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An Effective Adaptive Gain Dynamics for Time-Delay Control of Robot Manipulators

JUNYOUNG L[E](https://orcid.org/0000-0002-3593-0924)E^{©1}, (Member, IEEE), PYUN[G](https://orcid.org/0000-0002-8010-1359) HUN CHANG^{©2}, (Member, IEEE), BYEONGGI YU®2[, \(S](https://orcid.org/0000-0001-5420-9705)tudent Member, IEEE), KAP-HO SEO³, (Member, IEEE), AND MAOLIN JIN^{®1}, (Senior Member, IEEE)

1Human-Centered Robotics Research Center, Korea Institute of Robotics and Technology Convergence, Pohang 37553, South Korea ²Department of Robotics Engineering, Daegu–Gyeongbuk Institute of Science and Technology, Daegu 42988, South Korea ³ Interactive Robotics Research and Development Division, Korea Institute of Robotics and Technology Convergence, Pohang 37553, South Korea

Corresponding authors: Pyung Hun Chang (phchang@dgist.ac.kr) and Maolin Jin (mulimkim@kiro.re.kr)

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ABSTRACT The time-delay control (TDC) has recently been spotlighted as an effective solution owing to model-free, efficient, and robust properties thanks to a time-delay estimation (TDE) technique. The gain of TDC, usually denoted by \overline{M} , is crucial for its stability and performance, and it is reported that the constant gain of TDC does not always guarantee the best performance. To cope with this problem, this paper proposes an effective gain adaptation together with a nonlinear desired error dynamics and a new sliding variable. The resulting adaptive gain dynamics is combined with the TDC to form the proposed control, whose closed-loop stability is proved. Through simulation and experiment, we have shown that the proposed control enables to transfer M from an unstable initial value to a stable one, better than a best-tuned gain by trial and error. As a result, the proposed control is model-free, able to achieve time responses as fast as the inclusive enhanced TDC (IETDC) – arguably the fastest TDC – and tracking accuracy better than the IETDC. The proposed method has shown a strong potential to significantly relieve the burden of gain selection.

INDEX TERMS Adaptive control, robot manipulator, sliding mode control, time-delay estimation.

I. INTRODUCTION

Robot manipulators widely employed for various tasks [1]–[4] require accurate motion control, which is very challenging to control engineers. For robot manipulators inherently include highly nonlinear and coupled dynamics such as Coriolis and centrifugal torque, nonlinear friction, and gravitational torque [5], [6].

As an effective solution to this challenging problem, TDC has been drawing attention for its simplicity and robustness [7]–[9]. To estimate robot dynamics, the TDC adopts a time-delay estimation (TDE) technique, which intentionally employs time-delayed states at the previous sampling instant [10]–[12]. Thanks to the TDE technique, TDC becomes simple, efficient, and robust [10]–[12] and is widely applied to the various fields [13]–[19].

Despite its extraordinary strength and extensive applicability, the TDC and all TDE-based controls have an

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important issue – perhaps the most crucial one – in common: the selection of a control gain, usually denoted by M. The appropriate value of M is crucial because of its direct impact on the performance and stability of any TDC-based closedloop system. A larger gain generally yields faster response, a smaller gain slower response; an excessive one even causes instability. The selection issue is further complicated by the fact that even an appropriately chosen gain subsequently becomes either excessive or too small. These situations occur as the robot inertias vary owing to the changes in payloads or kinematic configurations. Furthermore, the gain needs to be selected by trial and error, unless robot inertias are precisely known. In practice therefore it is manually tuned by trial and error [20], [21], taking substantial time and effort.

No wonder, auto-gain tuning methods or gain adaptation methods have been reported as follows. A Nussbaum technique was applied to the TDC in order to secure its stability by auto-gain tuning and demonstrated in a one-link arm simulation [22]; however, the gain adaptation is restricted within a

region where the stability condition is already met, generating a small \overline{M} and providing weak performance.

As another research, an adaptive TDC with a linear sliding variable has been proposed and applied to leg joints of a humanoid [23] in order to improve the tracking accuracy. Although they have achieved an auto-tuning gain for the TDC, the adaptive control gain is intentionally bounded in a specific region by a saturation function to avoid an unstable response. For this reason, this control method requires a saturation function and a small initial value of the gain.

As different studies, in an attempt to improve tracking accuracy, two TDC methods have been proposed with respective adaptive rules using sliding variables [24], [25]. In these methods, M consists of two parts, a constant part called *bias* and a *variable* part to be adapted. Since the adaptation is made only by the variable part taking a very small portion of M , the gain variation is significantly restricted. Besides, the bias taking a major portion of M, it must be carefully tuned by trial ¯ and error; otherwise, the closed-loop system could become oscillatory or go unstable.

Recently, an adaptive TDC with a gain dynamics has improved accuracy and robustness under significant payload variations for robot manipulators [26]. However, this method also requires an arbitrarily selected small initial value for the gain M in order to avoid unstable or oscillatory response. ¯

To summarize, there have been five previous studies that employed adaptive M for the TDC. While [22] addressed the adaptation for stability, the rest aimed at improving the tracking error by adaptation, which nevertheless *imply* stability – the adaptation effort to minimize the tracking error includes the effort to avoid larger tracking errors coming from instability.

Among these studies, one of the essential differences lies in how to prepare the initial value of M. To elaborate, $[22]$ and [23] use a priori knowledge of the initial value: the former employs the restricted region within which the adaptation starts; the latter the upper bound and lower bound of the saturation function. References [24] and [25] obtain the initial values of the bias by trial and error. Reference [26], on the other hand, needs an arbitrary small initial value. One thing in common, though, is that they need to carefully select the initial values.

To prepare an initial value of \overline{M} is difficult and care demanding – usually, a sufficiently small value tends to be more stable – because a small value for one system is not small enough for other systems. Hence, it would be desirable to have a gain adaptation method that does not demand a stable initial value. To our knowledge, however, there has been no such method, yet.

In this paper, we propose a gain adaptation method that can start with an unstable M and change it to a stable value that also achieves an improved tracking performance. To this end, we are going to improve our previous work [27], by adopting the nonlinear desired error dynamics(DED) [28] for the faster response, by designing a new sliding variable based on this DED, and by analyzing its Lyapunov stability. The result will

The contribution we intend to make, therefore, is to propose an adaptive TDC that enables the stable gain adaptation regardless of its initial value, while achieving the fast response coming from the nonlinear DED and possessing the model-free property of the TDE. The intended capabilities will be tested through the simulation with a one-link arm and the experiment with a robot manipulator.

The rest of this paper is organized as follows. In Section II, the original TDC using a constant gain is reviewed with a focus on the stability condition. Section III presents the proposed adaptive TDC and its structure. In Section IV and Section V, we test the effectiveness of the proposed method through simulation and experiment. Finally, Section VI concludes this paper with the findings.

II. REVIEW OF TIME-DELAY CONTROL

In this section, we are going to make a brief review of the time-delay control (TDC) to the extent necessary to propose our new control approach. A more complete exposition will be found in [7], [8].

The dynamics of an n-degree-of-freedom (n-DOF) rigid robot manipulator can be written as follows:

$$
M(q)\ddot{q} + C(q, \dot{q}) + G(q) + F(q, \dot{q}) + \tau_d = \tau, \qquad (1)
$$

where $\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}} \in \mathbb{R}^n$ are vectors of the joint displacement, velocity, and acceleration, respectively; **M**(**q**) ∈ $\mathbb{R}^{n \times n}$ denotes the symmetric positive definite inertia matrix; $C(q, \dot{q}) \in \mathbb{R}^n$ is the vector of the Coriolis and centripetal torques; $G(q) \in \mathbb{R}^n$ is the gravitational vector; $F(q, \dot{q}) \in \mathbb{R}^n$ is the vector of friction torques; $\tau_d \in \mathbb{R}^n$ is the vector of disturbances; $\tau \in \mathbb{R}^n$ is the vector of applied joint torques. For simplicity, the time variable \bullet_t is omitted.

Using the positive diagonal gain matrix $\bar{\mathbf{M}} \in \mathbb{R}^{n \times n}$, we can reformulate the robot dynamics [\(1\)](#page-1-0) as

$$
\overline{\mathbf{M}}\ddot{\mathbf{q}} + \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) = \boldsymbol{\tau},\tag{2}
$$

where $N(q, \dot{q}, \ddot{q}) \in \mathbb{R}^n$ includes nonlinear terms such as the vectors of Coriolis/centripetal torques, the gravitational torques, the friction torques, and disturbances; it can be written as

$$
\mathbf{N} = [\mathbf{M} - \bar{\mathbf{M}}]\ddot{\mathbf{q}} + \mathbf{C} + \mathbf{G} + \mathbf{F} + \boldsymbol{\tau}_d, \tag{3}
$$

where the arguments \mathbf{q} , $\dot{\mathbf{q}}$, and $\ddot{\mathbf{q}}$ are omitted for simplicity.

A. TIME-DELAY ESTIMATION

Central to the TDC is the time-delay estimation (TDE), an ingenious scheme to estimate the nonlinear term **N** for compensation [10]–[12] as follows:

$$
\mathbf{N} \approx \hat{\mathbf{N}} = \mathbf{N}_{t-L} = \boldsymbol{\tau}_{t-L} - \bar{\mathbf{M}} \ddot{\mathbf{q}}_{t-L},
$$
\n(4)

where $\hat{\bf N}$ denotes the estimation of ${\bf N}$; \bullet_{t-L} a intentional time-delayed value of \bullet ; and *L* the predefined small timedelay, which is often set to the sampling period in digital

implementation. If **N** is continuous or piecewise continuous – which is true in most cases – the value of N at time *t* is close to that of **N** at time $t - L$ for sufficiently small time-delay L . It has been widely observed that the TDE scheme [\(4\)](#page-1-1) reduces computational complexity of robot dynamics, both effectively and efficiently.

B. TDC WITH LINEAR DESIRED ERROR DYNAMICS

The desired linear error dynamics for the TDC [7], [12] is written as

$$
\ddot{\mathbf{e}} + \mathbf{K}_D \dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e} = \mathbf{0},\tag{5}
$$

where $e \in \mathbb{R}^n$ represents the vector of joint errors, given as **e** = $\mathbf{q}_d - \mathbf{q}$; $\mathbf{q}_d \in \mathbb{R}^n$ is the desired joint displacement; and \mathbf{K}_D , $\mathbf{K}_P \in \mathbb{R}^{n \times n}$ are the positive diagonal feedback gain matrices for the desired damping and desired stiffness, respectively.

With [\(4\)](#page-1-1) and [\(5\)](#page-2-0), the control law of TDC is obtained as

$$
\tau = \underbrace{\tau_{t-L} - \bar{M}\ddot{q}_{t-L}}_{TDE} + \underbrace{\bar{M}(\ddot{q}_d + K_D \dot{e} + K_P e)}_{Injecting linear desired dynamics}.
$$
 (6)

First-term in the right-hand-side of [\(6\)](#page-2-1) compensates for the nonlinear robot dynamics and the second term inserts the linear desired error dynamics to the closed-loop system. Notice that the second part can be replaced with another formulation to provide fast response [25], [28], which is dealt with Section III.

C. GAIN **M**¯

Combining [\(2\)](#page-1-2)–[\(6\)](#page-2-1) leads to the following closed-loop dynamics:

$$
\ddot{\mathbf{e}} + \mathbf{K}_D \dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e} = \bar{\mathbf{M}}^{-1} [\mathbf{N} - \mathbf{N}_{t-L}]. \tag{7}
$$

[\(7\)](#page-2-2) displays that the constant diagonal matrix \overline{M} apparently affects the control performance. In fact it has been our observation that M is the most dominant and crucial parameter for the performance of the TDC-based system.

In addition to the performance, M is crucial to the stability of the closed-loop system, which is made plain by the well established condition for the stability of TDC [7], [8]:

$$
\left\| \mathbf{I} - \mathbf{M}^{-1}(\mathbf{q}) \bar{\mathbf{M}} \right\| < 1. \tag{8}
$$

When the stability condition [\(8\)](#page-2-3) is met, the right-hand side **N**−**N***t*−*^L* of [\(7\)](#page-2-2) is bounded [7], [20]. The bounded TDE error vector ε is defined as follows:

$$
\varepsilon \stackrel{\Delta}{=} N(q, \dot{q}, \ddot{q}) - N_{t-L}.
$$
 (9)

For this reason, the selection of M is very important for the closed-loop stability of the TDC-based system.

 \overline{M} can be theoretically derived from (8) (8) , provided that the exact value of \overline{M} is known. But in practice, the exact value of \overline{M} is difficult to estimate and \overline{M} is varying according to the posture of the robot. As a result, in practical applications \overline{M} is manually tuned by a trial-and-error. A well tuned \overline{M} guarantees stability, whereas improperly selected \overline{M} easily puts the

system at risk from instability. The gain selection becomes an intricate problem because even an appropriately chosen value gets excessive afterwards, depending on the variation of robot inertias. The effort and time for gain selection and its intricacy justify and call for the TDC with an adaptive gain dynamics.

III. ADAPTIVE GAIN DYNAMICS AND PROPOSED CONTROL

Out of the necessity for an adaptive \overline{M} that ensures stability, we are going to propose an adaptive gain dynamics, the core of this article, for that purpose. In conjunction, performance enhancement is attempted by introducing a nonlinear DED to speed up the response and a suppressing term to TDE error in [\(9\)](#page-2-4).

For the fast-tracking response, the linear desired error dynamics (DED) in [\(5\)](#page-2-0) is substituted with the nonlinear DED in [28] as follows:

$$
\ddot{\mathbf{e}} + \mathbf{K}_D \mathbf{sig}(\dot{\mathbf{e}})^\alpha + \mathbf{K}_P \mathbf{sig}(\mathbf{e})^\beta = 0, \tag{10}
$$

where

$$
\mathbf{sig}(\dot{\mathbf{e}})^{\alpha} = \begin{bmatrix} |\dot{e}_1|^{\alpha_1} \operatorname{sgn}(\dot{e}_1), \cdots, |\dot{e}_n|^{\alpha_n} \operatorname{sgn}(\dot{e}_n) \end{bmatrix}^T, \quad (11)
$$

$$
\mathbf{sig}(e)^{\beta} = \left[|e_1|^{\beta_1} sgn(e_1), \cdots, |e_n|^{\beta_n} sgn(e_n) \right]^T, \quad (12)
$$

with α , β denoting exponent vectors, respectively (α ^{*i*} > 0, β_i > 0). This DED being reduces to [\(5\)](#page-2-0) when $\alpha_i = 1$ and $\beta_i = 1$, it may be regarded as a *generalized* form of [\(5\)](#page-2-0). The details are referred in [28].

A. DESIGN OF ADAPTIVE GAIN DYNAMICS

Using the TDE (4) and the nonlinear DED (10) , we can design the control law as follows:

$$
\tau = \tau_{t-L} - \bar{M}\ddot{q}_{t-L} + \bar{M}[\ddot{q}_d + K_D sig(\dot{e})^\alpha + K_P sig(e)^\beta].
$$
 (13)

Since the gain matrix M is to be adapted, it is no more a constant, but a variable. Our approach is to make it a function of s , $M = M(s)$, where *s* denotes a sliding variable vector, which roughly stands for the tracking error. Thus, the adaptation is to be carried out, monitoring the tracking error and suppressing it.

Let us select a new sliding variable according to the DED in [\(10\)](#page-2-5) as follows:

$$
\mathbf{s} = \int \left[\ddot{\mathbf{e}} + \mathbf{K}_D \mathbf{sig}(\dot{\mathbf{e}})^\alpha + \mathbf{K}_P \mathbf{sig}(\mathbf{e})^\beta \right] dt \tag{14}
$$

where the initial value of *s* is set to zero. This variable *s* represents fast converging dynamics, differently from the *s* based on the linear error dynamics [\(5\)](#page-2-0).

This sliding variable is applied to an adaptation scheme we previously proposed in [27], whose time derivative is written as

$$
\dot{\overline{M}}_{ii} = \gamma_{ii} \overline{M}_{ii}^2 |s_i| sgn(|s_i| - \frac{\overline{M}_{ii}^2}{\delta_i}).
$$
\n(15)

Here \bullet_i and \bullet_{ii} denote the *i*-th element of a vector and the *ii*-th element of a diagonal matrix, respectively; γ*ii* the element of

FIGURE 1. Block diagram of the proposed TDC with an effective gain dynamics for robot manipulators.

the positive diagonal matrix, which we term the *adaptation gain*; and δ_i a normalizing factor between s_i and $\overrightarrow{M}_{ii}^2$. The term \bar{M}_{ii}^2/δ_i is defined as the *acceptance layer*, which will be detailed later.

By introducing *s*in [\(14\)](#page-2-6), we have created in effect a version of adaptive gain dynamics for faster convergence, a gain adaption dynamics that pushes the $\overline{M}(s)$ toward a stable range while meeting the requirement of faster convergence.

Let us elaborate on the adaptation mechanism embedded in [\(15\)](#page-2-7). Facing large tracking errors, s_i becomes larger so that $|s_i| > \bar{M}_{ii}^2/\delta_i$, making *sgn*() positive and increasing \bar{M}_{ii} , until it becomes large enough to compensate for the tracking errors. When the errors are small enough, on the other hand, *s*^{*i*} becomes smaller, leading to $|s_i| < M_{ii}^2/\delta_i$, making *sgn*() negative, and decreasing the value of \overline{M}_{ii} to avoid excessively large gain. This procedure is very much like the concept of variable boundary layers in sliding mode control (SMC). Consequently, as the adaptation proceeds, the variable $|s_i|$ converges to the vicinity of $\overline{M}_{ii}^2/\delta_i$.

Clearly, the term $\overline{M}_{ii}^2/\delta_i$ sets the limit of *s* for the adaptation and determines the tracking error *e*. Notice that the larger δ is, the smaller both *s* and *e* become. In this paper, $\overline{M}_{ii}^2/\delta_i$ is used to get an appropriate gain $\overline{M}(s)$ that meets the stability condition.

On the other hand, the adaptive gain \overline{M}_{ii} is bounded to be $\bar{M}_{ii} < \sqrt{|s_i| \delta_i}$ when \bar{M}_{ii} increases – $|s_i| > \bar{M}_{ii}^2/\delta_i$, and $\bar{M}_{ii} >$ $\sqrt{|s_i|\delta_i}$ when \bar{M}_{ii} decreases – $|s_i| < \bar{M}_{ii}^2/\delta_i$ by sgn() in [\(15\)](#page-2-7). Thus, \bar{M}_{ii} is bounded by the proposed gain dynamics [\(15\)](#page-2-7).

B. THE PROPOSED CONTROL

Before we propose the final control, let us explain the additional term in it to suppress the TDE error. It was demonstrated in [\(7\)](#page-2-2) that the closed-loop dynamics due to the linear

DED is affected by the TDE error [\(9\)](#page-2-4). In the same way, it is easy to prove that the one from the nonlinear DED, too, is influenced by the TDE error. Specifically, just as the linear DED yields the closed-loop dynamics in [\(7\)](#page-2-2), so does the nonlinear DED the following closed-loop dynamics:

$$
\ddot{\mathbf{e}} + \mathbf{K}_D \mathbf{sig}(\dot{\mathbf{e}})^\alpha + \mathbf{K}_P \mathbf{sig}(\mathbf{e})^\beta = \bar{\mathbf{M}}(\mathbf{s})^{-1} \boldsymbol{\varepsilon},\tag{16}
$$

where the LHS is equivalent to *s* according to [\(14\)](#page-2-6). Thus,

$$
\dot{\mathbf{s}} = \bar{\mathbf{M}}^{-1}(s) \cdot \boldsymbol{\varepsilon}.\tag{17}
$$

Clearly, [\(17\)](#page-3-0) show that the TDE error ε has a direct effect on both *s* and the tracking error and that its suppression is important for better tracking accuracy. To suppress the TDE error, the additional term λ**sig**(**s**) ^ψ is combined with [\(13\)](#page-2-8), absorbing the residual energy perturbed by the TDE error [28].

As a result, the *proposed enhanced control law* can be written as

$$
\tau = \underbrace{\tau_{t-L} - \bar{M}(s)\ddot{q}_{t-L}}_{\text{TDE}} + \underbrace{\bar{M}(s)}_{\text{Adaptive Gain Suppression of TDE error}} + \bar{M}(s) \cdot \underbrace{\bar{q}_{d} + \mathbf{K}_{D}\text{sig}(e)^{\alpha}}_{\text{Nonlinear DED}} + \mathbf{K}_{P}\text{sig}(e)^{\beta} \tag{18}
$$

and the closed-loop error dynamics becomes

$$
\dot{\mathbf{s}} + \lambda \mathbf{sig}(\mathbf{s})^{\psi} = \bar{\mathbf{M}}^{-1}(s) \cdot \varepsilon. \tag{19}
$$

One can clearly see the proposed control does not use a robot model at all; it is model-free. Notice that the effect of the TDE error is attenuated since [\(19\)](#page-3-1) can be regarded as a first-order low pass filter – the input ε and output *s*. Illustrated in Fig. [1](#page-3-2) is a closed-loop system with the TDE, the gain dynamics, the nonlinear DED, and the TDE error correction.

C. STABILITY ANALYSIS

We are going to prove that the closed-loop system with gain dynamics [\(15\)](#page-2-7) is uniformly ultimately bounded.

Assumption 1: The upper bound of ε_i is denoted by ε_i^+ *i* . It has been proven in [9], [29] that the TDE error ε_i is bounded, if the gain \overline{M}_{ii} meets the sufficient condition for the stability [\(8\)](#page-2-3).

Assumption 2: The sliding variable *sⁱ* is close to zero at the time $t = 0$ since it is a common practice that the desired trajectory is generated based on the initial posture of a robot manipulator.

Since s_i and \overline{M}_{ii} are interrelated, we employed the above two assumptions. When $s_i \approx 0$ at $t = 0$, \overline{M}_{ii} is rapidly two assumptions. When $s_i \approx 0$ at $i = 0$, M_{ii} is rapidly approaching $\sqrt{|s_i| \delta_i}$ by γ_{ii} of the gain dynamics [\(15\)](#page-2-7) and s_i is governed by the closed-loop error dynamics [\(19\)](#page-3-1).

Theorem 1: The closed-loop system is uniformly ultimately bounded if the following condition is satisfied such that

$$
\varepsilon_i^+ < \gamma_{ii}.\tag{20}
$$

proof: First, the range is assumed as $\forall |s_i| > \bar{M}_{ii}^2/\delta_i$. With consideration of [\(15\)](#page-2-7), a Lyapunov-like candidate is defined as

$$
V = \frac{1}{2}\mathbf{s}^{\mathrm{T}}\mathbf{s} + \frac{1}{2}\sum_{i=1}^{n} \left(\frac{1}{\bar{M}_{ii}}\right)^{2}.
$$
 (21)

where $\bar{M}_{ii} > 0$.

According to [\(15\)](#page-2-7), when $\forall |s_i| > \overline{M}_{ii}^2/\delta_i$, the adaptive gain \overline{M}_{ii} increases and the time derivative of *V* can be rewritten as

$$
\dot{V} = \mathbf{s}^{\mathrm{T}} \dot{\mathbf{s}} - \sum_{i=1}^{n} (\bar{M}_{ii}^{-3}) \dot{\bar{M}}_{ii}
$$
\n
$$
= \mathbf{s}^{T} (-\lambda \cdot |\mathbf{s}|^{\Psi} \cdot \text{sgn}(\mathbf{s}) + \bar{M}^{-1} \mathbf{\varepsilon}) - \sum_{i=1}^{n} [\gamma_{ii} \bar{M}_{ii}^{-1} |s_{i}|]
$$
\n
$$
\leq \sum_{i=1}^{n} [-\lambda_{ii} |s_{i}| \cdot |s_{i}|^{\psi} + |s_{i}| \bar{M}_{ii}^{-1} \{|\varepsilon_{i}| - \gamma_{ii}\}]. \tag{22}
$$

If $|\varepsilon_i| < \gamma_{ii}$, \dot{V} is negative definite. Thus, the closed loop system is Lyapunov stable when $\varepsilon_i^+ < \gamma_{ii}$.

When $\forall |s_i| < \bar{M}_{ii}^2/\delta_i$, the solution of a given closed-loop system is close to a sliding manifold, but it does not exactly reach $|s_i| = 0$, owing to the acceptance layer \bar{M}_{ii}^2/δ_i . Since the closed-loop system is uniformly ultimately bounded, \dot{V} is not considered in $\forall |s_i| < \bar{M}_{ii}^2/\delta_i$.

IV. SIMULATION

In this section and next, we are going to present simulations and experiments in a *complementary* way in order to show both the characteristics and effectiveness of the proposed adaptation method. Through simulations on a simple one-link arm, we intend to display the essential nature of our method in a transparent way; experiments on a three-degrees of freedom spatial robot demonstrate its capability and performance in more realistic and complicated environments.

FIGURE 2. (a) One-link robot manipulator. (b) Desired trajectory.

FIGURE 3. Simulation results of the proposed control: (a) adaptive gain (solid) and the upper bound (dashed) by stability condition (8). (b) Tracking error. (c) Control input.

A. SIMULATION SETUP

The simulation with a one-link arm shown in Fig. [2](#page-4-0) (a) is conducted to investigate if the proposed adaptation transfers the gains to the stable range determined by [\(8\)](#page-2-3). The arm dynamics is given as follows:

$$
\tau = I\ddot{q} + G(q) + F(\dot{q}),\tag{23}
$$

where $I = ml^2$, $G(q) = mlgcos(q)$, and $F(\dot{q}) = f_V \dot{q} + g$ $f_C sgn(\dot{q})$ where f_V and f_C denote the coefficients for the viscous friction and Coulomb friction, respectively. The param-eters in [\(23\)](#page-4-1) are set to be $m = 1.0$ kg, $l = 1.0$ m, $f_V =$ 5.0 N·m·s, $f_C = 5.0$ N·m, and $g = 9.8$ m/ s^2 , respectively. The desired displacement is shown in Fig. [2](#page-4-0) (b).

The nonlinear DED is determined as $\ddot{e} + 20 \cdot \text{sig}(\dot{e})^{95/100} +$ $100 \cdot sig(e)^{95/105} = 0$. For the suppression of TDE error, λ and ψ are tuned to be $\lambda = 10$ and $\psi = 0.8$, respectively. The

FIGURE 4. The phase portrait: (a) The proposed adaptive TDC. (b) The TDC with constant gains [28].

parameters γ and δ are tuned by trial and error as $\gamma = 1800.0$ and $\delta = 140.0$ for the gain dynamics [\(15\)](#page-2-7), respectively. Notice that δ has to be varied from a small positive value to a large value because $|s_i|$ is approaching to $\dot{\bar{M}}_{ii}^2/\delta$.

The initial value of $\overline{M}(s)$ is intentionally set to be $\overline{M} = 4$ $Kg \cdot m^2$ which is an unstable gain far outside the stable range, $0 < \overline{M} < 2$ determined by [\(8\)](#page-2-3) – note the dashed line in Fig. [3](#page-4-2) (a), which denotes the upper bound for the stable range of M . The upper bound is also used to discern if the initial value of $M(s)$ indeed belongs to the unstable range.

To indicate the significance of the proposed control, the gain M is tuned as constant values – 0.5, 2.1, 2.5, 3.0, and 4.0, respectively. Notice that the proposed method becomes the same as the one in [28] when the \overline{M} is set as a constant value.

B. SIMULATION RESULTS

Simulation results are shown in Fig. [3.](#page-4-2) Fig. [3](#page-4-2) (a) displays how the gain adaptation changes from the unstable initial value toward a stable one. The tracking error and the control input are shown in Fig. [3](#page-4-2) (b) and (c), respectively. When large tracking errors occur mainly due to the Coulomb friction, they are compensated by the enlarged $\overline{M}(s)$, which has been increased by the gain adaptation. In this way, the adaptive gain

FIGURE 5. (a) PUMA-type robot (Faraman-AT2). (b) Desired trajectory of joints 1, 2, and 3.

dynamics first *transfers* an unstable gain to the stable range, and then makes it *remain* there by resiliently adjusting the gain according to the variation of tracking errors.

To obtain further insight into the adaptive gain dynamics in [\(15\)](#page-2-7), we have plotted the phase portraits of the sliding variable *s* and the gain M . Fig. [4](#page-5-0) (a) shows the gain adaptation of the proposed control, whereas Fig. [4](#page-5-0) (b) the five gains of the TDC with fixed gains. Fig. [4](#page-5-0) (a) displays that the gain dynamics brings the initially unstable M to the stable range, the same trend observed in Fig. [3](#page-4-2) (a).

On the other hand, the five constant gains of the TDC with fixed gains remain *unchanged*, as shown in Fig. [3](#page-4-2) (b), producing respective response, either stable or unstable, according to the value of each gain. Specifically, while \overline{M} with respective values of 0.5, 2.1, 2.5, 3.0, and 4.0 remains the same, all the gains cause unstable response on the phase portrait, *except* the gain with 0.5 exhibits stable response.

V. EXPERIMENT

A. EXPERIMENTAL SETUP

The effectiveness of the proposed enhanced adaptive TDC has been demonstrated through the experiment with a PUMA-type robot in Fig. [5](#page-5-1) (a). The maximum continuous torques of AC servo motors are 0.637, 0.319, and 0.319 Nm, and each harmonic drive provides gear ratios of 120:1 for joints 1, 2, and 3, respectively. Resolution of each encoder is 2048 pulses/rev, which is equivalent to an angular resolution is 3.66×10^{-4} deg at each joint (quadrature encoder). The control system was operated under Linux RTAI, a realtime operating system environment. The desired trajectory is shown in Fig. [5](#page-5-1) (b) for joints 1, 2, and 3, respectively. The sampling period *L* is set as 0.001 *s*.

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FIGURE 6. Experimental comparison of proposed TDC with adaptive gain dynamics and the IETDC. (a)–(c) Gains M. (d)–(f) Tracking errors. (g)–(i) Control inputs of joints 1, 2, and 3 (dotted lines: the upper bound by IETDC (upper.); solid lines: proposed TDC with a small initial value–proposed. (small); dashed lines: proposed TDC with a unstable initial value–proposed. (large)).

1) SCENARIO 1.STABLE GAIN ADAPTATION

For the stable gain adaptation with the proposed enhanced adaptive TDC [\(18\)](#page-3-3), the nonlinear DED is selected as \ddot{e} + $20 sig(\dot{e})^{95/100} + 100 sig(e)^{95/105} = 0$, and parameters λ and ψ are tuned to be $\lambda = 20$ **I** and $\psi = [0.8, 0.8, 0.8]^T$, respectively. For the gain dynamics, γ and δ in [\(15\)](#page-2-7) are tuned as $\gamma = \text{diag}(5.0, 3.0, 2.8) \times 10^4$ and $\delta = [0.015, 0.008,$ 0.008 ^T, respectively. We have selected two sets of initial values for $\bar{M}(s)$: $\bar{M}_{large} = \text{diag}(1.273, 1.273, 0.636)$, which is in the unstable range; and $\bar{M}_{small} = \text{diag}(0.051, 0.051, 0.025)$ in the stable range.

In order to show that the adaptive gain using [\(15\)](#page-2-7) converges to the stable range, the applicable largest constant gain \overline{M} is intentionally tuned as $\overline{\dot{M}}_{upper} = \text{diag}(1.069, 0.890, 0.484)$. It was tuned by trial-and-error because the inertia matrix is difficult to estimate. Of the gains thus estimated, the largest one is assumed as the upper bound, which is used to confirm that the initial value starts from the unstable range.

2) SCENARIO 2. COMPARISON WITH A PREVIOUS METHOD The proposed adaptive TDC is compared with the inclusive enhanced TDC (IETDC) [28], a generalized formulation of TDC-based controllers with a constant \overline{M} . Note that the

proposed method become the same as the IETDC, when the adaptive gain $\overline{M}(s)$ becomes a constant. The constant \overline{M} of the IETDC is set as $\bar{M} = diag(0.3818, 0.3055, 0.153)$ which was a best-tuned gain in [28].

B. EXPERIMENTAL RESULTS

The experimental results of Scenario 1 are shown in Fig. [6.](#page-6-0) As shown in Figs. 6 (a)-(c), the adaptive gain $M(s)$ is automatically adjusted by proposed gain dynamics [\(15\)](#page-2-7). Comparison of the gain matrices, $\overline{M(s)}$ and $\overline{M_{upper}}$, as shown in Figs. [6](#page-6-0) (a)-(c), reveals that the proposed adaptive gain approaches stable range of M by the gain dynamics (15) (15) . The dotted lines by \overline{M}_{upper} in Figs. [6](#page-6-0) (a)-(c) can be regarded as the upper bound value found by trial and error. Figs. 6 (d)-(f) and (g)-(i) displays satisfactory tracking accuracy is achieved without noticeable chattering with the proposed method owing to the appropriate gain adaptation of $M(s)$ regardless of initial values in $M(s)$.

To help gain more insights, the phase portraits of \bar{M}_{ii} and s_i are shown in Fig [7.](#page-7-0) The proposed method using the gain dynamics converges on the stable range of \overline{M}_{ii} even though it starts at an unstable initial gain. The proposed TDC with the gain dynamics has provided more appropriate gains according to system states.

FIGURE 7. Proposed adaptive TDC with unstable initial condition. The phase portraits of $\bar{M}_{\bar{i}i}$ and s_i of joints 1, 2, and 3, respectively.

Fig. [8](#page-7-1) demonstrates how the adaptive gain goes to the stable range. The absolute value of the sliding variable s_i is converging to the acceptance layer \bar{M}_{ii}^2/δ_i , which is a similar concept with the variable boundary layer of adaptive SMC. When the sliding variable becomes larger than the acceptance layer, the gain increases by gain dynamics [\(15\)](#page-2-7), and when the sliding variable is relatively small, the gain decreases. As a result, the adaptive gain is regulated without unnecessary high gain.

In order to test the tracking accuracy of the proposed method, we have compared it with that of the best-tuned

FIGURE 8. Experimental results of proposed adaptive TDC. (a)–(c) Absolute values of "s $_i$ " (dotted) and acceptance layers (AL) "(M)²/ δ_i " (solid) for joints 1, 2, and 3, respectively.

TABLE 1. RMS values of the tracking errors (×10⁻³ deg).

Controller	Joint 1	Joint 2	Joint 3
IETDC [28]	5.8595	8.4020	9.0724
Proposed. (small)	5.8507	7.9217	8.5936
Proposed. (large)	5.8493	7.9013	8.5065
			(1 s to 10 s)

IETDC [28], In doing so, we used two initial values for $M(s)$: an unstable value, denoted by 'Proposed. (large)'; and a stable one, 'Proposed. (small)'.

Fig. [9](#page-8-0) shows the comparison results. More specifically, in Figs. [9](#page-8-0) (a)-(c), the two adaptive gains starting from two different initial values are converging to each other and staying in the vicinity of the IETDC gain. Their corresponding tracking errors are displayed in Figs. [9](#page-8-0) (d)-(f). The tracking errors appear very similar with respect to time, showing that the proposed control produces the response as fast as the IETDC, which claimed to achieve the fastest response [28]. The magnitude of errors, too, looks very similar, requiring a close comparison based on the root-mean-square (RMS)

FIGURE 9. Experimental comparison of the proposed adaptive TDC with small initial values (proposed. (small)) and unstable initial values (proposed. (large)), and the IETDC with the best-tuned gains (IETDC). (a)–(c) Gains M. (d)–(f) Tracking errors of joints 1, 2, and 3 (dotted lines: proposed. (small); ¯ solid lines: proposed. (large); dashed line: IETDC).

error, which is listed in Table [1.](#page-7-2) Table [1](#page-7-2) reveals that the proposed method achieves better accuracy than the IETDC, regardless of the initial values.

This comparison clearly supports the efficacy of the proposed method. It is remarkable that, wherever the initial value may lie, be it in the stable range or unstable one, the adaptation dynamics enables to find a gain that is not just stable but better than a best-tuned gain by trial and error.

VI. CONCLUSION

We have proposed an adaptive gain dynamics together with the TDC for the robot manipulator. We have shown through simulation and experiment that the proposed control enables to transfer $M(s)$ from an unstable initial value to a stable one better than a best-tuned gain. As a result, the proposed control, which is also model-free, achieves responses as fast as the IETDC, whereas the tracking accuracy achieved is slightly better than IETDC. Thanks to the proposed adaptation, the gain selection has become care-free.

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JUNYOUNG LEE (Member, IEEE) received the B.S. degree in electronics engineering from Keimyung University, Daegu, South Korea, in 2011, and the M.S. and Ph.D. degrees in robotics engineering from the Daegu–Gyeongbuk Institute of Science and Technology (DGIST), Daegu, in 2013 and 2019, respectively.

He is currently a Senior Researcher with the Human-Centered Robotics Research Center, Korea Institute of Robotics and Technology Con-

vergence (KIRO), Pohang, South Korea. His research interests include rehabilitation robotics, rescue robotics, the adaptive and robust control of nonlinear systems, and impedance control, including dual-arm manipulators.

PYUNG HUN CHANG (Member, IEEE) received the M.Sc. and Ph.D. degrees in mechanical engineering from the Massachusetts Institute of Technology (MIT), Cambridge, in 1984 and 1987, respectively.

He was a Professor at the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, South Korea, for 24 years. From March 2011 to February 2017, he was a Professor of the Department of Robotics Engineering,

Daegu–Gyeongbuk Institute of Science and Technology (DGIST), Daegu, South Korea, where he was also the Vice President of academic affairs, from April 2015 to February 2017. He is currently an Emeritus Professor with the Department of Robotics Engineering, DGIST. He has authored more than 83 articles in international journals, 23 articles in domestic journals, and 100 papers in conference proceedings. His current research interests include the rehabilitation robotics and the robust control of nonlinear systems, including kinematically redundant manipulators and the impedance control of dual-arm manipulators.

Dr. Chang received the Best Paper Award from the International Workshop of Human-Welfare Robotic Systems, in 2006, and the Best Research Award from the Agency for Defense Development, in 2010. He was a Plenary Speaker of the Fifth IEEE RAS/EMBS International Conference on Biomedical Robotics and Biomechatronics, in 2014.

BYEONGGI YU (Student Member, IEEE) received the B.S. degree in aerospace engineering from Korea Aerospace University, Goyang, South Korea, in 2013, and the M.S. degree in robotics engineering from the Daegu–Gyeongbuk Institute of Science and Technology (DGIST), Daegu, South Korea, in 2015, where he is currently pursuing the Ph.D. degree with the Department of Robotics Engineering.

His research interests include rehabilitation robotics, neurorehabilitation, and neurophilosophy.

KAP-HO SEO (Member, IEEE) received the B.S. degree in electrical engineering from Korea University, Seoul, South Korea, in 1999, and the M.S. and Ph.D. degrees in electrical engineering from the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, South Korea, in 2001 and 2009, respectively.

He was a Senior Researcher with the Korea Institute of Robotics and Technology Convergence (KIRO), Pohang, South Korea, from 2009 to

March 2016. He is currently working as the Director and a Chief Researcher with the Interactive Robotics Research and Development Division, KIRO. He is currently the Director and an Adjunct Professor with the Department of Mechanical Engineering, Pohang University of Science and Technology (POSTECH), Pohang. His current research interests include embedded systems, including HILS, intelligent control, wearable robotics, and force interaction between humans and robots, applied to disaster response robots and rehabilitation robot systems.

MAOLIN JIN (Senior Member, IEEE) received the B.S. degree in material science and mechanical engineering from the Yanbian University of Science and Technology, Jilin, China, in 1999, and the M.S. and Ph.D. degrees in mechanical engineering from the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, South Korea, in 2004 and 2008, respectively.

He was a Postdoctoral Researcher with the Mechanical Engineering Research Institute, KAIST, in 2008. From Novem-

ber 2008 to February 2016, he was a Senior Researcher with the Research Institute of Industrial Science and Technology, Pohang, South Korea. He is currently the Director and a Chief Researcher with the Human-Centered Robotics Research Center, Korea Institute of Robotics and Technology Convergence (KIRO), Pohang. His research interests include the robust control of nonlinear plants, time-delay control, robot motion control, electro-hydraulic actuators, winding machines, disaster robotics, and factory automation.

Dr. Jin serves as an Associate Editor for the *International Journal of Control, Automation, and Systems* (IJCAS), the *Journal of Drive and Control*, and the *Journal of the Korean Society for Precision Engineering*.

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