{i=1}^{I}4(\mathbb{p}-1)\alpha_{si}/(\mathbb{p}r_i^2)+\pi-\mathbb{p}d_s+\sum_{k=1}^{n}(\mathbb{p}-1)(|a_{sk}|F_{1k}+|b_{sk}|F_{2k}+|c_{sk}|F_{3k}+(\mathbb{p}-2)(\tau_{sk}+\eta_{sk})/2)+\sum_{k=1}^{n}\varepsilon_k/\varepsilon_s((\mathbb{p}-1)\tau_{ks}+|a_{ks}|F_{1s}+(|b_{ks}|F_{2s}+(\mathbb{p}-1)\eta_{ks})\mathbf{e}^{\pi\lambda_1}/(1-\lambda_2)+|c_{ks}|F_{3s}\int_0^{+\infty}\mathbf{e}^{\pi\omega}K_{ks}(\omega)\mathrm{d}\omega))/\mathbb{p}$ with constant $\varsigma_s>0$, it follows

$$\pounds V(v(u,t),t)\leq-\int_{\Pi}\sum_{s=1}^{n}\varepsilon_s\mathbf{e}^{\pi t}\varsigma_s|v_s(u,t)|^{\mathbb{p}}\mathrm{d}u\leq 0.\tag{23}$$

By taking the mathematical expectation of both sides of (15), we can get $\frac{\mathrm{d}\mathcal{E}V(v(u,t),t)}{\mathrm{d}t}\leq 0$. Thus, for any $t\in\mathcal{R}_+$,

$$\mathcal{E}V(v(u,t),t)\leq\mathcal{E}V(v(u,0),0),\tag{24}$$

i.e.,

$$\begin{aligned}&\mathcal{E}\left[\int_{\Pi}\sum_{s=1}^{n}|v_s(u,t)|^{\mathbb{p}}\mathrm{d}u\right]\\&\leq\mathcal{E}\Bigg[\frac{\bar{\varepsilon}}{\underline{\varepsilon}}\mathbf{e}^{-\pi t}\int_{\Pi}\sum_{s=1}^{n}\Bigg(|v_s(u,0)|^{\mathbb{p}}+\frac{\mathbb{p}}{2\gamma_s}(\beta_s(u,0)+\tilde{\beta}_s)^2\\&\quad+\sum_{k=1}^{n}\frac{(|b_{ks}|F_{2s}+(\mathbb{p}-1)\eta_{ks})}{1-\lambda_2}\\&\quad\times\int_{-\lambda_1}^{0}\mathbf{e}^{\pi(\lambda_1+\omega)}|v_s(u,\omega)|^{\mathbb{p}}\mathrm{d}\omega+\sum_{k=1}^{n}|c_{ks}|F_{3s}\\&\quad\times\int_{-\infty}^{0}\int_{\theta}^{0}\mathbf{e}^{\pi(\omega-\theta)}K_{ks}(-\theta)|v_s(u,\omega)|^{\mathbb{p}}\mathrm{d}\omega\mathrm{d}\theta\Bigg)\mathrm{d}u\Bigg]\\&\leq\tilde{\pi}\mathcal{E}\left[\int_{\Pi}\sup_{-\infty<\omega\leq 0}\sum_{s=1}^{n}|\phi_s(u,\omega)|^{\mathbb{p}}\mathrm{d}u\right]\mathbf{e}^{-\pi t}\end{aligned}\tag{25}$$

where $\underline{\varepsilon}=\min_s\{\varepsilon_s\}$, $\bar{\varepsilon}=\max_s\{\varepsilon_s\}$, and $\tilde{\pi}=\bar{\varepsilon}/\underline{\varepsilon}\max_s\{1+\nu+\sum_{k=1}^{n}((|b_{ks}|F_{2s}+(\mathbb{p}-1)\eta_{ks})\mathbf{e}^{\pi\lambda_1}\lambda_1)/(1-\lambda_2)+\sum_{k=1}^{n}(|c_{ks}|F_{3s}\int_0^{+\infty}\mathbf{e}^{\pi\omega}K_{ks}(\omega)\mathrm{d}\omega)\}$. ν is a constant such that $\sum_{s=1}^{n} \mathbb{p}(\beta_s(u,0) + \tilde{\beta}_s)^2/(2\gamma_s) \leq \nu \sup_{-\infty<\omega\leq 0} \sum_{s=1}^{n} |\phi_s(u,\omega)|^{\mathbb{P}})$.

Thus,

$$\mathcal{E}\|v(u,t)\|^{\mathbb{P}} \leq \tilde{\pi}\mathcal{E}\|\phi(u,\omega)\|^{\mathbb{P}}\mathbf{e}^{-\pi t}. \tag{26}$$

In light of Definition 1, systems (1) and (5) are globally exponentially synchronized under the adaptive controller (6). The proof is completed. ∎

Remark 4: In [15]–[19], [32], [37]–[44], authors discussed the dynamics for a class of RDNNs. Since the infinite distributed delays and stochastic disturbance are all taken into consideration in the RDNNs model of this paper, the model is generalized compared to the RDNNs without distributed delays and stochastic disturbance. Moreover, our results are also generalized and can be extend to other RDNNs with or without distributed delays and stochastic disturbance.

Remark 5: Our results show the superiority of the ones in [32], [41]–[44]. On the one hand, the synchronization criteria in this paper are more simpler compared to the ones in [32], [41], [43], [44] on account of the complicated calculation for the criteria in [32], [43], [44]. On the other hand, simple synchronization conditions are obtained for stochastic RDNNs via adaptive control approach in [42]. It is worth noting that our criteria are derived with infinite distributed delays while the ones in [42] are with the finite distributed delays.

If the control gain $\beta_s(u,t) = \beta_s$, then the adaptive control turns out to be the linear one. Then we can get the following Corollary 1 from Theorem 1.

Corollary 1: Suppose that assumptions H1-H4 hold, if there exists constant $\beta_s < 0$ such that

$$\begin{aligned}
&\mathbb{p}\beta_s - \sum_{i=1}^{I} 4(\mathbb{p}-1)\alpha_{si}/(\mathbb{p}r_i^2) + \pi - \mathbb{p}d_s \\
&+ \sum_{k=1}^{n}(\mathbb{p}-1)\Big(|a_{sk}|F_{1k} + |b_{sk}|F_{2k} + |c_{sk}|F_{3k} \\
&+ \frac{\mathbb{p}-2}{2}(\tau_{sk}+\eta_{sk})\Big) + \sum_{k=1}^{n}\frac{\varepsilon_k}{\varepsilon_s}\Big((\mathbb{p}-1)\tau_{ks} \\
&+ |a_{ks}|F_{1s} + \frac{(|b_{ks}|F_{2s} + (\mathbb{p}-1)\eta_{ks})\mathbf{e}^{\pi\lambda_1}}{1-\lambda_2} \\
&+ |c_{ks}|F_{3s}\int_0^{+\infty}\mathbf{e}^{\pi\omega}K_{ks}(\omega)\mathrm{d}\omega\Big) < 0,
\end{aligned} \tag{27}$$

then systems (1) and (5) are globally exponentially synchronized via the controller $\Delta_s(u,t) = \beta_s v_s(u,t)$.

Remark 6: It is obvious from Corollary 1 that the criteria via the adaptive control approach is simpler and less conservative compared to the criteria via the linear control approach. In the following part, we will show that our results still hold for stochastic RDNNs with finite distributed delays.

If the delay kernel function satisfies

$$K_{sk}(t) = \begin{cases} 1, & 0 \leq t \leq \rho, \\ 0, & t > \rho, \end{cases}$$

where ρ is a positive constant. Then the drive and response RDNNs turn out to be the following two systems.

$$\begin{aligned}
\mathrm{d}x_s(u,t) = \Big[&\sum_{i=1}^{I}\frac{\partial}{\partial u_i}\Big(\alpha_{si}\frac{\partial x_s(u,t)}{\partial u_i}\Big) - d_s x_s(u,t) \\
&+ \sum_{k=1}^{n} a_{sk}f_{1k}(x_k(u,t)) \\
&+ \sum_{k=1}^{n} b_{sk}f_{2k}(x_k(u,t-\lambda_k(t))) \\
&+ \sum_{k=1}^{n} c_{sk}\int_{t-\rho}^{t} f_{3k}(x_k(u,\omega))\mathrm{d}\omega\Big]\mathrm{d}t
\end{aligned} \tag{28}$$

and

$$\begin{aligned}
\mathrm{d}y_s(u,t) = \Big[&\sum_{i=1}^{I}\frac{\partial}{\partial u_i}\Big(\alpha_{si}\frac{\partial y_s(u,t)}{\partial u_i}\Big) - d_s y_s(u,t) \\
&+ \sum_{k=1}^{n} a_{sk}f_{1k}(y_k(u,t)) + \Delta_s(u,t) \\
&+ \sum_{k=1}^{n} b_{sk}f_{2k}(y_k(u,t-\lambda_k(t))) \\
&+ \sum_{k=1}^{n} c_{sk}\int_{t-\rho}^{t} f_{3k}(y_k(u,\omega))\mathrm{d}\omega\Big]\mathrm{d}t \\
&+ \sum_{k=1}^{n} \varrho_{sk}(v_k(u,t), v_k(u,t-\lambda_k(t)))\mathrm{d}B_k(t)
\end{aligned} \tag{29}$$

with the control input

$$\begin{aligned}
\Delta_s(u,t) &= \bar{\beta}_s(u,t)v_s(u,t), \\
\frac{\partial\bar{\beta}_s(u,t)}{\partial t} &= -\bar{\gamma}_s\mathbf{e}^{\bar{\pi}t}v_s^2(u,t)
\end{aligned} \tag{30}$$

where the parameters are similar defined as in (5)-(7).

Corollary 2: Suppose that assumptions H1, H2 and H4 hold, then systems (28) and (29) are globally exponentially synchronized via the adaptive controller (30).

Corollary 3: Suppose that assumptions H1-H3 hold and $\varrho_{sk}(\cdot,\cdot) = 0(s,k \in n^{\sharp})$, then systems (1) and (5) are globally exponentially synchronized via the adaptive controller (6).

Remark 7: Authors in [42] investigated the synchronization problem for RDNNs with finite distributed delays and stochastic disturbance via adaptive control approach. The results in [42] can be obtained directly from Corollary 2.

IV. NUMERICAL SIMULATIONS

One example is provided to show the effectiveness of results and the control approach.

Example 1: Consider the RDNNs with infinite distributed delays.

$$\begin{aligned} \mathrm{d}x_s(u,t) = \Bigg[&\alpha_s \frac{\partial^2 x_s(u,t)}{\partial u^2} - d_s x_s(u,t) \\ &+ \sum_{k=1}^{2} a_{sk} f_{1k}(x_k(u,t)) \\ &+ \sum_{k=1}^{2} b_{sk} f_{2k}(x_k(u,t-\lambda_k(t))) \\ &+ \sum_{k=1}^{2} c_{sk} \int_{-\infty}^{t} K_{sk}(t-\omega) f_{3k}(x_k(u,\omega))\mathrm{d}\omega \Bigg]\mathrm{d}t \end{aligned} \tag{31}$$

where $u \in \Pi = [-4, 4], \alpha_1 = \alpha_2 = 0.1, d_1 = d_2 = 1, a_{11} = 2, a_{12} = -0.1, a_{21} = -5, a_{22} = 2.5, b_{11} = -1.3, b_{12} = -0.1, b_{21} = 1, b_{22} = -0.5, c_{11} = -0.03, c = 0.15, c_{21} = -0.2, c_{22} = -0.1$, the delays $\lambda_k(t) = \mathbf{e}^t/(1 + \mathbf{e}^t), K_{sk}(\omega) = \mathbf{e}^{-\omega}$, and activation functions $f_{jk}(\cdot) = \tanh(\cdot)$ for $j = 1, 2, 3, s, k = 1, 2$.

Then under the stochastic disturbance and external control input, the response system is

$$\begin{aligned} \mathrm{d}y_s(u,t) = \Bigg[&\alpha_s \frac{\partial^2 y_s(u,t)}{\partial u^2} - d_s y_s(u,t) \\ &+ \sum_{k=1}^{2} a_{sk} f_{1k}(y_k(u,t)) + \Delta_s(u,t) \\ &+ \sum_{k=1}^{2} b_{sk} f_{2k}(y_k(u,t-\lambda_k(t))) \\ &+ \sum_{k=1}^{2} c_{sk} \int_{-\infty}^{t} K_{sk}(t-\omega) f_{3k}(y_k(u,\omega))\mathrm{d}\omega \Bigg]\mathrm{d}t \\ &+ \sum_{k=1}^{n} \varrho_{sk}(v_k(u,t), v_k(u,t-\lambda_k(t)))\mathrm{d}B_k(t) \end{aligned} \tag{32}$$

in which $\varrho_{11}(v_1(u,t), v_1(u,t-\lambda_1(t))) = 0.2\ v_1(u,t) + 0.3\ v_1(u,t-\lambda_1(t)), \varrho_{12}(v_2(u,t), v_2(u,t-\lambda_2(t))) = \varrho_{21}(v_1(u,t), v_1(u,t-\lambda_1(t))) = 0, \varrho_{22}(v_2(u,t), v_2(u,t-\lambda_2(t))) = 0.3\ v_2(u,t) + 0.2\ v_2(u,t-\lambda_2(t))$. The other parameters are the same as in the drive system (31).

Fig. 1 show the phase plot of drive system (31). It is noted that the chaotic attractors are reflected in this figure. Fig. 2 depicts the phase plot of system (32) with $u = -2$ and $\Delta_s(u,t) = 0$. The trajectories of the state variables $y_1(u,t)$ and $y_2(u,t)$ of system (31) without any controller are shown in Figs. 3 and 4. It is obviously that the response system is also a chaotic system, and it is not stable without any controller.

Then to ensure the synchronization of drive and response systems (31) and (32), the following controller is given as

$$\begin{aligned} \Delta_s(u,t) &= \beta_s(u,t)v_s(u,t), \\ \frac{\partial \beta_s(u,t)}{\partial t} &= -5\mathbf{e}^{0.005t}|v_s(u,t)|^2, \end{aligned} \tag{33}$$

for $s = 1, 2$. Then the globally exponentially synchronization of systems (31) and (32) via the controller (33) are

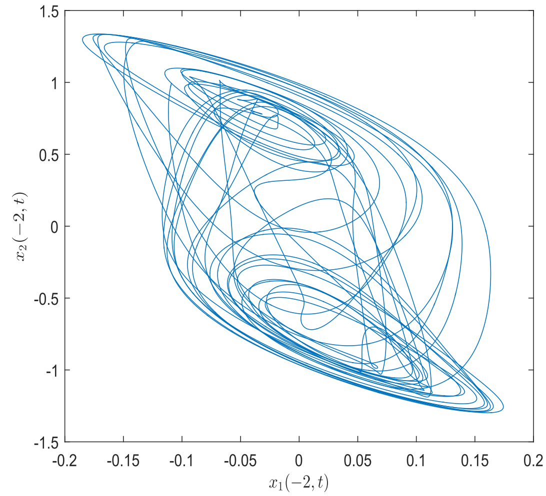

FIGURE 1. Phase plot of system (31) with $u = -2$.

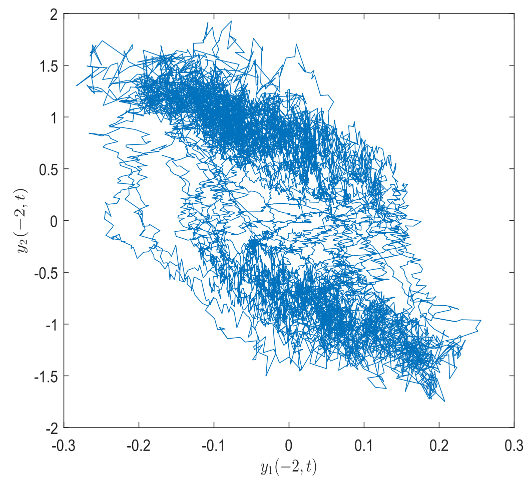

FIGURE 2. Phase plot of system (32) with $u = -2$ and $\Delta_s(u,t) = 0$.

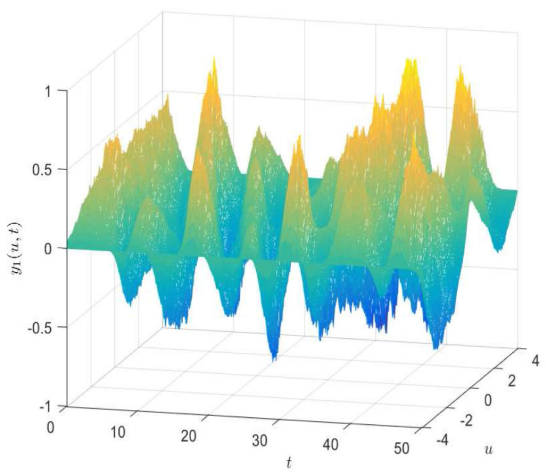

FIGURE 3. The trajectories of $y_1(u,t)$ of system (32) without any controller.

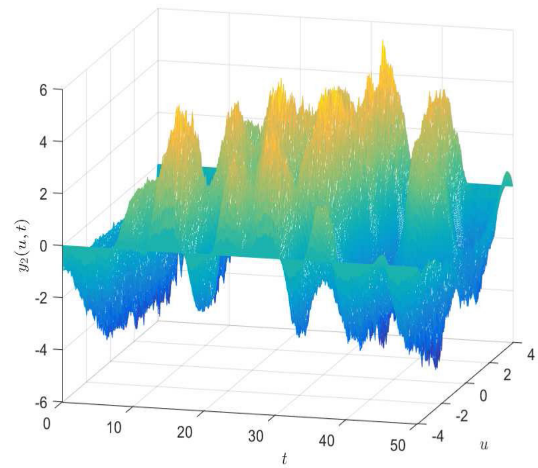

FIGURE 4. The trajectories of $y_2(u, t)$ of system (32) without any controller.

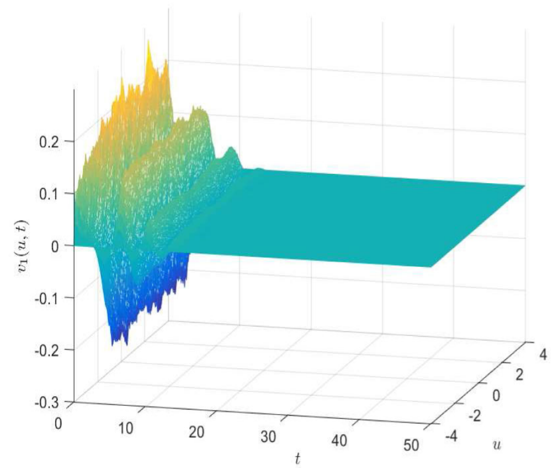

FIGURE 5. The trajectories of error state $v_1(u, t)$ between systems (31) and (32) via the controller (33).

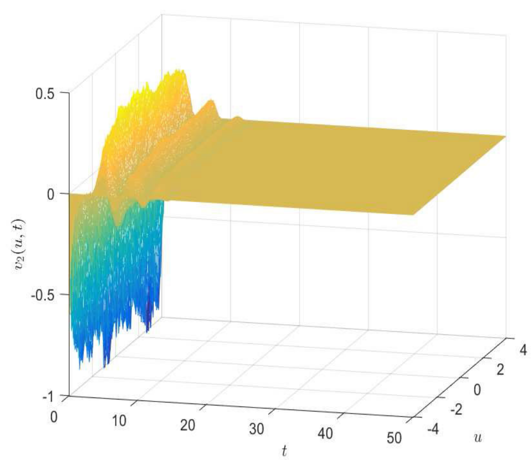

FIGURE 6. The trajectories of error state $v_2(u, t)$ between systems (31) and (32) via the controller (33).

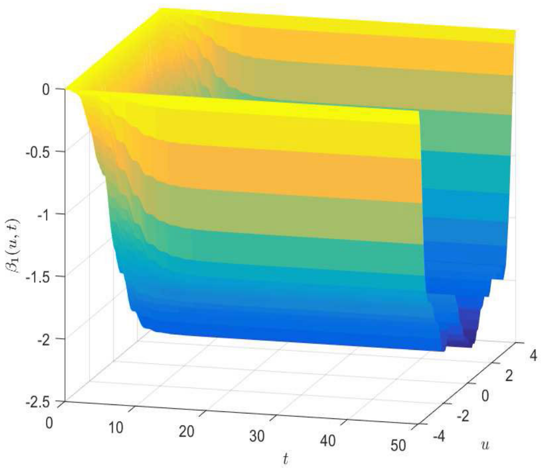

FIGURE 7. The control gain $\beta_1(u, t)$ of the adaptive controller (33).

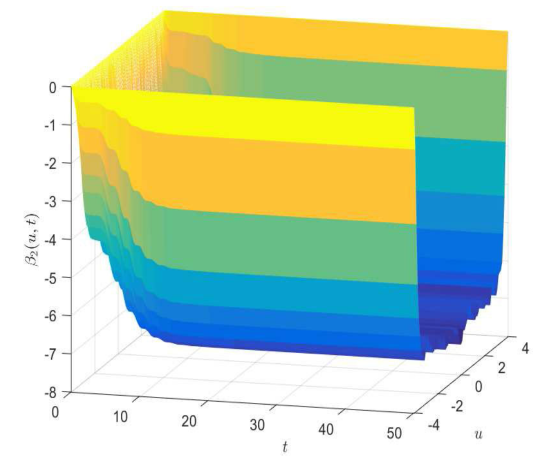

FIGURE 8. The control gain $\beta_2(u, t)$ of the adaptive controller (33).

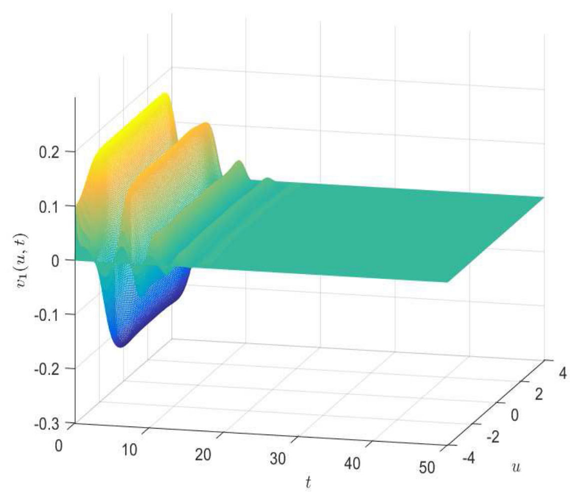

FIGURE 9. Under $\varrho_{sk}(\cdot, \cdot) = 0(s, k = 1, 2)$, the trajectories of error state $v_1(u, t)$ between systems (31) and (32) via the controller (33).

guaranteed based on the results of Theorem 1. The trajectories of synchronization errors $v_1(u, t)$ and $v_2(u, t)$ via the adaptive controller (33) are shown in Figs. 5 and 6. From Figs. 5 and 6, it is easy to see that the error states tend to zero exponentially

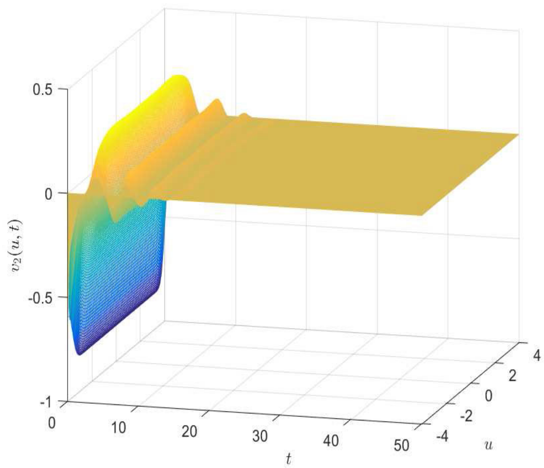

FIGURE 10. Under $\varrho_{sk}(\cdot,\cdot) = 0(s, k = 1, 2)$, the trajectories of error state $v_2(u, t)$ between systems (31) and (32) via the controller (33).

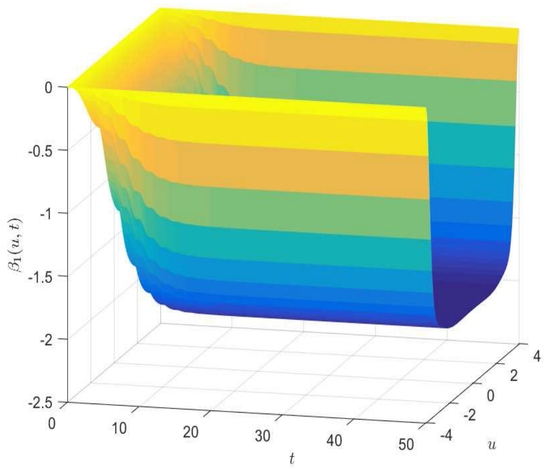

FIGURE 11. Under $\varrho_{sk}(\cdot,\cdot) = 0(s, k = 1, 2)$, the control gain $\beta_1(u, t)$ of the adaptive controller (33).

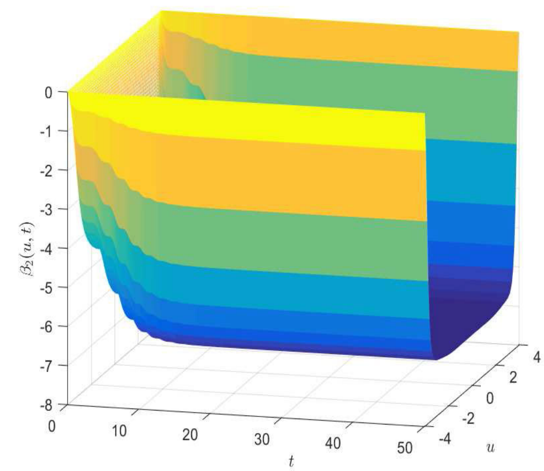

FIGURE 12. Under $\varrho_{sk}(\cdot,\cdot) = 0(s, k = 1, 2)$, the control gain $\beta_2(u, t)$ of the adaptive controller (33).

as time goes to infinite, which also shows the synchronization performance of the drive and response systems. The control gains $\beta_1(u, t)$ and $\beta_2(u, t)$ of the adaptive controller (33) are given in Figs. 7 and 8.

Moreover, if $\varrho_{sk}(\cdot,\cdot) = 0(s, k = 1, 2)$, then the synchronization of systems (31) and (32) can also be ensured on the strength of Corollary 3. The trajectories of synchronization errors are shown in Figs. 9 and 10. The control gains of the controller are shown in Figs. 11 and 12 on account of the condition $\varrho_{sk}(\cdot,\cdot) = 0(s, k = 1, 2)$.

V. CONCLUSION

This paper has discussed the global exponential synchronization problem for a class of RDNNs with distributed delays. Compared with some existing determined models, the stochastic disturbance has been taken into consideration and the distributed delays are infinite. By employing an adaptive controller, sufficient simple and easily checked criteria have been concluded for the addressed drive and response networks under stochastic disturbance. Finally, one RDNNs model with chaotic attractors has been presented to show the feasibility and validity of the proposed adaptive control approach. Since the parameters of the drive and response systems can not be identical in light of the complicated environment of practical application, future work may study the synchronization problem of RDNNs with mismatched parameters.

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