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# PWM-Based Adaptive Trajectory Tracking of Switched Buck Power Converters

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**ABSTRACT** Based on pulse width modulation (PWM) technique, adaptive trajectory tracking of switched Buck power converters with parameter uncertainty is investigated in this article. By the aid of the backstepping technique, an equivalent continuous adaptive tracking controller is designed to guarantee the asymptotic stability of the closed-loop error continuous system. For the switched Buck converter with unknown parameters, PWM-based adaptive tracking with the equivalent adaptive continuous control input is proposed such that the closed-loop error digital system is practically asymptotically stable, and the tracking error converges to an arbitrarily small neighborhood of the origin. Simulation results are shown to illustrate the effectiveness of the proposed new control schemes.

**INDEX TERMS** PWM control, adaptive backstepping, tracking, switched buck converters.

## I. INTRODUCTION

Switched DC-to-DC power converters, described by the multiple circuit topologies associated with the regulating switch position [1], is used to realize the voltage conversion of power electronic devices, which have widespread applications in DC voltage transformation, DC motor drives, switching power supply, and electric vehicles systems. The basic topologies of switched DC-to-DC power converters can be divided into are buck converter, boost converter and buck-boost converter. With the development of modern electronic technology, the increase targets of DC-to-DC power converters are required with output voltage stability and rapid system regulation. The research on control theory of these power converters has been paid great attention [2]–[4]. As we known, the switching signal is designed or regulated for changing the motion state of switched systems without external control input. Some switching strategies have been proposed for stability of switched DC-to-DC power converters on the basis of switched systems theory [5], such as switching rule based on combination of sliding mode control and equivalent control input [8]–[11], state-dependent switching on the basis of variable-structure control [7], optimal switching instants by means of a numerical optimization approach [6]. Remark that the existing results obtained in above literature only focused on the switching signal as the control variable directly.

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In particular, the output-voltage regulation problem for PWM-based DC-DC power converters has recently been the most important consideration. As the sampling frequency approaches to infinity, the limit approximation model of the PWM-based controlled system is referred to as the average model, which is a continuous control system, and the control input (i.e., duty ratio) is a continuous signal. For continuous average models of PWM-based power converters, a large amount of research results have been established on point regulating based on continuous control theory [12]. For example, the point regulating problem was investigated by passivity-based control method [13]–[15], energy shaping control with damping injection scheme [16], [17], and state feedback indirect control with non-minimum phase approach [18], [19]. Because of the effect of thermal noise in circuit systems, the circuit system performance can be reduced by the uncertainty, which is produced from variations of supply voltage, changes of load resistances, and electromagnetic interference generated by the switching actions. In order to achieve a satisfactory control effect, a controller is required to have stronger capacity of resisting disturbance, smaller steady-state error, faster dynamical response, and so on. To enhance output-voltage performance and estimate the uncertain parameters, many control approaches have been developed with better robustness and adaptability, such as adaptive backstepping technique [20], [21], sliding mode control method [22], [23], adaptive Chebyshev neural network technique [24], output feedback control with reduced-order observer [25], [26], and

robust control [27]. Remark that these advanced control methods are only considered for point regulating of continuous average models. However, the existing results mentioned above have two shortcomings: first, the stability relationship is not discussed between continuous average models and PWM-based DC-DC power converters, and second, the existing performance control schemes would be incapable for trajectory tracking of PWM-based DC-DC power converters. Therefore, for PWM-based DC-DC power converters, a natural and nontrivial control problem is that, how does one design a switched controller and analyze system stability for trajectory tracking and system uncertainties?

In this article, the adaptive trajectory tracking is investigated for switched Buck power converters with unknown parameters, where the system input is a digital switch control. The digital switch control is designed based on combination of adaptive backstepping method and PWM technique. Compared with the literature [20], [21], an equivalent continuous adaptive backstepping controller is designed such that all signals of the closed-loop error continuous system are asymptotically stable. For the switched Buck converter, PWM-based adaptive tracking with the equivalent control input is proposed such that the closed-loop error digital system is practically asymptotically stable, and the tracking error tends to an arbitrarily small neighborhood of the origin by tuning design parameters in contrast with [24], [26], [27].

In summary, the main contributions of the paper are highlighted as the following twofold:

1) *A novel framework on adaptive trajectory tracking is established for switched Buck power converters.* Because the existing results (i.e., [20]–[27]) focused on point regulating problem of continuous average models, where the control input is continuous signal, and hence these existing control methods are ineffective for switched power converters. In this article, a novel framework on PWM-based adaptive trajectory tracking is established for switched Buck power converters with unknown parameters, where the system input is a digital switch control.

2) *The stability relationship is discussed between continuous average models and switched Buck power converters compared with the existing results.* The stability analysis in the existing literature [13]–[27] was only discussed for the corresponding continuous average models with respect to point regulating problem. For trajectory tracking in this article, we not only testify asymptotical stability of the closed-loop continuous system, but also prove that the corresponding closed-loop digital system is practically asymptotically stable if continuous control is replaced by digital control.

The paper is organized as follows. Some preliminaries are presented in Section II; Problem formulation and system description are proposed in Section III; PWM-based adaptive tracking control is researched in Section IV; Simulation results for Buck converters are given by Simscape in Section V; The paper is concluded in Section VI.

*Notations:* For a vector  $x$ ,  $|x|$  denotes its usual Euclidean norm and  $x^T$  denotes its transpose;  $\mathbb{R}_+$  denotes the set of all

nonnegative real numbers;  $\mathbb{R}^n$  denotes the real  $n$ -dimensional space;  $\mathbb{R}^{n \times r}$  denotes the real  $n \times r$  matrix space;  $\mathcal{C}^i$  denotes the set of all functions with continuous  $i$ -th partial derivative;  $\mathcal{K}$  stands for the set of all functions:  $\mathbb{R}_+ \rightarrow \mathbb{R}_+$ , which are continuous, strictly increasing and vanishing at zero;  $\mathcal{K}_\infty$  denotes the set of all functions which are of class  $\mathcal{K}$  and unbounded;  $\mathcal{KL}$  stands for the set of all functions  $\beta(s, t) : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which is of class  $\mathcal{K}$  for each fixed  $t$ , and decreases to zero as  $t \rightarrow \infty$  for each fixed  $s$ ;  $AB$  denotes the distance between  $A$  and  $B$ ;  $S_{ABCD}$  denotes the area of a rectangle  $ABCD$ .

## II. PRELIMINARIES

In this section, we give some results about nonlinear systems and switched Lagrange systems.

### A. STABILITY OF NONLINEAR SYSTEMS

Consider a nonlinear system described by

$$\dot{x} = f(x, t), \quad x(t_0) = x_0, \quad (1)$$

where  $x \in \mathbb{R}^n$  is the state, function  $f : \mathbb{R}^n \times [t_0, \infty) \rightarrow \mathbb{R}^n$  is locally Lipschitz in  $x \in \mathbb{R}^n$  and piecewise continuous in  $t$  for all  $t \geq t_0$ , and  $f(0, 0) = 0$ .

The following definitions and Lemma will be used in stability analysis.

*Definition 1* [30]: System (1) is said to be globally asymptotically stable (GAS), if there exists a class- $\mathcal{KL}$  function  $\beta(\cdot, \cdot)$ , such that for all  $t \geq t_0$ ,

$$|x(t)| \leq \beta(x_0, t - t_0), \quad \forall x_0 \in \mathbb{R}^n. \quad (2)$$

*Definition 2* [31]: System (1) is said to be globally practically asymptotically stable (GPAS), if for a given constant  $\varrho > 0$ , there exist a class- $\mathcal{KL}$  function  $\beta(\cdot, \cdot)$  and a class- $\mathcal{K}$  function  $\gamma(\cdot)$ , such that for all  $t \geq t_0$ ,

$$|x(t)| \leq \beta(x_0, t - t_0) + \gamma(\varrho), \quad \forall x_0 \in \mathbb{R}^n. \quad (3)$$

*Remark 1:* For GPAS, as  $\varrho \rightarrow 0$ ,  $\gamma(\varrho) \rightarrow 0$ , then the role of GPAS is identical to GAS. Compared with boundedness of [31], GPAS here is more accurate to describe state trajectory of systems. If  $\varrho$  becomes smaller,  $\gamma(\varrho)$  of GPAS turns to be smaller and even tends toward zero.

*Lemma 1* [31]: For system (1), if there exist a function  $V : \mathbb{R}^n \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is continuously differentiable, such that for  $\forall t \geq 0, \forall x \in \mathbb{R}^n$ ,

$$\begin{aligned} \gamma_1(|x|) &\leq V(x, t) \leq \gamma_2(|x|), \\ \dot{V} &= \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(x, t) \leq -W(x) \leq 0, \end{aligned} \quad (4)$$

where  $\gamma_1$  and  $\gamma_2$  are class  $\mathcal{K}_\infty$  functions and  $W$  is a continuous function. Then, all solutions of  $\dot{x} = f(x, t)$  are globally bounded and satisfy

$$\lim_{t \rightarrow \infty} W(x(t)) = 0. \quad (5)$$

In addition, if  $W(x)$  is positive definite, then the equilibrium  $x = 0$  is globally asymptotically stable.

**B. SWITCHED LAGRANGE EQUATION FOR ELECTRICAL SYSTEMS**

Based on the Lagrange approach, the lagrangian dynamics of electrical systems is classically characterized by the following nonlinear differential equation [28]

$$\frac{d}{dt} \left( \frac{\partial \mathcal{L}(q, \dot{q})}{\partial \dot{q}} \right) - \frac{\partial \mathcal{L}(q, \dot{q})}{\partial q} + \frac{\partial \mathcal{F}(\dot{q})}{\partial \dot{q}} = \mathcal{Q}, \quad (6)$$

where  $q \in \mathbb{R}^n$  is the vector of electric charges constituting the generalized coordinates for the circuit,  $\mathcal{L}(q, \dot{q}) = \mathcal{T}(q, \dot{q}) - \mathcal{V}(q)$  is the lagrangian function,  $\mathcal{T}(q, \dot{q})$  is the magnetic co-energy in inductors,  $\mathcal{V}(q)$  is the electric energy in capacitors,  $\mathcal{F}(\dot{q})$  is the Rayleigh dissipation function in resistors,  $\mathcal{Q}$  is the generalized force caused by the voltage associated with the generalized coordinates.

The Lagrangian modeling of electrical systems can be rewritten as

$$\frac{d}{dt} \left( \frac{\partial \mathcal{T}(q, \dot{q})}{\partial \dot{q}} \right) - \frac{\partial \mathcal{T}(q, \dot{q})}{\partial q} + \frac{\partial \mathcal{V}(q)}{\partial q} + \frac{\partial \mathcal{F}(\dot{q})}{\partial \dot{q}} = \mathcal{Q}. \quad (7)$$

Following [29], we refer to the set of functions  $(\mathcal{T}, \mathcal{V}, \mathcal{F}, \mathcal{Q})$  as the Euler-Lagrange parameters of the Lagrangian system (7). In other words, the system (7) can be simply expressed as

$$\Sigma = (\mathcal{T}, \mathcal{V}, \mathcal{F}, \mathcal{Q}). \quad (8)$$

In fact, electrical systems usually have multiple switch controls. For instance, there is a switch control in the electrical system, when the switch is open or closed, the electrical system presents different circuit topologies. The Euler-Lagrange parameters of the two circuit topologies are generated by the different switch position values. Let  $\sigma$  denote the switching signal,  $\sigma = 1$  if the switch is closed, and  $\sigma = 0$  if the switch is open. Then,  $\sigma(t) \in \{0, 1\}$ , we introduce the set of switched Euler-Lagrange parameters  $\Sigma_\sigma = \{\mathcal{T}_\sigma, \mathcal{V}_\sigma, \mathcal{F}_\sigma, \mathcal{Q}_\sigma\}$  with the switching signal  $\sigma$ , the switched electrical system can be presented nominally as

$$\frac{d}{dt} \left( \frac{\partial \mathcal{T}_\sigma(q, \dot{q})}{\partial \dot{q}} \right) - \frac{\partial \mathcal{T}_\sigma(q, \dot{q})}{\partial q} + \frac{\partial \mathcal{V}_\sigma(q)}{\partial q} + \frac{\partial \mathcal{F}_\sigma(\dot{q})}{\partial \dot{q}} = \mathcal{Q}_\sigma. \quad (9)$$

**III. PROBLEM FORMULATION AND SYSTEM MODELING**

**A. PROBLEM FORMULATION**

A typical switch-regulated Buck converter circuit is demonstrated in Figure 1, which is composed by a DC voltage source parameter  $E$ , an inductor with inductance  $L$ , a capacitor with capacitance  $C$ , a resistor with resistance  $R$ , a diode  $VD$ , a PWM gate drive digital switch  $VT$ , and  $v_C$  respects the capacitor voltage. Introduce the control variable  $\sigma$  to denote the switch state,  $\sigma = 1$  when  $VT = \text{ON}$ , and  $\sigma = 0$  when  $VT = \text{OFF}$ . That is, such a control input takes values in the discrete set  $\{0, 1\}$ .

If the Buck converter is a ideal circuit, the system parameters are perfectly known. However, when this is not the case, namely, the converter model is not completely known in

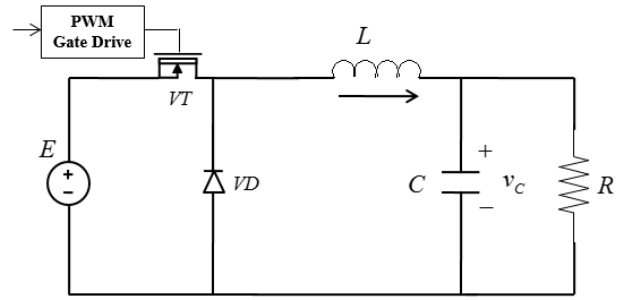


FIGURE 1. The Buck converter circuit.

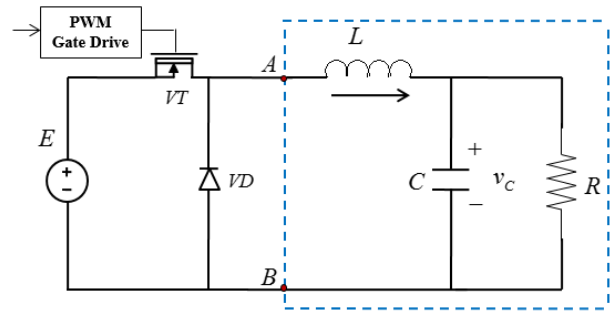


FIGURE 2. The equivalent circuit of Figure 1.

practical situations, some parameters need to be appropriately estimated for some control objective.

This article takes consideration of the case that the DC voltage source  $E$  and load resistance  $R$  are uncertain parameters. The control objective is to design a switching signal  $\sigma$  for the Buck converter, such that the capacitor voltage  $v_C$  can be driven to track a given twice continuously differentiable bounded reference voltage  $v_r(t)$  as closely as possible.

**B. MODELING OF SWITCHED BUCK CONVERTER**

In the following, the Lagrangian dynamics approach will be used for deriving the mathematical equations of switch regulated Buck converters.

Let the inductive current  $\dot{q}_L$  respect the derivative of the circulating electric charge  $q_L$ ,  $q_C$  is the electrical charge stored in the capacitor,  $q_C/C$  is the capacitor voltage,  $(q_L, q_C)^T \in \mathbb{R}^2$  is the generalized coordinate for the circuit. For the switch state with the cases of  $\sigma = 1$  and  $\sigma = 0$ , the Euler-Lagrange parameters of the two situations generated by the switch ON and OFF cases result in identical magnetic co-energy, electric filed energy, and Rayleigh dissipation function. The switching action merely changes the generalized force for the dashed box in Figure 2. Therefore, the generalized forcing structure of the system is the only one directly affected by the switch state  $\sigma$ . For the description of the switched Buck converter system, the switched Euler-Lagrange parameters are presented in the following.

$$\begin{cases} \mathcal{T}_\sigma = \frac{1}{2}L(\dot{q}_L)^2, & \mathcal{V}_\sigma = \frac{1}{2C}(q_C)^2, & \mathcal{F}_\sigma = \frac{1}{2}R[\dot{q}_L - \dot{q}_C]^2, \\ v_{AB}^\sigma = \sigma E, & \mathcal{Q}_\sigma = \mathcal{M}v_{AB}^\sigma, \end{cases} \quad (10)$$

where  $v_{AB}^\sigma$  is the voltage between  $A$  and  $B$  in Figure 2,  $\mathcal{M} = [1, 0]^T$  is the transformation matrix relating the external input  $v_{AB}^\sigma$  to the generalized force  $Q_\sigma$ . Substitute parameters (10) into the switched Euler-Lagrange equation (9), results in the switch-state parameterized differential equation

$$\begin{aligned} \frac{q_C}{C} &= -R(\dot{q}_C - \dot{q}_L), \\ L\dot{q}_L &= -R(\dot{q}_L - \dot{q}_C) + \sigma E, \end{aligned} \quad (11)$$

then, it can be derived as

$$\begin{aligned} \dot{q}_C &= \dot{q}_L - \frac{q_C}{RC}, \\ \ddot{q}_L &= -\frac{q_C}{LC} + \frac{E}{L}\sigma. \end{aligned} \quad (12)$$

Let  $x_1 = v_C = q_C/C, x_2 = \dot{q}_L$ , the switched Buck power converter is presented as

$$\begin{aligned} \dot{x}_1 &= -\frac{1}{RC}x_1 + \frac{1}{C}x_2, \\ \dot{x}_2 &= -\frac{1}{L}x_1 + \frac{1}{L}E\sigma, \end{aligned} \quad (13)$$

where  $x_1$  and  $x_2$  are the capacitor voltage and the inductor current, the control input  $\sigma$  is a digital switch control which belongs to binary set  $\{0, 1\}$ ,  $E$  and  $R$  are unknown parameters. Then, the objective is changed to design the digital switch control  $\sigma$ , such that the tracking error  $x_1 - v_r$  as small as possible.

#### IV. PWM-BASED ADAPTIVE TRACKING

In this Section, based on the PWM technique and adaptive backstepping method, we discuss the PWM-based adaptive tracking control design for the switched Buck converter (13).

##### A. DESIGN OF CONTINUOUS ADAPTIVE CONTROLLER

The switched Buck converter (13) with equivalent continuous control  $\mu$  is rewritten as

$$\begin{aligned} \dot{x}_1 &= -\frac{1}{RC}x_1 + \frac{1}{C}x_2, \\ \dot{x}_2 &= -\frac{1}{L}x_1 + \frac{1}{L}E\mu. \end{aligned} \quad (14)$$

Let  $\theta = \frac{1}{R}$  and  $\varrho = \frac{1}{E}$ , they will be estimated in the adaptive design procedure. Suppose that  $\hat{\theta}$  is the estimate of  $\theta$ , and  $\hat{\varrho}$  is the estimate of  $\varrho$ , then,  $\tilde{\theta} = \theta - \hat{\theta}, \tilde{\varrho} = \varrho - \hat{\varrho}$ . The adaptive continuous controller will be designed for the control objective of trajectory tracking based on adaptive backstepping method.

*Step 1:* Suppose the tracking error

$$z_1 = x_1 - v_r. \quad (15)$$

From (14) and (15), deriving  $z_1$  with respect to time leads to be

$$\dot{z}_1 = \dot{x}_1 - \dot{v}_r = -\frac{\hat{\theta}}{C}x_1 + \frac{1}{C}x_2 - \dot{v}_r - \frac{\tilde{\theta}}{C}x_1. \quad (16)$$

Choose a Lyapunov function  $V_1 = \frac{1}{2}z_1^2$ , the derivative of  $V_1$  with respect to the system (16) satisfies

$$\dot{V}_1 = z_1(-\frac{\hat{\theta}}{C}x_1 + \frac{1}{C}x_2 - \dot{v}_r) - \frac{\tilde{\theta}}{C}x_1z_1. \quad (17)$$

From equation (16) and (17), we select the following virtual control law

$$\alpha_1 = -c_1z_1 + \frac{\hat{\theta}}{C}x_1 + \dot{v}_r, \quad (18)$$

where  $c_1 > 0$  is a design parameter. Substituting (18) into (17) yields

$$\dot{V}_1 = -c_1z_1^2 + z_1(\frac{1}{C}x_2 - \alpha_1) - \frac{\tilde{\theta}}{C}x_1z_1. \quad (19)$$

*Step 2:* Define the transformation

$$z_2 = \frac{1}{C}x_2 - \alpha_1. \quad (20)$$

Considering (14) and differentiating  $z_2$  with respect to time, we obtain

$$\begin{aligned} \dot{z}_2 &= \frac{1}{C}\dot{x}_2 - \dot{\alpha}_1 \\ &= -\frac{1}{LC}x_1 + \frac{E}{LC}\mu - c_1^2z_1 + c_1z_2 \\ &\quad + \frac{1}{C^2}x_1\hat{\theta}^2 - \frac{1}{C^2}x_2\hat{\theta} - \frac{1}{C}x_1\dot{\hat{\theta}} - \ddot{v}_r \\ &\quad + \frac{1}{C^2}x_1\hat{\theta}\tilde{\theta} - c_1\frac{1}{C}x_1\tilde{\theta}. \end{aligned} \quad (21)$$

For the second Lyapunov-like function candidate

$$V = V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2\gamma_\theta}\tilde{\theta}^2 + \frac{|E|}{2\gamma_\varrho}\tilde{\varrho}^2$$

where  $\gamma_\theta$  and  $\gamma_\varrho$  are positive design parameters, the derivative of  $V$  along the system (21) satisfies

$$\begin{aligned} \dot{V} &= -c_1z_1^2 - c_2z_2^2 + z_2[c_2z_2 + z_1 + \dot{z}_2] \\ &\quad - \frac{1}{C}x_1z_1\tilde{\theta} - \frac{1}{\gamma_\theta}\tilde{\theta}\dot{\hat{\theta}} - \frac{|E|}{\gamma_\varrho}\tilde{\varrho}\dot{\hat{\varrho}} \\ &= -c_1z_1^2 - c_2z_2^2 + z_2[c_2z_2 + z_1 - \frac{1}{LC}x_1 + \frac{E}{LC}\mu \\ &\quad - c_1^2z_1 + c_1z_2 + \frac{1}{C^2}x_1\hat{\theta}^2 - \frac{1}{C^2}x_2\hat{\theta} - \frac{1}{C}x_1\dot{\hat{\theta}} - \ddot{v}_r] \\ &\quad - \frac{1}{\gamma_\theta}\tilde{\theta}[\dot{\hat{\theta}} + \gamma_\theta\frac{1}{C}x_1z_1 + \gamma_\theta\frac{c_1}{C}x_1z_2 - \gamma_\theta\frac{1}{C^2}\hat{\theta}x_1z_2] \\ &\quad - \frac{|E|}{\gamma_\varrho}\tilde{\varrho}\dot{\hat{\varrho}}, \end{aligned} \quad (22)$$

where  $c_2 > 0$  is a design parameter. From (21) and (22), the second virtual control law is chose as

$$\begin{aligned} \alpha_2 &= (c_1^2 - 1)z_1 - (c_2 + c_1)z_2 + \frac{1}{LC}x_1 \\ &\quad - \frac{1}{C^2}x_1\hat{\theta}^2 + \frac{1}{C^2}x_2\hat{\theta} + \frac{1}{C}x_1\dot{\hat{\theta}} + \ddot{v}_r. \end{aligned} \quad (23)$$

Then, the adaptive continuous tracking control law and parameter update laws can be designed as

$$\begin{aligned} \mu &= \hat{\varrho}LC\alpha_2, \\ \dot{\hat{\varrho}} &= -\gamma_\varrho\text{sgn}(E)z_2\alpha_2, \\ \dot{\hat{\theta}} &= -\gamma_\theta\frac{1}{C}x_1\{z_1 + (c_1 - \frac{1}{C}\hat{\theta})z_2\}, \end{aligned} \quad (24)$$

where  $\gamma_\theta > 0, \gamma_\varrho > 0$  are design parameters. From (23) and (24) together with (22), one has

$$\dot{V} = -c_1 z_1^2 - c_2 z_2^2. \tag{25}$$

Furthermore,

$$\dot{V} = -c|z|^2, \tag{26}$$

where  $c = \min\{2c_1, 2c_2\}$ .

Up to now, combining (16), (18), (21) and (23), the error system can be given in the following.

$$\begin{aligned} \dot{z}_1 &= -c_1 z_1 + z_2 - \frac{1}{C} \tilde{\theta} x_1, \\ \dot{z}_2 &= -z_1 - c_2 z_2 - \alpha_2 + \frac{E}{LC} \mu + \frac{1}{C^2} x_1 \hat{\theta} \tilde{\theta} - c_1 \frac{1}{C} x_1 \tilde{\theta}. \end{aligned} \tag{27}$$

In view of (27) and (24), stability analysis will be given by using the Lyapunov function

$$V = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 + \frac{1}{2\gamma_\theta} \tilde{\theta}^2 + \frac{|E|}{2\gamma_\varrho} \tilde{\varrho}^2. \tag{28}$$

Based on the above control analysis, we give the stability result of the continuous model with the unknown parameters in the following.

*Theorem 1:* For the switched Buck converter (13) with unknown parameters, the closed-loop error continuous adaptive system (27) and (24) has a globally stable equilibrium at  $(z, \tilde{\theta}, \tilde{\varrho}) = 0$ , and  $\lim_{t \rightarrow \infty} z(t) = 0$ , which means, in particular, that global asymptotic tracking is achieved:

$$\lim_{t \rightarrow \infty} [x_1 - v_r] = 0. \tag{29}$$

Moreover, the equivalent continuous adaptive control (24) is bounded, and by choosing design parameters appropriately, it can be made  $\mu$  such that

$$\mu(t) \in [0, 1], \quad t \geq 0. \tag{30}$$

*Proof:* For  $z = (z_1, z_2)^T$ , the Lyapunov function  $V$  (28) is positive definite and radially unbounded, and satisfies the inequality (26), that is,  $\dot{V} = -c|z|^2$ , the equilibrium  $(z, \tilde{\theta}, \tilde{\varrho}) = 0$  of the closed-loop error systems (27) and (24) is globally stable. According to the LaSalle-Yoshizawa theorem (Lemma 1), it further follows that, as  $t \rightarrow \infty$ , all solutions converge to the manifold  $z = 0$ . Since  $\lim_{t \rightarrow \infty} z(t) = 0$ , then,  $\lim_{t \rightarrow \infty} z_1(t) = \lim_{t \rightarrow \infty} [x_1(t) - v_r(t)] = 0$ .

Furthermore, globally stability of the equilibrium  $(z, \tilde{\theta}, \tilde{\varrho}) = 0$  implies that  $z, \tilde{\theta}, \tilde{\varrho}$  are globally bounded. since  $\theta$  and  $\varrho$  are bounded, then  $\hat{\theta}, \hat{\varrho}$  are bounded. For the boundedness of  $v_r(t), \dot{v}_r(t), \ddot{v}_r(t)$ , it can be obtained that  $x_1, x_2, \alpha_1, \alpha_2$  are bounded. From (24),  $\mu(t)$  is bounded. Let the initial values  $x_1(0) = 0, x_2(0) = 0$ , there are design parameters  $E, c_1, c_2, \gamma_\theta, \gamma_\varrho$  selected appropriately, such that  $\mu(t) \in [0, 1], t \geq 0$ .  $\square$

## B. IMPLEMENTATION OF PWM CONTROL

Let function  $f(t) = [-c_1 z_1 + z_2 - \frac{1}{C} \tilde{\theta} x_1, -z_1 - c_2 z_2 - \alpha_2 + \frac{1}{C^2} x_1 \hat{\theta} \tilde{\theta} - c_1 \frac{1}{C} x_1 \tilde{\theta}]^T \in \mathbb{R}^{2 \times 1}$ , and constant matrix  $B = [0, E/(LC)]^T \in \mathbb{R}^{2 \times 1}$ , system (27) is rewritten as

$$\dot{z} = f(t) + B\mu(t), \tag{31}$$

where  $z = [z_1, z_2]^T \in \mathbb{R}^{2 \times 1}$ , and  $\mu \in \mathbb{R}$  is the continuous control (24). According to Theorem 1, there exists the continuous adaptive control law designed as (24), that is,

$$\begin{aligned} \dot{\hat{\theta}} &= -\gamma_\theta \frac{1}{C} x_1 \{z_1 + (c_1 - \frac{1}{C} \hat{\theta}) z_2\}, \\ \dot{\hat{\varrho}} &= -\gamma_\varrho \text{sgn}(E) z_2 \alpha_2, \\ \mu &= \hat{\varrho} LC \alpha_2 \in [0, 1], \end{aligned} \tag{32}$$

such that the closed-loop error system (31) with (32) is globally asymptotically stable.

For a switched system

$$\dot{z} = f(t) + B\sigma(t), \tag{33}$$

where system input  $\sigma(t) \in \{0, 1\}$  is the digital switch control, and keeping other states and matrices intact as (31). In the following, for system (33), we will design the digital control  $\sigma(t)$  to guarantee all signals in the resulting system are required to be practically asymptotically stable based on the PWM technique.

To implement the digital control, regarding  $\mu(t)$  of (32) as a duty ratio, from the mathematics perspective, a PWM control is a strategy defined as [32]

$$\begin{aligned} \sigma &= \bar{\sigma}_T(t) = \begin{cases} 1, & t_k \leq t < t_k + \mu(t_k)T, \\ 0, & t_k + \mu(t_k)T \leq t < t_k + T, \end{cases} \\ t_{k+1} &= t_k + T, \quad t_0 = 0, \quad k = 0, 1, 2, \dots \end{aligned} \tag{34}$$

where  $T$  represents the sampling period and parameter-tuning,  $t_k$  is a sampling instant. Obviously, it is hard to realize the PWM control 34. In practice, the PWM control is driven as a discrete gate pulse signal [33], comparing a equivalent continuous control signal with a fixed-frequency wave. An approximate physical implementation of PWM strategy (34) is described as

$$\sigma = \sigma_T(t) = \begin{cases} 1, & \mu(t) > u_0(t) \\ 0, & \mu(t) < u_0(t) \end{cases} \tag{35}$$

where  $\mu(t)$  is the continuous control as (32),  $u_0(t)$  is a triangular wave with magnitude  $A = 1$  and same period  $T$  as (34), see Figure 3.

*Remark 2:* For controls  $\mu(t)$  (32),  $\bar{\sigma}_T(t)$  (34) and  $\sigma_T(t)$  (35), they are always integrable everywhere, but they are not guaranteed to be differentiable everywhere. Furthermore, the integral relationships among them largely determine the stability relationships between continuous systems and digital systems. In the following, the stability relationships will be discussed compared with the existing results [13]–[27]. As the tuning parameter  $T$  gets smaller, the difference between the solutions may be smaller and even tends to zero.

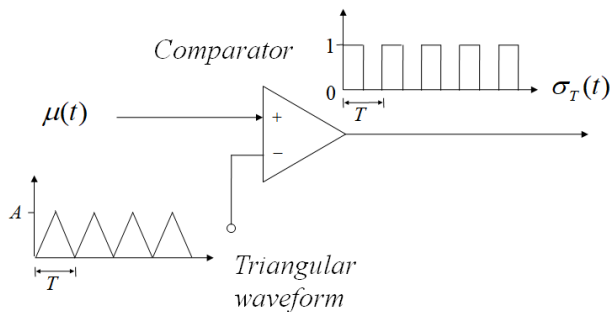


FIGURE 3. Producing pulse using comparator.

When digital switch control (i.e., PWM implement) (35) is substitute for the continuous control (32), the stability relationship will be discussed between the digital system (33) with (35) and the continuous system (31) with (32) as follows.

First, we discuss the integral relationship between  $\sigma_T(t)$  and  $\mu(t)$  on the interval  $[0, t]$ .

(1) If  $\mu(t) = a$  ( $a \in (0, 1)$ ) is a constant), one gets that (see Figure 4):

$$\begin{aligned} \int_0^{\frac{T}{2}} \mu(s)ds &= S_{OFGE} = \frac{aT}{2} = \frac{OF}{OA} \frac{T}{2} \\ &= \frac{OH}{OC} \frac{T}{2} = \frac{OD}{OE} S_{OACE} = S_{OABD} = \int_0^{\frac{T}{2}} \sigma_T(s)ds. \end{aligned} \quad (36)$$

Case 1):  $\exists k \in \mathbb{N}^+, t = kT$ , from (36),

$$\int_0^t \sigma_T(s)ds = \int_0^{kT} \sigma_T(s)ds = \int_0^{kT} \mu(s)ds = \int_0^t \mu(s)ds, \quad (37)$$

Case 2):  $\exists k \in \mathbb{N}^+, t \in (kT, (k + 1)T)$ , from (37) and the values of  $\mu(t)$  and  $\sigma_T(t)$ ,

$$\begin{aligned} \lim_{T \rightarrow 0} \int_0^t \sigma_T(s)ds &= \lim_{T \rightarrow 0} \int_0^t \mu(s)ds - \lim_{T \rightarrow 0} \int_{kT}^t \mu(s)ds \\ &\quad + \lim_{T \rightarrow 0} \int_{kT}^t \sigma_T(s)ds \\ &= \int_0^t \mu(s)ds. \end{aligned} \quad (38)$$

From two cases above, if  $\mu(t) = a$ , we have

$$\lim_{T \rightarrow 0} \int_0^t \sigma_T(s)ds = \int_0^t \mu(s)ds, \quad \forall t \geq 0. \quad (39)$$

(2) If  $\mu(t)$  is a continuous function, there always is simple function series  $\sum_{i=1}^n a_i I_{E_i}(t)$ , one obtains that that

$$\mu(t) = \lim_{n \rightarrow \infty} \sum_{i=1}^n a_i I_{E_i}(t),$$

where  $E_1, \dots, E_n$  are  $n$  disjoint closed sets of  $[0, \infty)$ . Let the length of  $E_i$  be less than that of  $T$ , then  $T \rightarrow 0$  implies that

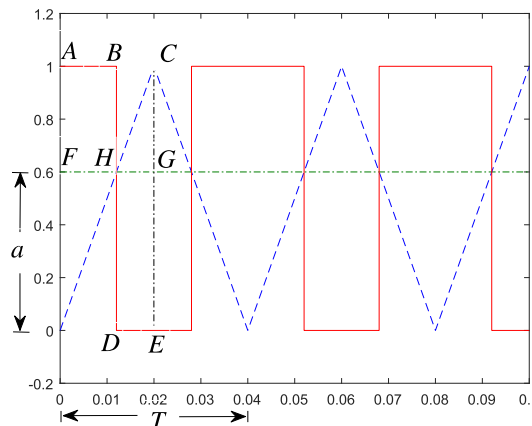


FIGURE 4. The pulse generated comparing  $\mu = a$  with a triangular wave.

$n \rightarrow \infty$ . From (39),

$$\begin{aligned} \int_0^t \mu(s)ds &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \int_0^t a_i I_{E_i}(s)ds \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \lim_{T \rightarrow 0} \int_0^t \sigma_T(s) I_{E_i}(s)ds \\ &= \lim_{T \rightarrow 0} \int_0^t \sigma_T(s)ds, \quad \forall t \geq 0. \end{aligned} \quad (40)$$

Second, let  $z_{\sigma_T}(t)$  respect the solution of digital system (33) with digital control (35),  $z_{\mu}(t)$  denotes the solution of continuous system (31) with continuous control (32). By formula (40), we can obtain that

$$\begin{aligned} \lim_{T \rightarrow 0} z_{\sigma_T}(t) &= z_0 + \int_0^t f(s)ds + B \lim_{T \rightarrow 0} \int_0^t \sigma_T(s)ds \\ &= z_0 + \int_0^t f(s)ds + B \int_0^t \mu(s)ds \\ &= z_{\mu}(t), \quad \forall t \geq 0. \end{aligned} \quad (41)$$

which implies that

$$z_{\sigma_T}(t) = z_{\mu}(t) + O_T, \quad \lim_{T \rightarrow 0} O_T = 0, \quad \forall t \geq 0. \quad (42)$$

Based on Theorem 1, the closed-loop error continuous system (31) with (32) is globally asymptotically stable. By Definition 1, there exists a  $\mathcal{KL}$  function  $\beta(\cdot, \cdot)$ , such that

$$|z_{\mu}(t)| \leq \beta(z_0, t), \quad \forall t \geq 0, \quad (43)$$

From (42) and (43), we have that

$$|z_{\sigma_T}(t)| \leq \beta(z_0, t) + |O_T|, \quad \lim_{T \rightarrow 0} O_T = 0, \quad \forall t \geq 0, \quad (44)$$

there exists a function  $\varepsilon_T$ , such that  $|O_T| \leq \varepsilon_T$ , and

$$|z_{\sigma_T}(t)| \leq \beta(z_0, t) + \varepsilon_T, \quad \lim_{T \rightarrow 0} \varepsilon_T = 0, \quad \forall t \geq 0. \quad (45)$$

According to Definition 2, the origin of closed-loop error digital system (33) and (35) is practically asymptotically stable.

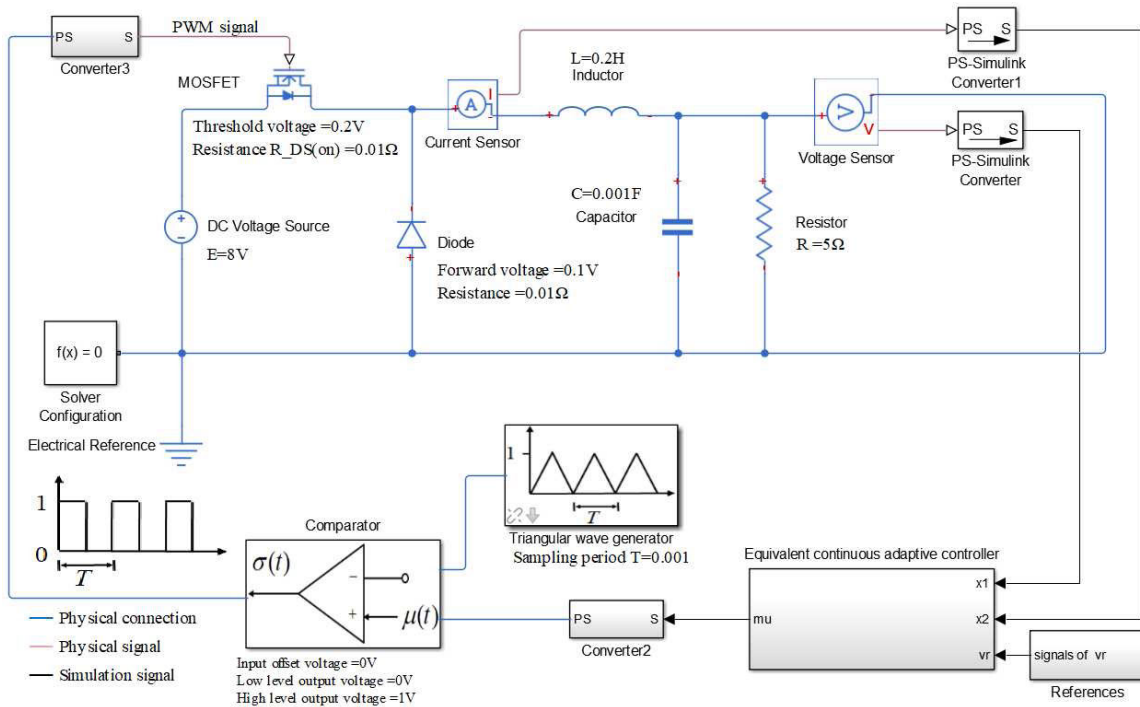


FIGURE 5. Simulation experiment platform based on Simscape.

From the above analysis, we can get some results as follows.

**Theorem 2:** For the switched Buck converter (13) with unknown parameters, if continuous control  $\mu(t)$  (24) is replaced by digital control  $\sigma(t)$  (35), then the closed-loop error digital system (33) with (35) is practically asymptotically stable. Furthermore, the tracking error  $x_1 - v_r$  can be made arbitrarily small for  $T$  small enough.

*Proof:* According to Theorem 1, the closed-loop error continuous system (31) with (24) is asymptotically stable. From (42) and (45), the closed-loop error digital system (33) with (35) is practically asymptotically stable.

From (45), as  $T \rightarrow 0$ ,  $\varepsilon_T \rightarrow 0$ . This implies that the sampling period  $T$  can be tuned smaller, the closed-loop error digital system (33) with (35) is closer to be asymptotically stable. In other words, all signals can be made arbitrarily small for  $T$  small enough. Moreover, when  $T$  is small enough, the tracking error  $z_1 = x_1 - v_r$  as a component of states  $z$  can be made arbitrarily small.  $\square$

### C. PWM-BASED ADAPTIVE TRACKING ALGORITHM

For the switched Buck converter (13) with unknown parameters, instructions for regulating digital control  $\sigma(t)$  are given in details as follows. The continuous tracking control problem is first solved for the continuous equivalent model, and the duty ratio is designed in the form of a state feedback function. It can be seen from the continuous adaptive backstepping tracking controller (32) (i.e., (24)),  $\mu(t)$  can be adjusted to keep in the interval  $[0, 1]$  by choosing appropriate design parameters  $E, c_1, c_2, \gamma_\theta, \gamma_\varrho$ . Second, selecting the switching period  $T$ , the digital control  $\sigma(t)$  can be established using the

duty ratio  $\mu(t)$  based on the PWM technique. The following PWM-based adaptive tracking algorithm is executed using  $\mu(t)$  (32) based on PWM implement (35) to realize the tracking target of the switched Buck converter (13) with unknown parameters in some sense.

**Algorithm 1 (PWM-Based Adaptive Tracking Algorithm):**  
 (1) The continuous adaptive controller  $\mu(t)$  is calculated as (32), that is,

$$\begin{aligned} \dot{\hat{\theta}} &= -\gamma_\theta \frac{1}{C} x_1 \{z_1 + (c_1 - \frac{1}{C} \hat{\theta}) z_2\}, \\ \dot{\hat{\varrho}} &= -\gamma_\varrho \operatorname{sgn}(E) z_2 \alpha_2, \\ \mu &= \hat{\varrho} L C \alpha_2 \in [0, 1], \end{aligned} \quad (46)$$

by selecting appropriate design parameters  $E, c_1, c_2, \gamma_\theta, \gamma_\varrho$ .

(2) The digital control  $\sigma(t)$  is constructed based on continuous control  $\mu(t)$  (46) and PWM implement (35), that is,

$$\sigma = \sigma_T(t) = \begin{cases} 1, & \mu(t) > u_0(t) \\ 0, & \mu(t) < u_0(t) \end{cases} \quad (47)$$

where  $u_0(t)$  is the triangular wave with period  $T$ , and period  $T$  is parameter-tuning.

**Remark 3:** For the switched Buck converter (13) with unknown parameters, Algorithm 1 is achievable according to the proof procedure of Theorem 2. The shorter the sampling period  $T$ , then the smaller the tracking error and the closer the asymptotic stability of error digital system (33) and (47) (i.e., (35)), by choosing appropriate design parameters  $c_1, c_2, \gamma_\theta, \gamma_\varrho$ .

**Remark 4:** In the existing results, the point regulating problem was only considered for continuous average models, where the control input is continuous control.

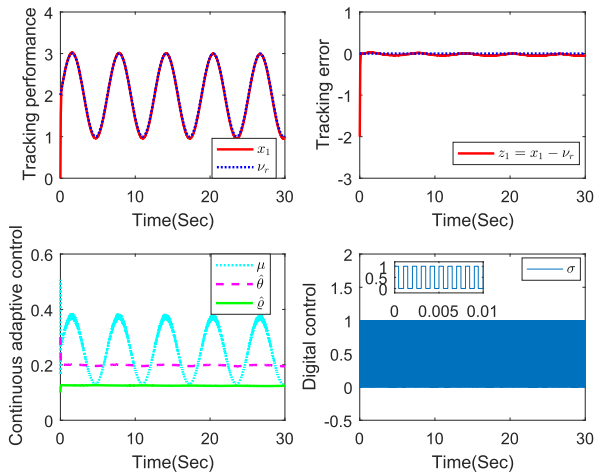


FIGURE 6. The response of closed-loop switched Buck converter (13) with digital control (47).

Compared with [20]–[27], based on combination of adaptive backstepping method and PWM technique, PWM-based adaptive trajectory tracking is proposed for switched Buck power converters with unknown parameters, where the system input is digital switch control.

*Remark 5:* In fact, due to the effect of circuit parameters and design parameters, the calculated value of the duty ratio  $\mu$  (i.e., equivalent continuous adaptive control (24)) probably would exceed the physical bounds of the actual value, which must be limited to take values on the interval  $[0, 1]$ . Therefore, the PWM-based adaptive tracking Algorithm 1 does not directly apply to the switched Buck converter (13). The supervisor (i.e., a saturation function) must be added to restrict the calculated value of  $\mu$ , after that, the obtained value is used to construct the digital switch  $\sigma(t)$ . Therefore, if  $\mu$  exceeds the physical bounds of the actual value  $[0, 1]$ , the results of this article may hold by using a saturation function to restrict  $\mu$  to  $[0, 1]$ . This case will be investigated in the further work.

### V. SIMULATION RESULTS

To investigate the effectiveness of the proposed control method, the physical model of Buck converter (13) is established in Figure 5 by Simscape, which is closer to the real physical model. For the experiment platform, Buck converter is composed by unknown DC voltage source, unknown resistor, inductor, capacitor, MOSFET, diode, voltage sensor, and current sensor. The circuit parameters  $E = 8 \text{ V}$ ,  $R = 5 \ \Omega$ ,  $L = 0.2 \text{ H}$ ,  $C = 0.001 \text{ F}$ , MOSFET with threshold voltage  $0.2 \text{ V}$  and resistance  $R_{\text{DS(on)}} 0.01 \ \Omega$ , diode with forward voltage  $0.1 \text{ V}$  and internal resistance  $0.01 \ \Omega$ , the inductor current and capacitor voltage are measured by current sensor and voltage sensor respectively.

The output voltage reference for tracking is  $v_r(t) = 2 + \sin t$ . The tracking algorithm based on the PWM technique requires the use of comparator, triangular wave generator and equivalent continuous adaptive controller. The unipolar triangular wave generator and the comparator is implemented

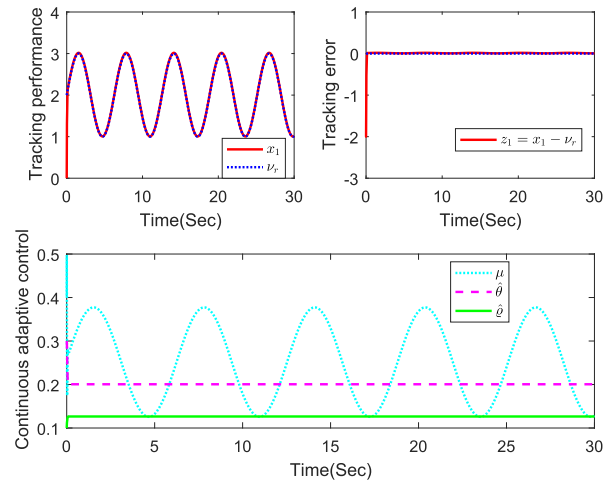


FIGURE 7. The response of closed-loop continuous model (14) with equivalent continuous control (46).

by means of analogue techniques, a detailed description of the procedure can be found in [34]. Based on Algorithm 1, the switching signal  $\sigma(t)$  is generated by the PWM technique comparing backstepping control signal with fixed frequency triangle wave by comparator.

In the simulation, the parameter to be estimated is  $\theta = 1/R = 1/5$ ,  $\varrho = 1/E = 1/8$ , choose the design parameters  $c_1 = 100$ ,  $c_2 = 100$ ,  $\gamma_\theta = 10^{-7}$ ,  $\gamma_\varrho = 3 * 10^{-7}$ , the initial values  $x_1(0) = q_C(0)/C = 0$ ,  $x_2(0) = \dot{q}_L(0) = 0$ ,  $\hat{\theta}(0) = 0.3$ ,  $\hat{\varrho}(0) = 0.1$ ; the sampling period  $T = 0.001$ .

To make comparisons of simulation results, we will take into consideration the responses of corresponding closed-loop system with two different cases:

(a) the switched Buck converter (13) based on PWM-based adaptive tracking Algorithm 1.

(b) the continuous average model (14) based on equivalent continuous adaptive controller (24) (i.e., (32), (46)).

Figure 6 and Figure 7 show the responses of the corresponding closed-loop system with the different cases. From Figure 6, it can be learned that the tracking error converges to a small neighborhood of the origin. Figure 7 implies that the tracking error tends to the origin. Compared with Figure 7, Figure 6 demonstrates the effectiveness of PWM-based adaptive tracking control of the closed-loop system.

### VI. CONCLUSION

The adaptive trajectory tracking of switched Buck converters with unknown parameters is considered in this article. Combined adaptive backstepping method and PWM technique, the digital control is designed for switched Buck converter, such that the origin of the closed-loop error digital system is practically asymptotically stable, and the tracking error tends to an arbitrarily small neighborhood of the origin. Simulation results are shown to illustrate the effectiveness of the proposed new control schemes.

One current work can be further generalized to stochastic digital control of switched Buck, Boost, Buck-boost power converters based on stochastic theory [35], there are other



further works to be considered such as fuzzy tracking control or second-order sliding mode control for switched power converters based on the methods in references [36]–[40].

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