IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received September 10, 2020, accepted September 16, 2020, date of publication September 28, 2020,
date of current version November 24, 2020.

*Digital Object Identifier 10.1109/ACCESS.2020.3027008*

# Preference-Inspired Coevolutionary Algorithm Based on Differentiated Resource Allocation Strategy

**QICANG QIU[1], WEI YU[2], LIPING WANG[3], HONG CHEN[4], AND XIAOTIAN PAN[3]**

[1]Zhejiang Laboratory, Hangzhou 310023, China
[2]Institute of Information Intelligence and Decision Optimization, Zhejiang University of Technology, Hangzhou, China
[3]College of Computer Science and Technology, Zhejiang University of Technology, Hangzhou, China
[4]College of Educational Science and Technology, Zhejiang University of Technology, Hangzhou, China

Corresponding author: Qicang Qiu (qiuqc@zhejianglab.com)

This work was supported in part by the Zhejiang Provincial Natural Science Youth Foundation of China under Grant LQ20F020014, in part by the National Natural Science Foundation of China under Grant 61379077 and Grant 61472366, and in part by the Zhejiang Provincial Natural Science Foundation of China under Grant LY17F020022 and Grant LQ20F020014.

**ABSTRACT** Preference-inspired co-evolutionary algorithms (PICEAs) consider the target vectors as the preferences, and then use the domination relationship between the candidate solutions and target vectors to increase their selection pressure. However, the size of dominating objective space varies with the different positions of candidate solutions and it leads to the imbalance of the evolutionary ability of whole population. To solve this problem, this paper proposes a preference-inspired coevolutionary algorithm based on a differentiated allocation strategy (PICEAg-DS). First, it sets up an external archive to save the nondominated solutions and then extracts the convergence and diversity information from it. Second, it divides the objective space into several subspaces and designs a space distance operator to evaluate their optimization difficulty. Finally, it dynamically assigns the target vectors and guides more computational resource to the difficult to optimize subspaces, and thus drives the whole population evolution. To prove the advantages of differentiated resource allocation strategy, the PICEAg-DS is compared with two classic coevolutionary algorithms (PICEAg, CMOPSO) and two classic MOEAs based on resource allocation strategy (EAG-MOEAD, MOEAD-DRA). The experimental results show that PICEAg-DS performs better than the other algorithms on many WFG test problems. To further analysis the effectiveness of PICEAg-DS, compare it with two MOEAs based on domination relationship (NSGAII, SPEA2) and two MOEAs based on decomposition (RVEA, MOEA/D-M2M) on MOP and UF test suite. The experimental results show the PICEAg-DS has a better convergence than the other comparison algorithms, especially on 3-objective MOP6-7 and UF8-9.

**INDEX TERMS** Coevolutionary, multiobjective optimization, objective space partition, resource allocation.

## I. INTRODUCTION

In many practical optimization problems, there are many optimization objectives that conflict with each other; these are called multiobjective optimization problems (MOPs). Many studies have shown that multi-objective evolutionary algorithms (MOEAs) can effectively solve MOPs. Because an evolutionary algorithm is actually a heuristic method that simulates biological evolution, it can develop a set of uniformly distributed solution sets in a simulation and obtain a set of trade-off Pareto-optimal (P-O) solutions that approximate to the Pareto front (PF).

In recent years, many evolutionary algorithms and their variants have been proposed, which can be divided into the three categories: 1) The MOEAs based on a Pareto-domination relationship, such as NSGA-II [1] and SPEA2 [2], have been proved that their ability often getting worse with the increase of the number of objectives [3], because their selection pressure decreased sharply in many-objectives optimization problems. 2) The MOEAs based on decomposition, which have an efficiency in balance the convergence and diversity by divided the objective space into

The associate editor coordinating the review of this manuscript and approving it for publication was Ioannis Schizas.

several subspaces and optimize them simultaneously, named MOEA/D [4], and in past decade, more variants have been proposed such as MOEA/D-M2M [5] and RVEA [6]. 3) The MOEAs based on indicator, which use a performance metric to guide the population evolution, such as ISDE+ [7] and HyPE [8], but their optimization results may only perform well on this performance metric.

The coevolutionary algorithms are different from the above three categories because their fitness values are obtained by cooperating with different individuals in different populations or other individuals in the same population [9], compared to the individuals in traditional evolutionary algorithms that obtain their fitness values by themselves. Therefore, the coevolutionary algorithm is an extension of traditional evolutionary algorithms. And many researchers have proven that coevolutionary algorithms have good performance in many MOPs [3], and these algorithms called coevolutionary multi-objective evolutionary algorithms (CMOEAs). With more effort into this framework, CMOEAs have expanded into the three branches of cooperative CMOEAs, competitive CMOEAs, and competitive-cooperative CMOEAs [10].

The cooperative CMOEAs decompose a MOP into several low-dimensional subproblems with partial variables of the original problem and then optimize these subproblems cooperatively, such as IBCCMOEA [11] and CCMOEA-HSU [12]. But how to determine the number of subproblems and choose an appropriate method to divide the decision variables have a great impact on their performance.

The competitive-cooperative CMOEAs consider both the cooperative and competitive relationships among the subpopulations and drive the whole population evolution, such as COMOEA [13] and CMOPSO [14].

The competitive CMOEAs divide the population into several subpopulations and use the competitive relationships to guide the population evolution. Competitive CMOEAs can be further divided into three categories: 1) based on predator-prey models, such as PPBBO [15] and MPP [16]; 2) based on moderate competition, such as C-RMOEA/D [17] and SPEA2-CE [18]; 3) based on the coevolution of solutions, such as CGA [19] and NNCA [20]. Of these, the CMOEAs based on the coevolution of solutions have attracted many attentions. Lohn *et al.* [19] proposed the CGA, which used target objective vectors (TOVs) as the preferred solutions and utilized the competition relationship between the TOVs and candidate solutions to guide the population evolution. Purshouse and Fleming [21] further improved the CGA and proposed preference-inspired coevolutionary algorithms (PICEAs), which also used the TOVs as the preference solutions and used the dominant relationship between the preference and candidate solutions to coevolve them.

The preference points in PICEAs are not the preference information of decision-makers in some sense, but they randomly generate for increasing the selection pressure of the candidate solutions. Wang *et al.* [22] proposed a preference-inspired coevolutionary algorithm for many-objective optimization (PICEAg), which is a popular one of PICEAs. It used the target vectors as preferences and adaptively generate to drive the population toward the PF. The simulation experiments have shown that the PICEAg outperforms many traditional evolutionary algorithms on many-objective optimization problems [23].

But we find the PICEAg has a disadvantage that in the course of evolution, the target vectors are randomly generated in objective spaces, but the sizes of the objective spaces dominated by the candidate solutions are different, which affects the fitness values obtained by the candidate solutions. It causes the search ability of different individuals to be different. Therefore, this paper proposes a collaborative evolutionary algorithm based on a differentiated resource allocation strategy (PICEAg-DS). In PICEAg-DS, an external archive is set up to save the non-dominated solutions; a space distance operator is designed to divide the objective space into several subspaces and measure the subspace hardness; then a differentiated resource allocation strategy is proposed to allocate target vectors dynamically and assigns more target vectors to the sparse subspace which denotes the subspace with few non-dominated solutions and poor convergence. It aims to increase the evolutionary ability in sparse subspaces and drive the whole population evolution. Besides, we use the IGD indicator to analyze the parameter setting and choose the optimal number of groups that divides the objective space. To prove the advantages of our proposed differentiated resource allocation strategy, we compare the PICEAg-DS with PICEAg, COMPSO, MOEAD-DRA and EAG-MOEAD. The first two are classic algorithms of CMOEAs, and last two are MOEAs based on resource allocation strategy. The five comparison algorithms are tested on 2- and 3- objective WFG1-9 test problems and utilize the SP, GD and IGD indicators to evaluate their distribution, convergence and comprehensive performance, respectively. To further verify the effectiveness of our proposed PICEAg-DS, we compare it with NSGAII, SPEA2, RVEA, MOEA/D-M2M on MOP1-7 and UF1-9 test problems. The first two are MOEAs based on domination relationship, and the last two are MOEAs based on decomposition. And utilize the GD indicators to evaluate their convergence performance.

The key contributions of this paper are as follows:

1) A novel method to partition the objective space, which designs a space distance operator to calculate the space distance of non-dominated solutions and then divides them into several uniform groups, and defines the maximum distance of each group as a subspace. Therefore, different subspace may have a different size, which can clearly show the distribution sparsity of non-dominated solutions in each subspace.

2) We propose a resource allocation method to solve the shortcoming of PICEAg that individuals in different position have a different evolutionary ability, which by allocating more target vector to sparse subspaces and enhance their evolutionary ability.

3) On the basis of the above strategy, a preference-inspired coevolutionary algorithm based on differentiated

resource allocation strategy, named PICEAg-DS, is designed for multi-objective optimization.

The rest of paper is organized as follows. In section II, we introduce the framework of PICEAg and our motivation. In section III, we present the proposed algorithm (PICEAg-DS) and describe the differentiated resource allocation strategy in detail. In section IV, we compare the performance of the proposed PICEAg-DS with PICEAg, COMPSO, MOEAD-DRA and EAG-MOEAD on 2- and 3- objective WFG1-9 test problems and NSGAII, SPEA2, RVEA, MOEA/D-M2M on MOP1-7 and UF1-9 test problems. In section V, we make a summary for this paper.

## II. RELATED WORK

### A. PICEAg

In PICEAg, the candidate solutions can obtain their fitness values by dominating the number of target vectors, as defined in equation (1), and the target vectors can also obtain their fitness values by candidate solutions, as defined in equation (2).

$$F_s = 0 + \sum\nolimits_{g \in G \uplus G_c | s \preccurlyeq g} \frac{1}{n_g} \tag{1}$$

$$F_g = \frac{1}{1+\alpha} \tag{2}$$

where

$$\alpha = \begin{cases} 1, & n_g = 0 \\ \dfrac{n_g - 1}{2N - 1}, & otherwise \end{cases} \tag{3}$$

where $G$ is the target vector set at current generation and $Gc$ is the offspring target vector of $G$; $s$ is the candidate solution; $N$ is the size of $s$; $g$ is a preference that dominated by $s$; $n_g$ is the number of solutions that dominate the $g$ and $F_s$ is the sum of the reciprocal of the $n_g$ that are dominated by $s$. If $s$ does not dominate any $g$, the $F_s$ is defined as 0.

### B. OBJECTIVE SPACE PARTITION METHODS

There are many methods to partition the objective space into several multiple small subspaces. In most of MOEA/D and their variants, the weight vectors are generated uniformly to decompose the complicated multi-objective optimization, such as, MOEA/D [4] uses a number of scalar subproblems to decompose the MOP into several simple subproblems and each subproblem is optimized by utilizing the information mainly from its several neighboring subproblems. In MOEA/D-M2M [5], it uses K unit vectors to partition the objective space into K subregions, and generate K subpopulations to search each subregion in order to enhance the population diversity. However, the uniformly distributed weight vectors cannot produce uniformly distributed P-O solutions when the PF is complex [24] or irregular [25]. Therefore, several works have adopted alternate ways of decomposition, such as, in RVEA [6], a reference vector adaptation method is proposed, which can generate a uniformly weight vector according to the ranges of the objective values. And in pa$\lambda$-MOEA/D [26], the weight vectors automatically adapt according to the geometrical properties of the PF. Generally, the uniformity of weight vectors can ensure the diversity of the P-O solutions, however, it cannot work as well when the target MOP has a complex PF (i.e., discontinuous PF or PF with sharp peak or low tail). To solve this problem, in MOEA/D-AWA [24], it firstly generates a set of predetermined weight vectors and then periodically remove them from the crowed part and added to sparse regions, which can effectively save the computing efforts that devoted to subproblems with duplicate optimal solution.

### C. RESOURCE ALLOCATION METHODS

It well known that some parts of the PF are difficult to converge than others [27]. Therefore, it is necessary to allocate different computational resources to different hardness subproblems in MOEAs based on decomposition. But how to determine the difficulty of different subproblems and how to guide the resource allocation are key issues in the optimization process. With increasing effort, the resource allocation methods can be divided into offline resource allocation (OFRA) and online resource allocation (ONRA).

The OFRA measures the subproblem difficulty in an offline manner. For example, it calculates the improvement value of different subproblems before and after 50 generations. The subproblem with a lower improvement value is regarded as difficult to optimize. However, the OFRA methods always have a low efficiency.

The ONRA methods are different from the OFRA methods, which dynamically measure the subproblem difficulty in the whole evolutionary process. Additionally, experimental studies have shown that the ONRA methods are more practical than the OFRA methods [28]. There are many studies of ONRA methods, in terms of decomposition based on MOEAs. Zhang proposed a dynamic resource allocation strategy based on MOEA/D, named MOEA/D-DRA [29], which designed a utility function to measure the subproblem difficulty and allocated more computing resources to the subproblem with the higher utility function value. Zhou and Zhang [28] further improved MOEA/D-DRA and proposed a generalized resource allocation strategy, named MOEA/D-GRA, which uses a probability of improvement (PoI) vector and determines whether a subproblem is chosen for invest according to its PoI vector and a random number. Cai *et al.* [30] proposed the EAG-MOEA/D to extract the convergence and diversity information from an external archive, which can identify the potential subproblems and then guide the population evolution. Lin *et al.* [31] proposed a diversity-enhanced resource allocation strategy, named MOEA/D-IRA; it assigns more computational resources to the subproblem with fewer solutions in its neighboring area and more relative improvement on the aggregated function value. Chen *et al.* [32] proposed an adaptive resource allocation strategy for objective space partition-based multi-objective optimization, named OPE-MOEA; it firstly partitions the objective space into $N$ subspaces evenly and then defines a metric to measure the contributions of subspaces to the population convergence, and according to the contributions to allocate computational resources. In terms of decomposition of decision variables, some studies allocate the computational resources to different subgroups by measuring their improvement [33], contribution [34]–[36] and the importance degree of their decision variables [37].

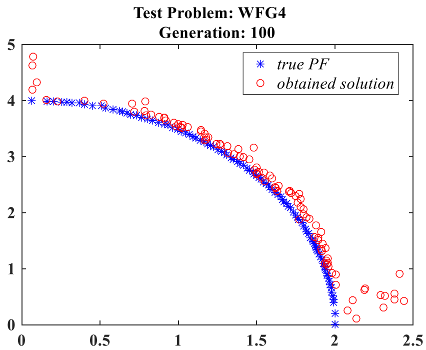



**FIGURE 1.** PICEAg execution on a 2-objective WFG4.

### D. THE SHORTCOMING ANALYSIS OF PICEAg

In PICEAg, we can find the individuals are concentrated in the center regions of objective space and also have a better convergence performance, while the individuals in the sharp and tail of PF are more difficult to converge. From Fig. 1, it can be seen that the individuals in the region of $f(x_1) \in [0, 1]$ are significantly more sparse than the individuals in the region of $f(x_1) \in [1, 2]$, and the number of target vectors in the region of $f(x_1) \in [0, 1]$ are also significantly less than that in the region of $f(x_1) \in [1, 2]$. In terms of the ordinate of $f(x_2)$, it can also be seen that individuals in the region of $f(x_2) \in [3.5, 4]$ are more sparse than the individuals in the region of $f(x_2) \in [0.5, 3.5]$, and the individuals in the region of $f(x_2) \in [0, 0.5]$ are more sparse than individuals in the region of $f(x_2) \in [0.5, 3.5]$. Additionally, the number of target vectors is consistent with the distribution of individuals.

The above phenomenon can be attributed to the individuals in different position that have different evolutionary ability. To further analyze this problem, the size of the objective space dominated by the candidate solutions is considered, as shown in Fig. 2. It can be seen that the candidate solutions in different position have different sized dominating spaces, whether it is a convex or concave optimization problem.

In Fig. 2(a), the candidate solution $P_1$ dominates the A + C space and $P_2$ dominates the B + C space. In Fig. 2(b), $P_1$ dominates the E + G space and $P_2$ dominates the F + G space. We can calculate the area of the dominating space, i.e., the space size of A + C is 2.6 * 0.4 = 1.04 and, using the same calculation, B + C = 1.12, E + G = 2.8 and F + G = 4. Additionally, we can see that the candidate solutions in the center position of the PF have a larger dominating space than those in the sharp and tail of the PF in both convex and concave optimization problems. In PICEAg, the fitness

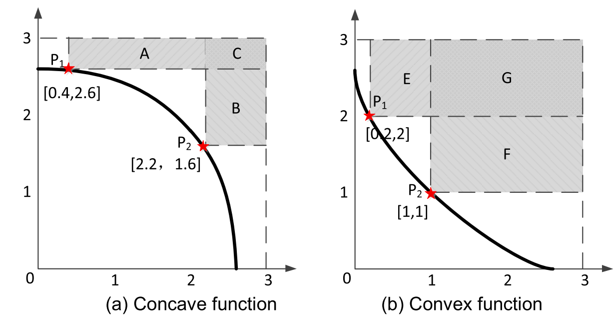



**FIGURE 2.** The size of objective spaces dominated by candidate solutions.

values of candidate solutions are related to the number of target vectors that are dominated by them, the more target vectors are dominated, the higher the fitness value. The candidate solution with a higher fitness value more easily survives. However, the size of the objective space dominated by candidate solutions is different and the target vectors are randomly distributed in the objective space; thus, this fitness calculation method is unfair. Additionally, it leads to different evolutionary abilities of candidate solutions in different positions.

Therefore, it is important to allocate more computational resources to the candidate solution with lower evolutionary ability and promote the evolution of whole population.

Inspired by the exist methods of objective space partition and resource allocation. This paper proposed a novel differentiated allocation strategy based on PICEAg and named it PICEAg-DS. The framework of PICEAg-DS can be seen in Fig. 3.

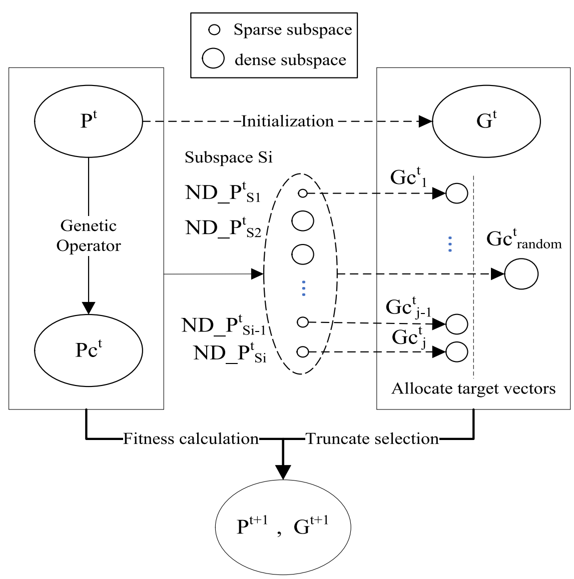



**FIGURE 3.** The framework of PICEA-DS.

As shown in Fig. 3., the main idea of PICEAg-DS is using the convergence and diversity information of nondominated solutions to divide the objective space, and then generating different numbers of target vectors in the subspaces. $p^t$ is the candidate solution at $t$ generation, $pc^t$ is the offspring of $p^t$, $NP\_p^t_{si}$ is the nondominated solution set in subspace $si$, $G^t$ is the target vector set at $t$ generations and $Gc^t$ is the offspring of $G^t$. A detailed description of PICEAg-DS is provided in section III.

## III. PREFERENCE-INSPIRED CO-EVOLUTIONARY ALGORITHMS BASED ON DIFFERENTIATED ALLOCATION STRATEGY

### A. THE MAIN FRAMEWORK OF PICEAg-DS

The pseudo code of the proposed MOEA/D-DS is described in Algorithm 1.

**Algorithm 1** The General Framework of PICEAg-DS

**Input:** The max generation: $Maxgen$, Population size: $N$, the number of target vectors is $Ngoal$ and the number of subspaces is $d$.
**Output:** The optimal solution $P$
1. Initiate the population: $P = [p_1, p_2, \ldots, p_N]$;
2. Randomly generate the target vectors in the objective space: $G = \left[g_1, g_2, \ldots, g_{Ngoal}\right]$;
3. Set up an external archive to save nondominated solutions: $Archive = [ND\_p_1, ND\_p_2, \cdots, ND\_p_i]$
4. ***For*** $t = 1$ ***To*** $Maxgen$ ***Do***
5. Generate the offspring population: $Pc_t = GeneticOperator\,[P_t, N]$;
6. Update the external archive by $Pc_t$: $Archive = [ND\_p^t_1, ND\_p^t_2, \cdots, ND\_p^t_j]$;
7. ***IF*** $size(Archive, 1) < N$ ***THEN***
8. Randomly generate the offspring target vectors in the objective space.
9. **ELSE**
10. Generate the offspring target vectors by a differentiated resource allocation strategy: $Gc_t = DifferentialSpace\_GenerateGoal\left[N_{goal}, Archive\right]$ in **Algorithm 2**.
11. ***End IF***
12. Calculate the fitness of $P_t \cup P_{ct}$, $G_t \cup Gc_t$ using equation (1) through (3).
13. Truncate the selected $P_{t+1}, G_{t+1}$ by the fitness values of $P_t \cup P_{ct}$ and $G_t \cup Gc_t$: $[P_{t+1}, G_{t+1}] = \text{truncateselected}[P_t \cup P_{ct}, G_t \cup Gc_t]$
14. ***End for***

As shown in Algorithm 1, the $N$ individuals and $Ngoal$ target vectors are randomly initialized (lines 1-2), and then set up an external archive to save the non-dominated solutions (line 3), which is used for extract the convergence and diversity information from the objective space. After initializing, the PICEAg-DS enters the main loop (lines 4-14), which includes two core functions: 1) Generate offspring (lines 5-8): generate offspring solutions (line 5) and update the external archive by these offspring solutions (line 6). Then, according to the space distance of non-dominated solutions in external archive (line 7) to decided generate target vectors by randomly (line 8) or differential allocation (Algorithm 2). 2) Update the population and target vectors (lines 12-13).

### B. THE DIFFERENTIATED RESOURCE ALLOCATION STRATEGY OF PICEAg-DS

To realize the dynamic resource allocation in PICEAg, this paper proposed a differentiated resource allocation strategy. The pseudo code of the differentiated resource allocation strategy of PICEAg-DS is provided in Algorithm 2.

**Algorithm 2** The Differentiated Resource Allocation Strategy

**Input:** The nondominated solutions in the external archive: $ND\_p^t_i$ $(i = 1, 2 \cdots, j | j \geq N)$, the number of target vectors: $Ngoal$ and the number of subspaces $d$.
**Output:** The offspring target vectors: $Gc = \left[gc_1, gc_2, \ldots, gc_{Ngoal}\right]$
1. Calculate the mean value of each objective of $P_t$ and $Pc_t$, and obtain the center point $A$.
2. Mapping the $A$ and $ND\_p^t$ to a 2-objective space, obtaining $ND\_p^{t'}$ and central point $A(a_1, a_2)$.
3. Calculate the Euclidean distance between the $ND\_p^{t'}$ and $A\ (a_1, a_2)$ using formulas (4) and (5).
4. Sort the distance in descending order, divide the $ND\_p^t$ into $d$ subgroups: $S_i$, and calculate the distribution distance of each subgroup: $SD_{S_i(i=1,2,\ldots,d)}$ using equation (6).
5. Divide the objective space into $d$ uniform subspaces: $SD_{mean}$ using formula (7).
6. ***FOR*** $i = 1$ ***TO*** $d$ ***DO***
7. ***IF*** $SD_{S_i} > SD_{mean}$ ***THEN***
8. Generate $Ngoal/d$ target vectors in the subspace $S_i$.
9. ***END IF***
10. ***END FOR***
11.Calculate the number of target vectors that have been allocated: $Ngoal_{exist}$.
12.Randomly generate $Ngoal - Ngoal_{exist}$ target vectors in the objective space using equation (8).

As shown in Algorithm 2, there are four core functions: 1) it designs a space distance operator to calculate the distance between the nondominated solutions with central point (lines 1-3); 2) divides the non-dominated solutions into $d$ uniform groups and calculate the subspace distance $SD_{si}$ of each group (line 4); 3) calculates $d$ uniformly partitioned subspace distances: $SD_{mean}$ (line 5); 4) compare the $SD_{si}$ with $SD_{mean}$, and generate different number of target vectors in different subspace (lines 6-12). The detailed calculation methods are defined as follows:

1) Mapping the nondominated solutions $ND\_p^t$ and center point $A$ to a 2-objective space. For example, for the 3-objective optimization in Fig. 4, the center point $A(a_1, a_2, a_3)$ mapping to the 2-objective space is $A'(a_2, a_3)$; in the same manner, map the $ND\_p^t_i$ to a 2-objective space.

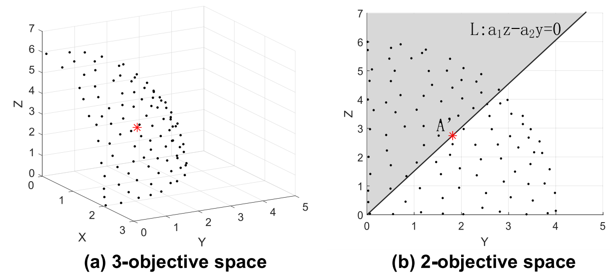



**FIGURE 4.** Mapping the center point and population to a 2-objective space.

2) Calculate the Euclidean distance between the $ND\_p^t$ and $A(a_1, a_2)$. In Fig. 3, a straight line passing through the two points of $A'(a_2, a_3)$ and $O(0, 0)$ can be defined as follows:

$$a_3 z - a_2 y = 0 \tag{4}$$

Then, calculate the Euclidean distance between each $ND\_p_i^t$ $(i = 1, 2 \cdots, j | j \geq N)$ and $A'(a_2, a_3)$, as follows:

$$vd_i = \frac{a_1 z - a_2 y}{\sqrt{a_1^2 + (-a_2)^2}} \tag{5}$$

When $vd < 0$, the nondominated solution $ND\_p^t$ is below $L$, which is in the nonshaded region of Fig. 4(b). When $vd > 0$, the nondominated solution $ND\_p^t$ is above $L$, which in the shaded region in Fig. 4(b).

3) Divide the nondominated solutions $ND\_p_i^t$ into $d$ subgroups, and calculate the distribution distance of each subgroup $S_i (i = 1, 2, \cdots, d)$, as follows:

$$SD = |Vd_{S_i}^{upper} - Vd_{S_i}^{lower}| \tag{6}$$

where the $Vd_{S_i}^{upper}$ is the maximum distance of the nondominated solutions in $S_i$ and $Vd_{S_i}^{lower}$ is the minimum distance of the nondominated solutions in $S_i$.

4) Divide the objective space into $d$ subspaces and calculate the space distance of each subspace, as follows:

$$SD_{mean} = |Vd_{max} - Vd_{min}| / d \tag{7}$$

where $Vd_{max}$ is the maximum of $vd_i (i = 1, 2, \cdots, N) and$ $Vd_{min}$ is the minimum of $vd_i (i = 1, 2, \cdots, N)$.

## IV. EXPERIEMENT AND ANALYSIS

### A. THE EXPERIMENT PARAMETER SETTINGS

The simulation experiments are executed in Matlab R2016a PlatEMOv1.3. To verify the effectiveness of differential resource allocation in PICEAg-DS, we choose the WFG test suite [38] as the test problems. The WFG is not only a scalable test suite, but also contains many function attributes, which can be seen in Table 1.

We choose two classic coevolutionary algorithms (PICEAg and CMOPSO) and two classic evolutionary algorithms based on resource allocation strategy (EAG-MOEAD and MOEAD-DRA) as the comparison algorithms for our proposed PICEAg-DS. For each test problem, the five algorithms run 20 times, and their means and variants are calculated as their final results. To ensure the fairness of the comparison experiment, the parameters of each comparison algorithm are consistent except for their specific parameters. The experiment parameter settings are shown in Table 2.

**TABLE 1.** The attributes of the WFG test suites.

| Pro. | Attributes |
|---|---|
| WFG1 | Convex, mixed, biased, separable |
| WFG2 | Convex, discontinuous, indecomposable |
| WFG3 | Linear, degenerate, single-modal, indecomposable |
| WFG4 | Concave, multimodal, decomposable |
| WFG5 | Concave, deceptive, decomposable |
| WFG6 | Concave, monomodal, indecomposable |
| WFG7 | Concave, monomodal, biased, decomposable |
| WFG8 | Concave, monomodal, biased, indecomposable |
| WFG9 | Concave, multimodal, deceptive, biased, indecomposable |

### B. PERFORMANCE INDICATORS

The optimization results of MOEAs are a group of optimal solutions that approximate to the PF, but it is difficult to measure the quality of the nondominated solutions. Therefore, this paper uses the GD [39], SP [40] and IGD [41] as the performance measuring methods to evaluate the convergence, distribution and comprehensive performance of the obtained solutions.

#### 1) GENERATION DISTANCE (GD)

It calculates the average Euclidean distance between each solution and its nearest true solution. The smaller the GD value, the better convergence of the obtained solution set.

#### 2) SPACING (SP)

It calculates the mean square error of the distance between every two adjacent solutions. The smaller the SP value, the better the distribution of the obtained solution set.

#### 3) INVERSE GENERATION DISTANCE (IGD)

It calculates the average distance between the true solutions and every obtained solution. The smaller the IGD value, the better the comprehensive performance of the obtained solution set.

### C. PARAMETER ANALYSIS

To further analyze the number of subspaces $d$ that influence the performance of PICEAg-DS and determine the optimal number of subspaces $d$, this paper sets the comparison experiments to $d = 8$, $d = 9$, $d = 10$, $d = 11$ and $d = 12$, which correspond to the algorithms of DS8, DS9, DS10, DS11 and DS12, respectively. The parameter comparison

**TABLE 2. The experiment parameters of the comparison algorithms.**

| Algorithms | Common parameters | Specific parameters |
|---|---|---|
| PICEAg-DS | (1) $MaxGen = 300$<br>(2) $N = 100$<br>(3) The crossover probability $Pc = 1$<br>(4) The mutation probability $P_m = 1/n$<br>($n$ is the number of decision variables)<br>(5) The distribution of polynomial mutation: $\eta_m = 20$<br>(6) The distribution of simulated binary crossover: $\eta_c = 20$ | $Ngoal = 100$<br>$\alpha = 1.2$<br>$d = 10$ |
| PICEAg | | $Ngoal = 100$<br>$\alpha = 1.2$ |
| CMOPSO | | *none* |
| EAG-MOEAD | | Learning algebra:<br>$LGS = 8$ |
| MOEAD-DRA | | Update $P_i$ : $\Delta_{gen}$= 10 |

experiments are tested on the WFG1-9 test problems. And set the $N = 100$, $N_{gaol} = 100$ and $Maxgen = 250$. The number of decision variables in 2-objective test problem is $D = 12$ and in 3-objective test problem is $D = 13$. The experiment results are shown in Table 3, and the bold in Table 3 indicates the IGD results of DS8-12 that are better than PICEAg.

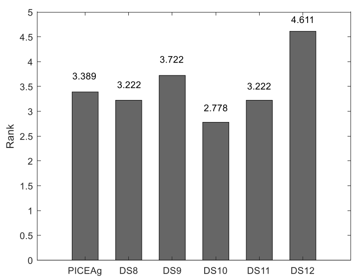



**FIGURE 5. The rank of the parameter comparison algorithms on 2- and 3-objective WFG2-9 test problems when d = 8, d = 9, d = 10, d = 11 and d = 12.**

To further calculate the average ranking of each parameter setting in Fig. 5., we can find the average ranking of $DS10$ performs better than others, therefore, the optimal group number of subspaces in PICEAg-DS is $d = 10$.

### D. THE PERFORMANCE ANALYSIS OF PICEAg-DS ON WFG TEST SUITE

To prove the effectiveness of the differentiated resource allocation strategy in PICEA-DS, we use the GD, SP and IGD indicators to measure the convergence, solution distribution and comprehensive performance of the solutions obtained by PICEAg-DS, PICEAg, CMOPSO, EAG-MOEAD and MOEAD-DRA are compared. And in Table 4 to Table 7, the "-", "+", and "=" indicate that the performance of the comparison algorithm is significantly worse than, better than, and not significantly different than that of PICEAg-DS with rank sum test.

#### 1) THE DISTRIBUTION ANALYSIS

To analyze the solution distribution of the comparison algorithms, the five comparison algorithms are executed on the WFG1-9 test problems 20 times. The means of the SP values are recorded in Table 4. The smaller SP value, the better the distribution of the obtained solutions.

In Table 4, it can be seen that the PICEAg-DS has an improvement in solution distribution on 2-objective WFG4, 6-9 and 3-objective WFG2, 8 functions, and its SP value is significantly better than CMOPSO, EAG-MOEAD and MOEA/D-DRA. The reason is that differentiated resource allocation strategy of PICEAg-DS divides the objective space according to the distribution of individuals and then allocates more target vectors to the sparse region to enhance population evolution in the sparse region, Therefore, it can balance the solution distribution both in dense and sparse subspaces.

To observe the change in SP values in the process of optimization. The Fig. 6. shows the curve of SP values of each algorithm on the 2-objective WFG2-9 functions; each node is the recorded SP value every ten generations. From Fig. 6(a)-(h), we can see that the SP value of PICEAg-DS declines sharply in the early stages of evolution, which means the solution distribution improved greatly. Because of the population random initialization, the distribution performance of each algorithm is poor in the early stage. However, the differentiated resource allocation strategy in PICEAg-DS can effectively allocate resources according to the individual distribution and improve the individual evolution in the sparse subspace, which can balance the distribution of different subspaces to some extent. Note that there always a fluctuation in the curve of SP values in Fig. 6(a)-(e). This is because the fitness evaluation method in PICEAg-DS selects the optimal solutions based on their objective functions but ignores their distribution; this causes the SP values to decline for a short time.

#### 2) THE CONVERGENCE ANALYSIS

From Table 5, it can be seen that the PICEAg-DS has an improvement in convergence on 2-objective WFG1, 3-4, 7-9 and 3-objective WFG4, 8 functions, and its GD value

**TABLE 3. The IGD results of the parameter setting of *d*.**

| Pro | | PICEAg | DS8 | DS9 | DS10 | DS11 | DS12 |
|---|---|---|---|---|---|---|---|
| WFG1 | 2 | 1.1808 e-1(3.85 e-2) | **1.1013 e-1(2.33 e-2)** | **1.1218 e-1(3.53 e-2)** | **1.0933 e-1(4.07 e-2)** | 1.3663 e-1(3.94 e-2) | **1.1765 e-1(3.22 e-2)** |
| | 3 | 3.9734 e-1(6.61 e-2) | 4.1462 e-1(4.95 e-2) | **3.8598 e-1(6.09 e-2)** | **3.8500 e-1(7.05 e-2)** | 4.0823 e-1(5.42 e-2) | 4.0835 e-1(6.19 e-2) |
| WFG2 | 2 | 1.6058 e-1(3.71e-3) | 1.6142 e-1(4.71e-3) | **1.6004 e-1(4.49e-3)** | **1.5790 e-1(3.56e-3)** | **1.5475 e-1(3.19e-3)** | **1.5520 e-1(3.68e-3)** |
| | 3 | 1.1872 e-2(6.20e-4) | 1.2078 e-2(6.22e-4) | 1.2079 e-2(4.79e-4) | 1.1973 e-2(4.40e-4) | 1.2141 e-2(5.40e-4) | 1.2438 e-2(7.54e-4) |
| WFG3 | 2 | 5.4082 e-2(4.35e-3) | **5.2931 e-2(4.34e-3)** | **5.3060 e-2(4.98e-3)** | **5.1069 e-2(3.70e-3)** | **5.3452 e-2(4.17e-3)** | 7.4044 e-2(3.80 e-2) |
| | 3 | 1.3713 e-2(6.87e-4) | **1.3670 e-2(7.14e-4)** | **1.3696 e-2(7.52e-4)** | **1.3713 e-2(7.02e-4)** | 1.3803 e-2(7.18e-4) | 1.4174 e-2(9.94e-4) |
| WFG4 | 2 | 2.1288 e-1(1.90e-3) | **2.1267 e-1(1.92e-3)** | 2.1336 e-1(1.76e-3) | **2.1235 e-1(2.38e-3)** | **2.1287 e-1(2.90e-3)** | 2.1699 e-1(2.77e-3) |
| | 3 | 1.4818 e-2(8.62e-4) | 1.4842 e-2(8.50e-4) | 1.4935 e-2(1.13e-3) | **1.4565 e-2(8.88e-4)** | **1.4291 e-2(6.17e-4)** | **1.4152 e-2(5.41e-4)** |
| WFG5 | 2 | 2.2398 e-1(1.80e-3) | **2.2219 e-1(1.48e-3)** | **2.2297 e-1(1.41e-3)** | **2.2248 e-1(1.78e-3)** | **2.2191 e-1(2.32e-3)** | **2.2771 e-1(2.49e-3)** |
| | 3 | 6.3946 e-2(2.09e-4) | 6.3962 e-2(1.96e-4) | 6.4301 e-2(1.41e-3) | 6.4012 e-2(7.70e-4) | **6.3903 e-2(1.53e-4)** | 6.4080 e-2(8.28e-4) |
| WFG6 | 2 | 2.3960 e-1(1.19e-2) | 2.4103 e-1(1.03e-2) | 2.4096 e-1(7.23e-3) | **2.3793 e-1(8.50e-3)** | 2.4426 e-1(9.95e-3) | 2.5092 e-1(1.42e-2) |
| | 3 | 8.6194 e-2(2.37 e-2) | 9.6562 e- 2(2.30e-2) | 9.2116 e-2(2.19e-2) | 9.7892 e-2(1.82e-2) | 9.0543 e-2(2.07 e-2) | 8.8633 e-2 (2.08 e-2) |
| WFG7 | 2 | 2.1397 e-1(2.83e-3) | 2.1492 e-1(2.58e-3) | **2.1277 e-1(2.57e-3)** | 2.1592 e-1(3.19e-3) | 2.2177 e-1(7.41e-3) | 2.3557e-1(7.25e-3) |
| | 3 | 1.3769 e-2(3.42e-4) | **1.3564 e-2(1.96e-4)** | **1.3568 e-2(2.93e-4)** | **1.3494 e-2(3.02e-4)** | **1.3391 e-2(2.63e-4)** | **1.3571 e-2(3.85e-4)** |
| WFG8 | 2 | 3.1189 e-1(4.97e-3) | **3.0897 e-1(5.91e-3)** | **3.0789 e-1(4.97e-3)** | **3.0867 e-1(5.48e-3)** | **3.0723 e-1(4.22e-3)** | **3.0883 e-1(4.68e-3)** |
| | 3 | 1.1423 e-1(2.03e-3) | 1.1471 e-1(2.17e-3) | 1.1698 e-1(5.55e-3) | 1.1502 e-1(1.81e-3) | 1.1680 e-1(4.79e-3) | 1.1576 e-1(3.86e-3) |
| WFG9 | 2 | 2.0969 e-1(1.71e-3) | 2.0980 e-1(2.08e-3) | 2.1025 e-1(2.72e-3) | 2.1052 e-1(1.87e-3) | 2.1220 e-1(2.02e-3) | 2.2414 e-1(7.55e-3) |
| | 3 | 2.1795 e-2(2.77e-3) | **2.0875 e-2(3.06e-3)** | **2.1572 e-2(2.86e-3)** | **2.0939 e-2(2.79e-3)** | **1.9264 e-2(2.39e-3)** | **2.0432 e-2(4.51e-3)** |
| Total (superior than PICEAg) | | | **8** | **10** | **12** | **9** | **7** |

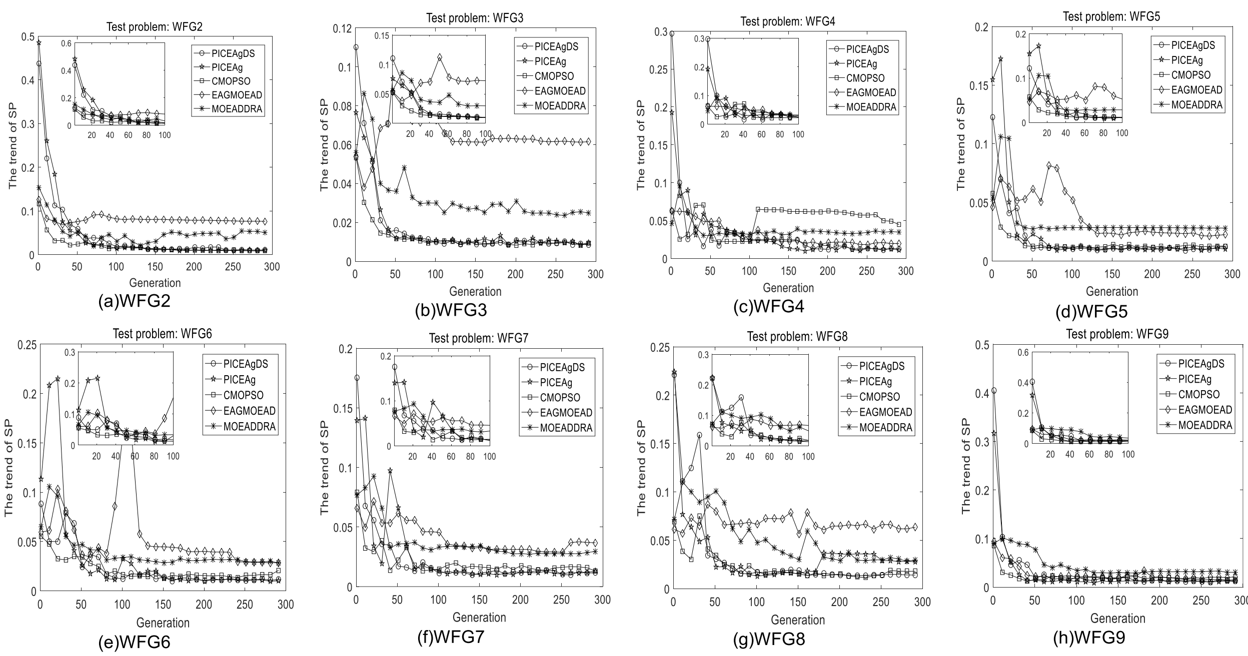



**FIGURE 6. The curves of the SP values of the five comparison algorithms on the 2-objective WFG2-9 test problems.**

is significantly better than CMOPSO, EAG-MOEAD and MOEA/D-DRA. And the its GD value is significantly better than PICEAg on 2-objective WFG1, 3, 4, 7, 9 and 3-objective WFG 4, 8.

From Figs. 7(c) and 7(h), we can see the GD values of PICEAg-DS decline sharply at *gen* = [0, 50], and its convergence speed is greater than the other algorithms. However, it is well known that WFG4 and WFG9 are two

**TABLE 4.** The SP results of the five comparison algorithms on the WFG1-9 test problems.

| Problems | | PICEAg-DS Mean (variance) | PICEAg Mean (variance) | CMOPSO Mean (variance) | EAG-MOEAD Mean (variance) | MOEAD-DRA Mean (variance) |
|---|---|---|---|---|---|---|
| WFG1 | 2 | 1.2957 e-1(2.37 e-2) | 1.2031 e-1(5.36 e-2) = | 2.1153 e-1(3.07 e-2) - | **5.3544 e-2(4.39 e-2)** + | 3.2007 e-1(8.25 e-2)- |
| | 3 | 2.9411 e-1(2.51 e-2) | 2.9887 e-1(9.56 e-2) = | 3.4073 e-1(6.10 e-2) = | **2.2395 e-1(1.57 e-1)** = | 5.1753 e-1(1.58 e-1) - |
| WFG2 | 2 | 9.7520 e-3(1.58 e-3) | 9.6893 e-3(1.63 e-3) = | **8.8779 e-3(1.27 e-3) =** | 3.5220 e-2(2.23 e-2) - | 5.2824 e-2(3.94e-3) - |
| | 3 | **8.0314 e-2(7.06e-3)** | 8.3463 e-2(7.58e-3) = | 1.5615 e-1(4.04e-2) - | 3.1093 e-1(5.55e-2) - | 2.4387 e-1(1.01 e-1) - |
| WFG3 | 2 | 9.6006 e-3(7.36e-4) | 9.6513 e-3(1.29 e-3) = | **9.2427 e-3(1.02 e-3) =** | 6.2908 e-2(1.62 e-2) - | 2.4731 e-2(8.56e-4) - |
| | 3 | 5.6811 e-2(7.62e-3) | **5.5269 e-2(8.09e-3) =** | 6.9241 e-2(6.34e-3) - | 1.7866 e-1(1.45e-2) - | 2.3771 e-1(4.11e-2) - |
| WFG4 | 2 | **1.2397 e-2(2.45e-3)** | 1.3527 e-2(2.55e-3) = | 3.6402 e-2(1.35 e-2) - | 2.2485 e-2(3.65e-3) - | 5.6121 e-2(2.41 e-2) - |
| | 3 | 9.6774 e-2(9.10e-3) | **9.5979 e-2(8.70e-3) =** | 1.4597 e-1(1.31e-2) - | 2.5359 e-1(2.41e-2) - | 3.6819 e-1(2.07e-2) - |
| WFG5 | 2 | 1.0786 e-2(9.82e-4) | **1.0375 e-2(9.44e-4) =** | 1.3517 e-2(1.40e-3) - | 2.1986 e-2(2.32e-3) - | 2.8519 e-2(7.78e-4) - |
| | 3 | 9.4349 e-2(1.09 e-2) | **9.3987 e-2(5.84e-3) =** | 1.2905 e-1(1.58e-2) - | 2.4121 e-1(2.84e-2) - | 3.5544 e-1(1.22e-2) - |
| WFG6 | 2 | **1.0906 e-2(1.06e-3)** | 1.1117 e-2(9.45e-4) = | 1.4263 e-2(1.39e-3) - | 7.0525 e-2(3.66 e-2) - | 2.8856 e-2(3.62e-3) - |
| | 3 | 9.8796 e-2(8.83e-3) | **9.7707 e-2(7.71e-3) =** | 1.4403 e-1(1.36e-2) - | 2.6669 e-1(4.14e-2) - | 3.9001 e-1(3.71e-2) - |
| WFG7 | 2 | **1.0966 e-2(1.18e-3)** | 1.1136 e-2(1.01e-3) = | 1.6453 e-2(2.92e-3) - | 6.7359 e-2(2.28 e-2) - | 2.7059 e-2(9.27e-4) - |
| | 3 | 9.7071 e-2(7.87e-3) | **9.5264 e-2(9.57e-3) =** | 1.4265 e-1(1.17e-2) - | 2.5273 e-1(3.95e-2) - | 4.0234 e-1(4.18e-2) - |
| WFG8 | 2 | **1.5059 e-2(6.23e-3)** | 1.5591 e-2(9.69e-3) = | 1.9466 e-2(5.06e-3) - | 1.0380 e-1(4.23e-2) - | 2.8920 e-2(4.29e-3) - |
| | 3 | **1.0604 e-1(9.76e-3)** | 1.0633 e-1(9.04e-3) = | 1.2940 e-1(1.30e-2) - | 2.9646 e-1(3.27e-2) - | 4.2958 e-1(3.86e-2) - |
| WFG9 | 2 | **1.1919 e-2(2.05e-3)** | 1.3621 e-2(4.15e-3) = | 1.5192 e-2(2.13e-3) - | 2.0356 e-2(2.85e-3) - | 2.7897 e-2(1.90e-3) - |
| | 3 | 1.0136 e-1(9.30e-3) | **9.3226 e-2(1.01 e-2)** + | 1.3019 e-1(1.27e-2) - | 2.2945 e-1(2.41e-2) - | 3.5236 e-1(1.69e-2) - |
| **Total (+/-/=)** | | | **1/0/17** | **0/15/3** | **1/16/1** | **0/18/0** |

multimodal functions, which is easy to fall into the local optimal and it must be iterated many times in the early stage to convergence. But PICEAg-DS performs well on these two test problems, which reflects its advantage on multimodal problems. Moreover, WFG7-9 are biased functions, which makes them difficult to improve population convergence and maintain diversity. However, the convergence performance of PICEAg-DS on the WFG7-9 functions is better than the other algorithms, which reflects the advantage of PICEAg-DS to improve the convergence performance on discontinuous, biased or multimodal test problems to some extent. We can also see that the convergence performance of PICEAg-DS is inferior to that of the other algorithms in the early stage of optimizing the WFG2-4 test problems. That is, because of the population random initialization after the differentiated resource allocation strategy in PICEAg-DS, which allocates more target vectors to the sparse subspaces and promotes their convergence. Therefore, the convergence of PICEAg-DS is faster than the other algorithms. For the WFG7-9 test problems, the convergence speed of PICEAg-DS is slightly lower than that of PICEAg and CMOPSO when $gen = [1, 50]$. That is because the PICEAg-DS focus on the convergence in some subspaces that difficult to optimizing, which ignored the convergence of the entire objective space to some extent. However, in the later stage, PICEAg-DS is faster than the PICEAg and CMOPSO, and finally outperforms the other comparison algorithms.

### 3) THE COMPREHENSIVE PERFORMANCE ANALYSIS

From Table 6, we can find the IGD values of PICEAg-DS outperform the other algorithms on WFG1-5 and WFG7-9 in 2-objective optimization problems and WFG1-3, WFG4 and WFG8 in 3-objective optimization problems. However, it is well known that WFG2, WFG3 and WFG6 are indecomposable functions, which are more difficult to optimize than the decomposable functions of WFG4 and WFG5. WFG7-9 are biased functions, which makes it difficult to improve their diversity, and WFG9 is a complex problem that has the characteristics of decomposable, multimodal, deceptive and biased. Therefore, WFG9 is more difficult than the other test problems. Moreover, the variance of PICEAg-DS is better than the other algorithms in most test problems, which means the PICEAg-DS has a good robustness. In summary, the PICEAg-DS has a good comprehensive

**TABLE 5. The GD results of the five comparison algorithms on the WFG1-9 test problems.**

| Problems | | PICEAg-DS Mean (variance) | PICEAg Mean (variance) | CMOPSO Mean (variance) | EAG-MOEAD Mean (variance) | MOEAD-DRA Mean (variance) |
|---|---|---|---|---|---|---|
| WFG1 | 2 | **6.2904 e-3(2.57 e-3)** | 6.9792 e-3(3.01 e-3) - | 9.2394 e-2(1.03 e-2) - | 7.2639 e-2(2.08 e-2) - | 1.0726 e-1 (2.03 e-2) - |
| | 3 | 2.4134 e-2(5.28 e-3) | **2.0763 e-2(5.40 e-3) =** | 1.4484 e-1(1.61 e-3) - | 4.8230 e-2(1.49 e-2) - | 1.8428 e-1(1.81 e-2) - |
| WFG2 | 2 | **3.0605 e-4(8.63e-5)** | 3.3234 e-4(5.84e-5) = | 3.5281 e-4(4.03e-5) - | 1.3291 e-3(1.17 e-3) - | 9.7692 e-4(2.17 e-4) - |
| | 3 | 6.5477 e-3(1.71 e-3) | **6.3047 e-3(1.10 e-3) =** | 2.0301 e-2(7.47e-3) - | 4.2791 e-2(1.10 e-2) - | 1.2656 e-2(9.69e-3) - |
| WFG3 | 2 | **5.2871 e-4(6.74e-5)** | 6.3928 e-4(1.27 e-4) - | 6.5357 e-4(9.09e-5) - | 9.6796 e-3(2.80 e-3) - | 1.1945 e-3(2.27e-4) - |
| | 3 | **5.2977 e-2(6.09e-3)** | 5.4831 e-2(8.45e-3) = | 96469 e-2(2.14e-3) - | 1.1693 e-1(4.11e-3) - | 1.1256 e-1(2.48e-3) - |
| WFG4 | 2 | **4.6426 e-4(9.13e-5)** | 5.8248 e-4(1.25 e-4) - | 4.0386 e-3(1.45 e-3) - | 5.5906 e-4(1.59 e-4) - | 8.8950 e-3(9.31e-4) - |
| | 3 | **3.4126 e-3(2.18e-4)** | 3.9224 e-3(3.46e-4) - | 1.4995 e-2(6.70e-4) - | 8.1323 e-3(3.31 e-3) - | 1.4046 e-2(9.64e-4) - |
| WFG5 | 2 | 6.3182 e-3(3.09e-5) | 6.3145 e-3(3.61e-5) = | 6.6043 e-3(1.01e-4) - | 6.4540 e-3(6.86e-5) - | **6.0731 e-3(1.55e-5) +** |
| | 3 | 8.8989 e-3(6.02e-4) | 8.3141 e-3(2.70e-4) + | 13010 e-2 (9.43e-4) - | 9.7134 e-3(2.13 e-3) = | **7.9615 e-3(2.38e-4) +** |
| WFG6 | 2 | 7.1659 e-3(1.86 e-3) | 7.9421 e-3(2.22 e-3) = | **1.3797 e-3(9.47e-4) +** | 1.2654 e-2(2.88e-3) - | 9.9763 e-3(7.55 e-3) = |
| | 3 | 1.1625 e-2(2.47e-3) | 1.1380 e-2(2.41e-3) = | **1.0860 e-2(1.32e-3) =** | 4.8282 e-2(6.23e-3) - | 2.2677 e-2(4.93e-3) - |
| WFG7 | 2 | **3.7599 e-4(7.30e-5)** | 4.8456 e-4(7.67e-5) - | 86368 e-4(7.43e-5) - | 5.6129 e-3(2.47 e-3) - | 4.9864 e-4(1.35 e-4) - |
| | 3 | **3.6874 e-3(3.30e-4)** | 3.7443 e-3(2.13e-4) = | 1.0793 e-2(1.08e-3) = | 2.9610 e-2(4.37e-3) - | 7.1999 e-3(1.33 e-3) - |
| WFG8 | 2 | **1.1606 e-2(1.75e-4)** | 1.1674 e-2(1.31e-4) = | 1.2104 e-2(4.17e-4) - | 3.1512 e-2(1.97 e-2) - | 1.2082 e-2(1.04e-3) - |
| | 3 | **2.3521 e-2(7.70e-4)** | 2.4014 e-2(5.86e-4) - | 2.7619 e-2(1.34e-3) - | 5.5672 e-2(5.11e-3) - | 3.8078 e-2(9.34e-3) - |
| WFG9 | 2 | **1.3250 e-3(4.90e-4)** | 1.6420 e-3(3.12e-4) - | 2.1474 e-3(1.38e-4) - | 5.7850 e-3(3.39 e-3) - | 6.0249 e-3(8.27 e-3) - |
| | 3 | 5.4654 e-3(4.46e-4) | **4.9394 e-3(3.11e-4) +** | 7.9621 e-3(9.61e-4) - | 1.2790 e-2(7.75e-3) - | 9.3194 e-3(6.21 e-3) - |
| **Total (+/-/=)** | | | **2/7/9** | **1/15/2** | **0/17/1** | **2/15/1** |

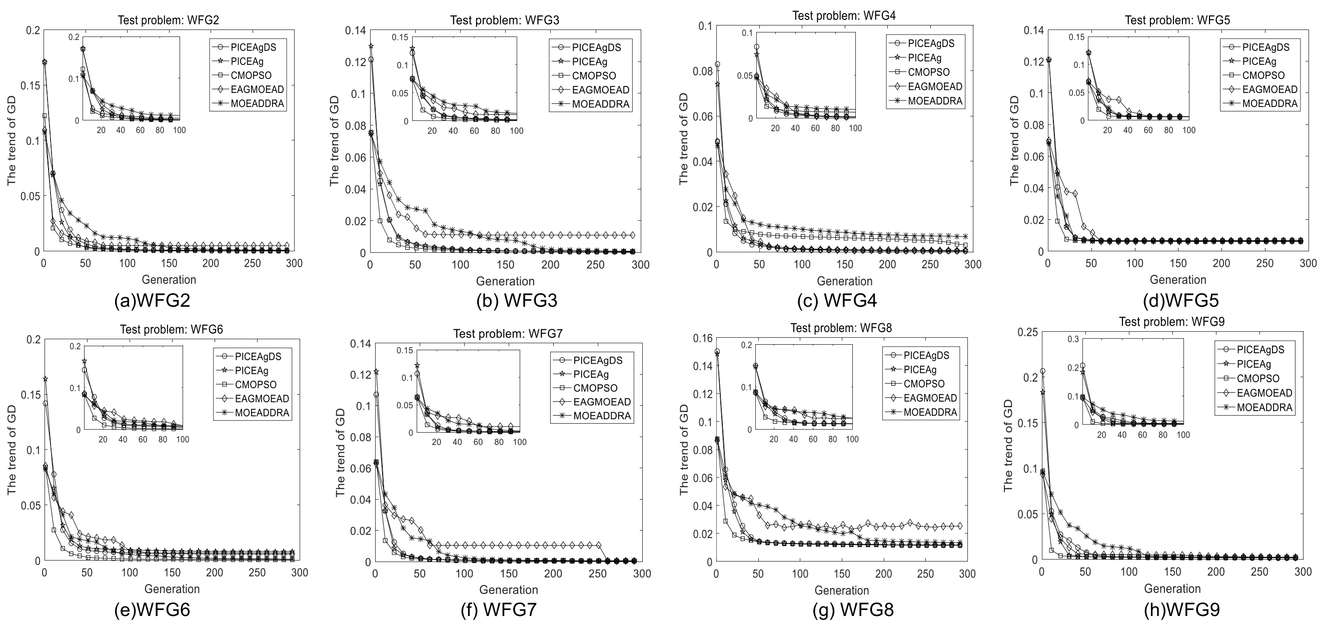



**FIGURE 7. The curves of the GD values of the five comparison algorithms on the 2-objective WFG2-9 test problems.**

performance in most of WFG test problems. It owes to the contribution of the differentiated resource allocation strategy, which allocates more target vectors to the individual sparse subspace that not only balances the solution distribution in both the sparse and dense subspaces but also enhances the evolution ability of individuals in the sparse

**TABLE 6. The IGD results of the five comparison algorithms on the WFG1-9 test problems.**

| Problem | | PICEAg-DS Mean (variance) | PICEAg Mean (variance) | CMOPSO Mean (variance) | EAG-MOEAD Mean (variance) | MOEAD-DRA Mean (variance) |
|---|---|---|---|---|---|---|
| WFG1 | 2 | **9.9579 e-2(2.33 e-2)** | 1.1942 e-1(2.98 e-2) - | 9.5639 e-1(7.04 e-2) - | 7.3956 e-1(1.91 e-1) - | 9.5375 e-1(8.22 e-2) - |
| | 3 | **3.1497 e-1(4.34 e-2)** | 3.3004 e-1(4.83 e-2) = | 1.5028 e+0(1.49 e-2) - | 8.2797 e-1(2.01 e-1) - | 1.5842 e+0(8.10 e-2) - |
| WFG2 | 2 | **1.1648e-2** (3.86e-4) | 1.1972e-2 (6.36e-4) = | 1.1888e-2 (2.45e-4) - | 5.1594e-2 (3.75e-2) - | 2.5769e-2 (3.36e-3) - |
| | 3 | **1.5897e-1 (3.48e-3)** | 1.6058e-1 (3.71e-3) = | 1.8010e-1 (4.35e-3) - | 3.0443e-1 (2.71e-2) - | 3.4989e-1 (3.03e-2) - |
| WFG3 | 2 | **1.3188e-2 (3.62e-4)** | 1.3774e-2 (7.61e-4) - | 1.3804e-2 (4.54e-4) - | 1.8588e-1 (3.28e-2) - | 1.9351e-2 (1.72e-3) - |
| | 3 | **5.2068e-2 (5.24e-3)** | 5.4082e-2 (4.35e-3) = | 1.5492e-1 (1.37e-2) - | 3.1640e-1 (5.61e-2) - | 1.9687e-1 (2.79e-2) - |
| WFG4 | 2 | **1.4129e-2 (6.73e-4)** | 1.4702e-2 (7.52e-4) - | 4.5275e-2 (1.44e-2) - | 1.6805e-2 (1.78e-3) - | 8.2307e-2 (9.64e-3) - |
| | 3 | **2.1279**e-1 (2.32e-3) | 2.1288e-1 (1.90e-3) = | 2.6342e-1 (3.78e-3) - | 3.2382e-1 (1.60e-2) - | 3.8713e-1 (1.32e-2) - |
| WFG5 | 2 | **6.3919e-2 (1.58e-4)** | 6.3947e-2 (1.90e-4) = | 6.7618e-2 (2.71e-3) - | 6.5599e-2 (5.96e-4) - | 6.9735e-2 (7.41e-5) - |
| | 3 | 2.2436e-1 (2.88e-3) | **2.2398e-1 (1.80e-3)** = | 2.4887e-1 (5.58e-3) - | 3.7606e-1 (1.91e-2) - | 3.3676e-1 (3.16e-3) - |
| WFG6 | 2 | 7.2078e-2 (1.78e-2) | 7.9654e-2 (2.14e-2) = | **2.0009e-2 (7.40e-3)** + | 2.4227e-1 (3.32e-2) - | 1.0126e-1 (7.46e-2) = |
| | 3 | 2.3993e-1 (1.18e-2) | **2.3960e-1 (1.19e-2)** = | 2.4034e-1 (7.18e-3) = | 6.2858e-1 (3.70e-2) - | 4.4043e-1 (2.49e-2) - |
| WFG7 | 2 | **1.3346e-2 (2.43e-4)** | 1.3795e-2 (3.65e-4) - | 1.6742e-2 (8.44e-4) - | 1.5730e-1 (3.38e-2) - | 1.5252e-2 (7.09e-4) - |
| | 3 | 2.1634e-1 (3.84e-3) | **2.1397e-1 (2.83e-3)** + | 2.3963e-1 (5.56e-3) - | 5.5977e-1 (3.27e-2) - | 3.6851e-1 (8.28e-3) - |
| WFG8 | 2 | **1.1362e-1 (1.86e-3)** | 1.1490e-1 (2.07e-3) - | 1.1855e-1 (3.82e-3) - | 3.1854e-1 (2.96e-2) - | 1.1448e-1 (9.05e-3) = |
| | 3 | 3.0862e-1 (5.81e-3) | 3.1189e-1 (4.97e-3) = | 3.3841e-1 (8.95e-3) - | 6.5143e-1 (2.98e-2) - | 4.7304e-1 (6.01e-2) - |
| WFG9 | 2 | 1.9503e-2 (3.95e-3) | 2.1979e-2 (2.77e-3) - | 2.6534e-2 (1.84e-3) - | 5.9040e-2 (3.10e-2) - | 7.1048e-2 (8.37e-2) - |
| | 3 | 2.1137e-1 (2.51e-3) | **2.0969e-1 (1.71e-3)** + | 2.2301e-1 (5.44e-3) - | 3.2680e-1 (3.99e-2) - | 3.4718e-1 (2.85e-2) - |
| **Total (+/-/=)** | | | **2/6/10** | **1/16/1** | **0/18/0** | **0/16/2** |

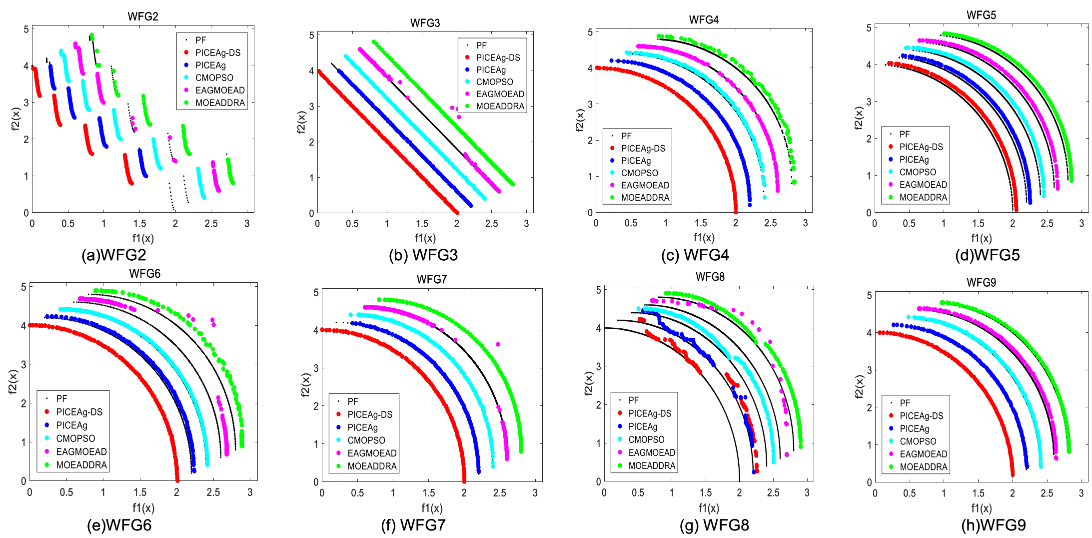



**FIGURE 8. The PF obtained by the five algorithms on the 2-objective WFG2-9 test problems.**

subspace, thus improving the overall convergence of the population.

### 4) THE PF COMPARATIVE ANALYSIS

To intuitively show the optimization results of the five comparison algorithms, Fig. 8. shows the PF obtained by the comparison algorithms on the 2-objective WFG2-9 test problems. From Fig. 8, we can see that the PF obtained by PICEAg-DS all converge to the true PF on most of the test problems (except for EFG5 and WFG8). On the WFG2 test problem, although the PF obtained by MOEAD-DRA and EAG-MOEAD all converge to the true

**TABLE 7.** The GD results of the five comparison algorithms on the UF1-9 and MOP1-7 test problems.

| Problem | M | PICEAg-DS Mean (variance) | RVEA Mean (variance) | MOEA/D-M2M Mean (variance) | NSGAII Mean (variance) | SPEA2 Mean (variance) |
|---|---|---|---|---|---|---|
| UF1 | 2 | **1.5880e-4 (2.40e-4)** | 1.3315e-2 (1.18e-2) - | 2.1949e-3 (4.19e-3) - | 5.4240e-4 (7.02e-4) - | 3.1334e-4 (6.15e-4) = |
| UF2 | 2 | 7.1059e-4 (4.13e-4) | 5.0003e-3 (7.79e-4) - | 1.3953e-3 (3.12e-3) = | 5.0478e-4 (1.11e-4) = | **4.4918e-4 (1.53e-4) +** |
| UF3 | 2 | 2.7884e-3 (2.59e-3) | 3.4364e-3 (3.05e-3) = | 3.9992e-3 (3.79e-3) = | **8.9897e-4 (8.16e-4) +** | 1.1723e-3 (8.68e-4) + |
| UF4 | 2 | **2.4133e-3 (1.10e-5)** | 1.0646e-2 (7.90e-4) - | 2.5719e-3 (6.84e-5) - | 2.4514e-3 (1.63e-5) - | 2.4380e-3 (1.44e-5) - |
| UF5 | 2 | 2.4543e-2 (1.30e-2) | 5.3660e-2 (1.33e-2) - | 4.0668e-2 (2.41e-2) = | 2.8974e-2 (1.64e-2) = | **1.8410e-2 (1.31e-2) =** |
| UF6 | 2 | 1.1231e-2 (9.87e-3) | 2.8299e-2 (1.93e-2) - | 1.5362e-2 (9.41e-3) = | **2.5312e-3 (3.47e-3) +** | 6.1149e-3 (9.69e-3) = |
| UF7 | 2 | 4.1861e-4 (3.94e-4) | 4.3916e-3 (1.09e-3) - | 8.0511e-4 (3.88e-4) - | **2.6886e-4 (1.45e-4) =** | 3.2066e-4 (2.30e-4) = |
| UF8 | 3 | **1.3770e-3 (3.12e-4)** | 3.2129e-1 (4.92e-2) - | 1.3578e-1 (2.36e-2) - | 1.7714e-1 (2.46e-2) - | 1.7500e-1 (5.61e-2) - |
| UF9 | 3 | **9.6551e-3 (6.06e-3)** | 1.1407e-1 (9.02e-2) - | 1.7980e-1 (3.15e-2) - | 5.4821e-2 (3.51e-2) - | 4.0794e-2 (1.57e-2) - |
| MOP1 | 2 | 2.0885e-4 (7.79e-5) | 2.5107e-2 (3.88e-3) - | 1.8416e-3 (1.05e-4) - | **1.9076e-4 (1.22e-4) =** | 2.1997e-4 (1.05e-4) = |
| MOP2 | 2 | 1.1324e-17 (7.09e-18) | 2.6131e-2 (6.86e-3) - | 1.9850e-3 (1.73e-3) - | 8.518 e-18 (5.82e-18) = | **0.0000e+0 (0.00e+0) +** |
| MOP3 | 2 | 7.0767e-3 (7.06e-3) | 7.7508e-2 (2.11e-2) - | **1.1482e-3 (7.86e-4) +** | 3.0919e-3 (1.31e-3) = | 2.0731e-2 (9.09e-3) - |
| MOP4 | 2 | **2.6697e-4 (1.36e-4)** | 9.9165e-3 (4.20e-3) - | 6.2069e-4 (2.54e-4) - | 8.3589e-4 (6.43e-4) = | 4.3075e-4 (4.38e-4) = |
| MOP5 | 2 | 7.1401e-2 (2.04e-2) | 4.8865e-2 (9.15e-3) + | **2.2974e-3 (2.24e-4) +** | 4.5987e-2 (2.21e-2) + | 4.0923e-2 (2.51e-2) + |
| MOP6 | 3 | **1.6941e-4 (1.66e-6)** | 1.6510e-2 (2.44e-3) - | 6.4606e-3 (1.91e-3) - | 1.6952e-4 (2.66e-6) = | 1.7005e-4 (2.05e-6) = |
| MOP7 | 3 | **1.9969e-4 (2.51e-6)** | 8.2615e-3 (3.37e-3) - | 3.3284e-3 (6.19e-4) - | 2.1396e-4 (4.49e-5) = | 2.1562e-4 (4.47e-5) = |
| **Total (+/-/=)** | | | **1/14/1** | **3/9/4** | **2/10/4** | **4/4/8** |

PF, their solution distribution is poor and some regions have no solutions. On the WFG3 test problem, the PF obtained by PICEA-DS, CMOPS and MOEAD-DRA all converge to the true PF, while the PF obtained by PICEAg and EAG-MOEAD are not the true PF; thus, their solution distribution is poor. On the WFG4 test problem, the convergence of PICEA-DS and PICEAg are better than the others. The solutions obtained by EAG-MOEAD and MOEAD-DRA both have an even distribution, and CMOPSO has no convergence and poor distribution in some areas. On the WFG5 test problem, the five algorithms do not fully converge to the true PF, but their distribution is fairly good. On the WFG6 test problem, the solutions obtained by PICEAg-DS, PICEAg and CMOPSO all converge to the true PF, but the EAG-MOEAD and MOEAD-DRA do not convergence and their solution distributions are poor. On the WFG7 test problem, except for the EAG-MOEAD, the other algorithms all converge to PF. On the WFG8 test problem, all the algorithms do not fully convergence, but the PF obtained by MOEAD-DRA and CMOPSO perform better than the others. On the WFG9 test problem, PICEAg-DS and PICEAg fully converge to the true PF, while the EAG-MOEAD does not convergence and CMOPSO and MOEAD-DRA do not converge in some areas.

In summary, except for the WFG5 and WFG8 test problems, the solutions obtained by PICEAg-DS all converge to the true PF and have a good distribution on the other test problems. It can be seen that the differentiated resource allocation strategy proposed in this paper can effectively improve the convergence and distribution of the population.

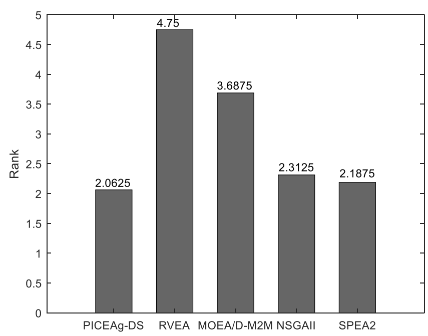



**FIGURE 9.** The average ranking in GD metric of five algorithms on MOP1-7and UF1-9 test problems.

## E. THE PERFORMANCE ANALYSIS OF PICEAg-DS ON MOP AND UF TEST SUITE

To further analysis the performance of PICEAg-DS on MOPs with complicated PF shape, such as UF test suite [29] and a combination test suite, such as MOP test suite [5]. We compare PICEAg-DS with some popular algorithms,

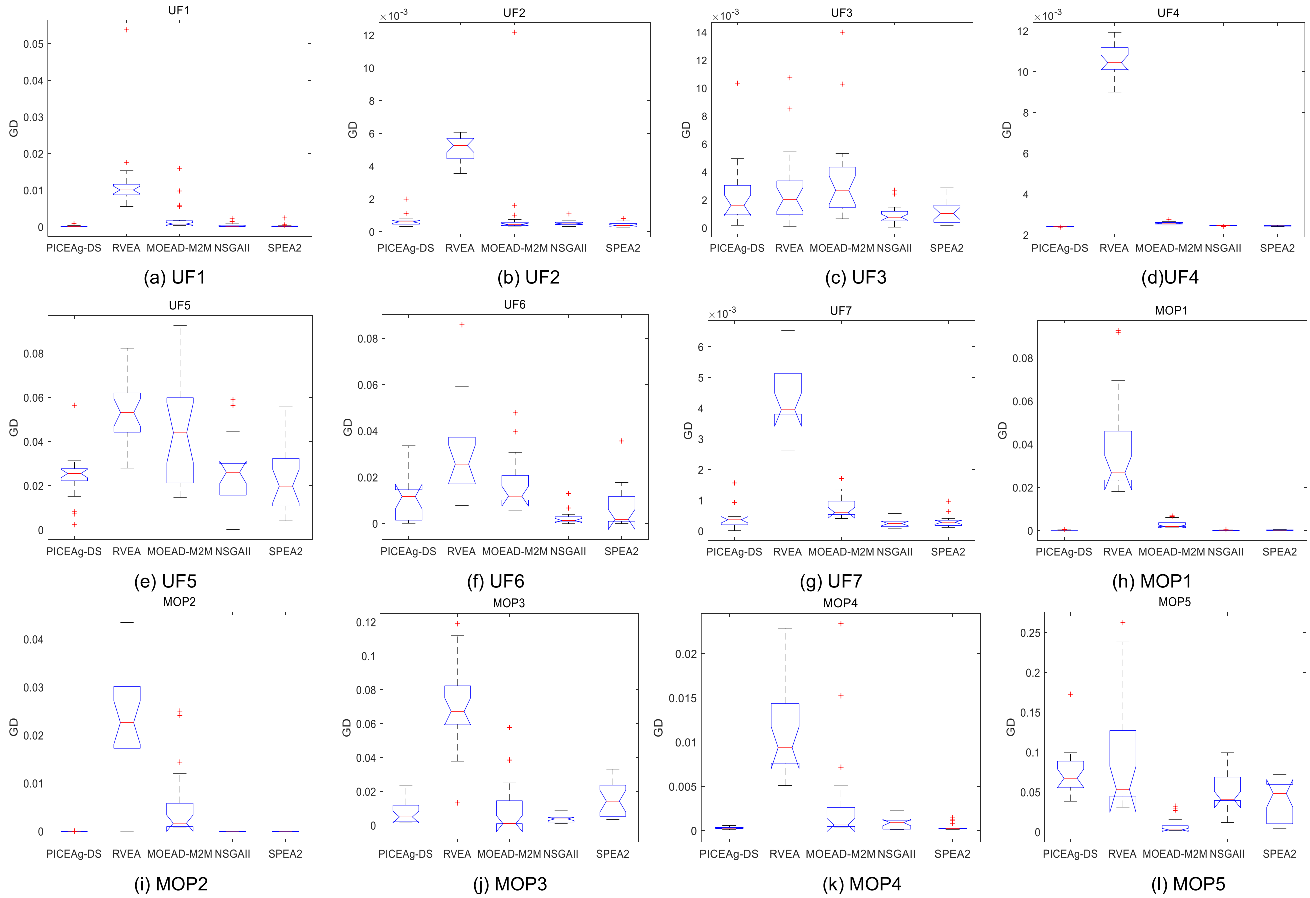
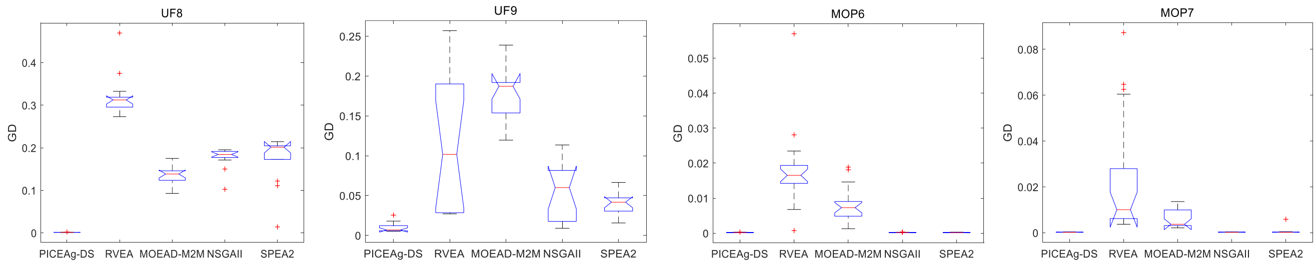



**FIGURE 10.** **The box charts of GD values for the five comparison algorithms on the 2-objective UF1-7 and MOP1-5 test problems.**

such as NSGA-II, SPEA2, RVEA, MOEA/D-M2M on these test problems. And using the GD indicator to evaluation their convergence performance. The NSGA-II and SPEA2 are MOEAs based on a domination relationship and the RVEA and MOEA/D-M2M are MOEAs based on decomposition, which are all representative MOEAs. The parameter of simulated binary crossover (SBX) and polynomial mutation are same setting with Table 2. Besides, the number of subproblems in MOEA/D-M2M is $K = 10$; the control parameter of RVEA is $\alpha = 2$ and set the $N = 100$, $Maxgen = 3000$ for 2-objective MOP1-5 and $N = 300$, $Maxgen = 3000$ for 3-objective MOP6-7; $N = 300$, $Maxgen = 1000$ for 2-objective UF1-7 and $N = 300$, $Maxgen = 1000$ for 3-objective UF8-9. Besides, the differential resource allocation in PICEAg-DS aims to promote the evolution in whole objective space. Therefore, we using the GD indicator to evaluate their convergence. To ensure the fairness of the comparison experiment, each algorithm runs on each test problem for 20 times, and then calculate their mean and variance as the final results, which can be seen in Table 7.

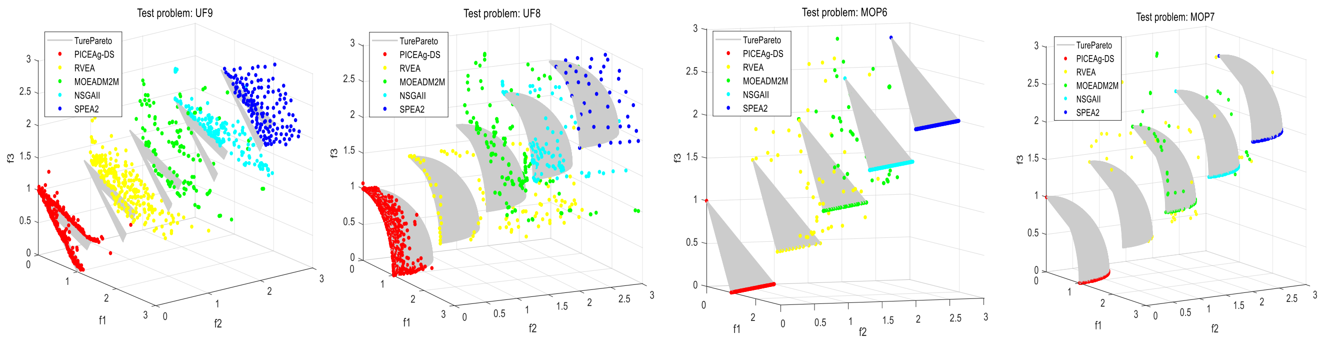
In Table 4, it can be seen that the PICEAg-DS has an improvement in convergence on 2-objective UF1,4, MOP4 and 3-objective UF8-9, MOP6-7 functions, and its GD value in UF4,8-9 is significantly better than the other four comparison algorithms. And the average ranking in GD metric of five comparison algorithms is shown in Fig. 9. And we can find the proposed PICEAg-DS ranking first, and followed SPEA2, NSGAII, MOEA/D-M2M and RVEA. It proved that the PICEAg-DS also has a good convergence in both MOPs with complicated PS shape and combinational test suite. It not only contributes the PICEAg-DS that combine the advantage of decomposition and domination strategies, but also has a differential resource allocation to balance the evolutionary ability of the whole population.

Figs. 10 and 11 show the box charts of the GD values of the five algorithms in 2-objective UF1-7, MOP1-5 and 3-objective UF8-9, MOP6-7 test problems. In these box charts, "+" represents an abnormal value and the five horizontal lines from top-to-bottom represent the maximum value, the upper quartile, median, the lower quartile and the minimum value of the GD values for 20 times. From the Fig. 10, it can be seen that PICEAg-DS has a low column height and few outliers in most of 2-objective MOP and UF test problems, it proves that PICEAg-DS has a robust convergence performance in these test problems. From the




(a) UF8 (b) UF9 (c) MOP6 (d) MOP7

**FIGURE 11.** **The box charts of GD values for the five comparison algorithms on the 3-objective UF8-9 and MOP6-7 test problems.**




(a) UF8 (b) UF9 (c) MOP6 (d) MOP7

**FIGURE 12.** **The PF obtained by the five comparison algorithms on the 3-objective UF8-9 and MOP6-7 test problems.**

Fig. 11, we can see the PICEAg-DS has the minimum mean value of GD and lowest column height in all 3-objective MOP and UF test problems. Therefore, PICEAg-DS has a better convergence performance and robustness in these test problems than MOEA/D-M2M, NSGAII and SPEA2. It also shows the PICEAg-DS can maintain the convergence performance with the number of objective increased.

To intuitively show the optimization results of the five comparison algorithms in 3-objective test problems. The Fig. 12. shows the PF obtained by each comparison algorithm on UF8-9 and MOP6-7 test problems. From the Fig. 12, the PF obtained by PICEAg-DS converges to the true PF that is superior than the other algorithms, and it can be clearly observed in Fig. 12(a) and (b). It is proved the effectiveness of differential resource allocation in PICEAg-DS that can enhance the evolutionary ability of whole population and promote the convergence. But we also find the PICEAg-DS has a poor performance in diversity, the reason is that some regions of true PF are difficult to convergence, when the algorithms are not fully convergence, the solutions are well-distributed in objective space, and it can be seen the SPEA2 in Fig. 12(a)-(b). But when they approach to the true PF, the solutions tend to concentrate in the easy optimization region, such as the PICEAg-DS, NSGAII and SPEA2 in Fig. 12(c)-(d). Thus, a larger population or more evolutionary algebra may help to enhance the diversity. But we also find PICEAg-DS has a fast convergence ability than the other comparison algorithms under the same experiment condition.

## V. CONCLUSION

To solve the problem of the imbalance in evolutionary ability of the whole population, this paper proposes a preference-inspired coevolutionary algorithm based on a differentiated resource allocation strategy (PICEAg-DS). In PICEAg-DS, a space distance operator is designed to divide the objective space into several subspaces and evaluate the sparsity of each subspace. Based on this, it realizes the dynamically resource allocation and assigns more target vectors to the sparse subspaces to increase the selection pressure and thus improve the evolutionary ability. The effectiveness of differentiated resource allocation strategy and PICEAg-DS are proved in a series of simulation experiments. In the future work, we will consider an adaptive resource allocation strategy in different stage and further improve the performance on many-objective problems.

## REFERENCES

[1] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Trans. Evol. Comput.*, vol. 6, no. 2, pp. 182–197, Apr. 2002, doi: 10.1109/4235.996017.

[2] E. Zitzler, M. Laumanns, and L. Thiele, "SPEA2: Improving the strength Pareto evolutionary algorithm for multiobjective optimization," in *Proc. Evol. Methods Design, Optim. Control With Appl. Ind. Problems*, 2002, vol. 3242, no. 103, pp. 95–100.

[3] R. C. Purshouse and P. J. Fleming, "On the evolutionary optimization of many conflicting objectives," *IEEE Trans. Evol. Comput.*, vol. 11, no. 6, pp. 770–784, Dec. 2007, doi: 10.1109/TEVC.2007.910138.

[4] Q. Zhang and H. Li, "MOEA/D: A multiobjective evolutionary algorithm based on decomposition," *IEEE Trans. Evol. Comput.*, vol. 11, no. 6, pp. 712–731, Dec. 2007, doi: 10.1109/TEVC.2007.892759.

[5] H.-L. Liu, F. Gu, and Q. Zhang, "Decomposition of a multiobjective optimization problem into a number of simple multiobjective subproblems," *IEEE Trans. Evol. Comput.*, vol. 18, no. 3, pp. 450–455, Jun. 2014, doi: 10.1109/TEVC.2013.2281533.

[6] R. Cheng, Y. Jin, M. Olhofer, and B. Sendhoff, "A reference vector guided evolutionary algorithm for many-objective optimization," *IEEE Trans. Evol. Comput.*, vol. 20, no. 5, pp. 773–791, Oct. 2016, doi: 10.1109/TEVC.2016.2519378.

[7] T. Pamulapati, R. Mallipeddi, and P. N. Suganthan, "$I_{SDE}$+—An indicator for multi and many-objective optimization," *IEEE Trans. Evol. Comput.*, vol. 23, no. 2, pp. 346–352, Apr. 2019, doi: 10.1109/TEVC.2018.2848921.

[8] J. Bader and E. Zitzler, "HypE: An algorithm for fast hypervolume-based many-objective optimization," *Evol. Comput.*, vol. 19, no. 1, pp. 45–76, Mar. 2011, doi: 10.1162/EVCO_a_00009.

[9] C. K. Goh, K. C. Tan, D. S. Liu, and S. C. Chiam, "A competitive and cooperative co-evolutionary approach to multi-objective particle swarm optimization algorithm design," *Eur. J. Oper. Res.*, vol. 202, no. 1, pp. 42–54, Apr. 2010, doi: 10.1016/j.ejor.2009.05.005.

[10] L. M. Antonio and C. A. C. Coello, "Coevolutionary multiobjective evolutionary algorithms: Survey of the state-of-the-art," *IEEE Trans. Evol. Comput.*, vol. 22, no. 6, pp. 851–865, Dec. 2018, doi: 10.1109/TEVC.2017.2767023.

[11] L. M. Antonio and C. A. C. Coello, "Indicator-based cooperative coevolution for multi-objective optimization," in *Proc. IEEE Congr. Evol. Comput. (CEC)*, Vancouver, BC, Canada, Jul. 2016, pp 991–998.

[12] M. Gong, H. Li, E. Luo, J. Liu, and J. Liu, "A multiobjective cooperative coevolutionary algorithm for hyperspectral sparse unmixing," *IEEE Trans. Evol. Comput.*, vol. 21, no. 2, pp. 234–248, Apr. 2017, doi: 10.1109/TEVC.2016.2598858.

[13] C. A. C. Coello and M. R. Sierra, "A coevolutionary multi-objective evolutionary algorithm," in *Proc. Congr. Evol. Comput. (CEC)*, Canberra, ACT, Australia, 2003, pp. 482–489, doi: 10.1109/CEC.2003.1299614.

[14] X. Zhang, X. Zheng, R. Cheng, J. Qiu, and Y. Jin, "A competitive mechanism based multi-objective particle swarm optimizer with fast convergence," *Inf. Sci.*, vol. 427, pp. 63–76, Feb. 2018, doi: 10.1016/j.ins.2017.10.037.

[15] M. de A. Costa e Silva, L. dos S. Coelho, and L. Lebensztajn, "Multiobjective biogeography-based optimization based on predator-prey approach," *IEEE Trans. Magn.*, vol. 48, no. 2, pp. 951–954, Feb. 2012, doi: 10.1109/TMAG.2011.2174205.

[16] C. Souma, D. George, and M. Ramon, "Modified predator-prey algorithm for constrained and unconstrained multi-objective optimization," *Int. J. Math. Model. Numer. Optim.*, vol. 1, no. 1, pp. 1–38, Jan. 2009, doi: 10.1504/IJMMNO.2009.030085.

[17] I. R. Meneghini, F. G. Guimaraes, and A. Gaspar-Cunha, "Competitive coevolutionary algorithm for robust multi-objective optimization: The worst case minimization," in *Proc. IEEE Congr. Evol. Comput. (CEC)*, Vancouver, BC, Canada, Jul. 2016, pp. 586–593, doi: 10.1109/CEC.2016.7743846.

[18] T. G. Tan, J. Teo, and H. K. Lau, "Competitive coevolution with K-random opponents for Pareto multiobjective optimization," in *Proc. 3rd Int. Conf. Natural Comput. (ICNC)*, Haikou, China, 2007, pp. 63–67, doi: 10.1109/ICNC.2007.309.

[19] J. D. Lohn, W. F. Kraus, and G. L. Haith, "Comparing a coevolutionary genetic algorithm for multiobjective optimization," in *Proc. Congr. Evol. Comput. (CEC)*, Honolulu, HI, USA, vol. 2, 2002, pp. 1157–1162, doi: 10.1109/CEC.2002.1004406.

[20] C. Mu, L. Jiao, Y. Liu, and Y. Li, "Multiobjective nondominated neighbor coevolutionary algorithm with elite population," *Soft Comput.*, vol. 19, no. 5, pp. 1329–1349, May 2015, doi: 10.1007/s00500-014-1346-1.

[21] R. C. Purshouse and P. J. Fleming, "Preference-driven co-evolutionary algorithms show promise for many-objective optimisation," in *Proc. Int. Conf. Evol. Multi-Criterion Optim.* Ouro Preto, Brazil: Springer-Verlag, 2011, pp. 136–150, doi: 10.1007/978-3-642-19893-9_10.

[22] R. Wang, R. C. Purshouse, and P. J. Fleming, "Preference-inspired coevolutionary algorithms for many-objective optimization," *IEEE Trans. Evol. Comput.*, vol. 17, no. 4, pp. 474–494, Aug. 2013, doi: 10.1109/TEVC.2012.2204264.

[23] K. Li, R. Wang, T. Zhang, and H. Ishibuchi, "Evolutionary many-objective optimization: A comparative study of the state-of-the-art," *IEEE Access*, vol. 6, pp. 26194–26214, 2018, doi: 10.1109/ACCESS.2018.2832181.

[24] Y. Qi, X. Ma, F. Liu, L. Jiao, J. Sun, and J. Wu, "MOEA/D with adaptive weight adjustment," *Evol. Comput.*, vol. 22, no. 2, pp. 231–264, Jun. 2014.

[25] H. Li, M. Ding, J. Deng, and Q. Zhang, "On the use of random weights in MOEA/D," in *Proc. IEEE Congr. Evol. Comput. (CEC)*, May 2015, pp. 978–985, doi: 10.1109/CEC.2015.7256996.

[26] J. Siwei, C. Zhihua, Z. Jie, and O. Yew-Soon, "Multiobjective optimization by decomposition with Pareto-adaptive weight vectors," in *Proc. 7th Int. Conf. Natural Comput. (ICNC)*, vol. 3, Jul. 2011, pp. 1260–1264, doi: 10.1109/ICNC.2011.6022367.

[27] H. L. Liu, L. Chen, K. Deb, and E. D. Goodman, "Investigating the effect of imbalance between convergence and diversity in evolutionary multiobjective algorithms," *IEEE Trans. Evol. Comput.*, vol. 21, no. 99, pp. 408–425, Jun. 2017, doi: 10.1109/TEVC.2016.2606577.

[28] A. Zhou and Q. Zhang, "Are all the subproblems equally important? Resource allocation in decomposition-based multiobjective evolutionary algorithms," *IEEE Trans. Evol. Comput.*, vol. 20, no. 1, pp. 52–64, Feb. 2016, doi: 10.1109/TEVC.2015.2424251.

[29] Q. Zhang, W. Liu, and H. Li, "The performance of a new version of MOEA/D on CEC09 unconstrained MOP test instances," in *Proc. IEEE Congr. Evol. Comput.*, Trondheim, Norway, May 2009, pp. 203–208, doi: 10.1109/CEC.2009.4982949.

[30] X. Cai, Y. Li, Z. Fan, and Q. Zhang, "An external archive guided multiobjective evolutionary algorithm based on decomposition for combinatorial optimization," *IEEE Trans. Evol. Comput.*, vol. 19, no. 4, pp. 508–523, Aug. 2015, doi: 10.1109/TEVC.2014.2350995.

[31] Q. Lin, G. Jin, Y. Ma, K.-C. Wong, C. A. C. Coello, J. Li, J. Chen, and J. Zhang, "A diversity-enhanced resource allocation strategy for decomposition-based multiobjective evolutionary algorithm," *IEEE Trans. Cybern.*, vol. 48, no. 8, pp. 2388–2401, Aug. 2018, doi: 10.1109/TCYB.2017.2739185.

[32] H. Chen, G. Wu, W. Pedrycz, P. N. Suganthan, L. Xing, and X. Zhu, "An adaptive resource allocation strategy for objective space partition-based multiobjective optimization," *IEEE Trans. Syst., Man, Cybern. Syst.*, early access, Mar. 12, 2019, doi: 10.1109/TSMC.2019.2898456.

[33] B. Kazimipour, M. N. Omidvar, X. Li, and A. K. Qin, "A sensitivity analysis of contribution-based cooperative co-evolutionary algorithms," in *Proc. IEEE Congr. Evol. Comput. (CEC)*, Sendai, Japan, May 2015, pp. 417–424, doi: 10.1109/CEC.2015.7256920.

[34] G. A. Trunfio, P. Topa, and J. Wąs, "A new algorithm for adapting the configuration of subcomponents in large-scale optimization with cooperative coevolution," *Inf. Sci.*, vol. 372, pp. 773–795, Dec. 2016, doi: 10.1016/j.ins.2016.08.080.

[35] M. Yang, M. N. Omidvar, C. Li, X. Li, Z. Cai, B. Kazimipour, and X. Yao, "Efficient resource allocation in cooperative co-evolution for large-scale global optimization," *IEEE Trans. Evol. Comput.*, vol. 21, no. 4, pp. 493–505, Aug. 2017, doi: 10.1109/TEVC.2016.2627581.

[36] M. N. Omidvar, B. Kazimipour, X. Li, and X. Yao, "CBCC3—A contribution-based cooperative co-evolutionary algorithm with improved exploration/exploitation balance," in *Proc. IEEE Congr. Evol. Comput. (CEC)*, Vancouver, BC, Canada, Jul. 2016, pp. 3541–3548, doi: 10.1109/CEC.2016.7744238.

[37] S. Mahdavi, S. Rahnamayan, and M. E. Shiri, "Multilevel framework for large-scale global optimization," *Soft Comput.*, vol. 21, no. 14, pp. 4111–4140, Feb. 2016, doi: 10.1007/s00500-016-2060-y.

[38] S. Huband, P. Hingston, L. Barone, and L. While, "A review of multiobjective test problems and a scalable test problem toolkit," *IEEE Trans. Evol. Comput.*, vol. 10, no. 5, pp. 477–506, Oct. 2006, doi: 10.1109/TEVC.2005.861417.

[39] D. A. Van Veldhuizen and G. B. Lamont, "On measuring multiobjective evolutionary algorithm performance," in *Proc. Congr. Evol. Comput. (CEC)*, La Jolla, CA, USA, vol. 1, 2000, pp. 204–211, doi: 10.1109/CEC.2000.870296.

[40] E. Zitzler, L. Thiele, M. Laumanns, C. M. Fonseca, and V. G. da Fonseca, "Performance assessment of multiobjective optimizers: An analysis and review," *IEEE Trans. Evol. Comput.*, vol. 7, no. 2, pp. 117–132, Apr. 2003, doi: 10.1109/TEVC.2003.810758.

[41] J. R. Schott, "Fault tolerant design using single and multicriteria genetic algorithm optimization," *Cellular Immunol.*, vol. 37, no. 1, pp. 1–13, May 1995, doi: 10.1016/0008-8749(78)90168-5.

**QICANG QIU** received the B.S. degree from Zhejiang University, Zhejiang, China, in 2015, and the M.S. degree from San Jose State University, CA, USA, in 2018. He is currently an Engineer with the Zhejiang Laboratory, China. His main research interests include computational intelligence and network security.

**WEI YU** received the B.S. degree from the Henan University of Science and Technology, Luoyang, China, in 2017. She is currently pursuing the degree in management science and engineering with the Zhejiang University of Technology, Hangzhou. She is with the School of Management, Zhejiang University of Technology. Her main research interests include intelligent evolutionary algorithms and decision optimization.

**LIPING WANG** received the B.S. degree from East China Normal University, Shanghai, China, in 1986, and the M.S. and Ph.D. degrees from Zhejiang University, Hangzhou, China, in 1997 and 2005, respectively. She is currently a Full Professor with the College of Computer Science and Technology, Zhejiang University of Technology, China. She is also the Director of the Institute of Information Intelligent and Decision Optimization, Zhejiang University of Technology. She is a CCF Member (98913M). Her main research interests include computational intelligence, decision optimization, and computer vision.

**HONG CHEN** received the B.S. and M.S. degrees from the Zhejiang University of Technology, Hangzhou, China, in 2007 and 2010, respectively, where he is currently pursuing the Ph.D. degree in business administration. His main research interests include decision optimization and computational intelligence.

**XIAOTIAN PAN** received the B.S. degree from Nankai University Binhai College, Tianjin, China, in 2015, and the M.S. degree from the Zhengzhou University of Aeronautics, Zhengzhou, China, in 2018. She is currently pursuing the Ph.D. degree with the Zhejiang University of Technology, Hangzhou, China. She is a CCF Student Member (97732G). Her main research interests include computational intelligence and decision optimization.